Chapter 2

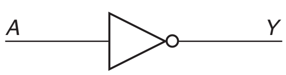
Boolean Algebra and Minimization of Functions

LEARNING OBJECTIVES

- *Logic gates*
- *Boolean algebra*
- *AXIOMS and Laws of Boolean algebra*
- *Properties of Boolean algebra*
- *Conversion from Min term to Max term*
- *Minimization of Boolean function*
- *K-map method*
- *Prime implicant*
- *Implementation of function by using NAND-NOR Gates*
- *EX-OR, EX-NOR GATE*

Logic Gates

1. **Inverter or NOT gate (7404 IC):** The inverter performs a basic logic operation called inversion or complementation. The purpose of the inverter is to change one logic level to the opposite level. In terms of digital circuits, it converts 1 to 0 and 0 to 1.



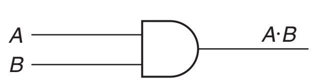
Symbol

$Y = \overline{A}$

Table 1 *Truth Table*

Input	Output
A	**Y**
0	1
1	0

2. **AND gate (logical multiplier 7408 IC):** The AND gate performs logical multiplication more commonly know as AND function. The AND gate is composed of 2 or more inputs and a single output



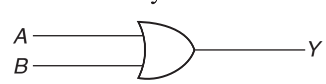
$Y = A{\cdot}B$

Figure 1 2 input AND gate

Table 2 *Truth Table*

Input		Output
A	**B**	**Y**
0	0	0
0	1	0
1	0	0
1	1	1

3. **OR gate (logical adder 7432 IC):** The OR gate performs logical, addition commonly known as OR function.



Symbol

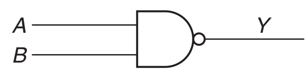
$Y = A + B$

Figure 2 2 input OR gate

Table 3 *Truth Table*

Input		Output
A	**B**	**Y**
0	0	0
0	1	1
1	0	1
1	1	1

4. NAND gate (7400 IC): The NAND gate's function is basically AND + NOT function.



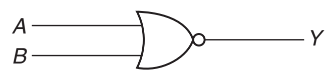
$Y = \overline{A \cdot B}$

Figure 3 2 input NAND gate

Table 4 *Truth Table*

Input			Output
A	**B**	**A·B**	$\overline{A+B}$ **(Y)**
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

5. NOR gate (7402 IC): The NOR gate is basically OR + NOT function.



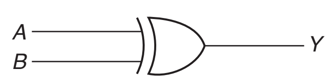
$Y = \overline{A + B}$

Figure 4 2 input NOR gate

Table 5 *Truth Table*

Input			Output
A	**B**	**A + B**	$\overline{A+B}$ **(Y)**
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

6. Exclusive OR gate X-OR (7486 IC): X-OR is a gate in which unequal inputs create a high logic level output and if both inputs are equal, the output will be low. Other name for EX-OR gate is unequivalent gate.
2 input X-OR Gate



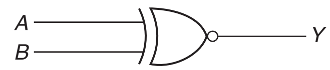
Symbol
$Y = A \oplus B = A\overline{B}\,\overline{A}B$

Figure 5 2 input X-OR Gate

Table 6 *Truth Table*

A	**B**	**Y**
0	0	0
0	1	1
1	0	1
1	1	0

7. Exclusive NOR gate (X-NOR): X-NOR is a gate in which equal inputs create a high logic level output; and if both inputs are unequal, then the output will be low. Other name for X-NOR gate is equivalent gate.



Symbol
$Y = A \odot B = AB + \overline{A}\,\overline{B}$

Figure 6 2 input X-NOR Gate

Table 7 *Truth Table*

A	**B**	**Y**
0	0	1
0	1	0
1	0	0
1	1	1

X-NOR Gate is complement of X-OR Gate.

Boolean Algebra

Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators OR and AND and one unary operator NOT. The Boolean algebra is governed by certain well-developed rules and laws.

AXIOMS and Laws of Boolean Algebra

1. AXIOMS

(a) AND operation

(1) $0 \cdot 0 = 0$
(2) $0 \cdot 1 = 0$
(3) $1 \cdot 0 = 0$
(4) $1 \cdot 1 = 1$

(b) OR operation

(5) $0 + 0 = 0$
(6) $0 + 1 = 1$
(7) $1 + 0 = 1$
(8) $1 + 1 = 1$

(c) NOT operation

(9) $\overline{1} = 0$
(10) $\overline{0} = 1$

2. Laws

(a) Complementation law

(1) $\overline{0} = 1$
(2) $\overline{1} = 0$
(3) If $A = 0$, then $\overline{A} = 1$
(4) If $A = 1$, then $\overline{A} = 0$
(5) $\overline{\overline{A}} = A$

(b) AND laws

(1) $A \cdot 0 = 0$ (NULL Law)
(4) $A \cdot 1 = A$ (Identity Law)
(3) $A \cdot A = A$
(4) $A \cdot \overline{A} = 0$

(c) OR laws

(1) $A + 0 = A$ (NULL Law)

(2) $A + 1 = 1$ (Identity Law)

(3) $A + A = A$

(4) $A + \overline{A} = 1$

(d) Commutative laws

(1) $A + B = B + A$

(2) $A \cdot B = B \cdot A$

(e) Associative laws

(1) $(A + B) + C = A + (B + C)$

(2) $(A \cdot B)C = A(B \cdot C)$

(f) Distributive laws

(1) $A(B + C) = AB + AC$

(2) $A + BC = (A + B)(A + C)$

(g) Redundant literal rule (RLR)

(1) $A + \overline{A}B = A + B$

(2) $A(\overline{A} + B) = AB$

(h) Idempotence laws

(1) $A \cdot A = A$

(2) $A + A = A$

(i) Absorption laws

(1) $A + A \cdot B = A$

(2) $A(A + B) = A$

3. Theorems

(a) Consensus theorem

Theorem 1:

$$AB + \overline{A}C + BC = AB + \overline{A}C$$

Proof:

$$\begin{aligned}\text{LHS} &= AB + \overline{A}C + BC \\ &= AB + \overline{A}C + BC(A + \overline{A}) \\ &= AB + \overline{A}C + BCA + BC\overline{A} \\ &= AB(1 + C) + \overline{A}C(1 + B) \\ &= AB(1) + \overline{A}C(1) \\ &= AB + \overline{A}C \\ &= \text{RHS.}\end{aligned}$$

Theorem 2:

$$(A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C)$$

Proof:

$$\begin{aligned}\text{LHS} &= (A + B)(\overline{A} + C)(B + C) \\ &= (A\overline{A} + AC + B\overline{A} + BC)(B + C) \\ &= (AC + BC + \overline{A}B)(B + C) \\ &= ABC + BC + \overline{A}B + AC + BC + \overline{A}BC \\ &= AC + BC + \overline{A}B\end{aligned}$$

$$\begin{aligned}\text{RHS} &= (A + B)(\overline{A} + C) \\ &= A\overline{A} + AC + BC + \overline{A}B \\ &= AC + BC + \overline{A}B \\ &= \text{LHS.}\end{aligned}$$

(b) Transposition theorem

$$AB + \overline{A}C = (A + C)(\overline{A} + B)$$

Proof:

$$\begin{aligned}\text{RHS} &= (A + C)(\overline{A} + B) \\ &= A\overline{A} + C\overline{A} + AB + CB \\ &= 0 + \overline{A}C + AB + BC \\ &= \overline{A}C + AB + BC(A + \overline{A}) \\ &= AB + ABC + \overline{A}C + \overline{A}BC \\ &= AB + \overline{A}C \\ &= \text{LHS}\end{aligned}$$

(c) De Morgan's theorem

Law 1: $\overline{A + B} = \overline{A} \cdot \overline{B}$

This law states that the complement of a sum of variable is equal to the product of their individual complements.

Law 2: $\overline{AB} = \overline{A} + \overline{B}$

This law states that the complement of the product of variables is equal to the sum of their individual complements.

Example 1: Simplify the Boolean function $Y = A(A + \overline{B})$

$$Y = A \cdot A + A \cdot \overline{B}$$

Solution:

$$\begin{aligned}Y &= A + A\overline{B} \\ &= A(1 + \overline{B}) \\ &= A\end{aligned}$$

Example 2: Simplify the Boolean function $Y = A + \overline{A}B$

Solution:

$$\begin{aligned}Y &= A \cdot (B + 1) + \overline{A} \cdot B \\ &= A \cdot B + A + \overline{A}B \\ &= B(A + \overline{A}) + A \\ &= A + B\end{aligned}$$

Example 3: Simplify the Boolean function

$$Y = A(A + B) + B(\overline{A} + B)$$

Solution:

$$\begin{aligned}Y &= A \cdot A + A \cdot B + B \cdot \overline{A} + B \cdot B \\ &= A + B(A + \overline{A}) + B \\ &= A + B \cdot 1 + B \\ &= A + B + B \\ &= A + B\end{aligned}$$

Example 4: Simplify the Boolean function

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C}$$

Solution:

$$\begin{aligned}Y &= \overline{A}\,\overline{C}(\overline{B} + B) + A\overline{C}(B + \overline{B}) \\ &= \overline{A}\,\overline{C} + A\overline{C} \\ &= \overline{C}(\overline{A} + A) \\ &= \overline{C}\end{aligned}$$

Example 5: Simplify the Boolean function

$$Y = \overline{\overline{A}BC + \overline{A}B\overline{C} + \overline{A}\,\overline{B}C}$$

Solution:

$$\begin{aligned}&= \overline{\overline{A}C(B + \overline{B}) + \overline{A}B\overline{C}} \\ &= \overline{\overline{A}C + \overline{A}B\overline{C}} \\ &= \overline{\overline{A}(C + B\overline{C})} \\ &= \overline{\overline{A}(C + B)} \\ &= A + \overline{C}\,\overline{B}\end{aligned}$$

Example 6: Simplify the Boolean function

$$Y = AB + C\bar{B} + CA + ABD$$

Solution: $Y = AB(1 + D) + C\bar{B} + CA$

$$= AB + C\bar{B} + CA$$

$$= AB + C\bar{B}$$

Properties of Boolean Algebra

With n variables, maximum possible distinct functions $= 2^{2^n}$.

Duality consider the distributive law

1. $x(y + z) = xy + xz$
2. $x + yz = (x + y)(x + z)$

Second one can be obtained from the first law if the binary operators and the identity elements are interchanged. This important property of Boolean algebra is called the duality principle.

The dual of an algebraic expression can be written by interchanging OR and AND operators, 1s by 0, and 0s by 1s.

Example 7: $x + x' = 1 \xleftrightarrow{\text{Dual}} x \cdot x' = 0$

Solution: $xy + xy' = x \xleftrightarrow{\text{Dual}} (x + y)(x + y') = x$

$$x + x'y = x + y \xleftrightarrow{\text{Dual}} x(x' + y) = xy$$

Example 8: The dual of $F = xy + xz + yz$ is?

Solution: Dual of $F = (x + y)(x + z)(y + z)$
$= (x + xz + xy + yz)(y + z) = xy + yz + xz$

So dual of $xy + xz + yz$ is same as the function itself; For N variables maximum possible self-dual functions $= 2^{2^{n-1}} = 2^{(2^n/2)}$

Example 9: Which of the following statement/s is/are true

S_1: The dual of NAND function is NOR
S_2: The dual of X-OR function is X-NOR

(A) S_1 and S_2 are true
(B) S_1 is true
(C) S_2 is true
(D) None of these

Solution: (A)

NAND $= (xy)' = x' + y'$
Dual of NAND $= (x + y)' = x'y'$
X-OR $= xy' + x'y$
Dual of X-OR $= (x + y')(x' + y) = xy + x'y' =$ X-NOR
Both S_1 and S_2 are true

Operator precedence The operator precedence for evaluating Boolean expression is

1. Parentheses
2. NOT
3. AND
4. OR

So the expression inside the parentheses must be evaluated before all the operations. The next operation to be performed is the complement and then follows AND and finally the OR.

Complement of function The complement of a function F is F' is obtained from an interchange of 0s for 1s and 1s for 0s in the value of F. The complements of a function may be derived algebraically through De Morgan's theorems.

$$(x_1 . x_2 . x_3 \ldots x_n)' = x_1' + x_2' + x_3' + \cdots + x_n'$$
$$(x_1 + x_2 + x_3 + \cdots + x_n)' = x_1' . x_2' . x_3' . x_4' \ldots x_n'$$

Example 10: The complement of function $F = a(b'c + bc')$ is?

Solution: $(F)' = [a(b'c + bc')]'$
$= a' + (b'c + bc')'$
$= a' + (b'c)' \cdot (bc')'$
$= a' + (b + c)'(b' + c)$
$F' = a' + bc + b'c'$

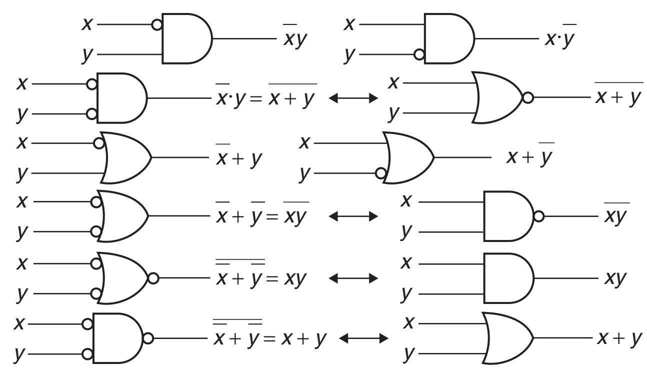


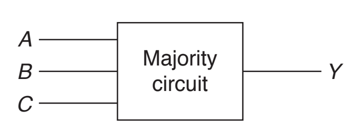
Figure 7 Gates with inverted inputs

Boolean Functions, Min Terms and Max Terms

The starting point for designing most logic circuits is the truth table, which can be derived from the statement of problem. The truth table is then converted into a Boolean expression and finally create the assembly of logic gates accordingly.

Let us consider the example of majority circuit. This circuit takes three inputs (A, B, C) and have one output (Y) which will give the majority of the inputs, i.e., if A, B, C are having more number of zeros. $Y = 0$ else if A, B, C are having more number of 1s, $Y = 1$.

So from the statements we can derive the truth table as follows:



As we are using three Boolean variables A, B, C, total number of combinations in truth table are $2^3 = 8$.

Similarly for n variables, the truth table will have total of 2^n combinations, for a Boolean function.

	Input			Output	
Sl. no.	A	B	C	Y	
1	0	0	0	0 ⟶	$Y = 0$, If inputs are having more zeros.
2	0	0	1	0	
3	0	1	0	0	
4	0	1	1	1 ⟶	$Y = 1$, If inputs are having more 1's
5	1	0	0	0	
6	1	0	1	1	
7	1	1	0	1	
8	1	1	1	1	

For some combinations, output $Y = 1$, and for others $Y = 0$. The input combinations for which output $Y = 1$ are called as min terms.

Similarly the input combinations for which output $Y = 0$ are called as max terms.

Min terms are expressed as product terms, Similarly, max terms are expressed as sum terms.

The output $Y = 1$, only in rows 4, 6, 7, 8.

So the min terms combinations are 011, 101, 110, 111 in Boolean Algebra, 1 input will be written as A, B, C and 0 input will be written as $\bar{A}, \bar{B}, \bar{C}$ in complement form, we express these min terms as product terms, $\bar{A}BC, A\bar{B}C, AB\bar{C}, ABC$.

To express Y as Boolean expression, we can write it as sum of the min terms.

$$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

We know that AND operation is a product while OR is sum. So the above equation is a sum of the products (SOP), (or) min terms expression.

The other way of expressing Y is $Y = \Sigma m\ (3, 5, 6, 7)$.

$$Y = m_3 + m_5 + m_6 + m_7.$$

The min term numbers are the decimal equivalent of input binary combinations.

Similar to SOP we can have product of sums (POS) Boolean expression.

The output $Y = 0$ for the input combinations 000, 001, 010, 100. For max terms 1 input will be indicated as $\bar{A}, \bar{B}, \bar{C}$ in complement form, 0 input will be indicated as A, B, C and max terms are expressed as sum terms.

$$A + B + C,\ A + B + \bar{C},\ A + \bar{B} + C,\ \bar{A} + B + C$$

Any function can be expressed as product of max terms.

$$\text{So } Y = (A + B + C)\,(A + B + \bar{C})\,(A + \bar{B} + C)\,(\bar{A} + B + C)$$

The above equation is a product of sum expression (POS) or max terms expression.

In other way $Y = \pi M\ (0, 1, 2, 4)$

$$= M_0 \cdot M_1 \cdot M_2 \cdot M_4$$

The max term numbers are decimal equivalents of corresponding input binary combinations.

Min Term and Max Term

All the Boolean expressions can be expressed in a standard sum of product (SOP) form or in a standard product of sum (POS) form.

- A standard SOP form is one in which a number of product terms, each contains all the variables of the function either in complement or non-complement form are summed together.
- A standard POS form is one in which a number of sum terms, each one of which contain all the variable of he function either in complemented or non-complemented form are multiplied together.
- Each of the product term in standard SOP form is called a min term.
- Each of the sum term in the standard POS form is called a max term.

Conversion from min terms to max terms representation

$$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$Y' = (\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC)'$$

$$= (\bar{A}BC)'(A\bar{B}C)'(AB\bar{C})'(ABC)'$$

$$(Y')' = [(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})]'$$

$$= [\pi(3, 5, 6, 7)]^1 = \pi(0, 1, 2\ ,4)$$

$$Y = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + C)$$

$$\text{or } Y = \Sigma(3, 5, 6, 7) = \pi(0, 1, 2, 4)$$

Conversion from normal SOP/POS form to canonical SOP/POS

Let us consider $f(A, B, C) = A + BC + \bar{A}C$

The above function is in normal (minimized) SOP from, to convert this function to standard SOP(or) canonical SOP form, include missing variable in each and every term, to make it complete. First term A, Missing literals are B, C. Consider $A\ X\ X$, so possible combinations are $\bar{A}BC, A\bar{B}C, AB\bar{C}, ABC$ or we can write

$$A = A = A(B + \bar{B})(C + \bar{C}) = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

Second term BC-missing literal is A. Consider $XBC \Rightarrow$ So possible combinations are ABC, $\bar{A}BC$ or we can write

$$BC = (A + \bar{A})BC$$

$$= ABC + \bar{A}BC$$

Third term $\bar{A}C$ = missing literal is B. Consider $\bar{A}XC \rightarrow$ so possible combinations are $\bar{A}BC, \bar{A}, \bar{B}C$ or we can write

$$\bar{A}C = \bar{A}(B + \bar{B})C$$

$$= \bar{A}BC + \bar{A}\bar{B}C$$

Now, $f(A,B,C) = ABC + AB\overline{C} + A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C$, after removing the redundant terms.

Now consider

$$f(A,B,C) = (A+B)(\overline{A}+C)$$

To convert this expression to canonical form or standard POS form we can write

$$f(A,B,C) = (A+B+C\cdot\overline{C})(\overline{A}+B\cdot\overline{B}+C)$$

Here the C variables is absent from first term and B from second term. So add $C\cdot\overline{C} = (0)$ to first, and $B\cdot\overline{B}$ to second, and using distributive law arrive at the result.

$$f(A,B,C) = (A+B+C)(A+B+\overline{C})(\overline{A}+B+C)(\overline{A}+\overline{B}+C)$$

Minimization of Boolean Functions

Simplification Procedure

- Obtain truth table, and write output in canonical form or standard form
- Generate K-map!
- Determine Prime implicants.
- Find minimal set of prime implicants.

Karnaugh Map (K-map) Method

Karnaugh map method is a systematic method of simplifying the Boolean expression. K-map is a chart or a graph composed of an arrangement of adjacent cell, each representing a particular combination of variable in sum or product form. (i.e., min term or max term).

Two-variable K-map

x	y	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

m_0	m_1
m_2	m_3

$x \backslash y$	0	1
0	$x'y'$	$x'y$
1	xy'	xy

Three-variable K-map

A three-variable map will have eight min terms (for three variables $2^3 = 8$) represented by 8 squares

x	y	z	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

$x \backslash yz$	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
1	$xy'z'$	$xy'z$	xyz	xyz'

3-variable K-map

Four-variable K-maps

The K-map for four variables is shown here, 16 min terms are assigned to 16 squares.

$wx \backslash yz$	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

The map is considered to lie on a surface with the top and bottom edges as well as the right and left edge touching each other to form adjacent squares.

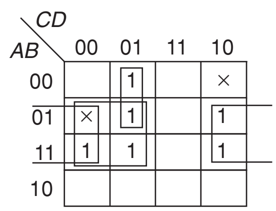
- One square → a min term of four literals
- Two adjacent square → a term of three literals
- Four adjacent square → a term of two literals
- Eight adjacent square → a term of one literal
- Sixteen adjacent square → The constant one

Don't-care Combinations

It can often occur that for certain input combinations, the value of the output is unspecified either because the input combination are invalid or because the precise value of the output is of no consequence. The combination for which the values of the expression are not specified are called don't-care combinations. During the process of design using an SOP, K-map, each don't-care is treated as a 1 if it is helpful in Map Reduction, otherwise it is treated as a 0 and left alone. During the process of design using a POS K-map, each Don't-care is treated as a 0 if it is useful in Map Reduction, otherwise it is treated as a 1 and left alone.

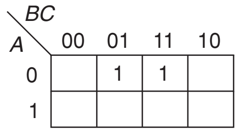
Example 11: Find the Minimal expression $\Sigma m\,(1, 5, 6, 12, 13, 14) + d\,(2, 4)$

Solution:



$$\therefore \quad F = B\overline{C} + B\overline{D} + \overline{A}\,\overline{C}D$$

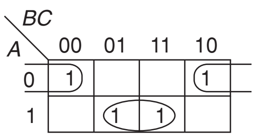
Pairs, Quads and Octets



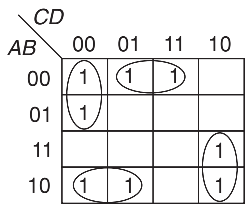
The map contains a pair of 1s those are horizontally adjacent. Those cells represent $\bar{A}\bar{B}C, \bar{A}BC$.

For these two min terms, there is change in the form of variable B. By combining these two cells we can form a pair, which is equal to $\bar{A}\bar{B}C + \bar{A}BC = \bar{A}C(\bar{B} + B) = \bar{A}C$.

If more than one pair exists on K-map, OR the simplified products to get the Boolean function.



$F = \bar{A}\bar{C} + AC$

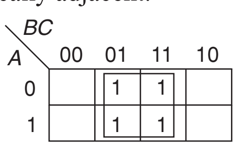


$F = \bar{A}\bar{C}\bar{D} + \bar{A}BD + AB\bar{C} + AC\bar{D}$

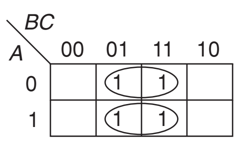
So Pair eliminates one variable by minimization.

Quad

Quad is a group of four 1s those are horizontally or vertically adjacent.

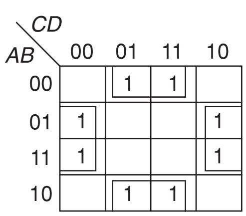


$F = C$



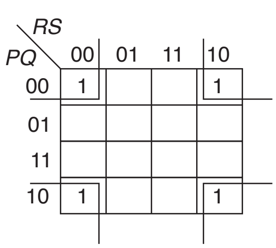
$F = \bar{A}C + AC = (\bar{A} + A)C = C$

By considering two pairs also it will be simplified to C. Quad eliminates two variables from the function



$F = B\bar{D} + \bar{B}D$

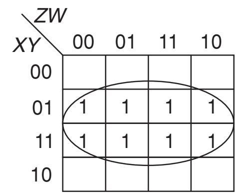
Corner min terms can from a Quad



$F = \bar{Q}\bar{S}$

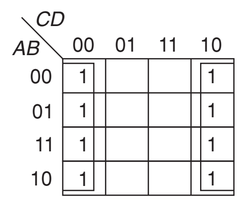
Octet

The group of eight cells will form one octet.



$F = Y$

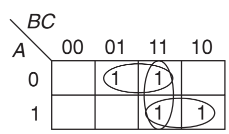
Other variable X, Z, W changes their form in octet. Octet can eliminate three variables and their complements.



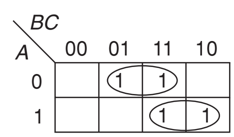
$F = \bar{D}$

Other variable A, B, C are vanished.

Eliminating Redundant Groups

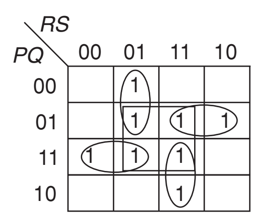


$AB + \bar{A}C + BC$

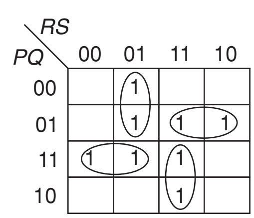


$AB + \bar{A}C$

Here BC is redundant pair, which covers already covered min terms of $AB, \bar{A}C$.



This K-map gives fourpairs and one quad.

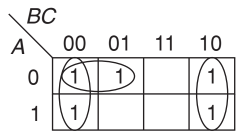


But only four pairs are enough to cover all the min times, Quad is not necessary.

$\bar{P}\bar{R}S + \bar{P}QR + PQ\bar{R} + PRS$ is minimized function.

Prime Implicant

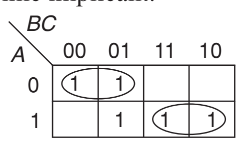
The group of adjacent min terms is called a Prime Implicant, i.e., all possible pairs, quads, octets, etc.



Prime implicants are $\overline{B}\,\overline{C}$, $B\overline{C}$, $\overline{C}$, $\overline{A}\,\overline{B}$. Minimized function is $\overline{C}+\overline{A}\,\overline{B}$

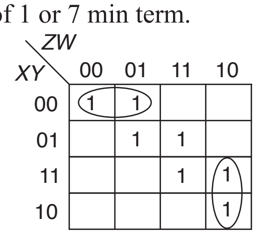
Essential Prime Implicant

The prime implicant which contains at least one min term which cannot be covered by any other prime implicant is called Essential prime implicant.



Min term 0, 6 can be grouped with only one pair each.

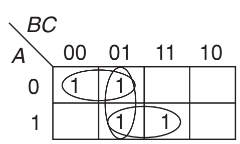
The total possible prime implicants are $\overline{A}\,\overline{B}$, $\overline{B}C$, AC, AB but min term 0, 6 can be covered with $\overline{A}\,\overline{B}, AB$. So we call them as essential prime implicants. Min term 5 can be paired with any of 1 or 7 min term.



The essential prime Implicants are $XZ\overline{W}$, $\overline{X}\,\overline{Y}\,\overline{Z}$

Redundant Prime Implicant

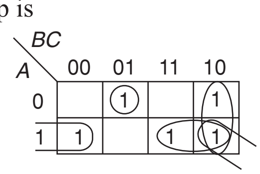
The prime implicant whose min terms are already covered by at least one min term is called redundant prime implicants.



Here prime implicants are $\overline{A}\,\overline{B}, AC, \overline{B}C$. But $\overline{B}C$ is already covered by other min terms So $\overline{B}C$ is redundant prime implicant.

Example 12: Find the minimal expression for $\Sigma m(1, 2, 4, 6, 7)$ and implement it using Basic gates.

Solution: K-map is



$$F = A\overline{C} + AB + B\overline{C} + \overline{B}C + \overline{A}\,\overline{B}C$$

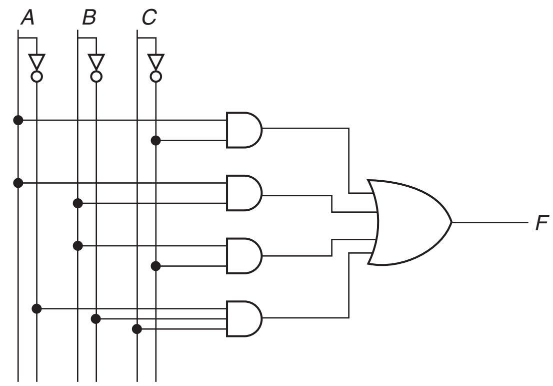
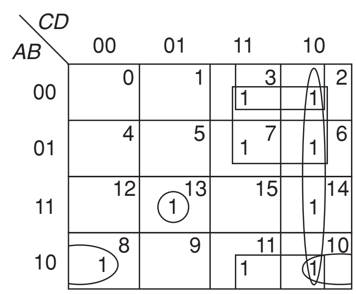


Figure 8 Logic diagram

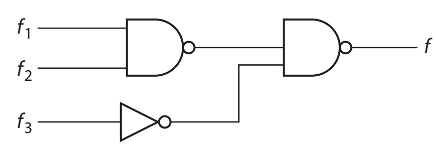
Example 13: Find the minimal expression for Σm (2, 3, 6, 7, 8, 10, 11, 13, 14)

Solution: K-map is:



$$\therefore \quad F(A,B,C,D) = AB\overline{C}D + A\overline{B}\,\overline{D} + \overline{A}C + \overline{B}C + C\overline{D}$$

Example 14: Three Boolean functions are defined as below $f_1 = \Sigma m(0, 1, 3, 5, 6)$, $f_2 = \Sigma m(4, 6, 7)$ $f_3 = \Sigma m(1, 4, 5, 7)$, then find f.



Solution: When two Boolean functions are ANDed, the resultant will contain the common min terms of both of the functions (like, intersection of min terms). If two Boolean functions are ORed, then resultant is the combination of all the min terms of the functions (like union of min terms)

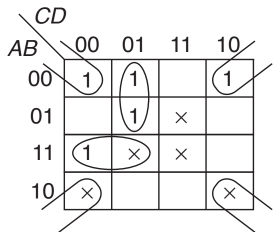
Here $f = \overline{\overline{f_1 f_2} \cdot \overline{f_3}} = f_1 f_2 + f_3$

Here $f_1 \cdot f_2$ = Common min terms in f_1 and $f_2 = \Sigma m(6)$

$f_1 \cdot f_2 + f_3$ = Combination of min terms of $f_1 \cdot f_2$ and f_3 $= \Sigma(1, 4, 5, 6, 7)$

Example 15: What is the literal count for the minimized SOP, and minimized POS form for the following function? $f(A, B, C, D) = \Sigma m(0, 1, 2, 5, 12) + \phi d(7, 8, 10, 13, 15)$

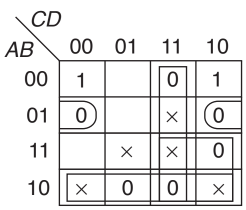
Solution: $f(A, B, C, D) = \Sigma m(0, 1, 2, 5, 12) + \phi(7, 8, 10, 13, 15)$



$f = 1$ quad + 2 pairs

Literal count $= 1 \times 2 + 2 \times 3 = 8$

$f(A, B, C, D) = \pi M(3, 4, 6, 9, 11, 14) + \phi(7, 8, 10, 13, 15)$



f will consists of 3 quads + 1pair

$= 3 \times 2 + 1 \times 3 = 6 + 3 = 9$

Implementation of Function by Using NAND–NOR Gates

NAND or NOR gates are called as universal gates, because any function can be implemented by using only NAND gates or only using NOR gates.

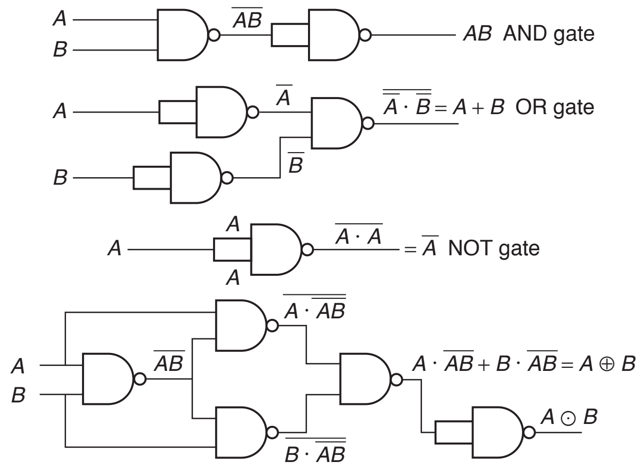


Figure 9 Implement of basics gates by using NAND gates

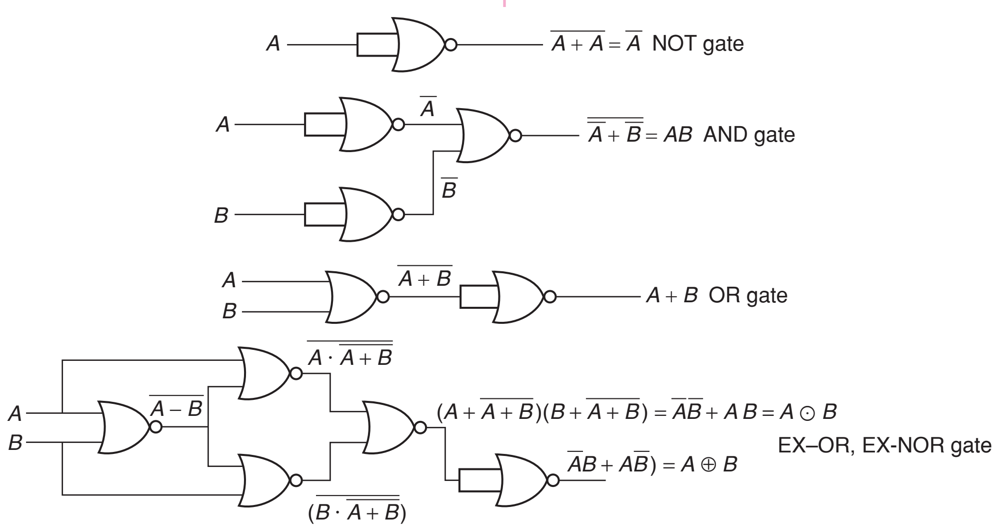
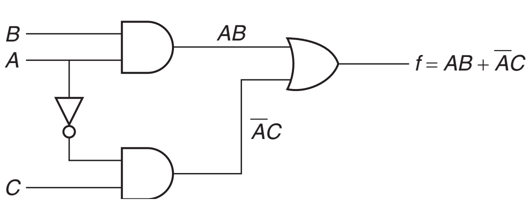


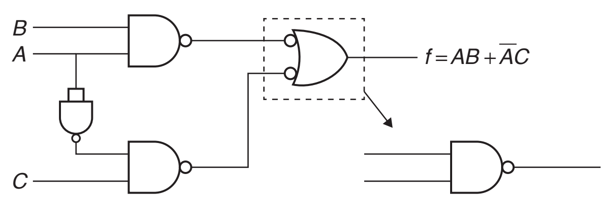
Figure 10 Implementation of basic gates by using NOR gates

Any function which is in the SOP form can be implemented by using AND-OR gates, which is also equivalent to NAND–NAND gates.

$F = AB + \overline{A}C$



By considering bubble at AND gate output and OR gate input, and by changing NOT gates to NAND gates the circuit becomes as,



Now the circuit is in completely in NAND – NAND form

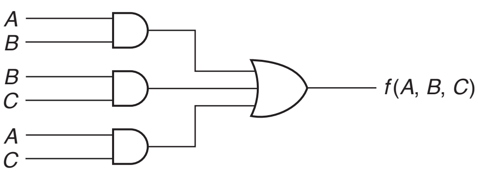
So the functions expressed in SOP form, can be implemented by using AND – OR, (or) NAND–NAND gates.

Any function in POS from, can be implemented by using OR–AND gates, which is similar to NOR–NOR gate.

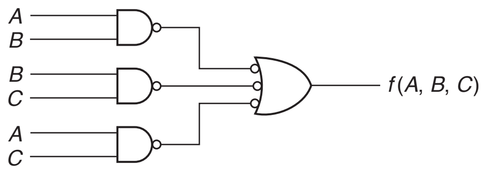
Example 16: How many number of NAND gates are required to implement $f(A, B, C) = AB + BC + AC$

(A) 3 (B) 4 (C) 5 (D) 6

Solution:



By considering bubbles at output of AND gate and input of OR gate.

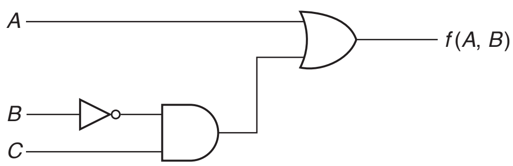


So four NAND gates are required.

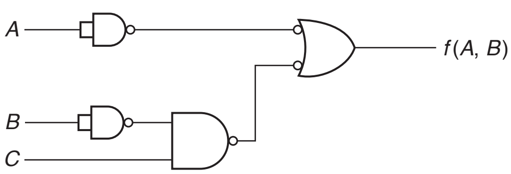
Example 17: Number of NAND gates required for implementation of $f(A,B) = A + \bar{B}C$ is

(A) 3 (B) 4 (C) 5 (D) 6

Solution:



To convert the all gates into NAND gates, place bubble output of AND, and inputs of OR gates. Now, the circuit can be drawn as



Four NAND gates are required.

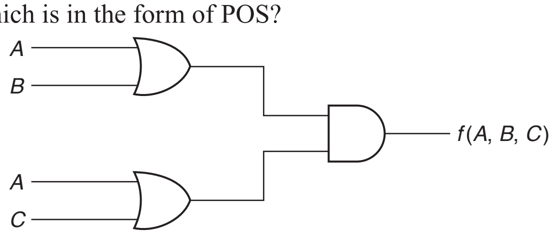
Example 18: $f = A + BC$, the number of NOR gates required to implement *f*, are?

(A) 3 (B) 4 (C) 5 (D) 2

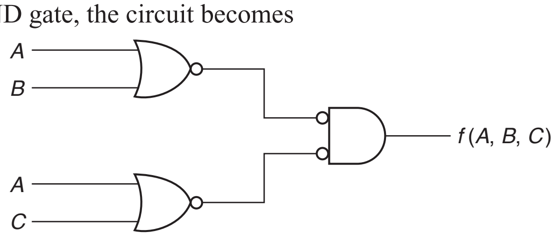
Solution: $A + BC$ is in SOP form.

To implement this function by using NOR gates, we can write $f(A, B, C) = A + BC = (A + B)(A + C)$

Which is in the form of POS?



By including bubbles at output of OR gate, and input of AND gate, the circuit becomes



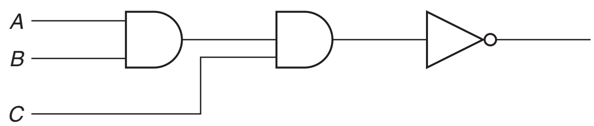
Now the circuit consists of all NOR gates. Three NOR Gates are required.

Example 19: How many number of two-input NAND–NOR gates are required to implement three-input NAND–NOR gates respectively?

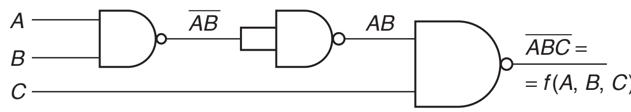
(A) 2, 2 (B) 2, 3
(C) 3, 2 (D) 3, 3

Solution: $f(A,B,C) = \overline{ABC} = \overline{AB} + \bar{C}$

(1) Implement above function by using two-inputs gates

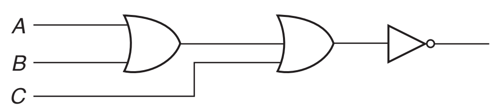


Now convert each gate to NAND gate

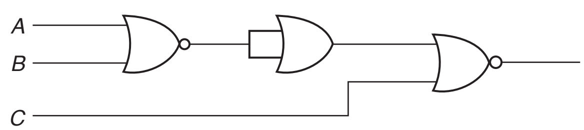


Three two-input NAND gates are required.

(2) $G(A,B,C) = \overline{A+B+C}$ Implement it by using two-input gates



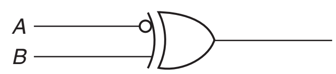
Now convert each gate to NOR gate



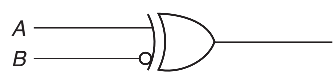
Three two-input, NOR gates are required.

EX-OR, EX-NOR GATES

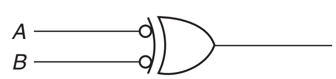
Inverted inputs for EX OR, EX-NOR gates



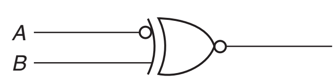
$\bar{A} \oplus B = \overline{\bar{A}}B + \bar{A}\,\bar{B} = AB + \bar{A}\,\bar{B} = A \odot B$



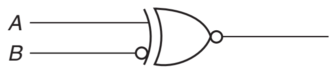
$A \oplus \bar{B} = A\overline{\bar{B}} + \bar{A}\,\bar{B} = AB + \bar{A}\,\bar{B} = A \odot B$



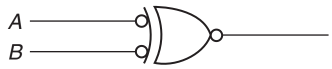
$\bar{A} \oplus \bar{B} = \bar{A}\,\overline{\bar{B}} + \overline{\bar{A}}\,\bar{B} = \bar{A}B + A\bar{B} = A \oplus B$



$\bar{A} \odot B = \bar{A}B + \overline{\bar{A}}\,\bar{B} = \bar{A}B + A\bar{B} = A \oplus B$



$$A \odot \overline{B} = A\overline{B} + \overline{A}\,\overline{\overline{B}} = A\overline{B} + \overline{A}B = A \oplus B$$

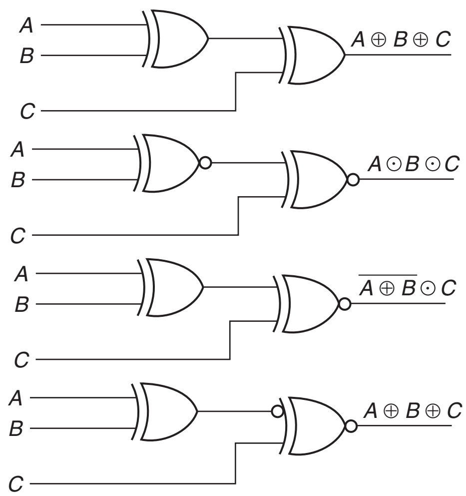


$$\overline{A} \odot \overline{B} = \overline{A}\,\overline{B} + \overline{\overline{A}}\,\overline{\overline{B}} = \overline{A}\,\overline{B} + AB = A \odot B$$

From the above discussions we can conclude that inverted input EXOR gate is EX-NOR gate.

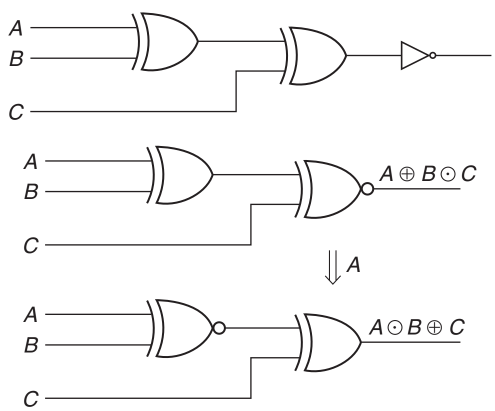
Similarly, inverted input EX-NOR gate is EX-OR gate. If both inputs are inverted the EX-OR / EX-NOR will remain as it is.

Consider a three-inputs X-OR gates by using two-input XOR gates.



So we can conclude that $A \oplus B \oplus C = A \odot B \odot C$

$$\overline{A \oplus B \oplus C} = \overline{A \odot B \odot C}$$



$\overline{A \oplus B \oplus C} = \overline{A \odot B \odot C} = A \oplus B \odot C$

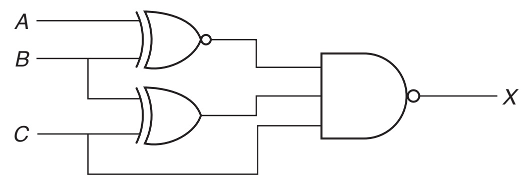
$= A \odot B \oplus C$

$A \oplus B \oplus C \oplus D = A \oplus B \odot C \odot D = A \odot B \odot C \oplus D = A \odot B \oplus C \odot D$

$A \odot B \odot C \odot D = \overline{A \oplus B \oplus C \oplus D}$

$A \odot B \odot C \odot D = A \oplus B \oplus C \odot D = A \oplus B \odot C \oplus D$

Example 20: For the logic circuit shown in figure, the required input condition (A, B, C) to make the output $X = 0$ is?



(A) 1, 1, 1 (B) 1, 0, 1
(C) 0, 1, 1 (D) 0, 0, 1

Solution: (D)

To get output $X = 0$, all inputs to the NAND gate should be 1, so $C = 1$.

When $C = 1$, the output of X-OR gate $B \oplus C = 1$ only when $B = 0$.

If $B = 0$ the output of X-NOR gate $A \odot B = 1$.

Only when $A = 0$

So $X = 1$, only when $(A, B, C) = (0, 0, 1)$.

Example 21: The minimized expression of

$(A+\overline{B})(A\overline{B}+AC)(\overline{A}\,\overline{C}+\overline{B})$ is

Solution: $(A+\overline{B})(A\overline{B}+AC)(\overline{A}\,\overline{C}+\overline{B})$

$$= (A+\overline{B})(A\overline{B}\cdot\overline{A}\,\overline{C} + A\overline{B}\cdot\overline{B} + AC\cdot\overline{A}\,\overline{C} + AC\cdot\overline{B})$$

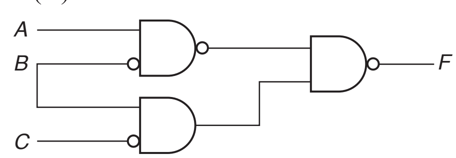
$$= (A+\overline{B})(A\overline{B} + A\overline{B}C) = (A+\overline{B})A\overline{B}(1+C)$$

$$= A\overline{B} + A\overline{B} = A\overline{B}$$

Example 22: The Boolean function f is independent of

(A) a (B) b
(C) c (D) None of these

Solution: (A)

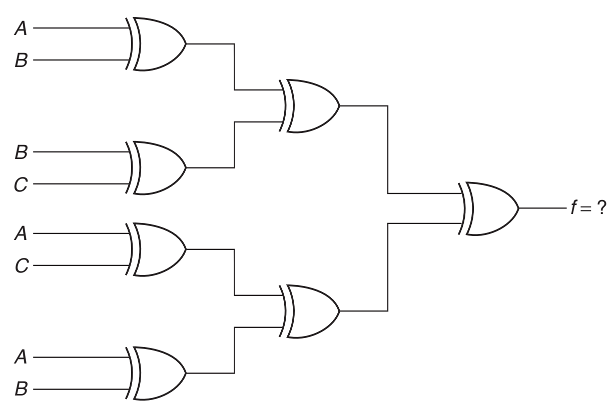


$$F = \overline{\overline{ab}\cdot\overline{bc}}$$

$$= ab + \overline{\overline{b}c} = ab + b + \overline{c}$$

$= b + \overline{c}$ is independent of 'a'.

Example 23:



Solution: $f = \{A \oplus B \oplus B \oplus C\} \oplus \{A \oplus C \oplus B \oplus A\}$

$= \{A \oplus 0 \oplus C\} \oplus \{0 \oplus C \oplus B\}$

$= A \oplus C \oplus C \oplus B = A \oplus 0 \oplus B = A \oplus B$

Solved Examples

Example 1: Simplify the Boolean function, $x\,y + x'z + y\,z$

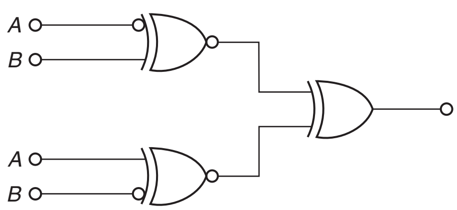
Solution: $x\,y + x'\,z + y\,z$

By using consensus property

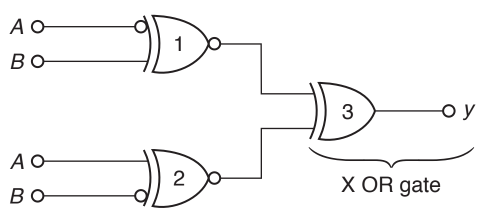
$xy + x'z + yz = xy + x'z$

$Y = xy + x'z$

Example 2: The output of the given circuit is equal to



Solution: $\overline{A} \odot B = \overline{A}B + A\overline{B}$



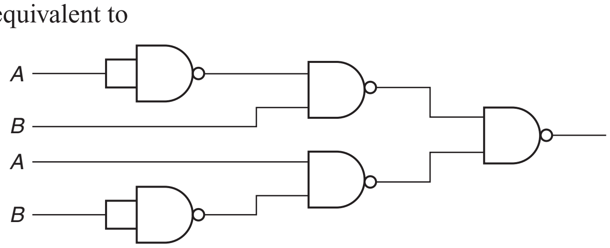
$$A \odot \overline{B} = \overline{A}B + A\overline{B}$$

So the output of above circuit is '0'. As two inputs are same at third gate.

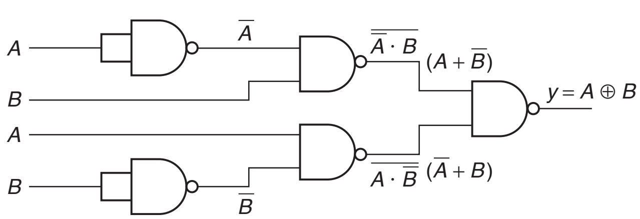
Output of XOR gate with two equal inputs is zero.

$$\therefore\ y = 0$$

Example 3: The circuit shown in the figure is functionally equivalent to



Solution:



$Y = \overline{\overline{\overline{A}\cdot B}\cdot\overline{A\overline{B}}} = \overline{(A+\overline{B})(\overline{A}+B)} \quad \because \quad (\overline{A\cdot B} = \overline{A}+\overline{B})$

$= \overline{(A+\overline{B})} + \overline{(\overline{A}+B)} = \overline{A}\cdot\overline{\overline{B}} + \overline{\overline{A}}\cdot\overline{B}$

$= \overline{A}\cdot B + A\cdot\overline{B} = A \oplus B$

Example 4: Simplify the Boolean function $A \oplus \overline{A}B \oplus \overline{A}$

Solution: $A \oplus \overline{A}B \oplus \overline{A}$

Associativity

$= 1 \oplus \overline{A}B = \overline{\overline{A}B}$

$= A + \overline{B}$ ($\because$ De Morgan's)

Example 5:

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	×	×	1
11	×	×	1	×
10	1	0	1	1

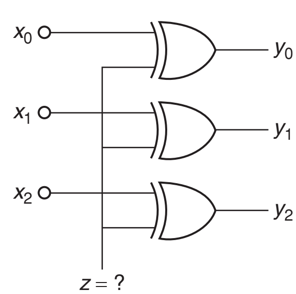
The minimized expression for the given K–map is

Solution:

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	×	×	1
11	×	×	1	×
10	1	0	1	1

$= A + \overline{B}C$

Example 6: In the figure shown, y_2, y_1, y_0 will be 1s complement of $x_2\,x_1\,x_0$ if $z = ?$

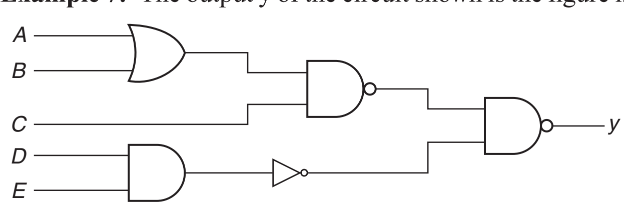


Solution: We are using X-OR gate

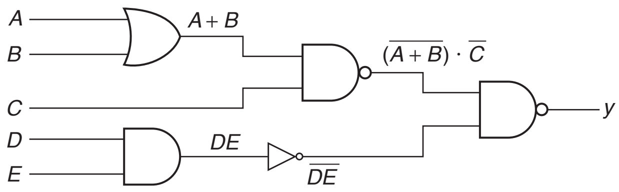
$\therefore$ XOR out-put is complement of input only when other input is high.

$\therefore Z = 1$

Example 7: The output y of the circuit shown is the figure is



Solution:



$$y = \overline{\overline{(A+B)\cdot C}\cdot\overline{DE}} = \overline{\overline{(A+B)\cdot C}} + \overline{\overline{DE}}$$

$$= (A+B)C + DE(\overline{x\cdot y} = \bar{x} + \bar{y})$$

Example 8: Simplify the following function

$$f = \overline{\overline{\overline{A}(\overline{AB})}\cdot\overline{\overline{B}(\overline{AB})}}$$

Solution: $\overline{\overline{\overline{A}(\overline{AB})}\cdot\overline{\overline{B}(\overline{A\cdot B})}}$

$$\overline{[A+(AB)]\cdot[B+(AB)]} = \overline{A+(AB)} + \overline{(B+AB)}$$

$$= \bar{A}\cdot(\overline{AB}) + \bar{B}\cdot(\overline{AB}) = \bar{A}\cdot(\bar{A}+\bar{B}) + \bar{B}(\bar{A}+\bar{B})$$

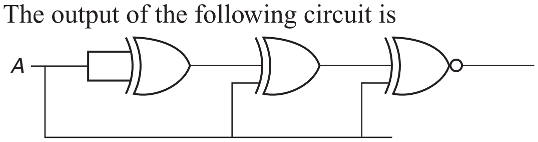
$$= \bar{A}\cdot\bar{A} + \bar{A}\,\bar{B} + \bar{B}\,\bar{A} + \bar{B}\cdot\bar{B} = \bar{A} + \bar{B} = \overline{AB}$$

Exercises

Practice Problems I

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The output of the following circuit is



(A) 0 (B) 1
(C) A (D) A'

2. The circuit which will work as OR gate in positive level will work as ___ gate in negative level logic
(A) NOR gate
(B) NAND gate
(C) Both NAND and NOR gate
(D) AND gate

3. Four logical expressions are given below:

(a) $\bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}\,\bar{E}\cdot\bar{F}\cdot\bar{G}\cdot\bar{H}$

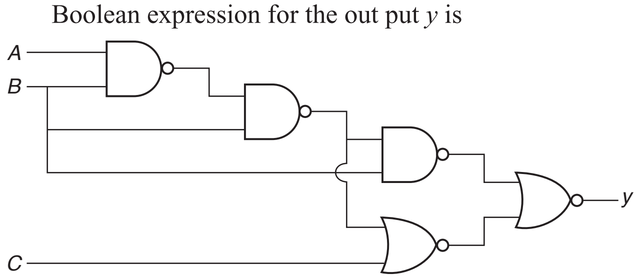
(b) $\overline{AB}\cdot\overline{CD}\cdot\overline{EF}\cdot\overline{GH}$

(c) $\bar{A}+\bar{B}+\bar{C}+\bar{D}+\bar{E}+\bar{F}+\bar{G}+\bar{H}$

(d) $(\bar{A}+\bar{B})(\bar{C}+\bar{D})(\bar{E}+\bar{F})(\bar{G}+\bar{H})$

Two of these expression are equal. They are
(A) c and d (B) b and d
(C) a and b (D) a and c

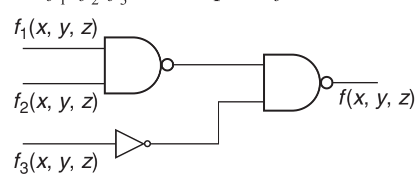
4. For the logic circuit shown in figure, the simplified Boolean expression for the out put y is



(A) $A + B + C$ (B) A
(C) ABC (D) $B\bar{C}$

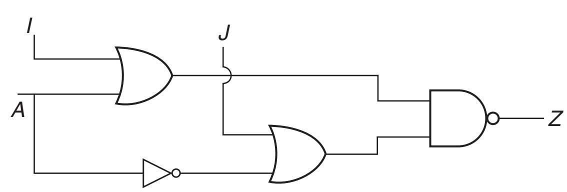
5. In a digital system, there are three inputs A, B and C. The output should be high when at least two inputs are high. The minimized Boolean expression for the out put is
(A) $AB + BC + AC$
(B) $ABC + AB\bar{C} + \bar{A}BC + A\bar{B}C$
(C) $AB\bar{C} + A\bar{B}C + \bar{A}BC$
(D) $A\bar{B} + B\bar{C} + \bar{A}C$

6. Consider the following logic circuit whose inputs are functions f_1, f_2, f_3 and output is f.



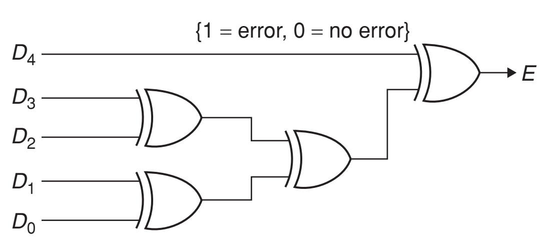
Given that $f_1(x, y, z) = \bullet(0, 1, 3, 5)$ $f_2(x, y, z) = \bullet(6, 7)$ and $f(x, y, z) = \bullet(1, 4, 5)$, then f_3 is
(A) $\bullet(1, 4, 5)$ (B) $\bullet(6, 7)$
(C) $\bullet(0, 1, 3, 5)$ (D) None of these

7. The circuit shown above is to be used to implement the function $z = f(A, B) = \bar{A} + B$ what values are to be selected for I and J?



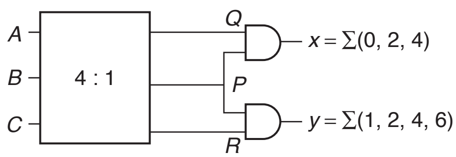
(A) $I = 0, J = B$ (B) $I = 1, J = B$
(C) $I = B, J = 1$ (D) $I = B, J = 0$

8. Parity checker output from the below figure, if input is 11111 ($D_4\,D_3\,D_2\,D_1\,D_0$) and 10000 ($D_4\,D_3\,D_2\,D_1\,D_0$).



(A) error, error
(B) error, no error
(C) no error, error
(D) no error , no error

9. For the given combinational network with three inputs A, B and C, three intermediate outputs P, Q and R, and two final outputs $X = P \cdot Q = \Sigma(0, 2, 4)$ and $Y = P \cdot R = \Sigma(1, 2, 4, 6)$ as shown below. Find the smallest function P(containing minimum number of min terms that can produce the output x and y)



(A) $\Sigma(2, 4)$
(B) $\Sigma(0, 1, 2, 4, 6)$
(C) $\Sigma(3, 5, 7)$
(D) $\Sigma(1, 2, 6)$

10. The standard form of expression $AB + ACD + \bar{A}C$ is

(A) $AB\bar{C}\bar{D} + ABC\bar{D} + AB\bar{C}D + ABCD + A\bar{B}CD$
$+\bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}\,\bar{B}CD + \bar{A}\bar{B}C\bar{D}$

(B) $AB + ACD + \bar{A}C$

(C) $AB\bar{C} + ABC + ABCD + \bar{A}CB + \bar{A}C\bar{B}\,\bar{D}$

(D) $\bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + ABCD + \bar{A}BC + AB\bar{D} + ABC$

11. Factorize $\bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + \bar{A}\,\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$

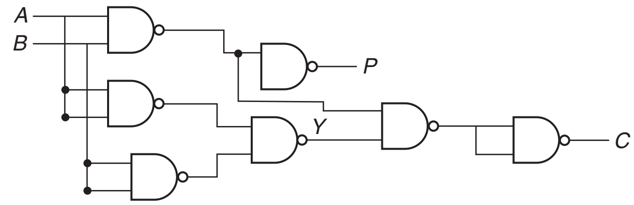
(A) $B + C$
(B) $AB + CD$
(C) $\bar{B}\bar{C}$
(D) AC

12. The K-map of a function is as shown. Find the function.

wx \ yz	00	01	11	10
	1			1
	1	1	1	1
	1			1
	1			1

(A) wx
(B) $\bar{z}$
(C) $\overline{w}(z + \bar{z}) + \bar{z}w$
(D) $\overline{wx} + \bar{z}$

13. The Boolean expression for P is



(A) AB
(B) $\overline{AB}$
(C) $\bar{A} + \bar{B}$
(D) $A + B$

14. The Boolean expression for the truth table is

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(A) $B(A+C)(\bar{A}+\bar{C})$
(B) $\bar{B}(A+\bar{C})(\bar{A}+\bar{C})$
(C) $B(A+\bar{C})(\bar{A}+C)$
(D) $\bar{B}(A+C)(\bar{A}+\bar{C})$

15. Simplify (d represents don't-care)

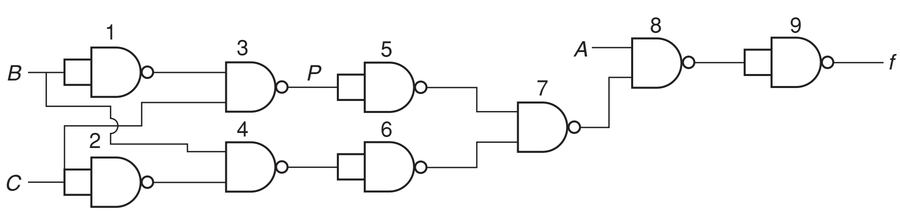
C \ AB	00	01	11	10
0	1	d		1
1	1	d		1

(A) $\bar{B}$
(B) $\bar{B} + C$
(C) $\bar{B} + \bar{A}$
(D) $A + \bar{C}$

16. Simplify $\overline{(AB + \bar{C})(\overline{A + B} + C)}$

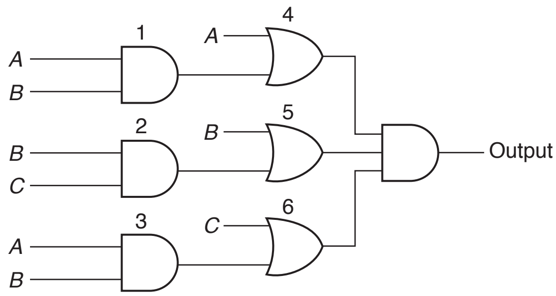
(A) $(\bar{A} + \bar{B} + \bar{C}) \cdot (A + B + C)$
(B) $(\bar{A} + B + C) \cdot (A + \bar{B} + \bar{C})$
(C) $(\bar{A} + \bar{B}) \cdot (A + B + C)$
(D) None of these

17. The point P in the figure is stuck at 1. The output f will be



(A) $\overline{ABC}$
(B) $\bar{A}$
(C) $AB\bar{C}$
(D) A

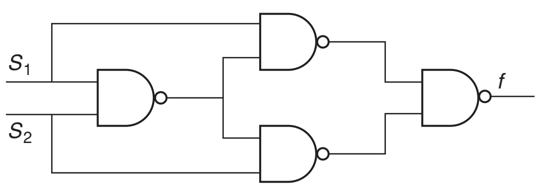
18. Find the function represented by the figure



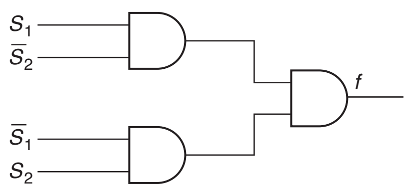
(A) $A + B + C$ (B) AB
(C) $AB + C$ (D) $B + C$

19. A staircase light is controlled by two switches, one is at the top of the stairs and other at the bottom of stairs. Realization of this function using NAND logic results in which of the following circuits? (Assume S_1 and S_2 are the switches)

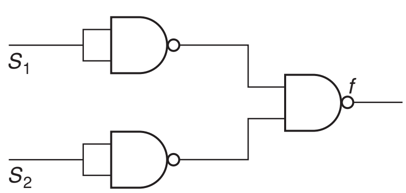
(A)



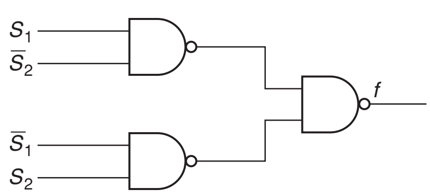
(B)



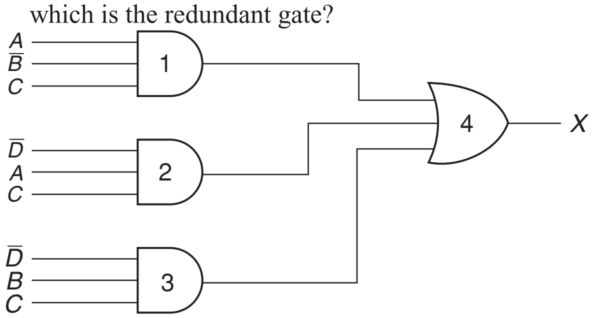
(C)



(D)



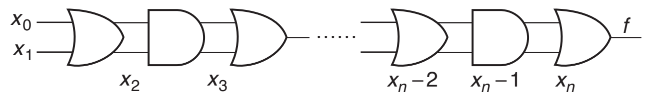
20. For the given figure simplify the expression and find which is the redundant gate?



(A) $ABC + DBC$, 4 (B) $A\bar{B}C + \bar{D}AC$, 3
(C) $\bar{D}AC + \bar{D}BC$, 1 (D) $A\bar{B}C + \bar{D}BC$, 2

21. The function $f = A \oplus B \oplus C \oplus D$ is represented as
(A) $f(A, B, C, D) = \Sigma(2, 6, 10, 11, 12, 13, 14)$
(B) $f(A, B, C, D) = \Sigma(3, 5, 7, 10, 11, 12, 13, 14)$
(C) $f(A, B, C, D) = \Sigma(1, 2, 6, 8, 10, 12, 13, 14)$
(D) $f(A, B, C, D) = \Sigma(1, 2, 4, 7, 8, 11, 13, 14)$

22. Find the function represented

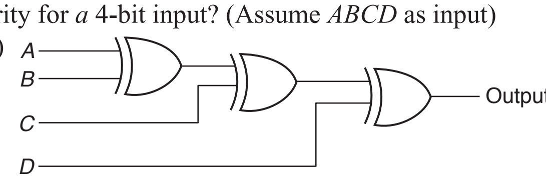


(A) $(x_0 + x_1)(x_2 + x_3)(x_4 + x_5) \ldots (x_{n-1} + x_n)$
(B) $x_0 + x_1 + x_2 + x_3 + \cdots + x_n$
(C) $x_0x_2x_4 \ldots x_n + x_1x_2 \ldots x_n + x_{n-1}x_n$
(D) $x_0x_1 + x_2x_3 + \cdots + x_{n-1}x_n$

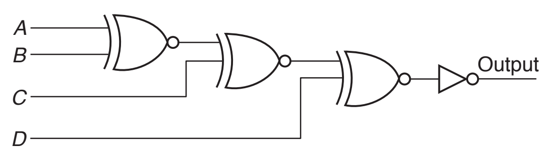
23. The minimum number of NAND gates required to implement $A \oplus B \oplus C$ is
(A) 8 (B) 10
(C) 9 (D) 6

24. Which of the following circuit will generate an odd parity for *a* 4-bit input? (Assume *ABCD* as input)

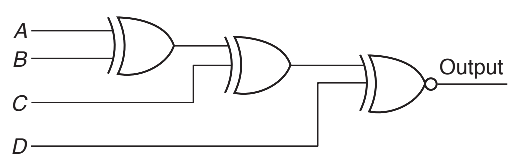
(A)



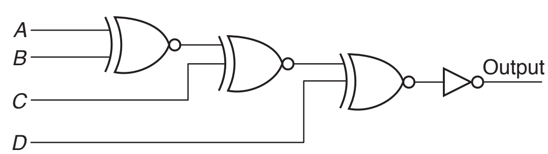
(B)



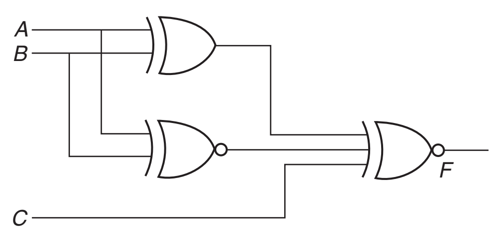
(C)



(D)



25. For the output F to be 1 in the circuit, the input combination should be



(A) $A = 1, B = 1, C = 0$ (B) $A = 1, B = 0, C = 0$
(C) $A = 0, B = 1, C = 0$ (D) $A = 0, B = 0, C = 1$

Practice Problems 2

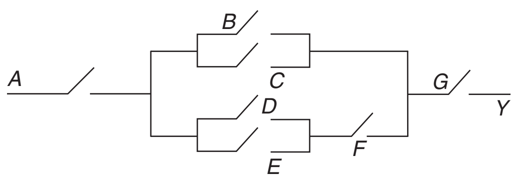
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. An OR Gate has six inputs. How many input words are there in its truth table?
(A) 6 (B) 36
(C) 32 (D) 64

2. Sum of product form can be implemented by using
(A) AND – OR
(B) NAND – NAND
(C) NOR – NOR
(D) Both A and B

3. Which one of the following is equivalent to the Boolean expression?
$Y = AB + BC + CA$
(A) $\overline{AB + BC + CA}$
(B) $(\bar{A} + \bar{B})(\bar{B} + \bar{C})(\bar{A} + \bar{C})$
(C) $\overline{(A + B)(B + C)(A + C)}$
(D) $\overline{(\bar{A} + \bar{B})(\bar{B} + \bar{C})(\bar{C} + \bar{A})}$

4. What Boolean function does the following circuit represents?



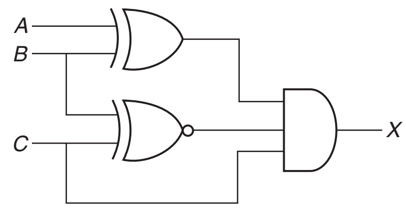
(A) $A\,[F + (B + C)\cdot(D + E)]\,G$
(B) $A + BC + DEF + G$
(C) $A\,[(B + C) + F\,(D + E)]\,G$
(D) $ABG + ABC + F\,(D + E)$

5. The minimum number of two input NOR gates are required to implement the simplified value of the following equation
$f(w, x, y, z) = \Sigma m(0, 1, 2, 3, 8, 9, 10, 11)$
(A) One (B) Two
(C) Three (D) Four

6. The out put of a logic gate is '1' when all inputs are at logic '0'. Then the gate is either
(1) NAND or X-OR gate
(2) NOR or X-OR gate
(3) NOR or X-NOR gate
(4) NAND or X-NOR gate
(A) 1 and 2 (B) 2 and 3
(C) 3 and 4 (D) 4 and 1

7. If the functions w, x, y, and z are as follows.
$w = R + \bar{P}Q + \bar{R}S$
$x = PQ\bar{R}\bar{S} + \bar{P}\bar{Q}\bar{R}\bar{S} + P\bar{Q}\bar{R}\bar{S}$
$y = RS + \overline{PR + P\bar{Q} + \bar{P}\cdot\bar{Q}}$
$z = R + S + PQ + \bar{P}\cdot\bar{Q}\cdot\overline{R + P\cdot\bar{Q}\cdot\bar{S}}$
(A) $w = z,\ x = y$ (B) $w = z,\ x = \bar{z}$
(C) $w = y$ (D) $w = y = z$

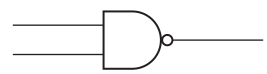
8. For the logic circuit shown in the figure, the required in put condition (A, B, C) to make the output (x) = 1 is

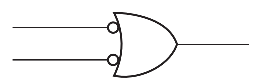


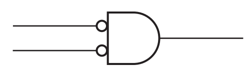
(A) 0, 0, 1 (B) 1, 0, 1
(C) 1, 1, 1 (D) 0, 1, 1

9. Which of the following is a basic gate?
(A) AND (B) X-OR
(C) X-NOR (D) NAND

10. Which of the following represent the NAND gate?

(a) 

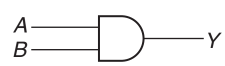
(b) 

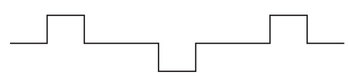
(c) 

(A) a only (B) a, b, c
(C) b, a (D) a, c

11. The universal gates are
(A) NAND and NOR (B) AND, OR, NOT
(C) X-OR and X-NOR (D) All of these

12. In the circuit the value of input A goes from 0 to 1 and part of B goes from 1 to 0. Which of the following represent output under a static hazard condition?



(a) 

(b) 

(c) 

(d) 

(A) Output a (B) Output b
(C) Output c (D) Output d

13. The consensus theorem states that
(A) $A + \bar{A}B = A + B$
(B) $A + AB = A$
(C) $AB + \bar{A}C + BC = AB + \bar{A}C$
(D) $(A + B)\cdot(A + \bar{B}) = A$

14. The dual form of expression
$AB + \bar{A}C + BC = AB + \bar{A}C$ is

(A) $(A+B)(\bar{A}+C)(B+C)=(A+B)(\bar{A}+C)$

(B) $(A+B)(\bar{A}+C)(B+C)=(\bar{A}+\bar{B})(A+\bar{C})$

(C) $(\bar{A}+\bar{B})(\bar{A}+\bar{C})(\bar{B}+\bar{C})=(\bar{A}+\bar{B})(A+\bar{C})$

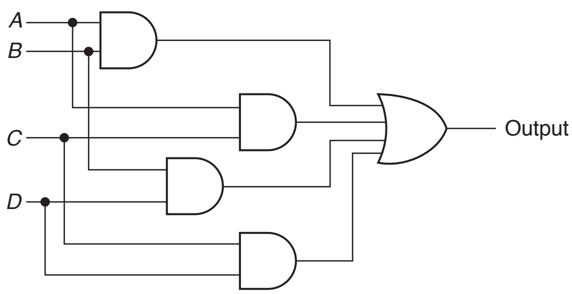
(D) $\bar{A}\bar{B}+A\bar{C}+\bar{B}\bar{C}=\bar{A}\bar{B}+A\bar{C}$

15. The max term corresponding to decimal 12 is

(A) $\bar{A}+\bar{B}+C+D$ (B) $A+B+\bar{C}+\bar{D}$

(C) $\bar{A}\bar{B}CD$ (D) $ABC\bar{D}$

16. The given circuit is equivalent to



(A) $(A+C)(B+D)$ (B) $AC+BD$

(C) $(A+D)(B+C)$ (D) $(\bar{A}+\bar{B})(\bar{C}+\bar{D})$

17. Minimized expression for Karnaugh map is

C \ AB	00	01	11	10
0	1			1
1	1			1

(A) $AB+C$ (B) $\bar{A}B+C$

(C) $\bar{B}$ (D) $\bar{B}+C$

18. An XOR gate will act as ________ when one of its input is one and as ________ when one of its input is zero.

(A) buffer, buffer (B) buffer, inverter

(C) inverter, buffer (D) inverter, inverter

19. The minimum number of two input NAND gates required to implement $A \odot B$ if only A and B are available

(A) 6 (B) 3

(C) 5 (D) 4

20. Negative logic in a logic circuit is one in which

(A) logic 0 and 1 are represented by GND and positive voltage respectively.

(B) logic 0 and 1 are represented by negative and positive voltage.

(C) logic 0 voltage level is lower than logic 1 voltage level.

(D) logic 0 voltage level is higher than logic 1 voltage level.

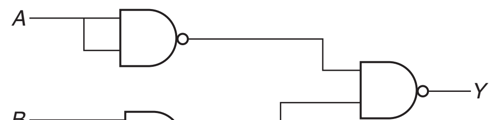
21. If the input to a gate is eight in number, then its truth table contains ________ input words.

(A) 128 (B) 8

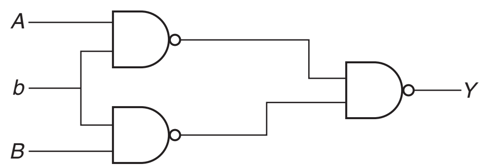
(C) 64 (D) 256

22. The X-OR gate implementation using NAND gate is

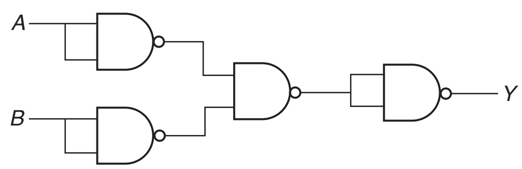
(A)



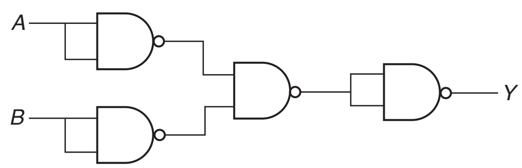
(B)



(C)



(D)

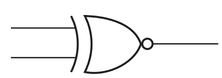
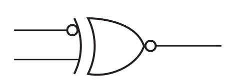


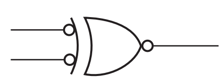
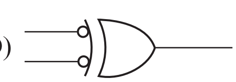
23. The equivalent of AND–OR logic circuit is

(A) NAND–NOR (B) NOR–AND

(C) NAND–NAND (D) NAND–OR

24. The X-OR is equivalent to

(A)  (B) 

(C)  (D) 

25. Simplify $A\bar{B}C+B+B\bar{D}+AB\bar{D}+\bar{A}C$

(A) B (B) $B+C$

(C) $C+A$ (D) $\bar{A}+B$

Previous Years' Questions

1. Let $f(w,x,y,z)=\Sigma(0,\ 4,\ 5,\ 7,\ 8,9,\ 13,\ 15)$. Which of the following expressions are NOT equivalent to f? **[2007]**

(P) $x'y'z' + w'xy' + wy'z + xz$
(Q) $w'y'z' + wx'y' + xz$
(R) $w'y'z' + wx'y' + xyz + xy'z$
(S) $x'y'z' + wx'y' + w'y$

(A) P only (B) Q and S
(C) R and S (D) S only

2. Define the connective $*$ for the Boolean variables X and Y as: $X * Y = XY + X'Y'$. Let $Z = X * Y$. Consider the following expressions P, Q and R. **[2007]**

$P: X = Y * Z$ $Q: Y = X * Z$
$R: X * Y * Z = 1$

Which of the following is TRUE?

(A) Only P and Q are valid.
(B) Only Q and R are valid.
(C) Only P and R are valid.
(D) All P, Q, R are valid.

3. In the Karnaugh map shown below, × denotes a don't-care term. What is the minimal form of the function represented by the Karnaugh map? **[2008]**

cd \ ab	00	01	11	10
00	1	1		1
01	×	1		
11	×			
10	1	1		×

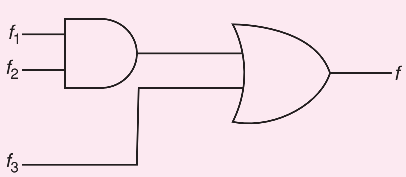
(A) $\bar{b}\cdot\bar{d}+\bar{a}\cdot\bar{d}$

(B) $\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{d}+\bar{a}\cdot b\cdot\bar{d}$

(C) $\bar{b}\cdot\bar{d}+\bar{a}\cdot b\cdot\bar{d}$

(D) $\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{d}+\bar{a}\cdot\bar{d}$

4. Given f_1, f_3 and f in canonical sum of products form (in decimal) for the circuit **[2008]**



$f_1 = \Sigma m(4, 5, 6, 7, 8)$
$f_3 = \Sigma m(1, 6, 15)$
$f = \Sigma m(1, 6, 8, 15)$

then f_2 is

(A) $\Sigma m(4, 6)$ (B) $\Sigma m(4, 8)$
(C) $\Sigma m(6, 8)$ (D) $\Sigma m(4, 6, 8)$

5. If P, Q, R are Boolean variables, then $(P+\bar{Q})(P\cdot\bar{Q}+P\cdot R)(\bar{P}\cdot\bar{R}+\bar{Q})$ simplifies to **[2008]**

(A) $P\cdot\bar{Q}$ (B) $P\cdot\bar{R}$
(C) $P\cdot\bar{Q}+R$ (D) $P\cdot\bar{R}+Q$

6. What is the minimum number of gates required to implement the Boolean function ($AB + C$), if we have to use only two-input NOR gates? **[2009]**

(A) 2 (B) 3
(C) 4 (D) 5

7. The binary operation □ is defined as follows **[2009]**

P	Q	P□Q
T	T	T
T	F	T
F	T	F
F	F	T

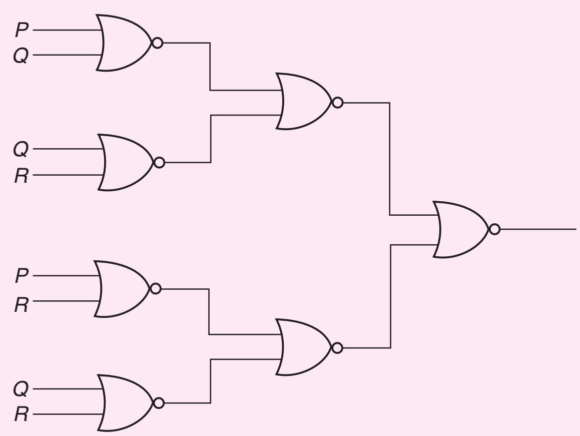
Which one of the following is equivalent to P□Q?

(A) ¬Q ¬P (B) P□¬Q
(C) ¬P□Q (D) ¬P□¬Q

8. The min term expansion of $f(P, Q, Q) = PQ + Q\bar{R} + P\bar{R}$ is **[2010]**

(A) $m_2 + m_4 + m_6 + m_7$
(B) $m_0 + m_1 + m_3 + m_5$
(C) $m_0 + m_1 + m_6 + m_7$
(D) $m_2 + m_3 + m_4 + m_5$

9. What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below? **[2010]**



(A) $\overline{Q+R}$ (B) $\overline{P+Q}$

(C) $\overline{P+R}$ (D) $\overline{P+Q+R}$

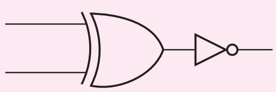
10. The simplified SOP (Sum of Product) form of the Boolean expression. **[2011]**

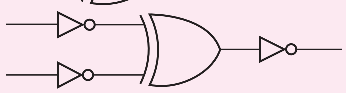
$(P+\overline{Q}+\overline{R})\,(P+\overline{Q}+R)\,(P+Q+\overline{R})$ is

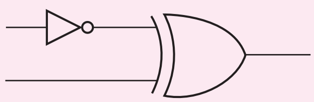
(A) $(PQ+\overline{R})$ (B) $(P+\overline{Q}\,\overline{R})$

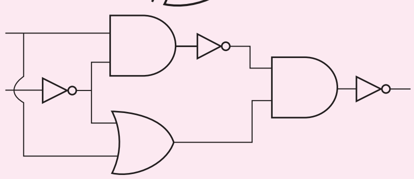
(C) $(\overline{P}Q+R)$ (D) $(PQ+R)$

11. Which one of the following circuits is NOT equivalent to a two-input X-NOR (exclusive NOR) gate? **[2011]**

(A) 

(B) 

(C) 

(D) 

12. The truth table **[2012]**

X	*Y*	*F(X, Y)*
0	0	0
0	1	0
1	0	1
1	1	1

represents the Boolean function

(A) X (B) $X+Y$

(C) $X\oplus Y$ (D) Y

13. What is the minimal form of the Karnaugh map shown below? Assume that × denotes a don't-care term. **[2012]**

cd \ ab	00	01	11	10
00	1	×	×	1
01	×			1
11				
10	1			×

(A) $\overline{b}\,\overline{d}$ (B) $\overline{b}\,\overline{d}+\overline{b}\,\overline{c}$

(C) $\overline{b}\,\overline{d}+a\overline{b}\,\overline{c}d$ (D) $\overline{b}\,\overline{d}+\overline{b}\,\overline{c}+\overline{c}\,\overline{d}$

14. Which one of the following expressions does not represent exclusive NOR of x and y? **[2013]**

(A) $xy+x'y'$ (B) $x\oplus y'$

(C) $x'\oplus y$ (D) $x'\oplus y'$

15. Consider the following Boolean expression for F:

$F(P,Q,R,S)=PQ+\overline{P}QR+\overline{P}Q\overline{R}S$

The minimal sum of products form of F is **[2014]**

(A) $PQ+QR+QS$ (B) $P+Q+R+S$

(C) $\overline{P}+\overline{Q}+\overline{R}+\overline{S}$ (D) $\overline{P}R+\overline{P}\,\overline{R}S+P$

16. The dual of a Boolean function $f(x_1, x_2, \ldots x_n, +, ., ')$, written as F^D, is the same expression as that of F with + and . swapped. F is said to be self-dual if $F = F^D$. The number of self-dual functions with n Boolean variables is **[2014]**

(A) 2^n (B) 2^{n-1}

(C) 2^{2^n} (D) $2^{2^{n-1}}$

17. Consider the following min term expression for F:

$F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 8, 10, 13, 15)$

The min terms 2, 7, 8 and 13 are 'don't-care terms. The minimal sum of products form for F is **[2014]**

(A) $Q\overline{S}+\overline{Q}S$

(B) $\overline{Q}\,\overline{S}+QS$

(C) $\overline{Q}\,\overline{R}\,\overline{S}+\overline{Q}R\overline{S}+Q\overline{R}S+QRS$

(D) $\overline{P}\,\overline{Q}\,\overline{S}+\overline{P}QS+PQS+P\overline{Q}\,\overline{S}$

18. The binary operator ≠ is defined by the following truth table

p	*q*	*p* ≠ *q*
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator ≠? **[2015]**

(A) Both commutative and associative
(B) Commutative but not associative
(C) Not commutative but associative
(D) Neither commutative nor associative

19. Consider the operations **[2015]**

$f(X, Y, Z) = X^1YZ + XY^1 + Y^1Z^1$ and

$g(X, Y, Z) = X^1YZ + X^1YZ^1 + XY$.

Which one of the following is correct?

(A) Both {f} and {g} are functionally complete
(B) Only {f} is functionally complete
(C) Only {g} is functionally complete
(D) Neither {f} nor {g} is functionally complete

20. The number of min-terms after minimizing the following Boolean expression is _______ **[2015]**

$[D^1 + AB^1 + A^1C + AC^1D + A^1C^1D]^1$

21. Let # be a binary operator defined as **[2015]**

$X \# Y = X^1 + Y^1$ where X and Y are Boolean variables.

Consider the following two statements.

(S1) $(P \# Q) \# R = P \# (Q \# R)$

(S2) $Q \# R = R \# Q$

Which of the following is/are true for the Boolean variables P, Q and R?

(A) Only S_1 is true
(B) Only S_2 is true

(C) Both S_1 and S_2 are true
(D) Neither S_1 nor S_2 are true

22. Given the function $F = P^1 + QR$, where F is a function in three Boolean variables P, Q and R and $P^1 = !P$, consider the following statements. **[2015]**

(S_1) $F = \Sigma(4, 5, 6)$

(S_2) $F = \Sigma(0, 1, 2, 3, 7)$

(S_3) $F = \pi(4, 5, 6)$

(S_4) $F = \pi(0, 1, 2, 3, 7)$

Which of the following is true?
(A) (S_1) – False, (S_2) – True, (S_3) – True, (S_4) – False
(B) (S_1) – True, (S_2) – False, (S_3) – False, (S_4) – True
(C) (S_1) – False, (S_2) – False, (S_3) – True, (S_4) – True
(D) (S_1) – True, (S_2) – True, (S_3) – False, (S_4) – False

23. The total number of prime implicants of the function $f(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 10)$ is ______ **[2015]**

24. Consider the Boolean operator # with the following properties: **[2016]**

$x \# 0 = x, x \# 1 = \bar{x}, x \# x = 0$ and

$x \# \bar{x} = 1$. Then $x \# y$ is equivalent to

(A) $x\bar{y} + \bar{x}y$
(B) $x\bar{y} + \bar{x}\bar{y}$
(C) $\bar{x}y + xy$
(D) $xy + \bar{x}\bar{y}$

25. Consider the Karnaugh map given below, where X represents *"don't care"* and blank represents 0.

dc \ ba	00	01	11	10
00		X	X	
01	1			X
11	1			1
10		X	X	

Assume for all inputs (a, b, c, d), the respective complements $(\bar{a}, \bar{b}, \bar{c}, \bar{d})$ are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is ________. **[2017]**

26. If w, x, y, z are Boolean variables, then which one of the following is INCORRECT? **[2017]**
(A) $wx + w(x + y) + x(x + y) = x + wy$
(B) $\overline{w\bar{x}(y + \bar{z})} + \bar{w}x = \bar{w} + x + \bar{y}z$
(C) $\left(w\bar{x}\left(y + x\bar{z}\right) + \bar{w}\bar{x}\right)y = x\bar{y}$
(D) $(w + y)(wxy + wyz) = wxy + wyz$

27. Given $f(w, x, y, z) = \sum m(0,1,2,3,7,8,10) + \sum d(5,6,11,15)$, where d represents the *don't-care* condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of $f(w, x, y, z)$? **[2017]**

(A) $f = (\bar{w} + \bar{z})(\bar{x} + z)$
(B) $f = (\bar{w} + z)(x + z)$
(C) $f = (w + z)(\bar{x} + z)$
(D) $f = (w + \bar{z})(\bar{x} + z)$

28. Let ⊕ and ⊙ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT? **[2018]**
(A) $\overline{P \oplus Q} = P \odot Q$
(B) $\bar{P} \oplus Q = P \odot Q$
(C) $\bar{P} \oplus \bar{Q} = P \oplus Q$
(D) $(P \oplus \bar{P}) \oplus Q = (P \odot \bar{P}) \odot \bar{Q}$

29. Consider the minterm list form of a Boolean function F given below.

$$F(P, Q, R, S) = \sum m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is ______. **[2018]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** D	**3.** B	**4.** C	**5.** A	**6.** A	**7.** B	**8.** A	**9.** B	**10.** A
11. C	**12.** D	**13.** A	**14.** A	**15.** A	**16.** A	**17.** D	**18.** B	**19.** A	**20.** D
21. D	**22.** C	**23.** A	**24.** C	**25.** D					

Practice Problems 2

1. D	**2.** D	**3.** D	**4.** C	**5.** A	**6.** C	**7.** B	**8.** D	**9.** A	**10.** C
11. A	**12.** D	**13.** A	**14.** A	**15.** A	**16.** C	**17.** C	**18.** C	**19.** C	**20.** D
21. D	**22.** C	**23.** C	**24.** B	**25.** B					

Previous Years' Questions

1. D	**2.** D	**3.** A	**4.** C	**5.** A	**6.** B	**7.** B	**8.** A	**9.** A	**10.** B
11. D	**12.** A	**13.** B	**14.** D	**15.** A	**16.** D	**17.** B	**18.** A	**19.** B	**20.** 1
21. B	**22.** A	**23.** 3	**24.** A	**25.** 1	**26.** C	**27.** A	**28.** D	**29.** 3	