CHAPTER 17

Circles (Chords & Tangents)

You may have come across many objects in daily life, which are round in shape, such as wheels of a vehicle, bangles, dials of many clocks, coins of denominations 50 paise, Re 1 and Rs 5, key rings buttons of shirts, etc. In a clock, you might have observed that the second's hand goes round the dial of the clock rapidly and its tip moves in a round path. This path traced by the tip of the second's hand is called a circle.

Take a compass and fix a pencil in it. Put its pointed leg on a point on a sheet of a paper. Open the other leg to some distance. Keeping the pointed leg on the same point, rotate the other leg through one revolution. As you know, it is a circle.

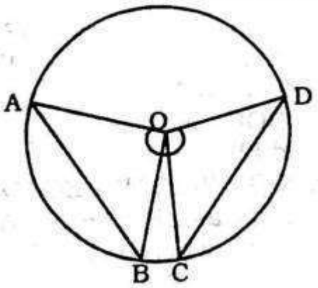
The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the **centre** of the circle and the fixed distance is called the **radius** of the circle.

Theorem 1. Equal chords of a circle subtend equal angles at the centre.

Proof : You are given two equal chords AB and CD of a circle with centre O. You want to prove that $\angle AOB = \angle COD$. In triangles AOB and COD,

OA = OC (Radii of a circle)
OB = OD (Radii of a circle)



AB = CD (Given)
Therefore, $\Delta AOB \cong \Delta COD$ (SSS rule)
This gives $\Delta AOB = \Delta COD$
(Corresponding parts of congruent triangles)

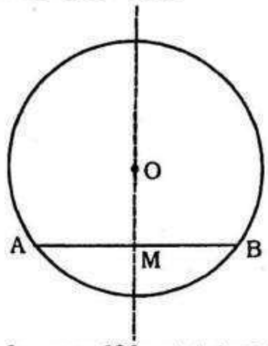
Remark : For convenience, the abbreviation CPCT will be used in place of 'Corresponding parts of congruent triangles.

Theorem 2. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

The above theorem is the converse of the Theorem 1.

Perpendicular from the Centre to a Chord

Draw a circle on a tracing paper. Let O be its centre. Draw a chord AB. Fold the paper along a line through O so that a portion of the chord falls on the other. Let the crease cut AB at the point M. Then, $\angle OMA = \angle OMB = 90°$ or OM is perpendicular to AB. The point B coincides with A. So MA = MB.

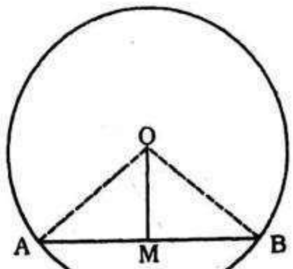


Give a proof yourself by joining OA and OB and proving the right triangles OMA and OMB to be congruent. This example is a particular instance of the following result:

Theorem 3. The perpendicular from the centre of a circle to a chord bisects the chord.

Given that the perpendicular from the centre of a circle to a chord is drawn and to prove that it bisects the chord. Thus in the converse, what the hypothesis is 'if a line from the centre bisects a chord of a circle' and what is to be proved is 'the line is perpendicular to the chord'. So the converse is:

Theorem 4. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Let AB be a chord of a circle with centre O and O is joined to the mid-point M of AB. You have to prove that $OM \perp AB$. Join OA and OB. In triangles OAM and OBM,

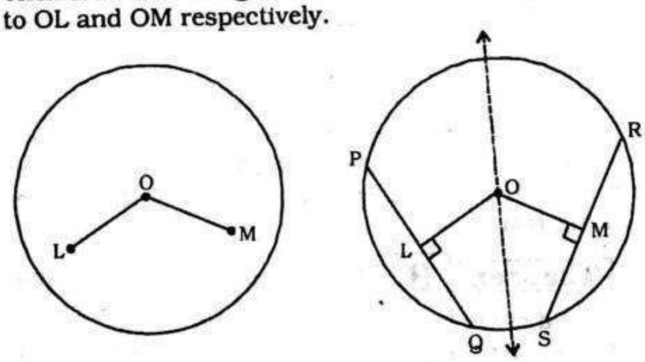
OA = OB
AM = BM
OM = OM (Common)
Therefore, $\Delta OAM \cong \Delta OBM$
This gives $\angle OMA = \angle OMB = 90°$

Theorem 5. There is one and only one circle passing through three given non-collinear points.

Remark : If ABC is a triangle, then by Theorem 5, there is a unique circle passing through the three vertices A, B and C of the triangle. This circle is called the **circumcircle** of the ΔABC. Its centre and radius are called respectively the circumcentre and the circumradius of the triangle.

Theorem 6. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).

Next, it will be seen whether the converse of this theorem is true or not. For this, draw a circle with centre O. From the centre O, draw two line segments OL and OM of equal length and lying inside the circle. Then draw chords PQ and RS of the circle perpendicular to OL and OM respectively.



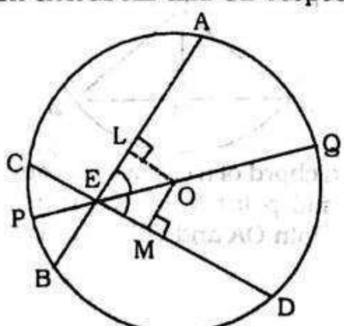
(i) (ii)

Measure the lengths of PQ and RS. Both are equal.

Theorem 7. Chords equidistant from the centre of a circle are equal in length.

Example 1 : If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Solution : Given that AB and CD are two chords of a circle, with centre O intersecting at a point E. PQ is a diameter through E, such that ∠AEQ = ∠DEQ. You have to prove that AB = CD. Draw perpendiculars OL and OM on chords AB and CD respectively. Now



∠LOE = 180° – 90° – ∠LOE = 90° – ∠LEO

(Angle sum property of a triangle)

= 90° – ∠AEQ = 90° – ∠DEQ

= 90° – ∠MEO = ∠MOE

In triangles OLE and OME,

∠LEO = ∠MEO (Proved above)

∠LOE = ∠MOE (Common)

EO = EO

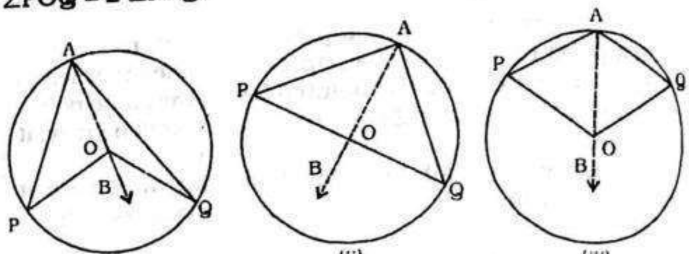
Therefore, ΔOLE ≡ ΔOME (CPCT)

This gives OL = OM

So, AB = CD

Theorem 8. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Proof : Given an arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle. We need to prove that ∠POQ = 2 ∠PAQ.



(i) (ii) (iii)

Consider the three different cases as given in the above figures. In (i), arc PQ is minor; in (ii), arc PQ is a semicircle and in (iii), arc PQ is major.

Let us begin by joining AO and extending it to a point B. In all the cases,

∠BOQ = ∠OAQ + ∠AQO

because an exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Also in ΔOAQ,

OA = OQ (Radii of a circle)

Therefore, ∠OAQ = ∠OQA (From Theorem)

This gives ∠BOQ = 2∠OAQ (1)

Similarly, ∠BOP = 2∠OAP (2)

From (1) and (2),

∠BOP + ∠BOQ = 2(∠OAP + ∠OAQ)

This is the same as ∠POQ = 2∠PAQ

For the case (iii), where PQ is the major arc, (3) is replaced by reflex angle POQ = 2∠PAQ

Remark : Suppose we join points P and Q and form a chord PQ in the above figures. Then ∠PAQ is also called the angle formed in the segment PAQP.

Theorem 9. Angles in the same segment of a circle are equal.

Again let us discuss the case (ii) of Theorem 8 separately. Here ∠PAQ is an angle in the segment, which is a semicircle. Also, $\angle PAQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 180° = 90°$.

If you take any other point C on the semicircle, again you get that

∠PCQ = 90°

Therefore, you find another property of the circle as:

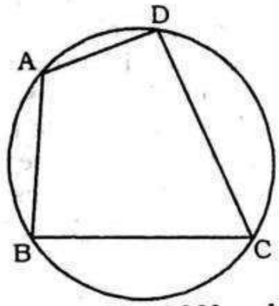
Angle in a semicircle is a right angle.

The converse of Theorem 9 is also true. It can be stated as:

Theorem 10. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

Cyclic Quadrilaterals

A quadrilateral ABCD is called **cyclic** if all the four vertices of it lie on a circle. You will find a peculiar property in such quadrilaterals. Draw several cyclic quadrilaterals of different sides and name each of these as ABCD. (This can be done by drawing several circles of different radii and taking four points on each of them.)



You find that $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$, neglecting the error in measurements. This verifies the following:

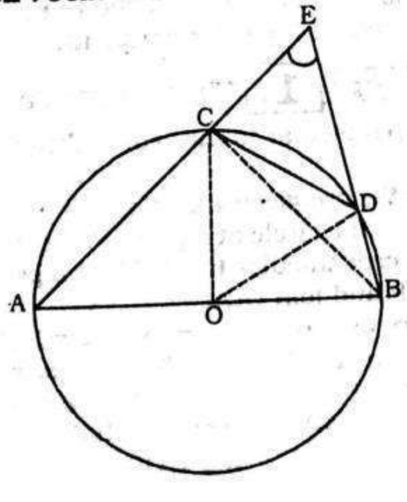
Theorem 11. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

In fact, the converse of this theorem, which is stated below is also true.

Theorem 12. If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

Example 2 : In the following figure, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that $\angle AEB = 60°$.

Solution : Join OC, OD and BC.



Triangle ODC is equilateral

Therefore, $\angle COD = 60°$

Now, $\angle CBD = \frac{1}{2}\angle COD$ (Theorem 8)

This gives $\angle CBD = 30°$

Again, $\angle ACB = 90°$

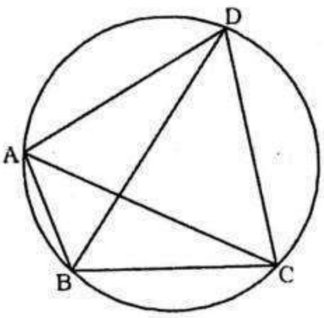
So, $\angle BCE = 180° - \angle ACB = 90°$

Which gives $\angle CEB = 90° - 30° = 60°$,

i.e. $\angle AEB = 60°$

Example 3 : In the following figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.

Solution : $\angle CAD = \angle DBC = 55°$ (Angles in the same segment)



Therefore, $\angle DAB = \angle CAD + \angle BAC$

$= 55° + 45° = 100°$

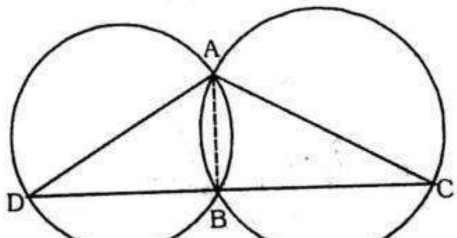
But $\angle DAB + \angle BCD = 180°$

(Opposite angles of a cyclic quadrilateral)

So, $\angle BCD = 180° - 100° = 80°$

Example 4 : Two circles intersect at two points A and B. AD and AC are diameters to the two circles. Prove that B lies on the line segment DC.

Solution : Join AB.



$\angle ABD = 90°$ (Angle in a semicircle)

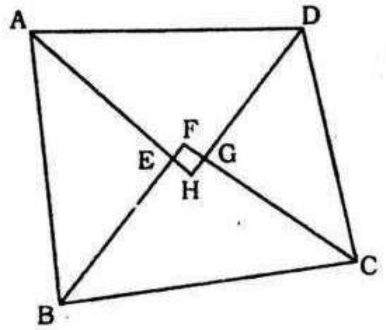
$\angle ABC = 90°$ (Angle in a semicircle)

So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

Therefore, DBC is a line. That is B lies on the line segment DC.

Example 5 : Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

Solution : In the following figure, ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.



Now, $\angle FEH = \angle AEB = 180° - \angle EAB - \angle EBA$

$= 180° - \frac{1}{2}(\angle A + \angle B)$

and $\angle FGH = \angle CGH = 180° - \angle GCD - \angle GDC$

$= 180° - \frac{1}{2}(\angle C + \angle D)$

Therefore, $\angle FEH + \angle FGH$

$= 180° - \frac{1}{2}(\angle A + \angle B) + 180° - \frac{1}{2}(\angle C + \angle D)$

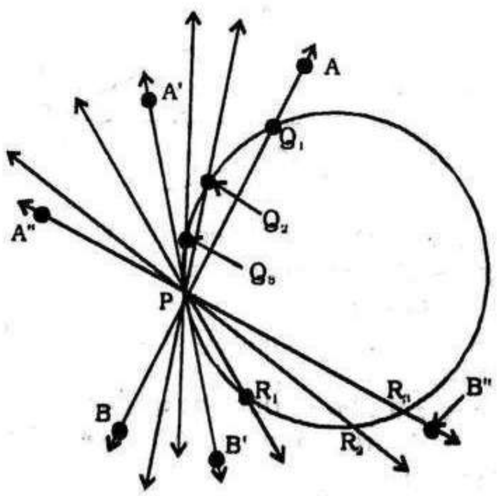
$= 360° - (\angle A + \angle B + \angle C + \angle D) = 360° - \frac{1}{2} \times 360°$

$= 360° - 180° = 180°$

Therefore, by Theorem the quadrilateral EFGH is cyclic.

A tangent to a circle is a line that intersects the circle at only one point.

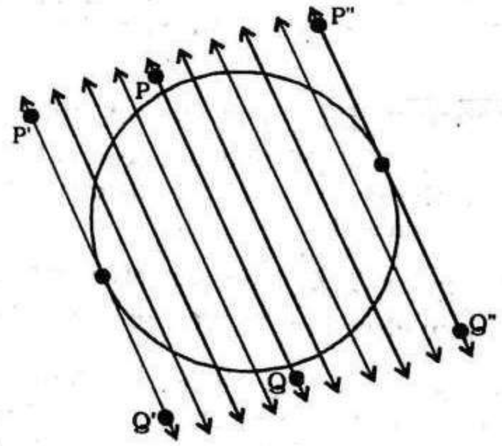
Take a circular wire and attach a straight wire AB at a point P of the circular wire so that it can rotate about the point P in a plane. Put the system on a table and gently rotate the wire AB about the point P to get different positions of the straight wire.



In various positions, the wire intersects the circular wire at P and at another point Q_1 or Q_2 or Q_3, etc. In one position, you will see that it will intersect the circle at the point P only (see position A'B' of AB). This shows that a tangent exists at the point P of the circle. On rotating further, you can observe that in all other positions of AB, it will intersect the circle at P and at another point, say R_1 or R_2 or R_3, etc. So, you can observe that there is only one tangent at a point of the circle.

The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.

On a paper, draw a circle and a secant PQ of the circle. Draw various lines parallel to the secant on both sides of it. You will find that after some steps, the length of the chord cut by the lines will gradually decrease, i.e., the two points of intersection of the line and the circle are coming closer and closer. In one case, it becomes zero on one side of the secant and in another case, it becomes zero on the other side of the secant. See the positions P' Q' and P" Q" of the secant in the figure. These are the tangents to the circle parallel to the given secant PQ. This also helps you to see that there cannot be more than two tangents parallel to a given secant.



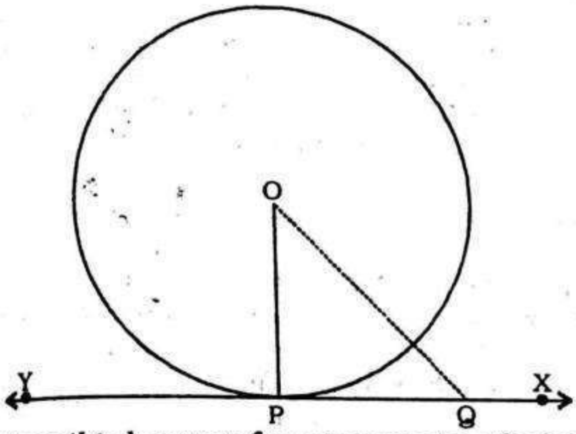
The common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.

Theorem 1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Proof : We are given a circle with centre O and a tangent XY to the circle at a point P. We need to prove that OP is perpendicular to XY. Take a point Q on XY other than P and join OQ.

The point Q must lie outside the circle. (Why? Note that if Q lies inside the circle, XY will become a secant and not a tangent to the circle). Therefore, OQ is longer than the radius OP of the circle. That is,

$$OQ > OP$$



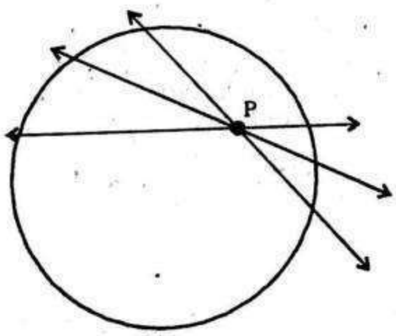
Since this happens for every point on the line XY except the point P, OP is the shortest of all the distances of the point O to the points of XY. So OP is perpendicular to XY.

Remarks :

1. By theorem above, we can also conclude that at any point on a circle there can be one and only one tangent.
2. The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

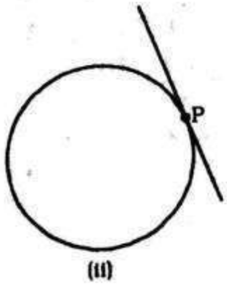
Number of Tangents from a Point on a Circle

Draw a circle on a paper. Take a point P inside it. Can you draw a tangent to the circle through this point? You will find that all the lines through this point intersect the circle in two points. So, it is not possible to draw any tangent to a circle through a point inside it.



(i)

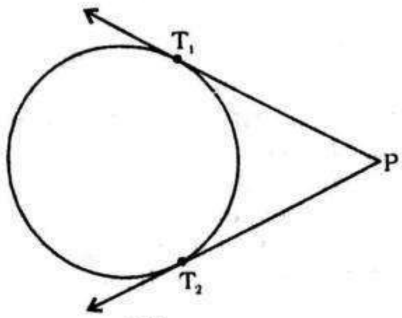
Next take a point P on the circle and draw tangents through this point. You have already observed that there is only one tangent to the circle at such a point.



(ii)

Finally, take a point P outside the circle and try to draw tangents to the circle from this point.

You will find that you can draw exactly two tangents to the circle through this point.



(iii)

We can summarise these facts as follows:

Case 1 : There is no tangent to a circle passing through a point lying inside the circle.

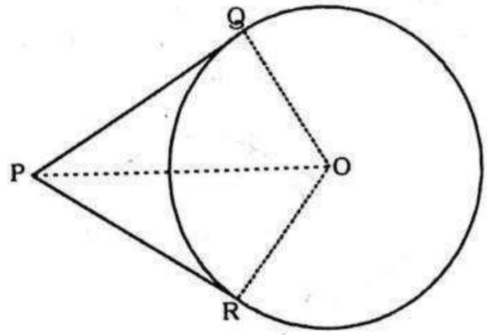
Case 2 : There is one and only one tangent to a circle passing through a point lying on the circle.

Case 3 : There are exactly two tangents to a circle through a point lying outside the circle. In the figure (iii), T_1 and T_2 are the points of contact of the tangents PT_1 and PT_2 respectively.

The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent from the point P to the circle.

Note that in figure (iii), PT_1 and PT_2 are the lengths of the tangents from P to the circle.

Theorem 2. The lengths of tangents drawn from an external point to a circle are equal.



Proof : We are given a circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P. We are required to prove that PQ = PR. For this, we join OP, OQ and OR. Then $\angle OQP$ and $\angle ORP$ are right angles, because these are angles between the radii and tangents, and according to Theorem 1 they are right angles. Now in right triangles OQP and ORP,

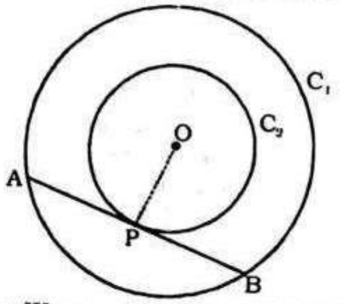
OQ = OR (Radii of the same circle)

OP = OP (Common)
Therefore, $\Delta OQP \cong \Delta ORP$ (RHS)
This gives PQ = PR (CPCT)

Remarks :

1. The theorem can also be proved by using the Pythagoras Theorem as follows:
$PQ^2 = OP^2 - OQ^2 = OP^2 - OR^2 = PR^2$ (As OQ = OR) which gives PQ = PR.
2. Note also that $\angle OPQ = \angle OPR$. Therefore, OP is the angle bisector of $\angle QPR$, i.e., the centre lies on the bisector of the angle between the two tangents. Let us take some examples.

Example 1 : Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.



Solution : We are given two concentric circles C_1 and C_2 with centre O and a chord AB of the larger circle C_1 which touches the smaller circle C_2 at the point P. We need to prove that AP = BP.

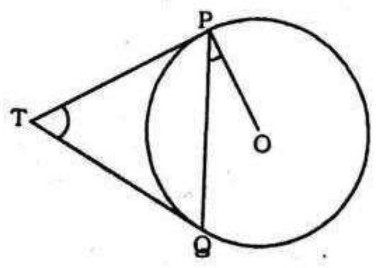
Let us join OP. Then, AB is a tangent to C_2 at P and OP is its radius. Therefore, by Theorem 1, OP AB

Now AB is a chord of the circle C_1 and OP $\perp$AB. Therefore, OP is the bisector of the chord AB, as the perpendicular from the centre bisects the chord,

i.e., AP = BP

Example 2 : Two tangents TP and TQ are drawn to a circle with centre O from an external point T.

Prove that $\angle PTQ = 2\angle OPQ$.



Solution : We are given a circle with centre O, an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact. We need to prove that

$\angle PTQ = 2\angle OPQ$

Let $\angle PTQ = \theta$

Now, by Theorem 2, TP = TQ. So, TPQ is an isosceles triangle.

Therefore, $\angle TPQ = \angle TQP = \frac{1}{2}(180° - \theta) = 90° - \frac{1}{2}\theta$

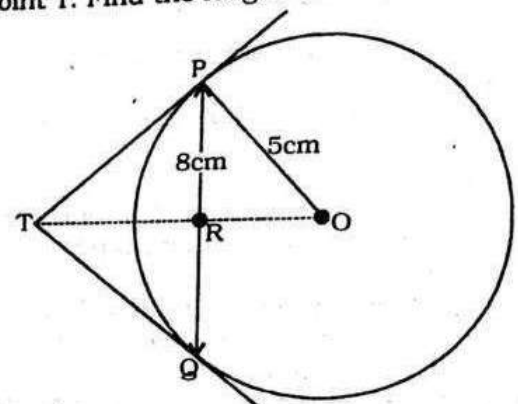
Also, by Theorem 1, $\angle OPT = 90°$

So, $\angle OPQ = \angle OPT - \angle TPQ$

$$= 90° - \left(90° - \frac{1}{2}\theta\right) = \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$$

This gives $\angle PTQ = 2\angle OPQ$

Example 3 : PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.



Solution : Join OT. Let it intersect PQ at the point R. Then ΔTPQ is isosceles and TO is the angle bisector of $\angle PTQ$. So, OT $\perp$ PQ and therefore, OT bisects PQ which gives PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}\text{cm} = 3\text{cm}$

Now, $\angle TPR + \angle RPO = 90°$

$= \angle TPR + \angle PTR$

So, $\angle RPO = \angle PTR$

Therefore, right triangle TRP is similar to the right triangle PRO by AA similarity.

This gives $\frac{TP}{PO} = \frac{RP}{RO}$

i.e. $\frac{TP}{5} = \frac{4}{3}$

or, $TP = \frac{20}{3}$ cm

Note : TP can also be found by using the Pythagoras Theorem, as follows:

Let TP = x and TR = y. Then

$x^2 = y^2 + 16$ (Taking right ΔPRT) (1)

$x^2 + 5^2 = (y + 3)^2$ (Taking right ΔOPT) (2)

Subtracting (1) from (2), we get

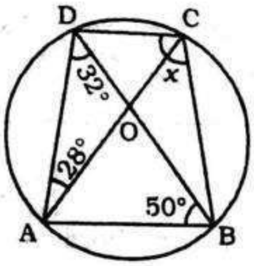
$25 = 6y - 7$ or $y = \frac{32}{6} = \frac{16}{3}$

Therefore, $x^2 = \left(\frac{16}{3}\right)^2 + 16 = \frac{16}{9}(16 + 9) = \frac{16 \times 25}{9}$ [From (1)]

or, $x = \frac{20}{3}$

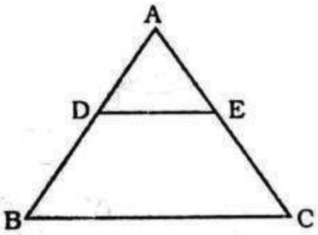
SOLVED OBJECTIVE QUESTIONS

1. If O is the centre of the circle, then 'x' is :



(1) 72° (2) 62°
(3) 82° (4) 52°

2. ABC is an isosceles triangle in which AB = AC. If D and E are the mid-points of AB and AC respectively. The point B, C, D, E are :



(1) collinear (2) non-collinear
(3) concyclic (4) None of these

3. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the outer circle which is outside the inner circle is of length.:
(1) $2\sqrt{2}$ cm (2) $3\sqrt{2}$ cm
(3) $2\sqrt{3}$ cm (4) $4\sqrt{2}$ cm

4. If two circles are such that the centre of one lies on the circumference of the other then the ratio of the common chord of the two circles to the radius of any one of the circles is :
(1) 2 : 1 (2) $\sqrt{3}$: 1
(3) $\sqrt{5}$: 1 (4) 4 : 1

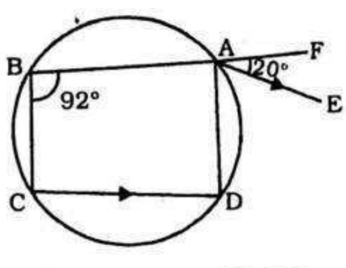
5. If one angle of a cyclic trapezium is triple of the other, then the greater one measures :
(1) 90° (2) 105°
(3) 120° (4) 135°

6. A circle of radius r has been inscribed in a triangle of area Δ. If the semi-perimeter of the triangle be s, then the correct relation is :
(1) $2r = \frac{\Delta}{s}$ (2) $r = \frac{\Delta}{s}$
(3) $r = \frac{s}{\Delta}$ (4) $2s = \Delta r$

7. In a cyclic quadrilateral ABCD, if ∠B − ∠D = 60° then the measure of the smaller of the two is :
(1) 60° (2) 40°
(3) 38° (4) 30°

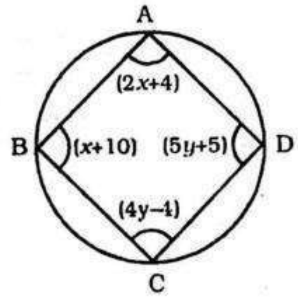
8. The incircle of a Δ ABC touches the sides AB, BC and AC at the points P, Q, R respectively then which of the following statements is/are correct:
I. AP + BQ + CR = PB + QC + RA
II. AP + BQ + CR = $\frac{1}{2}$ (perimeter of Δ ABC)
III. AP + BQ + CR = 3 (AB + BC + CA)
(1) I, II and III (2) only I
(3) II and III only (4) I and II

9. In the given figure, ABCD is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced. If ∠ABC = 92° and ∠FAE = 20°, then ∠BCD is equal to :



(1) 88° (2) 98°
(3) 108° (4) 72°

10. The values of x and y in the figure are measure of angles, then $x + y$ is equal to



(1) 90° (2) 85°
(3) 75° (4) 65°

11. Through any given set of four points P, Q, R, S it is possible to draw:
(1) atmost one circle (2) exactly one circle
(3) exactly two circles (4) exactly three circles

12. The number of common tangents that can be drawn to two given circles is at the most :
(1) one (2) two
(3) three (4) four

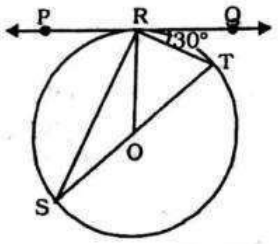
13. Two circles of radii R and r touch each other externally and PQ is the direct common tangent, Then PQ^2 is equal to:
(1) R − r (2) R + r
(3) 2Rr (4) 4Rr

14. ACB is a tangent to a circle at C. CD and CE are chords such that ∠ACE > ∠ACD. If ∠ACD = ∠BCE = 50°, then :
(1) CD = CE
(2) ED is not parallel to AB

(3) ED passes through the centre of the circle

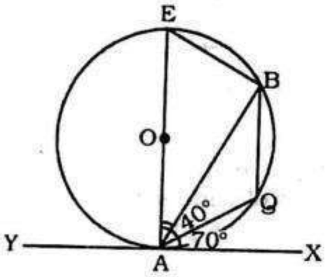
(4) Δ CDE is a right angled triangle

15. PQ is a tangent to the circle at R then m∠PRS is equal to :



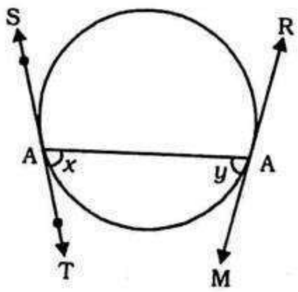
(1) 30° (2) 40°
(3) 60° (4) 80°

16. In the figure XQY is a tangent to the circle with centre O at A. If ∠BAX=70°, ∠BAQ = 40° then ∠ABQ is equal to :



(1) 20° (2) 30°
(3) 35° (4) 40°

17. If AB is the chord at the circle with centre O. SAT and RBM are the tangents at A and B respectively then which of the following is correct?



(1) $x > y$ (2) ∠SAB = y
(3) ∠SAB = y and ∠RBA = x
(4) $x = y$

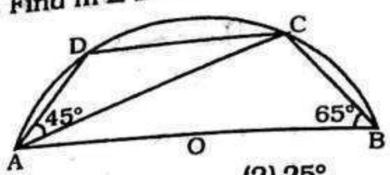
18. Any cyclic parallelogram having unequal adjacent sides is necessarily a :

(1) square (2) rectangle
(3) rhombus (4) trapezium

19. Which of the following statements is incorrect?

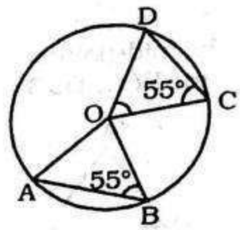
(1) A circle is symmetrical about the diameter
(2) Two circles are divided symmetrically by the line passing through their centres
(3) More than one circle can be drawn through three non-collinear points
(4) None of these

20. In the given figure, AB is diameter of the circle. C and D lie on the semicircle. ∠ABC = 65° and ∠CAD = 45°. Find m ∠ DCA :



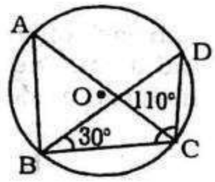
(1) 45° (2) 25°
(3) 20° (4) None of these

21. In the given figure, chords AB and CD are equal. If ∠OBA = 55°
then m∠COD is :



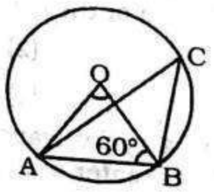
(1) 65° (2) 55°
(3) 70° (4) 50°

22. In the given figure, ∠BAC and ∠BDC are the angles of same segment. ∠DBC = 30° and ∠BCD = 110°. Find m ∠BAC :



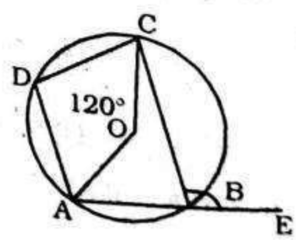
(1) 35° (2) 40°
(3) 55° (4) 60°

23. In the given figure, O is the centre of the circle. ∠ABO = 60°. Find the value of ∠ACB :



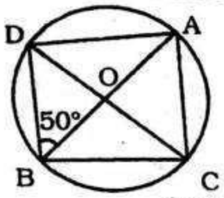
(1) 40° (2) 60°
(3) 50° (4) 30°

24. In the given figure, ∠AOC = 120°. Find m ∠CBE where O is the centre :



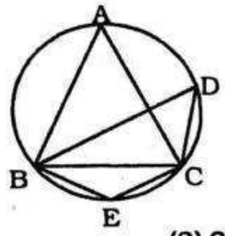
(1) 60° (2) 100°
(3) 120° (4) 150°

25. In the adjoining figure, O is the centre of the circle and ∠ OBD = 50°. Find the m ∠ BAD :



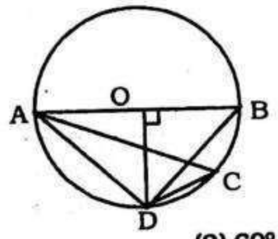
(1) 60° (2) 40°
(3) 80° (4) 45°

26. In the given figure, Δ ABC is an equilateral triangle. Find m ∠ BEC :



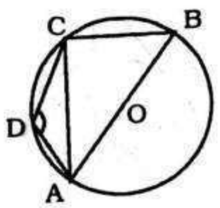
(1) 120° (2) 60°
(3) 80° (4) None of these

27. In the given figure, AB is the diameter of the circle. Find the value of ∠ ACD :



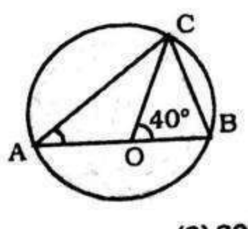
(1) 30° (2) 60°
(3) 45° (4) 25°

28. In the given figure, ABCD is a cyclic quadrilateral and AB is the diameter. ∠ ADC = 140°, then find m ∠ BAC :



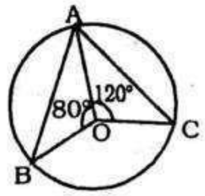
(1) 45° (2) 40°
(3) 50° (4) None of these

29. In the given figure, ∠ COB = 40°, AB is the diameter of the circle. Find m ∠ CAB :



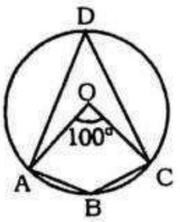
(1) 40° (2) 20°
(3) 30° (4) None of these

30. In the given figure, O is the centre of circle. ∠AOB = 80° and ∠ AOC = 120°. Find m ∠ BAC :



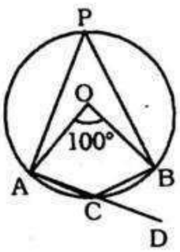
(1) 120° (2) 80°
(3) 100° (4) None of these

31. In the given figure, O is the centre of the circle and ∠ AOC = 100°. Find the ratio of m ∠ ADC : m ∠ ABC :



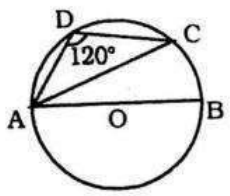
(1) 5 : 6 (2) 1 : 2
(3) 5 : 13 (4) 3 : 13

32. In the given figure, O is the centre of circle, ∠ AOB = 100°. Find m ∠ BCD :



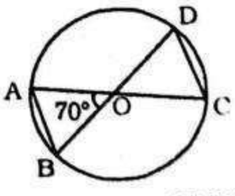
(1) 80° (2) 60°
(3) 50° (4) 40°

33. In the given figure, AB is the diameter of the circle. ∠ ADC = 120°. Find m ∠ CAB :



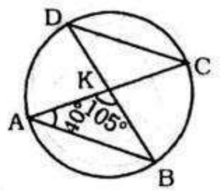
(1) 20° (2) 30°
(3) 40° (4) can't be determined

34. In the given figure, O is the centre of the circle. ∠ AOB = 70°. Find m ∠ OCD :



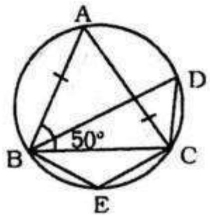
(1) 70° (2) 55°
(3) 65° (4) 110°

35. In the given figure, ∠ CAB = 40° and ∠ AKB = 105°. Find ∠ KCD :



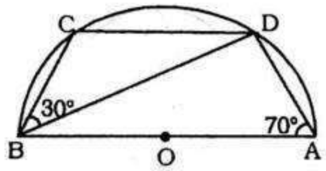
(1) 65° (2) 35°
(3) 40° (4) 72°

36. In the given figure, ABC is an isosceles triangle in which AB = AC and m ∠ABC = 50°, m ∠BDC = ?



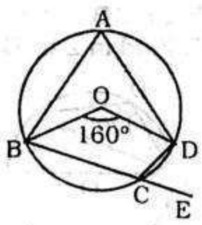
(1) 80° (2) 60°
(3) 65° (4) 100°

37. In the given figure, AB is the diameter. m ∠ BAD = 70° and m ∠ DBC = 30°. Find m ∠ BDC :



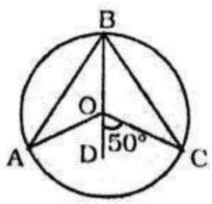
(1) 25° (2) 30°
(3) 40° (4) 60°

38. Find the value of ∠ DCE :



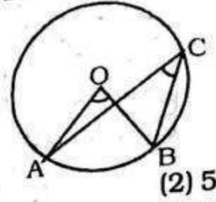
(1) 100° (2) 80°
(3) 90° (4) 75°

39. O is the centre of the circle, line segment BOD is the angle bisector of ∠ AOC, m ∠ COD = 50°. Find m ∠ABC :



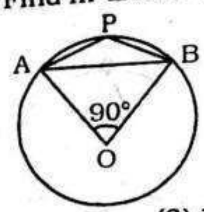
(1) 25° (2) 50°
(3) 100° (4) 120°

40. In the given figure, O is the centre of the circle and ∠ ACB = 25°. Find ∠ AOB :



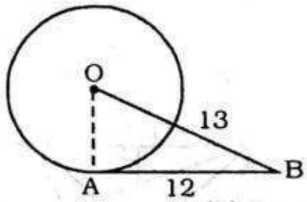
(1) 25° (2) 50°
(3) 75° (4) 60°

41. In the given figure, O is the centre of the circle. ∠AOB = 90°. Find m ∠APB :



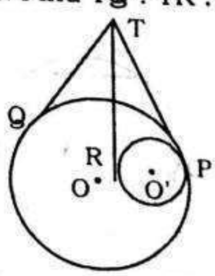
(1) 130° (2) 150°
(3) 135° (4) Can't be determined

42. In the given figure, O is the centre of the circle. AB is tangent. AB = 12 cm and OB = 13 cm. Find OA :



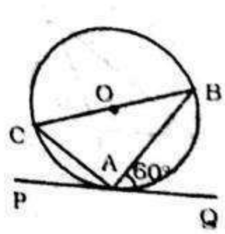
(1) 6.5 cm (2) 6 cm
(3) 5 cm (4) None of these

43. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR :



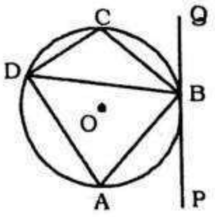
(1) 8 : 7 (2) 7 : 8
(3) 5 : 4 (4) 1 : 1

44. In the given figure, PAQ is the tangent. BC is the diameter of the circle. m ∠ BAQ = 60°, find m ∠ ABC :



(1) 25° (2) 30°
(3) 45° (4) 60°

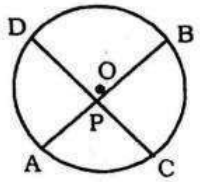
45. ABCD is a cyclic quadrilateral PQ is a tangent at B. If $\angle DBQ = 65^\circ$, then $\angle BCD$ is :



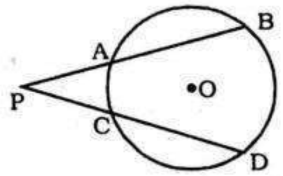
(1) 35° (2) 85°
(3) 115° (4) 90°

46. In the given figure, AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP :

(1) 6 cm (2) 4 cm
(3) 2 cm (4) 3 cm

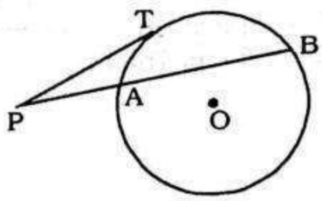


47. In the given figure, AP = 3 cm, BA = 5 cm and CP =2 cm. Find CD :



(1) 12 cm (2) 10 cm
(3) 9 cm (4) 6 cm

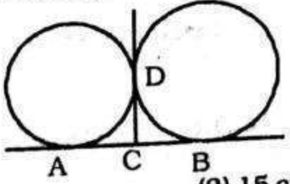
48. In the given figure, tangent PT = 5 cm, PA = 4 cm, find AB :



(1) $\frac{7}{4}$ cm (2) $\frac{11}{4}$ cm

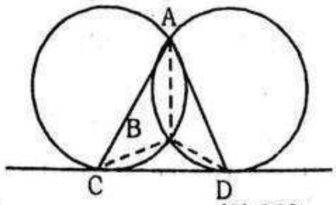
(3) $\frac{9}{4}$ cm (4) can't be determined

49. In the given figure, AB and CD are two common tangents to the two touching circle. If CD = 6 cm, then AB is equal to :



(1) 9 cm (2) 15 cm
(3) 12 cm (4) None of these

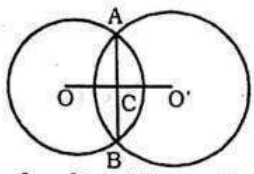
50. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then : $\angle CAD + \angle CBD = ?$



(1) 120° (2) 90°
(3) 360° (4) 180°

51. O and O' are the centres of circle of radii 20 cm and 30 cm. AB = 24 cm. What is the distance of OO' ?

(1) 51 cm (2) 45 cm
(3) 35 cm (4) 48 cm



52. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. the distance between the chords is 23 cm. If the length of one chord is 16 cm, then the lengh of the other is :

(1) 23 cm (2) 30 cm
(3) 15 cm (4) None of these

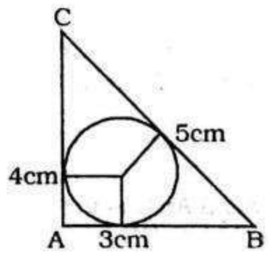
53. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :

(1) $\sqrt{3}:2$ (2) $\sqrt{3}:1$

(3) $\sqrt{5}:1$

(4) None of these

54. ABC is a right angled triangle AB = 3 cm, BC = 5 cm and AC = 4 cm, then the inradius of the circle is :



(1) 1 cm (2) 1.25 cm
(3) 1.5 cm (4) None of these

55. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length :

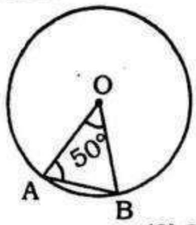
(1) 5 cm (2) 6 cm
(3) 8 cm (4) 10 cm

56. Three equal circles of unit radius touch each other. Then, the area of the circle circumscribing the three circles is :

(1) $6\pi(2+\sqrt{3})^2$ (2) $\frac{\pi}{6}(2+\sqrt{3})^2$

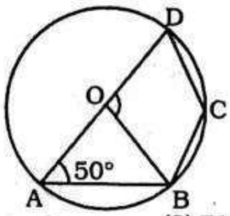
(3) $\frac{\pi}{3}(2+\sqrt{3})^2$ (4) $3\pi(2+\sqrt{3})^2$

57. In the given figure, O is the centre of a circle.Then, $\angle OAB = ?$



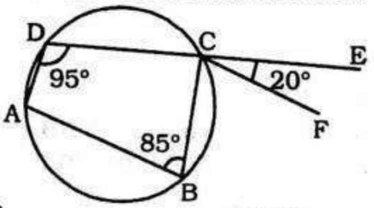
(1) 50° (2) 60°
(3) 55° (4) 65°

58. In the given figure, O is the centre of a circle and $\angle OAB = 50°$. Then, $\angle BOD = ?$



(1) 130° (2) 50°
(3) 100° (4) 80°

59. In the given figure, $ABCD$ is a cyclic quadrilateral in which DC is produced to E and CF is drawn parallel to AB such that $\angle ADC = 95°$ and $\angle ECF = 20°$.Then, $\angle BAD = ?$

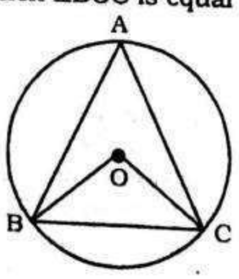


(1) 95° (2) 85°
(3) 105° (4) 75°

60. In a circle with centre O and radius 5 cm., AB is a chord of length 8 cm. If OM ⊥ AB, what is the length of OM ?

(1) 4 cm (2) 5 cm
(3) 3 cm (4) None of these

61. An equilateral Δ ABC is inscribed in a circle with centre O. Then ∠BOC is equal to :

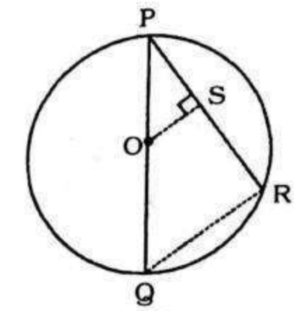


(1) 120° (2) 75°
(3) 180° (4) 60°

62. In which of the following is the lengths of diagonals equal ?

(1) Rhombus
(2) Rectangle
(3) Parallelogram
(4) Trapezium

63. In the following figure, PQ is the diameter of a circle with centre at O. OS is perpendicular to PR.
Then OS is equal to :



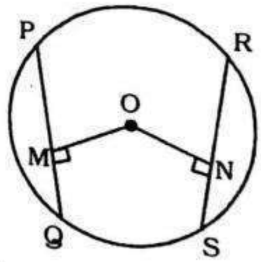
(1) $\frac{1}{4}$ QR (2) $\frac{1}{3}$ QR

(3) $\frac{1}{2}$ QR (4) QR

64. If two circles C_1 and C_2 have three points in common then :

(1) C_1 and C_2 are the same circle
(2) C_1 and C_2 are concentric
(3) C_1 and C_2 have different centres
(4) None of these

65. In the following figure OM and ON are the perpendiculars drawn on the chords PQ and RS. If OM = ON = 6 cm. Then :

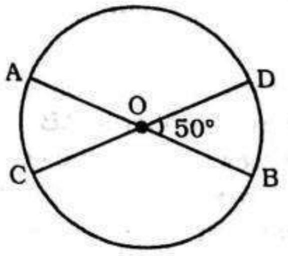


(1) PQ ≥ RS
(2) PQ < RS
(3) PQ ≤ RS
(4) PQ= RS

66. PQ and RS are two chords of a circle intersecting at O.
Then :

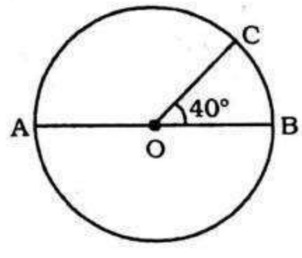
(1) ΔPOS ≅ ΔQOR
(2) ar (ΔPOS) =ar (ΔQOR)
(3) ΔPOS ~ ΔQOR
(4) both (1) and (2)

67. Diameter AB and CD of a circle intersect at O. If m BD = 50°, then m AD is



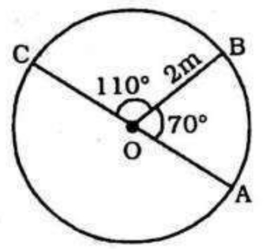
(1) 50° (2) 180°
(3) 130° (4) 310°

68. In the adjoining figure, AB is a diameter of a circle with centre O and ∠COB = 40° then 220° is :



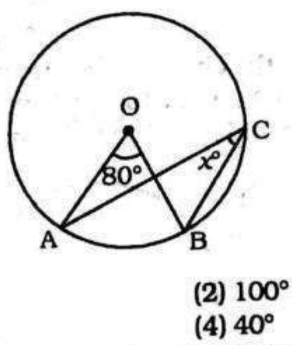
(1) minor BC (2) major BC
(3) minor AC (4) major AC

69. In the following figure, if m AB = 70° and m BC = 110° then m ABC is :



(1) 180° (2) 110°
(3) 220° (4) 90°

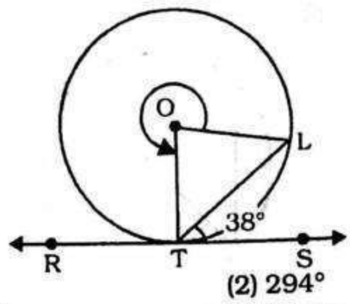
70. The value of $x°$ in the figure is :



(1) 20° (2) 100°
(3) 60° (4) 40°

71. In a circle with centre O, AOC is a diameter of the circle, BD is a chord and OB and CD are joined. If ∠AOB = 130° then ∠BDC = ?
(1) 30° (2) 25°
(3) 50° (4) 60°

72. ∠LTP = 38°, the measure of the reflex angle TOL is :

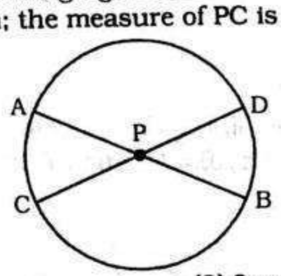


(1) 76° (2) 294°
(3) 274° (4) 284°

73. AB and CD are equal chords of a circle whose centre is O. When produced these chords meet at E. Then :
(1) EB = ED (2) EA = EC
(3) EA = ED (4) both (1) and (2)

74. It is not possible to draw a circle having its centre on a fixed straight line l and passing through two points A and B not on l if :
(1) l is parallel to $\overline{AB}$
(2) l is the perpendicular bisector of $\overline{AB}$
(3) l is perpendicular to $\overline{AB}$ but does not bisect it
(4) l is not perpendicular to $\overline{AB}$ but bisects it

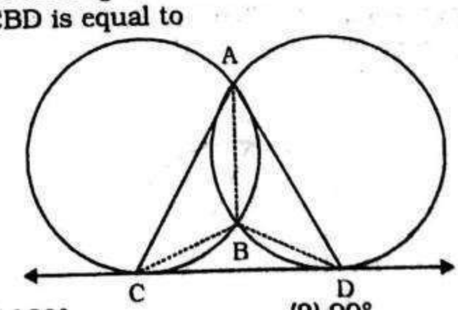
75. In the adjoining figure AB = 10 cm, PB = 4 cm, PD = 8 cm; the measure of PC is :



(1) 3 cm (2) 8cm
(3) 6 cm (4) 5 cm

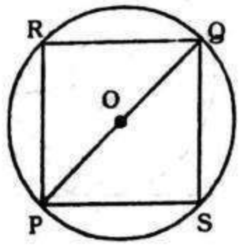
76. Δ ABC is inscribed in a circle ∠P, ∠Q and ∠R are angles inscribed in the arcs cut off by sides BC, AC and AB respectively. Then ∠P + ∠Q + ∠R is equal to :
(1) 180° (2) 360°
(3) 240° (4) None of these

77. CD is a direct common tangent to two circles intersecting each other at A and B. Then ∠CAD + ∠CBD is equal to



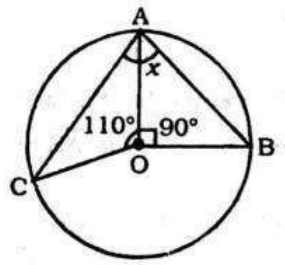
(1) 180° (2) 90°
(3) 360° (4) 120°

78. In the adjoining figure, POQ is the diameter of the circle, R and S are any two points on the circles. Then :



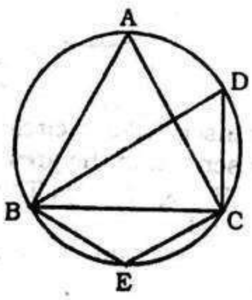
(1) ∠PRQ > ∠PSQ
(2) ∠PRQ < ∠PSQ
(3) ∠PRQ = ∠PSQ
(4) ∠PRQ = $\frac{1}{2}$ ∠PSQ

79. If O is the centre of the circle, the value of 'x' in the adjoining figure is :



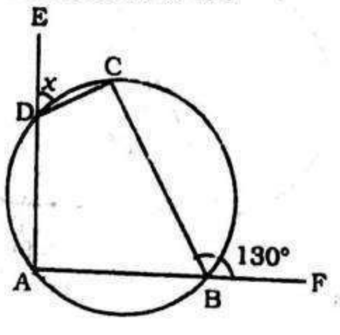
(1) 80° (2) 70°
(3) 60° (4) 50°

80. In the adjoining figure, ΔABC is an isosceles triangle. with AB = AC and ∠ABC = 50°. Then ∠BDC is :



(1) 110° (2) 90°
(3) 80° (4) 70°

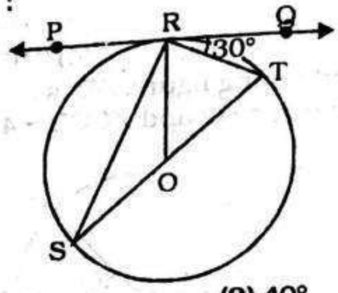
81. In the adjoining figure A, B, C, D are the concyclic points. The value of 'x' is :



(1) 50° (2) 60°
(3) 70° (4) 90°

82. ACB is a tangent to a circle at C. CD and CE are chords such that ∠ACE > ∠ACD. If ∠ACD = ∠BCE = 50°, then :
(1) CD = CE
(2) ED is not parallel to AB
(3) ED passes through the centre of the circle
(4) Δ CDE is a right angled triangle

83. PQ is a tangent to the circle at Rthen m∠PRS is equal to :

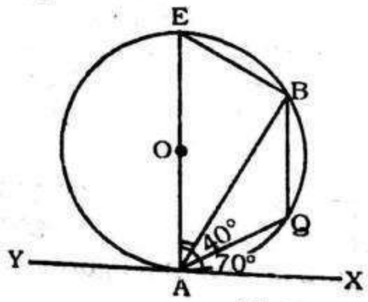


(1) 30° (2) 40°
(3) 60° (4) 80°

84. Two circles touch internally at a point P and form a point T on the common tangent at P, tangent segments TQ, TR are drawn to the two circles then:

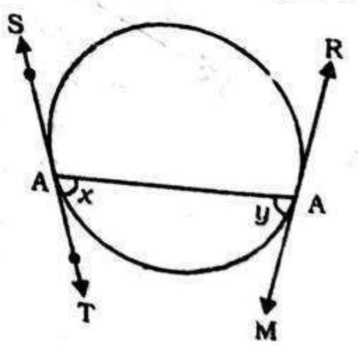
(1) TQ = TR (2) $TP^2 = \frac{1}{2}TR^2$
(3) TP > TR (4) TP < TR

85. In the figure XQY is a tangent to the circle with centre O at A. If ∠BAX=70°, ∠BAQ = 40° then ∠ABQ is equal to :



(1) 20° (2) 30°
(3) 35° (4) 40°

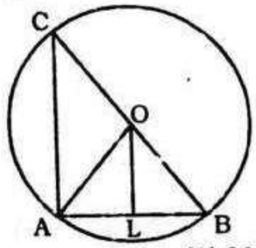
86. If AB is the chord at the circle with centre O. SAT and RBM are the tangents at A and B respectively then which of the following is correct?



(1) $x > y$
(2) $\angle SAB = y$
(3) $\angle SAB = y$ and $\angle RBA = x$
(4) $x = y$

87. If A, B,C,D are angles of a cyclic quadrilateral then $\cos A + \cos B + \cos C + \cos D$ is equal to :
(1) 1 (2) $2 \cos A$
(3) -1 (4) 0

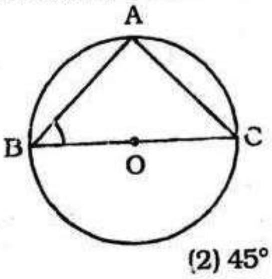
88. If O is the centre of a circle of radius r and AB is a chord of the cirlce at a distance $\frac{r}{2}$ from O, then $\angle BAO$ is :



(1) 15° (2) 30°
(3) 45° (4) 60°

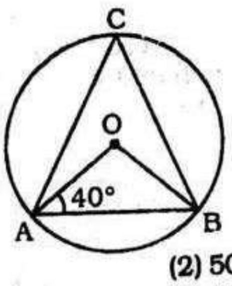
89. The chord AB of a circle of centre O subtends an angle θ with the tangent at A to the circle. $\angle ABO$ is :
(1) θ (2) $90° - \theta$
(3) $90° + \theta$ (4) $2(\pi - \theta)$

90. In the given figure, BOC is a diameter of a circle and $AB = AC$. Then, $\angle ABC = ?$



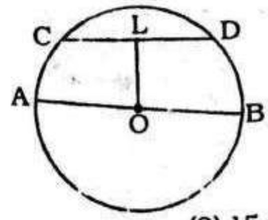
(1) 30° (2) 45°
(3) 60° (4) 90°

91. In the given figure, O is the centre of a circle. If $\angle OAB = 40°$ and C is a point on the circle, then $\angle ACB = ?$



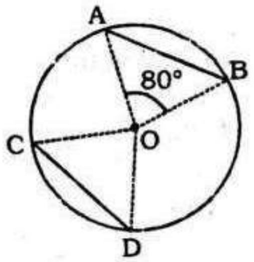
(1) 40° (2) 50°
(3) 80° (4) 100°

92. In the given figure, AOB is a diameter of a circle with centre O such that $AB = 34$ cm and CD is a chord of length 30 cm. Then, the distance of CD from AB is



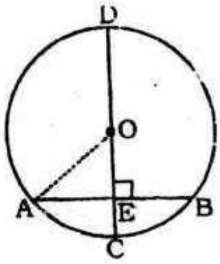
(1) 8 cm (2) 15 cm
(3) 18 cm (4) 6 cm

93. AB and CD are two equal chords of a circle with centre O such that $\angle AOB = 80°$, then $\angle COD = ?$



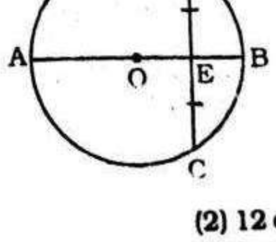
(1) 100° (2) 80°
(3) 120° (4) 40°

94. In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If $AB = 12$ cm and $CE = 3$ cm, then radius of the circle is



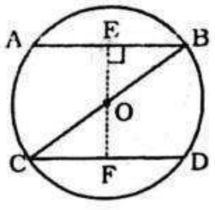
(1) 6 cm (2) 9 cm
(3) 7.5 cm (4) 8 cm

95. In the given figure, O is the centre of a circle and diameter AB bisects the chord CD at a point E such that $CE = ED = 8$ cm and $EB = 4$ cm. The radius of the circle is



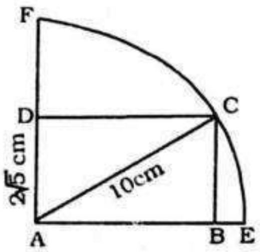
(1) 10 cm (2) 12 cm
(3) 6 cm (4) 8 cm

96. In the given figure, BOC is a diameter of a circle with centre O. If AB and CD are two chords such that $AB \parallel CD$. If $AB = 10$ cm, then $CD = ?$



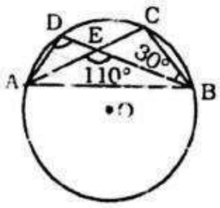
(1) 5 cm
(2) 12.5 cm
(3) 15 cm
(4) 10 cm

97. In the given figure, *ABCD* is a rectangle inscribed in a quadrant of a circle of radius 10cm. If $AD = 2\sqrt{5}$ cm, then area of the rectangle is



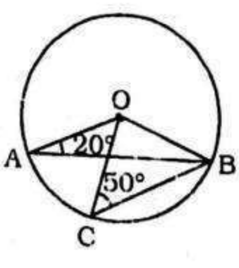
(1) 32 cm²
(2) 40 cm²
(3) 44 cm²
(4) 48 cm²

98. In the given figure, *O* is the centre of a circle and chords *AC* and *BD* intersect at *E*. If $\angle AEB = 110°$ and $\angle CBE = 30°$, then $\angle ADB$ = ?



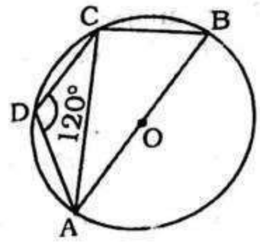
(1) 70°
(2) 60°
(3) 80°
(4) 90°

99. In the given figure, *O* is the centre of a circle in which $\angle OAB = 20°$ and $\angle OCB = 50°$. Then, $\angle AOC$ = ?



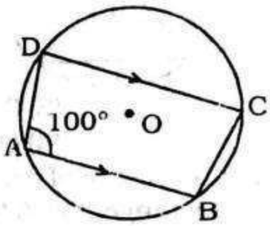
(1) 50°
(2) 70°
(3) 20°
(4) 60°

100. In the given figure, *AOB* is a diameter and *ABCD* is a cyclic quadrilateral. If $\angle ADC = 120°$, then $\angle BAC$ = ?



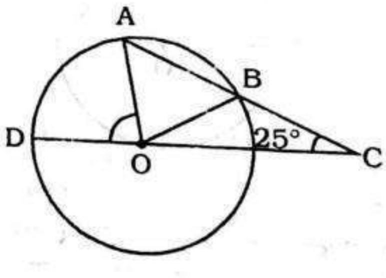
(1) 60°
(2) 30°
(3) 20°
(4) 45°

101. In the given figure *ABCD* is a cyclic quadrilateral in which $AB \parallel DC$ and $\angle BAD = 100°$. Then, $\angle ABC$ = ?



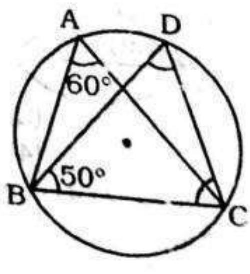
(1) 80°
(2) 100°
(3) 50°
(4) 40°

102. In the given figure, *AB* is a chord of a circle with centre *O* and *AB* is produced to *C* such that *BC* = *OB*. Also, *CO* is joined and produced to meet the circle in *D*. If $\angle ACD = 25°$, then $\angle AOD$ = ?



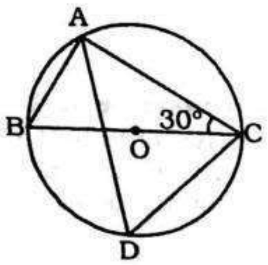
(1) 50°
(2) 75°
(3) 90°
(4) 100°

103. In the given figure, ΔABC and ΔDBC are inscribed in a circle such that $\angle BAC = 60°$ and $\angle DBC = 50°$. Then, $\angle BCD$ = ?



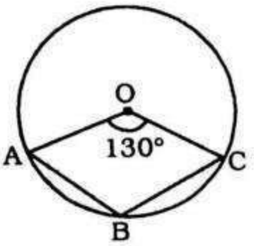
(1) 50°
(2) 60°
(3) 70°
(4) 80°

104. In the given figure, *BOC* is a diameter of a circle with centre *O*. If ∠*BCA* = 30°, then ∠*CDA* = ?



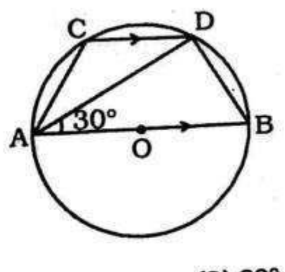
(1) 30° (2) 45°
(3) 60° (4) 50°

105. In the given figure, *O is* the centre of a circle and ∠*AOC* =130°. Then, ∠*ABC* =?



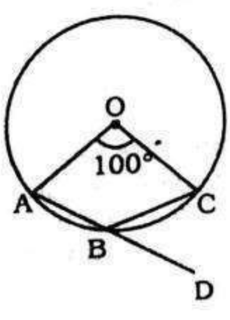
(1) 50° (2) 65°
(3) 115° (4) 130°

106. In the given figure, *AOB* is a diameter of a circle and *CD* || *AB*. If ∠*BAD* = 30°, then ∠*CAD* = ?



(1) 30° (2) 60°
(3) 45° (4) 50°

107. In the given figure, O is the centre of a circle in which ∠*AOC*=100°. Side *AB* of quadrilateral *OABC* has been produced to *D*. Then, ∠*CBD* = ?



(1) 50° (2) 40°
(3) 25° (4) 80°

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

108. O is the circum centre of the triangle ABC with circumradius 13 cm. Let BC = 24 cm and OD is perpendicular to BC. Then the length of OD is.
(1) 7 cm (2) 3 cm
(3) 4 cm (4) 5 cm
[SSC Graduate Level Tier-I Exam, 2012]

109. AB is a diameter of a circle with centre O. CD is a chord equal to the radius of the circle. AC and BD are produced to meet at P. Then the measure of ∠APB is :
(1) 120° (2) 30°
(3) 60° (4) 90°
[SSC Graduate Level Tier-I Exam, 2012]

110. R and r are the radius of two circles (R > r). If the distance between-the centre of the two circles be *d*, then length of common tangent of two circles is :
(1) $\sqrt{r^2 - d^2}$ (2) $\sqrt{d^2 - (R-r)^2}$
(3) $\sqrt{(R-r)^2 - d^2}$ (4) $\sqrt{R^2 - d^2}$
[SSC Graduate Level Tier-I Exam, 2012]

111. P is a point outside a circle and is 13 cm away from its centre. A secant drawn from the point P intersects the circle at points A and B in such a way that PA = 9 cm and AB = 7 cm. The radius of the circle is :
(1) 5.5 cm (2) 5 cm
(3) 4 cm (4) 4.5 cm
[SSC Graduate Level Tier-I Exam, 2012]

112. ABCD is a cyclic quadrilateral. Sides AB and DC, when produced meet at the point P and sides AD and BC, when produced meet at the point Q. If ∠ADC = 85° and ∠BPC = 40°, then ∠CQD is equal to
(1) 30° (2) 40°
(3) 55° (4) 85°
[SSC Graduate Level Tier-I Exam, 2012]

113. Two circles of radii 8 cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. Then length of PQ is equal to
(1) 2 cm (2) 3 cm
(3) 4 cm (4) 8 cm
[SSC Graduate Level Tier-I Exam, 2012]

114. A, B, C are three points on a circle. The tangent at A meets BC produced at T, ∠BTA = 40°, ∠CAT = 44°. The angle subtended by BC at the centre of the circle is
(1) 84° (2) 92°
(3) 96° (4) 104°
[SSC Graduate Level Tier-II Exam, 2011]

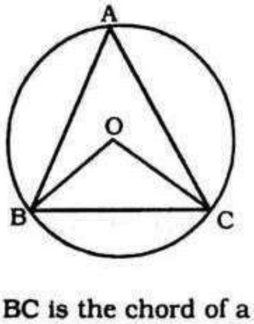
115. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is

(1) r_1r_2 (2) $2r_1r_2$

(3) $3r_1r_2$ (4) $4r_1r_2$

[SSC Graduate Level Tier-I Exam, 2012]

116.



BC is the chord of a circle with centre O. A is a point on major arc BC as shown in the above figure. What is the value of ∠BAC + ∠OBC ?

(1) 120° (2) 60°

(3) 90° (4) 180°

[SSC Graduate Level Tier-I Exam, 2012]

117. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then AP : AQ is

(1) 8 : 5 (2) 5 : 8

(3) 3 : 4 (4) 4 : 5

[SSC Graduate Level Tier-I Exam, 2012]

118. AB and CD are two parallel chords drawn on two opposite sides of their parallel diameter such that AB = 6 cm, CD = 8 cm. If the radius of the circle is 5 cm, the distance between the chords, in cm, is

(1) 2 (2) 8

(3) 5 (4) 3

[SSC Graduate Level Tier-I Exam, 2012]

119. A chord AB of length $3\sqrt{2}$ unit subtends a right angle at the centre O of a circle. Area of the sector AOB (in sq. units) is

(1) $\frac{9}{4}\pi$ (2) 5π

(3) 9π (d) $\frac{9}{2}\pi$

[SSC Graduate Level Tier-I Exam, 2012]

120. The radius of a circle is 6 cm. An external point is at a distance of 10 cm from the centre. Then the length of the tangent drawn to the circle from the external point upto the point of contact is

(1) 8 cm (2) 10 cm

(3) 6 cm (4) 12 cm

[SSC Graduate Level Tier-I Exam, 2012]

121. A triangle is inscribed in a circle and the diameter of the circle is its one side. Then the triangle will be

(1) right-angled (2) obtuse-angled

(3) equilateral (4) acute-angled

[SSC Graduate Level Tier-I Exam, 2012]

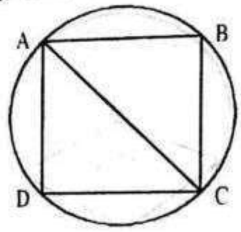
122. AB and BC are two chords of a circle with centre O. If P and Q are the mid-points of AB and BC respectively, then the quadrilateral OQBP must be

(1) a rhombus (2) concyclic

(3) a rectangle (4) a square

[SSC Graduate Level Tier-I Exam, 2012]

123. If the area of the circle in the figure is 36 sq. cm. and ABCD is a square, then the area of ΔACD, in sq. cm, is



(1) 12π (2) $\frac{36}{\pi}$

(3) 12 (4) 18

[SSC Graduate Level Tier-I Exam, 2012]

124. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of a square with one side PQ, is

(1) 97 sq.cm (2) 194 sq.cm

(3) 72 sq.cm (4) 144 sq.cm

[SSC Graduate Level Tier-I Exam, 2012]

125. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If ∠AOP = 60°, then ∠APB is

(1) 120° (2) 90°

(3) 60° (4) 30°

[SSC Graduate Level Tier-I Exam, 2012]

126. If the length of a chord of a circle, which makes an angle 45° with the tangent drawn at one end point of the chord, is 6cm, then the radius of the cirle is :

(1) $6\sqrt{2}$ cm (2) 5 cm

(3) $3\sqrt{2}$ cm (4) 6 cm

[SSC Graduate Level Tier-I Exam, 2012]

127. The length of the chord of a circle is 8 cm and perpendicular distance between centre and the chord is 3 cm. Then the radius of the circle is equal to :

(1) 4 cm (2) 5 cm

(3) 6 cm (4) 8 cm

[SSC FCI Assistant Grade-III Exam, 2012]

128. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the cirle respectively at the points R and S. T is any point on the major arc between the points R and S of the circle. If ∠BAC = 32°, ∠RTS = ?

(1) 32° (2) 74°
(3) 106° (4) 64°

[SSC Graduate Level Tier-I Exam, 2012]

129. The radius of a circle is 13 cm and XY is a chord which is at a distance of 12 cm from the centre. The length of the chord is :

(1) 15 cm (2) 12 cm
(3) 10 cm (4) 20 cm

[SSC Graduate Level Tier-I Exam, 2012]

130. Two equal circles pass through each other's centre. If the radius of each cirlce is 5 cm, what is the length of the common chord?

(1) 5 (2) $5\sqrt{3}$
(3) $10\sqrt{3}$ (4) $\frac{5\sqrt{3}}{2}$

[SSC Graduate Level Tier-I Exam, 2012]

131. SR is a direct common tangent to the circles of radii 8 cm and 3 cm respectively, their centres being 13 cm apart. If the points S and R are the respective points of contact, then the length of SR is

(1) 12 cm (2) 11 cm
(3) 17 cm (4) 10 cm

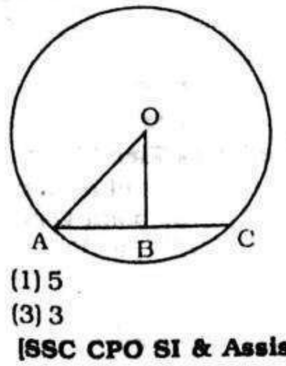
[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

132. PA and PB are two tangents drawn from and external point P to a circle with centre O where the points A and B are the points of contact. The quadrilateral OAPB must be

(1) a rectangle (2) a rhombus
(3) a square (4) concyclic

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

133. In the following figure, if OA = 10 and AC = 16, then OB must be



(1) 5 (2) 6
(3) 3 (4) 4

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

134. The radius of two concentric circles are 9 cm and 15 cm. If the chord of the greater circle be a tangent to the smaller circle, then the length of that chord is

(1) 24 cm (2) 12 cm
(3) 30 cm (4) 18 cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

135. O and C are respectively the orthocentre and circumcentre of an acute-angled triangle PQR. The points P and O are joined and produced to meet the side QR at S. If ∠PQS = 60° and ∠QCR = 130°, then ∠RPS =

(1) 30° (2) 35°
(3) 100° (4) 60°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

136. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to

(1) 30° (2) 45°
(3) 60° (4) 90°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

137. The ratio of the areas of the circumcircle and the incircle of an equilateral triangle is

(1) 2 : 1 (2) 4 : 1
(3) 8 : 1 (4) 3 : 2

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

138. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the centre of a circle. The distance between them is 1 cm. The radius of the circle is

(1) 5 cm (2) 4 cm
(3) 3 cm (4) 2 cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

139. Two chords AB and CD of cricle whose centre is O, meet at the point P and ∠ AOC = 50°, ∠ BOD = 40°. Then the value of ∠ BPD is

(1) 60° (2) 40°
(3) 45° (4) 75°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

140. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of the common chord is :

(1) $2\sqrt{3}$ cm (2) $4\sqrt{3}$ cm
(3) $2\sqrt{2}$ cm (4) 8 cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

141. One chord of a circle is known to be 10.1 cm. The radius of this circle must be :

(1) 5 cm (2) greater than 5 cm
(3) greater than or equal to 5 cm
(4) less than 5 cm

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

142. ABCD is a cyclic parallelogram. The angle ∠B is equal to :
(1) 30° (2) 60°
(3) 45° (4) 90°
[SSC FCI Assistant Grade Exam-III, 2012]

143. From four corners of a square sheet of side 4 cm, four pieces, each in the shape of arc of a circle with radius 2 cm, are cut out. The area of the remaining portion is :
(1) $(8-\pi)$ sq.cm.
(2) $(16-4\pi)$ sq.cm.
(3) $(16-8\pi)$ sq.cm.
(4) $(4-2\pi)$sq.cm.
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

144. If a chord of a circle of radius 5 cm is a tangent to a circle of radius 3 cm, both the circles being concentric, then the length of the chord is
(1) 10 cm (2) 12.5 cm
(3) 8 cm (4) 7 cm
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

145. The length of two chords AB and AC of a circle are 8 cm and 6 cm and ∠BAC = 90°, then the radius of circle is
(1) 25 cm (2) 20 cm
(3) 4 cm (4) 5 cm
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

146. If a chord of length 16 cm is at a distance of 15 cm from the centre of the circle, then the length of the chord of the same circle which is at a distance of 8 cm from the centre is equal to
(1) 10 cm (2) 20 cm
(3) 30 cm (4) 40 cm
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

147. Two circles touch each other externally at point A and PQ is a direct common tangent which touches the circles at P and Q respectively. Then ∠PAQ =
(1) 45° (2) 90°
(3) 80° (4) 100°
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

148. The tangents are drawn at the extremities of a diameter *AB* of a circle with centre *P*. If a tangent to the circle at the point *C* intersects the other two tangents at *Q* and *R*, then the measure of the ∠*QPR* is
(1) 45° (2) 60°
(3) 90° (4) 180°
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

149. *AB* is a chord to a circle and *PAT* is the tangent to the circle at *A*. If ∠*BAT* = 75° and ∠*BAC* = 45°, *C* being a point on the circle, then ∠*ABC* is equal to
(1) 40° (2) 45°
(3) 60° (4) 70°
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

150. PR is tangent to a circle, with centre O and radius 4 cm, at point Q. If ∠POR = 90°, OR = 5 cm and OP = $\frac{20}{3}$ cm, then, in cm, the length of PR is :
(1) 3 (2) $\frac{16}{3}$
(3) $\frac{23}{3}$ (4) $\frac{25}{3}$
[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

151. Chords AB and CD of a circle intersect externally at P. If AB = 6cm, CD = 3cm and PD = 5 cm, then the length of PB is
(1) 5 cm (2) 6.25 cm
(3) 6 cm (4) 4 cm
[SSC Delhi Police S.I. Exam, 19.08.2012]

152. O is the centre of a circle and arc ABC subtends an angle of 130° at O. AB is extended to P. Then ∠PBC is
(1) 75° (2) 70°
(3) 65° (4) 80°
[SSC Delhi Police S.I. Exam, 19.08.2012]

153. Circumcentre of ΔABC is O. If ∠BAC = 85°, ∠BCA = 80°, then ∠OAC is
(1) 80° (2) 30°
(3) 60° (4) 75°
[SSC Graduate Level Tier-I Exam, 2012]

154. If O is the circumcentre of ΔABC and ∠OBC = 35°, then the ∠BAC is equal to
(1) 55° (2) 110°
(3) 70° (4) 35°
[SSC Graduate Level Tier-II Exam, 2011]

155. If I is the incentre of Δ ABC and ∠ BIC = 135°, then Δ ABC is
(1) acute angled (2) equilateral
(3) right angled (4) obtuse angled
[SSC Graduate Level Tier-II Exam, 2011]

156. If S is the circumcentre of ΔABC and ∠A = 50°, then the value of ∠BCS is
(1) 20° (2) 40°
(3) 60° (4) 80°
[SSC Graduate Level Tier-I Exam, 2012]

157. The bisector of the angle BAC of a triangle ABC intersects the side BC at the point D and meets the circumcircle of the ΔABC at E. Then, it is always true that
AB . AC + DE . AE =
(1) AD^2 (2) AE^2
(3) CE^2 (4) CD^2
[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

158. The circumcentre of a triangle ABC is O. If ∠ BAC = 85° and ∠ BCA = 75°, then the value of ∠ OAC is

(1) 40° (2) 60°
(3) 70° (4) 90°

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

159. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle, in cm², is

(1) 450 (2) 308
(3) 154 (4) 77

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2011]

160. The area of a circle is increased by 22 cm² when its radius is increased by 1 cm. The original radius of the circle is

(1) 3 cm (2) 5 cm
(3) 7 cm (4) 9 cm

161. Two circles touch each other externally at P. AB is a direct common tangent to the two circles, A and B are points of contact and ∠ PAB = 35°. Then ∠ ABP is

(1) 35° (2) 55°
(3) 65° (4) 75°

162. The length of the common chord of two intersecting circles is 24 cm. If the diameters of the circles are 30 cm and 26 cm, then the distance between the centres in cm is

(1) 13 (2) 14
(3) 15 (4) 16

163. The area of the square inscribed in a circle of radius 8 cm is

(1) 256 sq. cm (2) 250 sq. cm
(3) 128 sq. cm (4) 125 sq. cm

164. X and Y are centres of circles of radii 9 cm and 2 cm respectively, XY = 17 cm. Z is the centre of a circle of radius r cm which touches the above circles externally. Given that ∠XZY = 90°, the value of r is

(1) 13 cm (2) 6 cm
(3) 9 cm (4) 8 cm

165. If the radii of two circles be 6 cm and 3 cm and the length of the transverse common tangent be 8 cm, then the distance between the two centres is

(1) $\sqrt{145}$ cm (2) $\sqrt{140}$ cm
(3) $\sqrt{150}$ cm (4) $\sqrt{135}$ cm

[SSC Graduate Level Tier-II Exam, 16.09.2012]

166. A chord of a circle is equal to its radius. The angle subtended by this chord at a point on the circumference in the major segment is

(1) 60° (2) 120°
(3) 90° (4) 30°

167. AB and CD are two parallel chords on the opposite sides of the centre of the circle. If $\overline{AB}$ = 10 cm, $\overline{CD}$ = 24 cm and the radius of the circle is 13 cm, the distance between the chords is

(1) 17 cm (2) 15 cm
(3) 16 cm (4) 18 cm

168. Two parallel chords are drawn in a circle of diameter 30 cm. The length of one chord is 24 cm and the distance between the two chords is 21 cm. The length of the other chord is

(1) 10 cm (2) 18 cm
(3) 12 cm (4) 16 cm

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (Ist Sitting)]

169. If two equal circles whose centres are O and O′, intersect each other at the points A and B, OO′ = 12 cm and AB = 16 cm, then the radius of the circles is

(1) 10 cm (2) 8 cm
(3) 12 cm (4) 14 cm

170. Two circles of radii 9 cm and 2 cm respectively have centres X and Y and $\overline{XY}$ = 17 cm. Circle of radius r cm with centre Z touches two given circles externally. If ∠XZY = 90°, find r.

(1) 18 cm (2) 3 cm
(3) 12 cm (4) 6 cm

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (IInd Sitting)]

171. Three circles of radii 4 cm, 6 cm and 8 cm touch each other pairwise externally. The area of the triangle formed, by the line-segments joining the centres of the three circles is

(1) $144\sqrt{13}$ sq. cm (2) $12\sqrt{105}$ sq. cm
(3) $6\sqrt{6}$ sq. cm (4) $24\sqrt{6}$ sq. cm

172. In a circle of radius 21 cm, an arc subtends an angle of 72° at the centre. The length of the arc is

(1) 21.6 cm (2) 26.4 cm
(3) 13.2 cm (4) 19.8 cm

173. The distance between the centres of two equal circles, each of radius 3 cm, is 10 cm. The length of a transverse common tangent is

(1) 8 cm (2) 10 cm
(3) 4 cm (4) 6 cm

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (Ist Sitting)]

174. Two chords AB and CD of a circle with centre O intersect each other at the point P. If ∠AOD = 20° and ∠BOC = 30°, then ∠BPC is equal to:

(1) 50° (2) 20°
(3) 25° (4) 30°

175. A unique circle can always be drawn through x number of given non-collinear points, then x must be :

(1) 2 (2) 3
(3) 4 (4) 1

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

176. The distance between the centres of two equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is

(1) 4cm (2) 6cm
(3) 8cm (4) 10cm

177. A chord of length 8 cm is at a distance 3cm from the centre of the circle. The length of the radius of the circle is

(1) $\sqrt{73}$ cm (2) $\sqrt{55}$ cm
(3) 5 cm (4) 10 cm

178. A circle (with centre at O) is touching two intersecting lines AX and BY. The two points of contact A and B subtend an angle of 65° at any point C on the circumference of the circle. If P is the point of intersection of the two lines, then the measure of ∠APO is

(1) 25° (2) 65°
(3) 90° (4) 40°

[SSC (10+2) Level Data Entry Operator and LDC Exam, 28.10.2012 (Ist Sitting)]

179. The length of radius of a circumcircle of a triangle having sides 3cm, 4cm and 5cm is :

(1) 2 cm (2) 2.5 cm
(3) 3 cm (4) 1.5 cm

[SSC (10+2) Level Data Entry Operator and LDC Exam, 04.11.2012 (Ist Sitting)]

ANSWERS

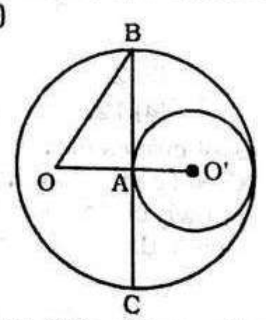
1. (3)	**2.** (3)	**3.** (4)	**4.** (2)	**5.** (4)
6. (2)	**7.** (1)	**8.** (4)	**9.** (3)	**10.** (4)
11. (1)	**12.** (2)	**13.** (4)	**14.** (1)	**15.** (3)
16. (2)	**17.** (4)	**18.** (2)	**19.** (3)	**20.** (3)
21. (3)	**22.** (2)	**23.** (4)	**24.** (3)	**25.** (2)
26. (1)	**27.** (3)	**28.** (3)	**29.** (2)	**30.** (2)
31. (3)	**32.** (3)	**33.** (2)	**34.** (2)	**35.** (2)
36. (1)	**37.** (3)	**38.** (2)	**39.** (2)	**40.** (2)
41. (3)	**42.** (3)	**43.** (4)	**44.** (2)	**45.** (3)
46. (2)	**47.** (2)	**48.** (3)	**49.** (3)	**50.** (4)
51. (1)	**52.** (2)	**53.** (2)	**54.** (1)	**55.** (4)
56. (3)	**57.** (4)	**58.** (3)	**59.** (3)	**60.** (3)
61. (1)	**62.** (2)	**63.** (3)	**64.** (1)	**65.** (4)
66. (4)	**67.** (3)	**68.** (4)	**69.** (1)	**70.** (4)
71. (2)	**72.** (4)	**73.** (4)	**74.** (3)	**75.** (1)
76. (2)	**77.** (1)	**78.** (3)	**79.** (1)	**80.** (3)
81. (1)	**82.** (1)	**83.** (3)	**84.** (1)	**85.** (2)
86. (4)	**87.** (4)	**88.** (2)	**89.** (2)	**90.** (2)
91. (2)	**92.** (1)	**93.** (2)	**94.** (3)	**95.** (1)
96. (4)	**97.** (2)	**98.** (3)	**99.** (4)	**100.** (2)
101. (2)	**102.** (2)	**103.** (3)	**104.** (3)	**105.** (3)
106. (1)	**107.** (1)	**108.** (4)	**109.** (3)	**110.** (2)
111. (2)	**112.** (1)	**113.** (4)	**114.** (4)	**115.** (4)
116. (3)	**117.** (2)	**118.** (*)	**119.** (1)	**120.** (1)
121. (1)	**122.** (2)	**123.** (2)	**124.** (4)	**125.** (3)
126. (3)	**127.** (2)	**128.** (2)	**129.** (3)	**130.** (2)
131. (1)	**132.** (4)	**133.** (2)	**134.** (1)	**135.** (2)
136. (1)	**137.** (2)	**138.** (1)	**139.** (3)	**140.** (2)
141. (2)	**142.** (4)	**143.** (2)	**144.** (3)	**145.** (4)
146. (3)	**147.** (2)	**148.** (3)	**149.** (3)	**150.** (4)
151. (*)	**152.** (3)	**153.** (2)	**154.** (1)	**155.** (3)
156. (2)	**157.** (2)	**158.** (3)	**159.** (3)	**160.** (1)
161. (2)	**162.** (2)	**163.** (3)	**164.** (2)	**165.** (1)
166. (4)	**167.** (1)	**168.** (2)	**169.** (1)	**170.** (4)
171. (4)	**172.** (2)	**173.** (1)	**174.** (3)	**175.** (2)
176. (3)	**177.** (3)	**178.** (1)	**179.** (2)	

EXPLANATIONS

1. (3) ∠ACB = ∠ADB = 32°
∠ACD = ∠ABD = 50°
∴ x = ∠BCD = ∠ACB + ∠ACD = 82°

2. (3) As DE ∥ BC, so ∠ADE = ∠ABC
Also, ∠ABC = ∠ACB
∴ ∠ADE = ∠ACB (∵ AB = AC)
∴ ∠ADE + ∠EDB = 180°
⇒ ∠ACB + ∠EDB = 180°
Hence, B, C, D and E are concyclic.

3. (4)



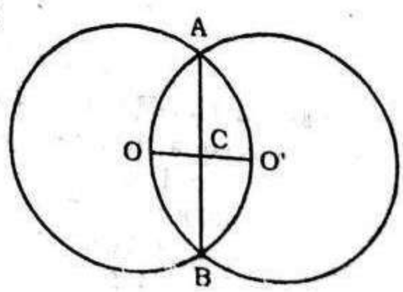
Here, OA = 10 cm, OB = 3 cm

∴ AB = $\sqrt{3^2 - 1^2} = \sqrt{8}$ cm

∴ Required length = BC = 2AB

= $2\sqrt{8}$ cm = $4\sqrt{2}$ cm

4. (2)



Here let O, O' be the centres of the circle.

As the centre of each lies on the circumference of the other, the two circles will have the same radius ame radius. Let it be r.

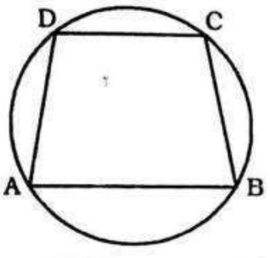
$\therefore\ OC = O'C = \frac{r}{2}$

$\therefore\ AC = \sqrt{OA^2 - OC^2} = \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}}{2}r$

$AB = \sqrt{3}\,r$

Hence, $\frac{\text{common chord}}{\text{radius}} = \sqrt{3}\,r : r = \sqrt{3} : 1$

5. (4)



Since ABCD is a cyclid trapezium then a cyclic trapezium being isosceles, so

$\angle DAB = \angle ABC$ and $\angle ADC = \angle BCD$

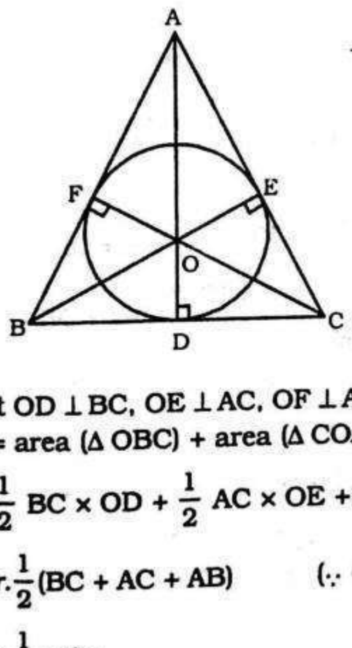
Let $\angle DAB = x^o$,

Then $\angle BCD = 3x^o$

$\therefore\ x + 3x = 180° \Rightarrow x = 45°$

$\therefore$ Largest angle $= 3x = 3 \times 45° = 135°$

6. (2)



Let $OD \perp BC$, $OE \perp AC$, $OF \perp AB$, then

Δ = area (Δ OBC) + area (Δ COA) + area (Δ AOB)

$= \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$

$= r.\frac{1}{2}(BC + AC + AB)$ $\quad (\because OD = OF = OE = r)$

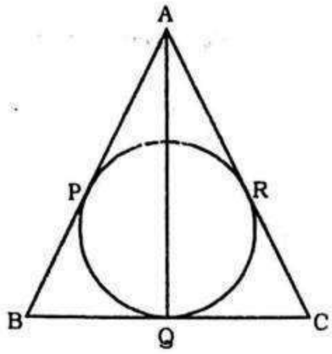
$= r.\frac{1}{2} \times 2s$

$\Delta = rs \Rightarrow r = \frac{\Delta}{s}$

7. (1) As $\angle B + \angle D = 180°$ and

$\angle B - \angle D = 60° \Rightarrow \angle B = 120°$ and $\angle D = 60°$

8. (4)



As the tangents drawn from an external point to a circle are equal.

$\therefore$ AP = AR, BQ = BP

and CR = QC

$\therefore$ AP + BQ + CR = BP + QC + RA

Here I is correct.

Perimeter of

Δ ABC = AB + BC + AC

= AP + PB + BQ + QC + RC + RA

= 2 (AP + BQ + CR)

$\therefore$ AP + BQ = CR = $\frac{1}{2}$ (Perimeter of Δ ABC)

II is correct.

9. (3) $\angle B + \angle D = 180°$

$\Rightarrow \angle D = 180 - \angle B = 180° - 92° = 88°$

$\angle DAE = \angle D = 88°$

$\angle FAD = 88° + 20° = 108°$

$\angle BCD = \angle FAD = 108°$

$\Rightarrow \angle BCD = 108°$

10. (4) As $\angle B + \angle D = 180°$ and

$\angle A + \angle C = 180°$

$x + 10 + 5y + 5 = 180°$

$\Rightarrow x + 5y = 165°$ ...(i)

$2x + 4 + 4y - 4 = 180°$

$\Rightarrow 2x + 4y = 180°$ ...(ii)

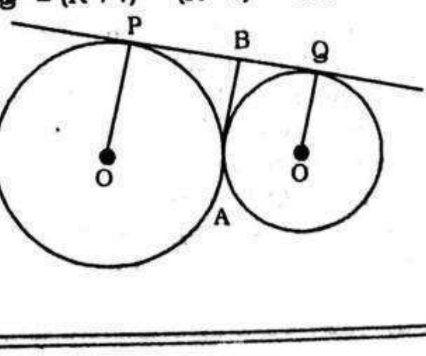
Solving, x and y are 40° and 25°

$\therefore x + y = 40° + 25° = 65°$

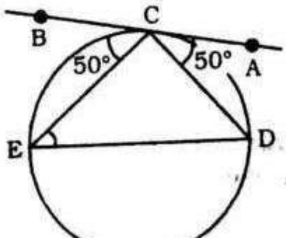
11. (1) Through four given points, we can draw at the most one circle.

12. (2) At the most two common tangents can be drawn to two circles.

13. (4) $PQ^2 = (R + r)^2 - (R - r)^2 = 4Rr$



14. (1)



Join ED, then

$\angle DEC = \angle ACD = 50°$

(angles in alternate segment)

$\angle EDC = \angle BCE = 50°$

(cyclic in alternate segment)

$\therefore \angle DEC = \angle EDC$

So, CD = CE

15. (3) $m\angle TRQ = m\angle RST$ [Angles in alternate segment]

But $m\angle TRQ = 30°$,

$m\angle RST = 30°$

also $m\angle SRT = 90°$

$\therefore m\angle RTS = 180° - (90° + 30°) = 60°$

$m\angle PRS = m\angle RTS$

$m\angle PRS = 60°$

16. (2) $\angle QAX = \angle BAX - \angle BAQ = 70° - 40° = 30°$

$\angle BAY = 180° - \angle BAX = 180° - 70° = 110°$

$\angle EBA = 90°$ [angle in semi-circle]

$\angle BAY = \angle AQB = 110°$

$\angle ABQ = 180° - (\angle BAQ + \angle AQB)$

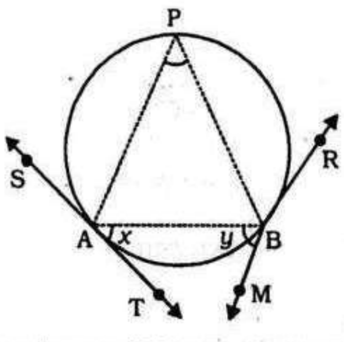
$= 180° - (40° + 110°) = 30°$

17. (4) Join AP and PB.

Now $\angle APB = x$ (angle is in alternate segment)

$\angle APB = y$ (angle is in alternate segment)

$\Rightarrow x = y$



18. (2) Any cyclic parallelogram having unequal sides is necessarily a rectangle.

19. (3) Clearly, not more than one circle can be drawn through three non-collinear points.

20. (3) It is necessarily a rectangle

$\angle ADC = 180° - 65° = 115°$

$\angle DCA = 180° - (115° + 45°) = 20°$

ADCB is a cyclic quadrilateral

21. (3) $\angle OBA = \angle OAB = 55°$

$\angle AOB = 180° - (55° + 55°) = 70°$

22. (2) $\angle BDC = 180° - (110° + 30°) = 40°$

$\angle BAC = \angle BDC = 40°$ (angles of the same segment)

23. (4) $\angle ABO = 60°$ $\angle BAO$

($\because AO = BO$, radii of the circle)

$\angle AOB = 180° - (60° + 60°) = 60°$

$\angle ACB = \frac{1}{2} \angle AOB = 30°$

24. (3) $\angle CBA = \frac{1}{2} \angle COA = 60°$

$\angle CBE = 180° - \angle CBA = 180° - 60° = 120°$

25. (2) $\angle BDA = 90°$

$\angle DBA = 50°$

$\angle DAB = 180° - (90° + 50°) = 40°$

26. (1) $\angle BAC = 60°$

$\angle BEC = 180° - 60° = 120°$

27. (3) $\angle ABD = 45°$

$\angle ACD = 45°$

28. (3) $\angle ADC = 140°$

$\therefore \angle ABC = 40°$

$\angle ACB = 90°$

$\therefore \angle BAC = 180° - (90° + 40°) = 50°$

29. (2) $\angle CAB = \frac{1}{2} \angle BOC = \frac{1}{2} \times 40° = 20°$

30. (2) $\angle OBA = \angle OAB = \left(\frac{180° - 80°}{2}\right) = 50°$ and

$\angle OCA = \angle OAC = \left(\frac{180° - 120°}{2}\right) = 30°$

$\therefore \angle BAC = \angle BAO + \angle OAC$

$= 50° + 30° = 80°$

31. (3) $\angle ADC = \frac{100°}{2} = 50°$

$\therefore \angle ABC = 180° - 50° = 130°$

$\therefore \frac{m\angle ADC}{m\angle ABC} = \frac{50°}{130°} = \frac{5}{13}$

32. (3) $\angle APB = 50°$

$\left(\because m\angle APB = \frac{1}{2} m\angle AOB\right)$

$\therefore \angle ACB = 180° - 50° = 130°$

$\therefore \angle BCD = 180° - 130° = 50°$

33. (2) $\angle ABC = 180° - 120° = 60°$

$\angle ACB = 90°$

$\therefore \angle CAB = 180° - (90° + 60°) = 30°$

34. (2) $\angle COD = \angle AOB = 70°$

$\therefore \angle OCD = \frac{180° - 70°}{2} = 55°$

35. (2) $\angle ABK = 180° - (105° + 40°) = 35°$

$\therefore \angle ACD = \angle ABD = 35°$

36. (1) $\angle BAC = 80°$ $(\therefore \angle ABC = \angle ACB = 50°)$

$\therefore \angle BDC = \angle BAC = 80°$

37. (3) $\angle ADB = 90°$

$\therefore \angle DBA = 20°$ $[180° - (90° + 70°)]$

$\therefore \angle CBA = 30° + 20° = 50°$

$\therefore \angle CDA = 180° - 50° = 130°$

$\therefore \angle CDB = 130° - 90° = 40°$

38. (2) $\angle BAD = 80°$

$\therefore \angle DCE = 80°$

39. (2) $\angle AOC = 2 \times 50 = 100°$

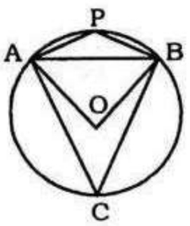
$\therefore \angle ABC = \frac{100}{2} = 50°$

40. (2) $\angle AOB = 2$

$\angle ACB = 2 \times 25 = 50°$

41. (3) $\angle APB = 180° - \frac{1}{2} \times 90° = 135°$

$$\left[\begin{array}{l}\because \angle ACB = 45° = \left(\frac{1}{2} \times 90°\right) \\ \text{and} \angle APB + \angle ACB = 180°\end{array}\right]$$



42. (3) $OA = \sqrt{(OB)^2 - (AB)^2}$

$OA = \sqrt{169 - 144} = 5$ cm

43. (4) TQ = TP and TP = TR

TQ = TP = TR

44. (2) $\angle BAC = 90°$

$\angle BCA = 60°$ $(\therefore \angle BCA = \angle BAQ)$

$\therefore \angle ABC = 180° - (90° + 60°)$

$\angle ABC = 30°$

45. (3) $\angle DBQ = 65°$

$\therefore \angle DAB = 65°$

$\therefore \angle DCB = 180° - 65° = 115°$

46. (2) $AP \times BP = DP \times CP$

$2 \times 6 = DP \times 3$

$\Rightarrow DP = 4$ cm

47. (2) $PA \times PB = PC \times PD$

$3 \times (3 + 5) = 2 \times PD$

$\therefore PD = 12$ cm

$\therefore CD = PD - PC = 12 - 2 = 10$cm

48. (3) $PT^2 = PA \times PB$

$5 \times 5 = 4 \times (4 + x)$

$\Rightarrow 25 = 16 + 4x$ $\Rightarrow 4x = 9$

$\Rightarrow x = \frac{9}{4}$cm, where AB = x

49. (3) CD = 6 cm

$\therefore$ AC = 6 cm and BC = 6 cm

($\because$ AC = CD and BC = CD. Two tangents from the same point are always equal.)

AB = 12 cm

50. (4) $\angle CAB = \angle BCD$ and $\angle DAB = \angle BDC$

$\therefore \angle CAD = \angle CAB + \angle DAB = \angle BCD + \angle BDC$

$\Rightarrow \angle CAD = \angle BCD + \angle BDC$

$\therefore \angle CAD + \angle CBD$

$= \angle BCD + \angle BDC + \angle CBD = 180°$

51. (1) AB = 24 cm

$\therefore$ AC = BC = 12 cm

OA = 20 cm

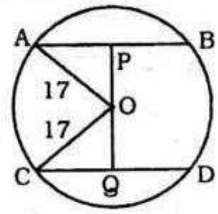
$\therefore OC = \sqrt{(OA)^2 - (AC)^2} = \sqrt{400 - 144} = 16$ cm

and O'A = 37cm

$\therefore O'C = \sqrt{(O'A)^2 - (AC)^2} = \sqrt{(37)^2 - (12)^2} = 35$cm

$\therefore OO' = OC + CO' = 16 + 35 = 51$ cm

52. (2) OA = OC = 17 cm



$AP = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8$ cm

$OP = \sqrt{OA^2 - AP^2} = 15$ cm

Now, PQ = OP + OQ = 23 cm

$\therefore$ 15 + OQ = 23 cm

$\therefore$ OQ = 8 cm

Again, $CQ = \sqrt{(OC)^2 - (OQ)^2}$

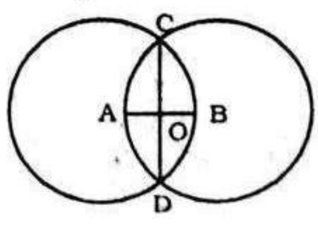
$CQ = \sqrt{(17)^2 - (8)^2}$

$\therefore$ CQ = 15 cm $\therefore$ CD = 2CQ = 30 cm

53. (2) AB = r (say)

then AC = BC = r, also

$\therefore OA = OB = \frac{r}{2}$ (CD is a common chord)



$\therefore OC = \sqrt{(AC)^2 - (OA)^2} = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \sqrt{\frac{3r^2}{4}} = \frac{\sqrt{3}}{2}r$

$\therefore CD = 2CO = 2 \times \frac{\sqrt{3}}{2}r = \sqrt{3}r$

$\therefore \frac{l(CD)}{l(AC)} = \frac{\sqrt{3}r}{r} = \frac{\sqrt{3}}{1}$

54. (1) In radius of an incircle of right angled triangle

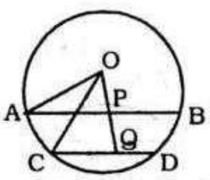
$= \frac{\text{Base + Altitude – Hypotenuse}}{2} = \frac{4+3-5}{2} = 1\text{ cm}$

Alternately : Inradius

$= \frac{\text{Area of riangle}}{\text{Simiperimeter of triangle}}$

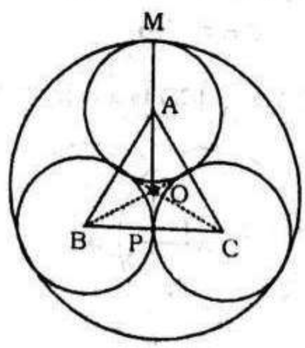
$= \frac{(3 \times 4)/2}{(3+4+5)/2} = \frac{6}{6} = 1\text{ cm}$

55. (4) Let AB and CD be the two chords, then AB = 8 cm and CD = 6 cm.



$\therefore AP = 4$ cm and $CQ = 3$ cm
and $PQ = 1$ cm (given)
Let $OP = x$
$\therefore AO^2 = AP^2 + OP^2$
$\therefore AO^2 = 4^2 + x^2 = 16 + x^2$ ...(i)
and $OC^2 = CQ^2 + OQ^2$
$= CQ^2 + (OP + PQ)^2 = (3)^2 + (x+1)^2$
$OC^2 = 9 + x^2 + 1 + 2x$ ...(ii)
But OA = OC, are the radii of the same circle.
$\therefore 16 + x^2 = 9 + x^2 + 1 + 2x$
$\Rightarrow x = 3$ cm
$\therefore OA^2 = 16 + x^2 = 16 + 9 = 25$
$\therefore OA = 5$ cm
$\therefore$ Diameter of the circle = 2(OA) = 10 cm

56. (3) AB = BC = AC = 2 cm
($\because$ radius of each circle = 1 cm)



$\therefore AP = \frac{\sqrt{3}}{2} \times 2$; $AP = \sqrt{3}$ cm
Let O be the centroid, then

$OA = \frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}}$ cm and AM = 1 cm

$\therefore OM = \frac{2}{\sqrt{3}} + 1 = \frac{2+\sqrt{3}}{\sqrt{3}}$ cm

OM is the radius of the larger circle.
$\therefore$ Area of the circumscribing circle = πR^2

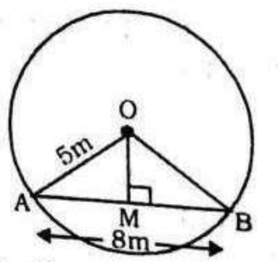
$= \pi \times \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)^2 = \frac{\pi}{3}(2+\sqrt{3})^2$

57. (4) $OA = OB$ (radii of the same circle)
$\Rightarrow \angle OAB = \angle OBA = x°$.
Then, $x + x + 50 = 180$
$\Rightarrow 2x = 130$
$\Rightarrow x = 65°$.

58. (3) $OA = OB \Rightarrow \angle OBA = \angle OAB = 50°$.
In ΔOAB, $\angle OAB + \angle OBA + \angle AOB = 180°$
$\Rightarrow 50° + 50° + \angle AOB = 180°$
$\therefore \angle AOB = 80°$
$\therefore \angle BOD = (180° - 80°) = 100°$

59. (3) $\angle ABC + \angle ADC = 180°$
$\Rightarrow \angle ABC + 95° = 180°$
$\Rightarrow \angle ABC = 85°$.
Now, $CF \parallel AB$ and CB is the transversal.
$\therefore \angle BCF = \angle ABC = 85°$
[Alternate Interior angles]
$\Rightarrow \angle BCE = (85° + 20°) = 105°$
$\Rightarrow \angle DCB = (180° - 105°) = 75°$.
Now, $\angle BAD + \angle BCD = 180°$
$\Rightarrow \angle BAD + 75° = 180°$
$\Rightarrow \angle BAD = 105°$.

60. (3)



OA = 5 cm

$AM = \frac{1}{2}AB$,

AM = 4 cm
$OM^2 = OA^2 - AM^2 = 5^2 - 4^2 = 9$
$\therefore OM = 3$ cm

61. (1) $\angle BOC = 90° + \frac{1}{2}\angle A = 90° + \frac{1}{2} \times 60$
$\Rightarrow \angle BOC = 120°$

62. (2) In rectangle the lengths of the diagonals are equal.

63. (3) As O is mid-point of PQ and $\angle PRQ = 90°$ (angle is semi-circle)

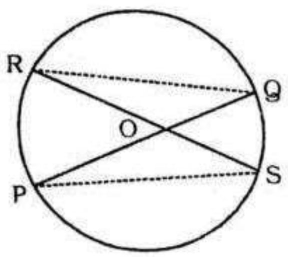
So $OS \parallel QR$ as both $\perp PR$

$\therefore \frac{PO}{PQ} = \frac{OS}{QR} \Rightarrow \frac{OS}{QR} = \frac{1}{2} \Rightarrow OS = \frac{1}{2}QR$

64. (1) Clearly two circles can intersect at two points only.

65. (4) PQ = RS as equal chords of circle are equidistant from the centre.

66. (4)



As in ΔPOS and ΔROQ

OR = OQ = OP = OS

and $\angle ROQ = \angle POS$

(Vertically opposite angles)

Also arcs are equal.

67. (3) m BD = 50°

$\Rightarrow \angle BOD = 50°$

$\Rightarrow \angle AOD = 180° - \angle BOD = 130°$

$\Rightarrow m AD = 130°$

68. (4) Major AC = 360° – minor AC

= 360° – 140° = 220°

$\Rightarrow$ Major AC = 220°

69. (1) m ABC = m (AB + BC)

= m AB + m BC = $\angle AOB + \angle BOC$

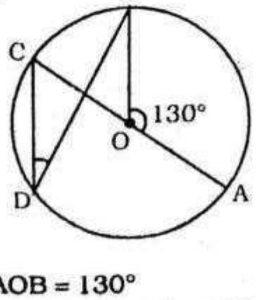
$\Rightarrow m ABC = 180°$

70. (4) Here $\angle AOB = 2 [\angle ACB]$

$80° = 2x$ (by theorem)

$\Rightarrow x = 40°$

71. (2)

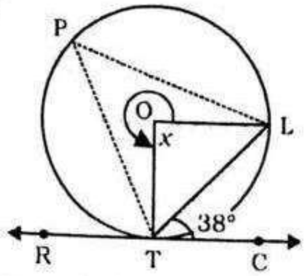


$\angle AOB = 130°$

$\Rightarrow \angle BOC = 180° - \angle AOB = 50°$

$\angle BDC = \frac{1}{2} \angle BOC = \frac{1}{2} \times 50° = 25°$

72. (4)



$\angle TPL = \angle LTS$

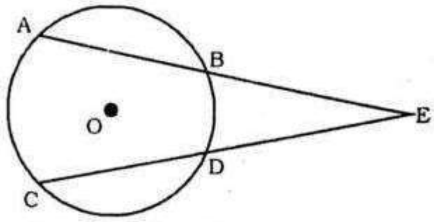
$\angle TPL = 38°$

Let

$x = \angle LOT = 2 \angle TPL = 60°$

$\therefore$ Reflex angle TOL = 360° – 76° = 284°

73. (4)



Clearly AE = EC and as AB = CD

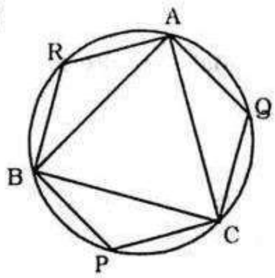
$\Rightarrow$ EB = ED

74. (3) Let P be a fixed point on l. Then, a circle can be drawn through the points P, A, B only when PA = PB. When $l \perp AB$ and l does not bisect AB, then $PA \neq PB$ so in this case, the circle cannot be drawn to pass through P, A and B.

75. (1) AP × PB = PC × PD

$\Rightarrow 6 \times 4 = 8 \times PC \qquad \Rightarrow PC = \frac{24}{8} = 3$ cm

76. (2)



$\angle R + \angle ACB = 180°$

$\angle P + \angle BAC = 180°$

$\angle Q + \angle ABC = 180°$

$\angle P + \angle Q + \angle R + \angle BAC + \angle ABC + \angle ACB = 540$

$\angle P + \angle Q + \angle R = 540° - 180°$

$\angle P + \angle Q + \angle R = 360°$

77. (1) Here $\angle CAB = \angle BCD$ (angles in alternate segments)

and $\angle DAB = \angle CDB$ (angles in alternate segments)

$\angle CAD = \angle CAB + \angle DAB = \angle BCD + \angle CDB$

$\Rightarrow \angle CAD + \angle CBD$

$= \angle BCD + \angle CDB + \angle CBD = 180°$

(angles of a Δ)

78. (3) $\angle PRQ = \angle PSQ = 90°$ (each angle in semi-circle)

79. (1) $\angle COB = 360° - (110° + 90°) = 160°$

$\therefore\ x = \angle CAB = \frac{1}{2}\angle COB = \frac{1}{2} \times 160° = 80°$

80. (3) As, AB = AC

$\Rightarrow \angle ACB = \angle ABC = 50°$

$\therefore\ \angle BAC = 180° - (50 + 50) = 80°$

$\therefore\ \angle BDC = \angle BAC = 80°$ (angles in the same segment)

81. (1) $\angle CBF = \angle CDA$

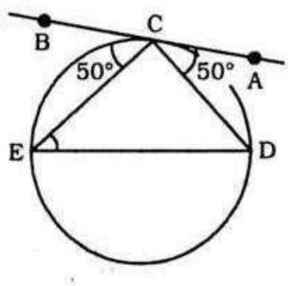
$\Rightarrow \angle CDA = 130°$

$\angle CDA + x = 180°$ (linear pair)

$\Rightarrow x = 180° - 130°$

$\Rightarrow x = 50°$

82. (1)



Join ED, then

$\angle DEC = \angle ACD = 50°$

(angles in alternate segment)

$\angle EDC = \angle BCE = 50°$

(cyclic in alternate segment)

$\therefore\ \angle DEC = \angle EDC$

So, CD = CE

83. (3) $m\angle TRQ = m\angle RST$

[Angles in alternate segment]

But $m\angle TRQ = 30°$,

$m\angle RST = 30°$

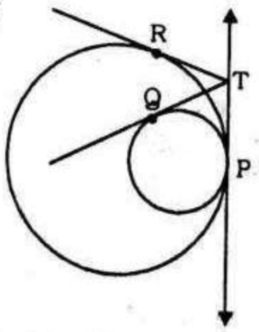
also $m\angle SRT = 90°$

$\therefore\ m\angle RTS = 180° - (90° + 30°) = 60°$

$m\angle PRS = m\angle RTS$

$m\angle PRS = 60°$

84. (1)



Here TP = TQ ...(i)

[$\because$ length of tangents drawn from an external point to a circle are equal]

Also TR = TP ...(ii)

From (i) and (ii), TQ = TR

85. (2) $\angle QAX = \angle BAX - \angle BAQ$

$= 70° - 40° = 30°$

$\angle BAY = 180° - \angle BAX = 180° - 70° = 110°$

$\angle EBA = 90°$ [angle in semi-circle]

$\angle BAY = \angle AQB = 110°$

$\angle ABQ = 180° - (\angle BAQ + \angle AQB)$

$= 180° - (40° + 110°) = 30°$

86. (4) Join AP and PB.

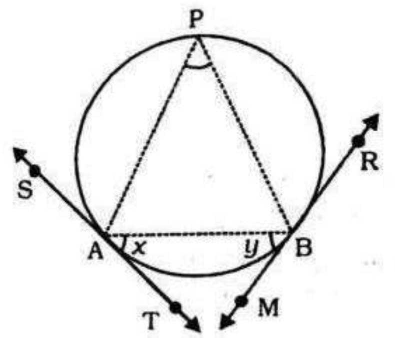
Now $\angle APB = x$

(angle is in alternate segment)

$\angle APB = y$

(angle is in alternate segment)

$\Rightarrow x = y$



87. (4) $\therefore$ As the opposite angles of a cyclic quadrilateral are supplementary,

$\therefore\ A + C = B + D = 180°$

$\therefore\ \cos A = \cos(180° - C) = -\cos C$

$\cos B = \cos(180° - D) = -\cos D$

$\therefore\ \cos A + \cos B + \cos C + \cos D$

$= \cos A + \cos B - \cos A - \cos B = 0$

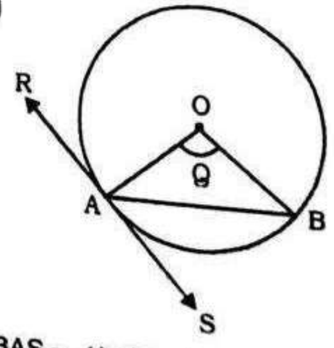
88. (2) Let $\angle BAO = \theta$, then

$\sin\theta = \frac{OL}{OA} = \frac{\frac{r}{2}}{r} = \frac{1}{2}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

89. (2)



$\angle BAS = \angle AOB = \theta$

(angles in alternate segments)

$\angle OAB = 90° - \angle BAS$

$\angle OAB = 90° - \theta$

$\angle OAB = \angle ABO$ [$\because$ OA = OB]

$\therefore \angle ABO = 90° - \theta$

90. (2) Since an angle in a semicircle is a right angle,

$\angle BAC = 90°$

$\therefore \angle ABC + \angle ACB = 90°$

Now, $AB = AC$

$\Rightarrow \angle ABC = \angle ACB = 45°$.

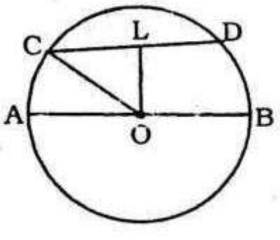
91. (2) $OA = OB$

$\Rightarrow \angle OBA = \angle OAB = 40°$

$\therefore \angle AOB = 180° - (40° + 40°) = 100°$

$\therefore \angle ACB = \frac{1}{2}\angle AOB = \left(\frac{1}{2}\times 100\right)^\circ = 50°$

92. (1) Join OC, Then, OC = radius = 17 cm.



$CL = \frac{1}{2}CD = \left(\frac{1}{2}\times 30\right)$ cm = 15cm

$OL^2 = OC^2 - CL^2 = (17)^2 - (15)^2 = (17+15)(17-15)$
$= (32 \times 2) = 64$

$\Rightarrow OL = \sqrt{64} = 8$ cm.

$\therefore$ Distance of CD from AB = 8 cm.

93. (2) Since equal chords of a circle subtend equal angles at the centre, so $\angle COD = \angle AOB = 80°$

94. (3) Let $OA = OC = r$ cm. Then, $OE = (r-3)$ cm and

$AE = \frac{1}{2}AB = 6$ cm.

Now, $OA^2 = OE^2 + AE^2$

$\Rightarrow r^2 = (r-3)^2 + 6^2$

$\Rightarrow 6r = 45 \Rightarrow r = \frac{45}{6} = 7.5$ cm

95. (1) Let the radius of the circle be r cm.

Then, $OD = OB = r$ cm,

OE = (r − 4) cm, ED = 8 cm.

Now, $OD^2 = OE^2 + ED^2$

$\Rightarrow r^2 = (r-4)^2 + 8^2$

$\therefore 8r = 80 \Rightarrow r = 10$ cm.

96. (4) Draw $OE \perp AB$ and $OF \perp CD$.

Then, $\Delta OEB \cong \Delta OFC$

[$\therefore OB = OC = r, \angle BOE = \angle COF$ (vertically opposite $\angle$s) and $\angle OEB = \angle OFC = 90°$]

$\therefore OE = OF$.

But, the chords equidistant from the centre are equal.

$\therefore$ CD = AB = 10 cm.

97. (2) $AB^2 = AC^2 - BC^2 = (AC^2 - AD^2) = \{(10)^2 - (2\sqrt{5})^2\}$

$= (100 - 20) = 80$

$\Rightarrow AB = \sqrt{80} = \sqrt{16\times 5} = 4\sqrt{5}$ cm.

$\therefore$ ar(rect. $ABCD$) = $(AB \times AD) = (4\sqrt{5} \times 2\sqrt{5})$ cm²

= 40 cm².

98. (3) $\angle AEB + \angle CEB = 180°$.

$\Rightarrow 110° + \angle CEB = 180°$

$\Rightarrow \angle CEB = 70°$.

In ΔCEB,

$\angle CEB + \angle EBC + \angle ECB = 180°$.

$\Rightarrow 70° + 30° + \angle ECB = 180°$

$\Rightarrow \angle ECB = 80°$.

$\therefore \angle ADB = \angle ACB = \angle ECB = 80°$

(Angles in the same segment).

99. (4) $OA = OB \Rightarrow \angle OBA = \angle OAB = 20°$.

In ΔOAB,

$\angle OAB + \angle OBA + \angle AOB = 180°$.

$\Rightarrow 20° + 20° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 140°$.

$OB = OC$

$\Rightarrow \angle OBC = \angle OCB = 50°$

In ΔOCB,

$\angle OCB + \angle OBC + \angle COB = 180°$,

$\Rightarrow 50° + 50° + \angle COB = 180°$

$\Rightarrow \angle COB = 80°$.

$\angle AOB = 140°$

$\Rightarrow \angle AOC + \angle COB = 140°$

$\Rightarrow \angle AOC + 80° = 140°$

$\Rightarrow \angle AOC = 60°$

100. (2) $\angle ABC + \angle ADC = 180°$

(opposite angles *of* cyclic quadrilateral)

$\Rightarrow \angle ABC + 120° = 180°$

$\Rightarrow \angle ABC = 60°$.

Also, $\angle ACB = 90°$ (angle in a semicircle).

In ΔABC,

$\angle BAC + \angle ACB + \angle ABC = 180°$

$\Rightarrow \angle BAC + 90° + 60° = 180°$

$\Rightarrow \angle BAC = 30°$.

101. (2) Since $ABCD$ is a cyclic quadrilateral, we have:

$\angle BAD + \angle BCD = 180°$

$\Rightarrow 100° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 80°$

(Now, $AB \parallel DC$ and CB is the transversal.

$\therefore \angle ABC + \angle BCD = 180°$

$\Rightarrow \angle ABC + 80° = 180°$

$\Rightarrow \angle ABC = 100°$

102. (2) $OB = BC$

$\Rightarrow \angle BOC = \angle BCO = 25°$.

Exterior $\angle OBA = \angle BOC + \angle BCO$

$= (25° + 25°) = 50°$.

$OA = OB$

$\Rightarrow \angle OAB = \angle OBA = 50°$.

In ΔAOC, side CO has been produced to D.

$\therefore$ Exterior $\angle AOD = \angle OAC + \angle ACO$

$= \angle OAB + \angle BCO = (50° + 25°) = 75°$.

103. (3) $\angle BDC = \angle BAC = 60°$ (Angles in the same segment of a circle).

In ΔBDC, $\angle DBC + \angle BDC + \angle BCD = 180°$.

$\therefore 50° + 60° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 70°$.

104. (3) $\angle BAC = 90°$ (angle in a semicircle).

In ΔABC, $\angle BAC + \angle ABC + \angle BCA = 180°$.

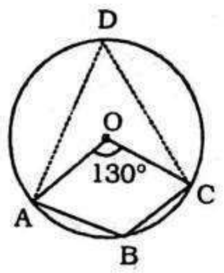
$\therefore 90° + \angle ABC + 30° = 180°$

$\Rightarrow \angle ABC = 60°$.

$\therefore \angle CDA = \angle ABC = 60°$

(angles in the same segment of a circle).

105. (3) Take a point D on the remaining part of the circumference. Join AD and CD. Then,



$\angle ADC = \frac{1}{2}\angle AOC = \left(\frac{1}{2} \times 130°\right) = 65°$.

In cyclic quadrilateral $ABCD$, we have:

$\angle ABC + \angle ADC = 180°$

$\Rightarrow \angle ABC + 65° = 180°$

$\Rightarrow \angle ABC = (180° - 65°) = 115°$.

106. (1) $\angle ADC = \angle BAD = 30°$

[Alternate Interior angles]

$\angle ADB = 90°$ (angle in a semicircle)

$\therefore \angle CDB = (30° + 90°) = 120°$.

But, $ABCD$ being a cyclic quadrilateral, we have:

$\angle BAC + \angle CDB = 180°$

$\Rightarrow \angle BAD + \angle CAD + \angle CDB = 180°$

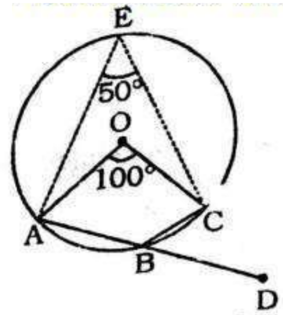
$\Rightarrow 30° + \angle CAD + 120° = 180°$

$\Rightarrow \angle CAD = 30°$.

107. (1) Take a point E on the remaining part of circumference of the circle. Join AE and CE.

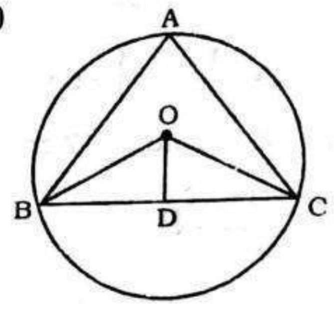
$\angle AEC = \frac{1}{2}\angle AOC$

$= \left(\frac{1}{2} \times 100°\right) = 50°$



Now, side AB of cyclic quadrilateral $ABCE$ has been produced to D. So, exterior $\angle CBD = \angle AEC = 50°$.

108. (4)



$BD = \frac{BC}{2} = 12$ cm

$OB = 13$cm

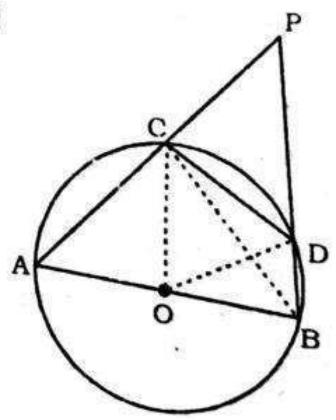
From Δ OBD,

$= OD = \sqrt{OB^2 - BD^2}$

$= \sqrt{13^2 - 12^2} = \sqrt{169 - 144}$

$= \sqrt{25} = 5$ cm

109. (3)



In Δ OCD,

$OC = OD = CD = r$

Δ OCD is an equilateral triangle.

$\angle COD = 60°$

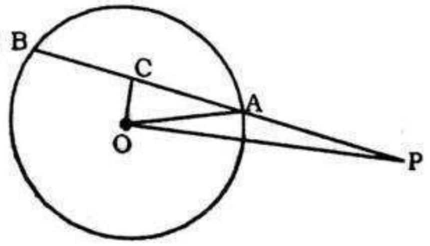
$\angle CBD = \frac{1}{2}$

$\angle COD = 30°$

$\angle ACB = 90°$
$\angle BCP = 180° - \angle ACB = 180° - 90° = 90°$
In Δ BCP,
$\angle BCP = 90°$, $\angle CBP = \angle CBD = 30°$
$\therefore \angle BCP + \angle CBP + \angle CPB = 180°$
$\Rightarrow 90° + 30° + \angle CPB = 180°$
$\Rightarrow \angle CBP = 60° \Rightarrow \angle APB = 60°$

110. (2) Length of common tangent $= \sqrt{d^2 - (R-r)^2}$

111. (2)



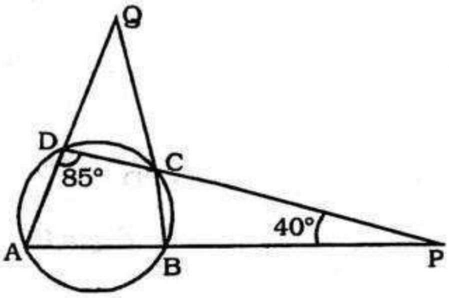
OC ⊥ AB
AC = BC = 3.5cm OP = 13 cm
PC = 9 + 3.5 = 12.5 cm

$\therefore OC = \sqrt{OP^2 - PC^2} = \sqrt{13^2 - 12^2} = \sqrt{12.75}$

$\therefore OA = \sqrt{OC^2 + CA^2} = \sqrt{12.75 + (3.5)^2}$

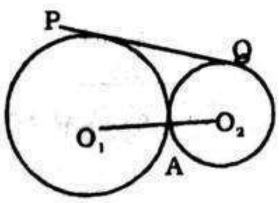
$= \sqrt{12.75 + 12.25} = \sqrt{25} = 5$ cm

112. (1)



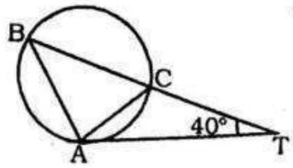
ABCD is a concyclic quadrilateral.
$\therefore \angle CBP = \angle ADC = 85°$
$\therefore \angle BCP = 180° - 40° - 85° = 55°$
$\therefore \angle DCB = 180° - 55° = 125°$
$\therefore \angle DCQ = 180° - 125° = 55°$
$\angle QDC = 180° - 85° = 95°$
$\therefore \angle DQC = 180° - 55° - 95° = 30°$

113. (4)



$\therefore PQ = \sqrt{4r_1r_2} = \sqrt{4 \times 8 \times 2} = 8$ cm

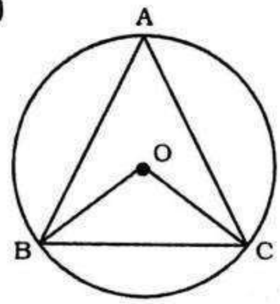
114. (4)



$\angle CAT = 44°$
$\angle BTA = 40°$
$\angle ACT = 180° - 44° - 40° = 96°$
$\angle CAT = \angle CBA = 44°$
$\angle BCA = 180° - 96° = 84$
$\therefore \angle BAC = 180° - 84° - 44° = 52°$
$\therefore$ Angle subtended by BC at centre
$= 2 \times 52° = 104°$

115. (4) $PQ^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2 = 4r_1r_2$

116. (3)

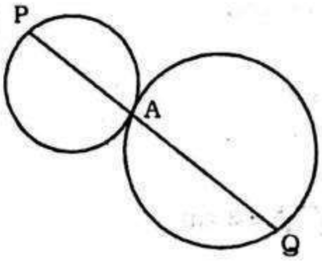


$\angle BOC = 2\angle BAC$
OB = OC
$\therefore \angle OBC = \angle OCB$

$\therefore \angle OBC = 90° - \dfrac{\angle BOC}{2}$

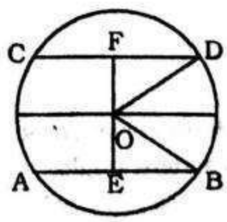
$= 90° - \angle BAC$
$\therefore \angle BAC + \angle OBC$
$= 90° - \angle BAC + \angle BAC = 90°$

117. (2)



$\therefore$ AP : AQ = 5 : 8

118. (*)



OE ⊥ AB

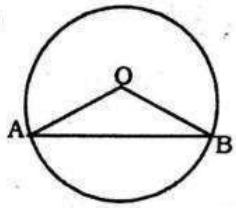
$\therefore$ BE = AE = 3 cm
and, OF $\perp$ CD
$\therefore$ FD = CF = 4 cm
From Δ OBE,

$OE = \sqrt{OB^2 - BE^2}$

$= \sqrt{5^2 - 3^2} = 4$ cm
From Δ OFD,

$OF = \sqrt{5^2 - 4^2} = 3$ cm
$\therefore$ EF = OE + OF = 4 + 3 = 7cm

119. (1)



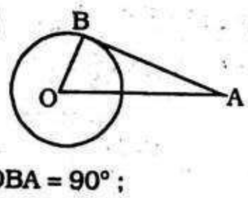
From Δ OAB,
$\angle AOB = 90°$
$OA^2 + OB^2 = AB^2$

$\Rightarrow 2r^2 = \left(3\sqrt{2}\right)^3 = 18$

$\Rightarrow r^2 = 9 \Rightarrow r = 3$ units
$\therefore$ Area of the sector AOB

$= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times 9 = \frac{9\pi}{4}$ sq. units

120. (1)



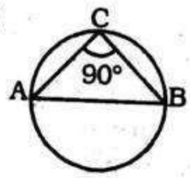
$\angle OBA = 90°$;
OA = 10 cm, OB = 6 cm
From Δ OAB,

$AB = \sqrt{OA^2 - OB^2}$

$= \sqrt{10^2 - 6^2}$

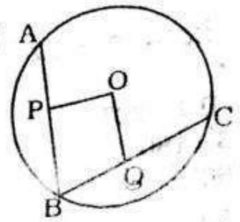
$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

121. (1)



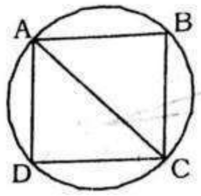
AB = Side of triangle = diameter of circle.
Angle of a semi-circle is a right angle.
i.e. $\angle ACB = 90°$
$\therefore$ ABC is a right angled triangle.

122. (2)



$\angle OPB = \angle OQB = 90°$
$\therefore \angle OPB + \angle OQB = 180°$
and, $\angle PBQ + \angle POQ = 180°$

123. (2)



$\pi r^2 = 36 \Rightarrow r^2 = \frac{36}{\pi}$

$r = \frac{6}{\sqrt{\pi}}$ cm

$\therefore$ AC = Diameter = $\frac{12}{\sqrt{\pi}}$ cm = Diagonal of square

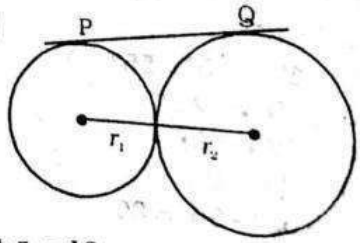
$\therefore$ Side of square = $\frac{1}{\sqrt{2}} \times$ Diagonal

$= \frac{1}{\sqrt{2}} \times \frac{12}{\sqrt{\pi}} = \frac{6\sqrt{2}}{\sqrt{\pi}}$ cm

$\therefore$ Area of Δ ACD = $\frac{1}{2} \times AD \times CD$

$= \frac{1}{2} \times \frac{6\sqrt{2}}{\sqrt{\pi}} \times \frac{6\sqrt{2}}{\sqrt{\pi}} = \frac{36}{\pi}$ sq.cm

124. (4)

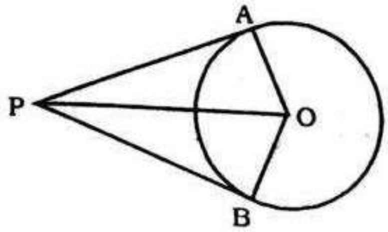


$r_1 + r_2 = 13$ cm
$r_2 - r_1 = 9 - 4 = 5$ cm

$PQ = \sqrt{\left(\text{distance between centres}\right)^2 - (r_2 - r_1)^2}$

$= \sqrt{\left(13^2 - 5^2\right)} = 12$ cm
$\therefore$ Area of square = 12 × 12 = 144 sq. cm.

125. (3)



In right Δs OAP and OPB,

AP = PB , OA = OB

OP = OP

∴ ΔOAP ≅ ΔOPB

∴ ∠AOP = ∠POB

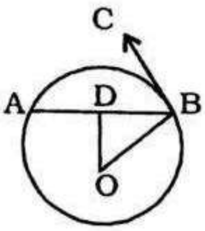
and ∠APO = ∠OPB

From Δ AOP,

∠APO = 180° – 90° – 60° = 30°

∴ ∠APB = 2 × 30 = 60°

126. (3)



∠ABC = 45°

⇒ ∠ABO = 45°

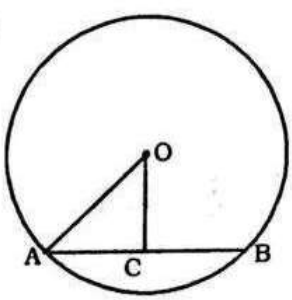
BD = 3 cm

$\cos OBD = \dfrac{BD}{OB}$

$\Rightarrow \cos 45° = \dfrac{3}{OB} \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{3}{OB}$

$\Rightarrow OB = 3\sqrt{2}$ cm

127. (2)



AC = CB = 4 cm

OC = 3 cm

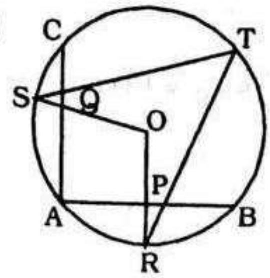
$\therefore OA = \sqrt{OC^2 + CA^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9+16} = \sqrt{25}$

= 5 cm

128. (2)



∠OQA = ∠OPA = 90°

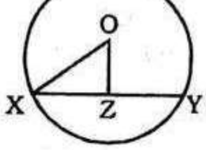
∠QOP + ∠QAP = 180°

⇒ ∠QOP = 180° – 32° = 148°

∠QOP = ∠SOR = 2 ∠STR

$\therefore \angle RTS = \dfrac{148}{2} = 74°$

129. (3)



XZ = ZY

OZ = 12 cm

OX = 13 cm

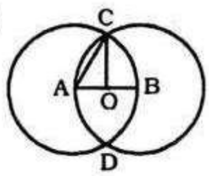
∴ XZ

$= \sqrt{OX^2 - OZ^2} = \sqrt{13^2 - 12^2}$

$= \sqrt{(13+12)(13-12)} = \sqrt{25} = 5$

∴ XY = 2 × 5 = 10 cm

130. (2)



$AO = OB = \dfrac{5}{2}$

AC = 5

$OC = \sqrt{AC^2 - OA^2}$

$= \sqrt{5^2 - \left(\dfrac{5}{2}\right)^2} = \sqrt{25 - \dfrac{25}{4}}$

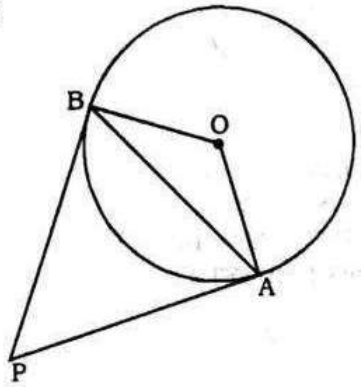
$= \sqrt{\dfrac{100-25}{4}} = \sqrt{\dfrac{75}{4}} = \dfrac{5\sqrt{3}}{2}$

$\therefore CD = 2\,OC = 2 \times \dfrac{5\sqrt{3}}{2} = 5\sqrt{3}$ cm

131. (1) $SR = \sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$

$= \sqrt{(13)^2 - (5)^2} = \sqrt{18 \times 8} = 12$ cm

132. (4)



OA⊥ AP and OB⊥ BP

$\angle OAP = 90°$ and $\angle OBP = 90°$

$\therefore$ $\angle OAP + \angle OBP = 90° + 90°$

$= 180°$

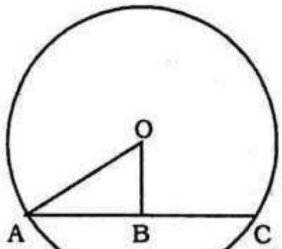
In quadrilateral OAPB,

$\angle OAP + \angle APB + \angle AOB + \angle OBP = 360°$

$\Rightarrow \angle APB + \angle AOB = 180°$

$\therefore$ The quadrilateral will be cyclic.

133. (2)



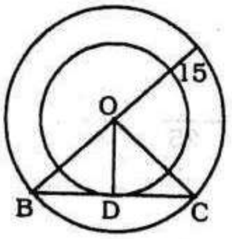
AB = BC = 8

OA = 10

$\therefore$ $OB = \sqrt{OA^2 - AB^2}$

$= \sqrt{10^2 - 8^2} = \sqrt{36} = 6$

134. (1)



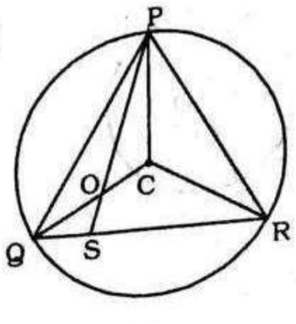
BO = OC = 15 cm.

OD = 9 cm.

$\therefore$ $BD = \sqrt{15^2 - 9^2} = \sqrt{24 \times 6} = 12$ cm

$\therefore$ BC = 2 × 12 = 24 cm.

135. (2)



$\angle PQS = 60°$

$\angle QCR = 130°$

$\therefore$ $\angle QPR = \frac{1}{2} \times 130° = 65°$

$\Rightarrow QPR = 180° - 60° - 65° = 55°$

$\therefore$ In Δ QCR

QC = CR

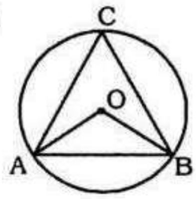
$\therefore$ $\angle CQR = \angle CRQ = 25°$

$\therefore$ $\angle PQC = \angle QPC = 35°$

$\angle CPR = 30°$

$\therefore$ $\angle RPS = 35°$

136. (1)



AO = OB = AB

$\therefore \angle AOB = 60°$

$\therefore \angle ACB = 30°$

137. (2) For the equilateral triangle of side a,

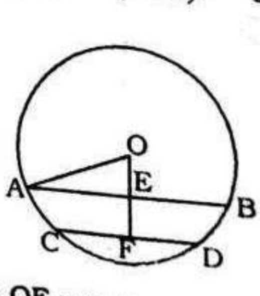
In radius = $\frac{a}{2\sqrt{3}}$

Circum-radius = $\frac{a}{\sqrt{3}}$

$\therefore$ Required ratio

$= \pi\left(\frac{a}{\sqrt{3}}\right)^2 : \pi\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{1}{3} : \frac{1}{12} = 4:1$

138. (1)



Let OE = x cm

$\therefore$ OF = $(x + 1)$ cm

OA = OC = r cm

AE = 4 cm; CF = 3 cm

From ΔOAE,

$OA^2 = AE^2 + OE^2$

$\Rightarrow r^2 = 16 + x^2$

$\Rightarrow x^2 = r^2 - 16$ .....(i)

From ΔOCF,

$(x + 1)^2 = r^2 - 9$ .... (ii)

By equation (ii) – (i),

$(x + 1)^2 - x^2 = r^2 - 9 - r^2 + 16$

$\Rightarrow 2x + 1 = 7$

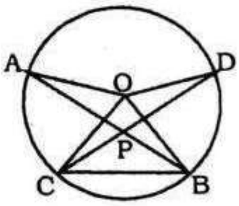
$\Rightarrow x = 3\text{cm}$

∴ From equation (i),

$9 = r^2 - 16 \Rightarrow r^2 = 25$

$\Rightarrow r = 5$

139. (3)



Join CB.

∠AOC + ∠BOD

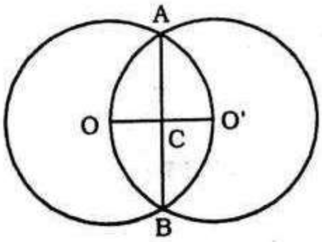
= 2∠ABC + 2∠BCD

(Exterior angles of triangle)

= 2 (∠ABC + ∠BCD) = 2∠BPD

$\therefore \angle BPD = \frac{1}{2}(50° + 40°) = 45°$

140. (2)



OC = 2cm

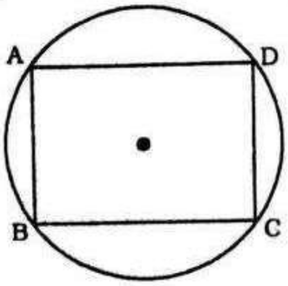
OA = 4cm

$\therefore AC = \sqrt{4^2 - 2^2} = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$

$\therefore AB = 4\sqrt{3}$ cm

141. (2) The largest chord of a circle is its diameter.

142. (4)



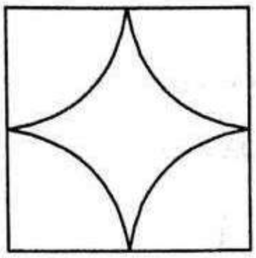
ABCD is a cyclic parallelogram.

∴ ∠B + ∠D = 180°

⇒ 2∠B = 180°

⇒ ∠B = 90°

143. (2)

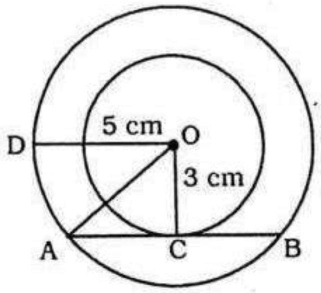


Area of sectors = $\pi r^2 = 4\pi$ sq.cm.

Area of square = 4 × 4 = 16 sq.cm.

Area of the remaining portion = $(16 - 4\pi)$ sq.cm.

144. (3)

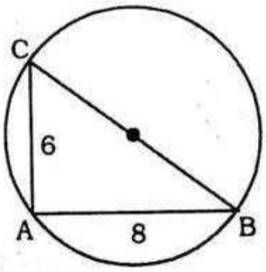


$AC = \sqrt{AO^2 - OC^2} = \sqrt{5^2 - 3^2}$

$= \sqrt{25-9} = \sqrt{16} = 4$ cm

∴ AB = 2 × 4 = 8 cm

145. (4)



∠BAC = 90°

∴ BC is the diameter of the circle.

$\therefore BC = \sqrt{AB^2 + AC^2} = \sqrt{8^2 + 6^2}$

$= \sqrt{64+36} = \sqrt{100} = 10$ cm

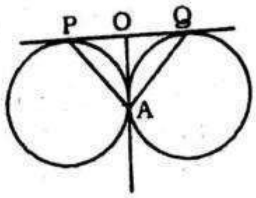
∴ Radius of the circle = 5 cm

146. (3) The chord nearer to the centre is larger.

$\therefore \frac{15}{8} = \frac{x}{16}$

$\Rightarrow x = \frac{15 \times 16}{8} = 30$ cm

147. (2)

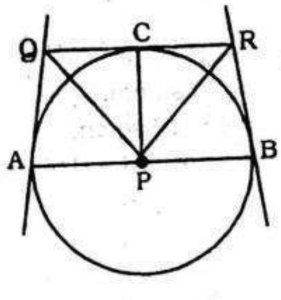


AO is perpendicular to PQ.

OA = OP = OQ.

$\angle OPA = \angle OAP = \angle OQA = 45°$

148. (3)



In ΔPCR and ΔRBP,

PC = PB (radii)

RC = RB

PR is common.

$\therefore\ \Delta PCR \cong \Delta RBP$

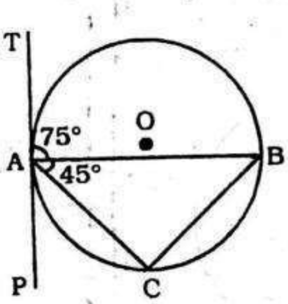
$\therefore\ \angle CPR = \angle RPB$

Similarly, $\angle CPQ = \angle QPA$

$\therefore \angle QPR = 90°$

because $\angle APB = 180°$

149. (3)

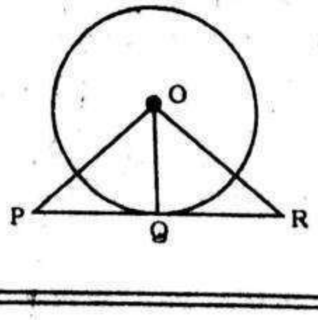


If a line touches a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respe. ctively to the angles formed in the corresponding alternate segments.

$\therefore\ \angle ACB = \angle BAT = 75°$

$\angle ABC = 180° - 45° - 75° = 60°$

150. (4)



$OQ \perp PR$

$\therefore$ From ΔOPQ,

$$PQ = \sqrt{OP^2 - OQ^2}$$

$$= \sqrt{\left(\frac{20}{3}\right)^2 - 4^2} = \sqrt{\frac{400}{9} - 16}$$

$$= \sqrt{\frac{400-144}{9}} = \sqrt{\frac{256}{9}} = \frac{16}{3} \text{ cm}$$

From ΔOQR,

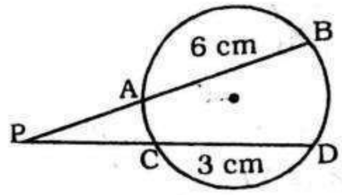
$$QR = \sqrt{OR^2 - OQ^2}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25-16}$$

$$= \sqrt{9} = 3 \text{ cm}$$

$$\therefore PR = PQ + QR = \frac{16}{3} + 3 = \frac{25}{3} \text{ cm}$$

151. (*)



AB = 6 cm; CD = 3 cm

PD = 5 cm; PB = ?

$PA \times PB = PC \times PD$

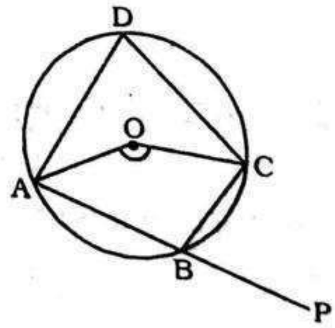
$\Rightarrow (PB - 6)\, PB = 2 \times 5$

$\Rightarrow PB^2 - 6PB - 10 = 0$

$$\Rightarrow PB = \frac{6 \pm \sqrt{36+40}}{2}$$

$$= \frac{6 \pm \sqrt{76}}{2} = \frac{6 + 8.7}{2} \approx 7.35$$

152. (3)

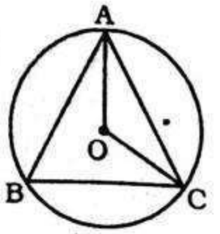


$\angle AOC = 130°$

$$\angle ADC = \frac{1}{2} \times 130° = 65°$$

$\angle PBC = \angle ADC = 65°$

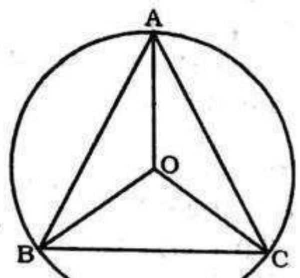
153. (2)



$\angle ABC = 180° - 85° - 80° = 15°$

$\Rightarrow \angle OAC = 2\angle ABC = 2 \times 15 = 30°$

154. (1) The point where the right bisectors of the sides meet, is called the circum-centre.



OB = OC = radius

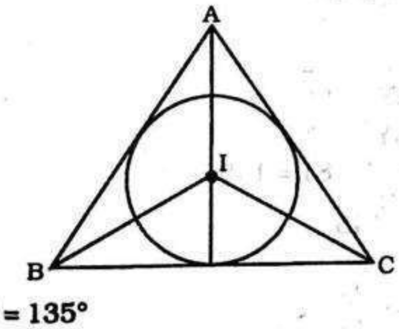
$\therefore \angle OBC = \angle OCB = 35°$

$\therefore \angle BOC = 180 - 70 = 110°$

$\therefore \angle BAC = 55°$

The angle subtended at the centre by an arc is twice to that at the circumference.

155. (3) The point where internal bisectors of angles of a triangle meet is called in–centre.



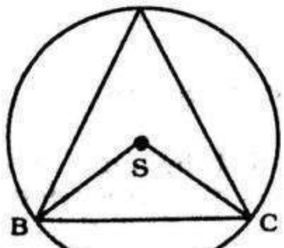
$\angle BIC = 135°$

$\therefore \frac{1}{2}(\angle B + \angle C) = 45°$

$\Rightarrow \angle B + \angle C = 90°$

$\therefore \angle A = 90°$

156. (2)



$\angle BAC = 50°$

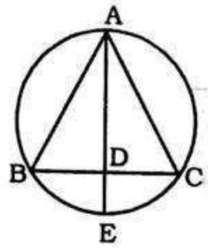
$\therefore \angle BSC = 100°$

BS = SC = radius

$\therefore \angle BCS = \frac{1}{2}(180 - 100)$

$= 40°$

157. (2)

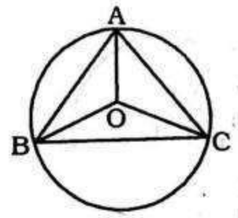


AB . AC + AE . DE = AE²

AB . AC = AE (AE – DE) = AE . AD.

158. (3) $\because \angle BAC = 85°$

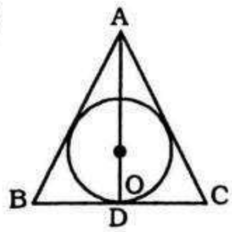
$\therefore \angle BOC = 2 \times 85° = 170°$



$\therefore \angle OBC = \angle OCB = 5°$

$\therefore \angle OCA = \angle OAC = 75° - 5° = 70°$

159. (3)



$BD = DC = 7\sqrt{3}$ cm

$AD = \sqrt{AB^2 - BD^2} = \sqrt{(14\sqrt{3})^2 - (7\sqrt{3})^2}$

$= \sqrt{(14\sqrt{3} + 7\sqrt{3})(14\sqrt{3} - 7\sqrt{3})}$

$= \sqrt{21\sqrt{3} \times 7\sqrt{3}} = 21$cm

$\therefore$ OD = Radius of circle $= \frac{1}{3} \times 21 = 7$cm

$\therefore$ Area of circle $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7 = 154$ sq.cm.

160. (1) $\pi(r+1)^2 - \pi r^2 = 22$

$\Rightarrow \pi(r^2 + 2r + 1 - r^2) = 22$

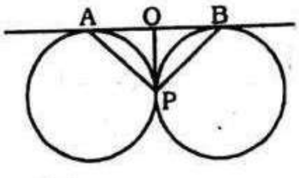
$\Rightarrow 2\pi r + \pi = 22$

$\Rightarrow \frac{22}{7}(2r+1) = 22$

$\Rightarrow 2r + 1 = 7$

$\Rightarrow 2r = 6 \Rightarrow r = 3$ cm.

161. (2)



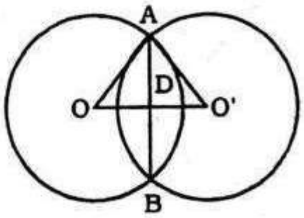
OA = OP

$\therefore \angle PAB = OPA = 35°$

$\therefore \angle AOP = 110° \Rightarrow \angle POB = 70°$

$\therefore \angle ABP = \frac{180°-70°}{2} = \frac{110}{2} = 55°$

162. (2)



$OD = \sqrt{15^2 - 12^2}$

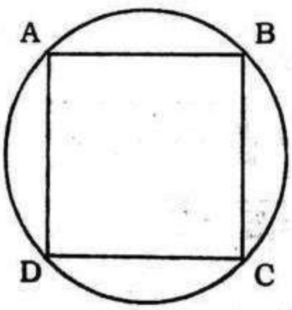
$= \sqrt{225-144} = \sqrt{81} = 9$

$O'D = \sqrt{13^2 - 12^2}$

$= \sqrt{169-144} = \sqrt{25} = 5$

$\therefore OO' = 9 + 5 = 14$ cm

163. (3)

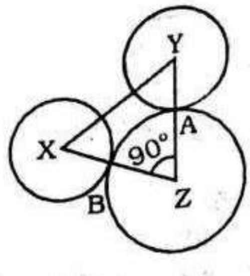


BD = Diagonal = 16 cm

Area of square $= \frac{1}{2} \times BD^2$

$= \frac{1}{2} \times 16 \times 16 = 128$ sq. cm.

164. (2) $XZ = r + 9$



$YZ = r + 2$

$\therefore XY^2 = XZ^2 + ZY^2$

$\Rightarrow 17^2 = (r+9)^2 + (r+2)^2$

$\Rightarrow 289 = r^2 + 18r + 81 + r^2 + 4r + 4$

$\Rightarrow 2r^2 + 22r + 85 - 289 = 0$

$\Rightarrow 2r^2 + 22r - 204 = 0$

$\Rightarrow r^2 + 11r - 102 = 0$

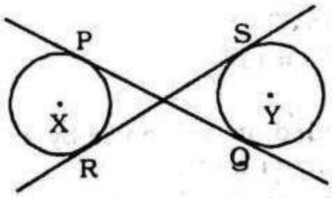
$\Rightarrow r^2 + 17r - 6r - 102 = 0$

$\Rightarrow r(r+17) - 6(r+17) = 0$

$\Rightarrow (r-6)(r+17) = 0$

$\Rightarrow r = 6$ cm

165. (1)



Length of transverse tangent

$= \sqrt{XY^2 - (r_1 + r_2)^2}$

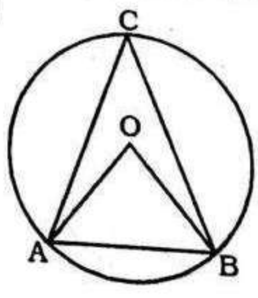
$\Rightarrow 8 = \sqrt{XY^2 - 9^2}$

$\Rightarrow 64 = XY^2 - 81$

$\Rightarrow XY^2 = 64 + 81 = 145$

$\Rightarrow XY = \sqrt{145}$

166. (4)



$\therefore OA = OB = AB$

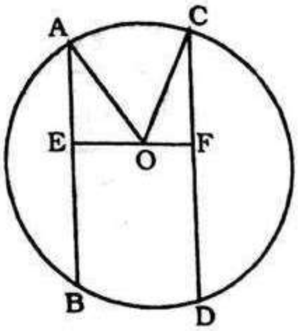
$\therefore \Delta OAB$ is an equilateral triangle.

$\therefore \angle AOB = 60$

The angle subtended by an arc at the circumference is half of that at the centre.

$\therefore \angle ACB = \frac{1}{2} \times 60° = 30°$

167. (1)



OE ⊥ AB and OF ⊥ CD
AE = EB = 5 cm
CF = FD = 12 cm
AO = OC = 13 cm
From Δ AOE,

$OE = \sqrt{13^2 - 5^2}$

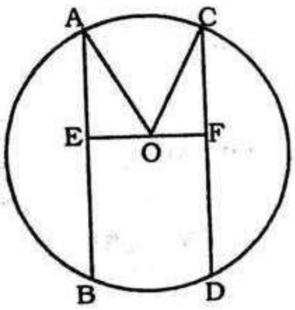
$= \sqrt{169 - 25} = \sqrt{144} = 12$ cm

From Δ COF,

$OF = \sqrt{13^2 - 12^2} = \sqrt{25} = 5$ cm

∴ EF = OE + OF = 17 cm

168. (2)



AB = 24 cm
AE = EB = 12 cm

$OE = \sqrt{OA^2 - AE^2}$

$= \sqrt{15^2 - 12^2}$

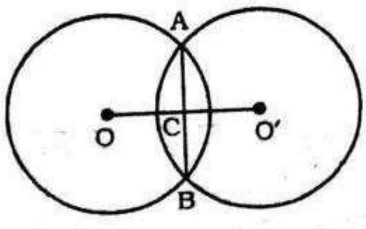
$= \sqrt{225 - 144} = \sqrt{81} = 9$ cm

∴ OF = 21 − 9 = 12 cm

∴ $CF = \sqrt{15^2 - 12^2} = 9$ cm

∴ CD = 2 × 9 = 18 cm

169. (1)



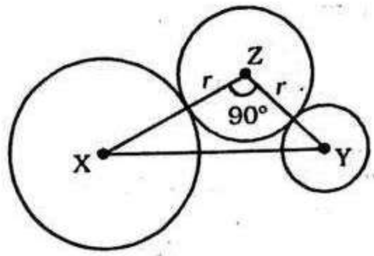
AB = 16
AC = BC = 8 cm
OC = CO′ = 6 cm

∴ $OA = \sqrt{OC^2 + CA^2}$

$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$

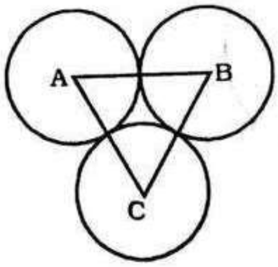
$= \sqrt{100} = 10$ cm

170. (4)



∠XZY = 90°
XY = (9 + r) cm,
YZ = (r + 2) cm
XY = 17 cm
∴ $XY^2 = XZ^2 + ZY^2$
⇒ $17^2 = (9 + r)^2 + (r + 2)^2$
⇒ $289 = 81 + 18r + r^2 + r^2 + 4r + 4$
⇒ $2r^2 + 22r - 204 = 0$
⇒ $r^2 + 11r - 102 = 0$
⇒ $r^2 + 17r - 6r - 102 = 0$
⇒ $r(r + 17) - 6(r + 17) = 0$
⇒ $(r - 6)(r + 17) = 0$
⇒ r = 6 cm

171. (4)



AB = 4 + 6 = 10 cm
BC = 6 + 8 = 14 cm
CA = 8 + 4 = 12 cm
∴ Semi-perimeter(s)

$= \frac{10 + 14 + 12}{2} = 18$ cm

∴ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-10)(18-14)(18-12)}$

$= \sqrt{18 \times 8 \times 4 \times 6}$

$= 3 \times 2 \times 2 \times 2\sqrt{6}$

$= 24\sqrt{6}$ sq.cm.

172. (2) $\theta = 72° = 72 \times \frac{\pi}{180}$ radians

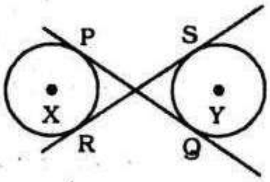
$= \frac{2\pi}{5}$ radians

$\therefore \theta = \frac{s}{r}$

$\Rightarrow s = \theta.r = \frac{2\pi}{5} \times 21$

$= \frac{2}{5} \times \frac{22}{7} \times 21 = \frac{132}{5} = 26.4$ cm.

173. (1)

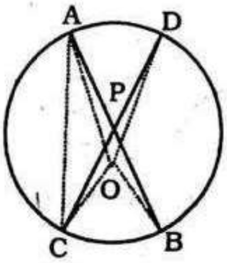


Transverse common tangent

$= \sqrt{\left(\begin{matrix}\text{Distance between}\\ \text{centres}\end{matrix}\right)^2 - (r_1 + r_2)^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{16 \times 4} = 8$ cm

174. (3)



$\angle BOC = 2\angle BAC$

$\angle AOD = 2\angle DCA$

$\therefore \angle BOC + \angle AOD$

$= 2(\angle BAC + \angle DCA) = 2\angle BPC$

$\therefore 2\angle BPC = 20° + 30° = 50°$

$\Rightarrow \angle BPC = 25°$

175. (2) One and only circle can pass through three non-collinear points.

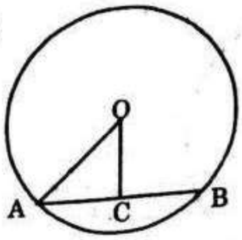
176. (3) Length of the transverse tangent

$= \sqrt{(\text{distance between the centres})^2 - (r_1 + r_2)^2}$

$= \sqrt{(10)^2 - (3+3)^2}$

$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

177. (3)



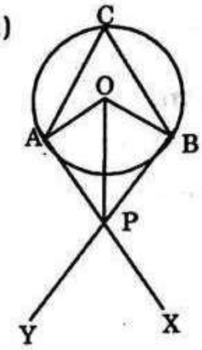
AC = BC = 4 cm.

OC = 3 cm.

$\therefore OA = \sqrt{AC^2 + OC^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ cm

178. (1)



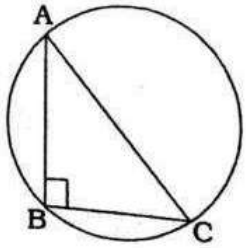
$\angle ACB = 65°$

$\angle AOB = 2 \times 65° = 130°$

$\angle OAP = 90°$; $\angle AOP = 65°$

$\therefore \angle APO = 180° - 90° - 65° = 25°$

179. (2)



$3^2 + 4^2 = 5^2$

Δ ABC is a right angled triangle.

$\angle B = 90°$ = angle at the circumference

$\therefore$ Diameter of circle = 5 cm

$\therefore$ Circum-radius = 2.5 cm

IMPORTANT POINTS : AT A GLANCE

LINES AND ANGLES

AXIOMS/POSTULATES : The basic facts which are taken for granted, without proof, are called axioms.

The postulates are:

- Space contains at Least two distinct points.
- A line contains infinitely many points and contains at least two distinct points.
- A plane is a set of many points and contains at least three non-collinear points.
- If two distinct plane intersect, then their intersection is a straight line.
- Halves of equals are equal.
- Through a given point, there pass an infinite number of lines.
- Given two distinct points *P* and *Q*, there is one and only one line that contains both the points.
- Three or more than three points are said to be collinear, then there is a line which contains them all.
- Two lines can intersect at the most at one point.
- Two lines having a common point are known as intersecting point.
- Two lines which are both parallel to the same line are parallel to each other.
- Point is represented by a dot on which length, breadth and height cannot be measured.

Line Segment: The straight path between two points *P* and *Q* is called a line segment $\overline{PQ}$.

- *P* and *Q* are end points of line segment.
- It has a definite length.
- Distance between *P* and *Q* is length of the line segment *PQ*.

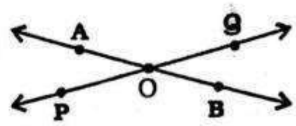
Ray: A ray extends indefinitely in one direction. This is exhibited by an arrow *i.e.*, $\vec{PQ}$.

- *P* is called initial point.
- It has no definite length.
- It cannot be drawn but can simply be represented on the plane of a paper.

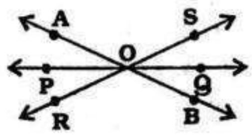
Line: A line segment $\overline{PQ}$ when extended indefinitely in both the directions is called line $\overline{PQ}$.

- Line has no end points.
- Line has no definite length.
- Line cannot be drawn.
- Line is a set of infinite points.

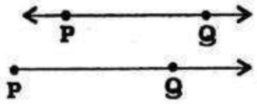
Intersecting Lines : Two lines having a common point are called intersecting lines. This common point is point of intersection *i.e.*, '*O*'



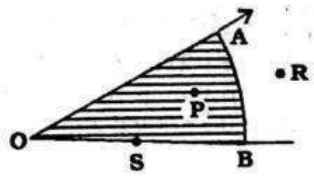
Concurrent Lines: Three or more lines intersecting at the same point are said to be concurrent.



Parallel Lines: Two lines *l* and *m* in a plane are said to be parallel. If they have no common point and we write $l \parallel m$.



Angles: A figure consisting of two rays with end points is called an angle.



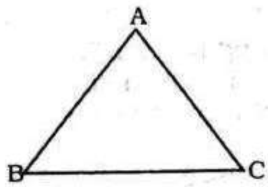
- *P* is a point in the interior of $\angle AOB$.
- *S* is a point on $\angle AOB$.
- *R* is a point in the exterior of $\angle AOB$.

An Angle of 360°: If a ray *OP* starting from its original position *OP*, rotates about *O*, in the anticlockwise direction and after making a complete revolution it comes back to its original position we say it has rotated through 360 degree Written as 360°

- $1° = 60$ minutes, written as $60'$
- $1' = 60$ seconds, written as $60''$
- If the sum of two angles is 90°, then they are complementary to each other.
- If the sum of two angles is 180°, they are supplementary to each other.

TRIANGLES

A plane (closed) figure bounded by three line segments is called a triangle. Triangles are denoted by Δ.



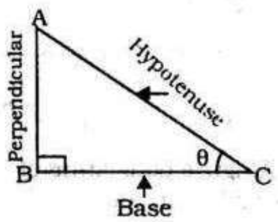
A Δ ABC has :

- three vertices, namely A, B and C
- three sides, AB, BC and AC.
- three angles, namely ∠A, ∠B and ∠C.

A triangle has six parts-three sides and three angles.

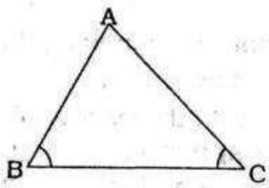
Types of Triangles On The Basis of Angles

(a) Right-angled Triangle : A triangle in which one of the angles measures 90° is called a right-angled triangle. The side opposite to the right angle is called its hypotenuse and the remaining two sides are called as perpendicular and base depending upon conditions.



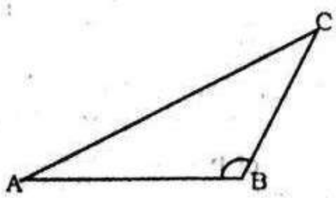
Here Δ ABC has ∠B = 90° and AC is hypotenuse.

(b) Acute-angled Triangle : A triangle in which every angle measures more than 0° and is less than 90° is called an acute-angled triangle.



Here Δ ABC is an acute-angled triangle.

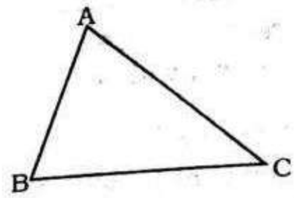
(c) Obtuse-angled Triangle : A triangle in which one of the angles measures more than 90° but less than 180° is called an obtuse-angled triangle.



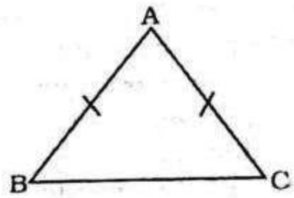
Here, Δ ABC is an obtuse-angled and ∠ABC is the obtuse angle.

Types of Triangles On the basis of Sides

(A) Scalene Triangle : A triangle in which all the sides are of different lengths is called a scalene triangle. Δ ABC is a scalene triangle as AB ≠ BC ≠ AC.

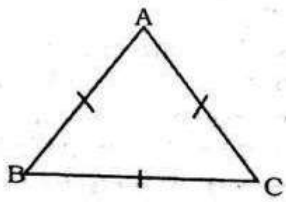


(B) Isosceles Triangle : A triangle in which two sides are equal is called an isosceles triangle. Here Δ ABC is an isosceles triangle as AB = AC.



- Angles opposite to equal sides are equal i.e., ∠B = ∠C

(C) Equilateral Triangle : A triangle having all sides equal is called an equilateral triangle. Here in Δ ABC, AB = BC = AC.



- All angles are equal and each is of measure 60°.

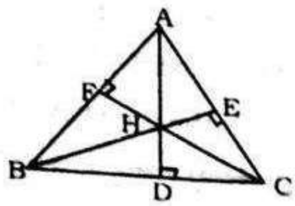
Perimeter of a Triangle : The sum of the lengths of three sides of a triangle is called its perimeter.

So in Δ ABC, perimeter

= AB + BC + AC.

Some Terminologies Related to A Triangle

Altitudes : The altitude of a triangle is a line segment perpendicularly drawn from vertex to the side opposite to it. The side on which the perpendicular is being drawn is called its base.

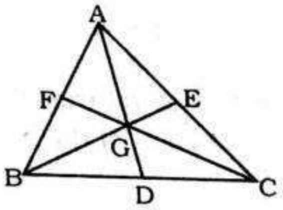


Here AD, BE and FC are altitudes drawn on BC, AC and AB respectively.

IMPORTANT POINTS : AT A GLANCE

- Altitudes of a triangle are concurrent.
- The point of intersection of all the three altitudes of a triangle is called its ortho-centre.

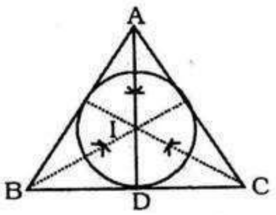
Medians : A line segment joining the mid-point of the side with the opposite vertex.



Here AD, BE and CF are medians.

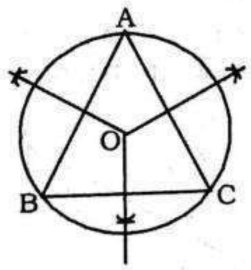
- The medians of a triangle are concurrent.
- The point of intersection of all the three medians is called its centroid.
- Centroid is denoted by G.

In-centre of a Triangle : The point of intersection of all the three angle bisectors of a triangle is called its in-centre.



The circle with centre I and radius ID is called incircle and radius is called as inradius denoted by 'r',

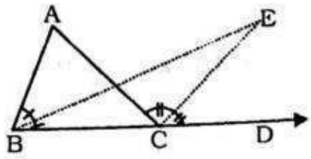
Circumcentre of a Triangle : The point of intersection of the perpendicular bisectors of the sides of a triangle is called its circumcentre



Circle through O and radius OA = OB = OC passing through A, B, C is called circumcircle. Radius of circumcircle is called circumradius denoted by R.

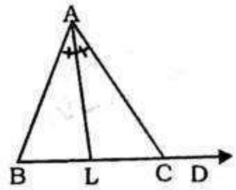
Some Useful Results on Triangles

- The sum of the angles of a triangle is 180°.
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.
- An exterior angle of a triangle is greater than either of the interior opposite angles.
- The internal bisector of one base angle and the external bisector of the other is equal to one half of the vertical angle.

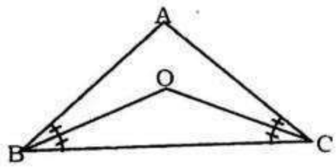


Here $\angle E = \frac{1}{2}\angle A$

- The side BC of Δ ABC is produced to D. The bisector of ∠A meets BC in L. Then ∠ABC + ∠ACD = 2 ∠ALC.

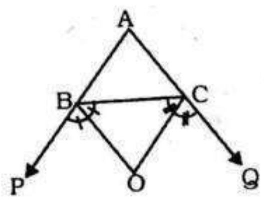


- In a Δ ABC the bisector of ∠B and ∠C intersect each other at a point O.



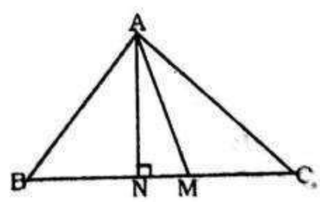
$\angle BOC = 90 + \frac{1}{2}\angle A$

- In a Δ ABC, the side AB and AC are produced to P and Q respectively. The bisectors of ∠PBC and ∠QCB intersect at a point O.



Then $\angle BOC = 90° - \frac{1}{2}\angle A$

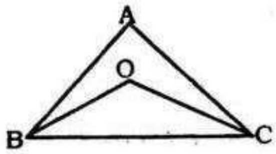
- In Δ ABC, ∠B > ∠C. If AM is the bisector of ∠BAC and AN ⊥ BC, then



$\angle MAN = \frac{1}{2} \angle B - \angle C$

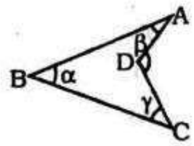
- The bisectors of the base angles of a triangle can never enclose a right angle.

  Here OB and OC are the bisectors of $\angle B$ and $\angle C$.

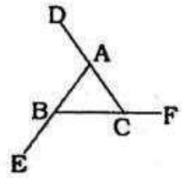


But $\angle BOC \neq 90°$.

- In figure $\angle ADC = \alpha + \beta + \gamma$

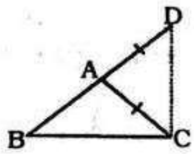


- If the three sides of a triangle be produced in order then the sum of all the exterior angles so formed is 360°.



So $\angle DAB + \angle EBC + \angle ACF = 360°$

- The sum of any two sides of a triangle is greater than its third side.



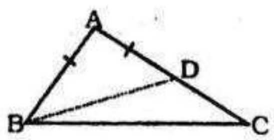
Here in ΔABC

$AB + AC > BC$

$AB + BC > AC$

$BC + AC > AB$.

- The difference between any two sides of a triangle is less than its third side.



Here in ΔABC,

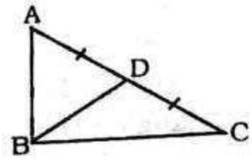
$AC - AB < BC$

$BC - AC < AB$.

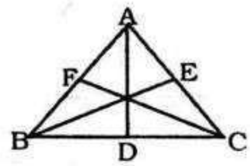
$BC - AB < AC$.

- If the bisector of the vertical angle of a triangle bisects the base, then that triangle is isosceles.
- If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles.
- The perpendiculars drawn from the vertices of equal angles of an isosceles triangle to the opposite sides are equal.
- Medians of equilateral triangle are equal.
- If D is the mid-point of the hypotenuse AC of a right-angled

  ΔABC. Then $BD = \frac{1}{2} AC$



- The sum of any two sides of a triangle is greater than twice the median drawn to the third side.
- Perimeter of a triangle is greater than the sum of its three medians.



So,

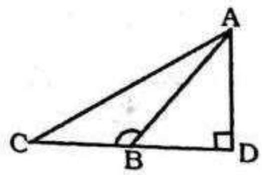
$AB + BC + AC > AD + BE + CF$

- In a ΔABC, $\angle B = 90°$, an obtuse angle or an acute angle accordingly

  $AC^2 = AB^2 + BC^2$ if $\angle B = 90°$

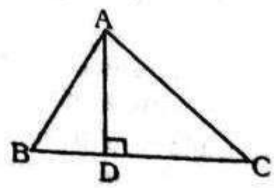
  $AC^2 > AB^2 + BC^2$ if $\angle B > 90°$

  $AC^2 < AB^2 + BC^2$ if $\angle B < 90°$



- In a ΔABC, $\angle B$ is obtuse then

  $AC^2 = AB^2 + BC^2 + 2BC\ BD$
- In ΔABC if $\angle B$ is acute then $AB^2 = BC^2 + AC^2 - 2BC \cdot CD$ and $AD \perp BC$.



Congruent Triangles : Two triangles are said to be congruent if both are exactly of same size i.e., all angles and sides are equal to corresponding angles and sides of other.

- Every triangle is congruent to itself $\Delta ABC \cong \Delta ABC$
- If $\Delta ABC \cong \Delta DEF$, then $DEF \cong \Delta ABC$
- If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta PQR$ then $\Delta ABC \cong \Delta PQR$

Sufficient Conditions For Congruence of Two Triangles

Theorem 1 : If two triangles have two sides and the included angle of the one are equal to the corresponding sides and the included angle of the other. (SAS)

Theorem 2 : If two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle. (ASA)

Theorem 3 : If three sides of one triangle are respectively equal to the three sides of the other. (SSS)

Theorem 4 : If the hypotenuse and other side of one triangle are respectively equal to the hypotenuse and the corresponding side of the other triangle. (RHS)

Some other Relations in a Triangle

- Angles opposite to two equal sides of a triangle are equal.
- If two angles of a triangle are equal then the sides opposite to them are also equal.
- If two sides of a triangle are unequal the longer side has greater angle opposite to it.
- In a triangle, the greater angle has the longer side opposite to it.

Congruent Figures : The geometrical figures having the same shape and size are known as congruent figures.

Congruent figures are just like photostat copies, which are alike in every respect.

Similar Figures : Geometric figures having the same shape but different sizes are known as similar figures.

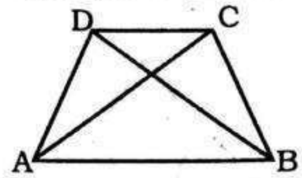
- The congruent figures are always similar but two similar figures need not be congruent. e.g., Any two circles are similar. Any two rectangles are similar.

Similar Triangles : Two triangles are said to be similar to each other if

- their corresponding sides are proportional.
- their corresponding angles are equal.

QUADRILATERALS

A plane, closed figure bounded by four line segments is called quadrilateral. There are different types of quadrilaterals so called trapezium, parallelogram, rhombus, rectangle, square.



- Sum of all the angles of a quadrilateral is 360°.
- Here ABCD is a quadrilateral. ($\angle A$, $\angle B$); ($\angle B$, $\angle C$); ($\angle C$, $\angle D$); ($\angle D$, $\angle A$) are four pairs of consecutive angles of quadrilateral ABCD.
- AC and BD are diagonals.
- (AB, BC); (BC, CD); (CD, DA) and (DA, AB) are four pairs of adjacent sides.

Various types of quadrilaterals

(A) Parallelogram : A quadrilateral in which opposite sides are equal and parallel then it is a parallelogram, written as || gm.

e.g. square, rectangle, rhombus.

Properties of Parallelogram

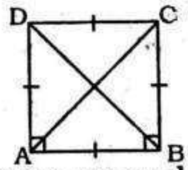
(a) Opposite sides are equal.

(b) Opposite angles are equal.

(c) The two diagonals bisect each other.

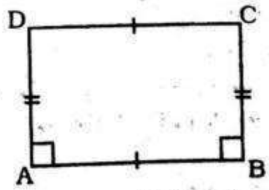
(d) Diagonal are equal in case of square and rectangle but not in rhombus.

Square : A parallelogram in which all sides are equal and are parallel. Here angle between the adjacent sides is 90°.



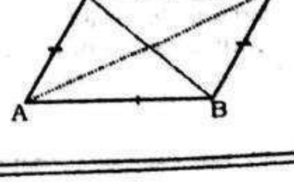
- Diagonals of square are equal. AC = BD
- Diagonals bisects the angle
- Diagonals are perpendicular to each other.

Rectangle :



The parallelogram in which only opposite side are equal and parallel, angle between adjacent sides is 90°.

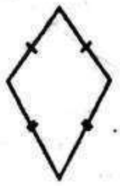
Rhombus :



A parallelogram in which all sides are equal is called a rhombus.

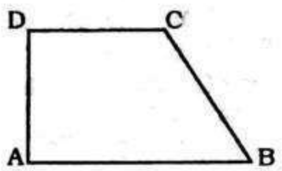
- Diagonals bisect each other at 90°
- Diagonals are not equal
- Sum of square of sides is equal to sum of the square of diagonal. i.e.
  $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Kite :



A quadrilateral in which two pairs of adjacent sides are equal is known as kite.

Trapezium :

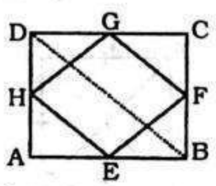


A quadrilateral in which two opposite sides are parallel and two opposite sides are not parallel. It is called a trapezium.

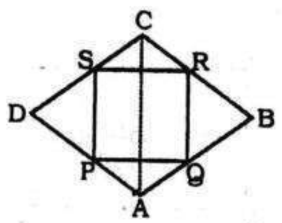
Here AB || DC

Some facts about parallelogram

- A diagonal of a parallelogram bisects one of the angles of the parallelogram, it also bisects the second angle, and then the two diagonals are perpendicular to each other.



- The quadrilateral formed by joining the mid-points of the consecutive sides of a rectangle is a rhombus. Here E, F, G, H are mid-points of AB, BC, CD, DA respectively then EFGH is a rhombus.



- The quadrilateral formed by joining the mid-point of the consecutive sides of a rhombus is a rectangle. Here PQRS will be a rectangle.
- The quadrilateral formed by joining the mid-points of the sides of a square, is also a square.
- The figure formed by joining the mid-points of the pairs of consecutive sides of a quadrilateral is a parallelogram.

POLYGONS

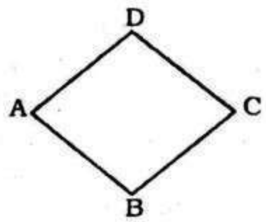
A polygon is a closed, plane figure bounded by 'n' straight lines ($n \geq 3$). Each of the n line segment forming the polygon is called its sides.

A polygon may be a triangle, quadrilateral, pentagon etc. Polygons are classified according to the number of sides as given below :

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Regular Polygon : A polygon is called a regular polygon if all its sides are equal and all angles are equal.

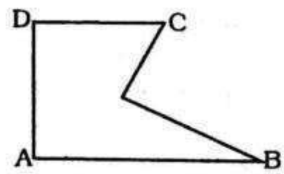
Convex Polygon :



A polygon is said to be a convex polygon, if each side of the polygon, the line containing that side has all the other vertices on the same side of it.

- None of its interior angle is more than 180°.

Concave Polygon :



A polygon in which at least one of the interior angle is more than 180°. Then the polygon is said to be a concave polygon.

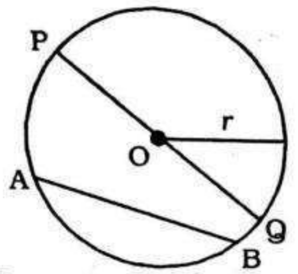
IMPORTANT RESULTS :

- If there is a polygon of n sides ($n \geq 3$) then we can cut it into ($n - 2$) triangles with common vertex. Then sum of the interior angles of a polygon of n sides is $(n - 2) \times 180°$ or $(2n - 4) \times 90°$.
- Each exterior angle of a regular polygon of n sides is $\left(\frac{360}{n}\right)^{\circ}$.
- Each interior angle of a regular polygon of n sides is $\frac{(n-2) \times 180°}{n}$ or interior angle = 180° – (exterior angle)
- The sum of all the exterior angle formed by producing the sides of a convex polygon in the same order is equal to 360°.
- Number of diagonals of a polygon of n sides is $\frac{n(n-1)}{2} - n$.

CIRCLE

A circle is a set of points which lie in a plane and at a constant distance from a fixed point in the plane.

Here the fixed point 'O' is called the **centre** of the circle and the constant distance is called **radius** of circle denoted by 'r'.



Chord : A line segment whose end points belong to the circle is a chord. So here AB is the chord.

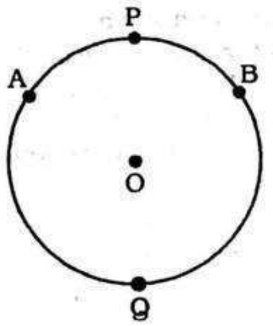
Diameter : The chord which passes through the centre of a circle is called a diameter of the circle. Here PQ is the diameter.

Diameter is the longest chord.

Diameter = 2 × radius.

Diameter divides a circle into two equal parts called a **Semi-circle**.

Arc : Any part of a circle is called an arc of the circle. Here AB is an arc of circle and written as AB.



If arc is less than semicircle it is called a **minor arc**, otherwise it is called as **major arc**.

APB is minor arc and AQB is major arc.

Segment : The area enclosed by an arc and its corresponding chord is called a segment of the circle.

A segment is minor if its arc is minor.

A segment is major if its arc is major.

Sector : The area enclosed by any two radii and the arc determined by the end points of the radii is called a sector of the circle.

Circumference : It is the distance travelled if you will go once around a circle.

Circumference = $2\pi r$

Concentric circles : In a plane two or more circle are called concentric if they have a common centre.

- Infinite number of circles can be drawn with same centre,

SOME RESULTS ON CIRCLES

Theorem 1. If two arcs of a circle arc congruent then the corresponding chords are equal.

Theorem 2. The perpendicular from the centre of a circle to a chord bisects the chord.

So here OD ⊥ AB then

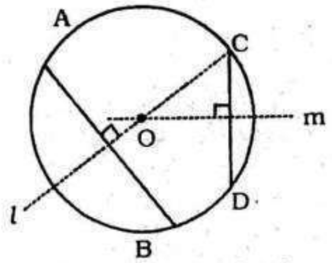
AD = DB

Theorem 3. The line joining the centre to the mid point of a chord is perpendicular to the chord.

Here if AD = DB then ∠ADO = ∠ODB = 90°

Theorem 4. The perpendicular bisectors of two chords of a circle intersect at its centre.

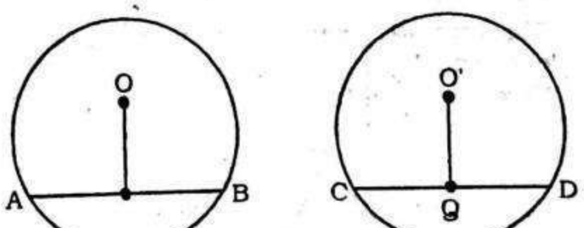
Here AB, CD are the chords and *l*, *m* are perpendicular bisector of AB and CD. So *l* and *m* meet at 'O'.



Theorem 5. There is one and only one circle passing through three non collinear points.

- An infinite number of circles can be drawn to pass through a single point.
- An infinite number of circles can be drawn to pass through two given points.
- A unique circle can be drawn to pass through three given non collinear points.

Theorem 6. Equal chords of congruent circles are equidistant from the corresponding centres.



Here it AB = CD then OP = O'Q

Theorem 7. Chords which are equidistant from the corresponding centres are equal.

So in above figure if OP = O'Q then AB = CD.

Theorem 8. Equal chords of a circle are equidistant from the centre.

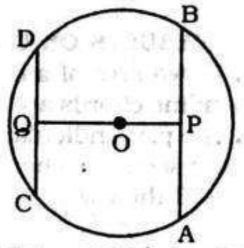
Here if AB and CD are equal chords of circle then OP = OQ. (conversely)

if OP = OQ then also AB = CD

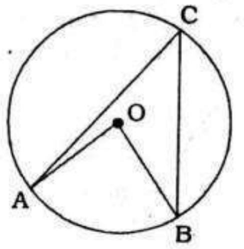
i.e., chords at equal distance for the centre are equal.

Theorem 9. Of any two chords of a circle, the greater chord is nearer to the centre.

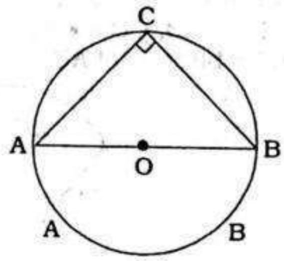
Here if OQ > OP then AB > CD, where AB and CD are chords.



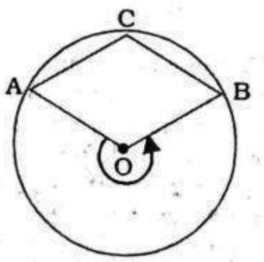
Theorem 10. The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the circumference of circle. Here three case arises.



(i)



(ii)



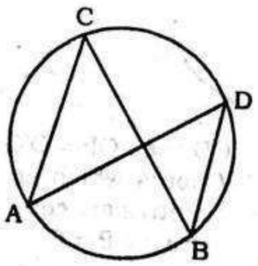
(iii)

Here in all the three cases ∠AOB = 2 ∠ACB

Theorem 11. The angle in a semi-circle is a right angle.

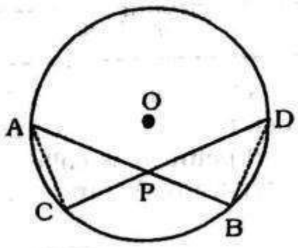
Theorem 12. Angle in the same segment of the circle are equal.

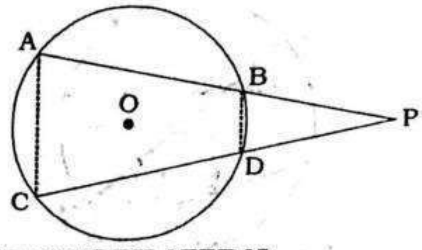
Here ∠ACB = ∠ADB



Theorem 13. If two chords AB and CD of a circle intersect inside or outside the circle when produced at a' point P.

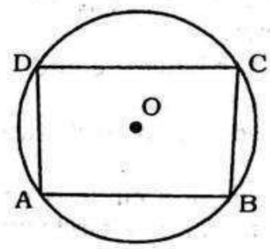
Then, AP × PB = DP × PC



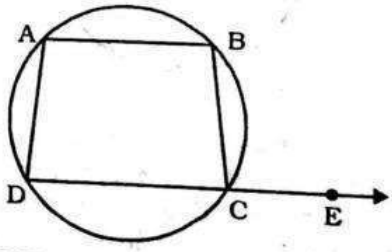


CYCLIC QUADRILATERAL

A quadrilateral whose vertices lie on a circle is called a cyclic quadrilateral.



- ∠A, ∠B, ∠C and ∠D are interior angles.
- ∠A and ∠C are opposite angles also ∠B and ∠D.
- ∠A + ∠C = 180° = ∠B + ∠D.
- The exterior angle, formed by producing a side of a cyclic quadrilateral is equal to the interior opposite angles.



Here ∠BAD = ∠BCE

Secant : A line which intersects a circle in two distinct points is called a secant of the circle. Here PAB is secant.

Tangent : A line meeting a circle only in one point is called a tangent to the circle. And the point where tangent meets is called point of contact. Here PT is a tangent and T is point of contact.