Mathematics
Class XII
Sample Paper – 6

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. A matrix has 12 elements. What are the possible orders it can have?

2. Differentiate $\sin(2x^2)$ w.r.t. x

3. Determine the order and degree of the following differential equation:
$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=e^t$$

4. Write the Cartesian equation of line passing through a point (2, -1, 4) and has direction ratios proportional to 1, 1, -2.

OR

Find the equation of a line passing through (1,-1, 0) and parallel to the line
$$\frac{x-2}{3}=\frac{2y+1}{2}=\frac{5-z}{1}$$

SECTION – B

5. (i) Is the binary operation *, defined on set N, given by
$$a*b=\frac{a+b}{2}\text{ for all } a,b\in N,\text{ commutative?}$$
(ii) Is the above binary operation * associative?

6. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find values of a and b.

7. Evaluate: $\int e^x \left(\dfrac{\sin 4x - 4}{1 - \cos 4x} \right) dx$

OR

Evaluate: $\int \dfrac{1 - x^2}{x(1-2x)} dx$

8. Evaluate: $\int \dfrac{2x}{\sqrt{(x^2+1)(x^2+3)}} dx$

9. Form differential equations of the family of curves represented by $c(y + c)^2 = x^3$, where c is a parameter

10. Find the angle between $\vec{a}$ and $\vec{b}$.
If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5$ & $|\vec{c}| = 7$

OR

Find λ if the vectors
$\vec{a} = \hat{i} - \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are perpendicular to each other.

11. A die is tossed thrice. Find the probability of getting an odd number at least once.

OR

A random variable X has the following probability distribution. Find

X	0	1	2	3	4	5
P(X)	0.1	K	0.2	2K	0.3	K

(i) The value of K (ii) $P(X \leq 1)$ (iii) $P(X > 3)$

12. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

SECTION – C

13. Let A = Q × Q, where Q is the set of all rational numbers, and * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a. b), (c, d) $\in$ A. Then find

(i) The identify element of * in A.

(ii) Invertible elements of A, and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

OR

Let f : W $\rightarrow$ W
be defined as

$$f\ n = \begin{cases} n-1, & \text{if n is odd} \\ n+1, & \text{if n is even} \end{cases}$$

Show that f is invertible and find the inverse of f. Here, W is the set of all whole numbers.

14. Solve the Equation:

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, (x > 0)$$

15. Show that x = 2 is a root of the equation formed by the following determinant

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

Hence, solve the equation.

16. If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $\left(1-x^2\right)\frac{dy}{dx}+y=0$

OR

Differentiate w.r.t. x

$\log_{10}x + \log_x 10 + \log_x x + \log_{10}10$

17. Differentiate w.r.t. x $y=e^{\cos^{-1}\sqrt{1-x^2}}$

18. Find the equation of tangent and normal to the curve $y=-3e^{5x}$ where it crosses the y-axis.

19. Evaluate:

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

20. Evaluate: $\int_0^2\left(x^2+2x+1\right)dx$ as limit of sum.

21. Solve the differential equation $(1 + e^{2x})\,dy + (1 + y^2)e^x\,dx = 0$ given that when x = 0, y = 1.

OR

Solve the differential equation $x(1 + y^2)\,dx - y(1 + x^2)dy = 0$, given that y = 0, when x = 1.

22. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$

23. A variable plane is at a constant distance p from the origin and meet the coordinate axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is
$x^{-2}+y^{-2}+z^{-2}=16p^{-2}$

SECTION – D

24. Let A = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that f(A) = 0. Use this result to find A^5.

OR

Let $f(x) = x^2 - 5x + 6$. Find f(A), if A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

25. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

26. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

27. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.

OR

Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$, $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

28. A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make item A and only half an hour to make item B. The maximum time available per day is 16 hours. The profit per item of A is Rs. 300 and Rs. 160 on one item of B. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

29. In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses it or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ be the probability that he copies it. Assuming that a student, who copies the answer, will be correct with the probability
¾, what is the probability that student knows the answer, given that he answered it
correctly?
Arjun does not know that answer to one of the questions in the test. Which value would Arjun violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years?

Mathematics
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SECTION – A

1. A matrix is of order m × n, then it has mn elements.

So, mn = 12

Which means we have to find values of m and n, such that it will satisfy the above condition

So all the possible cases are

(1, 12), (2, 6), (3, 4), (4, 3), (6, 2), (12, 1).

2.

Let $f(x) = \sin(2x^2)$

$$\frac{d}{dx}f(x)=\frac{d}{dx}\left(\sin\left(2x^2\right)\right)\frac{d}{dx}\left(2x^2\right)$$
$$=\cos\left(2x^2\right)(4x)$$
$$=4x\cos\left(2x^2\right)$$

3.

Sol:

Order: 3

Degree: 1

4.

The Cartesian form of a line is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Substituting point (2, -1, 4) and d. r. s. 1, 1, -2

We get the equation as

$$\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-4}{-2}$$

OR

The equation can be written as

$$\frac{x-2}{3}=\frac{y+\frac{1}{2}}{1}=\frac{z-5}{-1}$$

D.R.S. of this line is proportional to 3, 1,-1.

Also the line passes through (1,-1, 0)

$$\frac{x-1}{3}=\frac{y+1}{1}=\frac{z-0}{-1}$$

SECTION – B

5.

(i) For all $a,b \in N, a*b=\frac{a+b}{2}$

Now, $b*a=\frac{b+a}{2}=\frac{a+b}{2}=a*b$

Thus, the binary operation * is commutative.

(ii) Let a,b,c $\in$ N

$$a*\ \ b*c\ \ =a*\left(\frac{b+c}{2}\right)=\frac{a+\frac{b+c}{2}}{2}=\frac{2a+b+c}{4}$$

$$a*b\ \ *c=\left(\frac{a+b}{2}\right)*c=\frac{\frac{a+b}{2}+c}{2}=\frac{a+b+2c}{4}$$

$\therefore\ a*b\ \ *c \neq a*\ \ b*c$

Thus, the binary operation * is not associative.

6.

The corresponding values of two equal matrices are equal

a + b = 6 and ab = 8

Therefore, $b = \frac{8}{a}$

Substituting in first condition we get,

$$a + \frac{8}{a} = 6$$
$$a^2 - 6a + 8 = 0$$
$$(a-4)(a-2) = 0$$
$$a = 4 \text{ or } a = 2 \text{ and}$$
$$b = 2 \text{ or } b = 4 \text{ respectively.}$$

7.

$$\text{Let } I = \int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx$$
$$= \int e^x \left(\frac{\sin 2(2x) - 4}{1 - \cos 2(2x)} \right) dx$$
$$= \int e^x \left(\frac{2 \sin 2x \cos 2x - 4}{2 \sin^2 (2x)} \right) dx \qquad [\text{Using}, \sin 2x = 2 \sin x . \cos x \text{ and } 2 \sin^2 x = 1 - \cos(2x)$$

$$= \int e^x \left(\frac{2(\sin(2x)\cos(2x) - 4}{2 \sin^2 2x} \right) dx = \int e^x \left(\frac{\sin(2x)\cos(2x)}{\sin^2 2x} - \frac{2}{\sin^2 2x} \right) dx$$
$$= \int e^x \left(\cot(2x) - 2 \operatorname{cosec}^2 2x \right) dx$$

Now, let f(x) = cot(2x) then f'(x) = $-2\operatorname{cosec}^2 2x$

$$I = \int e^x (f(x) + f'(x)) dx$$

So, I = $e^x f(x) + C = e^x \cot 2x + C$, where C is a constant

Therefore, $\int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx = e^x \cot(2x) + C$

OR

$$\int \frac{1-x^2}{x(1-2x)}dx$$

Here $\frac{1-x^2}{x(1-2x)}$ is an improper rational fraction.

Reducing it to proper rational fraction gives

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right) \quad \ldots\ldots\ldots\ldots(1)$$

Now, let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

$$\Rightarrow \frac{2-x}{x(1-2x)} = \frac{A(1-2x)+Bx}{x(1-2x)} \Rightarrow 2-x = A - x(2A-B)$$

Equating the coefficients we get, $A = 2$ and $B = 3$

So, $\frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{(1-2x)}$

Substituting in equation (1), we get

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{(1-2x)}\right)$$

i.e $\int \frac{1-x^2}{x(1-2x)}dx = \int \left[\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{(1-2x)}\right)\right]dx$

$$= \int \frac{dx}{2} + \int \frac{dx}{x} + \frac{3}{2}\int \frac{dx}{(1-2x)} = \frac{x}{2} + \log|x| + \frac{3}{2} \times \frac{1}{(-2)}\log|1-2x| + C$$

$$= \frac{x}{2} + \log|x| - \frac{3}{4}\log|1-2x| + C$$

8.

$$I=\int\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx$$

Let $x^2=z$

$\therefore 2xdx=dz$

$$\therefore I=\int\frac{dz}{(z+1)(z+3)}$$

By partial fraction,

$$\frac{1}{(z+1)(z+3)}=\frac{A}{z+1}+\frac{B}{z+3}$$

$\Rightarrow 1=A(z+3)+B(z+1)$

Putting $z=-3$, we obtain:

$1=-2B$

$B=-\frac{1}{2}$

$\therefore A=\frac{1}{2}$

$$\therefore \frac{1}{(z+1)(z+3)}=\frac{\frac{1}{2}}{z+1}+\frac{\left(-\frac{1}{2}\right)}{z+3}$$

$$\Rightarrow \int\frac{dz}{(z+1)(z+3)}=\frac{1}{2}\int\frac{dz}{z+1}-\int\frac{dz}{z+3}$$

$$=\frac{1}{2}\log|z+1|-\frac{1}{2}\log|z+3|+C$$

$$\therefore \int\frac{2xdx}{\left(x^2+1\right)\left(x^2+3\right)}=\frac{1}{2}\log\left|x^2+1\right|-\frac{1}{2}\log\left|x^2+3\right|+C$$

$$=\frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right|+C$$

9.

The equation of family of curves is

$c(y + c)^2 = x^3$.......(1)

Differentiating w.r.t. x

$$\Rightarrow 2c(y+c)\frac{dy}{dx} = 3x^2 ......(2)$$

(1)/(2)

$$\Rightarrow \frac{c(y+c)^2}{2c(y+c)\frac{dy}{dx}} = \frac{x^3}{3x^2}$$

$$\Rightarrow c = \frac{2x}{3}\frac{dy}{dx} - y$$

putting value of c in (1)

$$\Rightarrow \left(\frac{2x}{3}\frac{dy}{dx} - y\right)\left(\frac{2}{3}x\frac{dy}{dx}\right)^2 = x^3$$

$$\Rightarrow \frac{4}{9}\left(\frac{dy}{dx}\right)^2\left(\frac{2x}{3}\frac{dy}{dx} - y\right) = x$$

$$\Rightarrow \frac{8}{27}x\left(\frac{dy}{dx}\right)^3 - \frac{4}{9}\left(\frac{dy}{dx}\right)^2 y = x$$

$$\Rightarrow 8x\left(\frac{dy}{dx}\right)^3 - 12\left(\frac{dy}{dx}\right)^2 y = 27x$$

10.

Here, $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow \left(\vec{a}+\vec{b}\right)^2=\left(-\vec{c}\right)^2$

$\Rightarrow \vec{a}^2+\vec{b}^2+2\vec{a}.\vec{b}=\vec{c}^2$

$\Rightarrow \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=\left|\vec{c}\right|^2$,

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\Rightarrow (3)^2+(5)^2+2(3)(5)\cos\theta=(7)^2$

$\Rightarrow 9+25+30\cos\theta=49$

$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2}$

$\Rightarrow \theta=\cos^{-1}\left(\frac{1}{2}\right)=60^{o}$

OR

$\vec{a}=\hat{i}-\lambda\hat{j}+3k$ and $\vec{b}=4\hat{i}-5\hat{j}+2k$

vectors are perpendicular if $\vec{a}\cdot\vec{b}=0$

$\vec{a}\cdot\vec{b}=\left(\hat{i}-\lambda\hat{j}+3k\right)\cdot\left(4\hat{i}-5\hat{j}+2k\right)$

$=1\times4+\left(-\lambda\right)\times\left(-5\right)+3\times2=4+5\lambda+6=10+5\lambda$

$\vec{a}\cdot\vec{b}=0\Rightarrow 10+5\lambda=0\Rightarrow \lambda=-2$

11.

S = {(x, y, z) : x, y, z ∈ {1, 2, 3, 4, 5, 6}}

S contains 6 × 6 × 6 = 216 cases

Let

E: an odd number appears at least once

and

E': an odd number appears not even once

i.e. E' an even number appears all three time

E' = {(x, y, z): x, y, z ∈ {2, 4, 6}}

E' contains 3 × 3 × 3 = 27 cases

Now , P(E') = 1 − P(E)

$$P(E) = 1 - \frac{27}{216} = \frac{7}{8}$$

OR

X	0	1	2	3	4	5
P(X)	0.1	K	0.2	2K	0.3	K

(i) Since

$$\sum_{i=1}^{n} P(X = x_i) = 1$$

(ii) So 0.1 + K + 0.2 +2K + 0.3 + K = 1

$\Rightarrow$ 4K = 0.4 $\Rightarrow$K = 0.1

(ii) P(X≤ 1) = P(0) + P(1) = 0.1 + 0. 1 = 0.2

(iii) P(X > 3)= P(4) + P(5) = 0.3 + 0.1 = 0.4

12.

Let X be the event of drawing an ace

X' be the event of not drawing an ace

$P(X) = \frac{4}{52} = \frac{1}{13};$

$P(S') = \frac{12}{13}$

X can take the values 0, 1 or 2

$P(0) = {}^2C_0 . \frac{12}{13} . \frac{12}{13} = \frac{144}{169};$

$P(1) = {}^2C_1 . \frac{1}{13} . \frac{12}{13} = \frac{24}{169};$

$P(2) = {}^2C_2 . \frac{1}{13} . \frac{1}{13} = \frac{1}{169}$

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

SECTION – C

13.

Let A = Q × Q, where Q is the set of rational numbers.
Given that * is the binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for
(a, b), (c, d) $\in$ A.

(i)

We need to find the identity element of the operation * in A.

Let (x, y) be the identity element in A.

Thus,

(a, b) * (x, y) = (x, y) * (a, b) = (a, b), for all (a, b) $\in$ A

$\Rightarrow$(ax, b + ay) = (a, b)

$\Rightarrow$ ax = a and b + ay =b

$\Rightarrow$ y = 0 and x = 1

Therefore, (1, 0) $\in$ A is the identity element in A with respect to the operation *.

(ii)

We need to find the invertible elements of A.

Let (p, q) be the inverse of the element (a, b)

Thus,

(a, b) * (p, q) = (1, 0)

$\Rightarrow$(ap, b + aq) = (1, 0)

$\Rightarrow$ ap = 1 and b + aq = 0

$\Rightarrow p = \frac{1}{a}$ and $q = -\frac{b}{a}$

Thus the inverse elements of (a, b) is $\left(\frac{1}{a}, -\frac{b}{a}\right)$

Now let us find the inverse of (5, 3) and $\left(\frac{1}{2}, 4\right)$

Hence, inverse of (5, 3) is $\left(\frac{1}{5}, -\frac{3}{5}\right)$

And inverse of $\left(\frac{1}{2}, 4\right)$ is $\left(2, \frac{-4}{\frac{1}{2}}\right) = (2, -8)$

OR

Let f: W→W be defined as

$$f(n) = \begin{cases} n - 1, \text{ if n is odd} \\ n + 1, \text{ if n is even} \end{cases}$$

We need to prove that 'f' is invertible.

In order to prove that 'f' is invertible it is sufficient to prove that f is a bijection.

A function f: A→B is a one-one function or an injection, if
f(x) = f(y) ⇒ x = y for all x, y ∈ A.

Case i:
If x and y are odd.

Let f(x) = f(y)

⇒x − 1 = y − 1

⇒x = y

Case ii:
If x and y are even,

Let f(x) = f(y)

⇒x + 1 = y + 1

⇒x = y

Thus, in both the cases, we have,

$f(x) = f(y) \Rightarrow x = y$ for all $x, y \in W$.

Hence f is an injection.

Let n be an arbitrary element of W.

If n is an odd whole number, there exists an even whole number $n - 1 \in W$ such that
$f(n - 1) = n - 1 + 1 = n$.

If n is an even whole number, then there exists an odd whole number $n + 1 \in W$ such that $f(n + 1) = n + 1 - 1 = n$.

Also, $f(1) = 0$ and $f(0) = 1$

Thus, every element of W (co-domain) has its pre-image in W (domain).

So f is an onto function.

Thus, it is proved that f is an invertible function.

Thus, a function $g: B \to A$ which associates each element $y \in B$ to a unique element $x \in A$ such that $f(x) = y$ is called the inverse of f.

That is, $f(x) = y \Leftrightarrow g(y) = x$

The inverse of f is generally denoted by f^{-1}.

Now let us find the inverse of f.

Let $x, y \in W$ such that $f(x) = y$

$\Rightarrow x + 1 = y$, if x is even

And

$x - 1 = y$, if x is odd

$$\Rightarrow x = \begin{cases} y-1, \text{ if } y \text{ is odd} \\ y+1, \text{ if } y \text{ is even} \end{cases}$$

$$\Rightarrow f^{-1}(y) = \begin{cases} y-1, \text{ if } y \text{ is odd} \\ y+1, \text{ if } y \text{ is even} \end{cases}$$

Interchange, x and y, we have,

$$\Rightarrow f^{-1}(x) = \begin{cases} x-1, \text{ if } x \text{ is odd} \\ x+1, \text{ if } x \text{ is even} \end{cases}$$

Rewriting the above we have,

$$\Rightarrow f^{-1}(x) = \begin{cases} x+1, \text{ if } x \text{ is even} \\ x-1, \text{ if } x \text{ is odd} \end{cases}$$

Thus, $f^{-1}(x) = f(x)$

14.

Given: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x \ (x>0)$

$$\Rightarrow 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1}x$$

$$\Rightarrow \tan\left[2\tan^{-1}\left(\frac{1-x}{1+x}\right)\right] = \tan\left[\tan^{-1}x\right]$$

$$\Rightarrow \frac{2\tan\left[\tan^{-1}\left(\frac{1-x}{1+x}\right)\right]}{1-\left(\tan\left[\tan^{-1}\left(\frac{1-x}{1+x}\right)\right]\right)^2} = x$$

$$\Rightarrow \frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} = x$$

$$\Rightarrow \frac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2} = x$$

$$\Rightarrow \frac{(1-x^2)}{2x} = x$$

$$\Rightarrow 3x^2 = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

15.

$$\text{Consider}\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}=0$$

$R_1 \to R_1 - R_2; R_2 \to R_2 - R_3$

$$\Rightarrow\begin{vmatrix} x-2 & 3x-6 & -x+2 \\ 5 & -5x & -5 \\ -3 & 2x & x+2 \end{vmatrix}=0$$

$$=5(x-2)\begin{vmatrix} 1 & 3 & -1 \\ 1 & -x & -1 \\ -3 & 2x & x+2 \end{vmatrix}=0$$

$\Rightarrow$ $(x-2)$ is a factor of the equation

$\Rightarrow x=2$ is a root of the equation

Further , performing the operation

$C_1 \to C_1 + C_3$, we get,

$$5(x-2)\begin{vmatrix} 0 & 3 & -1 \\ 0 & -x & -1 \\ x-1 & 2x & x+2 \end{vmatrix}=0$$

$$\Rightarrow 5(x-2)(x-1)\begin{vmatrix} 0 & 3 & -1 \\ 0 & -x & -1 \\ 1 & 2x & x+2 \end{vmatrix}=0$$

$\Rightarrow 5(x-2)(x-1)(x+3)=0$

$\Rightarrow x=-3,1,2$

16.

Differentiating w.r.t. x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}-1} \times \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \times \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1+x}{1-x}} \times \frac{(1+x)\frac{d}{dx}(1-x)-(1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{1+x}{1-x}} \times \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = \sqrt{\frac{1+x}{1-x}} \times \frac{-1}{(1+x)^2}(1-x^2)$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = -\sqrt{\frac{1-x}{1+x}}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = -y$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} + y = 0$$

OR

Let $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$

$$y = \frac{\log_e x}{\log_e 10} + \frac{\log_e 10}{\log_e x} + \frac{\log_e x}{\log_e x} + \frac{\log_e 10}{\log_e 10}$$

$$y = \frac{\log_e x}{\log_e 10} + \frac{\log_e 10}{\log_e x} + 2$$

then, differentiating w.r.t.

$$\frac{dy}{dx} = \frac{1}{x\log_e 10} + \frac{-\log_e 10}{(\log_e x)^2} \times \frac{1}{x} + 0$$

$$\frac{dy}{dx} = \frac{1}{x\log_e 10} - \frac{\log_e 10}{x(\log_e x)^2}$$

17.

$$y = e^{\cos^{-1}\sqrt{1-x^2}}$$

differentiating w.r.t. x

$$\frac{dy}{dx} = e^{\cos^{-1}\sqrt{1-x^2}}\left(\frac{d}{dx}\cos^{-1}\sqrt{1-x^2}\right)$$

$$\frac{dy}{dx} = e^{\cos^{-1}\sqrt{1-x^2}}\frac{-1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\left(\frac{d}{dx}\sqrt{1-x^2}\right)$$

$$\frac{dy}{dx} = e^{\cos^{-1}\sqrt{1-x^2}}\frac{-1}{\sqrt{1-\left(1-x^2\right)}}\left(\frac{1}{2}\left(1-x^2\right)^{\frac{1}{2}-1}\times\frac{d}{dx}\left(1-x^2\right)\right)$$

$$\frac{dy}{dx} = e^{\cos^{-1}\sqrt{1-x^2}}\frac{-1}{\sqrt{x^2}}\left(\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}\times(-2x)\right)$$

$$\frac{dy}{dx} = e^{\cos^{-1}\sqrt{1-x^2}}\times\frac{1}{\sqrt{1-x^2}}$$

18.

The given curve crosses the y-axis. When $x = 0$

So *y* becomes $y = -3e^0 = -3.$

So the curve intersects the y-axis at point (0, - 3)

Differentiating $y = -3e^{5x}$ with respect to *x*

$$\frac{dy}{dx} = -15e^{5x}$$

$$\left.\frac{dy}{dx}\right]_{(0,-3)} = -15e^0 = -15$$

Therefore, equation of tangent becomes

y + 3 = –15(x – 0)

which is 15x + y + 3 = 0

And also slope of the normal at (0, -3) would be $\frac{1}{15}$ so the equation of the normal is

$$y+3=\frac{1}{15}(x-0)$$
$$x-15y-45=0$$

19.

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

$$\text{Now}, 5x+3=A\frac{d}{dx}\left(x^2+4x+10\right)+B$$
$$\Rightarrow 5x+3=A\left(2x+4\right)+B$$
$$\Rightarrow 5x+3=2Ax+4A+B$$
$$\Rightarrow 2A=5 \text{ and } 4A+B=3$$
$$\Rightarrow A=\frac{5}{2}$$
$$\text{Thus, } 4\left(\frac{5}{2}\right)+B=3$$
$$\Rightarrow 10+B=3$$
$$\Rightarrow B=3-10=-7$$

On substituting the values of A and B, we get

$$\int \frac{(5x+3)}{\sqrt{x^2+4x+10}}dx=\int \frac{\left[\frac{5}{2}\frac{d}{dx}\left(x^2+4x+10\right)-7\right]}{\sqrt{x^2+4x+10}}dx$$
$$=\int \left[\frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}\right]dx$$
$$=\frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx-7\int \frac{dx}{\sqrt{x^2+4x+10}}$$
$$=\frac{5}{2}I_1-7I_2 \qquad ...(1)$$

$$I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$$

Put $x^2+4x+10 = z^2$

$(2x+4)dx = 2zdz$

Thus, $I_1 = \int \frac{2z}{z}dz = 2z = 2\sqrt{x^2+4x+10} + C_1$

$$I_2 = \int \frac{dx}{\sqrt{x^2+4x+10}}$$

$$= \int \frac{dx}{\sqrt{x^2+4x+4+6}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2+\left(\sqrt{6}\right)^2}}$$

$$= \log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C_2$$

Substituting I_1 and I_2 in (1), we get

$$\therefore \int \frac{5x+3}{\sqrt{x^2+4x+10}} = \frac{5}{2}(2\sqrt{x^2+4x+10} + C_1) - 7\left[\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C_2\right]$$

$$= 5\sqrt{x^2+4x+10} - 7\left[\log\left|(x+2) + \sqrt{x^2+4x+10}\right|\right] + \frac{5}{2}C_1 - 7C_2$$

$$= 5\sqrt{x^2+4x+10} - 7\left[\log\left|(x+2) + \sqrt{x^2+4x+10}\right|\right] + C, \quad \text{where } C = \frac{5}{2}C_1 - 7C_2$$

20.

On comparison with $\int_a^b f(x)dx$

We have a = 0, b = 2, f(x) = $x^2 + 2x + 1$

nh = b – a = 2

$$\therefore \int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+......+f\left(a+(n-1)h\right)\right]$$

$$\text{Now } f(a)=f(0)=0+0+1=1$$
$$f(a+h)=f(0+h)=f(h)=h^2+2h+1$$
$$f(a+2h)=f(0+2h)=f(2h)=(2h)^2+2.2h+1$$
$$=4h^2+4h+1$$

$$f\left(a+(n-1)h\right)=f\left(0+(n-1)h\right)$$
$$=f\left((n-1)h\right)$$
$$=(n-1)^2h^2+2(n-1)h+1$$

$$\int_a^b\left(x^2+2x+1\right)dx=\lim_{h\to 0} h\begin{bmatrix}1+\left(h^2+2h+1\right)\\ +\left(4h^2+4h+1\right)+....+(n-1)^2h^2+2(n-1)h+1\end{bmatrix}$$

$$=\lim_{h\to 0} h\left[n+h^2\left(1+4+....+(n-1)^2\right)+2h\left(1+2+.....+(n-1)\right)\right]$$

$$=\lim_{h\to 0} h\left[n+h^2.\frac{n(n-1)(2n-1)}{6}+2h.\frac{n(n-1)}{2}\right]$$

$$=\lim_{h\to 0}\left[nh+\frac{nh(nh-h)(2nh-h)}{6}+nh(nh-h)\right]$$

$$=\lim_{h\to 0}\left[2+\frac{2(2-h)(2.2-h)}{6}+2(2-h)\right]$$

$$=\lim_{h\to 0}\left[2+\frac{2(2-h)(4-h)}{6}+2(2-h)\right]$$

21.

Given that,

$\left(1+e^{2x}\right)dy+\left(1+y^{2}\right)e^{x}dx=0$

$\left(1+e^{2x}\right)dy=-\left(1+y^{2}\right)e^{x}dx$

$\frac{1}{1+y^{2}}dy=\frac{-e^{x}}{1+e^{2x}}dx$

$\int\frac{1}{1+y^{2}}dy=\int\frac{-e^{x}}{1+e^{2x}}dx$

$\int\frac{1}{1+y^{2}}dy=-\int\frac{1}{1+t^{2}}dt$, where $t=e^{x}$

$\tan^{-1}y=-\tan^{-1}t+c$

$\tan^{-1}y=-\tan^{-1}e^{x}+c$

given $x=0,\ y=1$

substituting

$\tan^{-1}1=-\tan^{-1}1+c$

$2\tan^{-1}1=c$

$c=\frac{\pi}{2}$

sub c back in equation

$\tan^{-1}y=-\tan^{-1}e^{x}+\frac{\pi}{2}$

$\tan^{-1}y=\cot^{-1}e^{x}$

$\tan^{-1}y=\tan^{-1}\left(\frac{1}{e^{x}}\right)$

$y=\frac{1}{e^{x}}$ ....which is required solution

OR

$$x\left(1+y^2\right)dx-y\left(1+x^2\right)dy=0$$

$$\Rightarrow x\left(1+y^2\right)dx=y\left(1+x^2\right)dy$$

$$\Rightarrow \frac{x}{\left(1+x^2\right)}dx=\frac{y}{\left(1+y^2\right)}dy$$

$$\Rightarrow \int\frac{x}{\left(1+x^2\right)}dx=\int\frac{y}{\left(1+y^2\right)}dy$$

$$\Rightarrow \int\frac{2x}{\left(1+x^2\right)}dx=\int\frac{2y}{\left(1+y^2\right)}dy$$

$$\Rightarrow \log\left|1+x^2\right|=\log\left|1+y^2\right|+\log c$$

$$\Rightarrow \log\left|\frac{1+x^2}{1+y^2}\right|=\log c$$

$$\frac{1+x^2}{1+y^2}=c$$

given that $x=1$, then $y=0$

substituting we get

$c=2$

$\frac{1+x^2}{1+y^2}=2$.......which is required solution.

22.

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\therefore \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$

$\vec{a} \times \vec{c} = \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x)$ ...(1)

Now, $\vec{a} \times \vec{c} = \vec{b}$

$\vec{b} = \hat{j} - \hat{k}$ ...(2)

Comparing (1) and (2), we get :

$z - y = 0 \Rightarrow z = y$ ...(3)

$z - x = -1$ ...(4)

$y - x = -1$ ...(5)

Also, given that

$\vec{a}.\vec{c} = 3$

$\therefore (\hat{i} + \hat{j} + \hat{k}).(x\hat{i} + y\hat{j} + z\hat{k}) = 3$

$x + y + z = 3$

Using (3), we get, $x + 2y = 3$ ...(6)

Adding (5) and (6), we get

$3y = 2 \Rightarrow y = \frac{2}{3}$

$\therefore z = \frac{2}{3} \quad (\because z = y)$

From (6), we have,

$x = 3 - 2y$

$\Rightarrow x = 3 - \frac{2 \times 2}{3}$

$\Rightarrow x = \frac{9 - 4}{3}$

$\Rightarrow x = \frac{5}{3}$

$\therefore \vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

Thus, the required vector $\vec{c}$ is $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$.

23.

Let the variable plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1\text{.....................}(1)$$

This meet the coordinate axes at A(a,0,0), B(0,b,0) and C(0,0,c) respectively.

Let (α, β, γ)be the coordinates of the centroid of tetrahedron OABC. Then,

$$\alpha=\frac{0+a+0+0}{4}=\frac{a}{4},\ \beta=\frac{0+0+b+0}{4}=\frac{b}{4},\ \gamma=\frac{0+0+0+c}{4}=\frac{c}{4}\text{........}(ii)$$

the plane in (i) is at constant distance p from the origin.

$\therefore p=$ length of perpendicular from $(0,0,0)$ to plane(i)

$$\Rightarrow p=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow p=\frac{1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow \frac{1}{p^2}=\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2$$

eliminating variables a, b and c from (ii)

$$\Rightarrow \frac{1}{p^2}=\left(\frac{1}{4\alpha}\right)^2+\left(\frac{1}{4\beta}\right)^2+\left(\frac{1}{4\gamma}\right)^2$$

$$\Rightarrow 16p^{-2}=\alpha^{-2}+\beta^{-2}+\gamma^{-2}$$

Hence the locus is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$

SECTION – D

24. We have, $f(x) = x^2 - 4x + 7$

$f(A) = A^2 - 4A + 7I$

Now,

$$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$
$$-4A = \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} \text{ and, } 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$f(A) = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$f(A) = A^2 - 4A + 7I = 0$

$A^2 = 4A - 7I$

$A^3 = A^2A = (4A - 7I)A = 4A^2 - 7A = 4(4A - 7I) - 7A = 9A - 28I$

$A^4 = A^3A = (4A^2 - 7A)A = 4A^3 - 7A^2 = 8A - 63I$…………..(put values of A^2 and A^3)

$A^5 = A^4A = (8A - 63I)A = 8A^2 - 63A = -31A - 56I$

$$A^5 = -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -62 & -93 \\ 31 & -62 \end{bmatrix} + \begin{bmatrix} -56 & 0 \\ 0 & -56 \end{bmatrix}$$
$$= \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$

OR

We have, $f(x) = x^2 - 5x + 6$

$f(A) = A^2 - 5A + 6I$

Now,

$$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$-5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

and

$$6I = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

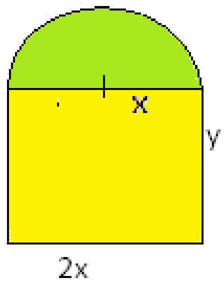
$$f(A) = A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

25.

Let the length of the window be 2x units and height of the rectangular portion of the window be *y* units



Perimeter or length of boundary of window is 10

So $10 = 2x + 2y + \pi x$ .....(i)

Area of the window is given as $A = 2xy + \frac{\pi x^2}{2}$ .....(ii)

From (i), $y = \frac{10 - 2x - \pi x}{2}$ using it in (ii), we get

$$A = 2x\left(\frac{10 - 2x - \pi x}{2}\right) + \frac{\pi x^2}{2}$$

$$A = 10x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$$

$$A = 10x - 2x^2 - \frac{\pi x^2}{2}$$

Differentiating with respect to x

$$\frac{dA}{dx} = 10 - 4x - \pi x$$

For maximum or minimum we have,

$$\frac{dA}{dx} = 0$$

$$10 = (4 + \pi)x$$

$$x = \frac{10}{4 + \pi}$$

$$\frac{d^2A}{dx^2} = -4 - \pi < 0$$

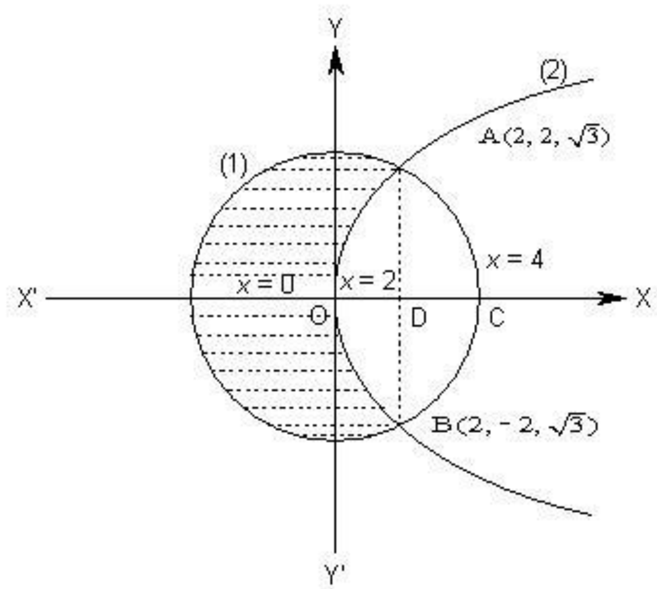
So $x = \frac{10}{4+\pi}$ and $y = \frac{10}{4 + \pi}$

So the length of the rectangular portion must be $\frac{10}{4 + \pi}$ to admit maximum sunlight.

26.

$x^2 + y^2 = 16$ ......(1)

$y^2 = 6x$ ......(2)



Points of intersection of curve (i) and (ii) is

$$x^2 + 6x - 16 = 0$$

$$\Rightarrow \quad (x+8)(x-2) = 0$$

$$\Rightarrow \quad x = 2 \qquad (\because x \neq -8)$$

$y^2 = 12$

$y = \pm 2\sqrt{3}$

$\therefore \quad A\left(2,\ 2\sqrt{3}\right)$ and $B\left(2,-2\sqrt{3}\right)$

Also $\quad C(4,0)$.

Area $\quad OBCAO = 2$ (Area ODA + Area DCA)

$$=2\left[\int_0^2 y_2 dx + \int_2^4 y_1 dx\right]$$

$$=2\left[\int_0^2 \sqrt{6x}\ dx + \int_2^4 \sqrt{16-x^2}\ dx\right]$$

$$=2\left[\sqrt{6}.\left\{\frac{2}{3}x^{3/2}\right\}_0^2 + \left\{\frac{x\sqrt{16-x^2}}{2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right\}_2^4\right]$$

$$=2\left[\frac{2\sqrt{6}}{3}.2\sqrt{2} + \left\{0+8\sin^{-1}1\right\} - \left\{\frac{2.2\sqrt{3}}{2} + 8\sin^{-1}\frac{1}{2}\right\}\right]$$

$$=\frac{16\sqrt{3}}{3} + 16.\frac{\pi}{2} - \left(4\sqrt{3} + 16.\frac{\pi}{6}\right)$$

$$=\left(\frac{4\sqrt{3}}{3} + \frac{16}{3}\pi\right)\text{sq. units.}$$

Required, Area = Area of circle $-\left(\frac{4\sqrt{3}}{3} + \frac{16}{3}\pi\right)$

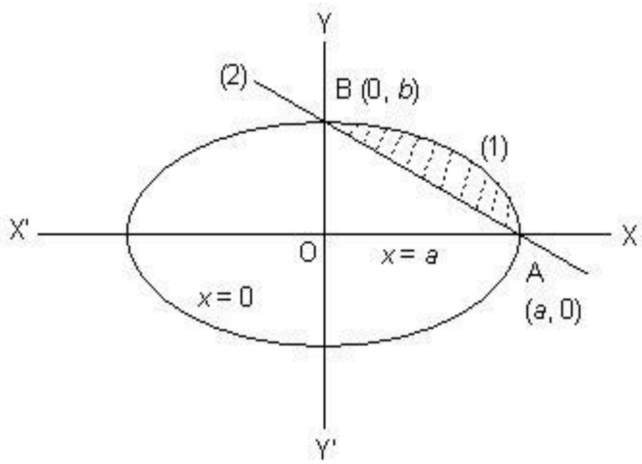
$$=16\pi - \frac{4\sqrt{3}}{3} - \frac{16}{3}\pi$$

$$=-\frac{32}{3}\pi - \frac{4\sqrt{3}}{3} = \frac{4}{3}\left(8\pi - \sqrt{3}\right)\text{sq. units}$$

OR

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

$$\frac{x}{a}+\frac{y}{b}=1 \qquad ...(2)$$



Equation (i) gives y = $b\sqrt{1-\frac{x^2}{a^2}}$

Equation (ii) gives y = $b\left(1-\frac{x}{a}\right)$

(Area of the smaller region)

$$=\int_0^a \left(y_1-y_2\right)dx$$

$$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx - \int_0^a b\left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}dx - b\int_0^a \left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - b\left[x-\frac{x^2}{2a}\right]_0^a$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right] - b\left[a-\frac{a^2}{2a}\right]$$

$$= \frac{ab}{2}.\frac{\pi}{2}-\frac{ab}{2} = \frac{1}{4}ab(\pi-2) \text{ sq. units.}$$

27.

Let the equation of the plane be,

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

Plane passes through the point (-1, 3, 2)

$$\therefore A(x+1)+B(y-3)+C(z-2)=0\_\_(i)$$

Now applying the condition of perpendicularity to the plane (i) with planes

$x + 2y + 3z = 5$ and $3x + 3y + z = 0$, we have

$A + 2B + 3C = 0$

$3A + 3B + C = 0$

Solving we get

$$A+2B+3C=0$$
$$9A+9B+3C=0$$

By cross multiplication, we have,

$$\frac{A}{2\times3-9\times3}=\frac{B}{9\times3-1\times3}=\frac{C}{1\times9-2\times9}$$
$$\Rightarrow \frac{A}{6-27}=\frac{B}{27-3}=\frac{C}{9-18}$$
$$\Rightarrow \frac{A}{-21}=\frac{B}{24}=\frac{C}{-9}$$
$$\Rightarrow \frac{A}{7}=\frac{B}{-8}=\frac{C}{3}$$
$$\Rightarrow A=7\lambda;B=-8\lambda;C=3\lambda$$

By substituting A and C in equation (i), we get

Substituting the values of A, B and C in equation (i), we have,

$$7\lambda(x+1)-8\lambda(y-3)+3\lambda(z-2)=0$$
$$\Rightarrow 7x+7-8y+24+3z-6=0$$
$$\Rightarrow 7x-8y+3z+25=0$$

OR

The equations of the given planes are

$$\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right) - 4 = 0 \quad \dots (1)$$

$$\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right) + 5 = 0 \quad \dots (2)$$

The equation of the plane passing through the line of intersection of the given planes is

$$\left[\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4\right]+\lambda\left[\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5\right] = 0$$

$$\vec{r}.\left[\left(1+2\lambda\right)\hat{i}+\left(2+\lambda\right)\hat{j}+\left(3-\lambda\right)\hat{k}\right]+\left(-4+5\lambda\right) = 0 \quad \dots (3)$$

The plane in equation (3) is perpendicular to the plane,
$\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right) + 8 = 0.$

$$\therefore\ 5(1+2\lambda)+3(2+\lambda)-6(3-\lambda)=0$$

$$\Rightarrow 5+10\lambda+6+3\lambda-18+6\lambda=0$$

$$\Rightarrow 19\lambda - 7 = 0$$

$$\Rightarrow \lambda = \frac{7}{19}$$

Substituting $\lambda=\frac{7}{19}$ in equation (3),

$$\vec{r}.\left[\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right]-\frac{41}{19}\ 0$$

$$\Rightarrow \vec{r}.\left(33\hat{i}+45\hat{j}+50\hat{k}\right) - 41 = 0$$

This is the vector equation of the required plane.

28.

Let the number of items of type A and B produced be x and y respectively.

The L. P.P. is Maximize:

z = 300x + 160y

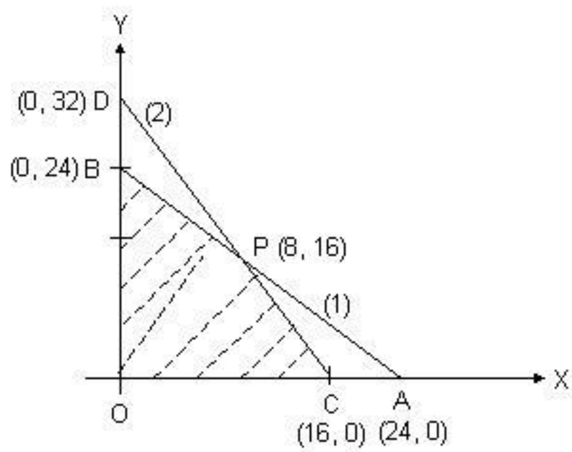
Subject to the constraints

$x + y \leq 24$

$x.1 + y.\frac{1}{2} \leq 16$

$x \geq 0,\ y \geq 0.$

Graphically



These meet at P (8, 16).

The feasible region is OCPB.

The value of z = 300x + 160y

at O(0, 0) is zero

at C(16, 0) is 4800

at B(0 ,24) is 3840

at P(8, 16) is 4960

Clearly value is max. at P(8, 16) ⇒ 8 items of type A and 16 of type B should be produced for maximum profit.

29.

Let E_1, E_2, E_3 and A be the events defined as follows:

Let E_1 be the event that the student knows the answer.

Let E_2 be the event that the student guesses the answer.

Let E_3 be the event that the student copies the answer.

Let A be the event that the answer is correct.

$P(E_1)=\frac{1}{2};P(E_2)=\frac{1}{4};P(E_3)=\frac{1}{4};$

Probability that he answers correctly given that he knew the answer is 1.

That is, $P(A|E_1)=1$

If E_2 has already occurred, then the student guesses. Since there are four choices out of which only one is correct, therefore, the probability that he answers correctly given that he has made a guess is $\frac{1}{4}$.

That is $P(A|E_2)=\frac{1}{4}$.

It is given that, $P(A|E_3)=\frac{3}{4}$

By Bayes Theorem, we have,

Required Probability=$P(E_1 | A)$

$$= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{2}\times 1}{\frac{1}{2}\times 1+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{16}+\frac{3}{16}}$$

$$= \frac{\frac{1}{2}}{\frac{8}{16}+\frac{1}{16}+\frac{3}{16}}$$

$$= \frac{\frac{1}{2}}{\frac{12}{16}}$$

$$= \frac{1}{2}\times\frac{16}{12}$$

$$= \frac{1}{2}\times\frac{4}{3}$$

$$= \frac{2}{3}$$

Thus, the probability that student knows the answer,

given that he answered it correctly is $\frac{2}{3}$.

Arjun is dishonest, as he copies from the other student(s).

Copying once may become habit forming as he may continue resort to dishonest means

in the coming years.