
Sample Paper-04 (solved)
Mathematics
Class – XI

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- d) Use of calculators is not permitted.

Section A

- 1. Find $\sin^{-1}\left(\sin\left(\frac{6\pi}{7}\right)\right)$
- 2. Write the coordinates of the point of intersections of the parabola represented by $y^2 = 4ax$ and its latus rectum
- 3. Find x and y if $(x+7, 8) = (10, x+y)$
- 4. Compute $(1+2i)i - \frac{3+2i}{1-i}$
- 5. Write the domain and range of the function $\cos^{-1} x$
- 6. Find the sign of y if $y = \sin(\cos^{-1} x)$

Section B

- 7. If α, β are the roots of the equation $x^2 - bx + c = 0$ find the value of $\alpha^2 + \beta^2$
 - 8. If P be the sum of the odd terms and Q the sum of the even terms in the expansion of $(x+a)^n$, prove that $P^2 - Q^2 = (x^2 - a^2)^n$
 - 9. Solve the inequality $\frac{x^2 - 3x + 6}{3 + 4x} < 0$
 - 10. Prove that $\cot(A+15) - \tan(A-15) = \frac{4 \cos 2A}{1 + 2 \sin 2A}$
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11. Find the domain of the function $f(x) = \sqrt{4 - x^2}$
 12. Evaluate $\frac{1}{2 + \cos \theta + \sin \theta}$ if $\tan \frac{\theta}{2} = 2$
 13. Find the limit $\lim_{x \rightarrow 0} \frac{\sin 5x}{x + x^3}$
 14. Solve $\sin^2 x + \sin^2 2x = 1$
 15. Find the value of $i^{30} + i^{40} + i^{60}$
 16. Determine whether the points $(0,0)$ and $(5,5)$ lie on different sides of the straight line $x + y - 8 = 0$ or on the same side of the straight line.
 17. Prove that $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
 18. Prove by mathematical induction that $11^{n+2} + 12^{2n+1}$ is divisible by 133 for all positive integer values of n
 19. A, B, C are 3 sets and U is the universal set such that $n(U) = 800, n(A) = 200, n(B) = 300, n(A \cap B) = 100$ Find $n(A' \cap B')$

Section C

20. Prove that $\frac{1}{\log_a b}, \frac{1}{\log_{2a} b}, \frac{1}{\log_{4a} b}$ form an AP
 21. On the average one person dies out of every 10 accidents find the probability that at least 4 will be safe out of 5 accidents.
 22. In the expansion $(1+x)^{40}$, the coefficients of T_{2r+1} and T_{r+2} are equal, find r
 23. Differentiate $\log_{10} x$ with respect to x
 24. How many 6 digits numbers can be formed with the digits 1, 2, 3, 4, 5, 6, 7 if the 10th, unit's places are always even and repetition is not allowed.
 25. Shift the origin to a suitable point so that the equation $x^2 + y^2 - 4x + 6y = 36$ representing a circle is transformed in to an equation of a circle with centre at origin in the new coordinate axes.
 26. The mean and variance of 7 observations are 8 and 19 respectively. If 5 of the observations are 2, 4, 12, 14, 11. Find the remaining observations.
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Class – XI

ANSWERS

Section A

1. Solution:

$$\begin{aligned}\sin^{-1}\left(\sin\left(\frac{6\pi}{7}\right)\right) &= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{7}\right)\right) \\ &= \sin^{-1}\left(\sin\left(\frac{\pi}{7}\right)\right) \\ &= -\frac{\pi}{2} \leq \frac{\pi}{7} \leq \frac{\pi}{2} \\ &= \frac{\pi}{7}\end{aligned}$$

2. Solution: $(a, 2a)$, $(a, -2a)$

3. Solution:

$$x + 7 = 10$$

$$x = 3$$

$$x + y = 8$$

$$y = 5$$

4. Solution:

$$\begin{aligned}\frac{3+2i}{1-i} &= \frac{3+2i}{1-i} \cdot \frac{1+i}{1+i} \\ &= \frac{3+2i+3i+2i^2}{1-i^2} = \frac{1}{2} + \frac{5}{2}i \\ (1+2i)i - \frac{3+2i}{1-i} &= (-2+i) - \left(\frac{1}{2} + \frac{5}{2}i\right) = -\frac{5}{2} - \frac{3}{2}i\end{aligned}$$

5. Solution:

$$\text{Domain} = [-1, 1] \quad \text{Range} = [0, \pi]$$

6. Solution :

$$0 \leq \cos^{-1} x \leq \pi$$

sin in this interval is positive and hence y is positive

Section B

7. Solution:

$$\alpha + \beta = b$$

$$\alpha\beta = c$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= b^2 - 2c$$

8. Solution:

$$(x + a)^n = P + Q$$

$$(x - a)^n = P - Q$$

$$(P + Q)(P - Q) = (x + a)(x - a)$$

$$P^2 - Q^2 = (x^2 - a^2)^n$$

9. Solution:

Discriminant of Numerator = $9 - 24 < 0$ and

coefficient of x^2 is positive. Hence Numerator is always positive

Hence dividing by the numerator on both sides of the inequality does not change the the sign of the inequality.

Hence we need only consider $\frac{1}{3x+4} < 0$

$$x < \frac{-4}{3}$$

$$x \in (-\infty, -\frac{4}{3})$$

10. Solution:

$$\cot(A + 15) - \tan(A - 15) = \frac{\cos(A + 15)}{\sin(A + 15)} - \frac{\sin(A - 15)}{\cos(A - 15)}$$

$$= \frac{\cos(A + 15)\cos(A - 15) - \sin(A + 15)\sin(A - 15)}{\sin(A + 15)\cos(A - 15)}$$

$$= \frac{\cos 2A}{\frac{1}{2}(\sin 2A + \frac{1}{2})}$$

$$= \frac{2\cos 2A}{\sin 2A + \frac{1}{2}}$$

$$= \frac{4\cos 2A}{1 + 2\sin 2A}$$

11. Solution :

$$4 - x^2 \geq 0$$

$$x^2 - 4 \leq 0$$

Domain of $x \in [-2, 2]$

$$y^2 = 4 - x^2$$

$$x^2 = 4 - y^2$$

$$x = \sqrt{4 - y^2}$$

$$4 - y^2 \geq 0$$

$$y^2 - 4 \leq 0$$

$$y \in [-2, 2]$$

Also for all values of $x \in [-2, 2]$

$$y = \sqrt{4 - x^2} \geq 0$$

Range $y \in [0, 2]$

12. Solution :

$$\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$\cos \theta = \frac{1 - 4}{1 + 4} = \frac{-3}{5}$$

$$\sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{2 \cdot 2}{1 + 4} = \frac{4}{5}$$

$$\frac{1}{2 + \cos \theta + \sin \theta} = \frac{1}{2 - \frac{3}{5} + \frac{4}{5}} = \frac{11}{5}$$

13. Solution:

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{x + x^3} = \lim_{x \rightarrow 0} \frac{5 \sin 5x}{5x(1 + x^2)}$$

$$= \lim_{x \rightarrow 0} \frac{5 \sin 5x}{5x} \lim_{x \rightarrow 0} \frac{1}{(1 + x^2)}$$

$$= 5 \cdot 1 \cdot 1$$

$$= 5$$

14. Solution:

$$\frac{1 - \cos 2x}{2} + \frac{1 - \cos 4x}{2} = 1$$

$$\cos 2x + \cos 4x = 0$$

$$2 \cos 3x \cos x = 0$$

$$\cos 3x = 0$$

$$x = \frac{\pi}{6} + \frac{\pi}{3}n$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} + \pi k = \frac{\pi}{6} + \frac{\pi}{3}n \quad n \text{ is integer}$$

15. Solution:

$$i^{30} + i^{40} + i^{60} = (i^4)^7 \cdot i^2 + (i^4)^{10} + (i^4)^{15}$$

$$i^4 = 1 = -1 + 1 + 1 = 1$$

16. Solution :

Substituting the points (0,0) and (5,5) on the given line

$$x + y - 8 = 0$$

$$0 + 0 - 8 = -8$$

$$5 + 5 - 8 = 2$$

Since the signs of the resulting numbers are different
the given points lie on opposite sides of the given line.

17. Solution :

$$\tan^{-1} x = A$$

$$\tan A = x$$

$$\cot^{-1} x = B$$

$$\cot B = x$$

$$\tan\left(\frac{\pi}{2} - B\right) = x$$

$$\tan^{-1} x = \frac{\pi}{2} - B$$

$$\tan^{-1} x = A$$

$$A = \frac{\pi}{2} - B$$

$$A + B = \frac{\pi}{2}$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

18. Solution:

$11^{n+2} + 12^{2n+1}$ is divisible by 133

$$n = 1$$

$$11^3 + 12^3 = (11+12)(11^2 - 11 \cdot 12 + 12^2)$$

$$= 23 \cdot 133$$

Let it be true for k

$11^{k+2} + 12^{2k+1}$ is divisible by 133

For $k = k + 1$

$$11^{k+3} + 12^{2k+3} = 11 \cdot 11^{k+2} + 12^2 \cdot 12^{2k+1}$$

$$= 11 \cdot 11^{k+2} + 144 \cdot 12^{2k+1}$$

$$= 11 \cdot 11^{k+2} + 133 \cdot 12^{2k+1} + 11 \cdot 12^{2k+1}$$

$$= 11 \cdot 11^{k+2} + 11 \cdot 12^{2k+1} + 133 \cdot 12^{2k+1}$$

Is divisible by 133 since $11^{k+2} + 12^{2k+1}$ is divisible by 133

19. Solution:

$$n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$$

$$= n(U) - [n(A) + n(B) - n(A \cap B)]$$

$$800 - [200 + 300 - 100]$$

$$= 400$$

Section C**20. Solution:**

$$\frac{1}{\log_a b} = \log_b a$$

$$\frac{1}{\log_{2a} b} = \log_b 2a$$

$$\frac{1}{\log_{4a} b} = \log_b 4a$$

$$\frac{\log_b a + \log_b 4a}{2} = \frac{\log_b (2a)^2}{2}$$

$$= 2 \frac{\log_b 2a}{2}$$

$$= \log_b 2a$$

Thus, $\frac{1}{\log_{2a} b}$ is the AM between $\frac{1}{\log_a b}$, $\frac{1}{\log_{4a} b}$

21. Solution:

$$\text{Probability of surviving} = \frac{9}{10}$$

Required to find out the probability of 4 are safe or 5 are safe

$$\text{Probability of 5 is safe} = \left(\frac{9}{10}\right)^5$$

$$\text{Probability of 4 is safe} = {}^5C_4 \left(\frac{9}{10}\right)^4 \frac{1}{10}$$

$$\text{Required Probability} = \left(\frac{9}{10}\right)^5 + 5 \left(\frac{9}{10}\right)^4 \frac{1}{10} = \frac{45927}{5000}$$

22. Solution:

$$T_{2r+1} = {}^{40}C_{2r}$$

$$T_{r+2} = {}^{40}C_{r+1}$$

$${}^{40}C_{2r} = {}^{40}C_{r+1}$$

$$2r + r + 1 = 40$$

$$3r = 39$$

$$r = 13$$

23. Solution :

$$y = \log_{10} x$$

$$x = 10^y$$

$$\log_e x = y \log_e 10$$

$$y = \frac{\log_e x}{\log_e 10}$$

$$\frac{dy}{dx} = \left(\frac{1}{\log_e 10} \right) \frac{1}{x}$$

24. Solution:

There are 3 even numbers 2, 4, 6

So the units place, 10th places can be filled in $3p_2$ ways

Remaining 5 digits can be used to fill 4 places in $5p_4$ ways.

Hence the total numbers satisfying the above condition is $3p_2 \times 5p_4 = 720$

25. Solution:

Let the origin be shifted to (h, k)

$$x = x' + h$$

$$y = y' + k$$

Then

$$(x' + h)^2 + (y' + k)^2 - 4(x' + h) + 6(y' + k) = 36$$

$$x'^2 + 2hx' + h^2 + y'^2 + 2ky' + k^2 - 4(x' + h) + 6(y' + k) = 36$$

$$x'^2 + y'^2 + x'(2h - 4) + y'(2k + 6) + h^2 + k^2 - 4h + 6k - 36 = 0$$

$$2h - 4 = 0$$

$$h = 2$$

$$2k + 6 = 0$$

$$k = -3$$

$$x'^2 + y'^2 + 2^2 + (-3)^2 - 8 - 18 - 36 = 0$$

$$x'^2 + y'^2 + 13 - 62 = 0$$

$$x'^2 + y'^2 = 49$$

26. Solution:

$$\frac{2 + 4 + 12 + 14 + 11 + x + y}{7} = 8$$

$$43 + x + y = 56$$

$$x + y = 13$$

$$\frac{2^2 + 4^2 + 12^2 + 14^2 + 11^2 + x^2 + y^2}{7} - (\text{mean})^2 = 19$$

$$\frac{4 + 16 + 144 + 196 + 121 + x^2 + y^2}{7} - 64 = 19$$

$$\frac{481 + x^2 + y^2}{7} = 83$$

$$481 + x^2 + y^2 = 581$$

$$x^2 + y^2 = 100$$

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

$$169 + (x - y)^2 = 200$$

$$(x - y)^2 = 31$$

$$x - y = 5.57$$

$$x + y = 13$$

$$x = 9.285$$

$$y = 3.715$$
