CBSE Class 10th Mathematics

Basic Sample Paper - 09

Maximum Marks:

Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. In a data, if l = 40, h = 15, f_1= 7, f_0 = 3, f_2 = 6, then the mode is

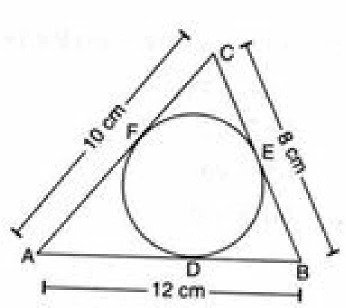
    a. 82

    b. 62

    c. 52

    d. 72

2. A circle is inscribed in $\triangle ABC$ having sides 8 cm, 10 cm and 12 cm as shown in the figure. Then the measure of AD and BE are...



a. AD = 8 cm, BE = 5 cm.

b. AD = 8 cm, BE = 6 cm

c. AD = 5 cm, BE = 7 cm

d. AD = 7 cm, BE = 5 cm

3. If ‘a’ and ‘b’ are both positive rational numbers, then $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ will be

a. neither rational nor rational number

b. a rational number

c. an irrational number

d. none of these

4. If a is rational and $\sqrt{b}$ is irrational, then $a + \sqrt{b}$ is:

a. an irrational number

b. an integer

c. a natural number

d. a rational number

5. The least number n so that 5^n is divisible by 3, where n is:

a. a whole number

b. a real number

c. a natural number

d. no natural number

6. If one of the zeroes of the quadratic polynomial $14x^2 - 42k^2x - 9$ is negative of the other, then the value of 'k' is

   a. 3

   b. 0

   c. 1

   d. 2

7. If' α' and 'β' are the zeroes of the polynomial $3x^2 + 11x - 4$, then the value of $\alpha^2 + \beta^2$ is

   a. $\frac{150}{9}$

   b. $\frac{145}{9}$

   c. $\frac{152}{9}$

   d. $\frac{144}{9}$

8. An unbiased die is thrown once. The probability of getting a prime or composite number is

   a. $\frac{1}{6}$

   b. $\frac{1}{2}$

   c. $\frac{5}{6}$

   d. 1

9. The co – ordinates of the point which is equidistant from the three vertices of a ΔAOB with vertices A(0, 2y), B(2x, 0) and O(0, 0) is

   a. $\left(\frac{x}{2}, \frac{y}{2}\right)$

b. (y, x)

c. (0, 0)

d. (x, y)

10. _________ is an algebraic tool for studying the geometry.

a. Statistics

b. None of these

c. Co – ordinate Geometry

d. Trigonometry

11. Fill in the blanks:

$\triangle$ABC and $\triangle$DEF are similar. Area of $\triangle$ABC is 9cm^2 and Area of $\triangle$DEF is 64cm^2. If DE = 5.1cm, then the value of AB is ________.

12. Fill in the blanks:

The mirror image of (3, 9) on x-axis is ________.

OR

Fill in the blanks:

Three points are said to be collinear, if area of triangle formed by these points is ________.

13. Fill in the blanks:

The ratio of the sides of a right triangle with respect to its acute angles, are called ________.

14. Fill in the blanks:

Two angles are said to be ________ if their sum is equal to 90^o.

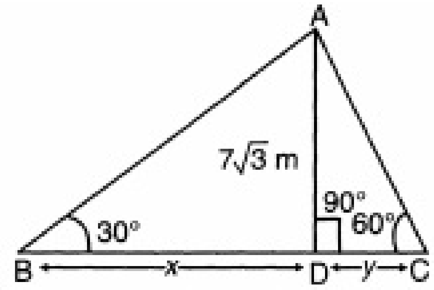
15. Fill in the blanks:

 The value of 3sin 30^{o} - 4$\sin^{3}$ 60^{o} is ________.

16. At some time of the day the length of the shadow of a tower is equal to its height. Find the sun’s altitude at that time.

OR

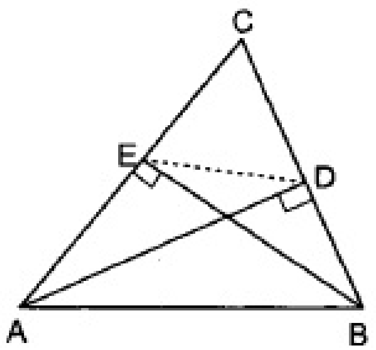
 In the given figure, if $AD = 7\sqrt{3}m$ then find the value of BC



17. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, \dots$.

18. The diameter of a wheel is 1.26 m. What is the distance covered in 500 revolutions?

19. In Fig. AD and BE are respectively perpendiculars to BC and AC. Show that $CD \times AB = CA \times DE$



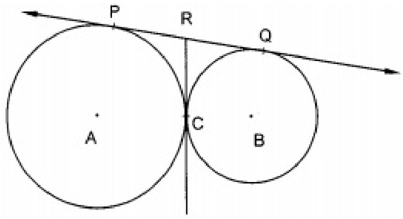
20. The faces of a red cube and a yellow cube are numbered from 1 to 6. Both cubes are rolled. What is the probability that the top face of each cube will have the same number?

Section B

21. If α and β are the zeros of the quadratic polynomial f(t) = t^2 - 4t + 3, find the value of

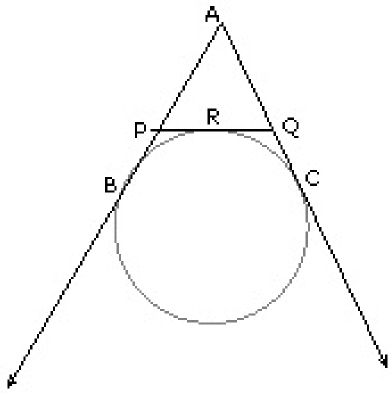
$\alpha^4\beta^3 + \alpha^3\beta^4$.

22. In figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



OR

If AB, AC and PQ are tangents in the given figure and AB = 25 cm, Find the perimeter of $\triangle$ APQ.

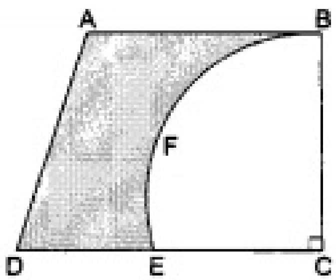


23. Prove the trigonometric identity:
$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

OR

Prove that $cosec\,(65^\circ+\theta) - \sec(25^\circ-\theta) - \tan(55^\circ-\theta) + \cot(35^\circ+\theta) = 0$

24. From a thin metallic piece in the shape of a trapezium ABCD in which $AB \| CD$ and $\angle BCD$ = 90°, a quarter circle BFEC is removed. Given, AB = BC =CE= 3.5 cm and DE = 2 cm, calculate the area of remaining (shaded) part of metal sheet.



25. A box contains 12 balls out of which 4 are red, 3 are black and 5 are white. A ball is taken out of the box at random. Find the probability that the selected ball is

i. not red
ii. black or red.

26. The probability of selecting a red ball at random form a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar.

Section C

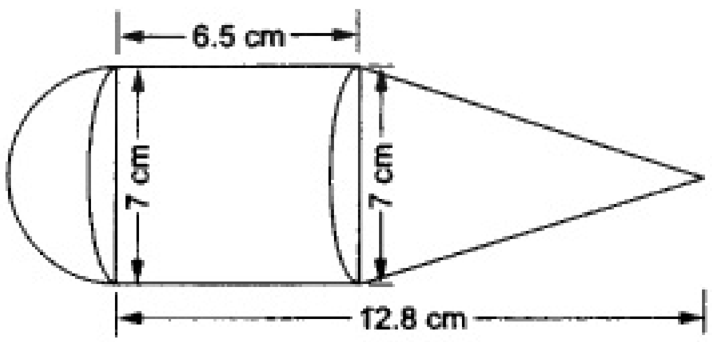
27. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a - b, a, a + b, find a and b.

28. Draw an isosceles $\triangle$ABC in which BC = 5.5 cm and altitude AL = 3 cm. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle$ABC.

OR

Construct a $\triangle$ABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle$ABC = 60°. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle$ABC.

29. The given figure represents a solid consisting of a cylinder surmounted by a cone at one end and a hemisphere at the other. Find the volume of the solid.



30. Prove the identity:

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A- cosec^3 A} = \sin^2 A \cos^2 A$$

OR

Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$ = cos A - sin A.

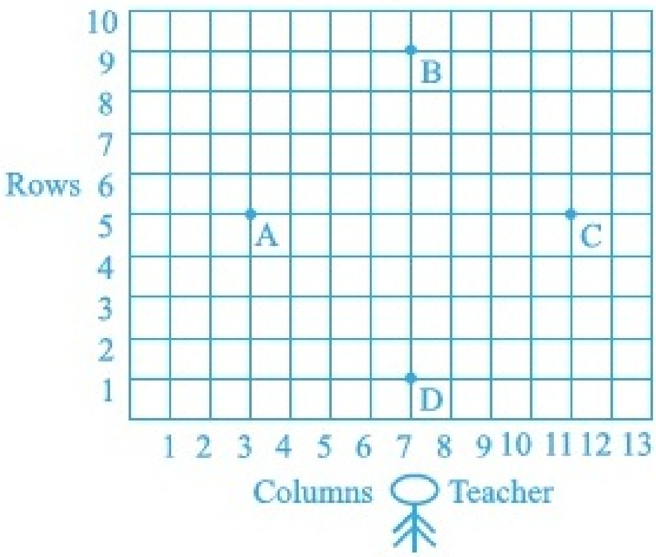
31. Prove that $3\sqrt{7}$ is irrational.

OR

Amita, Suneha and Raghav start preparing cards for greeting each person of an old age home on new year. In order to complete one card, they take 10, 16 and 20 minutes respectively. If all of them started together, after what time will they start preparing a new card together ? Why do you think there is a need to show elders that the young generation cares for them and remembers the contribution made by them in the prime of their life?

32. In two concentric circles, a chord of length 24 cm of larger circle becomes a tangent to the smaller circle whose radius is 5 cm. Find the radius of the larger circle.

33. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in figure.



i. Find the positions of the four students A, B, C and D and find the distance between them.
ii. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D?
iii. If so, what should be the Jaspal's position?

34. A path separates two walls. A ladder leaning against one wall rests at a point on the

path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall.

Section D

35. At t minutes past 2 p.m, the time needed by the minute hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

36. The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages of all the participants is 168 years, find the age of eldest participant in the painting competition.

OR

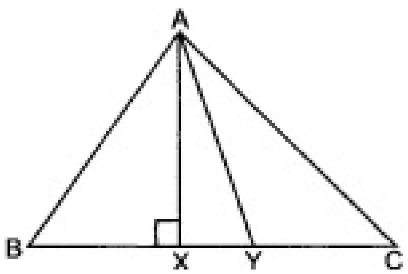
The sums of first n terms of three A.P.s are S_1, S_2, and S_3. The first term of each is 5 and their common differences are 2, 4 and 6 respectively. Prove that $S_1 + S_3 = 2S_2$.

37. Solve graphically the following pair of equations and show that the system has no solution.

$3x - 5y = 20$

$6x - 10y = -40$

38. in $\triangle$ABC, AX $\perp$ BC and Y is middle point of BC.



Prove that

i. $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$

ii. $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$

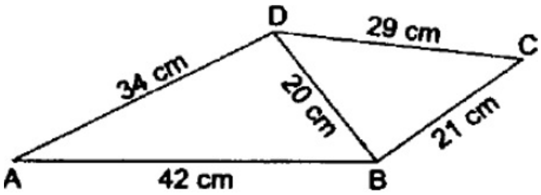
OR

Given a right-angled $\triangle ABC$ .The lengths of the sides containing the right angle are 6 cm and 8 cm. A circle is inscribed in $\triangle ABC$. Find the radius of the circle.

39. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub. (Use$\pi = \frac{22}{7}$)

OR

Find the area of the quadrilateral ABCD in which AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and diagonal BD = 20 cm.



40. Pocket expenses of the students of a class are shown in the following frequency distribution:

Pocket expenses (in Rs)	No. of students
0 - 200	33
200 - 400	74
400 - 600	170
600 - 800	88
800 - 1000	76
1000 - 1200	44
1200 - 1400	25

Find mean and median for the above data.

Solution

Section A

1. (c) 52

   Explanation:

   Mode = $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$
   = $40+\frac{7-3}{7\times2-3-6}\times15$
   = $40+\frac{4}{5}\times15$
   = 40 + 12
   = 52

2. (d) AD = 7 cm, BE = 5 cm

   Explanation:

   Let AD = x and BE = y
   $\therefore$ BD = 12 - x $\Rightarrow$ BE = y
   But BD = BE (Tangents to a circle from an external point B)
   $\Rightarrow$y = 12 - x $\Rightarrow$ x + y = 12 ....(i)
   Also, AF = x
   and CF = 10 - x
   and CE = 8 - y
   $\therefore$ 10 - x = 8 - y
   x - y = 2........(ii)
   On solving eq. (i) and (ii), we get
   x = 7 and y = 5
   Therefore AD = 7 cm and BE = 5 cm

3. (b) a rational number

   Explanation:

   $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ = $\left\{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2\right\}$

= $(a-b)$

Since a and b both are positive rational numbers,

therefore difference of two positive rational numbers is also rational.

4. (a) an irrational number
   Explanation:

   Let a be rational and $\sqrt{b}$ is irrational.

   If possible let $a+\sqrt{b}$ be rational.

   Then $a+\sqrt{b}$ is rational and a is rational.

   $\Rightarrow \left[\left(a+\sqrt{b}\right)-a\right]$ is rational [Difference of two rationals is rational]

   $\Rightarrow \sqrt{b}$ is rational.

   This contradicts the fact that $\sqrt{b}$ is irrational.

   The contradiction arises by assuming that $a+\sqrt{b}$ is rational.

   Therefore, $a+\sqrt{b}$ is irrational.

5. (d) no natural number
   Explanation:

   Since 5 is a prime number so it is not divisible by 3.

   Therefore there is no natural number n

   such that 5^{n} is divisible by 3.

6. (b) 0
   Explanation:

   Let one zero be αthen the other zero will be $(-\alpha)$
   $\therefore$Sum of the zeroes = $\frac{-b}{a}$
   $\Rightarrow\alpha+(-\alpha)$

= $\frac{42k^2}{14}$

$\Rightarrow 0 = \frac{42k^2}{14}$

$\Rightarrow 42k^2 = 0$

$\Rightarrow k = 0$

7. (b) $\frac{145}{9}$

   Explanation:

   Here $a = 3, b = 11, c = -4$

   Since $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

   = $\left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}$ = $\frac{b^2}{a^2} - \frac{2c}{a}$ = $\frac{b^2 - 2ac}{a^2}$

   Putting the values of a, b and c, we get = $\frac{(11)^2 - 2 \times 3 \times (-4)}{(3)^2}$

   = $\frac{121+24}{9}$

   = $\frac{145}{9}$

8. (c) $\frac{5}{6}$

   Explanation:

   prime numbers on a die are 2 , 3, 5
   composite numbers on a die are 4 , 6
   Prime and Composite numbers on a die = 2, 3, 4, 5, 6
   Number of possible outcomes = 5
   Number of Total outcomes = 6
   Required Probability = $\frac{5}{6}$

9. (d) (x, y)
   Explanation:

   AB = $\sqrt{(2x - 0)^2 + (0 - 2y)^2}$
   = $\sqrt{4x^2 + 4y^2}$ = $2\sqrt{x^2 + y^2}$ units

BO = $\sqrt{(0-2x)^2+(0-0)^2}$

= $\sqrt{4x^2}$ = 2x units

AO = $\sqrt{(0-0)^2+(0-2y)^2}$

= $\sqrt{4y^2}$ = $2y$ units

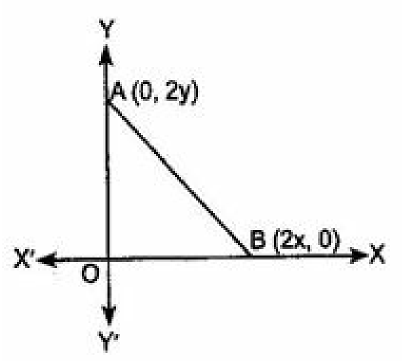
Now, $\mathrm{AB}^2 = \mathrm{AO}^2 + \mathrm{BO}^2 \Rightarrow \left(2\sqrt{x^2+y^2}\right)^2 = (2x)^2+(2y)^2$

$\Rightarrow 4\left(x^2+y^2\right) = 4\left(x^2+y^2\right)$

Therefore, triangle AOB is an isosceles right-angled triangle.

Since the coordinate of the point which is equidistant from the three vertices of a right-angled triangle is the coordinates of mid-point of its hypotenuse.

$\therefore$ Mid-point of AB = $\left(\frac{0+2x}{2}, \frac{2y+0}{2}\right)$ = (x,y)



10. (c) Co – ordinate Geometry

    Explanation:

    Co – ordinate Geometry is an algebraic tool for studying the geometry.

11. AB = 1.91cm

12. (-3, -9)

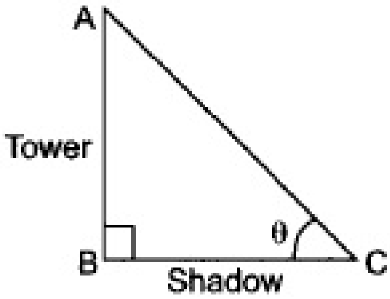
    OR

    zero

13. trigonometric ratios

14. complementary

15. $\frac{3(1-\sqrt{3})}{2}$

16. 

given, the length of the shadow of a tower is equal to its height i.e. AB=BC

In right $\triangle$ABC

$\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

OR

Let BD = x and DC = y

From $\triangle ABD$

$\frac{AD}{BD} = \tan 30^\circ$ using Pythagoras theorem

$\frac{7\sqrt{3}}{x} = \tan 30^\circ$

$\Rightarrow \quad \frac{7\sqrt{3}}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = 7\sqrt{3} \times \sqrt{3}$

= 21 m

From $\triangle ADC$,using Pythagoras theorem

$\frac{AD}{DC} = \tan 60^\circ$

$\frac{7\sqrt{3}}{y} = \tan 60^\circ$

$\frac{7\sqrt{3}}{y} = \sqrt{3}$

$\Rightarrow \quad 7\sqrt{3} = y\sqrt{3}$

$\Rightarrow$ y = 7 m,

BC = BD + DC

= 21 + 7 = 28 m

Hence, the value of BC = 28 m

17. Given sequence is an A.P.

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$

$= \sqrt{2}, 2\sqrt{2}, 3\sqrt{2} \ldots$

Where,$a=\sqrt{2}$, $d=2\sqrt{2}-\sqrt{2}=\sqrt{2}, n=10$

$a_n=a+(n-1)d$

or, $a_{10}=\sqrt{2}$ +(10 - 1)$\sqrt{2}$

$=\sqrt{2}+9\sqrt{2}$

$=10\sqrt{2}$

Hence, a_{10} = $\sqrt{200}$

18. Distance covered in 1 revolution = circumference of wheel

$=\pi d$

$=\pi\times 1.26\mathrm{m}.$

Distance covered in 500 revolutions = 500 × circumference of wheel

= 500 ×πd

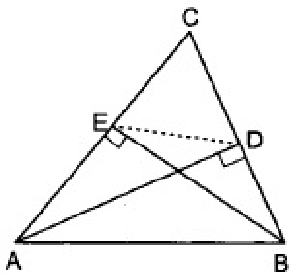
$=500\times\pi\times 1.26$

$=500\times\frac{22}{7}\times 1.26$

= 1980m

= 1.98 km (1000m = 1km)

Distance covered in 500 revolutions is 1.98 km.

19. 

$\angle$ADC = $\angle$BEC = 90o

$\angle$ACD = $\angle$BCE (Common)

$\triangle ADC \sim \triangle BEC$

Thus, $\frac{AC}{BC}=\frac{CD}{EC}$ ...(i)

In $\triangle$ABC and $\triangle$DEC,

From (i),

$\frac{AC}{CD}=\frac{BC}{EC}$

$\angle$DCE = $\angle$BCA (Common)

$\Delta ABC\sim\Delta DEC$ (By SAS Similarity)

$\Rightarrow \quad \frac{AB}{DE}=\frac{AC}{DC}\Rightarrow AB\times DC=AC\times DE\Rightarrow CD\times AB=CA\times DE$

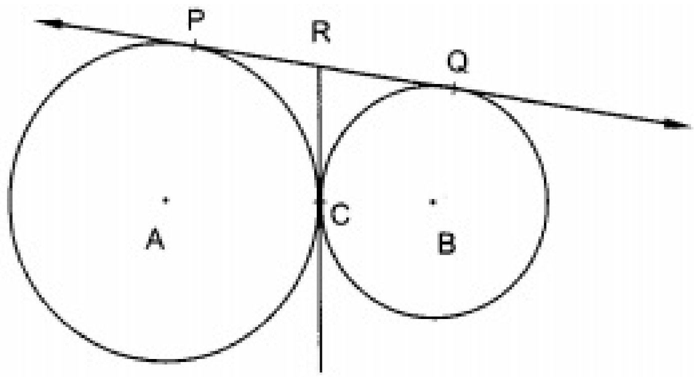
20. We know that Total number of outcomes for two cubes = 36

They are as follows:
{(1,1)(2,1)(3,1)(4,1)(5,1)(6,1)
(1,2)(2,2)(3,2)(4,2)(5,2)(6,2)
(1,3)(2,3)(3,3)(4,3)(5,3)(6,3)
(1,4)(2,4)(3,4)(4,4)(5,4)(6,4)
(1,5)(2,5)(3,5)(4,5)(5,5)(6,5)
(1,6)(2,6)(3,6)(4,6)(5,6)(6,6)}
Favourable outcomes that the top face of each cube will have the same number =
{(1,1),(2,2),(3,3), (4,4),(5,5),(6,6)}

Therefore, number of cases favourable to the event=6.
Hence,Probability of same number on both cube = $\frac{6}{36}=\frac{1}{6}$.

Section B

21. Since, α and β are the zeroes of the quadratic polynomial f(t) = t^2 - 4t + 3
Compare with F(x) = $ax^2 + bx + c$
a = 1 , b =-4 and c = 3
So, Sum of the zeroes = $\alpha+\beta = -\frac{b}{a} = -\frac{-4}{1}$ = 4
Product of the zeroes = $\alpha \times \beta = \frac{c}{a} = \frac{3}{1}$ = 3
Now,
$\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha+\beta)$
= $(\alpha\beta)^3(\alpha+\beta)$
= $(3)^3(4)$
= 108
Therefore $\alpha^4\beta^3 + \alpha^3\beta^4$ = 108

22. 

We know that the tangents drawn from an external point to a circle are equal.

$\therefore$ RP = RC and RC = RQ

$\Rightarrow$ RP = RQ

$\Rightarrow$ R is the mid-point of PQ.

OR

Perimeter of $\triangle$APQ = AP + AQ + PQ

= AP + AQ + PR + RQ

= AP + AQ + PB + CQ

= (AP + PB) + (AQ + QC)

= AB + AC

= 2AB = 2 $\times$ 25 = 50 cm

23. We have,

L.H.S $= \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$ $[\because 1-\sin^2\theta=\cos^2\theta]$

$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$ $[\because \frac{\cos\theta}{\sin\theta}=cot\theta]$

= R.H.S

Hence proved.

OR

LHS $=$ $cosec\,(65^\circ+\theta)-\sec(25^\circ-\theta)-\tan(55^\circ-\theta)+\cot(35^\circ+\theta)$

$$= \ cosec\,[90^\circ - (25^\circ - \theta)] - \sec(25^\circ - \theta) - \tan[90^\circ - (35^\circ + \theta)] + \cot(35^\circ + \theta)$$
$$= \sec(25^\circ - \theta) - \sec(25^\circ - \theta) - \cot(35^\circ + \theta) + \cot(35^\circ + \theta)$$
$$= 0$$
$$= RHS$$

24. Clearly, AB = BC = CE = 3.5 cm and DE = 2 cm

$\Rightarrow$ CD = DE + EC = 2 + 3.5

= 5.5 cm

$\therefore$ Area of the shaded part

= Area of trapezium ABCD - Area of quadrant BCE

$$= \left[\left\{\frac{1}{2}(AB + CD) \times BC\right\} - \left\{\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\}\right] \text{cm}^2$$
$$= \left[\left\{\frac{1}{2}(3.5 + 5.5) \times 3.5\right\} - \frac{77}{8}\right] \text{cm}^2$$
$$= \left[\left\{\frac{1}{2} \times 9 \times 3.5\right\} - \frac{77}{8}\right] \text{cm}^2$$
$$= [15.75 - 9.625]\text{cm}^2$$

= 6.125 cm^2

25. Total no. of balls = 12

Total no of outcomes = 12

i. Let R be the event of getting no red ball.

No of balls which are not red = 12 - 4 = 8

Favouring Outcomes = 8

P(R) = $\frac{8}{12} = \frac{2}{3}$

ii. Let K be the event of getting a black or red ball.

No. of balls red or black = 4 + 3 = 7

Outcomes favouring K = 7

P(K) = $\frac{7}{12}$

26. It is given that,

P(getting a red ball) = 14 and P(getting a blue ball) = 13

Let P(getting an orange ball) be x.

Since, there are only 3 types of balls in the jar, the sum of probabilities of all the three balls must be 1.

Therefore, $\frac{1}{4}$ + $\frac{1}{3}$ +x=1

$\Rightarrow$ x=1$-\frac{1}{4}-\frac{1}{3}$

$\Rightarrow$ x = $\frac{12-3-4}{12}$

$\Rightarrow$ x = $\frac{5}{12}$

Therefore, P(getting an orange ball) = $\frac{5}{12}$.

Let the total number of balls in the jar be n.

Therefore, P(getting an orange ball) = $\frac{10}{n}$

$\Rightarrow$ $\frac{10}{n}=\frac{5}{12}$

$\Rightarrow$ n = 24

Thus, the total number of balls in the jar is 24.

Section C

27. Since (a - b), a, (a + b) are the zeroes of the polynomial x^3-3x^2+x+1.

$\therefore \alpha+\beta+\gamma$ = a- b + a + a + b $=\frac{-(-3)}{1}$ = 3

$\Rightarrow$ 3a = 3

$\Rightarrow$ a = 1

And $\alpha\beta+\beta\gamma+\gamma\alpha$ = (a - b)a + a (a+ b) + (a + b)(a - b) $=\frac{1}{1}=1$

$\Rightarrow$ $a^2 - ab + a^2 + ab + a^2 - b^2 = 1$

$\Rightarrow$ $3a^2 - b^2 = 1$

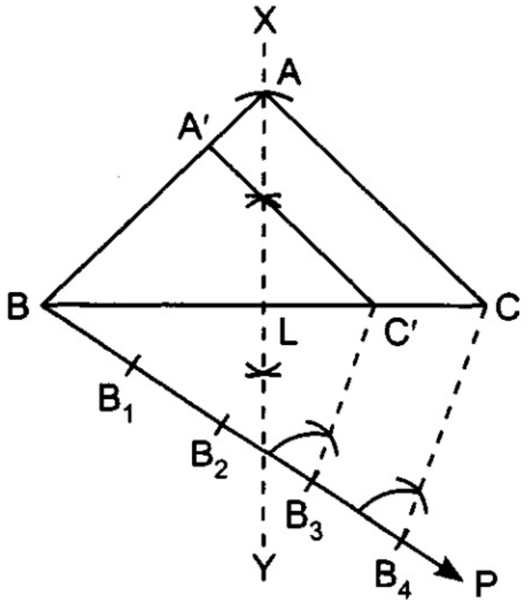
$\Rightarrow 3(1)^2-b^2=1[\because a=1]$

$\Rightarrow 3-b^2=1$

$\Rightarrow b^2=2$

$\Rightarrow b=\pm\sqrt{2}$

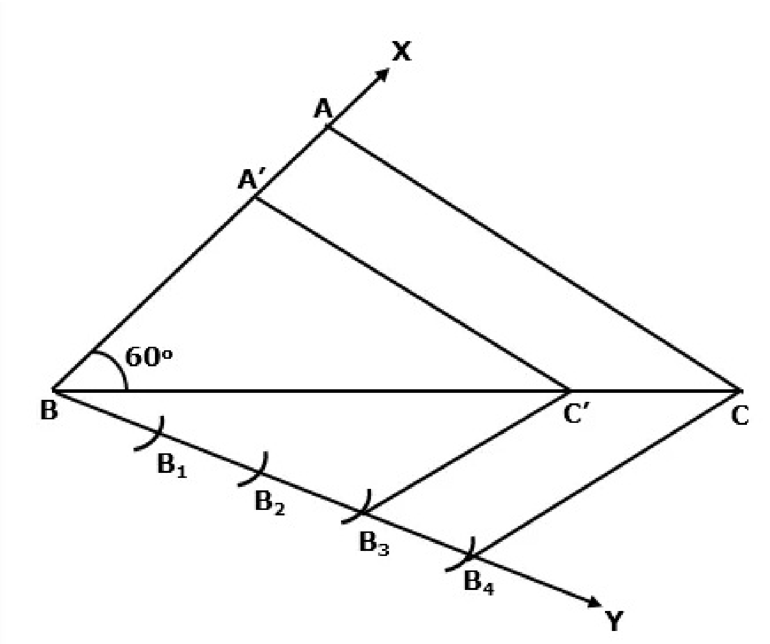
Hence a = 1 and $b=\pm\sqrt{2}$.

28. 

Steps of Construction:

i. Draw a line segment $BC = 5.5cm.$
ii. Perpendicular bisector XY of BC is drawn intersecting BC at L.
iii. Point A is marked on XL such that $AL = 3cm.$
iv. AB and AC are joined.
v. On BP, points B_1, B_2, B_3 and B_4 are marked such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
vi. B_4C is joined.
vii. B_3C' is drawn parallel to B_4C intersecting BC at C'.
viii. $C'A'$ is drawn parallel to CA intersecting BA at A'.
ix. $A'BC'$ is the required triangle.

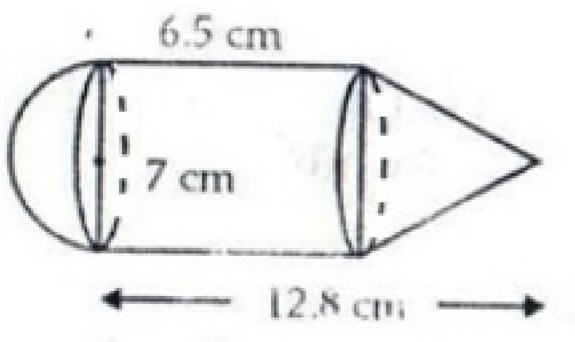
OR



Steps of construction:

1. Draw a line segment BC = 6.5 cm.
2. At B, construct $\angle$CBX = 60°.
3. With B as centre and radius 4.5 cm, draw an arc intersecting BX at A.
4. Join AC to obtain $\triangle$ABC
5. Below BC, make an acute $\angle$CBY
6. Along BY, mark off 4 points (greater of 3 and 4 in $\frac{3}{4}$)
   B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$
7. Join B_4C.
8. From point B_4, draw a line parallel to B_4C intersecting BC at C'.
9. From point C', draw a line parallel to AC intersecting AB at A'.

Thus, $\triangle$A'BC' is the required triangle.

29. 

According to the given figure,we have,

Height of cylinder= h_1 = 6.5 cm

Height of cone= h_2 =(12.8 - 6.5)cm = 6.3 cm

Radius of cylinder = radius of cone = radius of hemisphere

= $\left(\frac{7}{2}\right) \text{cm} = 3.5\ cm$

Therefore, Volume of solid = Volume of cylinder + Volume of cone + Volume of hemisphere

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3 = \pi r^2 \left(h_1 + \frac{1}{3}h_2 + \frac{2}{3}r\right)$

$= \left[\frac{22}{7} \times 3.5 \times 3.5 \times \left(6.5 + 6.3 \times \frac{1}{3} + \frac{2}{3} \times 3.5\right)\right]$

= [(38.5) $\times$(6.5 + 2.1 + 2.33)]cm^3

= (38.5 $\times$ 10.93) cm^3 = 420.80 cm^3

30. We have,

$$\text{LHS} = \frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{\left(\frac{1}{\cos^3 A}-\frac{1}{\sin^3 A}\right)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)}{\left(\frac{\sin^3 A-\cos^3 A}{\sin^3 A\cos^3 A}\right)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{1}{\sin A\cos A}\right)(\sin A-\cos A)\sin^3 A\cos^3 A}{(\sin^3 A-\cos^3 A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{(\sin A\cos A+1)(\sin A-\cos A)\sin^2 A\cos^2 A}{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}$$ [$\because$ a^3 - b^3 = (a - b) (a^2 + b^2 + ab)]

$$\Rightarrow \quad \text{LHS} = \frac{(\sin A\cos A+1)\sin^2 A\cos^2 A}{(1+\sin A\cos A)}$$ = $\sin^2$A $\cos^2$A = RHS

OR

LHS =

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$$

$$= \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A+\sin A} - \frac{\sin^2 A}{\sin A+\cos A}$$

$$= \frac{\cos^2 A-\sin^2 A}{(\sin A+\cos A)}$$

$$= \frac{(\cos A+\sin A)(\cos A-\sin A)}{\sin A+\cos A} \quad [\because (a^2-b^2) = (a+b)(a-b)]$$

= $cosA - sinA$

= RHS

Hence Proved.

31. Let $3\sqrt{7}$ be rational.

Its simplest from $3\sqrt{7} = \frac{a}{b}$, where a and b are positive integers having no common factor other than 1, then

$3\sqrt{7} = \frac{a}{b}$ ...(i)

$\sqrt{7} = \frac{a}{3b}$ ...(ii)

As a and 3b are integers .

So, $\frac{a}{3b}$ is rational number .

But $\sqrt{7}$ is not rational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $3\sqrt{7}$ is rational wrong .

Hence, $3\sqrt{7}$ is irrational.

OR

(i) The required number of minutes after which they start preparing a new card together = LCM of 10,16 and 20 minutes

Prime factorisation of 10 = 2 × 5

and prime factorisation of 16 = 2 × 2 × 2 × 2

and prime factorisation of 20 = 2 × 2 × 5

Now, LCM(10,16,20) = 2 × 2 × 2 × 2× 5 = 80

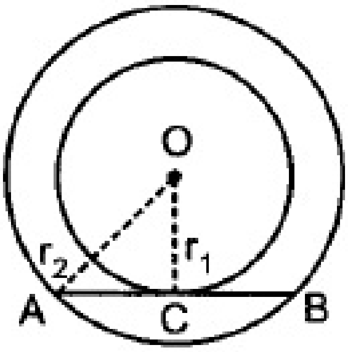
Therefore, Number of minutes after which they start preparing a new card together = 80 minutes.

(ii) Recognition and care for elders removes the loneliness due to age related diseases. Moreover they feel happy to help young minds through their experience.

32. Given,

$r_1 = 5cm,$

$AB = 24cm$



$\because$ AB is tangent to circle C(0,r_1) at C

$\therefore$ OC $\perp$ AB

In circle C(0,r_2), AB is a chord and

OC $\perp$ AB

$\therefore$ AC = BC

In right $\triangle$OCA,

$\mathrm{OC}^2 + \mathrm{AC}^2 = \mathrm{AO}^2$

$\Rightarrow\ 5^2 + (12)^2 = (r_2)^2$

$\Rightarrow\ 25 + 144 = (r_2)^2$

$\Rightarrow (r_2)^2 = 169$

r_2 = 13 cm

33. i. The positions of the students are (3, 5), B(7, 9), C(11, 5) and D(7, 1).

To find the distance between them, we use distance formula.

So,

$AB = \sqrt{(7-3)^2 + (9-5)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

$BC = \sqrt{(11-7)^2 + (5-9)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

$\mathrm{CD} = \sqrt{(7-11)^2 + (1-5)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

And $DA = \sqrt{(3-7)^2 + (5-1)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

ii. We see that, AB = BC = CD = DA i.e., all sides are equal.

Now, we find the length of both diagonals;

$AC = \sqrt{(11-3)^2 + (5-5)^2} = \sqrt{(8)^2 + 0} = 8$

and $BD = \sqrt{(7-7)^2 + (1-9)^2} = \sqrt{0 + (-8)^2} = 8$

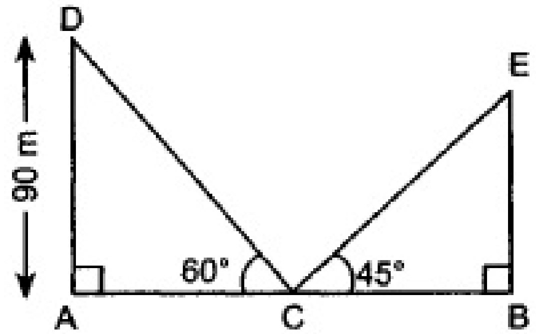
Here, AC = BD

Since AB = BC = CD = DA and AC = BD, so we can say that ABCD is a square.

Thus, it is possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D.

iii. As we also know that diagonals of a square bisect each other, so, let P be a position of Jaspal in which he is equidistant from each of the four students A, B, C and D.
Coordinates of P = Mid - point of AC
$= \left(\frac{3+11}{2}, \frac{5+5}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$
Hence, the required position of Jaspal is (7, 5).

34. Let AB is path



In rt. $\triangle$DAC, $\frac{DC}{AD} = \ cosec\, 60^{\circ}$
$\Rightarrow \quad \frac{\mathrm{DC}}{90} = \frac{2}{\sqrt{3}}$
$\mathrm{DC} = \frac{2}{\sqrt{3}} \times 90\mathrm{m} = \frac{180}{\sqrt{3}}\mathrm{m}$
Now, $DC = CE$
$\therefore \quad \mathrm{CE} = \frac{180}{\sqrt{3}}\mathrm{m}$
In rt. $\triangle$EBC,
$\frac{BE}{CE} = \sin 45^{\circ}$
$\Rightarrow \quad \mathrm{BE} = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}}\mathrm{m}$
$\Rightarrow \quad BE = 73.47\mathrm{m}$

Section D

35. Total time taken by minute hand from 2 p.m. to 3 p.m. is 60 min.

According to question,

$$t + \left(\frac{t^2}{4} - 3\right) = 60$$

$\Rightarrow 4t + t^2 - 12 = 240$

$\Rightarrow t^2 + 4t - 252 = 0$

$\Rightarrow t^2 + 18t - 14t - 252 = 0$

$\Rightarrow t(t + 18) - 14(t + 18) = 0$

$\Rightarrow (t + 18)(t - 14) = 0$

$\Rightarrow t + 18 = 0$ or $t - 14 = 0$

$\Rightarrow t = -18$ or $t = 14$ min.

As time can't be negative.

Therefore, $t = 14$ min.

36. $a = 8$, $d = 4$ months $= \frac{1}{3}$ years, $S_n = 168$

$S_n = \frac{n}{2}[2a + (n-1)d]$

or, $168 = \frac{n}{2}[2(8) + (n-1)\frac{1}{3}]$

or, $336 = n[16 + \frac{1}{3}n - \frac{1}{3}]$

or, $336 = 16n + \frac{1}{3}n^2 - \frac{1}{3}n$

or, $336 = \frac{48n+n^2-n}{3}$

or, $n^2 + 47n - 1008 = 0$

or, $n^2 + 63n - 16m - 1008 = 0$

or, $(n - 16)(n + 63) = 0$

or, $n = 16$ or $n = -63$

$n = 16$

(n cannot be negative So - 63 rejected)

Age of the eldest participant = $a + 15d = 13$ years

OR

$S_1 = \frac{n}{2}[2 \times 5 + (n-1) \times 2]$

$= \frac{n}{2}[10 + 2n - 2]$

$= \frac{n}{2}[8 + 2n] = n[4 + n] = 4n + n^2$

$S_2 = \frac{n}{2}[2 \times 5 + (n-1) \times 4]$

$= \frac{n}{2}[10 + 4n - 4]$

$= \frac{n}{2}[6 + 4n]$ = n[3 + 2n] = 3n + $2n^2$

S_3 = $\frac{n}{2}[2 \times 5 + (n - 1) \times 6]$

$= \frac{n}{2}[10 + 6n - 6]$

$= \frac{n}{2}[4 + 6n]$ = n[2 + 3n] = 2n + $3n^2$

Now,

$S_1 + S_3 = (4n + n^2) + (2n + 3n^2)$

$= 4n + n^2 + 2n + 3n^2$

$= 6n + 4n^2$

$= 2(3n + 2n^2)$

$2S_2$

37. The given equations are

3x - 5y = 20 .... (i)

And 6x - 10y = - 40 .... (ii)

Now, 3x - 5y = 20

⇒ 3x = 20 + 5y

$\Rightarrow x = \frac{5y+20}{3}$

When y = - 1 then, x = 5

When y = - 4 then, x = 0

Thus, we have the following table giving points on the line 3x - 5y = 20

x	5	0
y	-1	-4

Now, 6x - 10y = - 40

⇒ 6x = - 40 + 10y

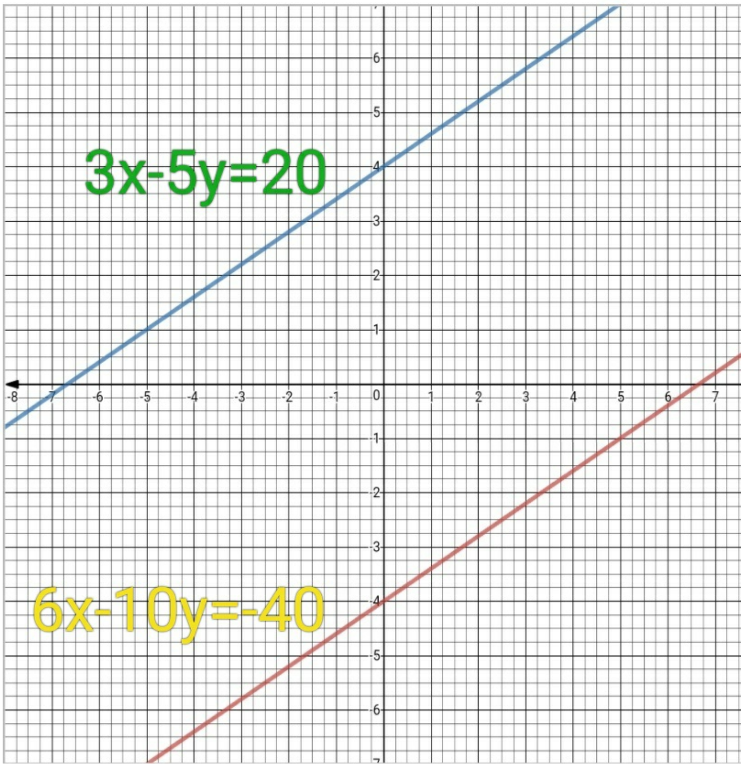
⇒ $x = \frac{10y-40}{6}$

When y = 4, then x = 0

When y = 1, then x = - 5

Thus, we have the following table giving points on the line 6x - 10y = - 40

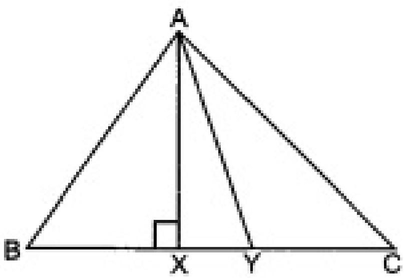
x	0	-5
y	4	1



Clearly, graph shows given pair of linear equations as parallel lines, which never intersect. Hence, there is no common point between these two lines.

Hence, given systems of equations is inconsistent i.e. no solution.

38. (i) In right $\triangle AXB$



$AB^2 = AX^2 + BX^2$ ..(1)

In right $\triangle AXY$

$AY^2 = AX^2 + XY^2$ ..(2)

subtract (2) from (1), we get

$AB^2 - AY^2 = AX^2 + BX^2 - AX^2 - XY^2$

$\Rightarrow AB^2 - AY^2 = BX^2 - XY^2$

$\Rightarrow$ AB^2 - AY^2 = (BX - XY)(BX + XY)

$\Rightarrow$ AB^2 - AY^2 = [(BY - XY) - XY](BY)

$\Rightarrow$ AB^2 - AY^2 = ($\frac{1}{2}$ BC - 2XY)$\frac{1}{2}$ BC [Y is the mid-point of BC]

$\Rightarrow$ AB^2 - AY^2 = $\frac{1}{4}$ BC^2 - BC.XY

$\Rightarrow$ AB^2 = AY^2 + $\frac{1}{4}$ BC^2 - BC.XY Hence proved

(ii) In right $\triangle$AXC

AC^2 = AX^2 + XC^2 ..(3)

In right $\triangle$AXY,

AC^2 - AY^2 = XC^2 - XY^2

(3) - (4), we get

AC^2 - AY^2 = XC^2 - XY^2

$\Rightarrow$ AC^2 - AY^2 = (XC - XY)(XC + XY)

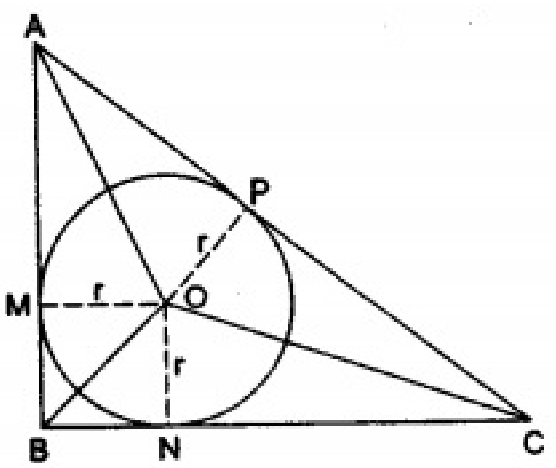
= YC(YC + XY + XY)

$\Rightarrow$ AC^2 - AY^2 = $\frac{1}{2}$ BC($\frac{1}{2}$ BC + 2XY) (Y is mid-point of BC)

$\Rightarrow$ AC^2 - AY^2 = $\frac{1}{4}$ BC^2 + BC.XY

$\Rightarrow$ AC^2 = + AY^2 + $\frac{1}{4}$ BC^2 + BC.XY Hence proved.

OR



In $\triangle ABC$, we have $\angle B = 90^\circ$ and BC = 8 cm.

A circle is inscribed in $\triangle ABC$.

Let O be its centre and M, N and P be the points where it touches the sides AB, BC and CA respectively.

Then, $OM \perp AB, ON \perp BC, OP \perp CA$

Let r cm be the radius of the circle.

Then, OM = ON = OP = r cm.

Now, $AB^2 + BC^2 = CA^2$ [by Pythagoras' theorem]

$\Rightarrow 6^2 \text{ cm}^2 + 8^2 \text{ cm}^2 = CA^2$

$\Rightarrow$ CA = 10 cm.

Now, $\operatorname{ar}(\triangle ABC) = \operatorname{ar}(\triangle AOB) + \operatorname{ar}(\triangle BOC) + \operatorname{ar}(\triangle COA)$

$\Rightarrow \quad \frac{1}{2} \times AB \times BC = \left(\frac{1}{2} \times AB \times OM\right) + \left(\frac{1}{2} \times BC \times ON\right)$

$+(\frac{1}{2} \times CA \times OP)$

$\Rightarrow \quad \frac{1}{2} \times 6 \times 8 = \left(\frac{1}{2} \times 6 \times r\right) + \left(\frac{1}{2} \times 8 \times r\right) + \left(\frac{1}{2} \times 10 \times r\right)$

$\Rightarrow$ r = 2

$\Rightarrow$ radius = 2 cm.

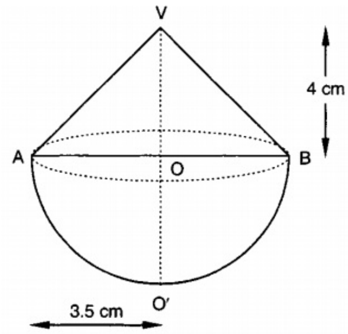
39. We have, radius of the hemisphere = 3.5 cm

Height of the cone = 4 cm

Radius of the cylinder = 5 cm

|Height of the cylinder = 10.5 cm

We have to find out the volume of water left in the cylindrical tub



$\therefore$ Volume of the solid = Volume of its conical part + Volume of its hemispherical part

$= \left\{\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 4 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3\right\} \text{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \{4 + 2 \times 3.5\} \text{cm}^3 = \left\{\frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \text{cm}^3$

Clearly, when the solid is submerged in the cylindrical tub the volume of water that flows out of the cylinder is equal to the volume of the solid.

Hence,

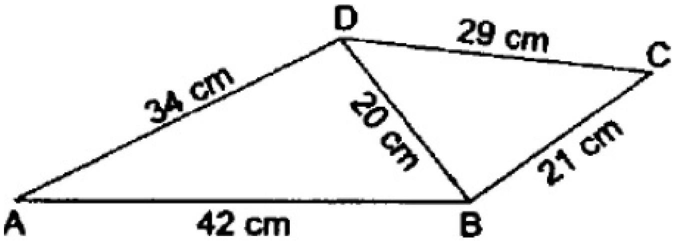
Volume of water left in the cylinder = Volume of cylinder - Volume of the solid

$= \left\{\frac{22}{7} \times (5)^2 \times 10.5 - \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \text{cm}^3$

$$= \left\{ \frac{22}{7} \times 25 \times \frac{21}{2} - \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11 \right\} \text{cm}^3$$
$$= \left(11 \times 25 \times 3 - \frac{1}{3} \times 11 \times \frac{7}{2} \times 11 \right) \text{cm}^3$$
= (825 - 141.16) cm^3 = 683.83 cm^3

OR

Area of quad. ABCD = Area of $\triangle$ABD + Area of $\triangle$DBC



For area of $\triangle$ABD

Let a = 42cm, b = 34 cm, and c = 20 cm

s = $\frac{a+b+c}{2} = \frac{(42+34+20)}{2}$ cm = 48

Then, (s - a) = 6, (s - b) = 14 and (s - c) = 28

Area of $\triangle$ABC = $\sqrt{s \times (s-a)(s-b)(s-c)}$

= $\sqrt{48 \times 6 \times 14 \times 28}$ cm^2

= 336 cm^2

For area of $\triangle$DBC

a = 29 cm, b = 21 cm, c = 20 cm

s = $\frac{a+b+c}{2} = \frac{(29+21+20)}{2}$ cm = 35 cm

Area of $\triangle$DBC = $\sqrt{s \times (s-a)(s-b)(s-c)}$ sq. units

= $\sqrt{35 \times 6 \times 14 \times 15}$ cm^2

= 210 cm^2

Area of quad. ABCD = Area of $\triangle$ABC + Area of $\triangle$DBC

= (336 + 210) cm^2 = 546 cm^2

40. **Calculation of mean:**

We may compute class marks (x_i) as per the relation

$$x_i = \frac{\text{Upper class limit} + \text{ lower class limit}}{2}$$

Now taking a = 700 as assumed mean, we may calculate d_i and f_id_i as following.

Pocket expenses (in Rs)	No. of students (f_i)	Class marks (x_i)	$d_i = x_i$ - 700	$f_i d_i$
0 - 200	33	100	-600	-19800
200 - 400	74	300	-400	-29600
400 - 600	170	500	-200	-34000
600 - 800	88	700	0	0
800 - 1000	76	900	200	15200
1000 - 1200	44	1100	400	17600
1200 - 1400	25	1300	600	15000
Total	$\sum f_i = 510$			$\sum f_i d_i$ = -35600

From the table, we may observe that,

$\sum f_i = 510, \sum f_i d_i$ = -35600, a = 700

Mean $\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 700 + \frac{-35600}{510}$

= 700 - 69.80

= 630.20

Hence mean of the given data is 630.20.

Calculation of median:

The cumulative frequency can be calculated as follows:

Pocket expenses (in Rs)	No. of students (f)	Cumulative frequency (cf)
0 - 200	33	33
200 - 400	74	33 + 74 = 107
400 - 600	170	107 + 170 = 277
600 - 800	88	277 + 88 = 365
800 - 1000	76	365 + 76 = 441
1000 - 1200	44	441 + 44 = 485

1200 - 1400	25	485 + 25 = 510

Now from table, we may observe that n = 510

Cumulative frequency (*cf*) just greater than $\left(\frac{n}{2}=255\right)$ is 277 belonging to interval 400 - 600.

So, median class = 400 - 600

Lower limit (l) of median class = 400

Class size (h) = 200

Frequency (f) of median class = 170

Cumulative frequency (cf) of class preceding median class = 107

$$\text{Median} = \mathrm{l} + \left(\frac{\frac{\mathrm{n}}{2} - \mathrm{cf}}{\mathrm{f}}\right) \times \mathrm{h} = 400 + \left(\frac{255-107}{170}\right) \times 200$$

$$= 400 + \frac{148}{170} \times 200 = 400 + 0.87 \times 200$$

= 400 + 174

= 574

So median of the given data is 574.