Dispersion and Spectra

Exercise Solutions

Solution 1:
Refractive index of flint glass = μ_f = 1.620
Refracting angle of flint prism = A_f = 6.0°
Refractive index of crown glass = μ_c =1.518

Now,
For zero net deviation of mean ray:
$(\mu_c - 1)A_c = (\mu_f - 1)A_f$

=> $A_c = [(\mu_f - 1)/(\mu_c - 1)]A_f$

= [1.620-1]/[1.518-1] x 6°

= 7.2°

Solution 2:

(a) Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$
Where μ_r = Refractive index of red light = 1.56
μ_y = Refractive index of yellow light = 1.60 and
μ_v = Refractive index of violet light = 1.68

=> ω = 0.2

(b) Angular dispersion = $\delta = [\mu_v - \mu_r]A$
Here, Refracting angle of prism = A = 6.0°

=> $\delta = (1.68 - 1.56)6^0 = 7.2°$

Solution 3:
The focal length of a lens is given by

$1/f = (\mu - 1)[1/R_1 - R_2]$

$$(\mu - 1) = \frac{1}{f}\frac{1}{(\frac{1}{R_1} - \frac{1}{R_2})}$$

Let $k = 1/[\ 1/R_1 – 1/R_2]$

Then, $\mu – 1 = k/f$

So, $\mu_r - 1 = k/100$ …(1)
$\mu_v - 1 = k/98$ …….(2) and
$\mu_v - 1 = k/96$ …..(3)

Where, μ_r = Refractive index for the red color
μ_v = Refractive index for the violet color
μ_y = Refractive index for the yellow color

Now,
Dispersive power = $\omega = [\mu_v - \mu_r]/[\ \mu_y - 1]$

$= [(\mu_v - \mu_r) – (\mu_r – 1)\ ]/[\ \mu_y - 1]$

$= [k/96 – k/100]/[k/98]$

$\Rightarrow \omega = 0.0408$

Solution 4:
$\mu_v - \mu_r = 0.014$ [given]

μ_y = [Real depth]/[Apparent depth] = 2/1.30 = 1.515

Dispersive power = $\omega = [\mu_v - \mu_r]/[\ \mu_y - 1]$

$= 0.014/[1.515\ -1]$

$= 0.027$

Solution 5:

Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$

Here, $\mu_v = 1.65$, $\mu_r = 1.61$, $\omega = 0.07$ and $\delta_y = 4^o$

$\Rightarrow 0.07 = [1.65 – 1.61]/[\mu_y - 1]$

$\Rightarrow \mu_y – 1 = 0.04/0.07 = 4/7$

Again, $\delta = (\mu - 1)A$

$\Rightarrow A = \delta_y/[\mu_y – 1] = 4/(4/7) = 7^o$

Solution 6:

Minimum Deviations by Red = $\delta_r = 38.4^0$

Minimum Deviations by Yellow = $\delta_y = 38.7^0$

Minimum Deviations by Violet = $\delta_v = 39.2^0$

Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$

We know, $\delta = (\mu - 1)A$

$\omega = [\mu_v - \mu_r]/[\mu_y - 1] = [(\mu_v -1) – (\mu_r -1)]/[\mu_y - 1]$

$= [(\delta_v/A) - (\delta_r/A)]/[(\delta_v/A)]$

$= [\delta_v – \delta_r]/\delta_v$

$= [39.2-38.4]/38.7$

$= 0.0204$

Solution 7: Two prisms of identical geometrical shape are combined.
Let A = Angle of the prisms.

$\delta = (\mu_{v1} - 1)A - (\mu_{v2} - 1)A$

Where, Refractive index of violet light through first prism = $\mu_{v1} = 1.52$
Refractive index of violet light from second prism = $\mu_{v2} = 1.62$
Deviation of violet light through prism = $\delta = 1^0$

Or, $\delta = (\mu_{v1} - \mu_{v2})A$
$\Rightarrow A = \delta/(\mu_{v1} - \mu_{v2})$
On substituting the values, we get
$A = 10^o$

Solution 8:
Total deviation for yellow ray produced by the prism combination:

$\delta_v - \delta_r = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2\delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{fy} - 1)A'$

Similarly, angular dispersion produced by the combination:
$\delta_v - \delta_r = 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$

(a) for net angular dispersion to be zero, $\delta v - \delta r = 0$

$\Rightarrow 2(\mu_{vc} - 1) A - (\mu_{vf} - 1) A' = 0$

$\Rightarrow 2(\mu_{vc} - 1) A = (\mu_{vf} - 1) A'$

$\Rightarrow A'/A = [2(\mu_{vc} - 1)]/[(\mu_{vf} - 1)]$

(b) For net deviation in the yellow ray to be zero, $\delta y = 0$
$2(\mu_{cy} - 1) A - (\mu_{fy} - 1) A' = 0$
$\Rightarrow A'/A = [2(\mu_{cy} - 1)]/[\mu_{fy} – 1]$

Solution 9:

A thin prism of crown glass (μ_r = 1.515, μ_v = 1.525) and a thin prism of flint glass ((μ_r = 1.612, μ_v = 1.632) are placed in contact with each other.

Since, they are similarly directed, the total deviation produced

$\delta = \delta_c + \delta_f = (\mu_c - 1)A + (\mu_f - 1)A$

$= (\mu_c + \mu_f - 2)A$

So, angular dispersion of the combination:

$\delta_v - \delta_r = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr})A$

$= (\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A$

$= (1.525 + 1.632 - 1.515 - 1.612)5$

$= 0.15^{o}$

Solution 10:

Here, For first prism:

$A_1 = 60$, $\omega_1 = 0.07$, $\mu_1 = 1.50$

For Second Prism

$A_2 = ?$, $\omega_2 = 0.08$ and $\mu_2 = 1.60$

The combination produces no deviation in the mean ray.

(a) $\delta = (\mu_2 - 1)A_2 - (\mu_1 - 1)A_1 = 0$

$\Rightarrow (1.6 – 1)A_2 – (1.5 - 1)6 = 0$

$\Rightarrow A_2 = 5^{o}$

(b) When a beam of white light passes through it,

Net angular dispersion = $(\mu_2 - 1)\omega_2 A_2 - (\mu_1 - 1)\omega_1 A_1$

$= (1.60 - 1)5^{o} - (1.60 - 1)5^{o}$

$= 0.03^{o}$

(c) If the prisms are similarly directed.

$\delta = (\mu_2 - 1)A_2 - (\mu_1 - 1)A_1 = 0$

$= (1.60 -1)5^{o} – (1.50 – 1)6^{o}$
$\Rightarrow \delta = 6^{o}$

(d) Similarly, if the prisms are similarly directed,
Net angular dispersion :
$\mu_v - \mu_r = (\mu_2 - 1)\omega_2 A_2 - (\mu_1 - 1)\omega_1 A_1$

$= (1.6 – 1)x 0.08 x 5 – (1.50-1)x0.07x6$

$= 0.45^{o}$

Solution 11:
Refractive index of a material M1 changes = $(\mu_{v1} – \mu_{r1}) = 0.014$
Refractive index of a material M2 changes = $(\mu_{v2} – \mu_{r2}) =0.024$
Prism angle of a material $M_1 = A_1 = 5.3^0$
Prism angle of a material $M_2 = A_2 = 3.7^0$

(a) When the prisms are oppositely directed,

Angular dispersion = $\delta = (\mu_{v2} – \mu_{r2})A_2 - (\mu_{v1} – \mu_{r1})A_1$

$\Rightarrow \delta = (0.024)x3.7^0 – (0.024)x5.3^{o} = 0.0146^{o}$

(b) When they are similarly directed,
Angular dispersion = $\delta = (\mu_{v2} – \mu_{r2})A_2 + (\mu_{v1} – \mu_{r1})A_1$

$\Rightarrow \delta = (0.024)x3.7^0 + (0.024)x5.3^{o} = 0.0163^{o}$