Verify the Algebraic Identity $a^3+b^3 = (a+b)(a^2-ab+b^2)$

OBJECTIVE

To verify the algebraic identity $a^3+b^3 = (a+b)(a^2-ab+b^2)$.

Materials Required

1. Acrylic sheets
2. Geometry box
3. Adhesive/Cello-tape
4. Cutter
5. Scissors

Prerequisite Knowledge

1. Concept of cuboid and its volume.
2. Concept of cube and its volume.

Theory

1. For concept of cuboid and its volume refer to Activity 7.
2. For concept of cube and its volume refer to Activity 7.

Procedure

1. Make a cube of side a units and another cube of side b units by using acrylic sheets and cello-tape/adhesive, (see Fig. 9.1 and 9.2)

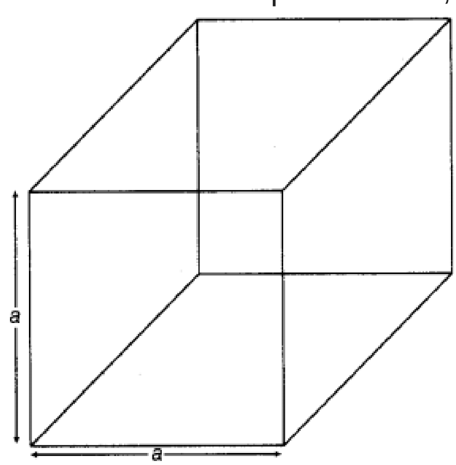

Fig. 9.1

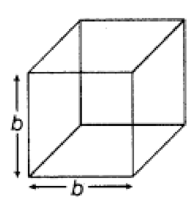

Fig. 9.2

2. Make a cuboid of dimensions a x a x b using acrylic sheet and cello-tape/adhesive, (see Fig. 9.3)

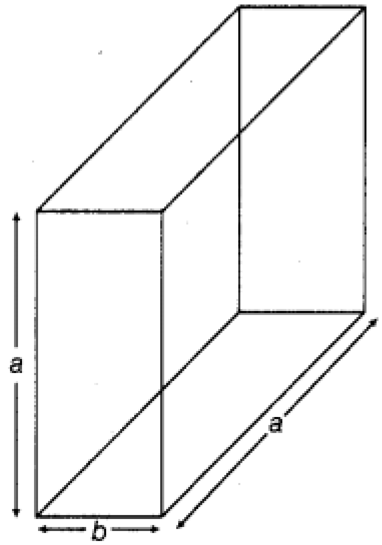

Fig. 9.3

3. Make a cuboid of dimensions a x b x b using acrylic sheet and cello-tape/adhesive, (see Fig. 9.4)

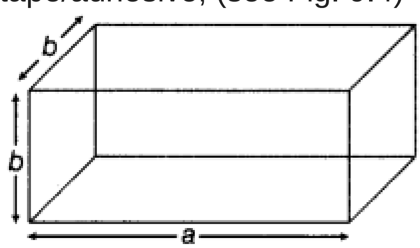

Fig. 9.4

4. Arrange these cubes and cuboids as shown in Fig. 9.5.

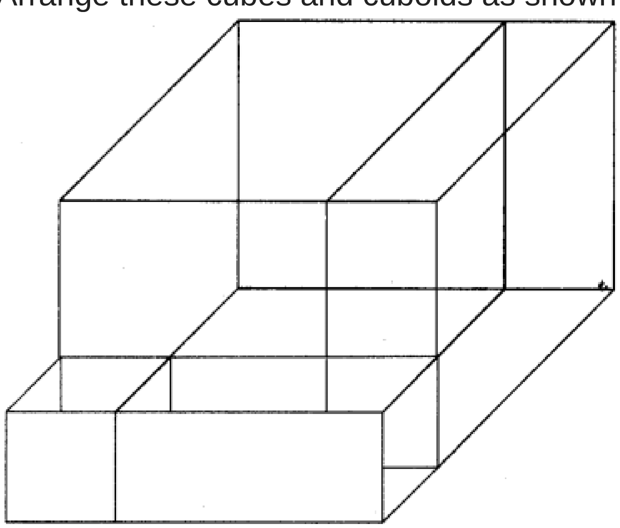

Fig. 9.5

5. On removing cuboids of dimensions a x a x b and a x b x b from the solid obtained in Fig. 9.5, to get another solid as shown in Fig. 9.6.

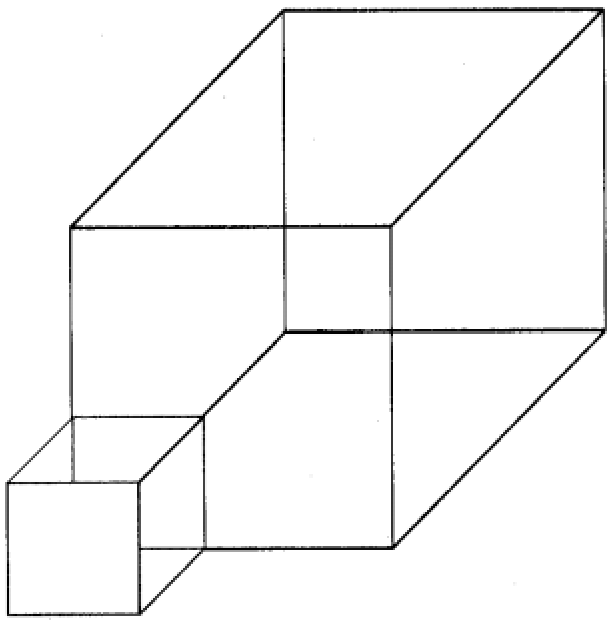

Fig. 9.6

Demonstration

In Fig. 9.1, volume of cube of side a units = a^3
In Fig. 9.2, volume of cube of side b units = b^3
In Fig. 9.3, volume of cuboid of dimensions a, a and b = a^2b
In Fig. 9.4, volume of cuboid of dimensions a, b and b = ab^2
Volume of the solid obtained in Fig. 9.5 = Total volume of all cubes and cuboids
$= a^3 + b^3 + ab^2 + a^2b = a^2(a + b) + b^2(a + b)$
$= (a + b)(a^2 + b^2)$

After removing cuboids of volume a^2b (i.e. a x a x b) and ab^2 (i .e. a x b x b) from solid obtained in Fig. 9.6.
So, volume of solid in Fig. 9.6 = $(a + b)(a^2 + b^2) - a^2b - ab^2$
$= (a + b)(a^2 + b^2) - ab(a + b) = (a + b)(a^2 + b^2 - ab)$ Also, volume of solid in Fig. 9.6 = $a^3 + b^3$ Hence, $a^3+b^3 = (a+b)(a^2-ab+b^2)$.
Here, volume is in cubic units.

Observation

On actual measurement, we get
a = ……….. , b = ……….. ,
So, a^3 = ……….. , b^3 = ……….. , a + b = ……….. ,
$(a + b)a^2$ = ……….. , $(a + b)b^2$ = ……….. ,
a^2b = ……….. , ab^2 = ……….. ,
$ab(a + b)$ = ……….. ,
Hence, $a^3+b^3 = (a+b)(a^2-ab+b^2)$.

Result

The algebraic identity $a^3+b^3 = (a+b)(a^2-ab+b^2)$ has been verified.

Application

The identity can be used in simplification and factorization of algebraic expressions.

Viva Voce

Question 1:
What is the expanded form of $a^3 + b^3$?
Answer:
Expanded form of $a^3+b^3 = (a+b)(a^2-ab+b^2)$.

Question 2:
What is the condition which satisfy the condition $a^3 + b^3 = 0$?
Answer:
$a = -b$

Question 3:
For evaluating $(90)^3 + (10)^3$, which identity you should follow?
Answer:
$a^3+b^3 = (a+b)(a^2-ab+b^2)$ will be appropriate.

Question 4:
How would you express $(a+b)(a^2 -ab+b^2)$ in shortest form?
Answer:
$a^3+ b^3$.

Question 5:
What is the degree of an algebraic identity $a^3 + b^3$?
Answer:
The degree of given algebraic identity is 3.

Question 6:
Let a and b be the edges of two cubes. Is $a^3 + b^3$ equal to the sum of volumes of cubes?
Answer:
Yes

Suggested Activity

By taking a = 3 and b = 7, verify the identity $a^3 + b^3$.