Permutations and Combinations

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. The number of six digit numbers, whose all digits are odd (*i.e.*, 1, 3, 5, 7, 9), is

(a) 6^5 (b) 5^6

(c) $\frac{6!}{2!}$ (d) None of these

2. Evaluate : (i) 5! (ii) 7!

(i) (ii) **(i) (ii)**

(a) 110, 5030 (b) 120, 5040

(c) 115, 5020 (d) 121, 5000

3. Compute $\frac{9!}{6!}$.

(a) 420 (b) 400 (c) 504 (d) 440

4. Evaluate $\frac{n!}{r!(n-r)!}$, when $n = 5$, $r = 2$.

(a) 10 (b) 12 (c) 14 (d) 16

5. If $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$, then find the value of x.

(a) 90 (b) 100 (c) 80 (d) 95

6. Evaluate 8! – 4!.

(a) 49206 (b) 49400 (c) 49000 (d) 40296

7. In a class, there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class for a function. In how many ways can the teacher make this selection?

(a) 378 (b) 377

(c) 375 (d) 379

8. Find the value of n such that $\frac{{}^{n}P_4}{{}^{n-1}P_4}=\frac{5}{3}, n>4$.

(a) 11 (b) 10 (c) 13 (d) 12

9. Find the value of n such that

${}^{n}P_5 = 42\ {}^{n}P_3,\ n > 4$.

(a) 10 (b) 15 (c) 12 (d) 20

10. Find the number of permutations of the letters of the word ALLAHABAD.

(a) 7530 (b) 7540 (c) 7560 (d) 7500

11. How many 4-digit numbers can be formed by using the digits 1 to 9, if repetition of digits is not allowed?

(a) 3024 (b) 3026 (c) 3040 (d) 3014

12. Find r, if $5\ {}^{4}P_r = 6\ {}^{5}P_{r-1}$.

(a) 2 (b) 4 (c) 3 (d) 5

13. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row, if the discs of the same colour are indistinguishable?

(a) 1200 (b) 1220 (c) 1240 (d) 1260

14. Seven different letters are given. Then the number of ways in which words of 5 letters can be formed such that atleast one of the letters is repeated, is

(a) ${}^{7}P_5$ (b) 14287 (c) 5^7 (d) 7^5

15. Find the number of arrangements of the letters of the word INDEPENDENCE when words begin with I and end in P.

(a) 12400 (b) 12420 (c) 12600 (d) 12620

16. Number of ways in which 3 boys and 3 girls (all are of different heights) can be arranged in a line so that boys as well as girls among themselves are in decreasing order of their heights (from left to right), is

(a) 6! (b) 3! × 3! × 2!

(c) 10 (d) 20

17. The number of ways in which the digits of the number 125453752 can be rearranged such that no two 5's come together, is

(a) $\frac{9!}{3!2!}$ (b) $\frac{7!}{3!2!}$

(c) $\frac{{}^{7}C_3\cdot 6!}{2!}$ (d) None of these

18. Find the number of permutations of the letters of the word AHMEDABAD.

(a) 31250 (b) 28540

(c) 30240 (d) 31500

19. How many different nine digit numbers can be formed from the number 223355888 by re-arranging its digits so that the odd digits occupy even positions?

(a) 16 (b) 36
(c) 60 (d) 180

20. Find the number of arrangements of the letters of the word INDEPENDENCE when vowels never occur together.

(a) 1646400 (b) 1646450
(c) 1640000 (d) 1646430

21. Four writers must write a book containing 17 chapters. The first and third writer must write 5 chapters each, the second writer must write 4 chapters and fourth writer must write three chapters. The number of ways that can be found to divide the book between four writers, is

(a) $\dfrac{17!}{(5!)^2 4!3!2!}$ (b) $\dfrac{17!}{5!4!3!2!}$
(c) $\dfrac{17!}{(5!)^2 4!3!}$ (d) $\dfrac{17!}{(5!)^2 \times 4 \times 3}$

22. A library has a copies of one book, b copies of each of two books, c copies of each of three books and single copies of d books. The total number of ways in which these books can be arranged in a shelf, is

(a) $\dfrac{(a+b+c+d)!}{a!\,b!\,c!}$ (b) $\dfrac{(a+2b+3c+d)!}{a!(b!)^2\,(c!)^3}$
(c) $\dfrac{(a+2b+3c+d)!}{a!\,b!\,c!}$ (d) None of these

23. Evaluate ${}^{12}C_2$.

(a) 66 (b) 65 (c) 60 (d) 64

24. If ${}^{10}C_{x-1} > 2\ {}^{10}C_x$, then set of values of x is

(a) $[1, 10]$ (b) $(7, \infty)$
(c) $\{1, 2, 3\}$ (d) $\{8, 9, 10\}$

25. If ${}^{2n+3}C_{2n} - {}^{2n+2}C_{2n-1} = 15(2n + 1)$, then $n =$

(a) 13 (b) 14 (c) 27 (d) 15

26. If ${}^nC_9 = {}^nC_8$, then find ${}^nC_{17}$.

(a) 1 (b) 2 (c) 0 (d) 3

27. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$, then

(a) $n > 5$ (b) $n > 6$
(c) $n > 7$ (d) None of these

28. Find the number of ways of choosing 4 cards from a pack of 52 playing cards when four cards belong to four different suits.

(a) 4^{13} (b) 13^3 (c) 13^5 (d) 13^4

29. In how many ways a committee consisting of 3 men and 2 women can be chosen from 7 men and 5 women?

(a) 45 (b) 350 (c) 4200 (d) 230

30. Find the number of ways of choosing 4 face cards from a pack of 52 playing cards.

(a) 495 (b) 493 (c) 490 (d) 492

31. A crocodile is known to have not more than 68 teeth. The total number of crocodiles with different set of teeth, are

(a) 68 (b) 68! (c) 16^{17} (d) 68^{68}

32. Find the number of ways of choosing two red cards and two black cards from a pack of 52 playing cards.

(a) 105620 (b) 105624
(c) 105625 (d) 105600

33. The value of $\dfrac{{}^{10}C_r}{{}^{11}C_r}$, when the numerator and denominator takes its greatest value, is

(a) $\dfrac{6}{11}$ (b) $\dfrac{5}{11}$ (c) $\dfrac{10}{6}$ (d) $\dfrac{10}{5}$

34. The number of ways you can find to pack 9 different books into five parcels if four of the parcels must contain two books each, is

(a) 945
(b) ${}^9C_2 \times {}^7C_2 \times {}^5C_2 \times {}^3C_2$
(c) ${}^9C_2 \times {}^7C_2 \times {}^5C_2 \times {}^3C_2 \times 5!$
(d) None of these

35. The number of diagonals of a polygon of 30 sides is

(a) 225 (b) 350 (c) 405 (d) 210

36. The number of circles that can be drawn out of 10 points of which 7 are collinear, is

(a) 120 (b) 113 (c) 85 (d) 86

37. If m points on one straight line are joined to n points on another straight line. The number of points of intersection of the line segments thus formed is

(a) $\dfrac{{}^mC_2 \cdot {}^nC_2}{4}$ (b) $\dfrac{mn(m-1)(n-1)}{4}$
(c) $\dfrac{{}^mC_2 \cdot {}^nC_2}{2}$ (d) ${}^mC_2 + {}^nC_2$

38. The triplet (x, y, z) is chosen from the set $\{1, 2, 3, \ldots n\}$, such that $x \le y < z$. The number of such triplets is

(a) n^3 (b) ${}^nC_3$
(c) ${}^nC_2$ (d) ${}^nC_2 + {}^nC_3$

39. ABC is a triangle. 4, 5, 6 points are marked on the sides AB, BC, CA respectively, the number of triangles having vertices on different sides is

(a) $(4 + 5 + 6)!$ (b) $(4 - 1)(5 - 1)(6 - 1)$
(c) $5!\ 4!\ 6!$ (d) $4 \times 5 \times 6$

40. There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one in each box, could be placed such that a ball does not go to box of its own colour, is

(a) 8 (b) 7 (c) 9 (d) 10

Case Based MCQs

Case I : Read the following passage and answer the questions from 41 to 45 .

Sumit works at a book shop. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.

41. In how many ways can he select either a History book or a maths book?

(a) 10 (b) 8
(c) 20 (d) 60

42. If he select 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made.

(a) 200 (b) 220
(c) 240 (d) 260

43. Find the number of ways, if the books of same subject are put together.

(a) $4! \cdot 2! \cdot 3!$ (b) $2! \cdot 3! \cdot 2! \cdot 5!$
(c) $5! \cdot 2! \cdot 4!$ (d) $3! \cdot 5! \cdot 3! \cdot 4!$

44. If we are given the number of arrangement of books as ${}^5P_2 \times {}^3P_1 \times {}^4P_1$, then the arrangement is in the manner

(a) 2 History books, 2 Maths books, 3 Science books respectively.
(b) 2 History books, 3 Maths books, 2 Science books respectively.
(c) 3 History books, 2 Maths books, 2 Science books respectively.
(d) None of these

45. Find the number of arrangements, if he select 3 History books, 2 Maths Books, 2 Science books are

(a) 4300 (b) 4320
(c) 4330 (d) 4380

Case II : Read the following passage and answer the questions from 46 to 50.

Reema and Seema are two friends. Reema decided to invite her friend for dinner. When she tried to call her for invitation first she forgot her mobile number. She had only first 4-digits i.e., 9, 7, 6 and 5.

46. If the repetition of the digits is not allowed, then find the number of ways to find her mobile number.

(a) 5! (b) 6!
(c) 7! (d) 8!

47. If the repetition is allowed, then the number of ways to find 10 digit mobile number is

(a) 5^6 (b) 7^6 (c) 9^6 (d) 10^6

48. If digits 3 and 4 come together, then number of ways to find mobile number (when repetition is not allowed), is

(a) 200 (b) 220
(c) 240 (d) 260

49. The number of ways to find 10 digit mobile number that digits 3 and 4 not occur together (when repetition is not allowed) is

(a) 720 (b) 240 (c) 400 (d) 480

50. Find the number of ways to find mobile number so that 4 become the unit digit and repetition is not allowed.

(a) 100 (b) 110 (c) 120 (d) 130

Case III : Read the following passage and answer the questions from 51 to 55.

Riya and her 5 friends went for a trip to Shimla. They stayed in a hotel. There were 4 vacant rooms A, B, C and D. Out of these 4 vacant rooms, two room A and B were double share rooms and two rooms C and D can contain one person each.

51. The number of ways in which room A can be filled is

(a) 10 (b) 15 (c) 20 (d) 25

52. If room A and B are already filled each, then find the number of ways in which room C can be filled.

(a) 2 (b) 4 (c) 6 (d) 8

53. The total number of ways of accommodating Riya and her friends in these 4 vacant rooms is

(a) 150 (b) 160

(c) 170 (d) 180

54. If room A is filled with 2 persons, then find the number in which rooms C and D can be filled.

(a) 4 (b) 12 (c) 8 (d) 10

55. $\begin{pmatrix} n \\ r-1 \end{pmatrix} + \begin{pmatrix} n \\ r \end{pmatrix} =$

(a) $\begin{pmatrix} n \\ r \end{pmatrix}$ (b) $\begin{pmatrix} n+1 \\ r \end{pmatrix}$

(c) $\begin{pmatrix} n-1 \\ r \end{pmatrix}$ (d) None of these.

Assertion & Reasoning Based MCQs

Directions (56 to 60) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.

(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.

(c) Assertion is correct statement but Reason is wrong statement.

(d) Assertion is wrong statement but Reason is correct statement.

56. Assertion : Number of lines formed by joining n points on a circle ($n \geq 2$) is $\frac{n(n-1)}{2}$.

Reason : $C(n, 2) = \frac{n(n-1)}{2}$.

57. Assertion : Product of five consecutive natural numbers is divisible by 4!

Reason : Product of n consecutive natural numbers is divisible by $(n+1)!$.

58. Assertion : Number of rectangles on a chess board is ${}^8C_2 \times {}^8C_2$.

Reason : To form a rectangle, we have to select any two of the horizontal line and any two of the vertical line.

59. Assertion : If n is a positive integer, then $n(n^2 - 1)(n+2)$ is divisible by 24.

Reason : Product of r consecutive whole numbers is divisible by $\underline{|r}$.

60. Assertion : The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Reason : The number of ways of choosing any 3 places, from 9 different places is ${}^9C_3$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

2. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

3. If ${}^nC_3 = {}^nC_4$, find ${}^nC_7$.

4. Determine n, if $C(2n, 3) : C(n, 3) = 11 : 1$.

5. If ${}^{n+1}C_2 = 45$. Find n.

6. Find n, if $C(25, n + 5) = C(25, 2n - 1)$.

7. If ${}^{10}C_x = {}^{10}C_{x+4}$, then find the value of x.

8. If ${}^nC_5 = {}^nC_7$, then find n.

9. If $C(n, 8) = C(n, 6)$, then find $C(n, 2)$.

10. If ${}^nC_{10} = {}^nC_3$, then find ${}^nC_3$.

Short Answer Type Questions (SA-I)

11. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

12. For a set of five true or-false questions, no student has written all the correct answers, and no two students have given the same sequence of answers. What is the maximum number of students in the class, for this to be possible?

13. How many words (with or without meaning) of three distinct English alphabets are there?

14. How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6, if the digits can be repeated?

15. Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. Find the total number of persons in the room.

16. A convex polygon has 65 diagonals. Find the number of sides of the polygon.

17. A polygon has 44 diagonals. If n denotes the number of vertices of polygon. Find the value of n.

18. How many words, each of 3 vowels and 2 consonants can be formed from the letters of the word 'INVOLUTE'?

19. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of one man and two women?

20. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and two different consonants can be formed from alphabets?

Short Answer Type Questions (SA-II)

21. If the letters of the word 'EXAMINATION' are arranged in all possible ways as listed in dictionary. How many words are there in the list in which the first word start with 'A'?

22. If there are 5 boys and 5 girls in a class, then in how many ways they can be seated in a row such that

(i) No two girls sit together?

(ii) All the girls never sit together?

23. The letters of the word 'WOMAN' are written in all possible orders and these words are written out as in a dictionary, then find the rank of the word 'WOMAN'.

24. (i) How many different numbers of 6-digit number can be formed with the digits 1, 2, 7, 0, 9, 5?

(ii) How many of them are divisible by 10?

(iii) How many of them will have zero in the ten's place?

25. How many words, with or without meaning can be made from the letters of the word 'MONDAY', assuming that no letter is repeated, if :

(i) 4 letters are used at a time,
(ii) All letters are used at a time?

26. How many automobile license plates can be made, if each plate contains two different letters (English alphabets) followed by three different digits?

27. How many numbers greater than 1000000 can be formed by using the digits 1, 2, 3, 4, 0, 5, 3?

28. How many numbers greater than 1000000 can be formed by using digits 1, 2, 0, 2, 4, 2, 4?

29. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then find the values of n and r.

30. A candidate is required to answer 7 questions out of 12 questions, which are divided into two sections A and B, each containing 6 questions. He is not permitted to attempt more than 5 questions from each section. Find the number of different ways of selecting the questions.

31. There are 15 points in a plane, out of which only 6 are in a straight line.
(i) How many different straight lines can be made?
(ii) How many triangles can be made?

32. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side of the table. Find the number of possible seating arrangements.

33. A committee of 7 has to be formed from 9 boys and 4 girls. In how may ways can this be done when committee consist of
(i) At least 3 girls (ii) At most 3 girls?

34. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that :
(i) Repetition of the digits is allowed?
(ii) Repetition of the digits is not allowed?

35. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8 books, he does not want to borrow mathematics part II unless mathematics part I is also borrowed. In how many ways can he choose the three books to be borrowed?

Long Answer Type Questions (LA)

36. If ${}^nC_r : {}^nC_{r+1} = 1 : 2$ and ${}^nC_{r+1} : {}^nC_{r+2} = 2 : 3$, then find the values of n and r.

37. How many words with or without meaning can be formed using the letters of the word 'DAUGHTER', if
(i) All vowels are never together
(ii) Vowels occupy odd places?

38. (i) How many different words can be formed with the letters of the word HARYANA?
(ii) How many of these begin with H and end with N?
(iii) In how many of these H and N are together?

39. From a class of 12 boys and 10 girls, 10 students are to be chosen for a competition, including atleast 4 boys and 4 girls. The two girls who won the prizes last year should be included. In how many ways can the selection be made?

40. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
(i) No girl?
(ii) Atleast one boy and one girl?
(iii) Atleast 3 girls?

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. (b) : Since six digit numbers, whose all digits are odd, are to be formed suggests that repetition of digits is must.
$\because$ Available digits are 1, 3, 5, 7 and 9
Hence required numbers is 5^6.

2. (b) : (i) $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$
(ii) $7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$

3. (c) : We have $\frac{9!}{6!} = \frac{9\times8\times7\times6!}{6!} = 9\times8\times7 = 504$

4. (a) : Since $n = 5$, $r = 2$

We have $\frac{5!}{2!(5-2)!} = \frac{5!}{2!\times3!} = \frac{4\times5}{2} = 10.$

5. **(b) :** We have $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$

$\Rightarrow \quad \frac{1}{8!}+\frac{1}{9\times 8!}=\frac{x}{10\times 9\times 8!}$

Therefore, $1+\frac{1}{9}=\frac{x}{10\times 9}$ or $\frac{10}{9}=\frac{x}{10\times 9}$

So, $x = 100$.

6. **(d) :** $8! - 4! = 40320 - 24 = 40296$.

7. **(a) :** Here the teacher has to perform two operations:

(i) Selecting a boy from among the 27 boys and

(ii) Selecting a girl from among 14 girls.

The first of these can be performed in 27 ways and second can be performed in 14 ways. By the fundamental principle of counting, the required number of ways $= 27 \times 14 = 378$.

8. **(b) :** Given that $\frac{{}^{n}P_4}{{}^{n-1}P_4}=\frac{5}{3}$

$\Rightarrow \quad 3n(n-1)(n-2)(n-3) = 5(n-1)(n-2)(n-3)(n-4)$

$\Rightarrow \quad 3n = 5(n-4)$ [as $(n-1)(n-2)(n-3) \neq 0$, $n > 4$]

$\Rightarrow \quad n = 10$

9. **(a) :** We have ${}^{n}P_5 = 42\ {}^{n}P_3$

$\Rightarrow \quad n(n-1)(n-2)(n-3)(n-4) = 42\, n(n-1)(n-2)$

$\Rightarrow \quad (n-3)(n-4) = 42$ [Since $n > 4$, so $n(n-1)(n-2) \neq 0$]

$\Rightarrow \quad n^2 - 7n - 30 = 0 \Rightarrow n^2 - 10n + 3n - 30 = 0$

$\Rightarrow \quad (n-10)(n+3) = 0 \Rightarrow n - 10 = 0$ or $n + 3 = 0$

$\Rightarrow \quad n = 10$ or $n = -3$

As n cannot be negative, so $n = 10$.

10. **(c) :** There are 9 letters in which there are 4A's, 2L's and rest all are different.

Therefore, the required number of permutations

$=\frac{9!}{4!\ 2!}=\frac{5\times 6\times 7\times 8\times 9}{2}=7560$

11. **(a) :** There will be as many 4 digit numbers as there are permutations of 9 different digits taken 4 at a time.

Therefore, the required 4 digit numbers

$={}^{9}P_4=\frac{9!}{(9-4)!}=\frac{9!}{5!}=9\times 8\times 7\times 6=3024.$

12. **(c) :** We have $5\ {}^{4}P_r = 6\ {}^{5}P_{r-1}$

$\Rightarrow \quad 5\times\frac{4!}{(4-r)!}=6\times\frac{5!}{(5-r+1)!} \Rightarrow (6-r)(5-r)=6$

$\Rightarrow \quad r^2 - 11r + 24 = 0 \Rightarrow r = 8$ or $r = 3$.

But $r \neq 8$, therefore $r = 3$.

13. **(d) :** Total number of discs are $4 + 3 + 2 = 9$. Out of 9 discs, 4 are of the first kind (red), 3 are of the second kind (yellow) and 2 are of the third kind (green).

Therefore, the number of arrangements $=\frac{9!}{4!\ 3!\ 2!}=1260.$

14. **(b) :** Required number of ways $= 7^5 - {}^{7}P_5 = 14287$.

15. **(c) :** Let us fix I and P at the extreme ends (I at the left end and P at the right end). Then, we will left with 10 letters in which there are 2 D's, 4 E's and 3 N's.

The required number of arrangements

$=\frac{10!}{3!\ 2!\ 4!}=12600$

16. **(d) :** Since order of boys and girls are to be maintained in any of the different arrangements.

$\therefore$ Required number $=\frac{6!}{3!3!}=20.$

17. **(c) :** Total number of digits = 9

5 occurs three times, 2 occurs two times.

Number of digits other than 5 = 6

$\therefore$ Required number $=\frac{6!}{2!}\cdot\frac{{}^{7}P_3}{3!}=\frac{6!}{2!}\cdot{}^{7}C_3$

18. **(c) :** There are 9 letters in which there are 3A's, 2D's and rest all are different.

Therefore, the required number of arrangements

$=\frac{9!}{3!\times 2!}=30240$

19. **(c) :** Number of digits = 9

Number of odd digits = 4, number of even digits = 5

Number of even places = 4

Odd digits can be arranged in even places in $\frac{4!}{2!\,2!}$ ways.

Even digits can be arranged in remaining 5 places in $\frac{5!}{2!3!}$ ways.

$\therefore$ Required number $=\frac{4!}{2!2!}\cdot\frac{5!}{2!3!}=60$

20. **(a) :** The required number of arrangements

= The total number of arrangements (without any restriction) – the number of arrangements where all the vowels occur together = 1663200 – 16800 = 1646400

21. **(c) :** Evidently (c) is correct option because we have to divide 17 into four groups each distinguishable into groups of 5, 5, 4 and 3.

22. **(b) :** Total number of books $= a + 2b + 3c + d$

$\therefore$ Total number of arrangements $=\frac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$

23. **(a) :** ${}^{12}C_2=\frac{12!}{(10)!2!}=\frac{12\times 11\times(10!)}{(10!)\times 2}=6\times 11=66$

24. **(d) :** We have, ${}^{10}C_{x-1} > 2\,{}^{10}C_x$

$$\frac{10!}{(x-1)!(11-x)!} > 2.\frac{10!}{(10-x)!x!},\ 1 \le x \le 10$$

$$\Rightarrow \quad \frac{1}{11-x} > 2\cdot\frac{1}{x} \Rightarrow x > 22 - 2x \Rightarrow x > \frac{22}{3}.$$

Hence, $x = 8, 9, 10$.

25. **(b) :** We have, ${}^{2n+3}C_{2n} - {}^{2n+2}C_{2n-1} = 15(2n + 1)$

$$\Rightarrow \quad \frac{\lfloor 2n+3}{\lfloor 2n \lfloor 3} - \frac{\lfloor 2n+2}{\lfloor 2n-1 \lfloor 3} = 15(2n+1)$$

$$\Rightarrow \quad \frac{(2n+2)(2n+1)}{2} = 15(2n+1) \Rightarrow n+1 = 15 \Rightarrow n = 14.$$

26. **(a) :** ${}^nC_p = {}^nC_k \Rightarrow n = p + k$ or $p = k$

$\therefore \quad {}^nC_9 = {}^nC_8 \Rightarrow n = 9 + 8 = 17$

Now ${}^{17}C_{17} = 1$

27. **(c) :** We have, ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$

$$\Rightarrow \quad {}^nC_4 > {}^nC_3 \Rightarrow \frac{n!}{(n-4)!4!} > \frac{n!}{(n-3)!\,3!}$$

$$\Rightarrow \quad \frac{1}{4} > \frac{1}{n-3} \Rightarrow n-3 > 4 \Rightarrow n > 7.$$

28. **(d) :** There are 13 cards in each suit.

$\therefore$ Required number of ways

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

29. **(b) :** Out of 7 men, 3 men can be chosen in ${}^7C_3$ ways and out of 5 women, 2 women can be chosen in ${}^5C_2$ ways. Hence, the committee can be chosen in ${}^7C_3 \times {}^5C_2 = 350$ ways.

30. **(a) :** There are 12 face cards and 4 are to be selected out of these 12 cards. This can be done in ${}^{12}C_4$ ways. Therefore, the required number of ways $= \frac{12!}{4!\,8!} = 495$.

31. **(c) :** The required number is equal to the number of all the subsets of a 68 elements set *i.e.* ${}^{68}C_0 + {}^{68}C_1 + {}^{68}C_2 + ... + {}^{68}C_{68} = 2^{68} = (2^4)^{17} = 16^{17}$.

32. **(c) :** There are 26 red cards and 26 black cards. Therefore, the required number of ways $= {}^{26}C_2 \times {}^{26}C_2$

$$= \left(\frac{26!}{2!\,24!}\right)^2 = (325)^2 = 105625$$

33. **(a) :** ${}^nC_r$ is greatest if $r = \frac{n}{2}$ or $\frac{n+1}{2}$

$\therefore$ The required value of $\frac{{}^{10}C_r}{{}^{11}C_r} = \frac{{}^{10}C_5}{{}^{11}C_6} = \frac{6}{11}$.

34. **(a) :** Here 9 things are to be divided into 5 groups out of which four groups have equal number of elements, *viz*. 2 and one group has only one element.

$\therefore$ Required number of ways $= \frac{9!}{2!2!2!2!4!} = 945.$

35. **(c) :** Required number $= {}^{30}C_2 - 30 = 405$.

36. **(c) :** Required number $= {}^3C_3 + {}^3C_2 \times {}^7C_1 + {}^7C_2 \times {}^3C_1$
$= 1 + 3 \times 7 + 21 \times 3 = 1 + 21 + 63 = 85.$

37. **(b) :** Each selection of 4 points, two on one line and two on the other will give one point of intersection as desired in the question.

$\therefore$ Required number $= {}^mC_2 \cdot {}^nC_2 = \frac{mn(m-1)(n-1)}{4}$

38. **(d) :** Number of selections when $x < y < z$ is ${}^nC_3$.

Number of selections when $x = y < z$ is ${}^nC_2$.

$\therefore$ Required number $= {}^nC_3 + {}^nC_2$

39. **(d) :** Required number of such triangles

$= {}^4C_1 \times {}^5C_1 \times {}^6C_1 = 4 \times 5 \times 6$

40. **(c) :** The number of ways in which four different balls can be placed in four different boxes

$= {}^4C_1 + {}^3C_1 + {}^2C_1 + {}^1C_1 = 4 + 3 + 2 + 1 = 10$

$\therefore$ Required number of ways = 10 – 1 = 9

(Since, only one way in which the same ball have a same box).

41. **(b) :** A History book can be selected in 5 ways and a Maths book can be selected in 3 ways.

Required number of ways = 5 + 3 = 8

[Using addition Principle]

42. **(c) :** Now, 2 History books can be chosen in ${}^5P_2$ ways, 1 Maths book can be chosen in ${}^3P_1$ ways and 1 Science book can be chosen in ${}^4P_1$ ways.

$\therefore$ Required number of ways $= {}^5P_2 \times {}^3P_1 \times {}^4P_1 = 240.$

43. **(d) :** Number of ways of arranging History books = 5!

Number of ways of arranging Maths books = 3!

Number of ways of arranging Science books = 4!

$\therefore$ Required number of way if the books of same subject are put together $= 3! \cdot 5! \cdot 3! \cdot 4!$.

44. **(d) :** The number of arrangement of books ${}^5P_2 \times {}^3P_1 \times {}^4P_1$ represents the arrangement of 2 History books, 1 Maths book and 1 Science book respectively.

45. **(b) :** Number of ways of choosing 3 History books $= {}^5P_3$

Number of ways of choosing 2 Maths books $= {}^3P_2$

and number of ways of choosing 2 Science books $= {}^4P_2$

$\therefore$ Total number of ways $= {}^5P_3 \times {}^3P_2 \times {}^4P_2 = 4320$

46. **(b) :** Since, the choices for remaining 6 digits are 0, 1, 2, 3, 4, 8 and repetition is not allowed.

$\therefore$ Required number of ways = 6 !.

47. **(d) :** If repetition is allowed, then the number of ways = 10 × 10 × 10 × 10 × 10 × 10 = 10^6.

48. **(c) :** If 3 and 4 occur together, they can be arranged in 2! ways and rest 4 digits can be arranged in 5! ways

$\therefore$ Required number of ways = 5! × 2! = 240

49. **(d) :** Total number of ways to find the remaining digits (when repetition is not allowed) = 6! = 720.

The number of ways when 3 and 4 come together = 240

$\therefore$ The number of ways that 3 and 4 never come together = 720 – 240 = 480.

50. **(c) :** If 4 is the unit digit, then required number of ways = 5! = 5 × 4 × 3 × 2 × 1 = 120

51. **(b) :** Total members = 6

$\because$ Room A is double share room.

$\therefore$ The number of ways in which room A can be filled

$$= \binom{6}{2} = \frac{6!}{2! \times 4!} = 15$$

52. **(a) :** Now, rooms A and B can be filled with 2 members each and room C can be filled with 1 person.

$\therefore$ Required number of ways = ${}^2C_1 = 2$

53. **(d) :** Required number of ways = $\binom{6}{2} \cdot \binom{4}{2}\binom{2}{1}\binom{1}{1}$

= 15 × 6 × 2 × 1 = 180

54. **(b) :** As, room A is filled with 2 persons

Now, the remaining persons = 4

Given that room C and D can occupy 1 person each.

$\therefore$ The number of ways in which rooms C and D can be filled = ${}^4C_1 \times {}^3C_1 = 12$

55. **(b) :** We have, $\binom{n}{r-1} + \binom{n}{r}$

$$= \frac{n!}{(r-1)!\,(n-r+1)!} + \frac{n!}{r!\,(n-r)!}$$

$$= n!\left[\frac{1}{(r-1)!\,(n-r+1)!} + \frac{1}{r!\,(n-r)!}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{n-r+1} + \frac{1}{r}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n+1}{r(n-r+1)}\right] = \frac{(n+1)!}{r!(n-r+1)!}$$

$$= \binom{n+1}{r}$$

56. **(c) :** Number of lines is ${}^nC_2 = \dfrac{n(n-1)}{2}$

$$C(n, 3) = \frac{n!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{6}$$

57. **(c) :** Product of n consecutive natural numbers

$= (m + 1)\,(m + 2)\,(m + 3) \ldots (m + n),\ m \in W$

$$= \frac{(m+n)!}{m!} = n! \times \frac{(m+n)!}{m!\,n!} = n! \times {}^{m+n}C_m$$

$\Rightarrow$ Product is divisible by $n!$ and so it is always divisible by $(n - 1)!$ but not by $(n + 1)!$

58. **(d) :** In a chess board, there are 9 horizontal and 9 vertical lines. Number of rectangles of any size are ${}^9C_2 \times {}^9C_2$.

59. **(a) :** $n(n^2 - 1)\,(n + 2) = (n - 1)\,n(n + 1)\,(n + 2)$

is the product of four consecutive whole numbers and hence it is divisible by $\lfloor 4$ = 24.

60. **(a) :** Let the number of ways of distributing n identical objects among r persons such that each person gets atleast one object is same as the number of ways of selecting $(r - 1)$ places out of $(n - 1)$ different places, *i.e.*, ${}^{n-1}C_{r-1}$.

SUBJECTIVE TYPE QUESTIONS

1. Here, the upper place of the flag can be filled in 4 ways by using the 4 flags of different colours. Now, the lower place of the flag can be filled in 3 ways by using the remaining 3 flags of different colours.

$\therefore$ Total number of signals can be generated

= 4 × 3 = 12

2. Here, 3-digit numbers are to be formed using digits 1 to 9 with no digit repeated.

This can be done in 9 × 8 × 7 = 504 ways.

3. Here, ${}^nC_3 = {}^nC_4$

$\Rightarrow$ $n = 3 + 4 = 7$ ($\because {}^nC_a = {}^nC_b \Rightarrow a = b$ or $n = a + b$)

$\therefore$ ${}^7C_7 = 1$

4. Here, $C(2n, 3) : C(n, 3) = 11 : 1$

$\Rightarrow$ ${}^{2n}C_3 : {}^nC_3 = 11 : 1$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2)(n-1)}{n(n-1)(n-2)} = \frac{11}{1}$$

$\Rightarrow$ $4(2n-1) = 11(n-2)$

$\Rightarrow$ $8n - 4 = 11n - 22 \Rightarrow 3n = 18 \Rightarrow n = 6.$

5. Here, ${}^{n+1}C_2 = 45 \Rightarrow \dfrac{(n+1)!}{2!(n-1)!} = 45$

$\Rightarrow$ $\dfrac{n(n+1)\,(n-1)!}{2(n-1)!} = 45$

$\Rightarrow$ $n(n+1) = 90 \Rightarrow n^2 + n - 90 = 0$

$\Rightarrow$ $(n+10)(n-9) = 0 \Rightarrow n = -10$ or 9

$\Rightarrow$ $n = 9$ [$\because$ $n = -10$ (Rejected)]

6. Here, $C(25, n+5) = C(25, 2n-1)$

$\Rightarrow$ ${}^{25}C_{n+5} = {}^{25}C_{2n-1}$

$\Rightarrow$ $25 = n + 5 + 2n - 1$ or $n + 5 = 2n - 1$

($\because$ ${}^nC_a = {}^nC_b \Rightarrow n = a + b$ or $a = b$)

$\Rightarrow$ $25 = 3n + 4$ or $n = 6$

$\Rightarrow$ $3n = 21$ or $n = 6$

Hence, $n = 7$ or $n = 6$

7. Given, ${}^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow$ $10 = x + x + 4 \Rightarrow 6 = 2x \Rightarrow x = 3.$

8. Given, ${}^nC_5 = {}^nC_7 \Rightarrow n = 5 + 7 = 12$

9. We have, ${}^nC_8 = {}^nC_6 \Rightarrow n = 6 + 8 = 14$

$\therefore$ $C(n, 2) = C(14, 2) = {}^{14}C_2 = \dfrac{14!}{2!\,12!} = 91$

10. We have ${}^nC_{10} = {}^nC_3 \Rightarrow n = 10 + 3 = 13$

Hence, ${}^{13}C_3 = \dfrac{13!}{3!10!} = \dfrac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 10!}$

$= 26 \times 11 = 286$

11. Let the consecutive even positive integers be $2n$, $2n + 2$

According to question, $2n > 5 \Rightarrow n > \dfrac{5}{2}$

and $2n + 2n + 2 < 23 \Rightarrow 4n < 21 \Rightarrow n < \dfrac{21}{4}$

$\therefore$ Possible values of n are 3, 4, 5.

Hence, possible pairs of consecutive even positive integers (6, 8), (8, 10), (10, 12).

12. Each of the five question can be answered in 2 ways.

Hence, total number of possible different answers

$= 2 \times 2 \times 2 \times 2 \times 2 = 32$

There is only one sequence of all correct answers. Thus, the total number of sequences are $32 - 1 = 31$. [Since no student has written all correct answers]

Now, as no two students have given the same sequence of answers, hence the maximum number of students in the class = 31.

13. There are 26 distinct English alphabets.

First alphabet can be chosen in 26 ways.

Second alphabet can be chosen in 25 ways.

Third alphabet can be chosen in 24 ways.

$\therefore$ Total number of three letter words

$= 26 \times 25 \times 24 = 15600$

14. Let 2 be fixed at unit's place. The ten's place can be filled up in 6 ways. The hundred's place can also be filled in 6 ways.

$\therefore$ Number of ways $= 6 \times 6 = 36$

When 4 is at unit's place, then no. of numbers that can be formed = 36.

Again when 6 is at the unit's place, then numbers that can be formed = 36

$\therefore$ The total ways in which three digit even numbers can be formed $= 36 \times 3 = 108$.

15. Let the total number of persons in the room be n.

$\therefore$ Required number of hand shakes $= {}^nC_2$

$= \dfrac{n!}{2!(n-2)!} = \dfrac{n(n-1)}{2}$

Given that, $\dfrac{n(n-1)}{2} = 66 \Rightarrow n(n-1) = 132$

$\Rightarrow$ $n^2 - n - 132 = 0 \Rightarrow (n-12)(n+11) = 0$

$\Rightarrow$ $n = 12$ ($\therefore$ n can't be negative)

16. Let number of sides or vertices be n

$\therefore$ Number of diagonals $= {}^nC_2 - n$

$\Rightarrow$ $65 = {}^nC_2 - n \Rightarrow 65 = \dfrac{n(n-1)}{2} - n$

$\Rightarrow$ $n^2 - 3n = 130$

$\Rightarrow$ $(n-13)(n+10) = 0$

$\Rightarrow$ $n = 13$ or $n = -10$ (Rejected)

17. Here, number of vertices or sides $= n$

$\therefore$ Number of diagonals $= {}^nC_2 - n$

$\Rightarrow$ $44 = {}^nC_2 - n \Rightarrow 44 = \dfrac{n(n-1)}{2} - n$

$\Rightarrow$ $n^2 - 3n - 88 = 0 \Rightarrow (n+8)(n-11) = 0$

$\Rightarrow$ $n = 11$ or $n = -8$ (Rejected)

18. Number of letters in the word = 8

Number of vowels in the word = 4 (I, O, U, E)

Number of consonants in the word = 4 (N, V, L, T)

Out of 4 vowels, we have to select 3.

Out of 4 consonants, we have to select 2.

Also, we have to arrange 3 vowels and 2 consonants.

$\therefore$ Required number of words $= C(4, 3) \cdot C(4, 2) \cdot 5!$

$= 4 \times \dfrac{4 \times 3}{2} \times 120 = 2880$

19. There are 2 men and 3 women in the group.

A committee of 3 persons can be selected in

${}^5C_3 = 10$ ways

A committee consisting of 1 man and 2 women

$= {}^2C_1 \times {}^3C_2 = 2 \times 3 = 6$ ways.

20. We have 5 vowels and 21 consonants our english alphabets.

$\therefore$ Number of words formed with 2 different vowels and 2 different consonants $= {}^5C_2 \times {}^{21}C_2$

$= 10 \times 210 = 2100$

Now, 4 selected letters can arrange themselves in 4! ways

$\therefore$ Total words formed $= 2100 \times 4! = 50400$

21. Total number of letters in the word 'EXAMINATION' are 11 with 2 A's, 2I's and 2N's.

Now when word starts with A *i.e.,* A is fixed at the beginning of the word. Then we have to arrange the remaining 10 letters with 2I's, 2N's.

Then words starting with $A = \dfrac{10!}{2!\,2!} = 907200$

$\therefore$ Required number of words = 907200

22. (i) Since no two girls sit together

$\therefore$ Seating arrangement should be

G	B	G	B	G	B	G	B	G	B	G

$\Rightarrow$ There are six positions of seating for girls.

$\therefore$ Required number of ways $= {}^5P_5 \times {}^6P_5 = 5! \times 6!$

(ii) Required ways

= Total arrangements – Number of ways when all the girls sit together = 10! – 5! 6!

23. Total number of ways of arranging the given word 'WOMAN' as in a dictionary

Word	A	M	N	O	WA	WM	WN	WOA	WOMAN
No. of ways	4!	4!	4!	4!	3!	3!	3!	2!	1!

$\therefore$ Required rank $= 4 \times 4! + 3 \times 3! + 2! + 1 = 117^{\text{th}}$

24. (i) Number of digits = 6

$\therefore$ Required number of 6-digit numbers

$= P(6,6) - P(5, 5) = 6! - 5! = 720 - 120 = 600.$

(ii) The numbers are divisible by 10 if 0 is in the unit place.

$\therefore$ The required numbers which are divisible by $10 = P(5, 5) = 5! = 120$

(iii) The numbers having 0 in the ten's place.

$\times\ \times\ \times\ \times\ 0\ \times$

$\therefore$ The required numbers $= P(5, 5) = 5! = 120$

25. MONDAY has 6 different letters, in which there are two vowels namely, A and O.

(i) If 4 letters are taken at a time, then,

Number of words $= {}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$

(ii) If all the letters of word MONDAY are taken at a time, then number of words = 6! = 720.

26. Total number of English alphabets = 26

Number of letter to be chosen = 2

$\therefore$ Number of ways of selecting two letters $= {}^{26}P_2$

$= 26 \times 25 = 650$

So, number of digits = 10

$\therefore$ Number of ways of selecting 3 digits $= {}^{10}P_3$

$= 10 \times 9 \times 8 = 720$

Hence, required license plates can be made

$= 650 \times 720 = 468000$

27. Digits are 1, 2, 0, 3, 4, 5, 3

Number greater than 10,00,000 by using the given digits are $= \dfrac{7!}{2!}$ ...(i)

But numbers starting with '0' are not greater than 10,00,000.

Total numbers starting with '0' are $= \dfrac{6!}{2!}$ ...(ii)

From (i) and (ii), numbers greater than 10,00,000 are

$= \dfrac{7!}{2!} - \dfrac{6!}{2!} = 2160$

28. Since, we have to form the numbers greater than 1000000 by using 1, 2, 0, 2, 4, 2, 4

So, numbers which begin with digit '1' $= \dfrac{6!}{3!2!} = 60$

Numbers that begin with digit '2' $= \dfrac{6!}{2!2!} = 180$

Numbers that begin with digit '4' $= \dfrac{6!}{3!} = 120$

$\therefore$ Required numbers = 60 + 180 + 120 = 360

29. Here, ${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$$\Rightarrow \frac{1}{n-r} = 1 \quad \Rightarrow n - r = 1 \qquad \text{...(i)}$$

Also, ${}^nC_r = {}^nC_{r-1}$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r+1)!(r-1)!}$$

$$\Rightarrow \frac{1}{(n-r)!\,r(r-1)!} = \frac{1}{(n-r+1)(n-r)!(r-1)!}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$$

$$\Rightarrow n - 2r = -1 \qquad \text{...(ii)}$$

From (i) and (ii), we get, $n = 3$ and $r = 2$

30. Since, candidate cannot attempt more than 5 questions from either section.

The number of questions attempted from each section is given in following table :

Section A	5	4	3	2
Section B	2	3	4	5

Hence, total number of possible ways

$= {}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3 + {}^6C_3 \times {}^6C_4 + {}^6C_2 \times {}^6C_5$

$= 2[{}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3]$

$= 2\,[90 + 300] = 2 \times 390 = 780$

31. (i) Number of straight lines formed joining the 15 points, taking 2 at a time $= {}^{15}C_2 = 105$

Number of straight lines formed by joining the 6 points, taking 2 at a time $= {}^6C_2 = 15$

But, 6 collinear points when joined pairwise give only line

$\therefore$ Required number of straight lines

$= 105 - 15 + 1 = 91$

(ii) Number of triangles formed by joining the points, taking 3 at a time $= {}^{15}C_3 = 455$

Number of triangles formed by joining the 6 points, taken 3 at a time $= {}^6C_3 = 20$

But, 6 collinear points cannot form a triangle when taken 3 at a time.

So, required number of triangles = 455 – 20 = 435.

32. Since, guests are to be seated half on each side *i.e.*, 9 to be seated on side *A* and rest on side *B*.

Now, out of 18 guests, 4 particular guests sit on one particular side and 3 on other side.

Now, we have 18 – 4 – 3 = 11 guests can be select 5 more for side *A* and 6 on side *B*.

Hence, selection can be done in ${}^{11}C_5$

and, a guests on each side of table can be seated in 9! × 9!

Thus, total arrangements $= {}^{11}C_5 \times 9! \times 9!$

33. Number of boys = 9, number of girls = 4

Total members in a committee = 7

(i) Number of ways forming a committee having at least 3 girls $= {}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3 = 504 + 84 = 588$

(ii) Number of ways forming a committee having at most 3 girls $= {}^4C_3 \times {}^9C_4 + {}^4C_2 \times {}^9C_5 + {}^4C_1 \times {}^9C_6 + {}^4C_0 \times {}^9C_7$

$= 504 + 756 + 336 + 36 = 1632$

34. (i) There are five digits, 1, 2, 3, 4 and 5. Each digit can be selected any number of times. Hence, we can select first digit in 5 ways. The second digit and third digit can also be selected in 5 ways.

$\therefore$ The number of ways in which the selection of three digits can be made = 5 × 5 × 5 = 125.

(ii) Under the restriction, first digit can be selected in 5 ways. After the selection of first digit four digits are left. Second digit can be selected in 4 ways and third digit can be selected in 3 ways.

$\therefore$ Required total no. of ways = 5 × 4 × 3 = 60

35. Case (i) When mathematics part I is borrowed, then part II is also borrowed. Hence, the number of ways of selecting 1 book from 6 books is ${}^6C_1 = 6$

Case (ii) When mathematics part I is not borrowed, then part II is not borrowed. So, he has to select three books out of remaining 6 books.

$\therefore$ Total number of ways of selection $= {}^6C_3 = 20$

Hence, total number of ways of selection = 6 + 20 = 26

36. ${}^nC_r : {}^nC_{r+1} = 1 : 2$ and ${}^nC_{r+1} : {}^nC_{r+2} = 2 : 3$

Now, $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{1}{2} \Rightarrow \dfrac{\dfrac{n!}{r!(n-r)!}}{\dfrac{n!}{(r+1)!(n-r-1)!}} = \dfrac{1}{2}$

$\Rightarrow \dfrac{(r+1)!(n-r-1)!}{r!(n-r)!} = \dfrac{1}{2} \Rightarrow \dfrac{(r+1)}{(n-r)} = \dfrac{1}{2}$

$\Rightarrow 2r + 2 = n - r \;\Rightarrow\; n = 3r + 2$ ...(i)

Similarly, $\dfrac{{}^nC_{r+1}}{{}^nC_{r+2}} = \dfrac{2}{3} \Rightarrow \dfrac{\dfrac{n!}{(r+1)!(n-r-1)!}}{\dfrac{n!}{(r+2)!(n-r-2)!}} = \dfrac{2}{3}$

$\Rightarrow \dfrac{(r+2)!(n-r-2)!}{(r+1)!(n-r-1)!} = \dfrac{2}{3} \Rightarrow \dfrac{r+2}{n-r-1} = \dfrac{2}{3}$

$\Rightarrow 3r + 6 = 2n - 2r - 2 \;\Rightarrow\; 2n = 5r + 8$ ...(ii)

Solving (i) and (ii), we get $r = 4$ and $n = 14$

37. The given word 'DAUGHTER' contains 8 different letters with vowels A, U and E.

$\therefore$ Total number of words formed by using all the 8 letters of the given word = 8! = 40320

(i) The total number of words in which vowels do not occur together = (Total number of words) –(number of words in which vowels are always together)

$= 40320 - 6! \times {}^3P_3$

= 40320 – 4320 = 36000

(ii) For vowels to occupy odd places, first arrange 3 vowels in 4 odd places. This can be done in ${}^4P_3$ ways and remaining 5 alphabets can be filled in 5! ways.

i.e.,

•		•		•		•	

$\therefore$ Required number of ways $= 5! \times {}^4P_3 = 2880$

38. (i) There are 7 letters in the word 'HARYANA' of which 3 are A's and remaining all are each of its own kind.

So, total number of words $= \dfrac{7!}{3!} = 840$.

(ii) After fixing H in first place and N in last place, we have 5 letters out of which three A's are alike.

So, total number of words $= \dfrac{5!}{3!} = 20$.

(iii) Considering H and N together we have $7-2+1 = 6$ letters out of which three A's are alike. These six letters can be arranged in $\dfrac{6!}{3!}$ ways. But H and N can be arranged among themselves in 2! ways.

$\therefore$ Required number of words $= \dfrac{6!}{3!} \times 2! = 240$.

39. There are 12 boys and 10 girls in the class. We have to select 10 students for a competition including atleast 4 boys and 4 girls. Two girls who were last year's winner are to be included. Since two girls are already selected, now we are left with 8 girls out of which atleast 2 girls are to be selected.

We can make selection in the following ways:

Choice	Boys	Girls (+2 particular girls)
I	4	4 + 2
II	5	3 + 2
III	6	2 + 2

First choice can be made ${}^{12}C_4 \times {}^{8}C_4$

$$= \frac{12\times11\times10\times9}{4\times3\times2\times1}\times\frac{8\times7\times6\times5}{4\times3\times2\times1} = 34650 \text{ ways}$$

Second choice can be made in ${}^{12}C_5 \times {}^{8}C_3$

$$= \frac{12\times11\times10\times9\times8}{5\times4\times3\times2\times1}\times\frac{8\times7\times6}{3\times2\times1} = 44352 \text{ ways}$$

Third choice can be made in ${}^{12}C_6 \times {}^{8}C_2$

$$= \frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}\times\frac{8\times7}{2\times1} = 25872 \text{ ways}$$

Hence, total number of possible selections

$= 34650 + 44352 + 25872 = 104874$

40. (i) A team consisting no girl can be selected ${}^{7}C_5$ ways *i.e.*, $\dfrac{7\times6}{2\times1} = 21$ ways

(ii) Number of ways of selecting at least one boy and one girl

$= {}^{7}C_1 \times {}^{4}C_4 + {}^{7}C_2 \times {}^{4}C_3 + {}^{7}C_3 \times {}^{4}C_2 + {}^{7}C_4 \times {}^{4}C_1$

$= 7 + 84 + 210 + 140 = 441$

(iii) A team having alteast 3 girls will consist of: 3 girls, 2 boys; 4 girls, 1 boy.

$\therefore$ Required no. of ways $= {}^{4}C_3 \times {}^{7}C_2 + {}^{4}C_4 \times {}^{7}C_1$

$= 4 \times 21 + 1 \times 7 = 91$ ways.