# Chapter
# Permutations and Combinations

## Topic-1: Factorials and Permutations

### 1 MCQs with One Correct Answer

1. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is [Adv. 2014]
   (a) 264 (b) 265
   (c) 53 (d) 67
2. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is [2009]
   (a) 55 (b) 66
   (c) 77 (d) 88
3. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is [2007 -3 marks]
   (a) 360 (b) 192
   (c) 96 (d) 48
4. If the LCM of $p, q$ is $r^2t^4s^2$, where $r, s, t$ are prime numbers and $p, q$ are the positive integers then the number of ordered pair (p, q) is [2006 - 3M, –1]
   (a) 252 (b) 254
   (c) 225 (d) 224
5. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is [2002S]
   (a) 40 (b) 60
   (c) 80 (d) 100
6. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ? [2000S]
   (a) 16 (b) 36
   (c) 60 (d) 180
7. A five-digit numbers divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is [1989 - 2 Marks]
   (a) 216 (b) 240
   (c) 600 (d) 3125
8. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated are [1982 - 2 Marks]
   (a) 69760 (b) 30240
   (c) 99748 (d) none of these

### 2 Integer Value Answer/ Non-Negative Integer

9. Five persons $A$, $B$, $C$, $D$ and $E$ are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is ___. [Adv. 2019]
10. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is ________. [Adv. 2018]
11. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is [Adv. 2014]

### 3 Numeric/ New Stem Based Questions

12. The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is ________. [Adv. 2022]

13. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is ______ [Adv. 2020]

### 4 Fill in the Blanks

14. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is ______. [1988 - 2 Marks]

15. In a certain test, $a_i$ students gave wrong answers to atleast i questions, where $i = 1, 2, \ldots, k$. No student gave more than $k$ wrong answers. The total number of wrong answers given is ................ [1982 - 2 Marks]

### 6 MCQs with One or More than One Correct Answer

16. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n- digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is [1998 - 2 Marks]
(a) 6 (b) 7 (c) 8 (d) 9

### 7 Match the Following

17. Consider all possible permutations of the letters of the word ENDEANOEL. Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS. [2008]

| Column I | Column II |
| --- | --- |
| (A) The number of permutations containing the word ENDEA is | (p) 5! |
| (B) The number of permutations in which the letter E occurs in the first and the last positions is | (q) 2 × 5! |
| (C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is | (r) 7 × 5! |
| (D) The number of permutations in which the letters A, E, O occur only in odd positions is | (s) 21 × 5! |

### 10 Subjective Problems

18. If total number of runs scored in $n$ matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k[th] match are given by k. $2^{n+1-k}$, where $1 \leq k \leq n$. Find $n$. [2005 - 2 Marks]

19. Prove by permutation or otherwise $\frac{(n^2)!}{(n!)^n}$ is an integer $(n \in I^+)$. [2004 - 2 Marks]

20. m men and n women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is $\frac{m!(m+1)!}{(m-n+1)!}$ [1983 - 2 Marks]

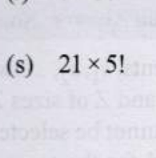
## Topic-2: Combinations and Dearrangement Theorem

### 1 MCQs with One Correct Answer

1. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 memoers) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is [Adv. 2016]
(a) 380 (b) 320 (c) 260 (d) 95

2. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is [2012]
(a) 75 (b) 150 (c) 210 (d) 243

3. A rectangle with sides of length $(2m - 1)$ and $(2n - 1)$ units is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is [2005S]



(a) $(m + n - 1)^2$ (b) $4^{m+n-1}$
(c) $m^2n^2$ (d) $m(m + 1)n(n + 1)$

**4.** Let $T_n$ denote the number of triangles which can be formed using the vertices of a regular polygon of $n$ sides. If $T_{n+1} - T_n = 21$, then $n$ equals **[2001S]**

(a) 5 (b) 7
(c) 6 (d) 4

**5.** Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4; and then the men select the chairs from amongst the remaining. The number of possible arrangements is **[1982 - 2 Marks]**

(a) ${}^6C_3 \times {}^4C_2$ (b) ${}^4P_2 \times {}^4P_3$
(c) ${}^4C_2 + {}^4P_3$ (d) none of these

**6.** The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to **[1982 - 2 Marks]**

(a) ${}^{47}C_5$ (b) ${}^{52}C_5$
(c) ${}^{52}C_4$ (d) none of these

**7.** ${}^nC_{r-1} = 36, {}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then $r$ is : **[1979]**

(a) 1 (b) 2
(c) 3 (d) None of these.

## 2 Integer Value Answer/ Non-Negative Integer

**8.** A group of 9 students, $s_1, s_2, ..... s_9$ is to be divided to form three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that $s_1$ cannot be selected for the team X, and $s_2$ cannot be selected for the team Y. Then the number of ways to form such teams, is ________. **[Adv. 2024]**

**9.** Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$ **[Adv. 2017]**

**10.** Let $n$ be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is **[Adv. 2015]**

**11.** Let $n \geq 2$ be an integer. Take $n$ distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of $n$ is **[Adv. 2014]**

## 3 Numeric/ New Stem Based Questions

**12.** An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that no two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is ________ **[Adv. 2020]**

## 4 Fill in the Blanks

**13.** Total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is ................ **[1988 - 2 Marks]**

**14.** The side $AB$, $BC$ and $CA$ of a triangle $ABC$ have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices is ................ **[1984 - 2 Marks]**

## 5 True / False

**15.** The product of any $r$ consecutive natural numbers is always divisible by $r!$. **[1985 - 1 Mark]**

## 6 MCQs with One or More than One Correct Answer

**16.** Let $S_1 = \{(i, j, k) : i, j, k \in \{1, 2, ..., 10\}\}$

$S_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10, i, j \in \{1, 2, ..., 10\}\}$,

$S_3 = \{(i, j, k, l) : 1 \leq i < j < k < l, i, j, k, l \in \{1, 2, .., 10\}\}$.

and $S_4 = \{(i, j, k, l) : i, j, k$ and $l$ are distinct elements in $\{1, 2, ..., 10\}\}$.

If the total number of elements in the set $S_r$ is $n_r$, $r = 1, 2, 3, 4$, then which of the following statements is (are) **TRUE**?

(a) $n_1 = 1000$ (b) $n_2 = 44$ **[Adv. 2021]**
(c) $n_3 = 220$ (d) $\frac{n_4}{12} = 420$

**17.** Let $S = \{1, 2, 3, ..., 9\}$. For $k = 1, 2, ..., 5$, let $N_k$ be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$ **[Adv. 2017]**

(a) 210 (b) 252 (c) 125 (d) 126

**18.** For non-negative integers $s$ and $r$, let

$$\binom{s}{r} = \begin{cases} \frac{s!}{r!(s-r)!} & \text{if } r \leq s, \\ 0 & \text{if } r > s. \end{cases}$$

For positive integers $m$ and $n$, let

$$g(m, n) = \sum_{p=0}^{m+n} \frac{f(m,n,p)}{\binom{n+p}{p}}$$

where for any non-negative integer $p$,

$$f(m, n, p) = \sum_{i=0}^{p} \binom{m}{i}\binom{n+i}{p}\binom{p+n}{p-i}.$$

Then which of the following statements is/are TRUE? **[Adv. 2020]**

(a) $g(m, n) = g(n, m)$ for all positive integers $m, n$

(b) $g(m, n+1) = g(m+1, n)$ for all positive integers $m, n$

(c) $g(2m, 2n) = 2g(m, n)$ for all positive integers $m, n$

(d) $g(2m, 2n) = (g(m, n))^2$ for all positive integers $m, n$

## 7 Match the Following

**19.** In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls $G_1, G_2, G_3, G_4, G_5$.

(i) Let $\alpha_1$ be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let $\alpha_2$ be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let $\alpha_3$ be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let $\alpha_4$ be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both $M_1$ and $G_1$ are **NOT** in the committee together.

| | **LIST - I** | | **LIST - II** |
|---|---|---|---|
| P. | The value of $\alpha_1$ is | 1. | 136 |
| Q. | The value of $\alpha_2$ is | 2. | 189 |
| R. | The value of $\alpha_3$ is | 3. | 192 |
| S. | The value of $\alpha_4$ is | 4. | 200 |
| | | 5. | 381 |
| | | 6. | 461 |

**[Adv. 2018]**

The correct option is:

(a) $P \to 4;\ Q \to 6;\ R \to 2;\ S \to 1$

(b) $P \to 1;\ Q \to 4;\ R \to 2;\ S \to 3$

(c) $P \to 4;\ Q \to 6;\ R \to 5;\ S \to 2$

(d) $P \to 4;\ Q \to 2;\ R \to 3;\ S \to 1$

## 8 Comprehension/Passage Based Questions

Let $a_n$ denote the number of all $n$-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let $b_n$ = the number of such $n$-digit integers ending with digit 1 and $c_n$ = the number of such $n$–digit integers ending with digit 0. **[2012]**

**20.** The value of $b_6$ is

(a) 7 (b) 8

(c) 9 (d) 11

**21.** Which of the following is correct?

(a) $a_{17} = a_{16} + a_{15}$

(b) $c_{17} \neq c_{16} + c_{15}$

(c) $b_{17} \neq b_{16} + c_{16}$

(d) $a_{17} = c_{17} + b_{16}$

## 10 Subjective Problems

**22.** A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in a committee? In how many of these committees **[1994 - 4 Marks]**

(a) The women are in majority?

(b) The men are in majority?

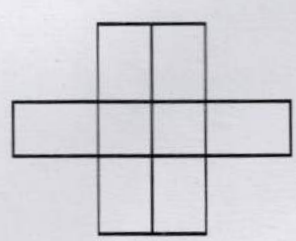
**23.** Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the sitting arrangements can be made. **[1991 - 4 Marks]**

**24.** A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw? **[1986 - 2½ Marks]**

**25.** 7 relatives of a man comprises 4 ladies and 3 gentlemen; his wife has also 7 relatives; 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of the wife's relatives? **[1985 - 5 Marks]**

**26.** Five balls of different colours are to be placed in there boxes of different size. Each box can hold all five. In how many different ways can we place the balls so that no box remains empty? **[1981 - 4 Marks]**

**27.** Six X's have to be placed in the squares of figure below in such a way that each row contains at least one X. In how many different ways can this be done. **[1978]**



# Answer Key

## Topic-1 : Factorials and Permutations

**1.** (c) **2.** (c) **3.** (c) **4.** (c) **5.** (a) **6.** (c) **7.** (a) **8.** (a) **9.** (30)

**10.** (625) **11.** (7) **12.** (569) **13.** (1080) **14.** (9) **15.** $\sum_{i=1}^{k} a_i$ **16.** (b) **17.** (A) $\rightarrow$ p; (B) $\rightarrow$ s; (C) $\rightarrow$ q; (D) $\rightarrow$ q

## Topic : 2 Combinations and Dearrangement Theorem

**1.** (1) **2.** (b) **3.** (c) **4.** (b) **5.** (d) **6.** (c) **7.** (c) **8.** (665) **9.** (5) **10.** (5)
**11.** (5) **12.** (495.00) **13.** (35) **14.** (205) **15.** (True) **16.** (a,b,d) **17.** (d) **18.** (a, b, d) **19.** (c) **20.** (b)
**21.** (a)

# 5 Permutations and Combinations

## Topic-1: Factorials and Permutations

1. **(c)** $\because$ Card numbered 1 is always placed in envelope numbered 2, we can consider two cases :
**Case I:** Card numbered 2 is placed in envelope numbered 1. Then it is dearrangement of 4 objects, which can be done in
$$4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)=9 \text{ ways}$$
**Case II:** Card numbered 2 is not placed in envelope numbered 1. Then it is dearrangement of 5 objects, which can be done in
$$5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)=44 \text{ ways}$$
$\therefore$ Total ways = 44 + 9 = 53

2. **(c)** We have to form 7 digit numbers, using the digits 1, 2 and 3 only, such that the sum of the digits in a number = 10. This can be done by taking 2, 2, 2, 1, 1, 1, 1, or by taking 2, 3, 1, 1, 1, 1, 1.
$\therefore$ Number of ways $=\frac{7!}{3!4!}+\frac{7!}{5!}=77.$

3. **(c)** The letter of word *COCHIN* in alphabetic order are *C, C, H, I, N, O*.
Fixing first and second letter as *C, C*, rest 4 can be arranged in 4! ways.
Similarly the words starting with each of *CH, CI, CN* are 4!
Then fixing first two letters as *CO* and next four places when filled in alphabetic order with remaining 4 letters give the word *COCHIN*.
$\therefore$ Numbers of words coming before *COCHIN*
$= 4 \times 4! = 4 \times 24 = 96$

4. **(c)** $\because$ r, s, t are prime numbers,
$\therefore$ Section of (p, q) can be done as follows

| p | q |
|---|---|
| $r^0$ | $r^2$ |
| $r^1$ | $r^2$ |
| $r^2$ | $r^0, r^1, r^2$ |

$\therefore$ r can be selected (1 + 1 + 3 = 5) ways
Similarly t and s can be selected in 9 and 5 ways respectivley.
$\therefore$ Total number of ordered pair $(p, q) = 5 \times 9 \times 5 = 225$

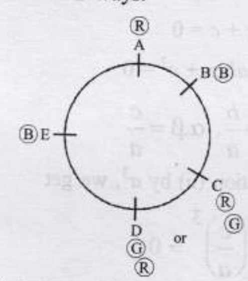
5. **(a)** Total number of ways of arranging the letters of the word BANANA is $\frac{6!}{2!3!}=60$. Number of words in which 2 N's come together is $\frac{5!}{3!}=20$
$\therefore$ the required number = 60 – 20 = 40

6. **(c)** $X-X-X-X-X$. The four digits 3, 3, 5, 5 can be arranged at (–) places in $\frac{4!}{2!2!}=6$ ways
The five digits 2, 2, 8, 8, 8 can be arranged at
$(X)$ places in $\frac{5!}{2!3!}=10$ ways
$\therefore$ Total number of arrangements = 6 × 10 = 60 ways

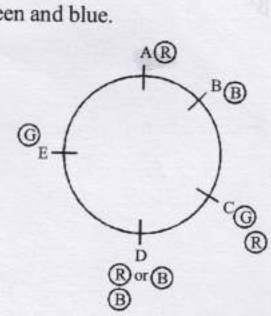
7. **(a)** We know that a number is divisible by 3 if the sum of its digits is divisibly by 3.
Now out of 0, 1, 2, 3, 4, 5 if we take 1, 2, 3, 4, 5 or 0, 1, 2, 4, 5 then the 5 digit numbers will be divisible by 3.
**Case I :** Number of 5 digit numbers formed using the digits 1, 2, 3, 4, 5 = 5! = 120
**Case II :** Taking 0, 1, 2, 4, 5 if we make 5 digit number then
1st place can be filled in 4 ways (0 can not come at I place)
2nd place can be filled in 4 ways
3rd place can be filled in 3 ways
4th place can be filled in 2 ways
5th place can be filled in 1 ways
$\therefore$ Total numbers = 4 × 4! = 96
Thus total numbers divisible by 3 are = 120 + 96 = 216

8. **(a)** Total number of words that can be formed using 5 letters out of 10 given different letters
= 10 × 10 × 10 × 10 × 10 (as letters can repeat)
= 1, 00, 000
Number of words that can be formed using 5 different letters out of 10 different letters
$={}^{10}P_5$ (none can repeat) $=\frac{10!}{5!}=30,240$
$\therefore$ Number of words in which at least one letter is repeated = total words–words with none of the letters repeated
= 1,00,000 – 30,240 = 69,760

9. **(30)** 5 persons A, B, C, D and E are seated in circular arrangement. Let A be given red hat, then there will be two cases.
**Case I :** B and E have same coloured hat blue/green. Say B and E have blue hat.
Then C and D can have either red and green or green and red i.e. 2 ways.




Similarly if B & E have green hat, there will be 2 ways for C & D.
Hence there are 2 + 2 = 4 ways.
**Case II :** B and E have different coloured hats blue and green or green and blue.



Let B has blue and E has green.
If C has green then D can have red or blue.
If C has red then D can have only blue.
∴ three ways.
Similarly 3 ways will be there when B has green and E has blue.
∴ there are 3 + 3 = 6 ways
On combining the two cases, there will be 4 + 6 = 10 ways
When similar discussion is repeated with A as blue and green hat, we get 10 ways for each.
Therefore, in all, there will be 10 + 10 + 10 = 30 ways

**10.** **(625)** The last 2 digits, in 5-digit number divisible by 4, can be 12, 24, 32, 44 or 52.
Also each of the first three digits can be any of {1, 2, 3, 4, 5}
∴ 5 options for each of the first three digits and total 5 options for last 2-digits
∴ Required number of 5 digit numbers are
= 5 × 5 × 5 × 5 = 625

**11.** **(7)** ∵ $n_1, n_2, n_3, n_4$ and $n_5$ are positive integers such that $n_1 < n_2 < n_3 < n_4 < n_5$
Then for $n_1 + n_2 + n_3 + n_4 + n_5 = 20$
If $n_1, n_2, n_3, n_4$ take minimum values 1, 2, 3, 4 respectively then $n_5$ will be maximum 10.
∴ Corresponding to $n_5 = 10$, there is only one solution
$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4$.
Corresponding to $n_5 = 9$, we can have, only one solution
$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 5$ i.e., one solution
Corresponding to $n_5 = 8$, we can have, only solution
$n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 6$
or $n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 5$
i.e., 2 solution
For $n_5 = 7$, we can have
$n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6$
or $n_1 = 1, n_2 = 3, n_3 = 4, n_4 = 5$
*i.e.* 2 solutions
For $n_5 = 6$, we can have
$n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5$
i.e., one solution
Thus there can be 7 solutions.

**12.** **(569)** Counting integers starting from 2
**Case I:** At unit's place we can fill 2/3/4/6/7
i.e., 2 0 2 [5] → 5 ways
At unit's place and ten's place we can fill digits as 3/4/6/7 and 0/2/3/4/6/7
or 2 0 [4] [6] → 24 ways
(Numbers except 0 or 2 in 3rd place)
**Case II:** If non-zero number on 2nd place
i.e., 2 [5] [6] [6] = 180 ways
Counting integers starting from 3
3 [6] [6] [6] = 216 ways
Counting integers starting from 4
**Case I:** If 0, 2 or 3 on 2nd place
i.e., 4 [3] [6] [6]
= 108 ways
**Case II:** If 4 on 2nd place
i.e., 4 4 [6] [6] = 36 ways
∴ Total 5 + 24 + 180 + 216 + 108 + 36 = 569 numbers

**13.** **(1080)** Groups can be possible in only 2, 2, 1, 1 way.
Number of ways of dividing persons in group

$$= \frac{6!}{(2!)^2(1!)^2(2!)^2}$$

Number of ways after arranging rooms $= \frac{6!}{(2!)^4} \cdot 4! = 1080$

**14.** We know that number of dearrangements of n objects

$$= n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots \frac{1}{n!}\right]$$

∴ No. of ways of putting all the 4 balls into boxes of different colour

$$= 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 4!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$$

$$= 24\left(\frac{12-4+1}{24}\right) = 9$$

**15.** Number of students who gave wrong answers to exactly one question $= a_1 - a_2$, Two questions $= a_2 - a_3$
Three questions $= a_3 - a_4$, $k$–1 question
$= a_{k-1} - a_k$, $k_{\text{question}} = a_k$
∴ Total number of wrong answers
$= 1\,(a_1 - a_2) + 2\,(a_2 - a_3) + 3\,(a_3 - a_4) + \ldots (k-1)(a_{k-1} - a_k) + k\,a_k$

$$= a_1 + a_2 + a_3 + \ldots a_k = \sum_{i=1}^{k} a_i$$

**16.** **(b)** Distinct $n$ digit numbers which can be formed using digits 2, 5 and 7 are $3^n$.
We have to find $n$ so that $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$
$\Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$. So the least value of $n$ is 7.

**17.** **(A) → p; (B) → s; (C) → q; (D) → q**
(A) For the permutations containing the word ENDEA we consider 'ENDEA' as single letter. Then we have total ENDEA, N, O, E, L i.e. 5 letters which can be arranged in 5! ways.
∴ (A) → (p)
(B) If E occupies the first and last position, the middle 7 positions can be filled by N, D, E, A, N, O, L. in

$$\frac{7!}{2!} = 7 \times 6 \times 5 \times 4 \times 3 = 21 \times 120 = 21 \times 5! \text{ ways.}$$

∴ (B) → (s)
(C) If none of the letters D, L, N occur in the last five positions then we should arrange D, L, N, N at first four positions and rest five i.e. E, E, E, A,O at last five positions. This can be done in

$$\frac{4!}{2!} \times \frac{5!}{3!} = 4 \times 3 \times \frac{5!}{3 \times 2} = 2 \times 5! \text{ ways}$$

∴ (C) → (q)
(D) As per question A, E, E, E, O can be arranged at 1st, 3rd, 5th, 7th and 9th positions and rest D, L, N, N at rest 4 positions. This can be done in

$$\frac{5!}{3!} \times \frac{4!}{2!} \text{ ways} = 2 \times 5! \text{ways} \qquad \therefore (D) \to (q)$$

**18.** Given :
Runs scored in $k^{th}$ match $= k.2^{n+1-k}$, $1 \leq k \leq n$
and runs scored in n matches $= \frac{n+1}{4}(2^{n+1} - n - 2)$

$$\therefore \sum_{k=1}^{n} k.2^{n+1-k} = \frac{n+1}{4}(2^{n+1} - n - 2)$$

$$\Rightarrow\ 2^{n+1}\left[\sum_{k=1}^{n}\frac{k}{2^k}\right]=\frac{n+1}{4}(2^{n+1}-n-2)$$

$$\Rightarrow\ 2^{n+1}\left[\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}\right]$$

$$=\frac{n+1}{4}(2^{n+1}-n-2) \quad \text{....(i)}$$

Let $S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{n}{2^n}$ ....(ii)

$$\therefore\ \frac{1}{2}S=\frac{1}{2^2}+\frac{2}{2^3}+....+\frac{n-1}{2^n}+\frac{n}{2^{n+1}} \quad \text{....(iii)}$$

On subtracting eq. (iii), from (ii), we get

i.e., $\frac{1}{2}S=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^n}-\frac{n}{2^{n+1}}$

$$\Rightarrow\ \frac{1}{2}S=\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}-\frac{n}{2^{n+1}}$$

$$\Rightarrow\ S=2\left[1-\frac{1}{2^n}-\frac{n}{2^{n+1}}\right] \quad \text{....(iv)}$$

From equation (i) and (iv),

$$2.2^{n+1}\left[1-\frac{1}{2^n}-\frac{n}{2^{n+1}}\right]=\frac{n+1}{4}\left[2^{n+1}-n-2\right]$$

$$\Rightarrow\ 2.\left[2^{n+1}-2-n\right]=\frac{n+1}{4}\left[2^{n+1}-2-n\right]$$

$$\Rightarrow\ \frac{n+1}{4}=2 \Rightarrow n=7$$

**19.** Let there be $n$ sets of different objects each set containing $n$ identical objects [eg (1, 1, 1 ... 1 ($n$ times) ), (2, 2, 2 ..., 2 ($n$ times)) ... ($n, n, n$ ... $n$ ($n$ times))]

Then the number of ways in which these $n \times n = n^2$ objects can be arranged in a row $=\frac{(n^2)!}{n!n!...n!}=\frac{(n^2)!}{(n!)^n}$

But these number of ways should be a natural number.

Hence $\frac{(n^2)!}{(n!)^n}$ is an integer. $(n \in I^+)$

**20.** Since, m men can be seated in m! ways creating $(m+1)$ places for ladies to sit.

$\therefore$ $n$ ladies out of $(m+1)$ places (as $n < m$) can be seated in ${}^{m+1}P_n$ ways

$\therefore$ Total ways $= m! \times {}^{m+1}P_n$

$$=m!\times\frac{(m+1)!}{(m+1-n)!}=\frac{(m+1)!m!}{(m-n+1)!}$$

## Topic-2: Combinations and Dearrangement Theorem

**1.** **(a)** Either one boy will be selected or no boy will be selected. Also out of four members one captain is to be selected.

$\therefore$ Required number of ways $= ({}^4C_1 \times {}^6C_3 + {}^6C_4) \times {}^4C_1$
$= (80 + 15) \times 4 = 380$

**2.** **(b)** $\because$ Each person gets at least one ball.

$\therefore$ 3 Persons can have 5 balls as follow.

| Person | No. of balls | No. of balls |
|---|---|---|
| I | 1 | 1 |
| II | 1 | 2 |
| III | 3 | 2 |

The number of ways to distribute balls 1, 1, 3 in first to three persons

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3$

Also 3, persons having 1, 1 and 3 balls can be arranged in $\frac{3!}{2!}$ ways.

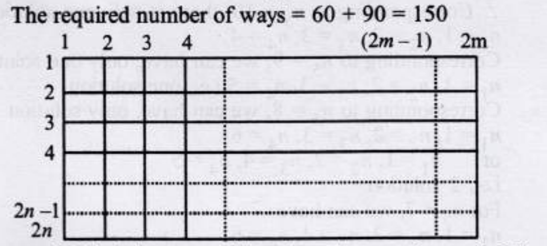
$\therefore$ Total no. of ways to distribute 1, 1, 3 balls to the three persons

$= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times \frac{3!}{2!} = 60$

Similarly, total no. of ways to distribute 1, 2, 2 balls to three persons $= {}^5C_1 \times {}^4C_2 \times {}^2C_2 \times \frac{3!}{2!} = 90$

$\therefore$ The required number of ways = 60 + 90 = 150

**3.** **(c)**




If we see the blocks in terms of lines then there are $2m$ vertical lines and $2n$ horizontal lines. To form the required rectangle, we must select two horizontal lines, one even numbered (out of 2, 4, .....$2n$) and one odd numbered (out of 1, 3....$2n$–1) and similarly two vertical lines.

The number of rectangles

$= {}^mC_1 . {}^mC_1 . {}^nC_1 . {}^nC_1 = m^2n^2$

**4.** **(b)** $T_n = {}^nC_3$ ; $T_{n+1} = {}^{n+1}C_3$

Now, $T_{n+1} - T_n = 21 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$$\Rightarrow\ \frac{(n+1)n(n-1)}{3.2.1}-\frac{n(n-1)(n-2)}{3.2.1}=21$$

$$\Rightarrow\ n(n-1)(n+1-n+2)=126$$

$$\Rightarrow\ n(n-1)=42 \Rightarrow n(n-1)=7\times 6,\ \therefore\ n=7$$

**5.** **(d)** $\overline{1}\,\overline{2}\,\overline{3}\,\overline{4}\,\overline{5}\,\overline{6}\,\overline{7}\,\overline{8}$

Two women can choose two chairs out of 1, 2, 3, 4, in ${}^4C_2$ ways and can arrange themselves in 2! ways. Three men can choose 3 chairs out of 6 remaining chairs in ${}^6C_3$ ways and can arrange themselves in 3! ways

$\therefore$ Total number of possible arrangements are
${}^4C_2 \times 2! \times {}^6C_3 \times 3! = {}^4P_2 \times {}^6P_3$

**6.** **(c)** $${}^{47}C_4+\sum_{j=1}^{5}{}^{52-j}C_3$$

$$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4)$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ({}^{48}C_3 + {}^{48}C_4)$$

[Using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

$= {}^{51}C_3 + {}^{50}C_3 + ({}^{49}C_3 + {}^{49}C_4)$

$= {}^{51}C_3 + ({}^{50}C_3 + {}^{50}C_4) = {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$

**7.** **(c)** ${}^nC_{r-1} = 36, {}^nC_r = 84, {}^nC_{r+1} = 126$

We know that

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{r}{n-r+1} \Rightarrow \frac{36}{84} = \frac{r}{n-r+1}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{7} \Rightarrow 3n - 10r + 3 = 0 \quad \text{....(i)}$$

Also, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r} = \frac{84}{126} = \frac{2}{3} \Rightarrow 2n - 5r - 3 = 0$ ....(ii)

On solving (i) and (ii), we get $n = 9$ and $r = 3$.

**8.** **(665)** Number of required ways

$$= \frac{9!}{2!3!4!} - (n(s_1 \in X) + n(s_2 \in Y) - n(s_1 \in X \text{ and } s_2 \in Y))$$

$$= \frac{9!}{2!3!4!} - \left(\frac{8!}{1!3!4!} + \frac{8!}{2!2!4!} - \frac{7!}{1!2!4!}\right) = 665$$

**9.** **(5)** $x = 10!$ and $y = {}^{10}C_1 \times {}^9C_8 \times \frac{10!}{2!} = 10 \times 9 \times \frac{10!}{2!}$

$$\therefore \quad \frac{y}{9x} = \frac{10 \times 9 \times \frac{10!}{2}}{9 \times 10!} = 5$$

**10.** **(5)** Here, $\_\_ B_1 \_\_ B_2 \_\_ B_3 \_\_ B_4 \_\_ B_5$

Out of 5 girls, 4 girls are together and 1 girl is separate. Now, to select 2 positions out of 6 positions between boys $= {}^6C_2$.... (i)

4 girls are to be selected out of 5 $= {}^5C_4$ .... (ii)

Now, 2 groups of girls can be arranged in 2! ways .... (iii)

Also, the group of 4 girls and 5 boys is arranged in 4! × 5! ways. .... (iv)

Now, total number of ways $= {}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!$

[from Eqs. (i), (ii), (iii) and (iv)]

$\therefore \quad m = {}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!$

and $n = 5! \times 6!$

$$\Rightarrow \quad \frac{m}{n} = \frac{{}^6C_2 \times {}^5C_4 \times 2! \times 4! \times 5!}{6! \times 5!} = \frac{15 \times 5 \times 2 \times 4!}{6 \times 5 \times 4!} = 5$$

**11.** **(5)** Number of adjacent lines $= n$

Number of non adjacent lines $= {}^nC_2 - n$

$$\therefore {}^nC_2 - n = n \Rightarrow \frac{n(n-1)}{2} - 2n = 0$$

$\Rightarrow n^2 - 5n = 0 \Rightarrow n = 0$ or 5 But $n \geq 2 \Rightarrow n = 5$

**12.** **(495.00)** We know that total number of ways of selection of r days out of $n$ days such that no two of them are consecutive $= {}^{n-r+1}C_r$.

$\therefore$ Selection of 4days out of 15 days such that no two of them are consecutive $= {}^{15-4+1}C_4 = {}^{12}C_4$

$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2} = 11 \times 5 \times 9 = 495$$

**13.** '+' signs can be put in a row in 1 way, creating 7 ticked places to keep '–' sign so that no two '–' signs occur together

√+√+√+√+√+√+√

Out of these 7 places 4 can be chosen in ${}^7C_4$ ways.

$\therefore$ Required no. of arrangements are

$$= {}^7C_4 = {}^7C_3 = \frac{7.6.5}{3.2.1} = 35$$

**14.** We have total 3 + 4 + 5 = 12 points out of which 3 fall on one line, 4 on second line and 5 on still other line. So number of Δ's that can be formed using 12 such points are

${}^{12}C_3 - {}^3C_3 - {}^4C_3 - {}^5C_3$

$$= \frac{12 \times 11 \times 10}{6} - 1 - 4 - \frac{5 \times 4}{2 \times 1} = 220 - 15 = 205$$

**15.** **(True)** Consider, $\frac{(n+1)(n+2)...(n+r)}{r!}$

$$= \frac{1.2.3...(n-1)n\ (n+1)(n+2)...(n+r)}{1.2.3...n.r!}$$

$$= \frac{(n+r)!}{n!r!} = {}^{n+r}C_r = \text{An integral value}$$

$\Rightarrow$ (n + 1) (n + 2) ...(n + r) is divisible by r!

Thus given statement is true.

**16.** **(a,b,d)** Number of elements is $S_1 = 10 \times 10 \times 10 = 1000$

Number of elements is $S_2 = 9\ (J = 8) + 8\ (J = 7) + 7\ (J = 6) + 6\ (J = 5) + 5\ (J = 4) + 4\ (J = 3) + 3\ (J = 2) + 2\ (J = 1) = 44$

Number of elements in $S_3 = {}^{10}C_4 = 210$

Number of elements in $S_4 = {}^{10}P_4 = 210 \times 4! = 5040$

So, options (a), (b), (d) are correct.

**17.** **(d)** Here set S contain 5 odd and 4 even numbers. Since each of $N_K$ containing five elements out of which exactly are odd.

$\therefore \quad N_1 = {}^5C_1 \times {}^4C_4 = 5$

$N_2 = {}^5C_2 \times {}^4C_3 = 40$

$N_3 = {}^5C_3 \times {}^4C_2 = 60$

$N_4 = {}^5C_4 \times {}^4C_1 = 20$

$N_5 = {}^5C_5 = 1 \quad \therefore \quad N_1 + N_2 + N_3 + N_4 + N_5 = 126$

**18.** **(a, b, d)** Given that:

$$f(m,n,p) = \sum_{i=0}^{p} \binom{m}{i} \cdot \frac{\lfloor\underline{n+i}}{\lfloor\underline{p}\lfloor\underline{n+i-p}} \cdot \frac{\lfloor\underline{p+n}}{\lfloor\underline{n+i}\lfloor\underline{p-i}}$$

$$\Rightarrow f(m,n,p) = \frac{\lfloor\underline{n+p}}{\lfloor\underline{p}} \sum_{i=0}^{p} \binom{m}{i} \cdot \frac{\lfloor\underline{1}}{\lfloor\underline{p-i}\lfloor\underline{n-(p-i)}}$$

$$f(m,n,p) = \frac{\lfloor\underline{n+p}}{\lfloor\underline{n}\lfloor\underline{p}} \sum_{i=0}^{p} {}^mC_i \, {}^nC_{p-i}$$

$$f(m,n,p) = \frac{\lfloor\underline{n+p}}{\lfloor\underline{n}\lfloor\underline{p}} \cdot {}^{m+n}C_p = {}^{n+p}C_p \cdot {}^{m+n}C_p$$

$$\left(\because \sum_{i=0}^{p} {}^mC_i \cdot {}^nC_{p-i} = {}^{m+n}C_p\right)$$

$$g(m,n) = \sum_{p=0}^{m+n} {}^{m+n}C_p = 2^{m+n}$$

So, options (a), (b) and (d) are true.

**19.** **(c)** Given 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls $G_1, G_2, G_3, G_4, G_5$

(i) $\alpha_1 \rightarrow$ Total number of ways of selecting 3 boys and 2 girls from 6 boys and 5 girls

i.e., ${}^6C_3 \times {}^5C_2 = 20 \times 10 = 200 \quad \therefore \quad \alpha_1 = 200$

(ii) $\alpha_2 \to$ Total number of ways selecting at least 2 member and having equal number of boys and girls
i.e., ${}^6C_1\,{}^5C_1 + {}^6C_2\,{}^5C_2 + {}^6C_3\,{}^5C_3 + {}^6C_4\,{}^5C_4 + {}^6C_5\,{}^5C_5$
$= 30 + 150 + 200 + 75 + 6 = 461 \Rightarrow \alpha_2 = 461$

(iii) $\alpha_3 \to$ Total number of ways of selecting 5 members in which at least 2 of them girls
i.e., ${}^5C_2\,{}^6C_3 + {}^5C_3\,{}^6C_2 + {}^5C_4\,{}^6C_1 + {}^5C_5\,{}^6C_0$
$= 200 + 150 + 30 + 1 = 381 \Rightarrow \alpha_3 = 381$

(iv) $\alpha_4 \to$ Total number of ways for selecting 4 members in which at least two girls such that $M_1$ and $G_1$ are not included together.
$G_1$ is included $\to {}^4C_1 \cdot {}^5C_2 + {}^4C_2 \cdot {}^5C_1 + {}^4C_3$
$= 40 + 30 + 4 = 74$
$M_1$ is included $\to {}^4C_2 \cdot {}^5C_1 + {}^4C_3 = 30 + 4 = 34$
$G_1$ and $M_1$ both are not included
${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2$
$1 + 20 + 60 = 81$
$\therefore$ Total number $= 74 + 34 + 81 = 189$
$\alpha_4 = 189$
Now, $P \to 4$; $Q \to 6$; $R \to 5$; $S \to 2$

**20.** **(b)** $\because$ $a_n$ = number of all $n$ digit +ve integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0.
and $b_n$ = number of such $n$ digit integers ending with 1
$c_n$ = number of such $n$ digit integers ending with 0
Clearly, $a_n = b_n + c_n$ ($\because$ $a_n$ can end with 0 or 1)
Also $b_n = a_{n-1}$
and $c_n = a_{n-2}$ [$\because$ if last digit is 0, second last has to be 1]
$\therefore$ We get $a_n = a_{n-1} + a_{n-2}, n \geq 3$
Also $a_1 = 1, a_2 = 2,$
Now by this recurring formula, we get
$a_3 = a_2 + a_1 = 3$
$a_4 = a_3 + a_2 = 3 + 2 = 5$
$a_5 = a_4 + a_3 = 5 + 3 = 8$
Also $b_6 = a_5 = 8$

**21.** **(a)** By recurring formula, $a_{17} = a_{16} + a_{15}$ is correct
Also $c_{17} \neq c_{16} + c_{15}$
$\Rightarrow a_{15} \neq a_{14} + a_{13}$ ($\because$ $c_n = a_{n-2}$) $\therefore$ Incorrect
Similarly, other parts are also incorrect.

**22.** Given that, there are 9 women and 8 men, a committee of 12 is to be formed including at least 5 women.
This can be doen in
= (5 women and 7 men) + (6 women and 6 men) + (7 women and 5 men) + (8 women and 4 men) + (9 women and 3 men) ways
Total number of ways of forming committee
$= ({}^9C_5 \cdot {}^8C_7) + ({}^9C_6 \cdot {}^8C_6) + ({}^9C_7 \cdot {}^8C_5) + ({}^9C_8 \cdot {}^8C_4) + ({}^9C_9 \cdot {}^8C_3)$
$= 1008 + 2352 + 2016 + 630 + 56 = 6062$
(i) The women are in majority $= 2016 + 630 + 56 = 2702$
(ii) The men are in majority = 1008 ways.

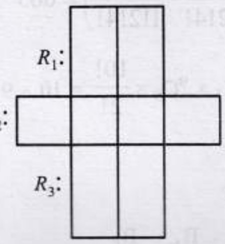
**23.** Out of 18 guests, 9 to be seated on side $A$ and rest 9 on side $B$. Now out of 18 guests, 4 particular guests desire to sit on one particular side say side $A$ and other 3 on other side $B$. Out of rest $18 - 4 - 3 = 11$ guests we can select 5 more for side $A$ and rest 6 can be seated on side $B$. Selection of 5 out of 11 can be done in ${}^{11}C_5$ ways and 9 guests on each sides of table can be seated in $9! \times 9!$ ways. Thus there are total ${}^{11}C_5 \times 9! \times 9!$ arrangements.

**24.** Number of ways of drawing at least one black ball
= 1 black and 2 other or 2 black and 1 other or 3 black
$= {}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3 = 3 \times 15 + 3 \times 6 + 1$
$= 45 + 18 + 1 = 64$

**25.** The possible cases are
**Case I :** A man invites 3 ladies and women invites 3 gentlemen.
Number of ways $= {}^4C_3 \cdot {}^4C_3 = 16$
**Case II :** A man invites (2 ladies, 1 gentleman) and women invites (2 gentlemen, 1 lady).
Number of ways $= ({}^4C_2 \cdot {}^3C_1) \cdot ({}^3C_1 \cdot {}^4C_2) = 324$
**Case III :** A man invites (1 lady, 2 gentlemen) and women invites (2 ladies, 1 gentleman).
Number of ways $= ({}^4C_1 \cdot {}^3C_2) \cdot ({}^3C_2 \cdot {}^4C_1) = 144$
**Case IV :** A man invites (3 gentlemen) and women invites (3 ladies).
Number of ways $= {}^3C_3 \cdot {}^3C_3 = 1$
$\therefore$ Total number of ways $= 16 + 324 + 144 + 1 = 485$

**26.** Since, each box can hold five balls.
$\therefore$ Number of ways in which balls could be distributed so that none is empty, are (2, 2, 1) or (3, 1, 1).
i.e. $({}^5C_2\,{}^3C_2\,{}^1C_1 + {}^5C_3\,{}^2C_1\,{}^1C_1) \times 3! = (30 + 20) \times 6 = 300$

**27.**




Here $R_1$ has 2 squares, $R_2$ has 4 squares and $R_3$ has 2 squares. The selection scheme is as follows :

| | $R_1$ | $R_2$ | $R_3$ |
|---|---|---|---|
| | 1 | 4 | 1 |
| or | 1 | 3 | 2 |
| or | 2 | 3 | 1 |
| or | 2 | 2 | 2 |

$\therefore$ Number of selections are
${}^2C_1 \times {}^4C_4 \times {}^2C_1 + {}^2C_1 \times {}^4C_3 \times {}^2C_2 +$
${}^2C_2 \times {}^4C_3 \times {}^2C_1 + {}^2C_2 \times {}^4C_2 \times {}^2C_2$
$= 4 + 8 + 8 + 6 = 26$