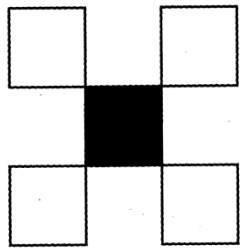
Fractions

Introduction

Fraction is a number which is used to represent a part of a whole. It is in the form P of $\frac{P}{Q}$. Where P and Q are natural numbers. P is called numerator of the fraction and Q is called denominator. For example, $\frac{5}{9}$ is a fraction, where 5 is numerator and 9 is denominator of the fraction.

Illustrative EXAMPLE

Represent the shaded part of the following figure as a fraction and write numerator and denominator of the fraction.



Explanation

The above figure has been divided into 5 equal parts. Out of 5 parts 1 part is shaded. Therefore, fractional representation of the shaded part $= \frac{1}{5}$ and numerator = 1, and denominator = 5.

Like Fraction

The fractions which have the same denominator are called like fractions. For example, $\frac{5}{7}, \frac{9}{7}, \frac{5}{7}$ are like fractions as they have the same denominator.

Illustrative EXAMPLE

Write the like fraction of $\frac{8}{21}$ whose numerator is 4.

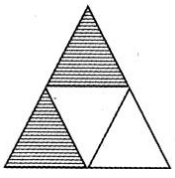
According to the question numerator should be 4 and like fractions have same denominator thus the required fraction will be $\frac{4}{21}$.

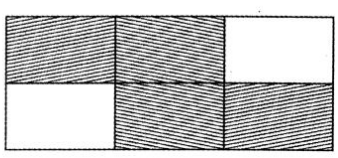
Unlike Fraction

The fractions which have different denominators are called unlike fractions. For example, $\frac{58}{87}, \frac{52}{75}, \frac{45}{88}$ are unlike fractions as they have different denominators.

Illustrative EXAMPLE

Represent the shaded part in the following figures into fractional form and check are they unlike fractions?





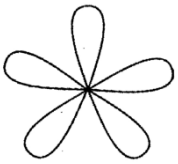
Fractional representation for the shaded part in first figure is $\frac{2}{4}$ and for the second figure is $\frac{4}{6}$. The fractions have different denominators. Therefore, they are unlike fractions.

Unit Fraction

The fractions which have the numerator 1 are called unit fractions. For example, $\frac{1}{5}, \frac{1}{8}, \frac{1}{12}$ are the unit fractions as each of them has the numerator 1.

Illustrative EXAMPLE

The following picture has been divided into 5 equal parts. How many parts of the figure should be shaded so that fractional representation of the shaded part of the figure is a unit fraction.



Solution:

A unit fraction has the numerator 1, therefore, only 1 part should be shaded.

Proper Fraction

If the numerator of a fraction is smaller than denominator, the fraction is called proper fraction. For example, $\frac{5}{7}$ is a proper fraction as 5 is smaller than 7.

Illustrative EXAMPLE

A flowering plant contains 19 flowers. 11 flowers fall down. What fraction of the total flower falls down? Is it a proper fraction?

Answer:

Total number of flower = 19

Number of flower which falls down = 11

Fractional representation of the flowers which fall down out of total flower $= \frac{11}{19}$ 11 is smaller than 19 thus $\frac{11}{19}$ is a proper fraction.

Improper Fraction

If the numerator of a fraction is greater than denominator, the fraction is called improper fraction. For example, $\frac{19}{17}$ is an improper fraction as 19 is greater than 17.

Illustrative EXAMPLE

11 kg of sweet is distributed among 5 persons. Represent the amount each of the person would get as a fraction and check it is a proper or improper fraction?

Solution:

Total amount of sweet = 11 kg

Number of person = 5

Total amount which each of the persons would get $= \frac{11}{5} kg$ is an improper fraction as numerator is greater than denominator in the fraction.

Mixed Fraction

Mixed fraction is a sum of a whole number and a proper fraction. Both the whole number and the fraction are written together but sign of the addition (+) remains disappeared between them. For example, $4\frac{1}{5}$ is a mixed fraction. Where 4 is a whole number and $\frac{1}{5}$ is a proper fraction.

Conversion of an Improper Fraction into a Mixed Fraction

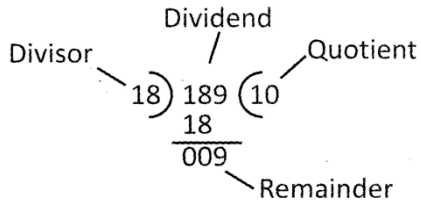
Divide the numerator by the denominator. The quotient represents the whole number, the remainder represents the numerator and the divisor represents the denominator for the fractional part in the mixed fraction.

Illustrative EXAMPLE

Change $\frac{189}{18}$ into a mixed fraction.

Solution:

Divide 189 by 18



Thus remainder = 9, Quotient = 10, and divisor = 18

Therefore, the mixed fraction for $\frac{189}{18} = 10\frac{9}{18}$.

Conversion of a Mined Fraction into an Improper Fraction

The whole number is multiplied with the denominator of the fractional part and the product is added by numerator. The sum represents numerator for the improper fraction and denominator of the improper fraction is same as the denominator of fractional part of the mixed fraction.

Illustrative EXAMPLE

Convert $4\frac{11}{17}$ into a mixed fraction.

Solution:

Multiply 4 with 17 and add the product by 11

= 4 x 17

= 68 + 11 = 79.

Thus 79 is numerator and 17 is denominator for the improper fraction.

Thus $\frac{79}{17}$ is the improper fraction for the mixed fraction $4\frac{11}{17}$.

Lowest or Simplest Form of a Fraction

When HCF of numerator and denominator of a fraction is 1, the fraction is in its simplest or lowest form. For example: The fraction $\frac{5}{7}$ is in its simplest form as HCF of 5 and 7 is 1.

Illustrative EXAMPLE

Is the fraction $\frac{27}{72}$ in its lowest form?

Solution:

HCF of 27 and 72 is 9. Therefore, the fraction $\frac{27}{72}$ is not in its simplest form.

How to Reduce a Fraction into Lowest Form

To reduce a fraction into its lowest form, numerator and denominator of the fraction is divided by their HCF. The resulting fraction is the reduced form of the given fraction.

Illustrative EXAMPLE

Reduce the fraction $\frac{16}{38}$ into its lowest form.

Solution:

HCF of 16 and 38 = 2

Now divide both numerator and the denominator by $2\frac{16\div 2}{38\div 2}=\frac{8}{19}$

Thus $\frac{8}{19}$ is the reduced form of $\frac{36}{38}$.

Equivalent Fractions

Two or more fractions are said to be equivalent fractions if they have the same value. In other word s when equivalent fractions are reduced into their simplest form, they give the same fraction. For example, $\frac{10}{15},\frac{20}{30},\frac{30}{45},\frac{40}{60}$... etc. are equivalent fractions.

Illustrative EXAMPLE

Are the fractions $\frac{24}{27}$ and $\frac{8}{9}$ equivalent fractions?

Solution:

The simplest form of $\frac{24}{27}=\frac{8}{9}$. Therefore, $\frac{24}{27}$ and $\frac{8}{9}$ are equivalent fractions.

How to Find Equivalent Fractions of a Given Fraction

Multiply both the numerator and denominator of the given fraction by a common number.

Illustrative EXAMPLE

Find three equivalent fractions of $\frac{6}{7}$.

Solution:

(a) $\frac{6}{7}=\frac{6\times2}{7\times2}=\frac{12}{14}$ (b) $\frac{6}{7}=\frac{6\times3}{7\times3}=\frac{18}{21}$

(c) $\frac{6}{7}=\frac{6\times4}{7\times4}=\frac{24}{28}$

Thus $\frac{12}{14},\frac{18}{21}$ and $\frac{24}{28}$ are equivalent fractions of $\frac{6}{7}$.

Reciprocal of a Fraction

Reciprocal of a fraction is the fraction by which if the fraction is multiplied the product is 1. The reciprocal of a fraction has reversed numerator and denominator.

For example, $\frac{Q}{P}$ is the reciprocal of the fraction $\frac{P}{Q}$.

Illustrative EXAMPLE

Find the reciprocal of $\frac{11}{13}$.

Solution:

Reciprocal of $\frac{11}{13}=\frac{13}{11}$.

Comparison of Unit Fractions

$\frac{1}{P}$ and $\frac{1}{Q}$ are unit fractions where P and Q are natural numbers. If P > Q then $\frac{1}{P}<\frac{1}{Q}$.

Illustrative EXAMPLE

Compare between $\frac{1}{5}$ and $\frac{1}{7}$. Which is greater?

Solution:

Compare between their denominators

5 < 7 (5 is smaller than 7)

Therefore, $\frac{1}{5}>\frac{1}{7}$.

Comparison of Like Fractions

Like fractions have same denominator. The fraction which has greater numerator is greater. For example, $\frac{5}{7}$ and $\frac{3}{7}$ are like fractions and 5 > 3, therefore, $\frac{5}{7}>\frac{3}{7}$.

Illustrative EXAMPLE

Is $\frac{24}{71}$ greater than $\frac{12}{71}$?

Solution:

Both the fractions have same denominator and 24 > 12Therefore, $\frac{24}{71} > \frac{12}{71}$.

Comparison of Unlike Fractions

To compare unlike fractions, unlike fractions are converted into like fractions, Numerator and denominator of one fraction is multiplied with the denominator of other fraction and vice-versa. For example, $\frac{P}{Q}$ and $\frac{R}{Q}$ are unlike fractions thus $\frac{P \times S}{Q \times S}$ and $\frac{R \times Q}{S \times Q}$ are like fractions.

Now compare the numerators.

Illustrative EXAMPLE

Compare between $\frac{3}{4}$ and $\frac{5}{7}$. which is greater?

Solution:

First convert $\frac{3}{4}$ and $\frac{5}{7}$ into like fractions

$\frac{3 \times 7}{4 \times 7} = \frac{21}{28}$ and $\frac{5 \times 4}{7 \times 4} = \frac{20}{28}$

Now compare numerators

21 > 20, therefore $\frac{21}{28} > \frac{20}{28}$ or $\frac{3}{4} > \frac{5}{7}$.

You Must KNOW

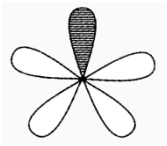
- Development of Egyptian fraction occurred in the middle kingdom of Egypt.
- The Hindus are believed to be the first group to represent fractions with numbers rather than words.
- Early Hindu mathematicians (Brahmagupta and bhaskara) wrote fractions as we do today.
- Earlier, in fractions bar was not used just one number above the other indicates a fraction.

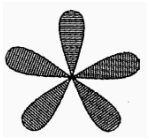
SUMMARY

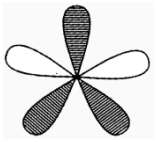
- Fraction is a part of a whole.
- Numerator of proper fraction is smaller than denominator.
- Numerator of improper fraction is greater than denominator.
- Unit fractions have the numerator 1.
- Like fractions have same denominator.
- Unlike fractions have different denominator.
- Equivalent fractions have same value.
- Reciprocal of a fraction has reversed numerator and denominator.
- Value of a proper fraction is less than 1.
- Value of an improper fraction is always greater than 1.
- Improper fraction is always greater than proper fraction.

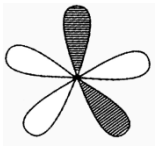
Commonly Asked QUESTIONS

In which one of the following figures does the unshaded part represent? $\frac{2}{5}$?

(a) 

(b) 

(c) 

(d) 

(e) None of these

Answer: (c)

Explanation

In the figure, which has been shown in the option C, 3 part out of the 5 part is shaded. Therefore, fractional representation of un shaded part is $\frac{2}{5}$.

Which one of the following options has correct fractional representation for the yellow part in the given figure?

(a) $\frac{2}{7}$

(b) $\frac{1}{7}$

(c) $\frac{4}{7}$

(d) $\frac{5}{7}$

(e) None of these

Answer: (b)

There are 2564577 fishes in a pond. If every third fish in the pond is rehu, what is the fractional representation of the number of rehu fishes out of the total fishes in the pond?

(a) $\frac{454855}{2564577}$ (b) $\frac{456487}{2564577}$

(c) $\frac{457858}{2564577}$ (d) $\frac{854859}{2564577}$

(e) None of these

Answer: (d)

Jack cuts a paper sheet into 8 equal pieces then he cuts each of the pieces into 8 equal pieces then again he cuts each of the pieces into 4 equal pieces. Now he colours 36 pieces with yellow colour, 108 pieces with red colour, and 54 pieces with blue colour. Which one of the following is the fractional representation for the coloured pieces out of total pieces?

(a) $\frac{194}{200}$ (b) $\frac{198}{256}$

(c) $\frac{200}{256}$ (d) $\frac{222}{256}$

(e) None of these

Answer: (b)

A question paper contains two types of questions objective and subjective.

If fractional representation for the number of subjective questions is $\frac{36}{80}$, find the number of objective questions the question paper contains.

(a) 44 (b) 116

(c) 36 (d) 80

(e) None of these

Answer: (a)

$\frac{a}{b}$ is a fraction where b > a. $\frac{a}{b}$ is a/an_____.

(a) Proper fraction (b) Improper fraction

(c) Mixed fraction (d) Unit fraction

(e) None of these

Answer: (a)

Explanation

According to the question denominator is greater than numerator, therefore, $\frac{a}{b}$ is a proper fraction.

Convert the fraction $\frac{15}{6}$ into mixed fraction and choose the correct option.

(a) $2\frac{1}{6}$ (b) $2\frac{2}{6}$

(b) $2\frac{3}{6}$ (d) $3\frac{2}{6}$

(e) None of these

Answer: (c)

Convert $4\frac{1}{5}$ into improper fraction and choose the correct option.

(a) $\frac{19}{5}$ (b) $\frac{20}{5}$

(c) $\frac{21}{5}$ (d) $\frac{22}{5}$

(e) None of these

Answer: (c)

Choose the proper fraction from the following: $\frac{22}{5}, \frac{12}{7}, \frac{8}{9}, \frac{7}{6}, \frac{5}{4}$

(a) $\frac{22}{5}$

(b) $\frac{5}{4}$

(c) $\frac{12}{7}$

(d) $\frac{8}{9}$

(e) None of these

Answer: (d)

Which one of the following is not true about the mixed fraction?

(a) Mixed fraction is a sum of a whole number and a proper fraction

(b) When a mixed fraction is converted into an improper fraction, denominator of the improper fraction and the fractional part in the mixed fraction is equal

(c) To convert a mixed fraction into an improper fraction, the whole number and the numerator of the fraction part is multiplied and the result is added by the denominator

(d) All of these

(e) None of these

Answer: (c)

Which one of the following fractions is in its lowest form?

(a) $\frac{35}{60}$ (b) $\frac{95}{171}$

(c) $\frac{96}{171}$ (d) $\frac{39}{190}$

(e) None of these

Answer: (d)

Explanation

HCF of 39 and 190 is 1. Therefore, $\frac{39}{190}$ is in its simplest form.

Which one of the following is the reciprocal of $7\frac{3}{16}$?

(a) $\frac{115}{16}$ (b) $\frac{16}{112}$

(c) $\frac{16}{115}$ (d) $7\frac{16}{3}$

(e) None of these

Answer: (c)

Explanation

First convert the mixed fraction $7\frac{3}{16}$ into an improper fraction

Thus $7\frac{3}{16}=\frac{115}{16}$. and reciprocal $\frac{115}{16}=\frac{16}{115}$.

Which one of the following statements is not true?

(a) If a fraction is in its lowest form, HCF of numerator and denominator is 1.

(b) Reciprocal of a fraction has reversed numerator and denominator.

(c) Product of a fraction and its reciprocal is 1.

(d) Two improper fractions can never be like fractions.

(e) None of these

Answer: (d)

By which one of the following numbers numerator and denominator of $\frac{1}{19}$ should be multiplied to get its one equivalent fraction?

(a) 7 (b) 5

(c) 19 (d) 11

(e) All of these

Answer: (e)

Find the fraction if numerator is 7 and denominator is one-fourth of 16.

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$

(c) $\frac{2}{7}$ (d) $\frac{7}{4}$

(e) None of these

Answer: (d)

Compare $\frac{11}{20}$ and $\frac{22}{9}$. which one is greater?

(a) $\frac{22}{9}$ (b) $\frac{11}{20}$

(c) Both are equal (d) All of these

(e) None of these

Answer: (a)

Explanation

$\frac{11\times9}{20\times9}=\frac{99}{180}$ and $\frac{22\times20}{9\times20}=\frac{440}{180}$

$\because 440>99$. Therefore, $\frac{11}{20}<\frac{22}{9}$

Which one of the following fractions is greater than $\frac{23}{55}$?

(a) $\frac{28}{63}$ (b) $\frac{22}{75}$

(c) $\frac{4}{11}$ (d) $\frac{13}{65}$

(e) None of these

Answer: (a)

Explanation

$\frac{23\times63}{55\times63}=\frac{1449}{3465}$ and $\frac{28\times55}{63\times55}=\frac{1540}{3465}$

$\because 1449<1540$. Therefore, $\frac{1540}{3465}>\frac{1449}{3465}$.

If, $\frac{1}{P} > \frac{1}{Q}$ Which one of the following is not true?

(a) $\frac{1}{P}$ and $\frac{1}{Q}$ are unit fractions

(b) Q is greater than P

(c) $\frac{1}{P \times Q}$ and $\frac{1}{Q \times P}$ are like fractions

(d) $\frac{1}{P}$ and $\frac{1}{Q}$ are equivalent fractions If 2P = Q

(e) None of these

Answer: (d)

The fraction which represents the shaded part in the figure 1 is smaller than the fraction which represents the shaded part in the figure 2.

Figure 1

Figure 2

The above statement is____.

(a)True

(b) False

(c) Partly true

(d) All of these

(e) None of these

Answer: (b)

A bag contains a number of balls of different colours. Jack finds number of blue balls is greater than number of red balls. It means:

(a) The fraction which represents red balls out of total balls is greater than that of blue balls

(b) The fraction which represents blue balls out of total balls is greater than that of red balls

(c) The fraction which represents red balls out of total balls is equivalent to the fraction which represents blue balls out of total balls

(d) All of these

(e) None of these

Answer: (b)

Self Evaluation TEST

Duration 10 Minutes

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1. **Which one of the following is a proper fraction?**

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$

(c) $\frac{3}{4}$ (d) $\frac{7}{5}$

(e) None of these

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2. **Which one of the following cannot be changed into mixed fraction?**

(a) $\frac{41}{8}$ (b) $\frac{42}{8}$

(c) $\frac{17}{2}$ (d) $\frac{11}{75}$

(e) None of these

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3. **Which one of the following is not correct?**

(a) Value of proper fraction is less than 1

(b) Value of improper fraction is greater than 1

(c) Value of equivalent fractions are same

(d) Value of a mixed fraction is less than 1

(e) None of these

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4. **Arrange the following fractions in ascending order.** $\frac{5}{7}, \frac{8}{7}, \frac{3}{7}, \frac{4}{7}$

(a) $\frac{5}{7}, \frac{8}{7}, \frac{3}{7}, \frac{4}{7}$ (b) $\frac{3}{7}, \frac{7}{7}, \frac{5}{7}, \frac{8}{7}$

(c) $\frac{8}{7}, \frac{5}{7}, \frac{4}{7}, \frac{3}{7}$ (d) $\frac{3}{7}, \frac{8}{7}, \frac{5}{7}, \frac{4}{7}$

(e) None of these

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5. **Change** $\frac{5}{2}, \frac{7}{6},$ **and** $\frac{5}{4}$ **into like fractions.**

(a) $\frac{15}{6}, \frac{7}{6}, \frac{10}{6}$ (b) $\frac{20}{8}, \frac{10}{8}, \frac{14}{8}$

(c) $\frac{10}{4}, \frac{5}{4}, \frac{7}{6}$ (d) $\frac{30}{12}, \frac{14}{12}, \frac{15}{12}$

(e) None of these

6. Which one of the following is the pair of equivalent fraction?

(a) $\frac{4}{6} and \frac{1}{3}$ (b) $\frac{5}{9} and \frac{4}{9}$

(c) $\frac{2}{3} and \frac{3}{9}$ (d) $\frac{1}{3} and \frac{3}{9}$

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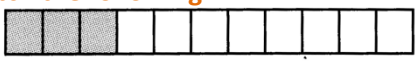
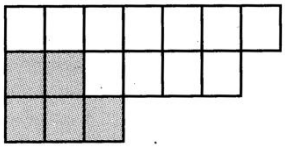
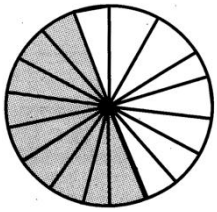
7. Match the following

(a)  (i) $\frac{9}{17}$

(b)  (ii) $\frac{3}{11}$

(c)  (iii) $\frac{5}{16}$

(a) $a-i, b-iii, c-ii$ (b) $a-ii, b-iii, c-i$

(c) $a-i, b-ii, c-iii$ (d) All of these

(e) None of these

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8. $\frac{P}{Q}$ is a fractions. P represents the number of prime numbers between 0 to 20 and Q represents the number of composite numbers between 0 to 20.

(a) Proper fraction (b) Improper fraction

(c) Mixed fraction (d) All of these

(e) None of these

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9. Change $\frac{7}{9}$ and $\frac{11}{12}$ into like fractions and choose the correct option.

(a) $\frac{28}{24} and \frac{33}{24}$ (b) $\frac{28}{36} and \frac{33}{36}$

(c) $\frac{28}{12} and \frac{11}{12}$ (d) $\frac{28}{9} and \frac{33}{9}$

(e) None of these

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10. **Which one of the following is not an equivalent fraction of $\frac{3}{5}$?**

(a) $\frac{6}{10}$ (b) $\frac{9}{15}$

(c) $\frac{10}{15}$ (d) $\frac{12}{20}$

(e) None of these

Answers – Self Evaluation Test

1.	C	**2.**	D	**3.**	D	**4.**	B	**5.**	D	**6.**	D	**7.**	B	**8.**	A	**9.**	B	**10.**	C

Self Evaluation Test

SOLUTIONS

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1. The fraction, which has greater denominator is $\frac{3}{4}$ proper fraction. Thus r- is a proper fraction.

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2. A proper fraction cannot be changed into a mixed fraction, jj^ is a proper fraction. $\frac{11}{75}$ is a proper fraction.

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3. Value of a mixed fraction is more than 1.

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4. Denominator of all the given fractions are equal. So the fraction having greater numerator is greater. Thus correct ascending order of the fraction is $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{8}{7}$

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5. LCM of 2, 6, and 4 = 12

Thus $\frac{5\times6}{2\times6}=\frac{30}{12}, \frac{7\times2}{6\times2}=\frac{14}{12}$, and $\frac{5\times3}{4\times3}=\frac{5}{12}$.

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6. $\frac{3}{9}=\frac{1}{3}$

Thus $\frac{1}{3}$ and $\frac{3}{9}$ are equivalent fractions.
