11. Solids

Questions Pg-199

1. Question

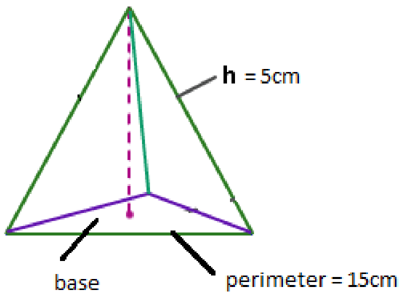
The base of a prism is an equilateral triangle of perimeter 15 centimetres square and its height is 5 centimetres. Calculate its volume.

Answer

Given: base of prism is equilateral triangle and perimeter of base “p” = $15cm^2$

Height of prism “h” = 5cm

To find : Volume “V” of the prism = ?



Procedure :

As we know that, base of prism is equilateral triangle and its perimeter is 3× side of triangle

⇒ p = 3× side

⇒ 15 = 3× side

∴ side of the triangular base of the prism = $\frac{15}{3}$ = 5cm

Now, area of the equilateral triangular base of the prism,

“a” = $\frac{\sqrt{3}}{4}\times side^2$

= $\frac{\sqrt{3}}{4}\times 5^2$

= $\frac{\sqrt{3}}{4}\times 25$

⇒ a = $10.83cm^2$

As we know that, Volume of the prism = height × area of base

∴ V = h × a

= 5 × 10.83

= $54.15cm^3$

∴ Volume of the given prism is $54.15cm^3$.

2. Question

A hexagonal hole of each side 2 metres is dug in the school ground to collect rain water. It is 3 metres deep. It now has water one metre deep. How much litres of water is in it?

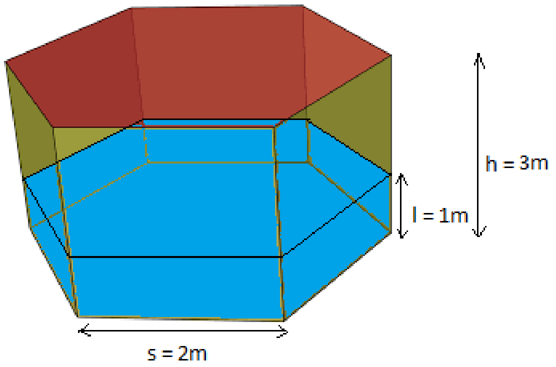
Answer

Given: side of hexagonal hole “s” = 2m

Height of the hole “h” = 3m

Level of water in the hole “l” = 1m

To find : water in hole in litres “W” = ?



Procedure :

Here, water filled in the hole will be, Volume of hole occupied by water in litres.

So, volume of water = height of water in hole × area of the base of the hexagonal base

⇒ volume of water = $1 \times \frac{3\sqrt{3}}{2} \times \text{side}^2$

$= \frac{3\sqrt{3}}{2} \times 2^2$

$= \frac{3\sqrt{3}}{2} \times 4$

$= 3\sqrt{3} \times 2$

$= 10.3923m^3$

Now, volume of water in the hole = $10.3923m^3$

As we know, one cubic meter = 1000 litres.

So, volume of water in the hole in litres = $10.3923m^3 \times 10^3L$

∴ volume of water in the hole in litres = 10392.3L

∴ there is 10392.3 Litres of water in the hexagonal hole.

3. Question

A hollow prism of base a square of side 16 centimetres contains water 10 centimetres high. If a solid cube of side 8 centimetres is immersed in it, by how much would the water level rise?

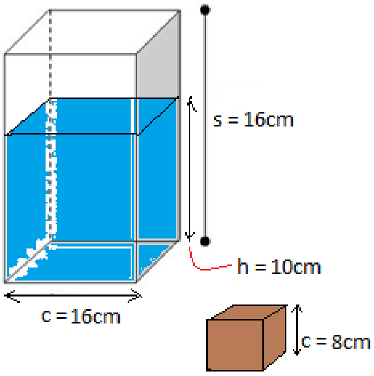
Answer

Given: side of base of the prism “s” = 16cm

Water is filled upto height “h” = 10cm

Side of cube “c” = 8cm

To find : rise in water level after inserting the cube in the prism = ?



Procedure :

First we will find the Volume of water filled in the prism before immersing the cube.

So, Volume of water V_{old} = area of base × height upto which water is filled

⇒ $V_{old} = (16\times16)\times10$

$= 2560cm^3$

Now, volume of the cube to be immersed $V_{cube} = 8\times8\times8$

⇒ $V_{cube} = 512\ cm^3$

Now, after immersing the cube, the total volume in which the water is present will be = old volume of water + volume of cube

So, $V_{new} = V_{old} + V_{cube}$

⇒ $V_{new} = 2560cm^3 + 512\ cm^3$

⇒ $V_{new} = 3072\ cm^3$

Now, this new volume will have an increase in height.

So to find the new height of water level,

⇒ V_{new} = area of base × new height upto which water is filled

⇒ $3072 = 16\times16\times h$

⇒ $h = \frac{3072}{16\times16}$

$= \frac{3072}{256}$

= 12cm

Now we have,

⇒ Old height upto which the water was present in the prism = 10cm

⇒ New height upto which the water was present in the prism = 12cm

∴ increase in height of water = 12-10

= 2cm

∴ The water rose by 2cm after immersing the cube in the prism.

Questions Pg-202

1. Question

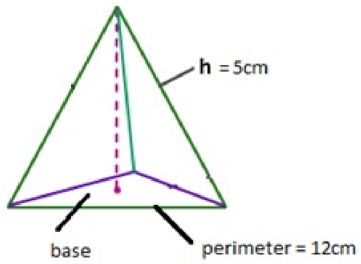
The base of a prism is an equilateral triangle of perimeter 12 centimetres and its height is 5 centimetres. What is its total surface area?

Answer

Given: base of a prism is an equilateral triangle and its perimeter “p” = 12cm

Height of the prism “h” = 5cm

To find : total surface area of the prism = ?



Procedure :

Now, we know that,

total surface area of the prism = lateral surface area of the prism + 2× (area of base of the prism)

⇒ first we will find the side of the triangular base of the prism.

So, perimeter of the equilateral triangle = 3×side

⇒ 12 = 3×side

⇒ Side = 4cm

now, to find the lateral surface area, we have

⇒ Lateral surface area =perimeter of base of prism × height of prism

= 12 × 5

= 60cm^2

Now, we can find the total surface area of the prism.

So, total surface area of the prism = lateral surface area of the prism + 2× (area of base of the prism)

⇒ total surface area of the prism = 60 + 2×(area of equilateral triangle)

⇒ total surface area of the prism = 60 + 2×($\frac{\sqrt{3}}{4}$× side^2)

= 60 + 2× ($\frac{\sqrt{3}}{4}$×4^2)

= 60 + 8√3

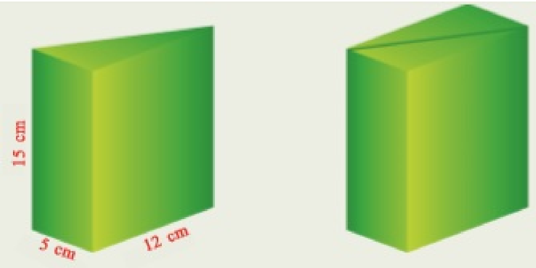
= 60 + 13.856

= 73.856cm^2

∴ Total surface area of the given prism is 73.856cm^2

2. Question

Two identical prisms with right triangles as base are joined to form a rectangular prism as shown below:



What is its total surface area?

Answer

Given: dimensions of triangular prism:

Height = 15cm

Smaller side of right triangular base = 5cm

bigger side of right triangular base = 12cm

To find: total surface area of the newly formed rectangular prism

Procedure:

After joining the 2 identical triangular prisms, we get a new rectangular prism.

⇒ The dimensions of the rectangular prism is:

Length of base = 12cm

Breadth of the base = 5cm

Height of the prism = 15cm

Now total surface area of the rectangular prism = lateral surface area + area of both the bases

We will have to find the lateral surface area of the rectangular prism.

⇒ Lateral surface area = base perimeter × height of prism

= (12+5) × 15

= 255cm^2

∴ total surface area of the rectangular prism = 255 + 2× (12× 5)

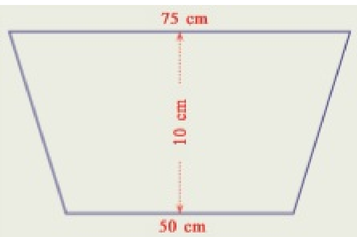
= 255 + 120

= 375cm^2

∴ Total surface area of the newly formed rectangular prism is 375cm^2

3. Question

A water trough in the shape of a prism has trapezoidal faces. The dimensions of a base are shown in this picture:



The length of the trough is 80 centimetres. It is to be painted inside and outside. How much would be the cost at 100 rupees per square metre?

Answer

Given: Length of the prism = 80cm

Height of trapezium base = 10cm

Shorter side of trapezium base = 50cm

Larger side of trapezium base = 75cm

Cost of painting = 100 rupees per square meter

To find: cost to paint the water trough from inside and outside

Procedure:

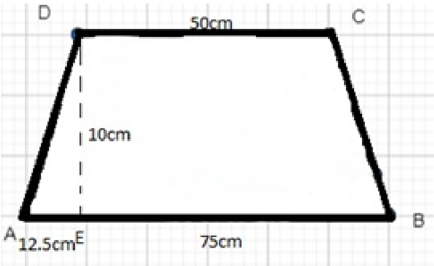
As it is the water trough, we will have only 5 sides to be painted from both sides.

From, the figure we can see that, the trough lies on the shorter side of the trapezium, so the side of prism which has the longer edge of the trapezium in it is absent, we will not consider it.

So, the area to be painted will be 2 × (area of both bases + area of 3 faces of the prism)

Now we have to find the length of non-parallel side of the prism.

ie. here length (AD)



⇒ So, by Pythagoras theorem, we have $AD^2 = AE^2 + ED^2$

⇒ $AE = \frac{1}{2}(75-50) = 12.5cm$

⇒ $AD^2 = (12.5)^2 + 10^2$

⇒ $AD^2 = 156.25 + 100$

$= 256.25$

SO, $AD = \sqrt{256.25}$

AD = 16cm

Here, we multiplied the area by 2 because we have to paint the prism from inside as well as outside.

So, area of prism to be painted = 2× (2× (area of trapezium) + (100×50)+(100×16) +(100×16))

⇒ area of prism to be painted = $2\times (2\times(10\times\frac{(75+50)}{2}) + (100\times50)+(100\times16) +(100\times16))$

⇒ area of prism to be painted = 2×(1250 + 5000 + 1600 + 1600)

⇒ area of prism to be painted = $18900cm^2 = 1.89m^2$

∴ cost of painting = area × cost per meter square

$= 1.89m^2\times 100$

= 189 rupees.

∴ cost of painting the trough from inside and outside is 189 rupees.

Questions Pg-204

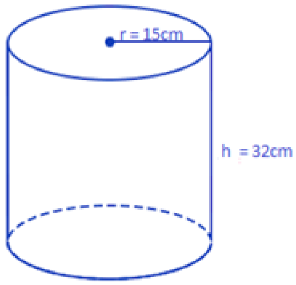
1. Question

The base radius of an iron cylinder is 15 centimetres and its height is 32 centimetres. It is melted and recast into a cylinder of base radius 20 centimetres. What is the height of this cylinder?

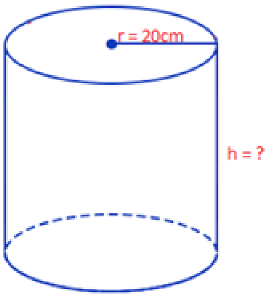
Answer

Given: radius of old cylinder “r_1” = 15cm

Height of old cylinder “h_1” = 32cm



Radius of new cylinder “r_2” = 20cm



$\prod = 3.14$

To find : height of the new cylinder “h_2” = ?

Procedure :

As we know that the new cylinder id formed by melting the old cylinder.

∴ volume of old cylinder = volume of new cylinder

Now, volume of old cylinder = Base area of old cylinder × height of old cylinder

⇒ volume of old cylinder = $\prod \times (r_1)^2 \times h_1$

$= \prod \times 15^2 \times 32$

Now, volume of new cylinder = $\prod \times (r_2)^2 \times h_2$

$= \prod \times (20)^2 \times h_2$

As we know, the volumes of the new and old cylinders is same, so we can now equate them.

⇒ volume of old cylinder = volume of new cylinder

⇒ $\prod \times 15^2 \times 32 = \prod \times (20)^2 \times h_2$

⇒ $15^2 \times 32 = 20^2 \times h_2$

⇒ $h_2 = \frac{15^2 \times 32}{20^2}$

$= \frac{225 \times 32}{400}$

$= \frac{7200}{400}$

= 18cm.

∴ the height of the newly formed cylinder is 18cm.

2. Question

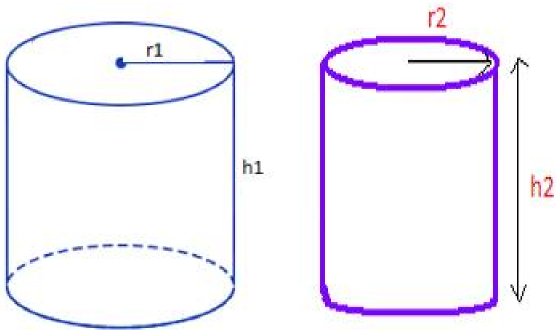
The base radii of two cylinders of the same height are in the ratio 3 : 4. What is the ratio of their volumes?

Answer

Given: ratio of radius of 2 cylinders is 3 : 4 ie. $r_1:r_2$

Height of the 2 cylinder is equal ie. $h_1=h_2$

To find : ratio of the volumes of the 2 cylinders ie. $V_1:V_2$



Procedure :

Volume of the cylinder = area of base × height

So, Volume of the cylinder = $\prod \times r^2 \times h$

⇒ Volume of the first cylinder $V_1 = \prod \times (r_1)^2 \times h_1$

⇒ Volume of the second cylinder $V_2 = \prod \times (r_2)^2 \times h_2$

So ratio of the volumes of these 2 cylinders $= \frac{V_1}{V_2}$

$\Rightarrow \frac{V_1}{V_2} = \frac{\prod \times (r_1)^2 \times h_1}{\prod \times (r_2)^2 \times h_2}$

$= \frac{(r_1)^2 \times h_1}{(r_2)^2 \times h_2}$

$= \frac{(r_1)^2}{(r_2)^2} \times \frac{h_1}{h_2}$

$= (\frac{r_1}{r_2})^2 \times \frac{h_1}{h_2}$

$= (\frac{3}{4})^2 \times 1$

$= \frac{9}{16}$

$\therefore \frac{V_1}{V_2} = \frac{9}{16}$

∴ the ratio of the volumes of both the cylinders is 9:16 ie. $V_1:V_2$

3 A. Question

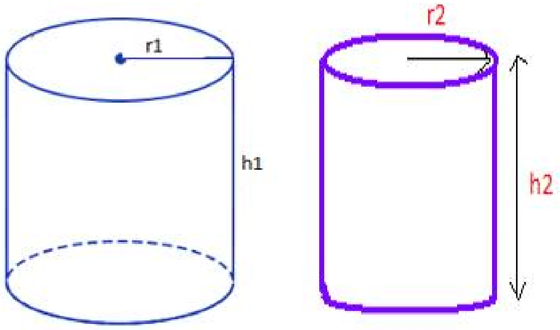
The base radii of two cylinders are in the ratio 2 : 3 and their height are in the ratio 5 : 4.

What is the ratio of their volumes?

Answer

Given: ratio of base radii of 2 cylinders is 2:3 ie. r_1:r_2

ratio of height of 2 cylinders is 5:4 ie. h_1:h_2



to find : i) ratio of the volumes of 2 cylinders

ii) volume of the second cylinder if first's volume is 720cm^3

procedure :

i)Volume of cylinder = area of base × height

So, Volume of the cylinder = $\prod \times r^2 \times h$

⇒Volume of the first cylinder V1= $\prod \times (r_1)^2 \times h_1$

⇒Volume of the second cylinder V2= $\prod \times (r_2)^2 \times h_2$

So ratio of the volumes of these 2 cylinders $= \frac{V_1}{V_2}$

$\Rightarrow \frac{V_1}{V_2} = \frac{\prod \times (r_1)^2 \times h_1}{\prod \times (r_2)^2 \times h_2}$

$= \frac{(r_1)^2 \times h_1}{(r_2)^2 \times h_2}$

$= \frac{(r_1)^2}{(r_2)^2} \times \frac{h_1}{h_2}$

$= (\frac{r_1}{r_2})2 \times \frac{h_1}{h_2}$

$= (\frac{2}{3})2 \times \frac{5}{4}$

$= \frac{4}{9} \times \frac{5}{4}$

$= \frac{5}{9}$

$\Rightarrow \frac{V_1}{V_2} = \frac{5}{9}$

∴ ratio of volumes of 2 cylinders = 5:9 ie. V_1:V_2

3 B. Question

The base radii of two cylinders are in the ratio 2 : 3 and their height are in the ratio 5 : 4.

The volume of the first cylinder is 720 cubic centimetres. What is the volume of the second?

Answer

volume of first cylinder = 720cm^3

now, as we know that, ratio of volumes of 2 cylinders is $\frac{V_1}{V_2}$

and, $\frac{V_1}{V_2} = \frac{\prod\times(r_1)^2\times h_1}{\prod\times(r_2)^2\times h_2}$

so, we can also say that

$\frac{V_2}{V_1} = \frac{\prod\times(r_2)^2\times h_2}{\prod\times(r_1)^2\times h_1}$

$\Rightarrow V_0 = \frac{\prod\times(r_2)^2\times h_2}{\prod\times(r_1)^2\times h_1} \times V_1$

$= \frac{(r_2)^2\times h_2}{(r_1)^2\times h_1} \times V_1$

$= \frac{(r_2)^2}{(r_1)^2}\times \frac{h_2}{h_1} \times V_1$

$= (\frac{r_2}{r_1})^2 \times\frac{h_2}{h_1} \times V_1$

$= (\frac{3}{2})^2\times \frac{4}{5}\times 720$

$= \frac{9}{4}\times \frac{4}{5}\times 720$

$= 9\times 144$

$= 1296cm^3$

$\therefore$ volume of the second cylinder is $1296cm^3$

Questions Pg-206

1. Question

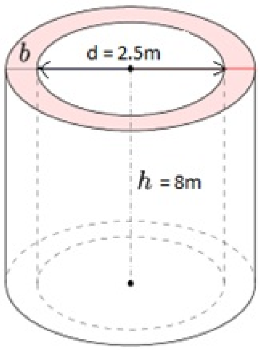
The inner diameter of a well is 2.5 metres and it is 8 metres deep. What would be the cost of cementing its inside at 350 rupees per square metre?

Answer

Given: inner diameter of a well "d" = 2.5m

Depth(height) of the well "h" = 8m

Cost of cementing = 350 rupees per square meter



To find : total cost of cementing the inner well

Procedure :

First, we will have to find the inner lateral surface area of the well.

And for that area we will have to find the cost of cementing.

So, lateral surface area of the well = perimeter of base of cylinder × height of cylinder

Now inner diameter = 2.5m

So, inner radius of the well = $\frac{2.5}{2}$ = 1.25m

Now, lateral surface area of the well = 2∏× radius × height of the well

⇒lateral surface area of the well = 2∏× 1.25 × 8

= 2× 3.14× 1.25× 8

lateral surface area of the well = 62.8m^2

now, the cost of cementing for this inner lateral surface area will be, lateral surface area of the well× cost per square meter

so, the total cost of cementing = 62.8× 350 = 21,980 rupees

∴ the total cost for cementing the inner walls of the well is 21,980 rupees.

2. Question

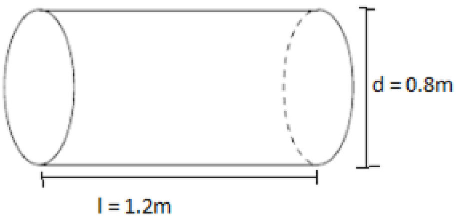
The diameter of a road roller is 80 centimetres and it is 1.20 metres long.

What is the area of leveled surface, when it rolls once?

Answer

Given: diameter of the cylindrical roller = 80cm

Length of the roller (height of the cylinder) = 1.2m = 120cm



To find: area of leveled surface, when it rolls once = ?

Procedure:

When the cylindrical roller will roll once on the road, the area of the leveled surface will be equal to the lateral surface area of the roller because on rolling once it will cover the distance equal to its lateral surface area.

So, now, lateral surface area of the roller = perimeter of base of cylinder × height of cylinder

⇒lateral surface area of the roller = (2∏r)× h

= (∏d)× h

= 3.14× 80× 120

lateral surface area of the roller = 30,144cm^2

∴ when the cylindrical roller rolls once, the area of the leveled surface is 30,144cm^2

3. Question

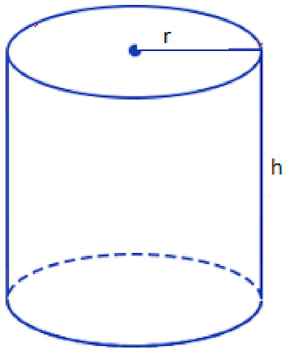
The base area and the curved surface area of a cylinder are equal. What is the ratio of the base radius and height?

Answer

Given: base area and the curved surface area of a cylinder are equal

r = radius of the cylinder

h = height of the cylinder



to find :

ratio of the base radius and height=?

Procedure :

We know that,

Curved surface area =perimeter of base of cylinder × height of cylinder

⇒Curved surface area = (2∏r)× h

Now, as,

base area and the curved surface area of a cylinder are equal

,we have

⇒$\prod r^2=(2\prod r)\times h$

⇒r = 2h

⇒$\frac{r}{h} = \frac{2}{1}$

∴ ratio of the base radius and height is 2:1 i.e. r:h