

STATISTICS

- Data (Raw Data, Frequency Distribution Table, Grouped Data)
- Central Tendency (mean, median, mode)
- Dispersion (Range, Mean Deviation, Standard Deviation)

Data:

Raw Data: marks (50, 45, 92, 73, 22, 73, 51)

Ungrouped Frequency Distribution Table.

Marks	Class No. of students.
0	2
5	7
10	15
15	12
20	3

} Frequency

Grouped Data:

- Continuous Data (Exclusive Data)
- Discontinuous Data (Inclusive Data)

(i) Continuous Data (Frequency)

Classes (marks)	No. of students
<u>0-10</u>	2
<u>10-20</u>	7
20-30	17
30-40	13
40-50	6

↓ ↓
Include Exclude

(ii) Discontinuous Data:

(Exclusive Data)

(Marks) Classes	(No. of students) frequency
1-10	2
11-20	7
21-30	12
31-40	9

Include Exclude.

e.g. Convert following discontinuous
Data into Continuous Data.

classes	Freq.		Classes	Freq.
1-10	2	$\text{Gap} = 1$ $\rightarrow$ $\frac{\text{Gap} = 1}{2 \times 2}$ $= 0.5$	0.5-10.5	2
11-20	7		10.5-20.5	7
21-30	12		20.5-30.5	12
31-40	9		30.5-40.5	9

(Continuous)

✓

Class Mark (in grouped Data)

Classes	CM(x_i) $= \frac{U.L. + L.L.}{2}$	Frequency (f_i)
0-10	$5 = x_1$	$2 = f_1$
10-20	$15 = x_2$	$7 = f_2$
20-30	$25 = x_3$	$12 = f_3$
30-40	$35 = x_4$	$9 = f_4$

0-10 $\rightarrow$ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
 10-20 $\rightarrow$ 10, 11, 19

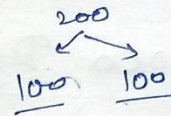
Class Size.

$$= (\text{Class mark}_2) - (\text{Class mark}_1)$$

$$= \text{Lower limit}_2 - \text{lower limit}_1$$

$$= \text{Upper limit}_2 - \text{Upper limit}_1$$

Central Tendency (Mean, median, Mode)



$$\underline{\text{Mean}} \text{ (Average)} = \frac{\text{Sum of observations}}{\text{total no. of observations}}$$

For Raw Data $x_1, x_2, x_3, \dots, x_n \leftarrow \text{Values}$

$$\text{Mean} = \bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

For Ungrouped Frequency Distribution Table.

(Marks) Values (x_i)	(No. of students) Frequency (f_i)	$x_i f_i$
x_1	f_1	$x_1 f_1$
x_2	f_2	$x_2 f_2$
$\vdots$	$\vdots$	$\vdots$
x_n	f_n	$x_n f_n$
	$n = \sum_{i=1}^n f_i$	$\sum_{i=1}^n x_i f_i$

$$\Rightarrow \text{mean} = \bar{x} = \frac{\sum x_i f_i}{n}$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \quad \star$$

Mean for Grouped Data. (Continuous or Discontinuous Data)

Classes	Class Mark $(x_i) = \frac{UL + LL}{2}$	Frequency (f_i)	$x_i f_i$	$d_i = x_i - A$ $\swarrow$ (d = deviation)	$f_i d_i$	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
$a_1 - a_n$	x_1	f_1	$x_1 f_1$	$d_1 = x_1 - A$	$f_1 d_1$	$u_1 = \frac{x_1 - A}{h}$	$f_1 u_1$
$b_1 - b_n$	x_2	f_2	$x_2 f_2$	$d_2 = x_2 - A$	$f_2 d_2$	u_2	$f_2 u_2$
$c_1 - c_n$	$x_3 = A$	f_3	$x_3 f_3$	$d_3 = x_3 - A$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$f_n d_n$	u_n	$f_n u_n$
$z_1 - z_n$	x_n	f_n	$x_n f_n$	$d_n = x_n - A$	$\Sigma f_i d_i$		$\Sigma f_i u_i$
		$N = n = \Sigma_{i=1}^n f_i$	$\Sigma_{i=1}^n x_i f_i$				

Direct

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

Assumed Mean = $A =$ (any no.) (Generally mid class mark)

$h =$ any number = (Generally class size)

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Assumed Mean method.

$$\bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

Step Deviation method

Median (middle most value)

median



For Raw Data → Ascending order → Middle
→ Descending order → middle

e.g. No. of Data = odd Data → 2, 5, 7, 8, 10 median = 7

No. of Data = Even. Data → 2, 5, 7, 8, 10, 15 median = $\frac{7+8}{2} = 7.5$

For Grouped Data. (only for Continuous Data)

classes	Frequency	Cumulative (C) Frequency (C)
$l_1 - u_1$	f_1	$C_1 = f_1$
$l_2 - u_2$	f_2	$C_2 = f_1 + f_2$
$l_3 - u_3$	f_3	$C_3 = f_1 + f_2 + f_3$
$\vdots$	$\vdots$	$\vdots$
$l - u$	f	$\vdots$
$\vdots$	$\vdots$	$\vdots$
$l_n - u_n$	f_n	$C_n = \sum f_i$
	$N = \sum f_i$	

Median class: the class whose cumulative freq. is just ~~greater~~ greater than $\frac{N}{2}$.

$$\text{median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

l = lower limit of median class

f = freq. _____

C = C.f. of preceding class.

h = class size

Mode (the most frequent value)

For Raw Data. e.g. 2, 3, 7, 2, 3, 5, 3, 3 (Mode = 3)

e.g. 2, 5, 10, 10, 10, 12, 15, 15, 15, 16 Mode = 10, 15

Frequency Distribution Table.

Values	freq.
5	2
10	7
15	10
20	6

mode = 15 ← 10 Highest freq. mode = 15

Modal class: the class with highest frequency.

$$\text{Mode} = l + \frac{f_m - f'}{2f_m - f' - f''} \times h$$

For Grouped Data. : (For Continuous Data)

Classes	Frequency
$l_1 - u_1$	f_1
$l_2 - u_2$	f_2
$l_3 - u_3$	f_3
$\vdots$	$\vdots$
$l_n - u_n$	f_n

l = lower limit of modal class

h = class size.

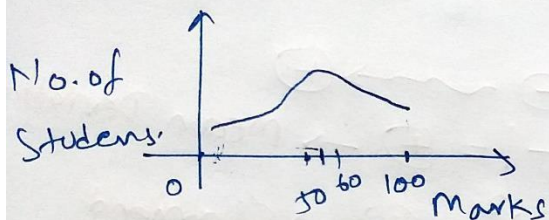
f_m = the highest frequency.

f' = freq. of preceding class

f'' = freq. of succeeding class.

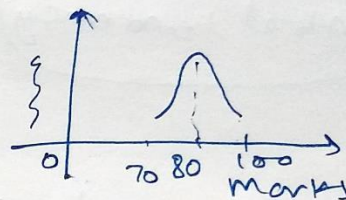
Dispersion (Degree of spreadness of Data) (थलता)

Combined class of 100 students

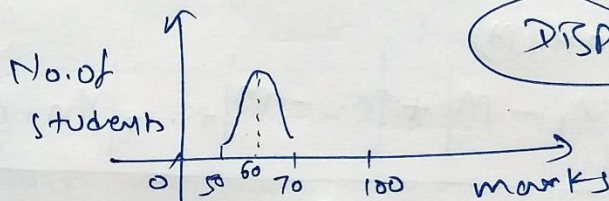


Dispersion ↑

(A) 50 Students



(B) 50 Students



Dispersion ↓

Measure of Dispersion

Range = Highest Data - lowest Data

→ Mean Deviation

→ Standard Deviation (Variance)

Note: Deviation (Difference)
↓
Always ⊕ve
⊕ ⊖
✓ ✓

eg. Data = 2, 3, 4

Deviation about 2.5

$$= 2 - 2.5, 3 - 2.5, 4 - 2.5$$

$$= -0.5, 0.5, 1.5$$

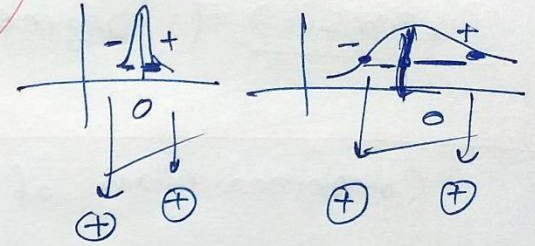
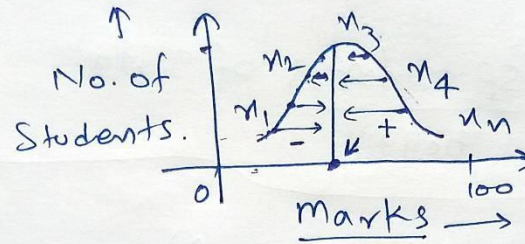
$$\text{mean} = \frac{2+3+4}{3} = 3 \checkmark$$

Deviation about (mean = 3) = $2-3, 3-3, 4-3$
= -1, 0, 1

Sum = 0

Mean Deviation

about any central tendency



Mean Deviation
about median (M)

$$= |x_1 - M| + |x_2 - M| + \dots + |x_n - M|$$

$\ominus \rightsquigarrow \oplus$
modulus $|-| = \oplus \rightarrow$ Mean Deviation
Square $(-)^2 = \oplus \rightarrow$ Standard Deviation

Mean Deviation

about
median (M)

$$= \frac{\sum_{i=1}^n |x_i - M|}{n}$$

★
Raw Data

mean Deviation
about
mean ($\bar{x}$)

$$= \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$$

Mean Deviation
about
median (M)

$$= \frac{\sum f_i |x_i - M|}{\sum f_i}$$

Grouped Data

Mean Deviation
about
mean $\bar{x}$

$$= \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

Standard Deviation. (variance) (we do Square instead of MODULUS)

$$\text{Data} = x_1, x_2, x_3, \dots, x_n$$



(Generally SD $\rightarrow$ mean $\bar{x}$ about)

$$\text{Mean} = \bar{x}$$

$$\text{Variance} = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}$$

(σ^2)

$$\boxed{\text{Variance} = \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}} \leftarrow \text{For Raw Data.}$$

$$\boxed{\text{Standard Deviation (SD)} = \sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\text{variance}}}$$

For Grouped Data.

$$\text{Variance} = \sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

(CM. above x_i)

$$\checkmark \text{ Standard Deviation (SD)} = \sigma = \sqrt{\text{variance}} = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}}$$

(CM. above x_i)

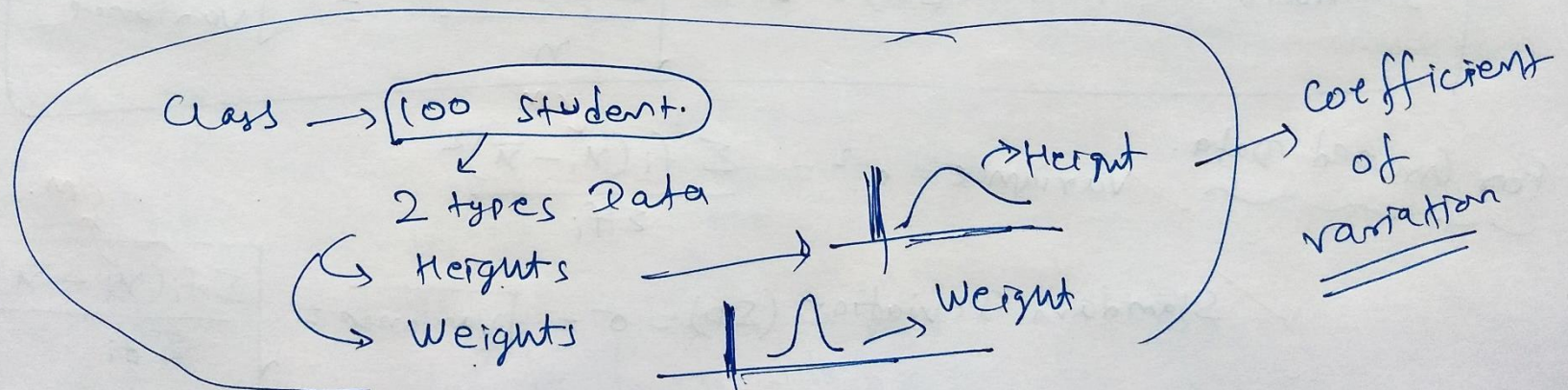
★ Mean Deviation (MD) = $\frac{\sum |x_i - M|}{n}$ $\text{or} \quad \frac{\sum f_i |x_i - M|}{n}$
 (generally about median) ($n = \sum f_i$)

★ ~~std~~ Variance (σ^2) = $\frac{\sum (x_i - \bar{x})^2}{n}$ $\text{or} \quad \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$
 (generally about mean)

$\sigma^2 = \text{Mean of } (x_i^2) - (\text{Mean of } x_i)^2$

Short cut
for
JEE Main

★ Standard Deviation (σ) = $\sqrt{\text{Variance}}$



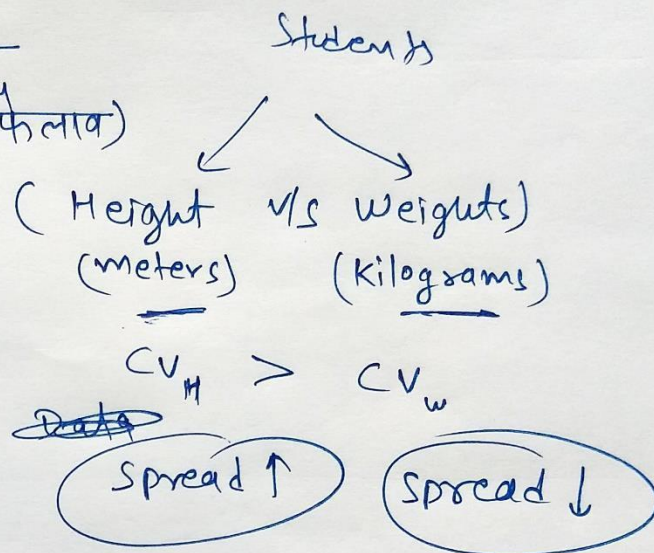
Coefficient of Variation (CV)

To compare the Dispersion (फलावरण) of Different kinds of Data (Height v/s weights)
(meters) (kilograms)

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

σ = Standard Deviation

$\bar{x}$ = mean



$$x_1, x_2, x_3, \dots, x_n$$

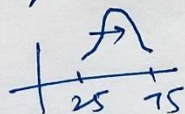
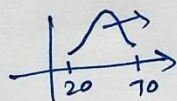
if K is added in each Data

Mean $\rightarrow +K$

Median $\rightarrow +K$

Mode $\rightarrow +K$

MD $\rightarrow$
 $\sigma^2 \rightarrow$
SD $\rightarrow$ } No change



if K is multiplied in each Data

Mean $\rightarrow \times K$

Median $\rightarrow \times K$

Mode $\rightarrow \times K$

MD $\rightarrow \times K$

$\sigma^2 \rightarrow \times K^2$

SD $\rightarrow \times K$

Exercise 13.1

Q.1 Mean Deviation about Mean

4, 7, 8, 9, 10, 12, 13, 17

$$\text{Mean} = \bar{x} = \frac{4+7+8+9+10+12+13+17}{8} \\ = \frac{80}{8} = 10$$

$$\underline{\underline{MD}} = \frac{\sum |x_i - \bar{x}|}{n} = \frac{|4-10| + |7-10| + \dots + |17-10|}{8}$$

$$MD(\bar{x}) = \frac{6+3+2+1+0+2+3+7}{8}$$

$$MD = \frac{24}{8} = 3 \checkmark$$

$$\frac{0.5 \times 12}{6}$$

Q.2 Mean Deviation about Median:

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Ascending order: $\uparrow \Rightarrow \uparrow$

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{median} = \frac{13+14}{2} = 13.5 = M$$

(middle most value)

$$MD(M) = \frac{\sum |x_i - M|}{n} \\ = \frac{3.5 + 2.5 + 2.5 + 1.5 + 0.5 + 0.5 + 0.5 + 2.5 + 2.5 + 3.5 + 3.5 + 4.5}{12}$$

$$= \frac{28}{12} = \frac{7}{3} = 2.33$$

Q.5 Mean Deviation about Mean

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

$$\bar{x} = \text{mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{350}{25} = 14$$

$$M_D = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{158}{25} = 6.32 \checkmark$$

Q.7 Mean Deviation about Median

x_i	f_i	$ x_i - m $	$f_i x_i - m $
5	8	2	16
7	6	0	0
9	2	2	4
10	2	3	6
12	2	5	10
15	6	8	48
	26		84

Median = the middle most value.

∴ Here no. of values = 26

$$\text{Median} = \frac{13^{\text{th}} + 14^{\text{th}} \text{ value}}{2} = \frac{7+7}{2} = 7$$

$5, 5, 5, 5, 5, 5, 5, 7, 7, 7, \overset{\uparrow}{7}, \overset{\uparrow}{7}, 9, 9, 10, 10$

$$MD = \frac{\sum f_i |x_i - M|}{\sum f_i} = \frac{84}{26} = \frac{42}{13} = 3.23$$

Q.12 Mean Deviation about Median

Age

16-20

21-25

26-30

31-35

36-40

41-45

46-50

51-55

Gap = 1

Gap = $\frac{1}{2}$

= 0.5

LL

0.5

UL

0.5

Given

Discontinuous

Age

15.5-20.5

20.5-25.5

25.5-30.5

30.5-35.5

35.5-40.5

40.5-45.5

45.5-50.5

50.5-55.5

Number (fi)

5

6

12

14

26 = f

12

16

9

$\Sigma f_i = 100$
 $N = \uparrow$

Cont'n.

Class Mark (x_i)

18

23

28

33

38

43

48

53

Cumulative freq. (cf)

5

11

23

37 = c

63

75

91

100

$|x_i - M|$

20

15

10

5

0

5

10

15

$f_i | x_i - M |$

100

90

120

70

0

60

160

135

735

Median class

35.5 - 40.5

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h = 35.5 + \left(\frac{50 - 37}{26} \right) \times 5 = 38 = M$$

h = class size = 5

$$M.D. (M) = \frac{\Sigma f_i |x_i - M|}{\Sigma f_i} = \frac{735}{100} = 7.35$$

Median class

$\frac{N}{2} < \text{C.F.}$
Just C.F.

$N = 100$

Exercise 13.2

Q.1 Mean & Variance.

6, 7, 10, 12, 13, 4, 8, 12

$$\bar{x} = \text{mean} = \frac{6+7+10+12+13+4+8+12}{8}$$

$$\bar{x} = \frac{72}{8} = 9$$

$$\sigma^2 = \text{Variance} = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$= \frac{9+4+1+9+16+25+1+9}{8}$$

$$= \frac{74}{8} = 9.25$$

100
50
25
10
5
2
1

100
50
25
10
5
2
1

Q.2

★ Mean & Variance of first n -natural numbers = ?

$$x_i = 1, 2, 3, 4, \dots, n$$

$$\text{mean} = \bar{x} = \frac{1+2+3+\dots+n}{n} = \frac{\sum x_i}{n}$$

$$= \frac{\left[\frac{n(n+1)}{2} \right]}{n} = \frac{n+1}{2} \checkmark$$

$$\text{Variance, } = \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i^2 - 2x_i\bar{x} + (\bar{x})^2)}{n}$$

$$= \frac{\sum_{i=1}^n (x_i^2) - 2\bar{x} \sum_{i=1}^n x_i + (\bar{x})^2 \sum_{i=1}^n 1}{n}$$

$$= \frac{\frac{n(n+1)(2n+1)}{6} - 2\left(\frac{n+1}{2}\right) \cdot \frac{n(n+1)}{2} + \left(\frac{n+1}{2}\right)^2 \cdot n}{n}$$

$$\text{Variance} = \sigma^2$$

$$= \frac{(n+1)(2n+1)}{6} - 2\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right)^2$$

$$= \frac{(n+1)}{2} \left\{ \frac{2n+1}{3} - \frac{(n+1)}{1} + \frac{n+1}{2} \right\}$$

$$= \frac{(n+1)}{2} \cdot \left\{ \frac{4n+2-6n-6+3n+3}{6} \right\}$$

$$= \left(\frac{n+1}{2}\right) \cdot \left(\frac{n-1}{6}\right)$$

$$\sigma^2 = \frac{n^2-1}{12}$$

Q.4

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
6	2	12	169	338
10	4	40	81	324
14	7	98	25	175
18	12	216	1	12
24	8	192	25	200
28	4	112	81	324
30	3	90	121	363
Σ	$n = 40$	760		1736

$$\bar{x} = \text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{760}{40} = 19$$

$$\sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \frac{1736}{40} = 43.5$$

Q.8

Class mark = $\frac{LL + UL}{2}$

Classes	x_i	f_i	$x_i f_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
0-10	5	5	25	484	2420
10-20	15	8	120	144	1152
20-30	25	15	375	4	60
30-40	35	16	560	64	1024
40-50	45	6	270	324	1944
		$\sum f_i = 50$	$\sum f_i x_i = 1350$		$\sum f_i (x_i - \bar{x})^2 = 6600$

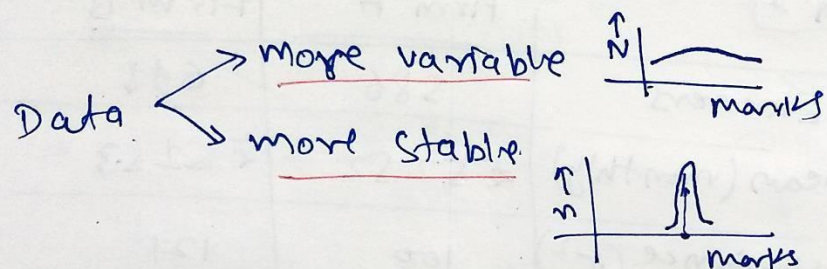
$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1350}{50} = 27$$

$$\text{Variance} = \sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{6600}{50} = 132 \checkmark$$

$$\text{Standard Deviation} = \sigma = \sqrt{\text{variance}} = \sqrt{132} = 11.489 \checkmark$$

Exercise- 13.3

(CV) = Coefficient of Variation.



$$CV \uparrow = \frac{\sigma \uparrow}{\bar{x}} \times 100$$

$\bar{x}$ = mean

$$CV \downarrow = \frac{\sigma \downarrow}{\bar{x}} \times 100$$

σ = Standard Deviation
 $= \sqrt{\text{var.}}$

(CM)

Q.1	Marks	f_A	f_B	x_i	$f_A x_i$	$f_B x_i$	$(x_i - \bar{x})^2$	$f_A (x_i - \bar{x})^2$	$f_B (x_i - \bar{x})^2$
	10-20	9	10	15	135	150	876.16	7885.44	8761.6
	20-30	17	20	25	425	500	384.16	6530.72	7683.2
	30-40	32	30	35	1120	1050	92.16	2949.12	2764.8
	40-50	33	25	45	1485	1125	0.16	5.28	4
	50-60	40	43	55	2200	2365	108.16	4326.4	4650.88
	60-70	10	15	65	650	975	46.16	461.6	6242.4
	70-80	9	7	75	675	525	924.16	8317.44	6469.12
		150	150		6690	6690		34179	36572

$$\bar{x}_A = \frac{\sum f_A x_i}{\sum f_A} = \frac{6690}{150} = 44.6 = \bar{x}_B$$

$$\sigma^2 = \frac{\sum f_i (\overset{\text{cm}}{x_i} - \bar{x})^2}{\sum f_i}$$

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100$$

$$\sigma_A^2 = \frac{34179}{150} = 227.86$$

$$\sigma_A = \sqrt{227.86} = 15.09 = \text{SD}_A$$

$$\sigma_B^2 = \frac{36572}{150} = 243.813$$

$$\sigma_B = \sqrt{243.813} = 15.61 = \text{SD}_B$$

$$\sigma_A < \sigma_B$$

15.09

15.61

$$\text{C.V.}_A < \text{C.V.}_B$$

'B' is more Variable.

Q.3

	Firm A	Firm B
Earners	586	648
mean (monthly)	₹ 5252	₹ 5253
Variance (σ^2)	100	121

(i) $\text{total Amount} \uparrow = \text{No. of workers} \times \text{Average pay.}$

Same

$\therefore$ Firm 'B' has more workers

$\therefore$ Firm 'B' pays larger amount.

(ii) Greater Variability. If mean is same, then the data with higher (σ) (B) has greater variability.

$$\text{C.V.} \uparrow = \frac{\sigma}{\bar{x}} \times 100$$

(Coeff. of Variation)

(B)

Q.4

Team - A

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

No. of Goals Scored (x_i)	No. of matches (f_i)	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
0	1	0	4	4
1	9	9	1	9
2	7	14	0	0
3	5	15	1	5
4	3	12	4	12
	25	50	Σ	30

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{50}{25} = 2 = \bar{x}_A$$

$$\text{Variance, } \sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \frac{30}{25} = 1.2$$

$$SD_A = \sigma_A = \sqrt{1.2} = \underline{\underline{1.095}}$$

For Team - B.

$$\bar{x}_B = 2$$

$$\sigma_B = 1.25$$

$$CV_A = \frac{\sigma_A}{\bar{x}_A} \times 100 = \frac{1.095}{2} \times 100$$

$$CV_B = \frac{\sigma_B}{\bar{x}_B} \times 100 = \frac{1.25}{2} \times 100$$

$$CV_A < CV_B$$

Variability of A ↓

⇒ Consistency of A ↑

Q.5 $\sum_{i=1}^{50} x_i = 212$

$\sum_{i=1}^{50} y_i = 261$

$\sum x_i^2 = 902.8$

$\sum y_i^2 = 1457.6$

$\bar{x} = \frac{\sum x_i}{n}$

$n = 50$

$\bar{y} = \frac{\sum y_i}{50}$

Variance = $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$\sigma^2 = \frac{\sum_{i=1}^{50} (x_i^2 - 2x_i\bar{x} + (\bar{x})^2)}{n}$

$= \frac{\sum x_i^2 - 2\bar{x} \cdot \sum x_i + (\bar{x})^2 \cdot \sum 1}{n}$

$= \frac{\sum x_i^2}{n} - 2\bar{x} \cdot \left(\frac{\sum x_i}{n}\right) + \frac{(\bar{x})^2 \cdot n}{n}$

$= \frac{\sum x_i^2}{n} - 2(\bar{x})^2 + (\bar{x})^2$

$= \frac{\sum x_i^2}{n} - (\bar{x})^2 \Rightarrow \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$
 $\downarrow$
Var.

variability

$C.V. \uparrow = \frac{\sigma}{\bar{x}} \times 100$

$\rightarrow S.D. = \sqrt{\text{Var.}}$

$\sigma_x = \sqrt{\text{Variance}(x)} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

$\sigma_x = \sqrt{\frac{902.8}{50} - \left(\frac{212}{50}\right)^2}$

$\sigma_x = 0.28$

$\bar{x} = 4.24$

Similarly

$\sigma_y = \sqrt{\frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n}\right)^2}$

$= \sqrt{\frac{1457.6}{50} - \left(\frac{261}{50}\right)^2} = 1.37$

$\bar{y} = (5.22)$

$y \rightarrow \text{weight}$

CV_x

$\frac{\sigma_x}{\bar{x}} \times 100$

$= 6.6$

CV_y

$\frac{\sigma_y}{\bar{y}} \times 100$

$= 26.24$

Miscellaneous Exercise - 13.4

Important for JEE - Main

Revision:

$$\bullet \text{ Mean} = \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x_i}{n}$$

$$\bullet \text{ Standard Deviation} = \sigma = \sqrt{\text{Variance}}$$

$$\bullet \text{ Variance} = \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\boxed{\sigma^2 = \frac{\sum (x_i^2)}{n} - \left(\frac{\sum x_i}{n} \right)^2} \quad \star$$

$\xrightarrow{\text{red arrow}} \bar{x}$

• Effect of multiplication on Values.

$$\{x_1, x_2, x_3, \dots, x_n\} \rightarrow \{kx_1, kx_2, \dots, kx_n\}$$

$$\text{Mean}_1 = \bar{x}$$

$$\rightarrow \text{Mean}_2 = k\bar{x}$$

$$\text{Variance}_1 = \sigma^2$$

$$\text{Variance}_2 = k^2 \sigma^2$$

$$\text{SD}_1 = \sigma$$

$$\rightarrow \text{SD}_2 = k \cdot \sigma$$

$$\boxed{\text{Q.1}} \quad \bar{x} = 9, \sigma^2 = 9.25$$

$$\underline{6, 7, 10, 12, 12, 13, x, y}$$

$$\text{mean} = \bar{x} = \frac{\sum x_i}{n} = 9$$

$$\Rightarrow \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow 60 + x + y = 72$$

$$\Rightarrow x + y = 12$$

$$\Rightarrow \boxed{y = 12 - x} \quad \text{--- (1)}$$

$$\sigma^2 = 9.25 \text{ (Given)}$$

$$\Rightarrow 9.25 = \frac{6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2}{8}$$

$$- (\bar{x})^2$$

$$\Rightarrow 9.25 = \frac{642 + x^2 + y^2}{8} - 81$$

$$\Rightarrow 90.25 = \frac{642 + x^2 + 42}{8}$$

$$\Rightarrow 722 = 642 + x^2 + (12-x)^2$$

$$\Rightarrow 80 = x^2 + 144 - 24x + x^2$$

$$\Rightarrow 2x^2 - 24x + 64 = 0$$

$$\Rightarrow x^2 - 12x + 32 = 0$$

$$\Rightarrow x^2 - 8x - 4x + 32 = 0$$

$$\Rightarrow x(x-8) - 4(x-8) = 0$$

$$\Rightarrow (x-4)(x-8) = 0$$

$$\begin{matrix} x=4 \\ y=8 \end{matrix}$$

$$\begin{matrix} x=8 \\ y=4 \end{matrix}$$

$$y = 12 - x$$

$$\underline{\underline{No. = 4, 8}}$$

Q.3 $x_1, x_2, x_3, \dots, x_6$
6 observations

$$\text{Mean} = 8$$

$$SD = \sigma = 4$$

$3x_1, 3x_2, 3x_3, \dots, 3x_6$
new observations

$$\text{Mean (New)} = 3 \times 8 = 24$$

$$SD (\text{New}) = 3 \times 4 = 12$$

Q.4 n -observations $\{x_1, x_2, \dots, x_n\} \rightarrow x_i$

$$\text{Mean} = \bar{x} = \frac{\sum x_i}{n}$$

$$\text{Variance} = \sigma^2 \rightarrow \frac{\sum (x_i - \bar{x})^2}{n}$$

New observations $\{ax_1, ax_2, \dots, ax_n\} \rightarrow ax_i$

$$\text{Mean}_N = \frac{ax_1 + ax_2 + \dots + ax_n}{n}$$

$$\text{Mean}_N = a \left(\frac{\sum x_i}{n} \right) = a \cdot \bar{x}$$

$$\text{Variance}_N = \frac{\sum (a x_i - a \bar{x})^2}{n}$$

$\uparrow$ new observations $\geq a x_i$
 $\uparrow$ New mean

new observations $\geq a x_i$
 Pattern

$$\text{Variance}_N = \frac{\sum_{i=1}^n a^2 (x_i - \bar{x})^2}{n}$$

$$= a^2 \cdot \frac{\sum (x_i - \bar{x})^2}{n}$$

$\rightarrow \sigma^2$

New
Variance = $a^2 \cdot \sigma^2$

Q.5

~~20 obs~~
 20 observations

$\nearrow$ Mean = $\bar{x} = 10$
 $\searrow$ SD = $\sigma = 2$
 Var. (σ^2) = 4

{ Among these 20 observations,
 one obs. (8) is wrong. }

(Find all information about
 all 19-correct observations.)

20-obs. $\rightarrow x_1, x_2, \dots, x_{19}, \underbrace{x_{20}}_{\substack{\uparrow \\ 8}}$

Correct

$$\text{Mean} = 10 = \bar{x}$$

$$\Rightarrow \frac{x_1 + x_2 + \dots + x_{19} + 8}{20} = 10$$

$$\Rightarrow \boxed{x_1 + x_2 + \dots + x_{19} = 192} \text{ --- (1)}$$

$$\sigma^2 = \text{Variance} = 4$$

$$\Rightarrow \frac{\sum (x_i^2)}{20} - \left(\frac{\sum x_i}{20} \right)^2 = 4$$

$\downarrow$ mean = 10

$$\Rightarrow \frac{x_1^2 + x_2^2 + \dots + x_{19}^2 + 64}{20} - 100 = 4$$

$$\Rightarrow x_1^2 + x_2^2 + \dots + x_{19}^2 = 2016 \quad \text{--- (2)} \quad \rightarrow \sum (x_i^2)$$

$$x_1 + x_2 + \dots + x_{19} = 192 \quad \text{--- (1)} \quad \rightarrow \sum (x_i)$$

(i) wrong obs. is deleted.

No. of obs. $\rightarrow 19$.

$$\text{mean} = \bar{x} = \frac{\sum_{i=1}^{19} x_i}{n}$$

(New)

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_{19}}{19}$$

$$\bar{x} = \frac{192}{19} = 10.1$$

$$\text{Variance} = \sigma^2 = \frac{\sum (x_i^2)}{n} - \left(\frac{\sum x_i}{n} \right)^2$$

$$\Rightarrow \sigma^2 = \frac{2016}{19} - \left(\frac{192}{19} \right)^2$$

$$\Rightarrow \sigma^2 = 3.9889$$

$$SD = \sigma = \sqrt{3.9889} = 1.99$$

(ii) wrong obs. is replaced by '12'

$$\text{No. of obs.} = 20 = n$$

$$x_{20}$$

$$\text{obs.} \rightarrow x_1, x_2, \dots, x_{19}, 12$$

$$n = 20$$

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n=20} x_i}{n}$$

$$= \frac{x_1 + x_2 + \dots + x_{19} + 12}{20}$$

$$\bar{x} = \frac{192 + 12}{20} = \frac{204}{20} = 10.2$$

$$\text{Var.} = \sigma^2 = \frac{\sum_{i=1}^{20} (x_i)^2}{20} - \left(\frac{\sum_{i=1}^{20} x_i}{20} \right)^2$$

$$\sigma^2 = \frac{x_1^2 + x_2^2 + \dots + x_{19}^2 + 12^2}{20} - (10.2)^2$$

$$\sigma^2 = \frac{2016 + 144}{20} - (10.2)^2 = 3.96$$

$$SD = \sigma = \sqrt{3.96} = 1.98$$

⑥ 50 Students.

Subject	M	P	C
$\bar{x}$	42	32	40.9
SD = σ	12	15	20

$$CV_M = \frac{\sigma_M}{\bar{x}_M} \times 100 = \frac{12}{42} \times 100 = 28.57$$

$$CV_P = \frac{15}{32} \times 100 = 46.875$$

$$CV_C = \frac{20}{40.9} \times 100 = 48.899$$

Descending

$$48.899 > 46.875 > 28.57 \leftarrow \text{CV}$$

Chemistry > Physics > Maths.



↑
Variability

Coefficient
of
variation

$\sigma = \text{S.D.}$

$\bar{x} = \text{Mean}$

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

C.V. ↑

Variability ↑