KVPY QUESTION PAPER-2018 (STREAM SX)

Part – I

One - Mark Questions

Date : 04 / 11 / 2018

MATHEMATICS

1. Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a real matrix with nonzero entries, ad – bc = 0, and A^2 = A. Then a + d equals

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **[A]**

Sol. $A^2 - A = 0$

$A(A - I) = 0$

$|A| = 0, |A - I| = 0$

$\begin{bmatrix} a-1 & b \\ c & d-1 \end{bmatrix} = 0$

$ad - a - d + 1 - bc = 0$

$a + d = 1$ $(ad - bc) = 0$

2. On any given arc of positive length on the unit circle |z| = 1 in the complex plane.

(A) there need not be any root of unity

(B) there lies exactly one root of unity

(C) there are more than one but finitely many roots of unity

(D) there are infinitely many roots of unity

Ans. **[D]**

Sol. n^{th} root of unity = $e^{i\frac{2\pi}{n}}, e^{i\frac{4\pi}{n}}$ …………

All these roots lie on the unit circle |z| = 1 and however small the arc of this circle, we may find infinite n for which the roots of unity lie on circle

3. For $0 < \theta < \frac{\pi}{2}$, four tangents are drawn at the four points (±3cosθ, ±2sinθ) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If A(θ) denotes the area of the quadrilateral formed by these four tangents, the minimum value of A(θ) is

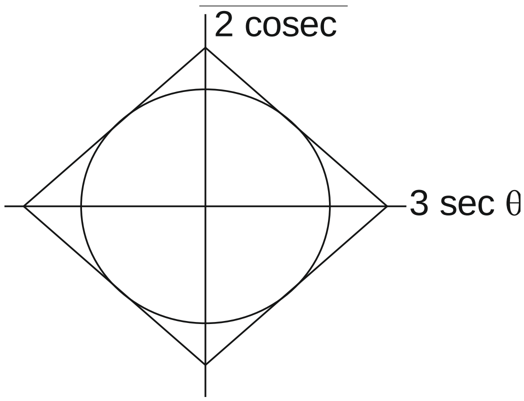
(A) 21 (B) 24 (C) 27 (D) 30

Ans. **[B]**

Sol. $\frac{x - 3\cos\theta}{9} + \frac{y - 2\sin\theta}{4} = 1$

At y = 0, x = 3secθ

At x = 0, y = 2cosec θ



$A(\theta) = 4 \times \frac{1}{2} \times 3\sec\theta \times 2\text{cosec}\,\theta$

$= \frac{24}{\sin 2\theta}$

Min. A = 24

4. Let $S = \{x \in R : \cos(x) + \cos(\sqrt{2}\,x) < 2\}$. Then

(A) $S = \phi$ (B) S is a non-empty finite set

(C) S is an infinite proper subset of $R \backslash \{0\}$ (D) $S = R \backslash \{0\}$

Ans. **[D]**

Sol. Cos x + cos($\sqrt{2}$ x) will always be less than 2 except when both cos x = 1 & cos ($\sqrt{2}$ x) = 1

$\cos x = 1 \Rightarrow x = 2n\pi$

$\cos(\sqrt{2}\,x) = 1 \Rightarrow x = \sqrt{2}\,m\pi$

both can simultaneously be 1 only when x = 0

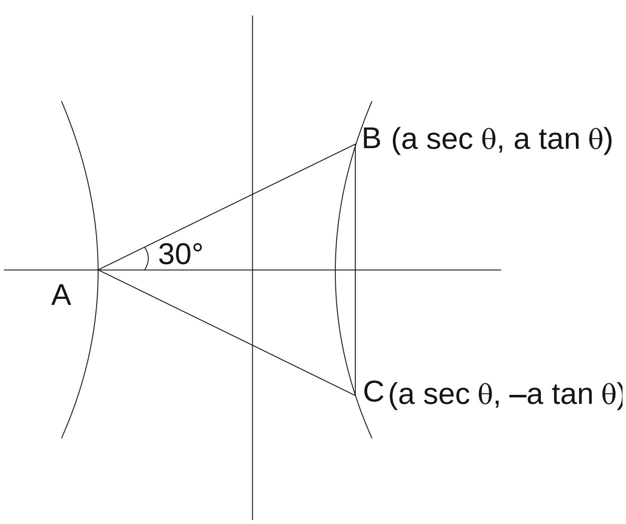
$\Rightarrow S = R - \{0\}$

5. On a rectangular hyperbola $x^2 - y^2 = a^2$, a > 0, three points A, B, C are taken as follows : A = (–a, 0); B and C are placed symmetrically with respect to the x-axis on the branch of the hyperbola not containing A. Suppose that the triangle ABC is equilateral. If the side-length of the triangle ABC is ka, then k lies in the interval

(A) (0, 2] (B) (2, 4] (C) (4, 6] (D) (6, 8]

Ans. **[B]**

Sol.



$AB^2 = BC^2$

$(2a \tan \theta)^2 = a^2(\sec \theta + 1)^2 + a^2(\tan^2 \theta)$

$4 \tan^2\theta = \sec^2 \theta + 1 + 2 \sec \theta + \tan^2 \theta$

$\tan^2 \theta = 1 + \sec \theta$

$\sec^2 \theta - \sec \theta - 2 = 0$

$(\sec \theta - 2)(\sec \theta + 1) = 0$

$\sec \theta = +2$, $\sec \theta = -1$ (not possible)

$\tan \theta = \sqrt{3}$ length $= 2a\sqrt{3}$

$\Rightarrow k = 2\sqrt{3}$

6. The number of real solutions x of the equation

$\cos^2(x \sin(2x)) + \frac{1}{1+x^2} = \cos^2 x + \sec^2 x$ is

(A) 0 (B) 1 (C) 2 (D) infinite

Ans. **[B]**

Sol. $\cos^2(x \sin 2x) + \frac{1}{1+x^2} = \cos^2 x + \sec^2 x$ (which is ≥ 2)

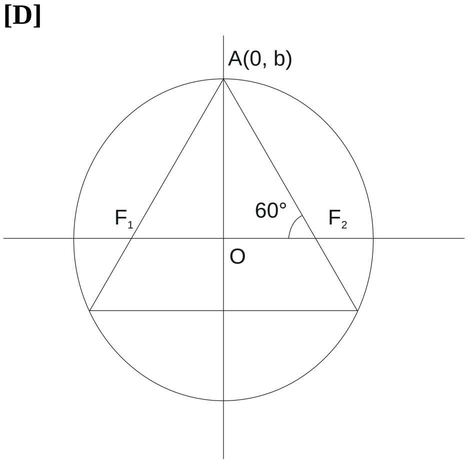
but $0 \leq \cos^2(x \sin 2x) \leq$ and $0 < \frac{1}{1+x^2} \leq 1$

$\Rightarrow$ only one solution at x = 0, when every term becomes 1

7. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b, be an ellipse with foci F_1 and F_2. Let AO be its semi-minor axis, where O is the centre of the ellipse. The lines AF_1 and AF_2, when extended, cut the ellipse again at points B and C respectively. Suppose that the triangle ABC is equilateral. Then the eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Ans. **[D]**



Sol.

$\frac{b}{ae} = \tan 60° = \sqrt{3}$

$\frac{a^2(1-e^2)}{e^2} = 3$

$3e^2 = 1 - e^2$

$e = 1/2$

8. Let a = cos 1° and b = sin 1°. We say that a real number is algebraic if it is a root of a polynomial with integer coefficients. Then

(A) a is algebraic but b is not algebraic
(B) b is algebraic but a is not algebraic
(C) both a and b are algebraic
(D) neither a nor b is algebraic

Ans. **[C]**

Sol. If $\cos 1° = p + \sqrt{q}$ (p, q $\in$ Q)

then it will be root of a quadratic equation whose other root is $p - \sqrt{q} \Rightarrow \cos 1°$ is algebraic

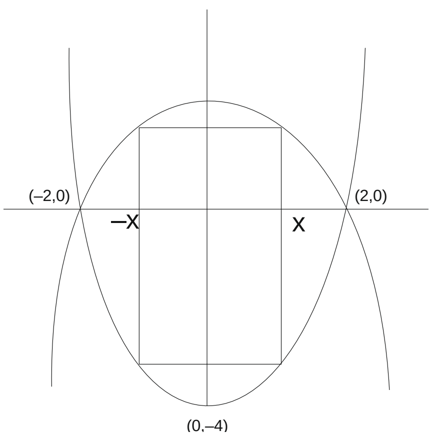
$\Rightarrow \sin 1°$ is algebraic

9. A rectangle with its sides parallel to the x-axis and y-axis is inscribed in the region bounded by the curves $y = x^2 - 4$ and $2y = 4 - x^2$. The maximum possible area of such a rectangle is closest to the integer

(A) 10
(B) 9
(C) 8
(D) 7

Ans. **[B]**

Sol.



$A = 2x[(2 - \frac{x^2}{2}) - (x^2 - 4)]$

$= x(12 - 3x^2)$

$= 12x - 3x^3$

$= \frac{dA}{dx} = 0 \Rightarrow 12 - 9x^2 = 0$

$\Rightarrow x^2 = \frac{4}{3}$

$\Rightarrow x = \frac{2}{\sqrt{3}}$

$A = 3 \times \frac{2}{\sqrt{3}}\left(4 - \frac{4}{3}\right)$

$= \frac{48}{3\sqrt{3}}$

10. Let f(x) = x|sin x|, $x \in$ R. Then

(A) *f* is differentiable for all x, except at x = nπ, n = 1, 2, 3,….

(B) *f* is differentiable for all x, except at x = nπ, n = ±1, ±2, ±3,….

(C) *f* is differentiable for all x, except at x = nπ, n = 0, 1, 2, 3,….

(D) *f* is differentiable for all x, except at x = nπ, n = 0, ±1, ±2, ±3,….

Ans. **[B]**

Sol. f(x) = x|sin x| (continuous fucntion)

$$f(x) = \begin{cases} x\sin x & x \in U(2m\pi,(2m+1)\pi) \\ -x\sin x & x \in U((2m+1)\pi,(2m+2)\pi) \end{cases}$$

critical points are x = nπ

$$f'(x) = \begin{cases} \sin x + x\cos x & x \in U(2m\pi,(2m+1)\pi) \\ -\sin x - x\cos x & x \in U((2m+1)\pi,(2m+2)\pi) \end{cases}$$

LHD $\neq$ RHD for x = nπ – {0}

11. Let f : [–1, 1] $\rightarrow$ R be a function defined by

$$f(x) = \begin{cases} x^2\left|\cos\left(\frac{\pi}{x}\right)\right| & \text{for } x \neq 0, \\ 0 & \text{for } x = 0 \end{cases}$$

The set of points where f is not differentiable is

(A) $\{x \in [-1,1] : x \neq 0\}$

(B) $\{x \in [-1, 1] : x = 0 \text{ or } x = \frac{2}{2n+1}, n \in Z\}$

(C) $\{x \in [-1, 1] : x = \frac{2}{2n+1}, n \in Z\}$

(D) [–1, 1]

Ans. **[C]**

Sol. Modulus changes it's definition at $x = \frac{2}{(2n+1)}$

$$f'(x) = \begin{cases} -\sin\frac{\pi}{x} + 2x\cos\left(\frac{\pi}{x}\right) & x \in U\left(\frac{2}{4n-1}, \frac{2}{4n+1}\right) \\ \sin\frac{\pi}{x} - 2x\cos\left(\frac{\pi}{x}\right) & x \in U\left(\frac{2}{4n+1}, \frac{2}{4n+3}\right) \end{cases}$$

LHD $\neq$ RHD for $x = \frac{2}{(2n+1)}$

And at x = 0, LHD = $\lim\limits_{h \to 0} \frac{h^2 \left|\cos\frac{\pi}{h}\right| - 0}{-h} = 0$

RHD = $\lim\limits_{h \to 0} = \frac{h^2 \left|\cos\frac{\pi}{h}\right| - 0}{h} = 0$

LHD = RHD, hence differentiable at zero.

12. The value of the integral $\int_0^{\pi}(1-|\sin 8x|)\,dx$ is

(A) 0 (B) $\pi - 1$ (C) $\pi - 2$ (D) $\pi - 3$

Ans. **[C]**

Sol. $\int_0^{\pi}(1-|\sin 8x|)\,dx$

$= \pi - \quad 8\int_0^{\frac{\pi}{8}}(\sin 8x)dx$

($\because$ $|\sin 8x|$ is periodic with period $\frac{\pi}{8}$)

$= \pi - 2$

13. Let ln(x) denote the logarithm of x with respect to the base e. Let $S \subset R$ be the set of all points where the function ln $(x^2 - 1)$ is well-defined. Then the number of functions $f : S \to R$ that are differentiable, satisfy $f'(x) = \ln(x^2 - 1)$ for all $x \in S$ and $f(2) = 0$, is

(A) 0 (B) 1 (C) 2 (D) infinite

Ans. **[D]**

Sol. $x^2 - 1 > 0$ $x \in (-\infty, -1) \cup (1, \infty)$

$f'(x) = \ln(x^2 - 1)$

$f(x) = \int 1.\ln(x^2-1)dx$

$= x\ln(x^2 - 1) - 2x - \log\left|\frac{x-1}{x+1}\right| + c$ (integrating by parts)

$$f(x)=\begin{cases} x\ln(x^2-1)-2x-\log\left(\frac{1-x}{1+x}\right)+c_1 & \text{when } x<-1 \\ x\ln(x^2-1)-2x-\log\left(\frac{x-1}{x+1}\right)+c_2 & \text{when } x>1 \end{cases}$$

While applying initial condition f (2) = 0 we will get only c_2 but not c_1.
hence , infinite function possible.

14. Let S be the set of real numbers p such that there is no nonzero continuous function $f : R \to R$ satisfying $\int_0^x f(t)dt = p\,f(x)$ for all $x \in R$. Then S is

(A) the empty set (B) the set of all rational numbers
(C) the set of all irrational numbers (D) the whole set R

Ans. **[D]**

Sol. $f(0) = 0$

$\int_0^x f(t)\,dt = pf(x) \Rightarrow Pf'(x) = f(x)$

$\int_0^x f(x-t)\,dt = pf(x) \Rightarrow Pf'(x) = f(0) \Rightarrow f'(x) = 0 \Rightarrow f'(x) = f(0) = 0$

Clearly f(x) is always a zero function.

15. The probability of men getting a certain disease is $\frac{1}{2}$ and that of women getting the same disease is $\frac{1}{5}$. The blood test that identifies the disease gives the correct result with probability $\frac{4}{5}$. Suppose a person is chosen at random from a group of 30 males and 20 females, and the blood test of that person is found to be positive. What is the probability that the chosen person is a man ?

(A) $\frac{75}{107}$ (B) $\frac{3}{5}$ (C) $\frac{15}{19}$ (D) $\frac{3}{10}$

Ans. **[A]**

Sol. Let E be the event that person is found to be positive, M be the event that chosen person is a man and W be the event that chosen person is a woman.

$$P\left(\frac{E}{M}\right)=\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times\frac{1}{5}=\frac{1}{2}$$

$$P\left(\frac{E}{W}\right)=\frac{1}{5}\times\frac{4}{5}+\frac{4}{5}\times\frac{1}{5}=\frac{8}{25}$$

By Baye's theorem,

$$P\left(\frac{M}{E}\right)=\frac{\frac{1}{2}\times\frac{3}{5}}{\frac{1}{2}\times\frac{3}{5}+\frac{8}{25}\times\frac{2}{5}} = \frac{75}{107}$$

16. The number of functions f : [0, 1] $\to$ [0, 1] satisfying |f(x) – f(y)| = |x – y| for all x, y in [0, 1] is

(A) exactly 1 (B) exactly 2

(C) more than 2, but finite (D) infinite

Ans. **[B]**

Sol. $\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|=1$

$\Rightarrow$ |f'(x)| = 1

f'(x) = ± 1

f (x) = x or f(x) = 1 – x ($\because$ f : [0, 1] $\to$ [0, 1])

17. Suppose A is a 3 × 3 matrix consisting of integer entries that are chosen at random from the set {–1000, –999, …, 999, 1000}. Let P be the probability that either $A^2 = -I$ or A is diagonal matrix, (where I is the 3 × 3 identity matrix). Then

(A) $P < \frac{1}{10^{18}}$ (B) $P = \frac{1}{10^{18}}$ (C) $\frac{5^2}{10^{18}} \le P \le \frac{5^3}{10^{18}}$ (D) $P \ge \frac{5^4}{10^{18}}$

Ans. **[A]**

Sol. Total matrices = 2001^9

favourable = 2001^3 ($\because$ $A^2 = -I \Rightarrow |A|^2 = -1 \Rightarrow$ not possible ; hence A can only be diagonal matrix)

$$P=\frac{2001^3}{2001^9}=\frac{1}{2001^6}<\frac{1}{1000^6}=\frac{1}{10^{18}}$$

18. Let x_k be real numbers such that $x_k \geq k^4 + k^2 + 1$ for $1 \leq k \leq 2018$. Denote $N = \sum_{k=1}^{2018} k$. Consider the following inequalities :-

I. $$\left(\sum_{k=1}^{2018} kx_k\right)^2 \leq N\left(\sum_{k=1}^{2018} kx_k^2\right)$$

II. $$\left(\sum_{k=1}^{2018} kx_k\right)^2 \leq N\left(\sum_{k=1}^{2018} k^2x_k^2\right)$$

Then

(A) both I and II are true
(B) I is true and II is false
(C) I is false and II is true
(D) both I and II are false

Ans. **[A]**

Sol. I. Variance for variable x_k with frequency k = $\dfrac{\sum kx_k^2}{\sum k} - \left(\dfrac{\sum kx_k}{\sum k}\right)^2 \geq 0$

II. Variance for variable kx_k with frequency one = $\dfrac{\sum (kx_k)^2}{\sum 1} - \left(\dfrac{\sum kx_k}{\sum 1}\right)^2 \geq 0$

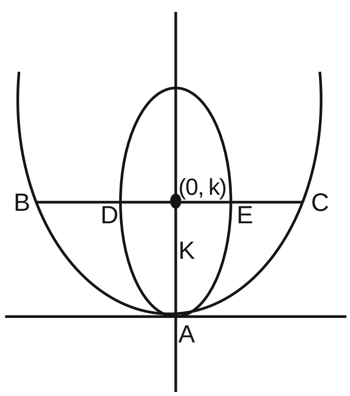
$\Rightarrow 2018 \sum k^2x_k^2 \geq (\sum kx_k)^2$ $\quad (\sum 1 = 2018)$

$\Rightarrow N \sum k^2x_k^2 \geq (\sum kx_k)^2$ $\quad (N \geq 2018)$

19. Let $x^2 = 4ky$, $k > 0$, be a parabola with vertex A. Let BC be its latus rectum. An ellipse with center on BC touches the parabola at A, and cuts BC at points D and E such that BD = DE = EC (B, D, E, C in that order). The eccentricity of the ellipse is

(A) $\dfrac{1}{\sqrt{2}}$ (B) $\dfrac{1}{\sqrt{3}}$ (C) $\dfrac{\sqrt{5}}{3}$ (D) $\dfrac{\sqrt{3}}{2}$

Ans. **[C]**



Sol.

a = k

$$DE = \frac{BC}{3} = \frac{4k}{3} \Rightarrow b = \frac{2k}{3}$$

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{4}{9} = \frac{5}{9}$$

$$e = \frac{\sqrt{5}}{3}$$

20. Let f : [0, 1] $\to$ [–1, 1] and g : [–1, 1] $\to$ [0, 2] be two functions such that g is injective and gof : [0, 1] $\to$ [0, 2] is surjective. Then

(A) f must be injective but need not be surjective

(B) f must be surjective but need not be injective

(C) f must be bijective

(D) f must be a constant function

Ans. **[B]**

Sol. f : [0, 1] $\to$ [–1, 1]

g : [1, 1] $\to$ [–0, 2]

g is injective & gof is surjective.

$\Rightarrow$ f must be surjective otherwise f(x) would not cover the whole co-domain [–1, 1] (which is also the domain of g) & then consecutively gof would not be able to cover the whole [0, 2] (as g is injective).

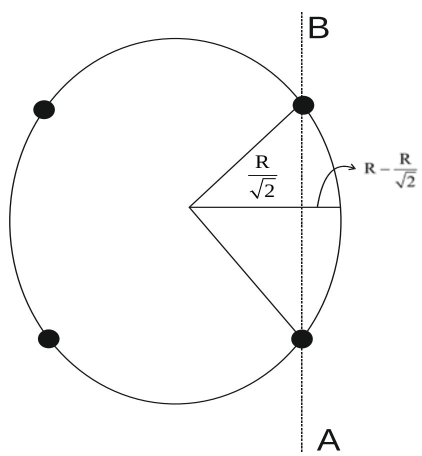
PHYSICS

21. A table has a heavy circular top of radius 1 m and mass 20 kg, placed on four light (considered massless) legs placed symmetrically on its circumference. The maximum mass that can be kept anywhere on the table without toppling it is close to

(A) 20 kg (B) 34 kg (C) 47 kg (D) 59 kg

Ans. **[C]**

Sol.



$$MgR\left(1-\frac{1}{\sqrt{2}}\right)=20g\frac{R}{\sqrt{2}}$$

$$M(\sqrt{2}-1)=20$$

M = 48.28

22. Air (density ρ) is being blown o a soap film (surface tension T) by a pipe of radius R with its opening right next to the film. The film is deformed and a bubble detaches from the film when the shape of the deformed surface is a hemisphere. Given that the dynamic pressure on the flim due to the air blown at speed v is $\frac{1}{2}\rho v^2$, the speed at which the bubble is formed is

(A) $\sqrt{\frac{T}{\rho R}}$ (B) $\sqrt{\frac{2T}{\rho R}}$ (C) $\sqrt{\frac{4T}{\rho R}}$ (D) $\sqrt{\frac{8T}{\rho R}}$

Ans. **[D]**

Sol. $\frac{1}{2}\rho v^2 = \frac{4T}{R}$

$v = \sqrt{\frac{8T}{R\rho}}$

23. For an ideal gas the internal energy is given by U = 5PV/2 + C, where C is a constant. The equation of the adiabats in the PV plane will be :-

(A) P^5V^7 = constant (B) P^7V^5 = constant (C) P^3V^5 = constant (D) P^5V^2 = constant

Ans. **[A]**

Sol. $U = \frac{5}{2}PV + C$

$dU = \frac{5}{2}PdV + \frac{5}{2}VdP$

For adiabatic process

dU = –dW = –PdV

$\frac{5}{2}PdV + \frac{5}{2}VdP = -PdV$

$\frac{7}{2}PdV = -\frac{5}{2}PVdP$

7PdV = –5VdP

$\int 7\frac{dV}{V} = \int -5\frac{dP}{P}$

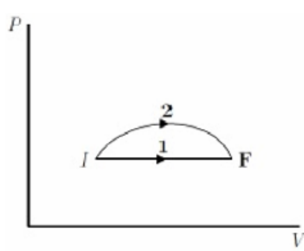
7 LnV = –5LnP + LnC

$LnV^7 = LnP^{-5}C$

$V^7 = P^{-5}C$

P^5V^7 = constant

24. An ideal gas undergoes change in its state from the initial state I to the final state F via two possible paths as shown. Then



(A) there is no change in internal energy along path 1

(B) heat is not absorbed by the gas in both paths

(C) the temperature of the gas first increases and then decreases for path 2

(D) work done by the gas is larger in path 1

Ans. **[C]**

Sol. $PV = nRT$

$P\Delta V + V\Delta P = NR\Delta T$

$$\frac{\Delta P}{\Delta V} = \frac{P}{nR} + \frac{V}{nR}\frac{\Delta P}{\Delta V}$$

$\frac{\Delta P}{\Delta V}$ is positive for 1st half & negative for 2nd half have temperature 1st increases then decreases.

25. A thermally insulated rigid container of one litre volume contains a diatomic ideal gas at room temperature. A small paddle installed inside the container is rotated from the outside such that the pressure rises by 10^5 Pa. The change in internal energy is close to

(A) 0 J (B) 67 J (C)150 J (D) 250 J

Ans. **[D]**

Sol. $\Delta U = n\,C_v(T_2 - T_1)$

$$= \frac{nR}{\gamma - 1}\left[\frac{P_2 V}{nR} - \frac{P_1 V}{nR}\right]$$

$$= \frac{(P_2 - P_1)V}{\gamma - 1}$$

$$= \frac{(2\times10^5 - 1\times10^5)\times10^{-3}}{\frac{7}{5} - 1}$$

$$= \frac{5}{2}\times100 = 250\,J$$

26. In a Young's double slit experiment the amplitudes of the two waves incident on the two slits are A and 2A. If I_0 is the maximum intensity, then the intensity at a spot on the screen where the phase difference between the two interfering waves is ϕ.

(A) $I_0\cos^2(\phi/2)$ (B) $\frac{I_0}{3}\sin^2(\phi/2)$ (C) $\frac{I_0}{9}(5 + 4\cos(\phi))$ (D) $\frac{I_0}{9}(5 + 8\cos(\phi))$

Ans. **[C]**

Sol. $I_1 = kA^2 = I$; $I_2 = 4kA^2 = 4I$

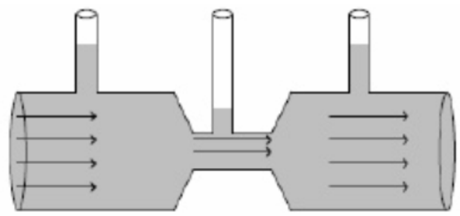
$I_0 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$; $I_0 = 9I$

$I = I + 4I + 2\sqrt{I + 4I}\cos\theta$

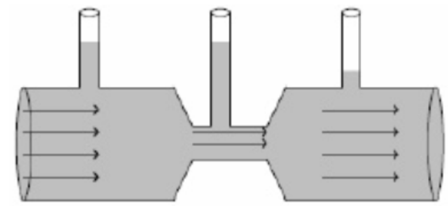
$= 5I + 4I\cos\theta = \frac{I_0}{9}(5 + 4\cos\theta)$

27. Figures below show water flowing through a horizontal pipe from left to right. Note that the pipe in the middle is narrower. Choose the most appropriate depiction of water levels in the vertical pipes.

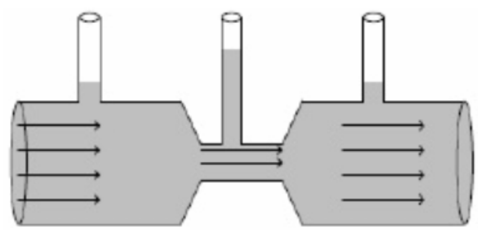
(A)



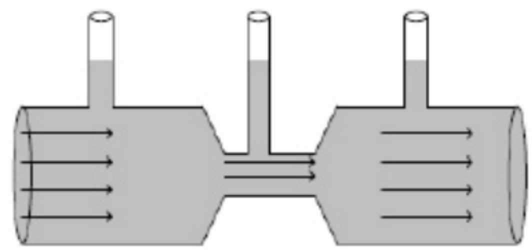
(B)



(C)



(D)

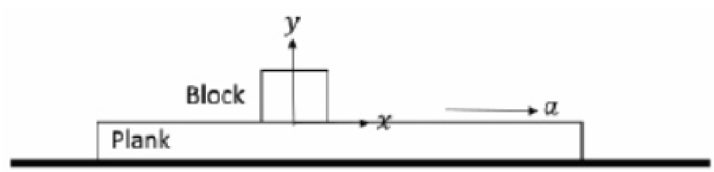


Ans. **[A]**

Sol. $P+\frac{1}{2}\rho v^2 = C$ $A_1V_1 = A_2V_2$

$V\uparrow = P\downarrow$

28. A plank is moving in a horizontal direction with a constant acceleration $a\hat{i}$. A uniform rough cubical block of side l rests on the plank, and is at rest relative to the plank.

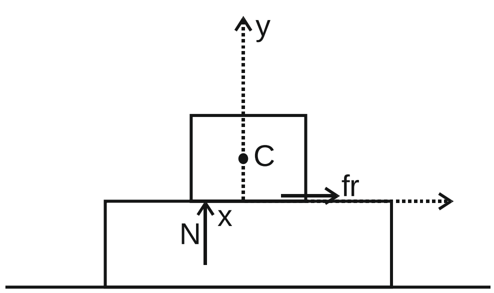


Let the center of mass of the block be at (0, l/2) at a given instant. If a = g/10, then the normal reaction exerted by the plank on the block at that instant acts at

(A) (0, 0) (B) (–l/20, 0) (C) (–l/10, 0) (D) (l/10, 0)

Ans. **[B]**

Sol.



Torque balance about com of block

$Nx = fr\ \frac{l}{2}$

$Nx = ma\ \frac{l}{2}$

$Nx = \frac{mg}{10} \times \frac{l}{2}$

$mg\ x = \frac{mg}{10} \times \frac{l}{2}$

$x = \frac{l}{20}$

29. Using the Heisenberg uncertainty principle, arrange the following particles in the order of increasing lowest energy possible.

(I) an electron in H_2 molecule

(II) a H atom in a H_2 molecule

(III) a proton in the carbon nucleus

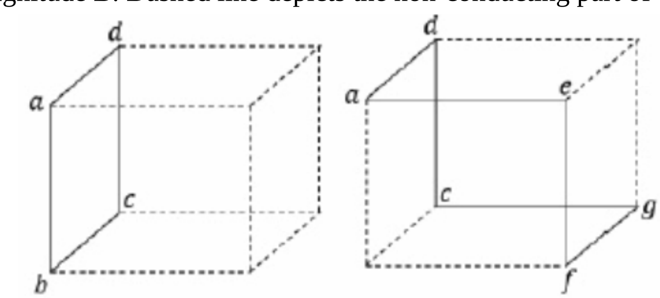
(IV) a H_2 molecule within a nanotube

(A) (I) < (III) < (II) < (IV)
(B) (IV) < (II) < (I) < (III)
(C) (II) < (IV) < (III) < (I)
(D) (IV) < (I) < (II) < (III)

Ans. **[B]**

Sol. By theory

30. The current is flowing along the path abcd of a cube (shown to the left) produces a magnetic field at the centre of cube of magnitude B. Dashed line depicts the non-conducting part of the cube.



Consider a cubical shape shown to the right which is identical in size and shape to the left. If the same current now flows in along the path *daefgcd*, then the magnitude of magnetic field at the centre will be :-

(A) zero
(B) $\sqrt{2}B$
(C) $\sqrt{3}B$
(D) B

Ans. **[C]**

Sol. From symmetry $\vec{B} = B_x\hat{i} + B_y(-\hat{j}) + B_z\hat{k}$ and $B_x = B_y = B_z = B$ and so $B_{net} = \sqrt{3}B$

31. A thin metallic disc is rotating with constant angular velocity about a vertical axis that is perpendicular to its plane and passes through its centre. The rotation causes the free electrons in the disc to redistribute. Assume that there is no external electric or magnetic field. Then

(A) a point o the rim of the disc is at a higher potential than the centre

(B) a point on the rim of the disc is at a lower potential than the centre

(C) a point on the rim of the disc is at the same potential as the centre

(D) the potential in the material has an extremum between center and the rim.

Ans. **[B]**

Sol. Due to contrifugal force electrons have a tendency to go towards the circumference.

32. One mole of a monatomic gas and one mole of a diatomic gas are initially in the same state. Both gases are expanded isothermally and then adiabatically such that they acquire the same final state. Choose the correct statement.

(A) work done by diatomic gas is more than that by monatomic gas

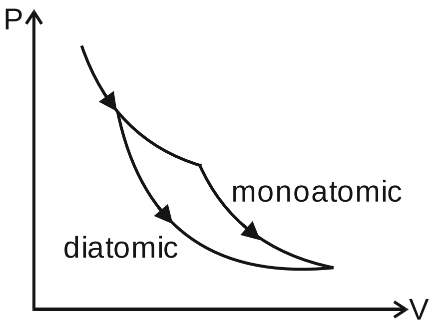
(B) work done by monatomic gas is more than that by diatomic gas

(C) work done by both the gases are equal

(D) change in internal energies of both the gases are equal

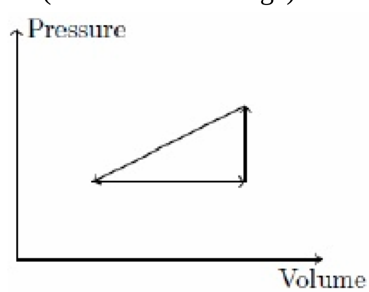
Ans. **[B]**

Sol.



Area under curve of diatomic is
Less hence work done is less

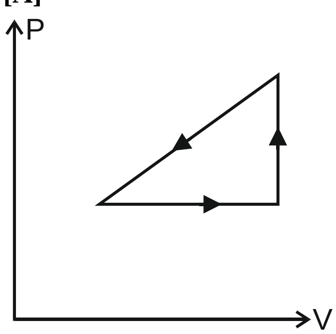
33. An ideal gas is made to undergo the cyclic process shown in the figure below. Let ΔW depict the work done. ΔU be the change in internal energy of the gas and Q be the heat added to the gas. Sign of each of these three quantities for the whole cycle will be (0 refers to no change)



(A) –, 0, – (B) +, 0, + (C) 0, 0, 0 (D) +, +, +

Ans. **[A]**

Sol.



W = –ive

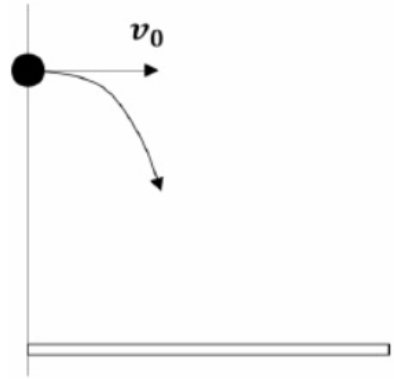
$\Delta U = 0$

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = 0 + \Delta W$

So ΔQ = –ive

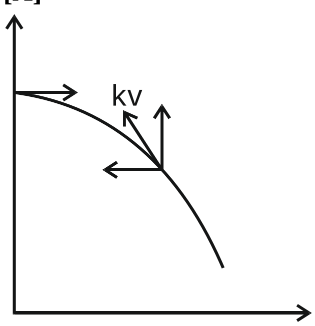
34. Two balls of mass M and 2M are thrown horizontally with the same initial velocity v_0 from top of a tall tower and experience a drag force of $-kv(k > 0)$, where v is the instantaneous velocity. Then



(A) the heavier ball will hit the ground further away than the lighter ball
(B) the heavier ball will hit the ground closer than the lighter ball
(C) both balls will hit the ground at the same point
(D) both balls will hit the ground at the same time

Ans. **[A]**

Sol.



$kv_x = ma_x$

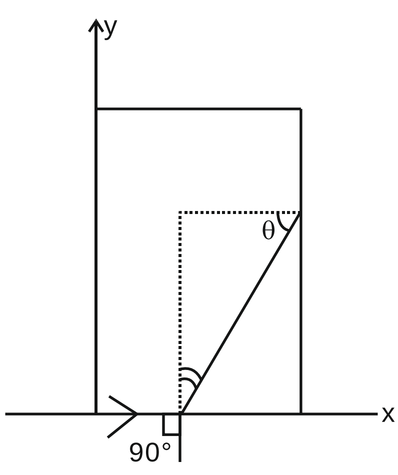
$a_x = \frac{kv_x}{m}$

35. Consider a glass cube slab of dielectric bound by the planes $x = 0$, $x = a$; $y = 0$, $y = b$; $z = 0$, $z = c$; with $b > a > c$. The slab is placed in air and has a refractive index of n. The minimum value for n such that all rays entering the dielectric at $y = 0$ reach $y = b$ is

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

Ans. **[B]**

Sol.



$\theta \geq i_c$

$90 - i_c \geq i_c$

$2i_c \leq 90^o$

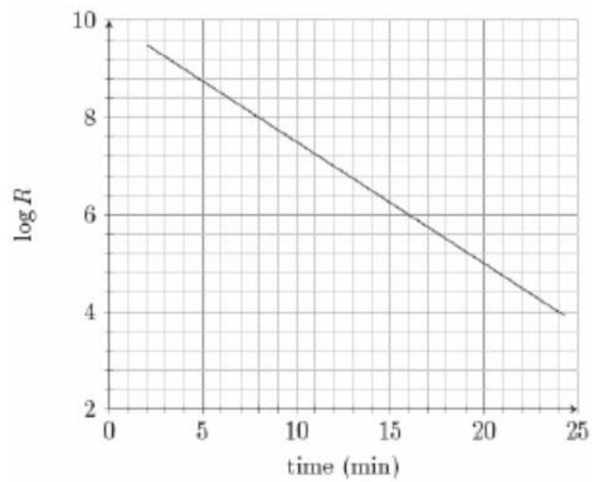
$i_c \leq 45^o$

$\sin i_c \leq \frac{1}{\sqrt{2}}$

$\frac{1}{\mu} \leq \frac{1}{\sqrt{2}}$

$\mu \geq \sqrt{2}$

36. The graph shows the log of activity (log R) of a radioactive material as a function of time t in minutes.



The half-life (in minutes) for the decay is closest to

(A) 2.1 (B) 3.0 (C) 3.9 (D) 4.4

Ans. **[B]**

Sol. $R = \lambda N_0 e^{-\lambda t}$

$\ln R = \ln(\lambda N_0)(e^{-\lambda t})$

$\ln R = -\lambda t + \ln(\lambda N_0)$

comparing $y = -mx + C$

$m = \lambda$

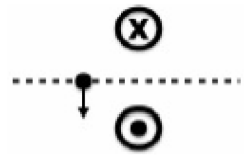
By graph :

$M = \frac{8-6}{16-8} = \frac{1}{4} = \lambda$

$T_{1/2} = \frac{\ln}{\lambda} = \frac{0.693}{1/4} = 0.277$

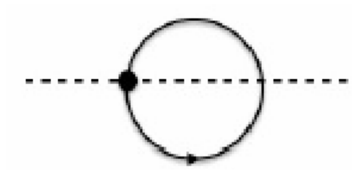
Closest value in 3.0.

37. The magnetic field is uniform for $y > 0$ and points into the plane. The magnetic field is uniform and points out of the plane for $y < 0$. A proton denoted by filled circle leaves $y = 0$ in the y-direction with some speed as shown below.

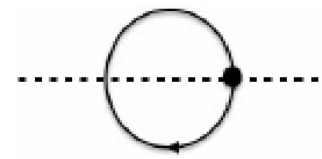


Which of the following best denotes the trajectory of the proton.

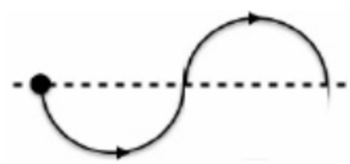
(A)



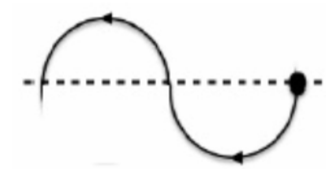
(B)



(C)

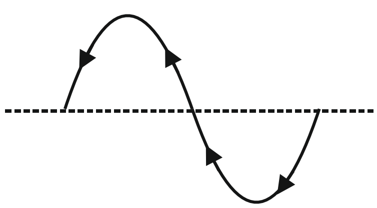


(D)



Ans. **[D]**

Sol. From $\vec{F} = q(\vec{v} \times \vec{B})$



38. The Hitomi satellite recently observed the Lyman alpha emission line (n = 2 to n = 1) of Hydrogen-like iron ion (atomic number of iron is 26) from the Perseus galaxy cluster. The wavelength of the line is closest to

(A) 2Å (B) 1Å (C) 50Å (D) 10Å

Ans. **[A]**

Sol. $\frac{1}{\lambda} = R(26-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\lambda \simeq 2A^o$

39. Assume that the drag force on a football depends only on the density of the air, velocity of the ball and the cross-sectional area of the ball. Balls of different sizes but the same density are dropped in an air column. The terminal velocity reached by balls of masses 250 g and 125 g are in the ratio :

(A) $2^{1/6}$ (B) $2^{1/3}$ (C) $2^{1/2}$ (D) $2^{2/3}$

Ans. **[A]**

Sol. $F \propto \rho^a v^b A^c$

$MLT^{-2} = M^a L^{-3a} L^b T^{-b} L^{2c}$

on solving a = 1, b = –2, c = 1

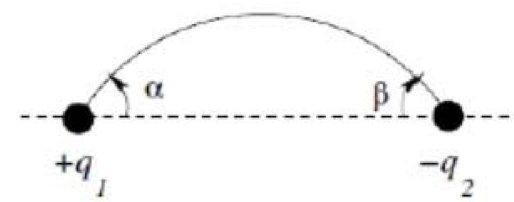
so $mg = \rho v^2 A$ since $m = v\rho$

$mg = \rho v^2 m^{2/3}$ $R \propto m^{1/3}$

$A \propto m^{2/3}$

$v \propto m^{1/6}$

40. An electrostatic field line leaves at an angle α from point charge q_1 and connects with point charge $-q_2$ at an angle β(q_1 and q_2 are positive) (see figure below). If $q_2 = \frac{3}{2}q_1$ and $\alpha = 30°$, then



(A) $0° < \beta < 30°$ (B) $\beta = 30°$ (C) $30° < \beta \le 60°$ (D) $60° < \beta \le 90°$

Ans. **[C]**

Sol. $$\frac{2\pi(1-\cos\alpha)}{2\pi(1-\cos\beta)} = \frac{q_1}{q_2}$$

$$\frac{2\sin^2\frac{\alpha}{2}}{2\sin^2\frac{\beta}{2}} = \frac{2}{3}$$

$\beta \simeq 40°$

CHEMISTRY

41. The amount (in mol) of bromoform ($CHBr_3$) produced when 1.0 mol of acetone reacts completely with 1.0 mol of bromine in the presence of aqueous NaOH is

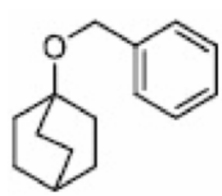
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) 2

Ans. **[A]**

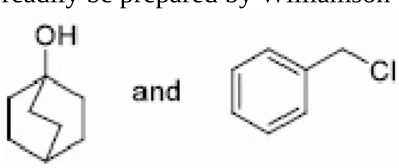
Sol. $$CH_3-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow[4NaOH]{3Br_2} CH_3-\overset{\overset{O}{\|}}{C}-ONa + CHBr_3$$

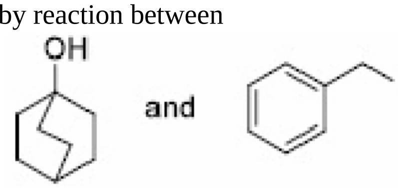
3 Br_2 Gives 1 mol of $CHBr_3$

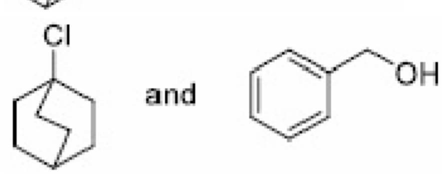
1 mol Br_2 ____________ $\frac{1}{3}$ $CHBr_3$

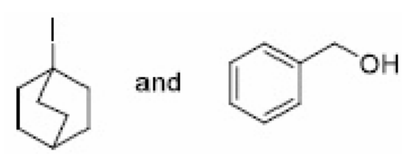
42. The following compound 

can readily be prepared by Williamson ether synthesis by reaction between

(A) 

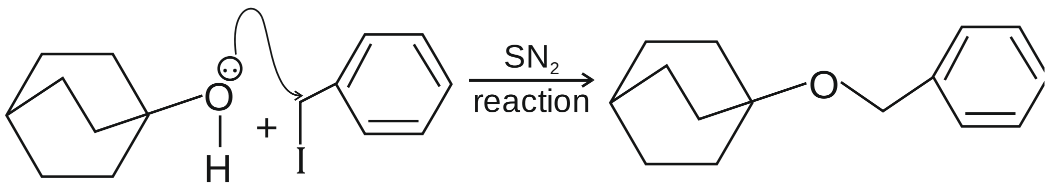
(B) 

(C) 

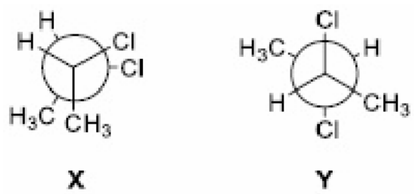
(D) 

Ans. **[B]**

Sol.



43. X and Y



are

(A) enantiomers (B) diastereomers (C) constitutional isomers (D) conformers

Ans. **[D]**

Sol. X & Y are identical that is why only option is conformed.

44. The higher stabilities of tert-butyl cation over isopropyl cation, and trans-2-butene over propene, respectively, are due to orbital interactions involving

(A) $\sigma \rightarrow \pi$ and $\sigma \rightarrow \pi^*$ (B) $\sigma \rightarrow$ vacant p and $\pi \rightarrow \pi^*$

(C) $\sigma \rightarrow \sigma^*$ and $\sigma \rightarrow \pi$ (D) $\sigma \rightarrow$ vacant p and $\sigma \rightarrow \pi^*$

Ans. **[D]**

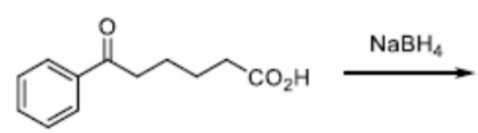
Sol. In t – butyl cation hyper conjugation takes place due to σ-P conjugation and in trans-2-butene hyper conjugation takes place between σ & π∗ orbitals.

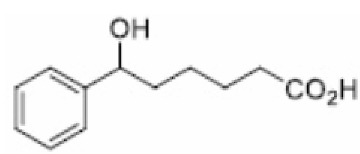
45. Benzaldehyde can be converted to benzyl alcohol in concentrated aqueous NaOH solution using

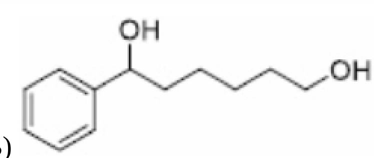
(A) acetone (B) acetaldehyde (C) formic acid (D) formaldehyde

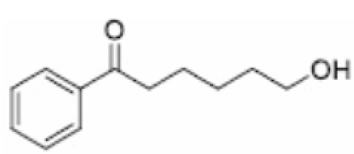
Ans. **[D]**

Sol. Ph – CHO with $\xrightarrow{\text{NaOH}}$ Ph –CH_2–OH + HCOONa

46. The major product of the following reaction  is ?

(A) 

(B) 

(C) 

(D) 

Ans. **[A]**

Sol. $NaBH_4$ only reduce carbonyl group not acid.

47. Among the following species, the H–X–H angle (X = B, N or P) follows the order

(A) $PH_3 < NH_3 < NH_4^+ < BF_3$
(B) $NH_3 < PH_3 < NH_4^+ < BF_3$
(C) $BF_3 < PH_3 < NH_4^+ < NH_3$
(D) $BF_3 < NH_4^+ < NH_3 < PH_3$

Ans. **[A]**

Sol.

Molecule	Bond angle
PH_3	90º
NH_3	107º
NH_4^+	109º. 28 min.
BF_3	120º

48. The ionic radii of Na^+, F^-, O^{2-}, N^{3-} follow the order

(A) $O^{2-} > F^- > Na^+ > N^{3-}$
(B) $N^{3-} > Na^+ > F^- > O^{2-}$
(C) $N^{3-} > O^{2-} > F^- > Na^+$
(D) $Na^+ > F^- > O^{2-} > N^{3-}$

Ans. **[A]**

Sol. For iso-electronic species

Ionic size $\propto$ –ve charge on anion

49. The oxoacid of phosphorus having the strongest reducing property is :

(A) H_3PO_3
(B) H_3PO_2
(C) H_3PO_4
(D) $H_4P_2O_7$

Ans. **[B]**

Sol. In H_3PO_2 2 reducing hydrogen atom are present.

50. Among C, S and P, the element(s) that produce(s) SO_2 on reaction with hot conc. H_2SO_4 is/are

(A) only S
(B) only C and S
(C) only S and P
(D) C, S and P

Ans. **[D]**

Sol. $H_2SO_4 \rightarrow H_2O + SO_2 + [0]$

(conc)

$C + [0] \rightarrow CO_2$

$S + [0] \rightarrow SO_2$

$P_4 + [0] \rightarrow P_4O_{10}$

51. The complex that can exhibit linkage isomerism is

(A) $[Co(NH_3)_5(H_2O)]Cl_3$
(B) $[Co(NH_3)_5(NO_2)]Cl_2$
(C) $[Co(NH_3)_5(NO_3)](NO_3)_2$
(D) $[Co(NH_3)_5Cl]SO_4$

Ans. **[B]**

Sol. NO_2 is an ambidonate ligand so can show linkage isomerism.

52. The tendency of X in BX_3 (X = F, Cl, OMe, NMe) to form a π bond with boron follows the order

(A) $BCl_3 < BF_3 < B(OMe)_3 < B(NMe_2)_3$
(B) $BF_3 < BCl_3 < B(OMe)_3 < B(NMe_2)_3$
(C) $BCl_3 < B(NMe_2)_3 < B(OMe)_3 < BF_3$
(D) $BCl_3 < BF_3 < B\ (NMe_2)_3 < B(OMe)_3$

Ans. **[A]**

Sol. Back bonding tendency decreases as Nme_2, Ome, F, Cl.

53. Consider the following statements about Langmuir isotherm :

(i) The free gas and adsorbed gas are in dynamic equilibrium

(ii) All adsorption sites are equivalent

(iii) The initially adsorbed layer can act as a substrate for further adsorption

(iv) The ability of a molecule to get adsorbed at a given site is independent of the occupation of neighboring sites

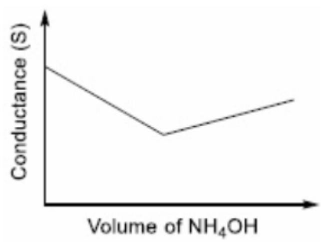
The correct statements are

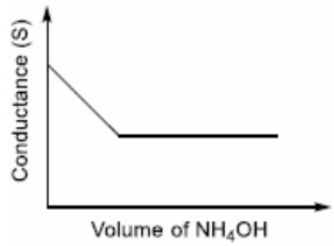
(A) (i), (ii), (iii) and (iv) B) only (i), (ii) and (iv) (C) only (i), (iii), and (iv) (D) only (i), (ii) and (iii)

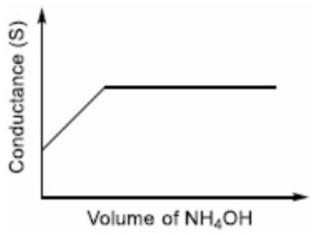
Ans. **[B]**

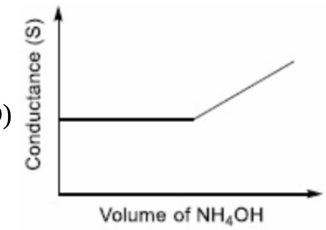
Sol. Fact

54. Among the following, the plot that correctly represents the conductometric titration of 0.05 M H_2SO_4 with 0.1M NH_4OH is :-

(A)


(B)


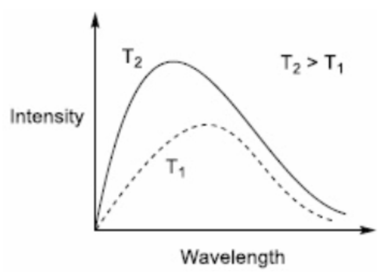
(C)


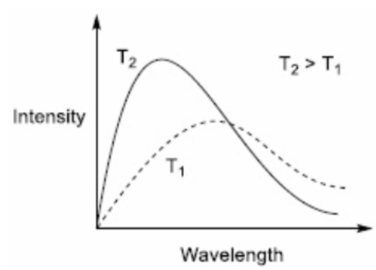
(D)


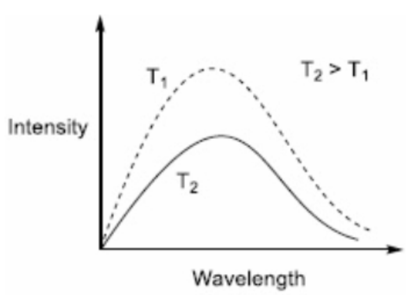
Ans. **[B]**

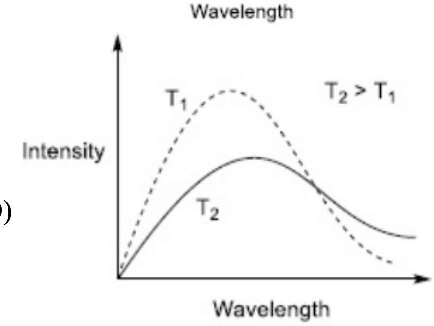
Sol. Initially H^+ ions are present due to complete dissociation of H_2SO_4. Upon addition of NH_4OH, H^+ ions decreases so conductivity decreases & becomes constant after salt formation.

55. The correct representation of wavelength-intensity relationship of an ideal blackbody radiation at two different temperatures T_1 and T_2 is :

(A)


(B)


(C)


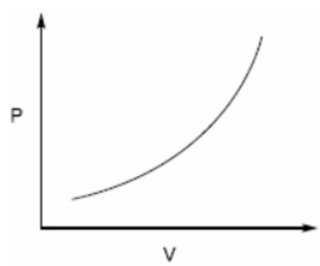
(D)


Ans. **[A]**

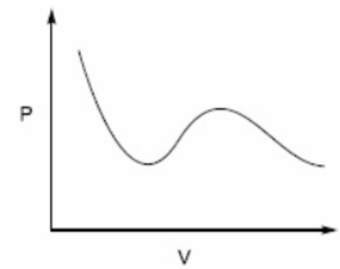
Sol. $\lambda_{max} \propto \frac{1}{T}$ So on increasing temp. Maximum wavelength shifts to left side.

56. The pressure (P)-volume (V) isotherm of a van der Waals gas, at the temperature at which it undergoes gas to liquid transition, is correctly represented by

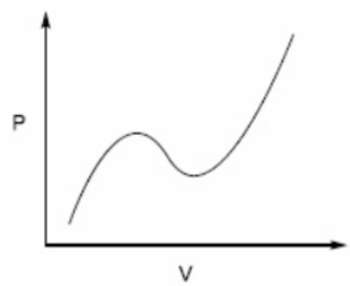
(A)



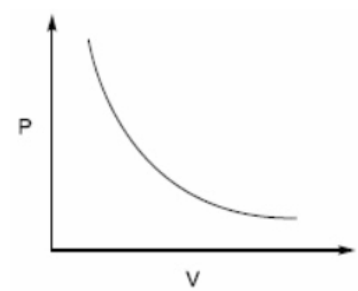
(B)



(C)

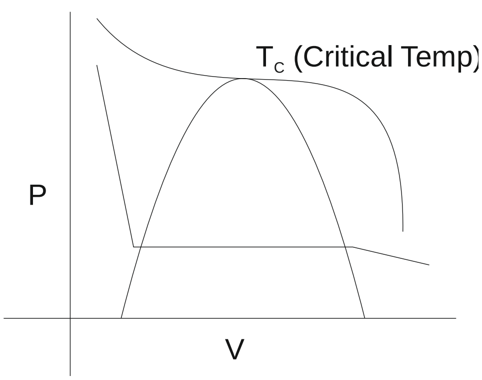


(D)



Ans. **[B]**

Sol.



57. A buffer solution can be prepared by mixing equal volumes of

(A) 0.2 M NH_4OH and 0.1 M HCl

(B) 0.2 M NH_4OH and 0.2 M HCl

(C) 0.2 M NaOH and 0.1 M CH_3COOH

(D) 0.1 M NH_4OH and 0.2 M HCl

Ans. **[A]**

Sol. $NH_4OH + HCl \longrightarrow NH_4Cl + H_2O$

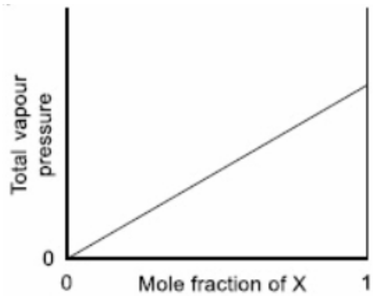
0.2M   0.1M

0.1   0.1

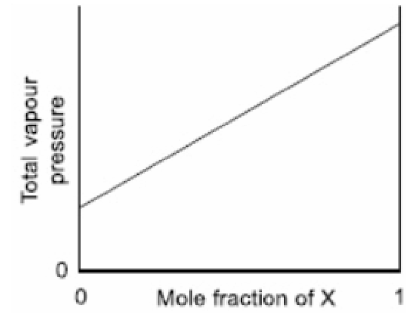
this forms basic buffer solution.

58. The plot of total vapour pressure as a function of mole fraction of the components of an ideal solution formed by mixing liquids X and Y is :

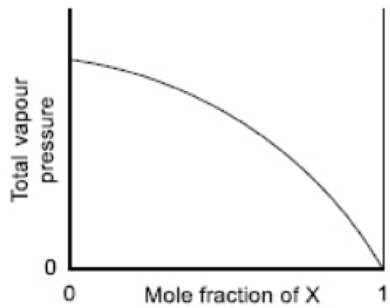
(A)



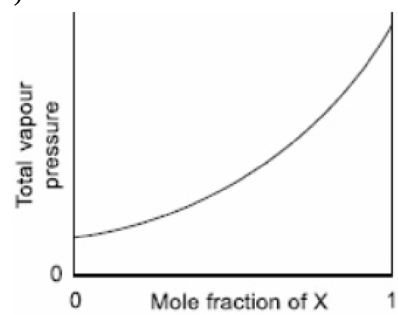
(B)



(C)



(D)



Ans. **[B]**

Sol. $P_T = P_A + P_B$

$P_T = P_A^{\circ} X_A + P_B^{\circ} X_B$

Total pressure (P_T) is always more than P_A & P_B

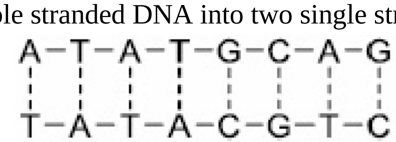
59. On complete hydrogenation, natural rubber produces

(A) polyethylene
(B) ethylene-propylene copolymer
(C) polyvinyl chloride
(D) polypropylene

Ans. **[B]**

Sol. Natural rubber forms due to polymerisation of isoprene and on hydrogenation provides polymer of ethylene, propylene.

60. The average energy of each hydrogen bond in A-T pair is x kcal mol^{-1} and that in G-C pair is y kcal mol^{-1}. Assuming that no other interaction exists between the nucleotides, the approximate energy required in kcal mol^{-1} to split the following double stranded DNA into two single strands is



[Each dashed line many represent more than one hydrogen bond between the base pairs]

(A) 10x + 9y
(B) 5x + 3y
(C) 15x + 6y
(D) 5x + 4.5 y

Ans. **[A]**

Sol. no. of hydrogen bond between A – T = 2

no. of hydrogen bond between a – c = 3

so total energy required = 10 x + 9y

BIOLOGY

61. What is the maximum number of oxygen atoms that a molecule of hemoglobin can bind ?

(A) 2 (B) 4 (C) 8 (D) 16

Ans. **[C]**

Sol. Haemoglobin molecule is a respiratory pigment, present in erythrocytes necessary for transport of O_2 & CO_2 gas. It consist of two part like haeme and globin. Haeme is Fe^{2+} contain porphyrin pigment which associate with for oxygen molecule reversibly (unstable binding) or 8 oxygen atom.

62. Bt toxin produced by *Bacillus thuringiensis* does not kill the producer because the toxin is.

(A) in an inactive protoxin form (B) rapidly secreted outside

(C) inactivated by an antitoxin (D) in unfolded form

Ans. **[A]**

Sol. Bt toxin is in inactive form in cotton but as the insect engulf the bud (recombinated) the cry gene mediated toxin activated in the alkaline pH (9.5), frequently such conditions are present inside insects gut so the insect die but not the producer (plant)

63. An angiosperm was identified with its endosperm of 6n. Assuming that this is a self-pollinating species, which ONE of the following is the correct ploidy of the parent ?

(A) 3n (B) 4n (C) 6n (D) 8n

Ans. **[B]**

Sol. Endosperm generally – 3n

But it is given 6n – so, n = 2n the polidy of mature plant is $2 \times 2n = 4n$.

64. Which ONE of the following statements is TRUE about viruses ?

(A)All viruses possess a protein coat around its genetic material at all stages of their life cycle

(B) All viruses contain RNA as genetic material

(C) All viruses contain DNA as genetic material

(D) All viruses replicate only within the host cell

Ans. **[D]**

Sol. Virus is non-autonomous outside a living host body, as the polymerase genes expressed early in the cycle of virus but capsid & tail proteins are expressed later. Virus either have DNA or RNA.

65. Mitochondrial cristae are infoldings of the

(A) outer membrane and they increase the surface area

(B) outer membrane and they decrease the surface area

(C) inner membrane and they increase the surface area

(D) inner membrane and they decrease the surface area

Ans. **[C]**

Sol. Cristae are finger like infoldings of mitochondrial inner membrane inner side of matrix. This increases surface area for faster production of ATP.

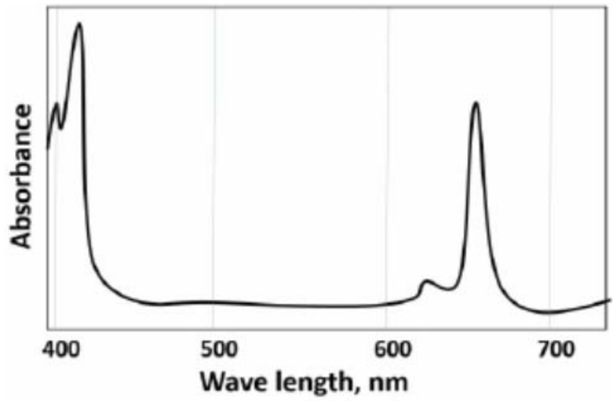
66. In biological nitrogen fixation, the enzyme nitrogenase converts

(A) nitrate to nitrite
(B) atmospheric nitrogen to nitrite
(C) nitrite to ammonia
(D) atmospheric nitrogen to ammonia

Ans. **[D]**

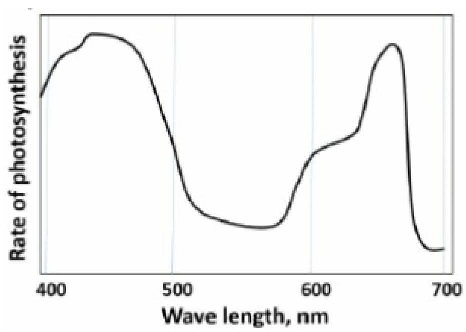
Sol. Nitrogenase enzyme under biological N-fixation reduce atmospheric N_2 ($N \equiv N$) into NH_3.

67. The graph below represents the absorption spectrum of a major pigment contributing to photosynthesis.

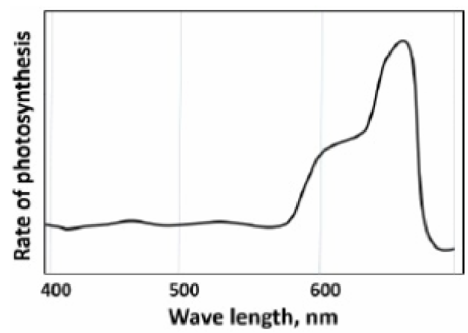


Which ONE of the following best represents the photosynthetic efficiency of

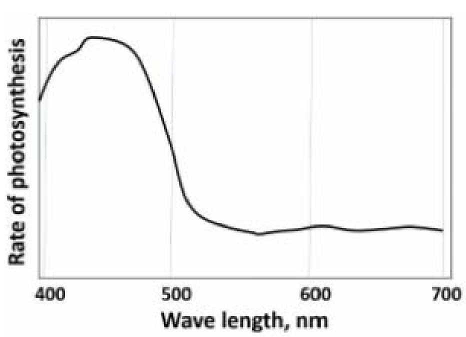
(A)



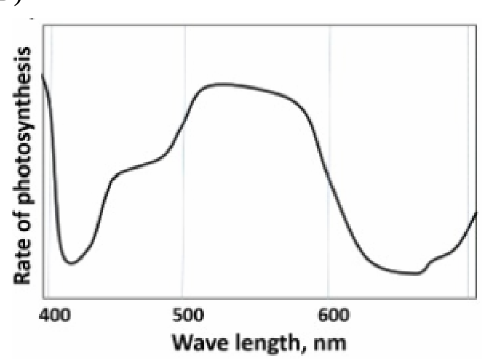
(B)



(C)



(D)



Ans. **[A]**

Sol. The above given absorbance graph relate the absorption spectrum of chlorophyll 'a'. Action spectrum of photosynthesis resemble the absorption spectrum of Chl 'a' but not completely overlape.

68. Which ONE of the following properties of normal cell is lost during its transition to cancerous cell ?

(A) Glutamine utilization (B) Contact inhibition

(C) Glucose utilization (D) Membrane fluidity

Ans. **[B]**

Sol. In normal mitotic cells they have contact inhibition because of that they do not get contact & not make a cluster of cells. But in cancerous cells they last the property and get attached with each other.

69. Which ONE of the following gases is produced during fermentation by yeast ?

(A) CO_2 (B) O_2 (C) H_2 (D) N_2

Ans. **[A]**

Sol. In fermentation yeast convert pyruvic acid into ethanol and CO_2 gas.

70. Serine proteases are called so because they

(A) require free serine for their activity (B) cleave after serine residues in the substrate

(C) are inhibited by the presence of free serine (D) have a serine residue at their active site

Ans. **[D]**

Sol. Serine proteases have serine residue at their active side. Serine proteases (serine endopeptidases) are enzymes that cleave peptide in protein, inwhich serine serves as the nucleophilic amino acid at the (enzymes) active site.

71. The maximum number of genotypes of the pollens produced by a tall pea plant with round, yellow seeds of the genotype TtRrYY, if the three loci are unlinked, would be

(A) 1 (B) 2 (C) 4 (D) 8

Ans. **[C]**

Sol. Given plant is dihybrid T t Rr YY so that total No. Of gamete 2^n, here n = 2, $2^2 = 4$ = max. no. of pollen

72. Which ONE of the following statements is TRUE with respect to human ovary ?

(A) Estrogen is secreted by Graafian follicles and progesterone by corpus luteum

(B) Estrogen is secreted by corpus luteum and progesterone by Graafian follicles

(C) Both estrogen and progesterone are secreted by Graafian follicles

(D) Both estrogen and progesterone are secreted by corpus luteum

Ans. **[A]**

Sol. During puberty age hypothalamus releases GnRH harmone that promote secretion of gonadotrophins (FSH and LH) from anterior pituitary gland.

FSH promotes growth and development of follicles in ovary two produce estrogen hormone, where as LH promotes ovulation and carpous leuteum formation which produces progesterone hormone that maintain endometrium of uterus.

73. Which ONE of the following statements is INCORRECT with respect to human antibodies ?

(A) They can neutralize microbes

(B) They are synthesized by T cells

(C) They are made up of four polypeptide chains

(D) Milk contains antibodies

Ans. **[B]**

Sol. Antibodies synthesized by ‘B’ cells not by T-cells. ‘B’ cells produced humoral immunity.

74. Concentration (%) of NaCl isotonic to human blood is

(A) 0.085 – 0.09% (B) 1.7 – 1.8% (C) 3.4 – 3.6% (D) 0.85 – 0.9%

Ans. **[D]**

Sol. 0.9% NaCl solution is isotonic for human blood cell.

75. Which ONE of the following statements is TRUE about the Golgi apparatus ?

(A) It is found only in animals

(B) It is found only in prokaryotes

(C) It modifies and targets proteins to the plasma membrane

(D) It is a site for ATP production

Ans. **[C]**

Sol. Golgi apparatus found in animals & also in plants & fungi as dictyosome. Golgi is respobsible for glycosylation protein and lipids. The glycosylated proteins are modified proteins of plams membrane.

76. Creutzfeldt Jakob Disease (CJD) is a transmissible disease caused by a

(A) virus (B) bacterium

(C) fungus (D) misfolded protein

Ans. **[D]**

Sol. It is degenerative fatal brain disorder, in the early stage of the deseases pupil have failing memory, behaviour changes and lack of coordination.

77. A researcher found petrified dinosaur faeces. Which ONE of the following is unlikely to be found in this fossil ?

(A) Decayed conifer wood (B) Bamboo

(C) Cycad (D) Giant fern

Ans. **[B]**

Sol. Dinosaur extinct in the cretaceous – tertiary extintion approx 66 mya. The estimated origin of the bamboo is 30 mya in the quaternary period of cretaceous.

78. Which ONE of the pairs of amino-acids contains two chiral centres ?

(A) Isoleucine and threonine (B) Leucine and valine

(C) Valine and isoleucine (D) Threonie and leucine

Ans. **[A]**

Sol. The amino acid form two stero isomers that are mirror image of each other. The structure are not supermaposable on each other.

79. In photosynthetic carbon fixation, which ONE of the following reacts with CO_2 ?

(A) Phosphoglycolate (B) 3-Phosphoglycerate

(C) Ribulose-1, 5-bisphosphate (D) Ribose-5-phosphate

Ans. **[C]**

Sol. Ribulose –1,5 – bis – phosphate is the 1^{st} CO_2 acceptor in C_3 cycle during photo synthesis and produce 3–PGA.

80. Match the diseases in **Column I** with the routes of infection in **Column II**. Choose the CORRECT combination.

Column I	**Column II**
P. Tuberculosis	i. Contaminated food and water
Q. Dysentry	ii. Inhalation of aerosol
R. Filariasis	iii. Contact via skin
S. Syphilis	iv. Sexual intercourse
	v. Mosquito bite

(A) P-ii, Q-i, R-v, S-iv
(B) P-ii, Q-i, R-iii, S-v
(C) P-i, Q-iii, R-v, S-iv
(D) P-ii, Q-iii, R-iv, S-v

Ans. **[A]**

Sol. Direct matching

Part – II

Two - Mark Questions

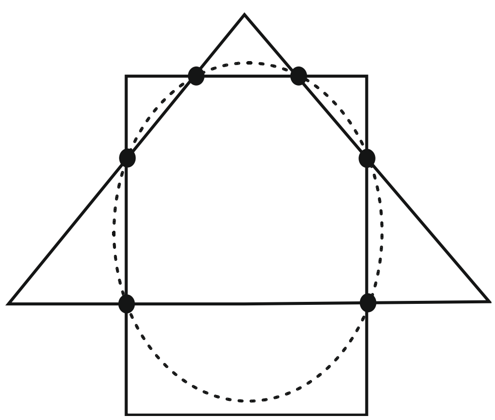
MATHEMATICS

81. Let R be a rectangle, C be a circle, and T be a triangle in the plane. The maximum possible number of points common to the perimeters of R, C, and T is

(A) 3 (B) 4 (C) 5 (D) 6

Ans. **[D]**

Sol.



82. The number of different possible values for the sum x + y + z, where x, y, z are real numbers such that $x^4 + 4y^4 + 16z^4 + 64 = 32xyz$ is

(A) 1 (B) 2 (C) 4 (D) 8

Ans. **[C]**

Sol. $S = x + y + z = ?$

$x^4 + 4y^4 + 16z^4 + 64 = 32\,xyz$ ……….(1)

$$\frac{x^4 + 4y^4 + 16z^4 + 64}{4} \geq (x^4 .4y^4 . 16z^4.64)^{1/4}$$

$$\frac{32xyz}{4} \geq 8xyz$$

$8xyz \geq 8xyz$

$\because$ AM = GM

$\therefore x^4 = 4y^4 = 16z^4 = 64$

$x = \pm 2\sqrt{2}$

$y = \pm 2$

$z = \pm \sqrt{2}$

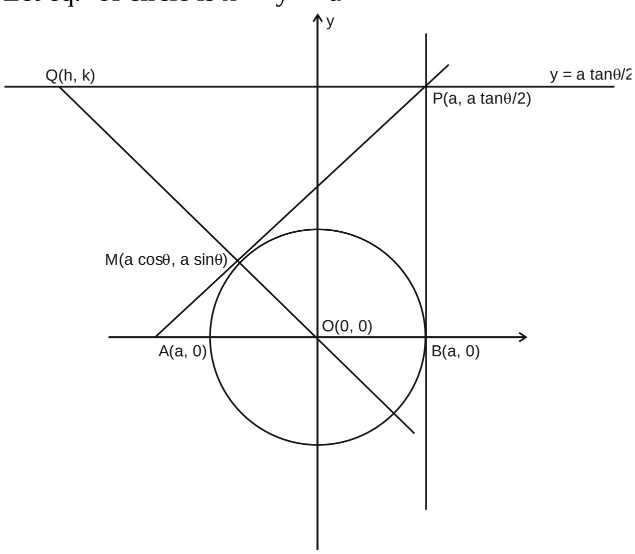
but 32xyz is positive. Hence, no. possible sums are 4.

83. Let Γ be a circle with diameter AB and centre O. Let ℓ be the tangent to Γ at B. For each point M on Γ different from A, consider the tangent t at M and let it intersect ℓ at P. Draw a line parallel to AB through P intersecting OM at Q. The locus of Q as M varies over Γ is

(A) an arc of a circle (B) a parabola (C) an arc of an ellipse (D) a branch of a hyperbola

Ans. **[B]**

Sol. Let eq.^{n} of circle is $x^2 + y^2 = a^2$



Eq.^{n} of tangent at M is $x \cos\theta + y \sin\theta = a$

Co-ordinate of $P\left(a, \frac{a(1-\cos\theta)}{\sin\theta}\right)$

$= P(a, a \tan \theta/2)$

Eq.^{n} of OM is $y = \frac{a\sin\theta}{a\cos\theta}x$

$y = x \tan\theta$

Let Q(h, k)

$k = a \tan\theta/2 \; : \; h = \frac{a\tan\theta/2}{\tan\theta}$

$h = \dfrac{a\tan\theta/2}{\left(\dfrac{2\tan\theta/2}{1-\tan^2\theta/2}\right)}$

$2h = a(1 - \tan^2\theta/2)$

$2h = a(1 - k^2/a^2)$

$\Rightarrow 2ax = a^2 - y^2$

which is parabola

84. The number of solutions x of the equation $\sin(x + x^2) - \sin(x^2) = \sin x$ in the interval [2, 3] is

(A) 0 (B) 1 (C) 2 (D) 3

Ans. **[C]**

Sol. $\sin(x + x^2) = \sin x + \sin x^2$

$$2\sin\left(\frac{x+x^2}{2}\right)\cos\left(\frac{x+x^2}{2}\right) = 2\sin\left(\frac{x+x^2}{2}\right)\cos\left(\frac{x-x^2}{2}\right)$$

$$2\sin\left(\frac{x+x^2}{2}\right)\left(\cos\left(\frac{x+x^2}{2}\right) - \cos\left(\frac{x-x^2}{2}\right)\right) = 0$$

$$-2\sin\left(\frac{x+x^2}{2}\right).2\sin\left(\frac{x}{2}\right)\sin\left(\frac{x^2}{2}\right) = 0$$

$$\Rightarrow \sin\frac{x}{2} = 0 \text{ or } \sin\frac{x^2}{2} = 0 \text{ or } \sin\left(\frac{x+x^2}{2}\right) = 0$$

$$\Rightarrow x = 2\pi \text{ or } x = \sqrt{2\pi} \text{ or } \frac{x+x^2}{2} = \pi$$

or $x^2 + x - 2\pi = 0$

or $x = \dfrac{\sqrt{1+8\pi}-1}{2} \in (2,3)$

and $x = \dfrac{\sqrt{1+8\pi}+1}{2} > 3$

So $x = \sqrt{2\pi}$ and $x = \dfrac{\sqrt{1+8\pi}-1}{2} \in (2,3)$

85. The number of polynomials $P : R \to R$ satisfying $P(0) = 0$, $P(x) > x^2$ for all $x \neq 0$, and $P''(0) = \frac{1}{2}$ is -

(A) 0 (B) 1 (C) more than 1, but finite (D) infinite

Ans. **[A]**

Sol. $P : R \to R$, $P(0) = 0$, $P(x) > x^2 \ \forall \ x \neq 0$

P(x) will be polynomial of even degree.

$P(x) = a_0x^{2n} + a_1x^{2n-1} + \ldots\ldots\ldots\ldots\ldots + a_{2n-2}x^2 + a_{2n-1}x$

$P''(0) = ½$

$a_{2n-2} = ¼$

$P(x) = a_0x^{2n} + a_1x^{2n-1} + \ldots\ldots\ldots + \frac{1}{4}x^2 + a_{2n-1}x > x^2 \ \forall \ x \neq 0$

$P(x) = a_0x^{2n} + a_1x^{2n-1} + \ldots\ldots\ldots - \frac{3}{4}x^2 + a_{2n-1}x > 0 \ \forall \ x \neq 0$

Which is not always true

Method II: $P(x) > x^2$

$P(x) = x^2 + f(x)$ $\quad f(x) > 0 \ \forall \ x \in R_0$

$\because P(0) = 0 \Rightarrow f(0) = 0$

Now, $P''(x) = 2 + f''(x)$

$f''(0)$ = negative

$\Rightarrow$ f(x) is concave down so f(x) can't be +ve always

$\Rightarrow$ which is contradiction

86. Suppose the limit $L = \lim_{n\to\infty}\sqrt{n}\int_0^1 \frac{1}{(1+x^2)^n}dx$, Exists and is larger than $\frac{1}{2}$. Then

(A) $\frac{1}{2} < L < 2$ (B) $2 < L < 3$ (C) $3 < L < 4$ (D) $L \geq 4$

Ans. **[A]**

Sol. $(1 + x^2)^n > 1 + nx^2$

$$\frac{1}{(1+x^2)^n} < \frac{1}{1+nx^2}$$

$$\int_0^1 \frac{1}{(1+x^2)^n} < \int_0^1 \frac{1}{1+nx^2}$$

$$= \frac{1}{\sqrt{n}}\tan^{-1}\sqrt{n}x\,\Big|_0^1 = \frac{1}{\sqrt{n}}\tan^{-1}\sqrt{n}$$

$$\therefore \lim_{n\to\infty}\sqrt{n}\int_0^1 \frac{1}{(1+x^2)^n}dx < \lim_{n\to\infty}\sqrt{n}\frac{1}{\sqrt{n}}\tan^{-1}\sqrt{n} = \pi/2 < 2$$

87. Consider the set A_n of points (x, y) such that $0 \leq x \leq n$, $0 \leq y \leq n$ where n, x, y are integers. Let S_n be the set of all lines passing through at least two distinct points from A_n. Suppose we choose a line ℓ at random from S_n. Let P_n be the probability that ℓ is tangent to the circle $x^2 + y^2 = n^2\left(1+\left(1-\frac{1}{\sqrt{n}}\right)^2\right)$. Then the limit $\lim_{n\to\infty} P_n$ is

(A) 0 (B) 1 (C) $1/\pi$ (D) $1/\sqrt{2}$

Ans. **[A]**

Sol. Total no. of points = $(n+1)^2 = m$

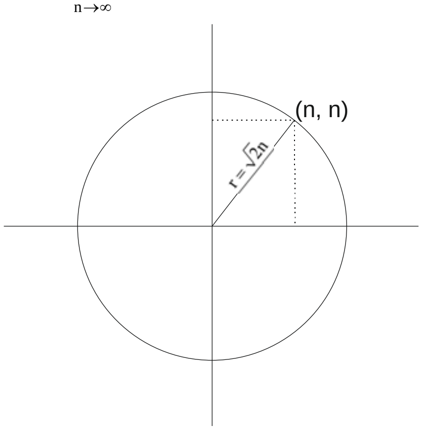
Total no. of lines = ${}^mC_2$

Circle $x^2 + y^2 = n^2\left(1+(1-\frac{1}{\sqrt{n}})^2\right)$

as $n \to \infty$, $r \to \sqrt{2}\,n$

It means only one tangent is possible.

$$\underset{n\to\infty}{P_n} = \frac{1}{\underset{n\to\infty}{{}^mC_2}} = 0$$



88. Let $f:[0, 1] \to R$ be an injective continuous function that satisfies the condition

$$-1 < f(0) < f(1) < 1$$

Then the number of functions $g : [-1, 1] \to [0, 1]$ such that $(g \circ f)(x) = x$ for all $x \in [0,1]$ is.

(A) 0 (B) 1 (C) more than 1, but finite (D) infinite

Ans. **[D]**

Sol. $f : [0, 1] \to R$ (injective)

$-1 < f(0) < (f(1) < 1$

now, if $gof(x) = x$

$(g \circ f)(x)$ is identity function & $g : [-1, 1] \to [0, 1]$

then range $(f) \subset [-1, 1]$

There may be infinite number of such f's and correspondingly infinite number of g's.

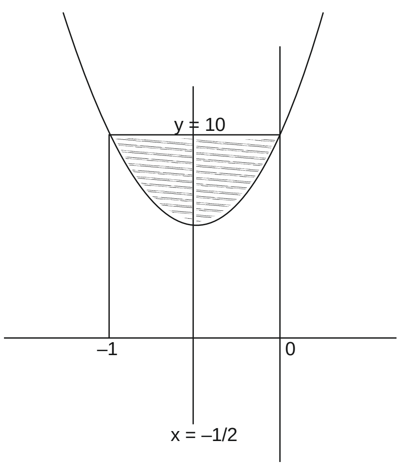
89. The maximum possible area bounded by the parabola $y = x^2 + x + 10$ and a chord of the parabola of length 1 is.

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Ans. **[B]**

Sol. By summetry, the maximum area will be bound by the horizontal chord.

$$A = \int_{-1}^{0} 10 - (x^2 + x + 10)dx = \frac{1}{6}$$



90. Suppose z is any root of $11z^8 + 20iz^7 + 10iz - 22 = 0$, where $i = \sqrt{-1}$. Then $S = |z|^2 + |z| + 1$ satisfies

(A) $S \leq 3$ (B) $3 < S < 7$ (C) $7 \leq S < 13$ (D) $S \geq 13$

Ans. **[B]**

Sol. **by theory**

PHYSICS

91. In steady state heat conduction, the equations that determine the heat current $\vec{J}(\vec{r})$ [heat flowing per unit time per unit area] and temperature $T(\vec{r})$ in space are exactly the same as those governing the electric field $\vec{E}(\vec{r})$ and electrostatic potential $V(\vec{r})$ with the equivalence given in the Table below :

Heat flow	Electrostatics
$T(\vec{r})$	$V(\vec{r})$
$\vec{j}(\vec{r})$	$\vec{E}(\vec{r})$

We exploit this equivalence to predict the rate $\dot{Q}$ of total heat flowing by conduction from the surface of spheres of varying radii, all maintained at the same temperature. If $\dot{Q} \propto R^n$, where R is the radius, then the value of n is

(A) 2 (B) 1 (C) –1 (D) –2

Ans. **[B]**

Sol. We know that $\frac{V}{E} = R$

So $\frac{T}{J} = R \; ; \; J = \frac{T}{R}$

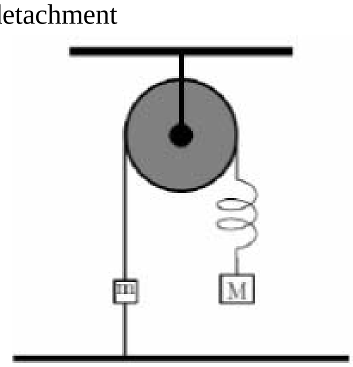
$$\frac{Q}{A \times t} = \frac{T}{R}$$

$$Q = \frac{T \times t \times A}{R} = \frac{T \times t \times 4\pi R^2}{R}$$

$$Q \propto R^1$$

$$n = 1$$

92. An arrangement of spring, strings, pulley and masses is shown in the figure. The pulley and the strings are massless and M > m. The spring is light with sprig constant k. If the string connecting m to the ground is detached, then immediately after detachment



(A) the magnitude of the acceleration of m is zero and that of M is g

(B) the magnitude of the acceleration of m is (M – m)g/m and that of M is zero.

(C) the accelerations of both masses are same

(D) the elongation in the spring is (M – m)g/k.

Ans. **[B]**

Sol. After string is cut :

Acceleration of 'm'

$$a = \frac{T - mg}{m} = \frac{(M - m)g}{m} \qquad (T = mg)$$

Acceleration of 'm'

$$a = \frac{Mg - Mg}{M} = 0$$

93. The potential due to an electrostatic charge distribution is

$$V(r) = \frac{qe^{-\alpha r}}{4\pi\varepsilon_0 r}$$

Where α is positive. The net charge within a sphere centered at the origin and of radius $1/\alpha$ is

(A) 2q/e (B) (1 – 1/e)q (C) q/e (D) (1 + 1/e)q

Ans. **[A]**

Sol. $V = \frac{Kqe^{-\alpha r}}{r}$

$$E = \frac{-dV}{dr} = \frac{Kq\alpha e^{-\alpha r}}{r} + \frac{Kqe^{-\alpha r}}{r^2}$$

$$r = \frac{1}{\alpha}$$

$E = Kq\,[\alpha^2 e^{-1} + \alpha^2 e^{-1}]$ at surface

$$E = \frac{2Kq\alpha^2}{e}$$

$$\phi = EA = \frac{Q}{\epsilon_0}$$

$$\phi = EA\,\epsilon_0 = \frac{2Kq\alpha^2}{e} \times 4\pi \times \frac{1}{\alpha^2} \times \epsilon_0$$

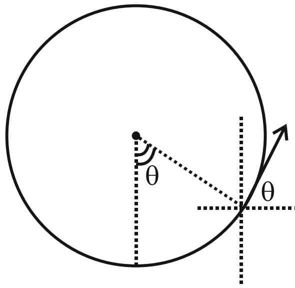
$$Q = \frac{2q}{e}$$

94. A wheel of radius R is trapped in a mud pit and spinning. As the wheel is spinning, it splashes mud blobs with initial speed u from various points on its circumference. The maximum height from the centre of the wheel, to which a mud blob can reach is

(A) $u^2/2g$ (B) $\frac{u^2}{2g} + \frac{gR^2}{2u^2}$ (C) $\frac{u^2}{2g}$ (D) $R + \frac{u^2}{2g}$

Ans. **[B]**

Sol.



$$H = \frac{u^2 \sin^2\theta}{2g} - R\cos\theta$$

$$\frac{dH}{d\theta} = \frac{u^2 \sin\theta\cos\theta}{g} + R\sin\theta$$

$$\frac{dH}{d\theta} = 0$$

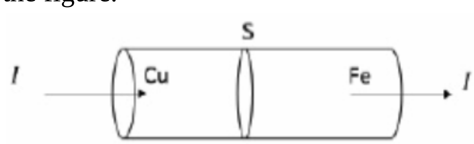
$$\cos\theta = \frac{-gR}{u^2}$$

$$H = \frac{u^2}{2g}\left[1 - \frac{g^2R^2}{u^4}\right] + \frac{gR^2}{u^2}$$

$$H = \frac{u^2}{2g} - \frac{gR^2}{2u^2} + \frac{gR^2}{u^2}$$

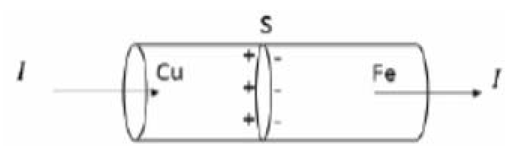
$$H = \frac{u^2}{2g} + \frac{gR^2}{2u^2}$$

95. Two rods of copper and iron with the same cross sectional area are joined at S and a steady current I flows through the rods as shown in the figure.

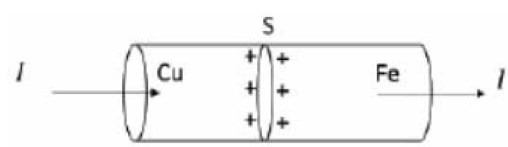


Choose the most appropriate representation of charges accumulated near the junction S.

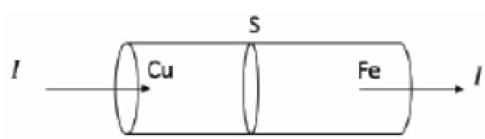
(A)



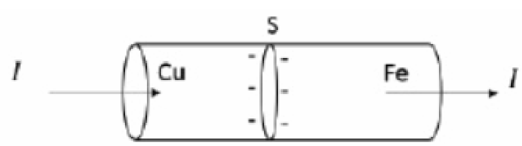
(B)



(C)

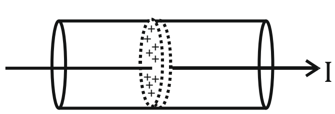


(D)



Ans. **[B]**

Sol.



Gauss Law

$$-E_{Cu}A + E_{Fe}A = \frac{q_{en}}{\epsilon_0}$$

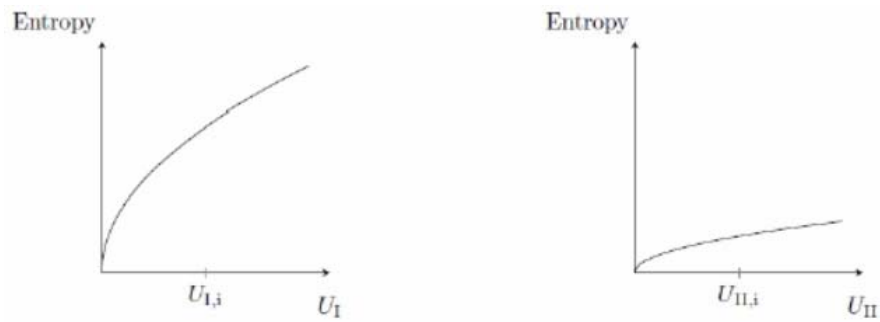
$$J = \sigma E$$

$$\frac{-IA}{\sigma_{Cu}A} + \frac{IA}{A\sigma_{Fe}} = \frac{q_{en}}{\epsilon_0}$$

$$\frac{I}{A} = \sigma E$$

$$I\epsilon_0\left[\frac{1}{\sigma_{Fe}} - \frac{1}{\sigma_{Cu}}\right] = q_{en}$$

96. Graphs below show the entropy vs energy (U) of two system I and Ii at constant volume. The initial energies of the systems are indicated by $U_{I,i}$ and $U_{II,i}$, respectively. Graphs are drawn to the same scale. The system are then brought into thermal contact with each other. Assume that at all times the combined energy of the two systems remains constant. Choose the most appropriate option indicating the energies of the two systems and the total entropy after they achieve the equilibrium.



(A) U_I increases and U_{II} decreases and the total entropy remains the same

(B) U_I decreases and U_{II} increases and the total entropy remains the same

(C) U_I increases and U_{II} decreases and the total entropy increases

(D) U_I decreases and U_{II} increases and the total entropy increases

Ans. **[C]**

Sol. $\Delta S_1 = \frac{\Delta Q_1}{T_1} \rightarrow \text{negative}$

$\Delta S_2 = \frac{\Delta Q_2}{T_2} \rightarrow \text{positive}$

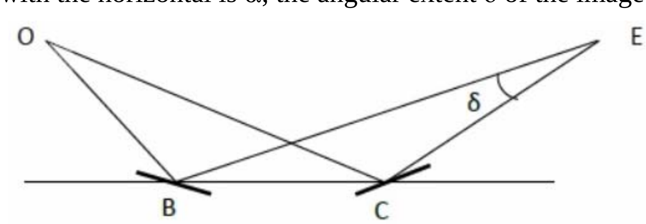
$\Delta Q_1 = \Delta Q_2$

$T_1 > T_2$

So $\Delta S_1 < \Delta S_2$

$\Delta S = \Delta S_1 + \Delta S_2 > 0$

97. The image of an object O due to reflection from the surface of a lake is elongated due to the ripples on the water surface caused by a light breeze. This is because the ripples act as tilted mirrors as shown. Consider the case where O and the observer E are at the same height above the surface of the lake. If the maximum angle that the ripples make with the horizontal is α, the angular extent δ of the image will be



(A) $\frac{\alpha}{2}$ (B) α (C) 2α (D) 4α

Ans. **[C]**

Sol. From geometry when mirros are rorated by angle α, reflected ray are rotated by an angle 2α.

98. A spiral galaxy can be approximated as an infinitesimally thin disk of a uniform surface mass density (mass per unit area) located at z = 0. Two stars A and B start from rest from heights $2z_0$ and z_0 (z_0 << radial extent of the disk), respectively, and fall towards the disk, cross over to the other side, and execute periodic oscillations. The ratio of time periods of A and B is

(A) $2^{-1/2}$ (B) 2 (C) 1 (D) $2^{1/2}$

Ans. **[D]**

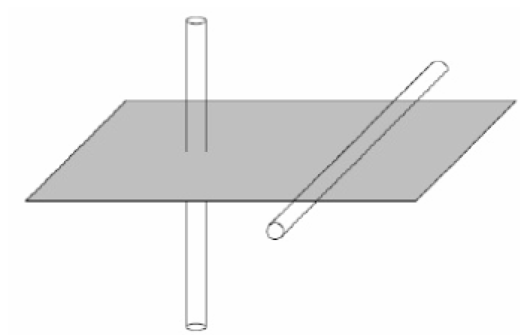
Sol. Assuming spiral galaxy as a large sheet, then accelration of star is constant

$a = c$

$v \propto t$

$z \propto t^2$

99. Two mutually perpendicular infinitely long straight conductors carrying uniformly distributed charges of linear densities λ_1 and λ_2 are positioned at a distance r from each other.

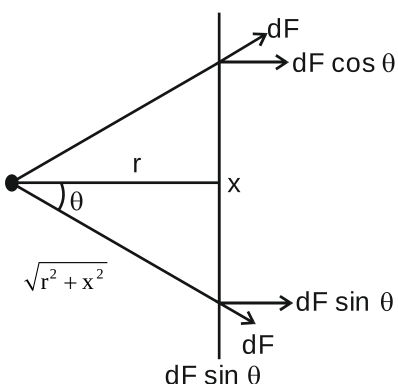


Force between the conductors depends on r as

(A) l/r (B) l/r^2 (C) r (D) 2^0

Ans. **[D]**

Sol.

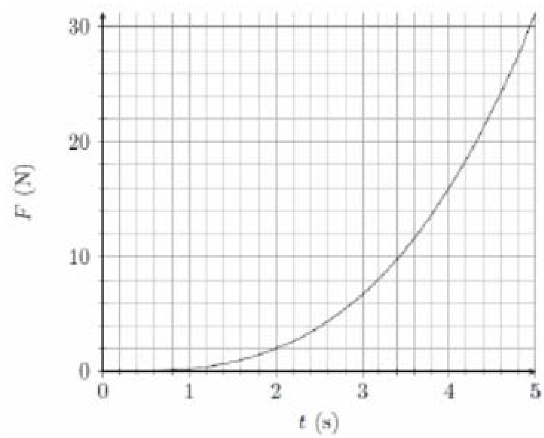


$$dF_{Net} = 2dF\cos\theta = 4K\lambda_1\lambda_2 \frac{r}{(r^2+x^2)}dx$$

$$F_{Net} = \int_0^\infty 4K\lambda_1\lambda_2 \frac{r}{(r^2+x^2)}dx$$

Solving t $F_{Net} \propto r^0$

100. The graph below shows the variation of a force (F) with time (t) on a body which is moving in a straight line. Dependence of force on time is $F \propto t^n$. Initially body is at rest



If the speed of the object is 2 m/s at 3 s, the speed at 4 s will be approximately (in m/s)

(A) 2.5 (B) 6.5 (C) 7.8 (D) 3.1

Ans. **[B]**

Sol. Suppose $F = Kt^n$

from graph $2 = K2^n$

$16 = K4^n = K(2^{2n})$

$2^n = 8$

$n = 3$

$\& \ K = \frac{1}{4}$

$F = \frac{1}{4}t^3$

$F = \frac{mdv}{dt}$

$\Delta V = \frac{1}{m}\int Fdt$

$\Delta V_1 = \frac{K}{m}\int_0^3 t^3dt = 2$

$\Delta V_2 = \frac{K}{m}\int_0^4 t^3dt$

$\Delta V_2 = \frac{2\int_0^4 t^3dt}{\int_0^3 t^3dt} = 6.5 m/s$

CHEMISTRY

101. For the electrochemical cell shown below

$Pt|H_2$ (P = 1 atm)$|H^+$(aq., x M)$||Cu^{2+}$ (aq., 1.0 M)$|Cu(s)$

The potential is 0.49 V at 298 K. The pH of the solution is closest to

[Given : Standard reduction potential, E° for Cu^{2+}/Cu is 0.34 V]

Gas constant, R is 8.31 J K^{-1} mol^{-1}

Faraday constant, F is 9.65×10^4 J V^{-1} mol^{-1}]

(A) 1.2 (B) 8.3 (C) 2.5 (D) 3.2

Ans. **[C]**

Sol. anode : $H_2 \rightarrow 2H^+ + 2e^-$

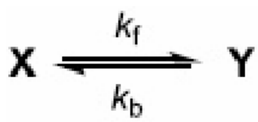
Cathode : $\dfrac{Cu^{2+} + 2e^- \rightarrow Cu(s)}{H_2 + Cu^{2+} \rightarrow Cu + 2H^+}$

$$E = E^o_{cell} - \frac{0.0591}{n}\log Q \Rightarrow 0.49 = 0.34 - \frac{0.06}{2}\log\frac{[x]^2}{1}$$

$$0.15 = 0.06\ pH$$

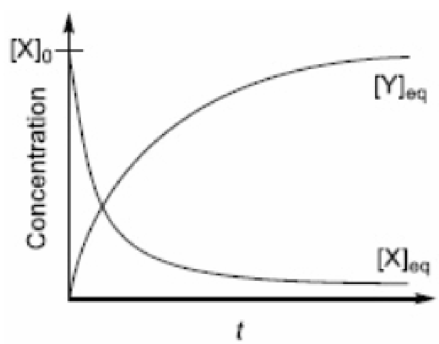
$$pH = 2.5$$

102. Consider the following reversible first-order reaction of X at an initial concentration $[X]_0$. The values of the rate constants are $k_f = 2s^{-1}$ and $k_b = 1s^{-1}$

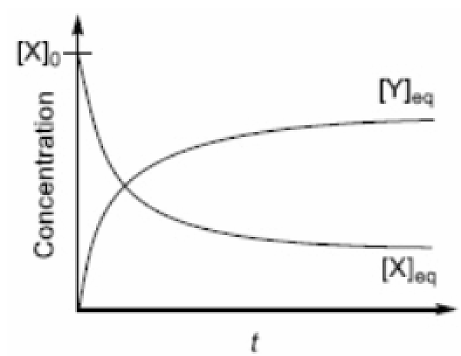


A plot of concentration of X and Y as function of time is

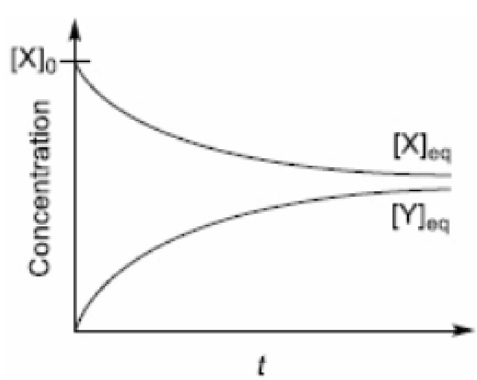
(A)



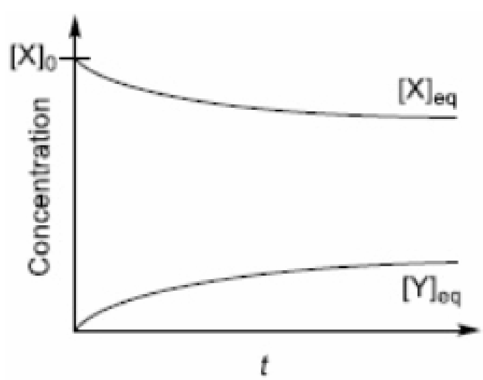
(B)



(C)



(D)



Ans. **[B]**

Sol. $X \underset{k_b}{\overset{k_f}{\rightleftharpoons}} Y$

t = 0 X_0

t = eq. $X_0 - \alpha \quad \alpha$

$$\frac{\alpha}{2x_0 - \alpha} = 2$$

$\alpha = 2x_0 - 2\alpha$

$3\alpha = 2x_0$

$\alpha = 2x_0 /3$

At equilibrium

$$[x] = \frac{x_0}{3}$$

$$[y] = \frac{2x_0}{3}$$

103. Nitroglycerine (MW = 227.1) detonates according to the following equation :

$2C_3H_5(NO_3)_3(l) \rightarrow 3N_2(g) + ½\ O_2(g) + 6\ CO_2(g) + 5\ H_2O(g)$

The standard molar enthalpies of formation, ΔH_f^o for all the compounds are given below :

$\Delta H_f^o\ [C_3H_5(NO_3)_3]$ = –364 kJ/mol

$\Delta H_f^o\ [CO_2(g)]$ = –393.5 kJ/mol

$\Delta H_f^o\ [H_2O(g)]$ = –241.8 kJ/mol

$\Delta H_f^o\ [N_2(g)]$ = 0 kJ/mol

$\Delta H_f^o\ [O_2(g)]$ = 0 kJ/mol

The enthalpy change when 10 g of nitroglycerine is detonated is

(A) –100.5 kJ (B) –62.5 kJ (C) –80.3 kJ (D) –74.9 kJ

Ans. **[B]**

Sol. $\Delta H^o{}_{f\,,\,r} = \Delta H^o{}_{f\,,\,P} - \Delta H^o{}_{f\,'R}$

$$= 3\Delta H^o{}_{f,\,N_2} + \frac{1}{2}\Delta H^o{}_{f,O_2} + 6\Delta H^o{}_{f,CO_2} + 5\Delta H^o{}_{f,H_2O} - 2\Delta H^o{}_{f\,,C_3H_5(NO_3)_3}$$

= 0 + 0 + 6 × –393.5 + 5 × –241.8 + 2 × 364

$\Delta H^o{}_{f,\,r}$ = –2842

when 1 mole of nitroglycerine detonate, $\Delta H^o{}_{f,\,r} = -1421\ \frac{kJ}{mol}$

for $\frac{10}{227.1}$ mole,

$$\Delta H^o{}_{f,\,r} = -1421 \times \frac{10}{227.1}$$

= –62.5 kJ

104. The heating of $(NH_4)_2Cr_2O_7$ produces another chromium compound along with N_2 gas. The change of the oxidation state of Cr in the reaction is .

(A) +6 to +2 (B) +7 to +4 (C) +8 to +4 (D) +6 to +3

Ans. **[D]**

Sol. $(NH_4)_2\ Cr_2O_7 \rightarrow N_2 + Cr_2O_3 + 4H_2O$

+6 +3

105. The complex having the highest spin-only magnetic moment is

(A) $[Fe(CN)_6]^{3-}$ (B) $[Fe(H_2O)_6]^{2+}$ (C) $[MnF_6]^{4-}$ (D) $[NiCl_4]^{2-}$

Ans. **[C]**

Sol. Fe^{3+} n = 1

Fe^{2+} n = 4

Mn^{2+} n = 5

Ni^{2+} n = 2

n = no. Of unpaired electrons

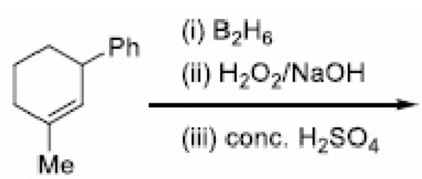
106. Among Ce($4f^1\ 5d^1\ 6s^2$), Nd($4f^4\ 6s^2$), Eu($4f^7\ 6s^2$) and Dy($4f^{10}\ 6s^2$), the elements having highest and lowest 3^{rd} ionization energies, respectively, are

(A) Nd and Ce (B) Eu and Ce (C) Eu and Dy (D) Dy and Nd

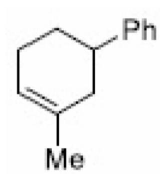
Ans. **[B]**

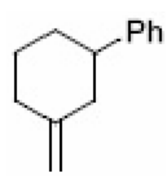
Sol. After removig 2 electrons in E, it acquires stable half –filled orbital whereas in ce 3^{rd} electron can easily removed from 5d sub-shell.

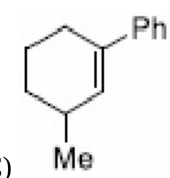
107. The major product of the following reaction sequence

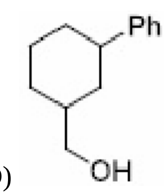


is

(A) 

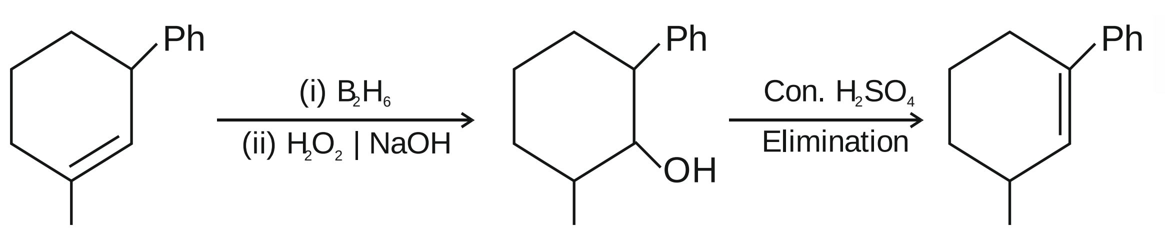
(B) 

(C) 

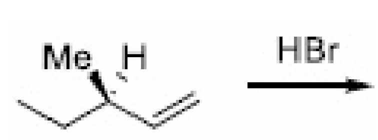
(D) 

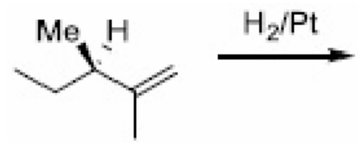
Ans. **[C]**

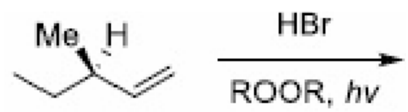
Sol.

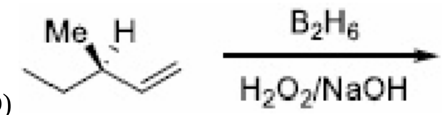


108. Among the following reactions, a mixture of diastereomers is produced from

(A) 

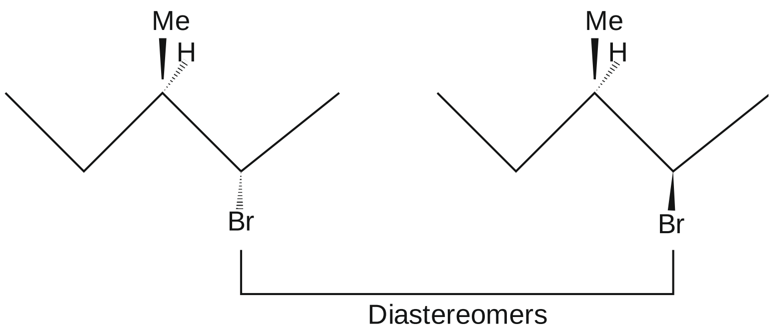
(B) 

(C) 

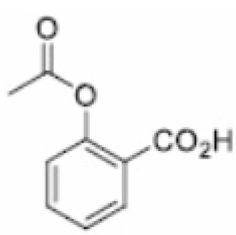
(D) 

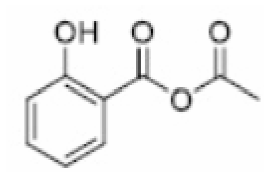
Ans. **[A]**

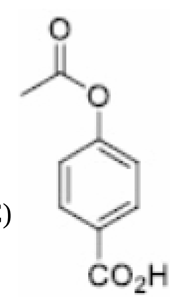
Sol.

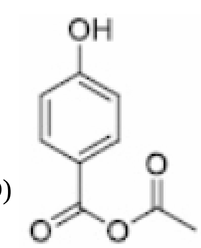


109. Reaction of phenol with NaOH followed by heating with CO_2 under high pressure, and subsequent acidification gives compound X as the major product, which can be purified by steam distillation. When reacted with acetic anhydride in the presence of a trace amount of conc. H_2SO_4, compound X produces Y as the major product. Compound Y is

(A) 

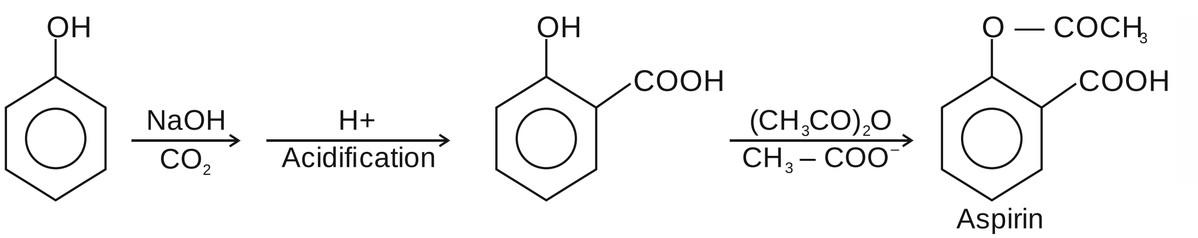
(B) 

(C) 

(D) 

Ans. **[A]**

Sol.



110. A tetrapeptide is made of naturally occurring alanine, serine, glycine and valine. If the C-terminal amino acid is alanine and the N-terminal amino acid is chiral, the number of possible sequences of the tetrapeptide is

(A) 12 (B) 8 (C) 6 (D) 4

Ans. **[D]**

Sol. Ser – Val – Gly – Ala

Ser – Gly – Val – Ala

Val – Ser – Gyl – Ala

Val – Gyl – Ser – Ala

BIOLOGY

111. What is the probability that a human individual would receive the entire haploid set of chromosomes from his/her grandfather ?

(A) 1/2 (B) $(1/2)^{23}$ (C) $(1/2)^{2}$ (D) $(1/2)^{46}$

Ans. **[B]**

Sol. Human have 46 chromosome in diploid cells while haploid cells have 23 chromosomes in gamete, every human receive ½ of 46 from each parent.

112. Which ONE among the following primer pairs would amplify the fragment of DNA given below ?

5' –CTAGTCGTCGAT-$(N)_{300}$-GACTGAGCTGAGCTG-3'

3' –GATCAGCAGCTA-$(N)_{300}$-CTGACTCGACTCGAC-5'

(A) 5' –CTAGTCGTCGAT-3' and 5' –GACTGAGCTGAGCTG-3'

(B) 5' –CTGACTCGACTCGAC-3' and 5' –CTAGTCGTCGAT-3'

(C) 5' –CTAGTCGTCGAT-3' and 5' –CAGCTCAGCTCAGTC-3'

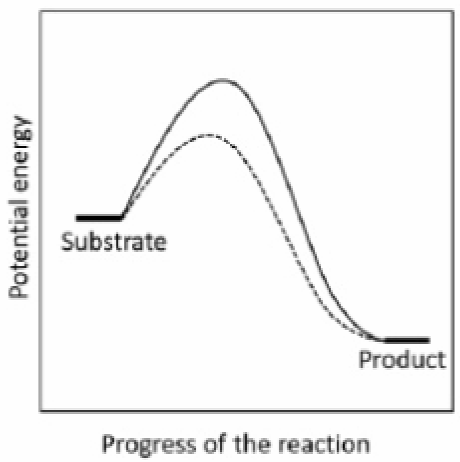
(D) 5' –CTAGTCGTCGAT-3' and 5' –GTCGAGTCGAGTCAG-3'

Ans. **[C]**

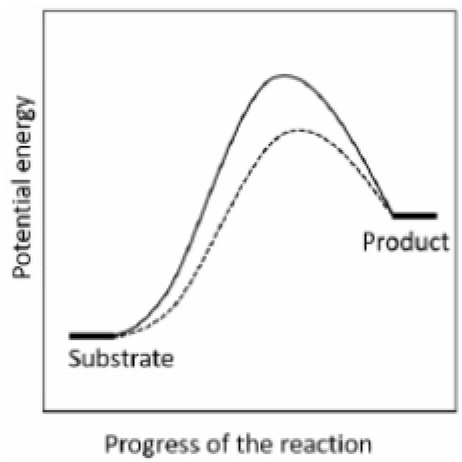
Sol. $5' \rightarrow 3'$ is the direction of primer.

113. The following graphs with the solid and dotted lines correspond to the reactions without and with enzyme, respectively. Which of the following graph(s) correctly represent the concept of activation energy ?

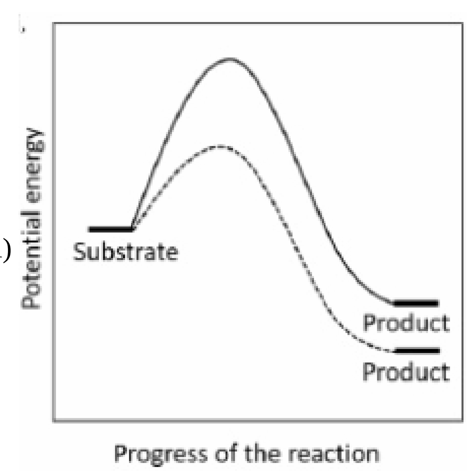
(i)



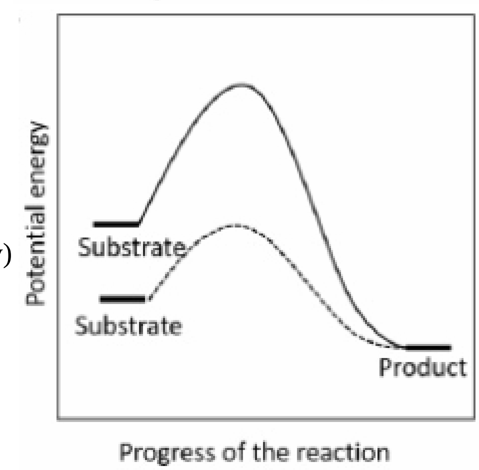
(ii)



(iii)



(iv)



(A) (i) only (B) (iii) and (iv) (C) (ii) only (D) (i) and (ii)

Ans. **[D]**

Sol. The activation enerfy of a chemical reaction is closely related to its rate. Specifically, the higher the activation energy, the slower the chemical reaction will be. This is because molecules can only complete the reaction once they have reached the top of the activation energy barrier. Each and every enzyme has active side at which substrate can bind, forming enzymes substrate complex to form product.

114. A novel species with double stranded genetic material consists of 5 bases namely P, Q, R, S, T, with percentages given below.

	P	Q	R	S	T
Percentage	22	28	22	12	16

Based on the above information, which ONE of the following inferences is NOT supported by the observations ?

(A) S base pairs with T, and Q base pairs with R

(B) S base pairs with Q, and T base pairs with Q

(C) P base pairs with R, and S base pairs with Q

(D) P base pairs with R, and T base pairs with Q

Ans. **[A]**

Sol. P = R = 22 so they bind together while Q = 28 bind with S = 12 & T = 16.

115. How many different blood groups are possible in a diploid species with ABCO blood grouping system involving I^A, I^B, I^C and I^O alleles (I^O is recessive and others are co-dominant) ?

(A) 4 (B) 6 (C) 7 (D) 8

Ans. **[C]**

Sol. ABCO blood group except 'O' all alleles are dominative so, there possible blood groups can be A, B, C, O, AB, AC, BC

116. Within the exponential phase of growth, if the initial surface area and the growth rate of a leaf are 10 mm^2 and 0.015 nm^2/hour respectively, the area of the leaf after 4 days would range from :

(A) 10 to 12 mm^2 (B) 20 to 24 mm^2 (C) 30 to 36 mm^2 (D) 40 to 48 mm^2

Ans. **[D]**

Sol. Equation of exponential growth in plants is $W_t = W_0 \times e^{rt}$

$W_0 = 10\ mm^2$

$r = 0.015\ mm^2/hour$

$t = 4\ days \rightarrow 4 \times 24 = 96\ hours$

$w_t = 10 \times e^{0.015 \times 96} = 42.2$

117. If the acidic, basic and hydrophobic residues of proteins are considered to be red, green and blue in color, respectively, then a globular protein in aqueous solution would have.

(A) red and blue on the surface and green at the core

(B) red and green on the surface and blue at the core

(C) blue on the surface and red ad green at the core

(D) blue and green on the surface and red at the core

Ans. **[B]**

Sol. Direct question.

118. A lysosome vesicle of 1 μm diameter has an internal pH of 5.0. The total number of H^+ ions inside this vesicle would range from

(A) 10^3 to 10^4 (B) 10^4 to 10^5 (C) 10^5 to 10^{10} (D) 10^{10} to 6.023×10^{23}

Ans. **[A]**

Sol. Lysosome radius is 0.5 μm = 0.5×10^{-6} m

Spherical lyosome volume is $\frac{4}{3}\pi r^3$

$\frac{4}{3}\pi\frac{1}{8}\times 10^{-18} m^3$ or $\frac{4}{3}\pi\frac{1}{8}\times 10^{-15} L$

$[H^+] = 10^{-pH}$ mole/lit

Number of moles of H^+ ions = $\frac{4}{3}\pi\frac{1}{8}\times 10^{-15}\times 10^{-5}$ mole

Number of H^+ ions = n × NA

n = number of moles

NA = avagadaro number

$[H^+]$ number = $\frac{4}{3}\pi\frac{1}{8}\times 10^{-20}\times 6.023\times 10^{+23}$

$[H^+]$ number = in between 10^3 to 10^4.

119. Match the vitamins listed in **Column I** with their respective coenzyme forms in **Column II**. Choose the CORRECT combination.

Column I	**Column II**
P. Vitamin B_1	i. Thiamine pyrophosphate
Q. Vitamin B_2	ii. Flavine adenine dinucleotide
R. Vitamin B_6	iii. Methylcobalamin
S. Vitamin B_{12}	iv. Coenzyme A
	v. Pyridoxal phosphate

(A) P-v, Q-iii, R-i, S-iv (B) P-iii, Q-iv, R-ii, S-i (C) P-i, Q-ii, R-v, S-iii (D) P-i, Q-iv, R-ii, S-iii

Ans. **[C]**

Sol. Vitamin B complex are necessary for vital functions and essential for formation of co-enzyme, necessary for enzymatic reaction . Example – Vitamin B_1 – also called thiamine – required for TPP (Thiamine Pyrophosphate), Vitamin B_2 also called riboflavin – required for FMN/FAD formation, Vitamin B_3 also called niacin or nicotinic acid – required for formation of NAD, NADP.

120. Two independent experiments related to photosynthesis were conducted – one with ^{18}O-labelled water (experiment P), and the other with ^{14}C-labelled CO_2 (experiment Q). Which ONE of the following options lists the first labeled products in experiments P and Q, respectively ?

(A) P : O_2, Q : 3-Phosphoglycerate
(B) P : 3-Phosphoglycerate, Q : NADPH
(C) P : O_2, Q : ATP
(D) P : 3-Phosphoglycerate, Q : 3-Phosphoglycerate

Ans. **[A]**

Sol. By the use of O^{18} radioisotope it was confirmed that O_2 exist from H_2O not from CO_2 during lighr reaction.
By the use of C^{14} radioisotope it was confirmed that C in $C_6H_{12}O_6$ comes from CO_2.