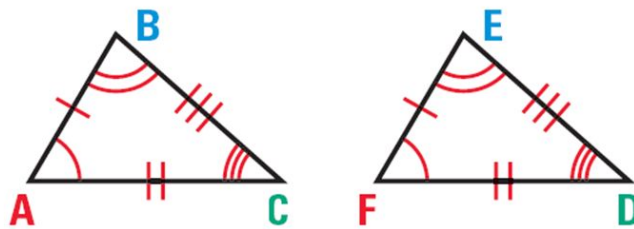


# Congruence

## Congruence Statement

When naming two congruent triangles, order is very impo



$\triangle ABC \cong \triangle FED$  or  $\triangle BCA \cong \triangle EDF$ .

Corresponding angles  $\angle A \cong \angle F$   $\angle B \cong \angle E$   $\angle C \cong \angle D$

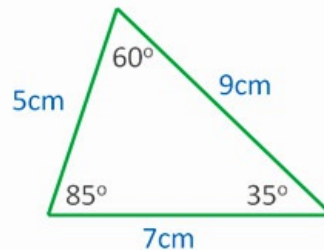
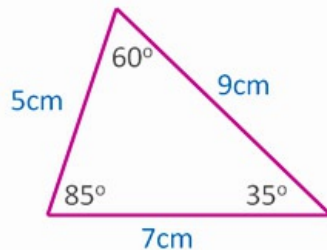
Corresponding sides  $\overline{AB} \cong \overline{FE}$   $\overline{BC} \cong \overline{ED}$   $\overline{AC} \cong \overline{FD}$

### Conditions for Congruence of Two Triangles

|                                                                                                |                                                                                                    |
|------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| <p>SSS (Side – Side – Side)</p> <p>3 sides are respectively equal</p>                          | <p>SAS (Side – Angle – Side)</p> <p>2 sides and the included angle are respectively equal</p>      |
| <p>ASA (Angle – Side – Angle)</p> <p>2 angles and the included side are respectively equal</p> | <p>RHS (Right angle – Hypotenuse – Side)</p> <p>Hypotenuse and one side are respectively equal</p> |

# Congruent Triangles

Identical Triangles have all three Sides, and all three Angles exactly the same sizes.



If we gave several people three sticks: 5cm, 7cm and 9cm long, they would all only be able to make the exact same Triangle.

Q1

Answer :

We have to state the correspondence between the vertices, sides and angles of the following pairs of congruent triangles.

(i)  $\triangle ABC \cong \triangle EFD$

Correspondence between vertices :

$A \leftrightarrow E, B \leftrightarrow F, C \leftrightarrow D$

Correspondence between sides :

$AB = EF, BC = FD, CA = DE$

Correspondence between angles :

$\angle A = \angle E, \angle B = \angle F, \angle C = \angle D$

(ii)  $\triangle CAB \cong \triangle QRP$

Correspondence between vertices :

$C \leftrightarrow Q, A \leftrightarrow R, B \leftrightarrow P$

Correspondence between sides :

$CA = QR, AB = RP, BC = PQ$

Correspondence between angles :

$\angle C = \angle Q, \angle A = \angle R, \angle B = \angle P$

(iii)  $\triangle XZY \cong \triangle QPR$

Correspondence between vertices :

$X \leftrightarrow Q, Z \leftrightarrow P, Y \leftrightarrow R$

Correspondence between sides :

$XZ = QP, ZY = PR, YX = RQ$

Correspondence between angles :

$\angle X = \angle Q, \angle Z = \angle P, \angle Y = \angle R$

(iv)  $\triangle MPN \cong \triangle SQR$

Correspondence between vertices :

$M \leftrightarrow S, P \leftrightarrow Q, N \leftrightarrow R$

Correspondence between sides :

$MP = SQ, PN = QR, NM = RS$

Correspondence between angles :

$\angle M = \angle S, \angle P = \angle Q, \angle N = \angle R$

Q2

Answer :

(i)  $\triangle ACB \cong \triangle DEF$

(SAS congruence property)

(ii)  $\triangle RPQ \cong \triangle LNM$

(RHS congruence property)

(iii)  $\triangle YXZ \cong \triangle TRS$

(SSS congruence property)

(iv)  $\triangle DEF \cong \triangle PNM$

(ASA congruence property)

(v)  $\triangle ACB \cong \triangle ACD$

(ASA congruence property)

Q3

Answer :

Given :

$$PL \perp OA$$

$$PM \perp OB$$

$$PL = PM$$

To prove :

$$\triangle PLO \cong \triangle PMO$$

Proof :

In  $\triangle PLO$  and  $\triangle PMO$  :

$$\angle PLO = \angle PMO \quad (90^\circ \text{ each})$$

$$PO = PO \quad (\text{common})$$

$$PL = PM \quad (\text{given})$$

By RHS congruence property :

$$\triangle PLO \cong \triangle PMO$$

Q4

Answer :

Given :

$$AD = BC$$

$$AD \parallel BC$$

We have to show that  $AB = DC$ .

Proof :

$$AD \parallel BC$$

$$\therefore \angle BCA = \angle DAC \quad (\text{alternate angles})$$

In  $\triangle ABC$  and  $\triangle CDA$  :

$$BC = DA \quad (\text{given})$$

$$\angle BCA = \angle DAC \quad (\text{proved above})$$

$$AC = AC \quad (\text{common})$$

By SAS congruence property :

$$\triangle ABC \cong \triangle CDA$$

$$\Rightarrow AB = CD \quad (\text{corresponding parts of the congruent triangles})$$

Q5

Answer :

Given :

$$AB = AC, BD = DC$$

To prove :  $\triangle ADB \cong \triangle ADC$

Proof :

(i) In  $\triangle ADB$  and  $\triangle ADC$  :

$$AB = AC \quad (\text{given})$$

$$BD = DC \quad (\text{given})$$

$$DA = DA \quad (\text{common})$$

By SSS congruence property :

$$\triangle ADB \cong \triangle ADC$$

$$\angle ADB = \angle ADC \quad (\text{corresponding parts of the congruent triangles}) \quad \dots(1)$$

$\angle ADB$  and  $\angle ADC$  are on the straight line.

$$\therefore \angle ADB + \angle ADC = 180^\circ$$

$$\angle ADB + \angle ADB = 180^\circ$$

$$\Rightarrow 2\angle ADB = 180^\circ$$

$$\Rightarrow \angle ADB = 90^\circ$$

From (1) :

$$\angle ADB = \angle ADC = 90^\circ$$

(ii)  $\angle BAD = \angle CAD$  (corresponding parts of the congruent triangles)

Q6

Answer :

Given :

$AD$  is a bisector of  $\angle A$ .

$$\Rightarrow \angle DAB = \angle DAC \quad \dots(1)$$

$AD \perp BC$

$$\Rightarrow \angle BDA = \angle CDA \quad (90^\circ \text{ each})$$

To prove :

$\triangle ABC$  is isosceles.

Proof :

In  $\triangle DAB$  and  $\triangle DAC$  :

$$\angle BDA = \angle CDA \quad (90^\circ \text{ each})$$

$$DA = DA \quad (\text{common})$$

$$\angle DAB = \angle DAC \quad (\text{from 1})$$

By ASA congruence property :

$$\triangle DAB \cong \triangle DAC$$

$$\Rightarrow AB = AC \quad (\text{corresponding parts of the congruent triangles})$$

Therefore,  $\triangle ABC$  is isosceles.

Q7

Answer :

Given :

$$AB = AD$$

$$CB = CD$$

To prove :

$$\triangle ABC \cong \triangle ADC$$

Proof :

In  $\triangle ABC$  and  $\triangle ADC$  :

$$AB = AD \quad (\text{given})$$

$$BC = DC \quad (\text{given})$$

$$AC = AC \quad (\text{common})$$

$$\therefore \triangle ABC \cong \triangle ADC \quad (\text{by SSS congruence property})$$

Q8

Answer :

Given :

$$PA \perp AB$$

$$QB \perp AB$$

$$PA = QB$$

To prove :  $\triangle OAP \cong \triangle OBQ$

Find whether  $OA = OB$ .

Proof :

In  $\triangle OAP$  and  $\triangle OBQ$  :

$$\angle POA = \angle QOB \quad (\text{vertically opposite angles})$$

$$\angle OAP = \angle OBQ \quad (90^\circ \text{ each})$$

$$PA = QB \quad (\text{given})$$

By AAS congruence property :

$$\triangle OAP \cong \triangle OBQ$$

$$\Rightarrow OA = OB \quad (\text{corresponding parts of the congruent triangles})$$

Q9

Answer :

Given :

Triangles  $ABC$  and  $DCB$  are right angled at  $A$  and  $D$ , respectively.

$$AC = DB$$

To prove :  $\triangle ABC \cong \triangle DCB$

In  $\triangle ABC$  and  $\triangle DCB$  :

$$\angle CAB = \angle BDC \quad (90^\circ \text{ each})$$

$$BC = BC \quad (\text{common})$$

$$AC = DB \quad (\text{given})$$

By R.H.S. congruence property :

$$\triangle ABC \cong \triangle DCB$$

Q10

**Answer :**

*Given :*

$\triangle ABC$  is an isosceles triangle in which  $AB = AC$ .

$E$  and  $F$  are midpoints of  $AC$  and  $AB$ , respectively.

To prove :

$$BE = CF$$

*Proof :*

$E$  and  $F$  are midpoints of  $AC$  and  $AB$ , respectively.

$$\Rightarrow AF = FB, AE = EC$$

$$AB = AC$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} AC$$

$$\Rightarrow FB = EC$$

$$\angle ABC = \angle ACB \quad (\text{angle opposite to equal sides are equal})$$

$$\Rightarrow \angle FBC = \angle ECB$$

Consider  $\triangle BCF$  and  $\triangle CBE$  :

$$BC = BC \quad (\text{common})$$

$$\Rightarrow \angle FBC = \angle ECB$$

Consider  $\triangle BCF$  and  $\triangle CBE$  :

$$BC = BC \quad (\text{common})$$

$$\angle FBC = \angle ECB \quad (\text{proved above})$$

$$FB = EC \quad (\text{proved above})$$

By SAS congruence property :

$$\triangle BCF \cong \triangle CBE$$

$$BE = CF \quad (\text{corresponding parts of the congruent triangles})$$

Q11

**Answer :**

*Given :*

$$AB = AC$$

$\triangle ABC$  is an isosceles triangle.

$$AP = AQ$$

To prove :

$$BQ = CP$$

*Proof :*

$$AB = AC \quad (\text{given})$$

$$AP = AQ \quad (\text{given})$$

$$AB - AP = AC - AQ$$

$$\Rightarrow BP = CQ$$

$$\angle ABC = \angle ACB \quad (\text{angle opposite to the equal sides are equal})$$

$$\Rightarrow \angle PBC = \angle QCB$$

In  $\triangle PBC$  and  $\triangle QCB$  :

$$PB = QC \quad (\text{proved above})$$

$$\angle PBC = \angle QCB \quad (\text{proved above})$$

$$BC = BC \quad (\text{common})$$

By SAS congruence property :

$$\triangle PBC \cong \triangle QCB$$

$$BQ = CP \quad (\text{corresponding parts of the congruent triangles})$$

Q12

**Answer :**

**Given :**

$ABC$  is an isosceles triangle.

$$AB = AC$$

$$BD = CE$$

To prove :

$$BE = CD$$

**Proof:**

$$AB + BD = AC + CE \quad (\text{As, } AB = AC, BD = CE)$$

$$\Rightarrow AD = AE$$

Consider  $\triangle ACD$  and  $\triangle ABE$  :

$$AC = AB \quad (\text{given})$$

$$\angle CAD = \angle BAE \quad (\text{common})$$

$$AD = AE \quad (\text{proved above})$$

By *SAS* congruence property :

$$\triangle ACD \cong \triangle ABE$$

$$\Rightarrow CD = BE \quad (\text{corresponding parts of the congruent triangles})$$

Q13

**Answer :**

**Given :**

$\triangle ABC$  is an isosceles triangle.

$$AB = AC$$

$$BD = CD$$

To prove :

$AD$  bisects  $\angle A$  and  $\angle D$ .

**Proof :**

Consider  $\triangle ABD$  and  $\triangle ACD$  :

$$AB = AC \quad (\text{given})$$

$$BD = CD \quad (\text{given})$$

$$AD = AD \quad (\text{common})$$

By *SSS* congruence property :

$$\triangle ABD \cong \triangle ACD$$

$$\Rightarrow \angle BAD = \angle CAD \quad (\text{by cpct})$$

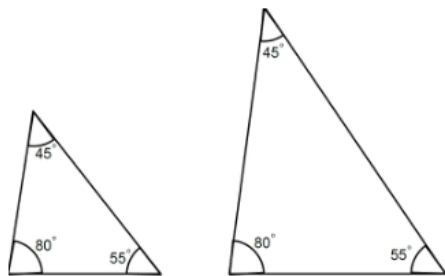
$$\Rightarrow \angle BDA = \angle CDA \quad (\text{by cpct})$$

Q14

**Answer :**

No, its not necessary. If the corresponding angles of two triangles are equal, then they may or may not be congruent.

They may have proportional sides as shown in the following figure:



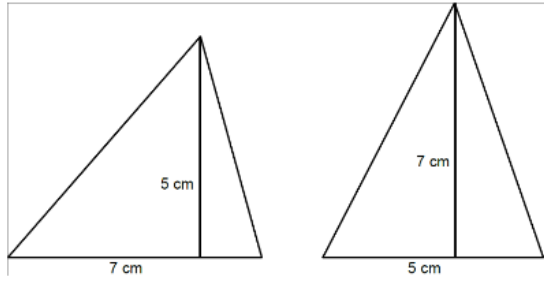
Q15

**Answer :**

No, two triangles are not congruent if their two corresponding sides and one angle are equal. They will be congruent only if the said angle is the included angle between the sides.

Q16

**Answer :**



Both triangles have equal area due to the the same product of height and base. But they are not congruent.

Q17

**Answer :**

- (i) the same length
- (ii) the same measure
- (iii) the same side length
- (iv) the same radius
- (v) the same length and the same breadth
- (vi) equal parts

Q18

**Answer :**

(i) False

This is because they can be equal only if they have equal sides.

(ii) True

This is because if squares have equal areas, then their sides must be of equal length.

(iii) False

For example, if a triangle and a square have equal area, they cannot be congruent.

(iv) False

For example, an isosceles triangle and an equilateral triangle having equal area cannot be congruent.

(v) False

They can be congruent if two sides and the included angle of a triangle are equal to the corresponding two sides and the included corresponding angle of another triangle.

(vi) True

This is because of the AAS criterion of congruency.

(vii) False

Their sides are not necessarily equal.

(viii) True

This is because of the AAS criterion of congruency.

(ix) False

This is because two right triangles are congruent if the hypotenuse and one side of the first triangle are respectively equal to the hypotenuse and the corresponding side of the second triangle.

(x) True