Chapter-(04)
Freq. domain analysis

* When any sys. is subjected to sinusoidal i/p the o/p is also sinusoidal having diff. magnitude & phase angle but same i/p freq. ω r/s.
* Freq. responce analysis emplies varying ω from $(0-\infty)$ & observing variation in magnitude & phase angle of the responce.

$$\xrightarrow[A\sin(\omega t \pm \phi_1)]{r(t)} \boxed{\begin{matrix}\text{LTI} \\ \text{System} \\ F(s)\end{matrix}} \xrightarrow[B\sin(\omega t \pm \phi_2)]{c(t)}$$

$$\boxed{\begin{aligned} B &= A\,|F(s)| \\ \phi_2 &= \pm\phi_1 + \angle F(s) \end{aligned}}$$

$$\text{T.F.} = F(s) = \frac{C(s)}{R(s)}$$

Put $s = j\omega$

$F(j\omega) =$ sinusoidal TF

$\triangleq$ sinusoidal Responce

$$F(j\omega) = |F(j\omega)|\,\angle F(j\omega) \longrightarrow \tan^{-1}\left(\frac{\text{I.P.}}{\text{r.p.}}\right)$$

$$|F(j\omega)| \downarrow \sqrt{(\text{r.p.})^2 + (\text{I.P.})^2}$$

Ex.:-
Chap.(06)
Q.(1)

$$\xrightarrow{x(t)} \boxed{\frac{1}{s+1}} \xrightarrow{y(t)}$$

$x(t) = \sin t$; find $y(t) = ?$

$$F(s) = \frac{Y(s)}{R(s)} = \frac{1}{s+1}$$

$$F(j\omega) = \frac{1}{j\omega + 1} = \frac{1+j0}{1+j\omega}$$

$$|F(j\omega)| = \frac{\sqrt{1^2+0^2}}{\sqrt{1^2+\omega^2}} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\underline{|F(j\omega)} = \frac{\tan^{-1}(0/1)}{\tan^{-1}\left(\frac{\omega}{1}\right)} = -\tan^{-1}(\omega)$$

$$F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \underline{|-\tan^{-1}(\omega)}$$

Given; $\sin t$

$x \sin\omega t$; $\omega = 1$ r/s

$$F(j\omega) = \frac{1}{\sqrt{2}} \cancel{\sin} \underline{|-45^\circ}$$

$$\boxed{y(t) = \frac{1}{\sqrt{2}} \sin(t-45^\circ)}$$

<u>Ex:-</u> Find $y(t)$ when $x(t) = 10\sin(t+30^\circ)$

$$\boxed{y(t) = \frac{10}{\sqrt{2}} \sin(t-15^\circ)}$$

<u>Q(2)</u>
$$G(s) = \frac{(s^2+9)(s+2)}{(s+3)(s+4)(s+5)}$$

<u>Sol^n→</u>
$$G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+3)(j\omega+4)(j\omega+5)} = 0$$

$\omega^2 = 9$, $\omega = 3$ r/s

* Freq. Responce plot →

(1) Polar plots →

Absolute values of $|F(j\omega)|$ Vs ω

& $\underline{|F(j\omega)}$ (degree)

(2) Bode plot →

decibal values (db) of $|F(j\omega)|$ Vs $\log\omega$

$[20\log|F(j\omega)|]$

& $\underline{|F(j\omega)}$ (degree)

Freq. responce analysis of 2nd order sys →

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$F(s) = \frac{1}{\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1}$$

Put $(s = j\omega)$

$$F(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2}+\frac{2\xi(j\omega)}{\omega_n}+1}$$

$$F(j\omega) = \frac{1}{-\frac{\omega^2}{\omega_n^2}+1+j2\xi\frac{\omega}{\omega_n}}$$

$$|F(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Its dB value;

$$-20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

Asymptotic approximation →

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

Case(1) Low freq.

$$1 \gg \left(\frac{\omega}{\omega_n}\right)^2$$

$$-20\log\sqrt{1} = 0\,\text{db}$$

Case(2) High freq.

$$\left(\frac{\omega}{\omega_n}\right)^2 \gg 1$$

$$-20\log\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2\right]^{2}}$$

$$-40\log\left(\frac{\omega}{\omega_n}\right) \quad \text{——— (i)}$$

$$-40\log\omega + 40\log\omega_n$$

$$m\ X\ +\ C$$

$$\boxed{\text{Slope}(m) = -40\frac{\text{db}}{\text{dec}}}$$

Corner freq. (ω_{cf})

$$0 = -40\log\left(\frac{\omega}{\omega_n}\right)$$

$$\log\left(\frac{\omega}{\omega_n}\right) = 0$$

$$\left(\frac{\omega}{\omega_n}\right) = \log^{-1}(0) = 1$$

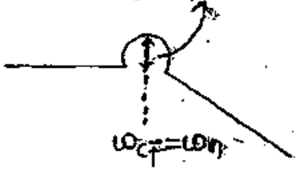
$$\omega = \omega_{cf} = \omega_n$$

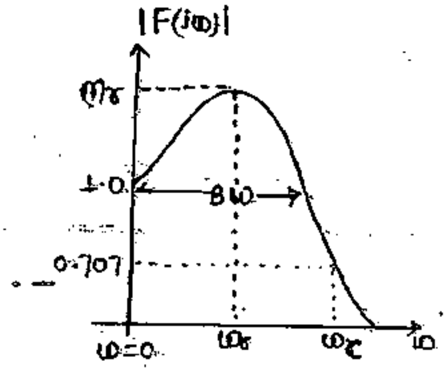
Error at ω_{cf} →

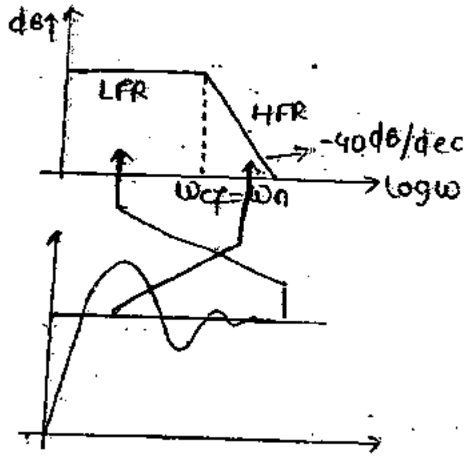
at $\omega = \omega_{cf} = \omega_n$

$$-20\log\sqrt{(1-1)^2+(2\xi)^2}$$

$$-20\log 2\xi$$







✱ Freq. domain specification →

(1.) Resonant freq (ω_r) → It is defined as the freq at which the magnitude has max^m value.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}}$$

where; $u = \frac{\omega}{\omega_n}$

at $\omega = \omega_r$ = resonant freq. ; $u = u_r = \frac{\omega_r}{\omega_n}$

$$\frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-1/2} = 0$$

$$-\frac{1}{2}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{-3/2} \frac{d}{du_r}\left[(1-u_r^2)^2 + (2\xi u_r)^2\right] = 0$$

$$2(1-u_r^2)(-2u_r) + 4\xi^2 2u_r = 0$$

$$(2-2u_r^2)(-2u_r) + 8\xi^2 u_r = 0$$

$$-4u_r + 4u_r^3 + 8\xi^2 u_r = 0$$

$$-1 + u_r^2 + 2\xi^2 = 0$$

$$u_r^2 = 1 - 2\xi^2$$

$$u_r = \sqrt{1-2\xi^2}$$

$$\boxed{\omega_r = \omega_n \sqrt{1-2\xi^2} \text{ r/s}}$$

- For 'ω_r' to be real & +ve

$$2\xi^2 < 1 \Rightarrow \boxed{\xi < \frac{1}{\sqrt{2}}}$$

It is corelerated with $\omega_d = \omega_n\sqrt{1-\xi^2}$ r/s

For 'ω_d' to be real & +ve

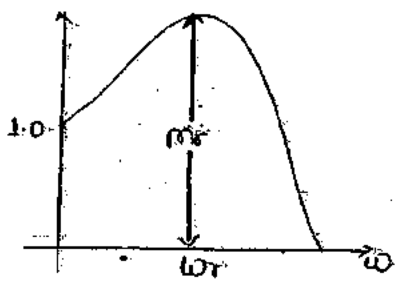
$$\xi^2 < 1 \Rightarrow \boxed{\xi < 1}$$

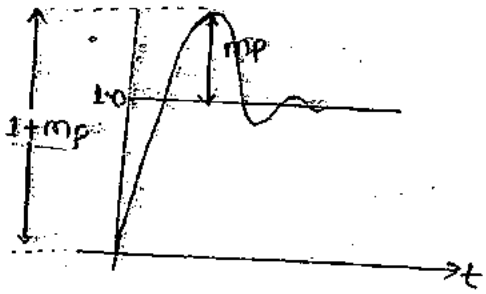
2.) Resonant peak (OR) peak magnitude (M_r) → It is the max^m value of magnitude occuring at resonant freq. ω_r.

$$|F(j\omega)|_{\omega=\omega_r} = M_r$$

$$M_r = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_n\sqrt{1-2\xi^2}}{\omega_n}\right)^2\right]^2 + \left(\frac{2\xi\,\omega_n\sqrt{1-2\xi^2}}{\omega_n}\right)^2}}$$

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$





$$\boxed{\begin{array}{l} \xi < \frac{1}{\sqrt{2}} \Rightarrow M_r > 1 \\ \xi = \frac{1}{\sqrt{2}} \Rightarrow M_r = 1 \\ \xi > \frac{1}{\sqrt{2}} \Rightarrow \text{No } M_r \end{array}}$$

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$$M(j\omega) = \frac{10}{100-\omega^2+10\sqrt{2}j\omega} = \frac{1}{1-\frac{\omega^2}{100}+\frac{10\sqrt{2}j\omega}{100}}$$

$$\frac{1}{1-\left(\frac{\omega}{10}\right)^2+j\frac{\sqrt{2}\omega}{10}} = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+2j\xi\frac{\omega}{\omega_n}}$$

$$\frac{2\xi\cancel{\omega}}{\omega_n} = \frac{\sqrt{2}\cancel{\omega}}{10} \quad ; \quad \boxed{M_r = 1}$$

(3.) Band width (BW) → It is the range of freq. over which the magnitude has a value of $1/\sqrt{2}$.

* It indicates the speed of responce of the sys.

* wider BW implies ~~faster~~ responce.

$$\boxed{\begin{array}{c} BW \propto \frac{1}{t_r} \\ \text{where; } t_r = \text{Rise time} \end{array}}$$

(4) Cut off freq: → It is the freq at which the magnitude has a value of $\frac{1}{\sqrt{2}}$

* It indicates the ability of a sys to distinguish signal from noise.

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}}$$

where; $u = \frac{\omega}{\omega_n}$

at $\omega = \omega_c$ = cut off freq;

$$u = u_c = \frac{\omega_c}{\omega_n}$$

$$\frac{1}{\sqrt{(1-u_c^2)^2+(2\xi u_c)^2}} = \frac{1}{\sqrt{2}}$$

$$(1-u_c^2)^2 + (2\xi u_c)^2 = 2$$

$$u_c^4 + 1 - 2u_c^2 + 4\xi^2 u_c^2 - 2 = 0$$

$$\Rightarrow u_c^4 + u_c^2(4\xi^2-2) - 1 = 0$$

$$\Rightarrow \frac{-(4\xi^2-2) \pm \sqrt{(4\xi^2-2)^2+4}}{2}$$

$$\Rightarrow 1-2\xi^2 \pm \frac{\sqrt{16\xi^4-16\xi^2+8}}{2}$$

$$= 1-2\xi^2 \pm \sqrt{4\xi^4-4\xi^2+2}$$

$$u_c^2 = 1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}$$

$$u_c = \sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}$$

$$\boxed{\begin{array}{l} \omega_c \text{ (or) BW} = \omega_n\sqrt{1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}}\ r/s \\ \text{linear approximation} \\ \omega_c \text{ (or) BW} = \omega_n(-1.19\xi + 1.85) \end{array}}$$

* Freq. Responce analysis of dead time (or) transportation lag →

$$F(s) = \frac{Y(s)}{X(s)} = e^{-Ts}$$

$$F(j\omega) = e^{-j\omega T} = \cos\omega T - j\sin\omega T$$

$$|F(j\omega)| = \sqrt{\cos^2\omega T + \sin^2\omega T} = 1$$

$$\angle F(j\omega) = \tan^{-1}\left(\frac{-\sin\omega T}{\cos\omega T}\right)$$

$$= \tan^{-1}(-\tan\omega T)$$

$$= -\omega T \text{ (radians)}$$

$$\boxed{e^{-j\omega T} = 1\angle -\omega T \text{ (radians)}}$$

$$\pi \text{ rad} = 180^\circ$$

$$-\omega T \text{ rad} = ?$$

$$\frac{-\omega T}{\pi} \times 180^\circ = -57.3\,\omega T \text{ (degree)}$$

$$\boxed{e^{-j\omega T} = 1\angle -57.3\,\omega T \text{ (degree)}}$$

* Stability from freq. responce plot →

$$1 + G(s)H(s) = 0$$

$$G(s)\cdot H(s) = -1$$

put $(s = j\omega)$

$$G(j\omega)\cdot H(j\omega) = -1 + j0 \quad \text{(critical point)}$$

Stability criteria →

(1) Gain cross over freq. (ω_{gc})

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1 \text{ (or) } 0\text{ db}$$

(2) Phase cross over freq. (ω_{pc})

$$\angle G(j\omega)\cdot H(j\omega)\,|_{\omega=\omega_{pc}} = -180^\circ$$

(3) Gain margin (Gm) :- It is the "allowable Gain"

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = X \quad ; \quad Gm(db) = 20\log\left(\frac{1}{X}\right)$$

$$Gm = \frac{1}{X}$$

(4.) Phase margin (PM) → It is the "Allowable phase lag"

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}} = \phi^\circ \; ; \quad PM \Rightarrow 180+\phi^\circ$$

$$\boxed{\begin{array}{l} \text{Stable} \Rightarrow GM \;\&\; PM = +ve \Rightarrow \omega_{gc} < \omega_{pc} \\ \text{Unstable} \Rightarrow GM \;\&\; PM = -ve \Rightarrow \omega_{gc} > \omega_{pc} \\ \text{marignaly stable} \Rightarrow GM = PM = 0 ; \; \omega_{gc} = \omega_{pc} \end{array}}$$

* GM & PM for 2nd order sys →

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega+2\xi\omega_n)} = \frac{\omega_n^2+j0}{(0+j\omega)(j\omega+2\xi\omega_n)}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{\omega^2+4\xi^2\omega_n^2}}$$

$$\angle G(j\omega) = \frac{0^\circ}{(90^\circ)\left(\tan^{-1}\frac{\omega}{2\xi\omega_n}\right)}$$

$$= -90^\circ - \tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)$$

$$G(j\omega) = \frac{\omega_n^2+j0}{-\omega^2+2\xi\omega\cdot\omega_n j}$$

$$\angle G(j\omega) = \frac{0^\circ}{180-\tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)}$$

$$= -180^\circ + \tan^{-1}\left(\frac{2\xi\omega_n}{\omega}\right)$$

at $\omega = \omega_{pc} = \infty$ r/s

$$\angle G(j\omega) = -180^\circ$$

$$|G(j\omega)|_{\omega=\omega_{pc}=\infty} = X$$

$$X = 0 \Rightarrow GM = \frac{1}{0} = \infty$$

$$\boxed{\begin{array}{l} \omega_{pc} = \infty \\ GM = \infty \end{array}}$$

at $\omega=\omega_{gc}$

$$\frac{\omega_n^2}{\omega\sqrt{\omega^2+4\xi^2\omega_n^2}}=1$$

$$\omega_n^4=\omega^2(\omega^2+4\xi^2\omega_n^2)$$

$$\omega^4+\omega^2 4\xi^2\omega_n-\omega_n^4=0$$

$$\omega^2=\frac{-4\xi^2\omega_n^2\pm\sqrt{16\xi^4\omega_n^4+4\omega_n^4}}{2}$$

$$=-2\xi^2\omega_n^2\pm\omega_n^2\sqrt{4\xi^4+1}$$

$$\omega^2=-2\xi^2\omega_n^2+\omega_n^2\sqrt{4\xi^4+1}$$

$$\boxed{\omega=\omega_{gc}=\omega_n\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}\ \text{r/s}}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}}=\phi=-90^\circ-\tan^{-1}\left(\frac{\omega_n\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}{2\xi\omega_n}\right)$$

$$pm=180^\circ+\phi$$

$$\boxed{pm=90^\circ-\tan^{-1}\left[\frac{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}{2\xi}\right]}$$

$$\angle G(j\omega)\Big|_{\omega=\omega_{gc}}=\phi=-180^\circ+\tan^{-1}\left[\frac{2\xi\omega_n}{\omega_n\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]$$

$$pm=180^\circ+\phi$$

$$\boxed{pm=\tan^{-1}\left[\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{4\xi^4+1}}}\right]}$$

linear approximation
$pm=100\xi$

(3/70) $\%mp = 50\%$

$mp = 0.5$

$e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.5$

$\xi = 0.215$

Period of oscillations $= 0.2s$.

$T = \frac{1}{fd} = 0.2s$

$fd = 5Hz$

$\omega_d = 2\pi fd$

$\omega_d = 2\pi \times 5 = 31.41\ r/s$

$\omega_n\sqrt{1-\xi^2} = 31.41$

$\omega_n\sqrt{1-(0.215)^2} = 31.41$

$\omega_n = 32.16\ r/s$

$\omega_r = \omega_n\sqrt{1-2\xi^2}$

$\omega_r = 32.16\sqrt{1-2(0.215)^2}$

$\boxed{\omega_r = 30.63}\ r/s.$

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$1 + \frac{k}{s(s+a)} = 0$

$s^2 + as + k = 0$

$k = \omega_n^2$

$a = 2\xi\omega_n$

given; $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04$

$\xi = 0.6\ ;\ \cancel{0.8}$

$\omega_r = \omega_n\sqrt{1-2(0.6)^2} = 11.55$

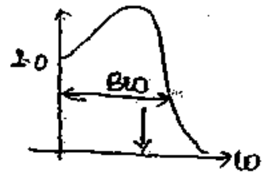
$\omega_n = 22.2\ r/s$

$k = (22.2)^2 = 492.84$

$a = 2 \times 0.6 \times 22.2 = 26.4$

(4/70) (c) (11/71) (a)

(4/70)



Sharp cut off c/s $\Rightarrow$ less BW $\Rightarrow$ more t_r
less stable

BW$\downarrow$, $M_r\uparrow$, stability$\downarrow$, $t_r\uparrow$

(5/70) $1 + \frac{100}{s(s+10)} = 0$

$s^2 + 10s + 100 = 0$

$\omega_n = 10\ r/s$

$2\xi \times 10 = 10$

$\xi = 0.5$

$\omega_r = 10\sqrt{10 - 2\times0.5^2} = 7.07\ r/s$

$BW = \omega_n(-1.19\xi + 1.85)$

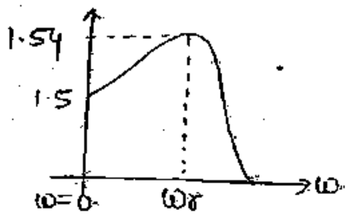
$= 10(-1.19\xi \times 0.5 + 1.85)$

$\boxed{BW = 12.7\ r/s}$

Que → The TF of 2nd order sys. is

$$\frac{k\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Its freq. responce is shown in fig.



The value of 'k' is (a) 1 (b) 1.54 (c) 1.5 (d) 1.04

Soln →

$$TF = \frac{k\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{k}{\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1}$$

(put $s=j\omega$);

$$TF = \frac{k}{1-\left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

$$= \frac{k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\boxed{\begin{array}{l} at\ \omega=0; \\ |F(j\omega)| = k = 1.5 \end{array}}$$

(6/70)

$$G(s) = \frac{2\sqrt{3}}{s(s+1)}$$

$$1+\frac{2\sqrt{3}}{s(s+1)}=0$$

$$s^2+s+3.46=0$$

$$\omega_n = \sqrt{3.46} = 1.86\ r/s$$

$$2\xi\times 1.86 = 1$$

$$\xi = 0.27$$

$$PM = 100\xi = 100\times 0.27 = 27^\circ \simeq 30^\circ$$

(7/70)

$$G(s) = \frac{as+1}{s^2}$$

$$1+\frac{as+1}{s^2}=0$$

$$s^2+as+1=0$$

$$\omega_n = 1\ r/s$$

$$2\xi\times 1 = a$$

Given; $PM = 45^\circ$, $\xi = \frac{PM}{100} = \frac{45}{100} = 0.45$

$$\therefore a = 2\times 0.45 = 0.9$$

ans(c)

(8/70)

$$G(s) = \frac{1}{s(s^2+s+1)}$$

$$G(j\omega) = \frac{1}{j\omega((j\omega)^2+j\omega+1)}$$

$$G(j\omega) = \frac{1}{j\omega(1-\omega^2+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

at $\omega = \omega_{pc}$

$$-90 - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -180^\circ$$

$$\therefore -\tan^{-1}\left(\frac{\omega}{1-\omega^2}\right) = -90^\circ$$

$$1-\omega^2 = 0$$

$$\omega = \omega_{pc} = 1\, r/s$$

$$|G(j\omega)|_{\omega=\omega_{pc}=1} = X$$

$$X = \frac{1}{1\sqrt{(1-1)^2+1^2}} = 1$$

$$Gm = \frac{1}{X} = \frac{1}{1} = 1$$

$$Gm(db) = 20\log 1 = 0db$$

$$\boxed{Gm(db) = 0db}$$

(9/71)

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3\times 1}{\omega\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = \frac{0^\circ(-57.3^\circ\times 2\omega)}{(90^\circ)(\tan^{-1}0.5\omega)}$$

$$= (-90^\circ) - 114.6\omega - \tan^{-1}(0.5\omega)$$

at $\omega = \omega_{gc}$

$$\frac{3}{\omega\sqrt{\omega^2+4}} = 1$$

$$9 = \omega^2(\omega^2+4)$$

$$\omega^4 + 4\omega^2 - 9 = 0$$

$$\omega^2 = \frac{-4 \pm \sqrt{16+36}}{2}$$

$$\omega^2 = -2 \pm 3.6$$

$$\omega^2 = 1.6, -5.6$$

$$\omega^2 = 1.6$$

$$\omega_{gc} = \sqrt{1.6} = 1.26\, r/s.$$

$$\angle G(j\omega)|_{\omega=\omega_{gc}=1.26\, r/s} = \phi$$

$$\phi = -90^\circ - 114.6\times 1.26 - \tan^{-1}0.5\times 1.26$$

$$\phi = -267.5^\circ$$

$$Pm = 180^\circ - 267.5^\circ$$

$$Pm = -87.5^\circ$$

By observation
$\omega_{pc} < \omega_{gc}$
$Gm = Pm = -ve$

12/71

$$G(s) = \frac{s+5}{s^2+4s+9}$$

$$G(j\omega) = \frac{j\omega+5}{9-\omega^2+4j\omega}$$

$$\angle G(j\omega)\Big|_{\omega<3} = \frac{\tan^{-1}(\omega/5)}{\tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)} = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$$\angle G(j\omega)\Big|_{\omega>3} = \frac{\tan^{-1}\omega/5}{180^\circ - \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)} = \tan^{-1}\left(\frac{\omega}{5}\right) - 180^\circ + \tan^{-1}\left(\frac{4\omega}{9-\omega^2}\right)$$

$\omega = 0$, $\angle G(j\omega) = 0^\circ$

$\omega = \infty$, $\angle G(j\omega) = 90^\circ - 180^\circ + 0^\circ = -90^\circ$

$$\boxed{0^\circ \longrightarrow 90^\circ}$$

16/72

correction

$\cancel{X}G \to G$ ans (a)

20/73

$$G(s) = \frac{e^{-Ts}}{s(s+1)}$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)}$$

$$\angle G(j\omega) = \frac{-\omega T}{\left(\frac{\pi}{2}\right)\left(\tan^{-1}\omega\right)}$$

$$= -\frac{\pi}{2} - \omega T - \tan^{-1}\omega$$

At $\omega = \omega_1$

$\angle G(j\omega) = 0$

$-\frac{\pi}{2} - \omega_1 T - \tan^{-1}\omega_1 = 0$

$-\tan^{-1}\omega_1 = \frac{\pi}{2} + \omega_1 T$

$-\omega_1 = \tan\left(\frac{\pi}{2} + \omega_1 T\right)$

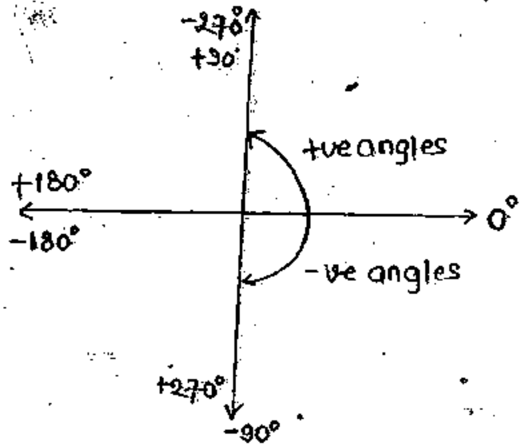
$-\omega_1 = -\cot(\omega_1 T)$

$$\boxed{\omega_1 = \cot(\omega_1 T)}$$

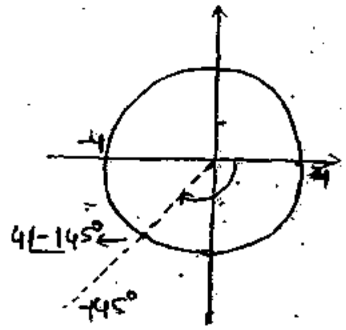
POLAR PLOT

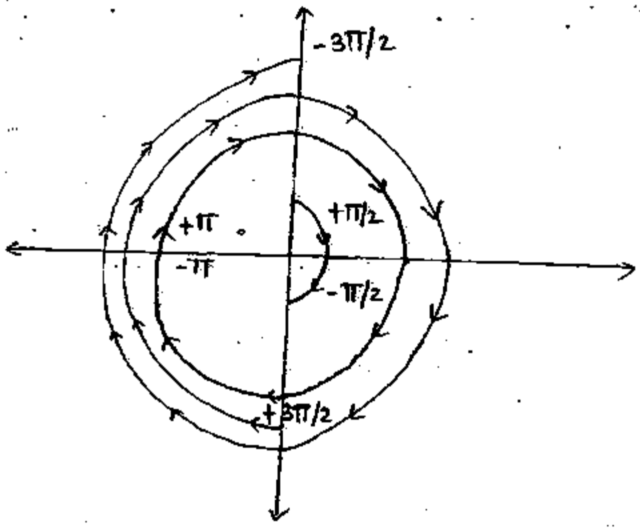
* It is a plot of absolute values of magnitude & phase angle in degrees of open loop TF $G(j\omega).H(j\omega)$ vs ω drawn on polar co-ordinates.

Polar co-ordinates :-



Eg:- $4\angle -45°$

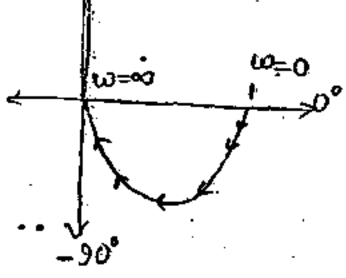


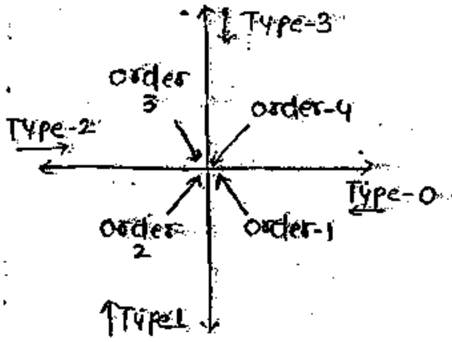


$$G(s) = \frac{1}{(s+1)} = \frac{1}{(1+s)} = \frac{1}{(1+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}} \; ; \; \angle G(j\omega) = -\tan^{-1}\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	1	0
$\angle G(j\omega)$	0°	-90°



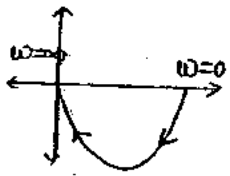


* General shapes of polar plot →

Type/order	Polar plot

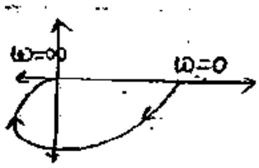
(1) Type-0/ order-1

$$G(s) = \frac{1}{(1+Ts)}$$



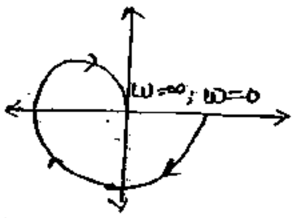
(2) Type-0/ order-2

$$G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)}$$



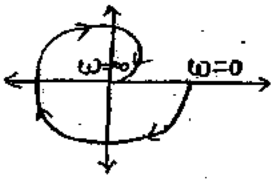
(3) Type-0/order-3

$$G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$$



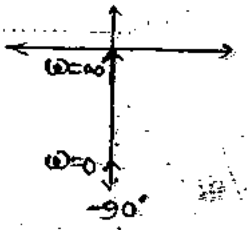
(4) Type-0/order-4

$$G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)}$$



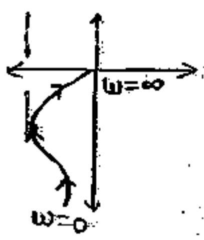
5) Type-1/ order-1

$$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega}\angle -90^\circ$$



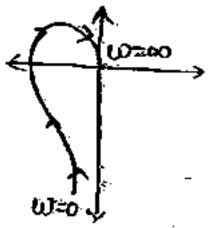
6) Type-1/ order-2

$$G(s) = \frac{1}{s(1+Ts)}$$



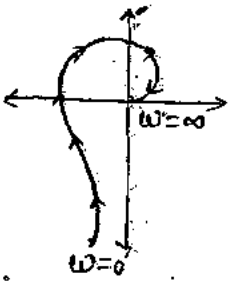
(7.) Type-1 / order-3

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$$



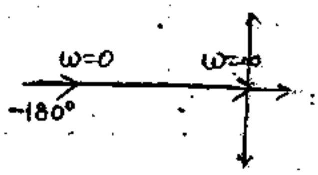
(8.) Type-1 / order-4

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)(1+T_3s)}$$



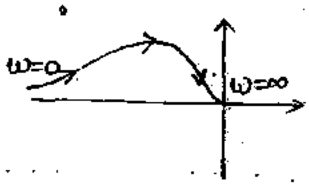
(9.) Type-2 / order-2

$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2}\angle -180^\circ$$



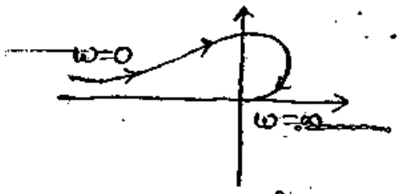
(10.) Type-2 / order-3

$$G(s) = \frac{1}{s^2(1+Ts)}$$



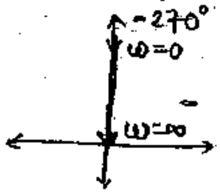
(11.) Type-2 / order-4

$$G(s) = \frac{1}{s^2(1+T_1s)(1+T_2s)}$$



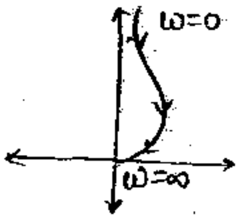
(12.) Type-3 / order-3

$$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3}\angle -270^\circ$$



(13.) Type-3 / order-4

$$G(s) = \frac{1}{s^3(1+Ts)}$$



*** Effect of adding zeros on the of the polar plot →** for min^m phase or Non-min^m phase f^n when zeros are added it is necessary to check the intersection of polar plot with -ve real axis b/w $\omega=0$ & $\omega=\infty$.

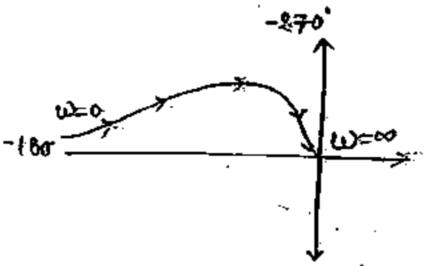
(4/75)

$$G(s) = \frac{1}{s^2(1+s)}$$

$\angle G(j\omega) = -180^\circ - \tan^{-1}(\omega)$

$\omega=0$, $|G(j\omega)|=\infty$; $\angle G(j\omega) = -180^\circ$

$\omega=\infty$, $|G(j\omega)|=0$; $\angle G(j\omega) = -270^\circ$

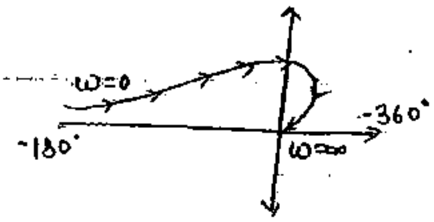


$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega$

$\omega=0$; $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega=\infty$; $|G(j\omega)| = 0$; $\angle G(j\omega) = -360^\circ$



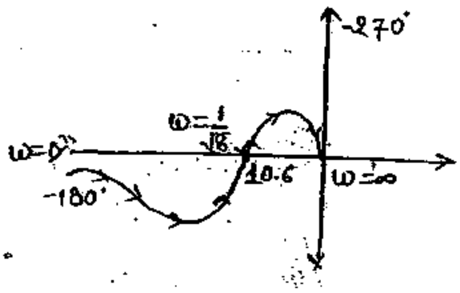
$$G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$

$\angle G(j\omega) = -180 - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega$

$\omega=0$, $|G(j\omega)| = \infty$; $\angle G(j\omega) = -180^\circ$

$\omega=\infty$, $|G(j\omega)| = 0$; $\angle G(j\omega) = -270^\circ$

Because of present of one zero, the polar plot will cut the -ve real axis at some value. So for finding that value,



$-180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega = -180^\circ$

$\tan^{-1}4\omega = \tan^{-1}\omega + \tan^{-1}2\omega$

$$4\cancel{\omega} = \frac{\cancel{\omega}+2\cancel{\omega}}{1-2\omega^2}$$

$4 - 8\omega^2 = 3$;

$\omega = \omega_{pc} = \frac{1}{\sqrt{8}}$ r/s

$|G(j\omega)|_{\omega=\omega_{pc}=\frac{1}{\sqrt{8}}} = X$

$$X = \frac{\sqrt{1+\left(\frac{4}{\sqrt{8}}\right)^2}}{\sqrt{1+\left(\frac{1}{\sqrt{8}}\right)^2}\sqrt{1+\left(\frac{2}{\sqrt{8}}\right)^2}} = 10.6$$

Conv-(4)
75

$$G(s) = \frac{s+2}{(s+1)(s-1)}$$

$$= \frac{2(1+0.5s)}{1(1+s)(-1)(1-s)}$$

$$G(s) = \frac{-2(1+0.5s)}{(1+s)(1-s)}$$

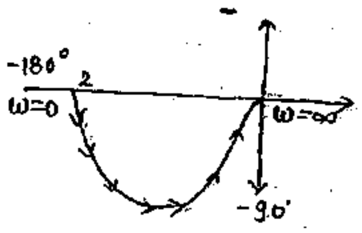
$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(1+j\omega)(1-j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{(1+\omega^2)}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(\tan^{-1}0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}\omega)}$$

$$= -180^\circ + \tan^{-1}0.5\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	2	0
$\angle G(j\omega)$	-180°	-90°



Note:- -ve gain $(-K+j0)$ contributes -180° for all ω.

$$G(s) = \frac{(s+2)}{(s+1)(s-4)} = \frac{-0.5(1+0.5s)}{(1+s)(1-0.25s)}$$

$$G(j\omega) = \frac{-0.5(1+0.5j\omega)}{(1+j\omega)(1-0.25j\omega)}$$

$$|G(j\omega)| = \frac{0.5\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}\sqrt{1+(0.25\omega)^2}}$$

$$\angle G(j\omega) = \frac{(-180^\circ)(+\tan^{-1}0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}0.25\omega)}$$

$$-180^\circ + \tan^{-1}0.5\omega + \tan^{-1}0.25\omega - \tan^{-1}\omega$$

ω	0	$\sqrt{2}$	∞
$\lvert G(j\omega)\rvert$	0.5	0.33	0
$\angle G(j\omega)$	-180°	-180°	-90°

$$-180^\circ + \tan^{-1}0.5\omega + \tan^{-1}0.25\omega - \tan^{-1}\omega = -180^\circ$$

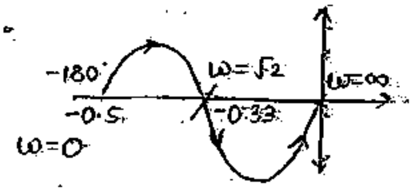
$$\tan^{-1}\omega = \tan^{-1}0.5\omega + \tan^{-1}0.25\omega$$

$$\omega = \frac{0.5\omega + 0.25\omega}{1 - 0.125\omega}$$

$$1 - 0.125\omega^2 = 0.75$$

$$\omega = \omega_{pc} = \sqrt{2}\ r/s$$

$$X = \frac{0.5\sqrt{1+(0.5\times\sqrt{2})^2}}{\sqrt{1+(\sqrt{2})^2}\ \sqrt{1+(0.25\times\sqrt{2})^2}} = 0.33$$



* find GM & PM from polar plots →

$GM = \frac{1}{X}$; $GM(db) = 20\log(\frac{1}{X}) = +ve$

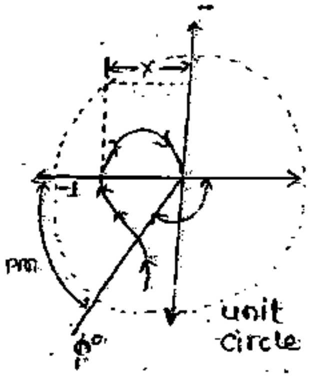
$PM = \phi - (-180°) = 180° + \phi = +ve$

$X = 1$; $GM = \frac{1}{X} = \frac{1}{1} = 1$, $GM(db) = 20\log 1 = 0\ db$

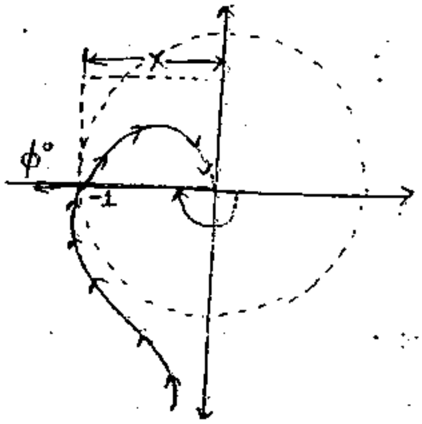
$\phi = -180°$; $PM = 180° - 180° = 0$

$GM = \frac{1}{X} = GM(db) = 20\log(\frac{1}{X}) = -ve$

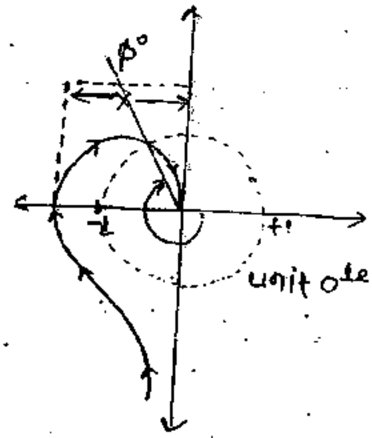
$PM = 180° + \phi = -ve$



stable

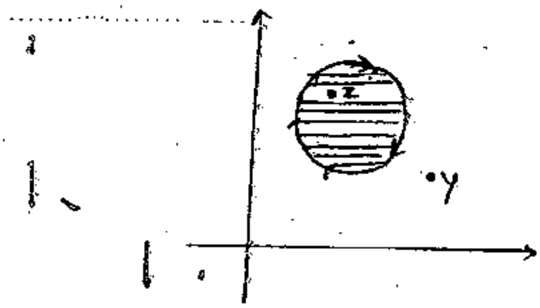


marginaly stable

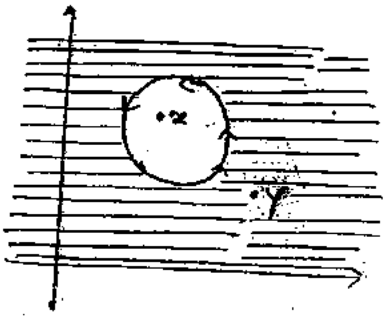


Unstable

Concept of enclosement & encirculement →



Fig(1)



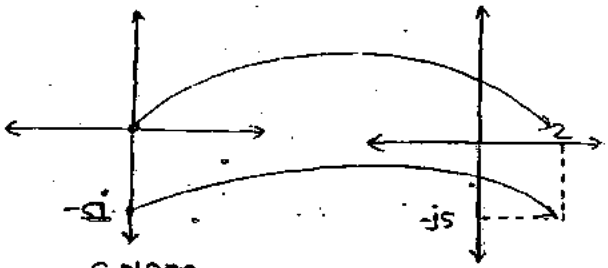
Fig(2)

* A point is said to be enclosed by a contour if it lies to the Right side of the dirn of contour.
* A point is said to be encircule if the contour is a closed path.
* In fig (2.1) point y is said to be enclosed where as point x is said to be - only encircule in Acw dirn
* In polar plots if the critical point $-1+j0$ is not enclosed than the sys. is said to be stable.

* Theory of Nyquist plots →

* The mapping theorem states that every point of s-plane will get mapped onto a corresponding point in p(s) plane where p(s) is any fn of s.

Principle of mapping →

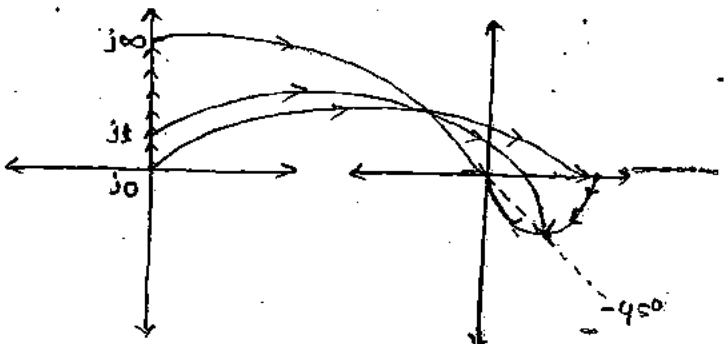


$P(S) = S+2$

$P(0) = 0+2 = 2$

$P(-jS) = 2-5j$

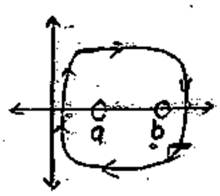
Eg:- Polar plot.



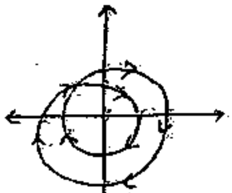
$P(S) = G(S)\cdot H(S) = \frac{1}{S+1} = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$

Principle of argument →

Case (i) →



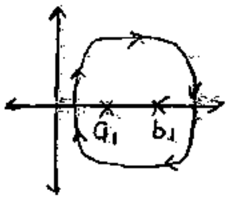
s-plane



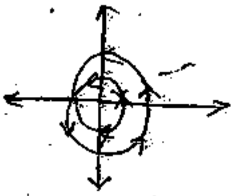
$P(s) = (s-a)(s-b)$

PA → pole ACW

Case (ii) →

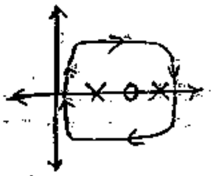


s-plane

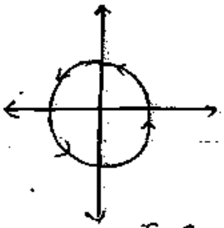


$P(s) = \frac{1}{(s-a_1)(s-b_1)}$

Case (iii) →



s-plane



$P(s) = \frac{s-c}{(s-a)(s-b)}$

$$\boxed{N = P - Z}$$

where; N = no. of encirclement
= +ve (ACW dirⁿ)
= −ve (CW dirⁿ)
P = No. of poles in RHS of s-plane
Z = No. of zeros in RHS of s-plane.

* The principle of argument may be stated as if the s-plane closed contour encloses p poles & z zeros ($p > z$) in RHS of s-plane, then the origin of P(s) plane in encircled $(P-Z)$ times in ACW dirⁿ.

* Nyquist stability criteria →

$$G(s)H(s) = \frac{K(s \pm z_i)}{s(s \pm p_i)}$$

$1 + G(s)H(s)$

$$1 + \frac{K(s \pm z_i)}{s(s \pm P_i)}$$

$$\frac{s(s \pm P_i) + K(s + z_i) \longrightarrow \text{CL poles}}{s(s \pm P_i) \longrightarrow \text{OL poles}} \quad \text{------ (i)}$$

UC → Upper closed loop pole

~~LO~~ → Lower open loop zeros

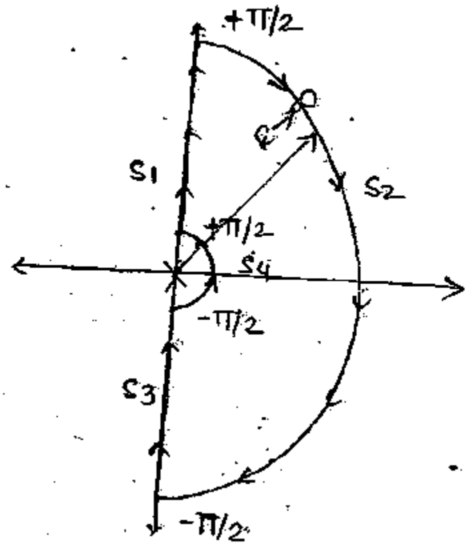
Applying $N = P - Z$ to eqn (i)

P = No. of OL poles in RHS of s-plane

Z = No. of CL poles in RHS of s-plane

(i) $Z = 0$, $N = P$

Nyquist Path →



To map S_1 ⇒ Polar plot

To map S_2 ⇒ put $s = \lim_{R \to \infty} Re^{j\theta}$

$\left(\theta = +\frac{\pi}{2} \to -\frac{\pi}{2}\right)$

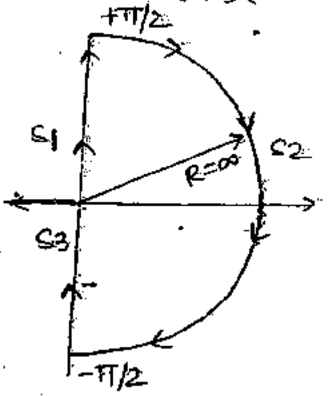
To map S_3 ⇒ Inverse polar plot

To map S_4 ⇒ put $s = \lim_{r \to 0} re^{j\theta}$

$\left(\theta = -\frac{\pi}{2} \to +\frac{\pi}{2}\right)$

Eg:- $G(s) = \frac{10}{(s+2)(s+4)}$

P = RHS pole
N = Encirclement

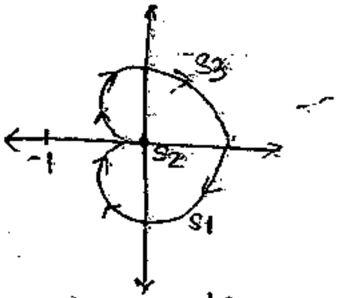


S-plane

$N = P - Z$

$0 = 0 - Z$

$\boxed{Z=0}$ Stable



$G(s) = \frac{10}{(s+2)(s+4)}$

To map $S_2 \rightarrow$

$$G(s) = \frac{10}{s^2} = \frac{10}{\lim\limits_{R\to\infty}(Re^{j\theta})^2} = \frac{10}{\lim\limits_{R\to\infty} R^2 e^{j2\theta}} = 0e^{2j\theta}$$

Que:- w/o constructing Nquist plot find the No of encirclement about $-1+j0$?

Sol$^n \rightarrow$

$$1 + \frac{10}{(s+2)(s+4)} = 0$$

$$s^2 + 6s + 18 = 0$$

s^2	1	18
s	6	0
s^0	18	0

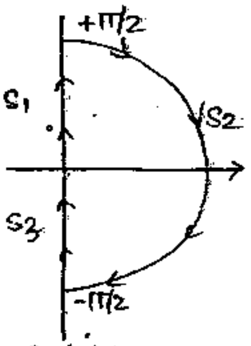
$N = P - Z$
$\downarrow \quad \downarrow$
$0 \quad 0$

$\boxed{N=0}$

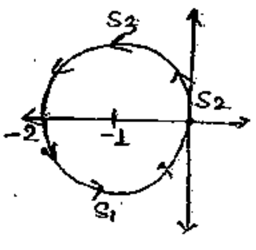
Z = sign change
P = open loop pole

Que:- $G(s) = \frac{(s+2)}{(s+1)(s-1)}$

Sol$^n \rightarrow$ for also this que section will 3, because of No. poles in origin.



S-plane



$G(s) = \frac{s+2}{(s+1)(s-1)}$

$N = P - Z$

$1 = 1 - Z$

$\boxed{Z=0}$

$$1+\frac{s+2}{s^2-1}=0$$

$$s^2-1+s+2=0$$

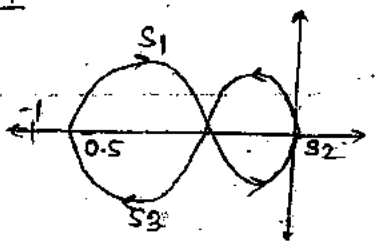
$$s^2+s+1=0$$

s^2	1	1
s^1	1	0
s^0	1	0

$$N=P-Z$$
$$\downarrow \quad \downarrow$$
$$1 \quad 0$$
$$\boxed{N=1}$$

Q→ $G(s)=\frac{(s+2)}{(s+1)(s+4)}$

Sol^n→



$$N=P-Z$$
$$0=1-Z$$
$$\boxed{Z=1} \quad \text{Unstable}$$

$$1+\frac{s+2}{(s+1)(s-4)}=$$

$$s^2+4s+s-4+s+2=0$$

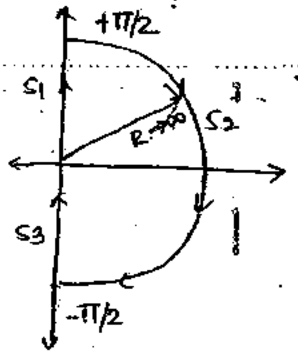
$$s^2-2s-2=0$$

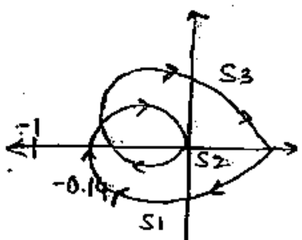
s^2	1	-2
s^1	-2	0
s^0	-2	0

$$N=P-Z$$
$$\downarrow \quad \downarrow$$
$$N=1-1$$
$$\boxed{N=0}$$

Q→ $G(s)=\frac{100}{(s+2)(s+4)(s+8)}$

Sol^n→





$$G(s)=\frac{100}{(s+2)(s+4)(s+8)}$$

$$N=P-Z$$
$$\downarrow \quad \downarrow$$
$$0 \quad 0-Z$$
$$\boxed{Z=0} \quad \text{Stable}$$

shortcut method →

$$\frac{100}{\cancel{(-\omega^2+6)j}} \qquad \frac{100}{(-\omega^2+6j\omega+8)(j\omega+8)}$$

$$\frac{100}{-j\omega^3-8\omega^2-6\omega^2+48j\omega+8j\omega+64}$$

$$\frac{100}{64-14\omega^2+j(56\omega-\omega^3)}$$

$$56\omega-\omega^3=0$$

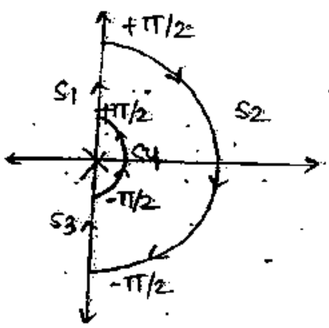
$$\omega^2=56$$

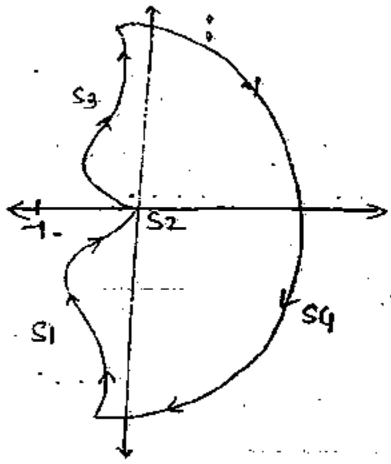
$$\omega=\omega_{pc}=\sqrt{56}\ r/s$$

$$X=\frac{100}{64-14(7.4)^2}=-0.14$$

Que → $G(s)=\frac{10}{s(1+5s)}$

Solⁿ →





$$G(s)=\frac{10}{s(1+5s)}$$

$N = P - Z$

$0 = 0 - Z$

$\boxed{Z=0}$ Stable

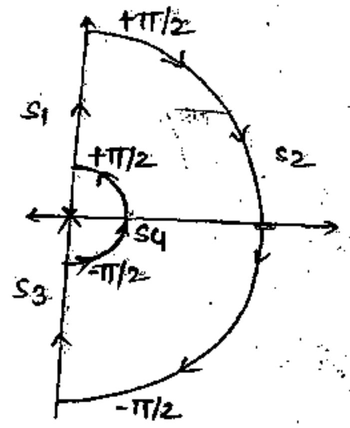
To map S4 →

$$G(s)=\frac{10}{s(1)}=\frac{10}{\lim\limits_{r\to 0} re^{j\theta}}=\infty e^{-j\theta}\ \left(\theta=-\frac{\pi}{2}\to+\frac{\pi}{2}\right)$$

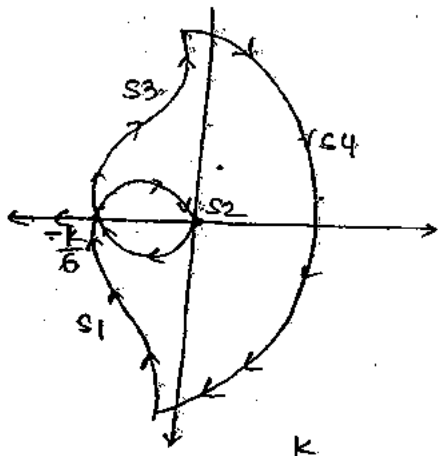
$$=\infty e^{j\pi/2}\to\infty e^{-j\pi/2}$$

Que:- $G(s) = \frac{K}{s(s+1)(s+2)}$

Sol^n →



s-plane



$G(s) = \frac{K}{s(s+1)(s+2)}$

∴ To map S_4 →

$$G(s) = \frac{K}{s(1)(2)} = \frac{0.5K}{\lim_{r \to 0} re^{j\theta}} = \infty e^{-j\theta} \quad \left(\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2}\right)$$

$$= \infty e^{j\pi/2} \longrightarrow \infty e^{-j\pi/2}$$

$$\frac{K}{j\omega(-\omega^2+3j\omega+2)}$$

$$\frac{K}{-3\omega^2 + j(2\omega-\omega^3)}$$

$2\omega - \omega^3 = 0$

$\omega = \omega_{pc} = \sqrt{2}$ r/s

$$\frac{K}{-3(\sqrt{2})^2} = \frac{-K}{6}$$

$$\boxed{\begin{aligned} X &= \frac{K}{6} \\ GM &= \frac{1}{X} = \frac{6}{K} \\ GM &\propto \frac{1}{K} \end{aligned}}$$

$\frac{-K}{6} = -1$; $K_{mar} = 6$

Case(1) $K > 6$

$-2 = 0 - Z$

$Z = 2$

Unstable

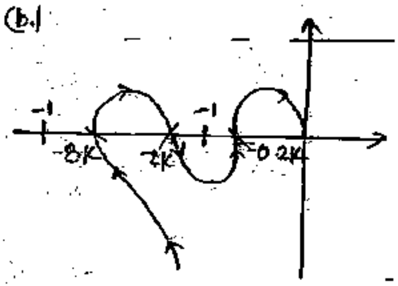
Case(2) $K < 6$

$0 = 0 - Z$

$Z = 0$

stable

29/74 (b)



(1) $-0.2K = -1$

$K_{max} = 5$

$K < 5 \rightarrow$ stable

(2) $-2K = -1$

$K_{max} = \frac{1}{2}$

$K > \frac{1}{2} \rightarrow$ stable

(3) $-8K = -1$

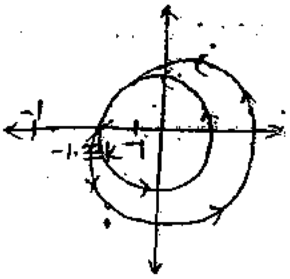
$K_{max} = \frac{1}{8}$

$K < \frac{1}{8} \rightarrow$ stab

$$\boxed{\frac{1}{2} < K < 5 \ \& \ K < \frac{1}{8}}$$

19/73

$$G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$$



$-1.33K = -1$

$K_{max} = \frac{1}{1.33} = (0.75)$

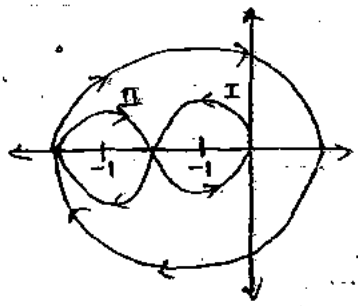
(i) $K < \frac{1}{1.33}$ $0 = 2 - Z$

$Z = 2$ (Unstable)

(ii) $K > \frac{1}{1.33}$

$2 = 2 - Z$; $Z = 0$ (stable)

18/72



given $P = 0$

Region (I)

$1 - 1 = 0 - Z$

$Z = 0$

Stable

Region (II)

$-2 = 0 - Z$

$Z = 2$

unstable

15/71

$$G(s) = \frac{1}{S(1+ST_1)(1+ST_2)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$= \cancel{\frac{1}{-\omega^2(\;\;}}$$

$$= \frac{1}{j\omega\left[1+j\omega(T_1-T_2)-\omega^2 T_1T_2\right]}$$

$$= \frac{1}{-\omega^2(T_1+T_2)+j(\omega-\omega^3T_1T_2)}$$

$$\omega-\omega^3T_1T_2=0$$

$$1-\omega^2T_1T_2=0$$

$$\boxed{\omega=\omega_{pc}=\frac{1}{\sqrt{T_1T_2}}\ r/s}$$

$$\frac{1}{-\left(\frac{1}{\sqrt{T_1T_2}}\right)^2(T_1+T_2)}$$

$$= \frac{-T_1T_2}{T_1+T_2}$$

$$X = \frac{T_1T_2}{T_1+T_2}$$

$$\boxed{G_m = \frac{T_1+T_2}{T_1T_2}}$$

$\frac{14}{71}$ (a)

$\frac{21}{73}$ If the sys is unstable then its pole will be RHS & its magnitude is not affected but the phase will be affected.

(A) $K=K_1$ $K\downarrow \Rightarrow \xi\uparrow,\ M_r\downarrow \longrightarrow 4$

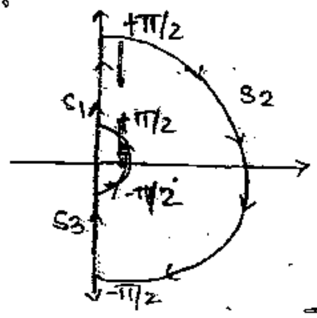
(B) $K=K_2$ $K\uparrow \Rightarrow \xi\downarrow,\ M_r\uparrow \longrightarrow 3$ ans(a)
$K_2>K_1$

(C) marginaly stable
(Responce is oscillatory $\Rightarrow \xi=0,\ M_r=\infty$) $\longrightarrow 2$

(d) Unstable sys
Phase c/s will not ~~tend to~~ tend to -270° $\longrightarrow 1$

Que $\rightarrow$ $\frac{10}{S(S-5)}$

Soln $\rightarrow$



To map S_1
Polar plot $\rightarrow$

$$G(s) = \frac{-2}{S(1-0.2S)}$$

$$G(j\omega) = \frac{-2}{j\omega(1-0.2j\omega)}$$

$$|G(j\omega)| = \frac{2}{\omega\sqrt{1+(0.2\omega)^2}}$$

$$\angle G(j\omega) = \frac{-180^\circ}{90(-\tan^{-1}0.2\omega)} = -180^\circ - 90 + \tan^{-1}0.2\omega = -270^\circ + \tan^{-1}0.2\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-270°	-180

To map $S_4 \rightarrow$

$$G(s) = \frac{10}{s(-5)} = \frac{2}{s(-1)}$$

$$= \frac{-2}{\lim_{r\to 0} re^{j\theta}e^{j\pi}} \quad (\because -1 = e^{j\pi})$$

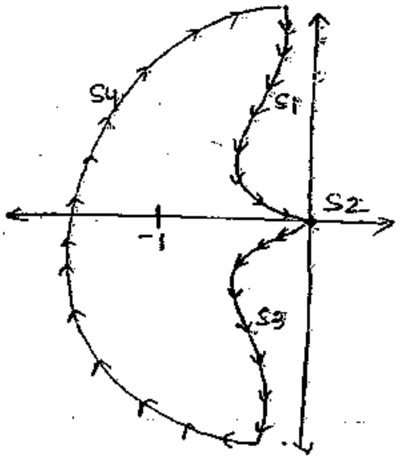
$$= \infty e^{-j(\theta+\pi)}$$

$$\theta = -\frac{\pi}{2} \text{ to } +\frac{\pi}{2}$$

$$\infty e^{-j(-\frac{\pi}{2}+\pi)} = \infty e^{-j\pi/2}$$

$$\infty e^{-j(\pi/2+\pi)} = \infty e^{-j3\pi/2}$$

$$\infty e^{-j\pi/2} \longrightarrow \infty e^{-j3\pi/2}$$



$$G(s) = \frac{10}{s(s-5)}$$

$N = P - Z$

$-1 = 1 - Z$

$Z = 2$ (Unstable)

Que → $G(s) = \frac{s+2}{s^2(s-1)}$

Sol^n → $G(s) = \frac{s+2}{s^2(s-1)}$

	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-360	-180

$$G(s) = \frac{2(1+0.5s)}{s^2(-1)(1-s)}$$

$$G(j\omega) = \frac{-2(1+0.5j\omega)}{(j\omega)^2(1-j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\omega^2\sqrt{1+\omega^2}}$$

$$\angle G(j\omega) = \frac{-180(\tan^{-1}0.5\omega)}{(180)(-\tan^{-1}\omega)}$$

$$= -180 - 180 + \tan^{-1}(0.5\omega) + \tan^{-1}(\omega)$$

Because of the presence of zero

$$-180 - 180 + \tan^{-1}(0.5\omega) + \tan^{-1}\omega = -180$$

$$\tan^{-1}(0.2\omega) + \tan^{-1}\omega = 180$$

$$\tan^{-1}\left(\frac{0.2\omega+\omega}{1-0.2\omega^2}\right) = 180$$

$$0.2\omega + \omega = 0$$

To map $S_4 \rightarrow$

$$G(s) = \frac{2}{s^2(-1)}$$

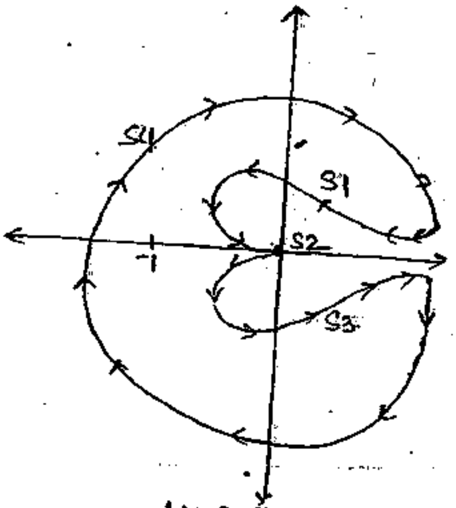
$$= \frac{2}{\lim_{r\to 0} (re^{j\theta})^2 e^{j\pi}}$$

$$= \frac{2}{\lim_{r\to 0} r^2 e^{j2\theta} e^{j\pi}}$$

$$\infty e^{-j(2\theta+\pi)}$$

$$\theta = -\frac{\pi}{2} \rightarrow +\frac{\pi}{2}$$

$$\infty e^{j0} \longrightarrow \infty e^{-j(2\pi)}$$



$N = P - Z$

$-1 = 1 - Z$ $\boxed{Z=2}$ (unstable)

$N = -1$ (Because of anticlock encirclement)

Check $\rightarrow$

$$1 + \frac{s+2}{s^2(s-1)} = 0$$

$$s^3 - s^2 + s + 2 = 0$$

s^3	1	0
s^2	-1	2
s^1	3	0
s^0	2	1

$N = P - Z$, $P = 1$, $Z = -2$

$\boxed{N=-1}$

Que $\rightarrow$ $$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

Soln $\rightarrow$ To map $S_4 \rightarrow$

$$G(s) = \frac{1}{s^2}$$

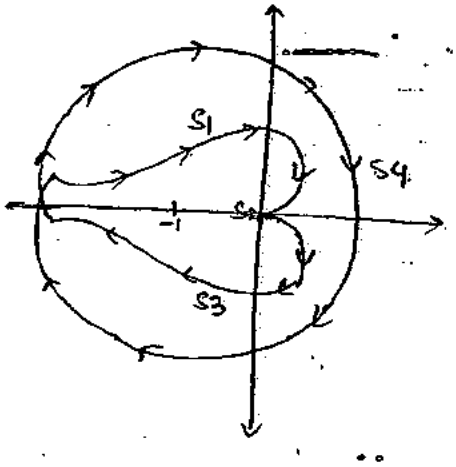
$$= \frac{1}{\lim_{r\to 0} (re^{j\theta})^2}$$

$$= \frac{1}{\lim_{r\to 0} r^2 e^{j2\theta}}$$

$$= \infty e^{-j2\theta}$$

$$\theta = -\pi/2 \rightarrow +\pi/2$$

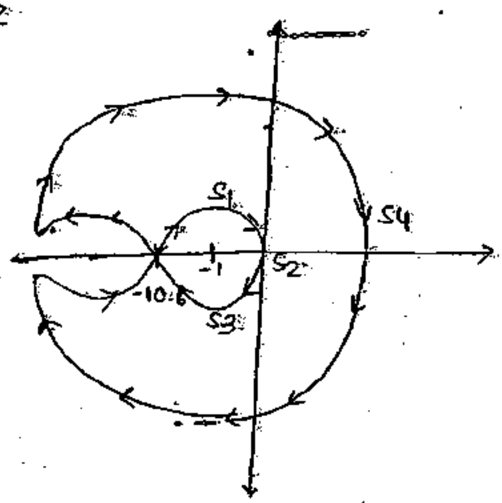
$$= \infty e^{j\pi} \rightarrow \infty e^{-j\pi}$$



$N = P - Z$

Q → $G(s) = \dfrac{1+4s}{s^2(1+s)(1+2s)}$

Sol^n →



$N = P - Z$

$-2 = 0 - Z$

$Z = 2$

Stable

$N = -2$ (No. of encirclement of -1 are 2 times in CW dir^n)

Conv(4) 75

$$G(s) = \frac{s}{1-0.2s}$$

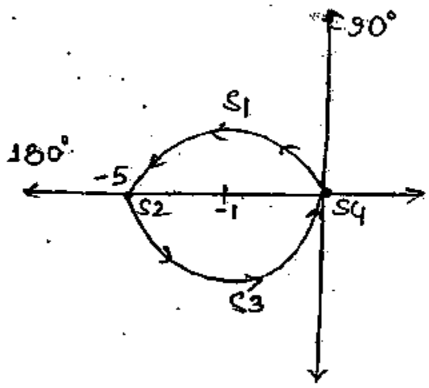
ω	0	∞
$\lvert G(j\omega)\rvert$	0	5
$\angle G(j\omega)$	90°	180°

To map of S_1 →

$$G(j\omega) = \frac{(j\omega)}{1-0.2(j\omega)}$$

$$|G(j\omega)| = \frac{\omega}{\sqrt{1+(0.2\omega)^2}}$$

$$\angle G(j\omega) = 90 + \tan^{-1}(0.2\omega)$$



To map S_2 →

$$G(s) = \frac{\cancel{s}}{-0.2\cancel{s}} = -5$$

To map S_4 →

$$G(s) = s$$

$$= \lim_{r\to 0} (re^{j\theta})$$

$$= 0e^{j\theta}$$

$$\boxed{\begin{aligned} N &= P - Z \\ 1 &= 1 - Z \\ Z &= 0 \end{aligned}}$$ (Stable)

Que → $G(s) = \frac{k(1+s)^2}{s^3}$

Solⁿ → To map S_1 →

$G(j\omega) = \frac{k(1+j\omega)^2}{(j\omega)^3}$

ω	0	1	∞
$\lvert G(j\omega)\rvert$	∞	2k	0
$\angle G(j\omega)$	$-270°$	$-180°$	-90

$|G(j\omega)| = \frac{k(1+\omega^2)}{\omega^3}$

$\angle G(j\omega) = -270° + 2\tan^{-1}\omega$

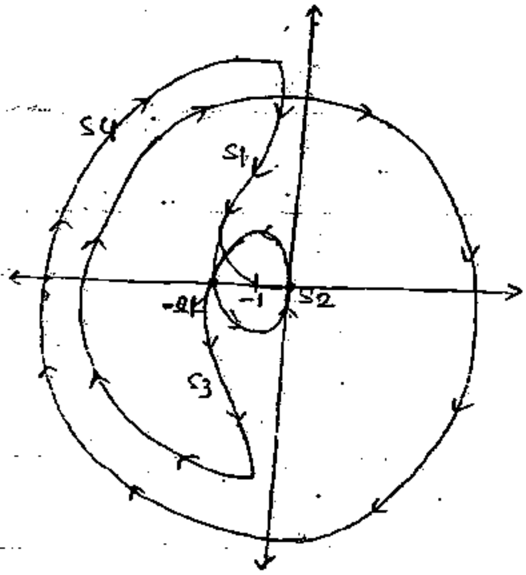
$-270° + 2\tan^{-1}\omega = -180°$

$2\tan^{-1}\omega = 90°$

$\tan^{-1}\omega = 45$

$\omega = \omega_{pc} = 1$ r/s

$X = \frac{k(1+1^2)}{1^3} = 2k$



To map S_4 →

$G(s) = \frac{k}{s^3}$

$= \frac{k}{\lim_{r \to 0} r^3 e^{j3\theta}}$

$\infty e^{-j3\theta} \left(\theta = -\frac{\pi}{2} \to +\frac{\pi}{2}\right)$

$\infty e^{j3\pi/2} \to \infty e^{-j3\pi/2}$

$-2k = -1$, $k_{mar} = 0.5$

$k > 0.5$

$1 - 1 = 0 - Z$

$\boxed{Z = 0}$ stable

Que → $G(s) = \frac{10e^{-s}}{s(s+5)}$

Solⁿ → To map s_1 →

polar plot

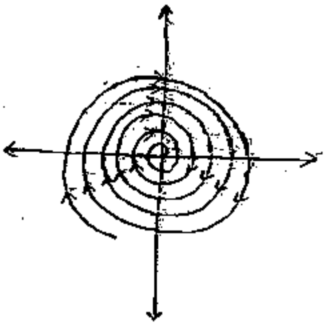
$$G(s) = \frac{2e^{-s}}{s(1+0.2s)}$$

$$G(j\omega) = \frac{2e^{-j\omega}}{(j\omega)(1+0.2j\omega)}$$

$$|G(j\omega)| = \frac{2\times 1}{\omega\sqrt{1+(0.2\omega)^2}}$$

$$\angle G(j\omega) = -90^\circ - 57.3\omega - \tan^{-1} 0.2\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	-90°	-90°



BODE PLOT

* It is a plot of the dB value of magnitude & phase angle in degrees vs $\log\omega$.

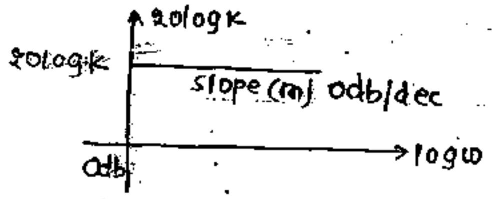
(1) System Gain ($\pm k$) →

$F(s) = \pm k$

$F(j\omega) = \pm k + j0$

$|F(j\omega)| = \sqrt{(\pm k)^2 + 0^2} = k$

Its dB value is



$\angle F(j\omega) = k + j0 \rightarrow 0^\circ$ for all 'ω'

$= -k + j0 \rightarrow -180^\circ$ for all 'ω'

(2) Integral & derivative factors ($s^{\pm n}$) →
(poles/zeros at origin)

$F(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$

$|F(j\omega)| = \left[\sqrt{0 + \omega^2}\right]^{\pm n} = (\omega)^{\pm n}$

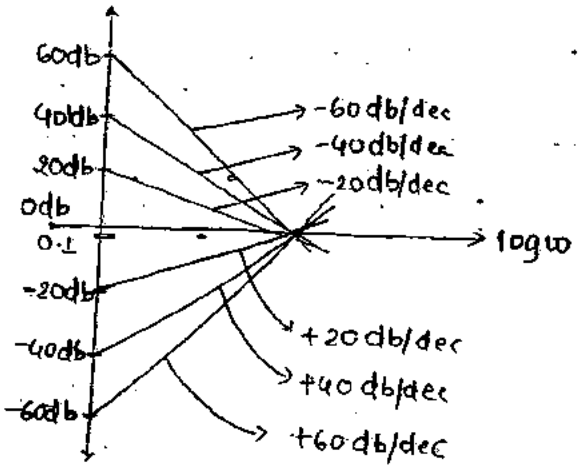
Its dB value is

$20\log(\omega)^{\pm n} \Rightarrow \pm 20 \cancel{\log} \times n \log\omega$ — (i)

$\boxed{\text{Slope}(m) = \pm 20 \times n \text{ db/dec}}$

$\cancel{\pm 20\log}\ \pm 20 \times n \log\omega = 0$

$\log\omega = 0,\ \omega = \log^{-1}(0) = 1\ r/s$



* **First Order Factors →** $(1 \pm Ts)^{\pm 1}$

$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$

$|F(j\omega)| = \left[\sqrt{1 + (\omega T)^2}\right]^{\pm 1}$

Its dB value

$-20\log\left[\sqrt{1+(\omega T)^2}\right]^{\pm 1}$

$\pm 20\log\sqrt{1+(\omega T)^2} \longrightarrow (i)$

Asymptotic approximations →

Case(i) → Low freq.

$1 >> (\omega T)^2$

$\pm 20\log\sqrt{1} = 0db$

Case(ii) High freq.

$(\omega T)^2 >> 1$

$\pm 20\log\sqrt{(\omega T)^{\cancel{2}}}$

$\pm 20\log(\omega T)$ ----- (2)

$\pm 20\log\omega \pm 20\log T$

$(mx + c)$

$$\boxed{\text{Slope}(m) = \pm 20\ db/dec}$$

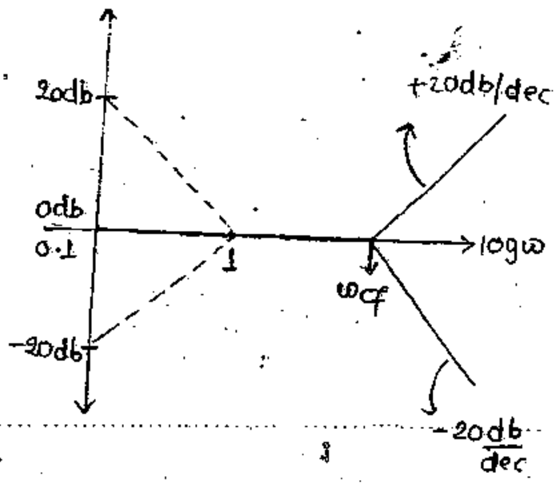
Corner Freq. (ω_{cf}) →

$\pm 20\log\omega T = 0$

$\log\omega T = 0$

$\omega T = \log^{-1}(0) = 1$

$$\boxed{\omega = \omega_{cf} = \frac{1}{T}\ r/s}$$



Ex:- $(s\pm 2)^{\pm 1}$

$\left(1\pm\frac{s}{2}\right)^{\pm 1}$

$T=\frac{1}{2} \Rightarrow \omega_{cf} = 2\ r/s$

Error at ω_{cf} →

At $\omega = \omega_{cf} = \frac{1}{T}$

$\pm 20\log\sqrt{1+\left(\frac{1}{T}\times T\right)^2} = \pm 20\log\sqrt{2} = \pm 3db$

* Quadratic factors → $(s^2+2\xi\omega_n s+\omega_n^2)^{\pm1}$

$$\left(\frac{s^2}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1\right)^{\pm1}$$

(put $s=j\omega$)

$$\left[1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\left(\frac{\omega}{\omega_n}\right)\right]^{\pm1}$$

$$\pm 20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+2\xi\left(\frac{\omega}{\omega_n}\right)^2}$$

LFR: 0db/dec

HFR: $\pm 40\log\left(\frac{\omega}{\omega_n}\right)$

$$\boxed{\text{slope } (m) = \pm 40\,db/dec}$$

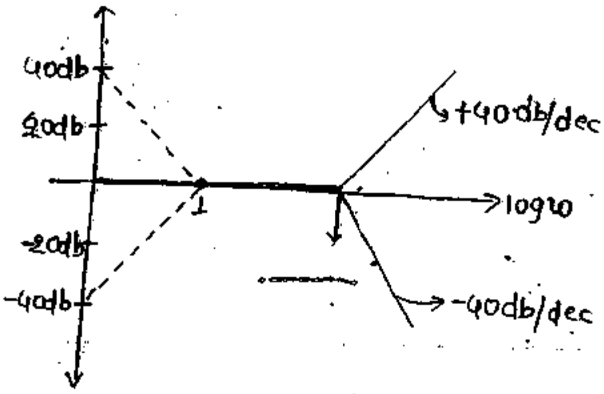
Corner freq. ($\omega_{cf}=\omega_n$ r/s).

$(s^2+4s+25)^{\pm1}$

$\omega_n^2=25$, $\omega_n=\omega_{cf}=5$ r/s.

error at ω_{cf} →

$\pm 20\log 2\xi$

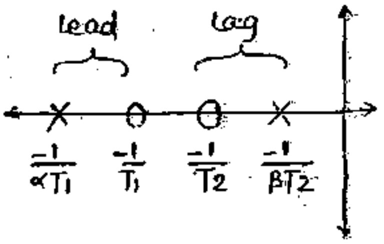


factors	corner freq.	magnitude	slope
k	–	$20\log k$	0db/dec
$(j\omega)^{\pm n}$	–	$\pm 20\times n\log\omega$	$\pm 20\times n\ \frac{db}{dec}$
$(1+j\omega T)^{\pm1}$	$\frac{1}{T}$	$\pm 20\log\omega T$	±20db/dec
$\left[1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\left(\frac{\omega}{\omega_n}\right)\right]^{\pm1}$	ω_n	$\pm 40\log\left(\frac{\omega}{\omega_n}\right)$	±40db/dec

* Bode plot for lag-lead compensator →

$$F(s) = \frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1 s)(1+T_2 s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}$$

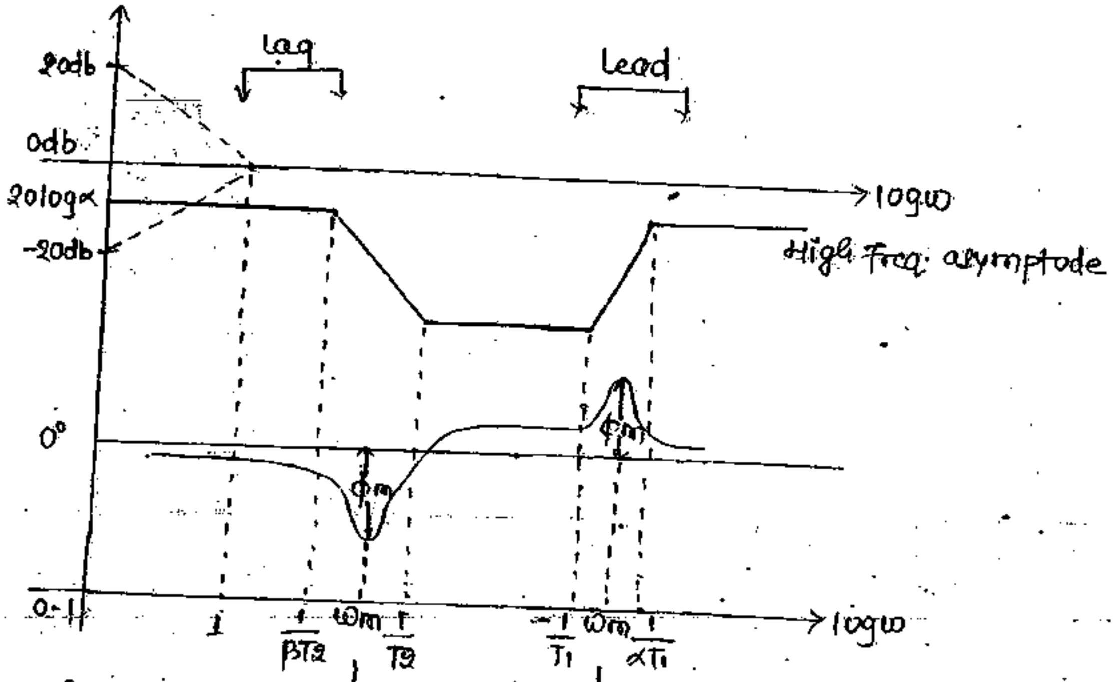
$$F(j\omega) = \frac{\alpha(1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega T_1)(1+j\omega\beta T_2)}$$



$$\underline{|F(j\omega)} = \tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1 + \tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2$$

magnitude table →

	Factor	Corner freq.		Magnitude
(1)	$K=\alpha$	—		$20\log\alpha$
(2)	$(j\omega)^{\pm n}$	—	0	Nill
(3)	$\frac{1}{1+j\omega\beta T_2}$	$\frac{1}{\beta T_2}$		-20 db/dec
(4)	$1+j\omega T_2$	$\frac{1}{T_2}$	(-20)	$+20$ db/dec
(5)	$1+j\omega T_1$	$\frac{1}{T_1}$	0	$+20$ db/dec
(6)	$\frac{1}{1+j\omega\alpha T_1}$	$\frac{1}{\alpha T_1}$	+20	-20 db/dec
			0	



$$\omega_m = \sqrt{\frac{1}{\beta T_2} \times \frac{1}{T_2}} = \frac{1}{T_2\sqrt{\beta}}$$

$$\omega_m = \sqrt{\frac{1}{T_1} \times \frac{1}{\alpha T_1}} = \frac{1}{T_1\sqrt{\alpha}}$$

for calculate High Freq. asymptode

* C/s of phase lead compensator → * The phase lead compensator shifts the gain cross over freq. to higher values where the desired phase margin is acceptable.

Hence it is effective when the slope of uncompensated sys. near the gain cross over freq. is low.

* The max^m phase lead occurs at the geometric mean of 2 corner freq.

$$\underline{|F(j\omega)} = \phi = \tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1$$

$$\tan\phi = \tan(\tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1)$$

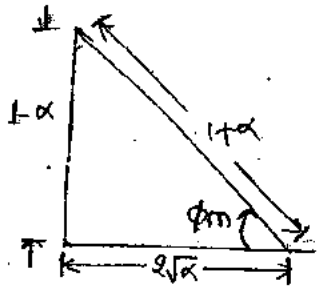
$$\tan\phi = \frac{\omega T_1 - \omega\alpha T_1}{1+(\omega T_1)^2\alpha} = \frac{\omega T_1(1-\alpha)}{1+(\omega T_1)^2\alpha}$$

At $\omega = \omega_m = \frac{1}{T_1\sqrt{\alpha}}$; $\phi = \phi_m$

$$\tan\phi_m = \frac{\frac{1}{T_1\sqrt{\alpha}} \cdot T_1 (1-\alpha)}{1 + \left(\frac{1}{T_1\sqrt{\alpha}} \cdot T_1\right)^2 \cdot \alpha} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right)}$$



$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)}$$

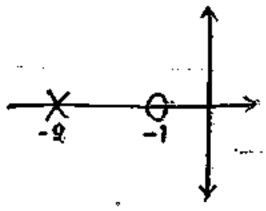
$$\sin\phi_m (1+\alpha) = 1-\alpha$$

$$\alpha \sin\phi_m + \alpha = 1 - \sin\phi_m$$

$$\alpha(1+\sin\phi_m) = 1 - \sin\phi_m$$

$$\boxed{\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

(3.) Polar plot →



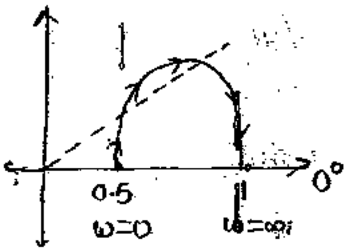
$$F(s) = \frac{s+1}{s+2} = \frac{0.5(1+s)}{(1+0.5s)}$$

$$F(j\omega) = \frac{0.5(1+j\omega)}{(1+0.5j\omega)}$$

$$|F(j\omega)| = \frac{0.5\sqrt{1+\omega^2}}{\sqrt{1+(0.5\omega)^2}}$$

$$\underline{/F(j\omega)} = \tan^{-1}\omega - \tan^{-1}0.5\omega$$

ω	0	1	∞
$\lvert F(j\omega)\rvert$	0.5		1
$\underline{/F(j\omega)}$	0°	18.4°	0°



* C/S of phase lag compensator → * Phase lag compensator shifts the gain cross over freq to lower values where the desired ϕ margin is acceptable.

Hence it is effective when the slope of the uncompensated sys. near gain cross over freq. is high.

* The max^{m} phase-lag occurs at the geometric mean of the 2 corner freq.

$$\angle F(j\omega) = \phi = \tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2$$

$$\tan\phi = \tan\left(\tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2\right)$$

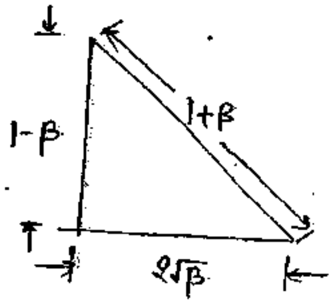
$$\tan\phi = \frac{\omega T_2 - \omega T_2\beta}{1+(\omega T_2)^2\cdot\beta} = \frac{\omega T_2(1-\beta)}{1+(\omega T_2)^2\beta}$$

At $\omega = \omega_m = \frac{1}{T_2\sqrt{\beta}}$; $\phi = \phi_m$

$$\tan\phi_m = \frac{\frac{1}{T_2\sqrt{\beta}} \times T_2(1-\beta)}{1+\left[\frac{1}{T_2\sqrt{\beta}}\cdot T_2\right]^2\cdot\beta} = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\tan\phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\boxed{\phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)}$$



$$\sin\phi_m = \frac{1-\beta}{1+\beta}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{1-\beta}{1+\beta}\right)}$$

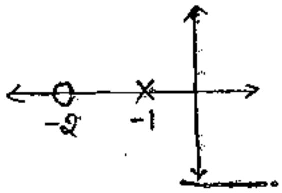
$$\sin\phi_m(1+\beta) = 1-\beta$$

$$\beta\sin\phi_m + \beta = 1-\sin\phi_m$$

$$\beta(1+\sin\phi_m) = 1-\sin\phi_m$$

$$\boxed{\beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$$

(3.) polar plot →



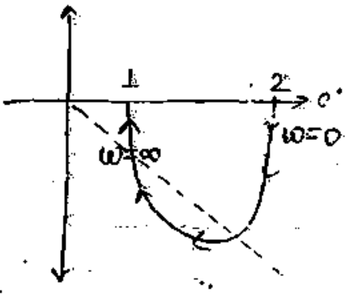
$$F(s) = \frac{s+2}{s+1} = \frac{2(1+0.5s)}{(1+s)}$$

$$F(j\omega) = \frac{2(1+0.5j\omega)}{(1+j\omega)}$$

$$|F(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}}$$

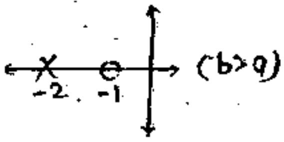
$$\angle F(j\omega) = \tan^{-1}0.5\omega - \tan^{-1}\omega$$

ω	0	1	∞
$\lvert F(j\omega)\rvert$	2		1
$\angle F(j\omega)$	0°	-18.4°	0°



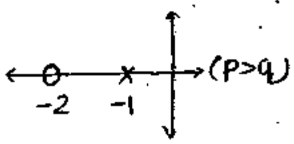
(1/76) lead :-

$$\frac{s+a}{s+b} = \frac{s+1}{s+2}$$



lag:-

$$\frac{s+p}{s+q} = \frac{s+2}{s+1}$$



(3/76) $\alpha = \frac{R_2}{R_1+R_2}$

$$\phi_m = \sin^{-1}\left(\frac{1-\frac{R_2}{R_1+R_2}}{1+\frac{R_2}{R_1+R_2}}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$$

(2/76) $G_c(s) = \frac{5(1+0.3s)}{(1+0.1s)}$

$$\frac{1+0.3s}{1+0.1s} = \frac{1+Ts}{1+kTs}$$

$T = 0.3$, $kT = 0.1$

$k \times 0.3 = 0.1$; $k = \alpha = \frac{1}{3}$

Lead compensator

$$\phi_m = \sin^{-1}\left(\frac{1-1/3}{1+1/3}\right)$$

$$\phi_m = \sin^{-1}(1/2) = 30^\circ$$

ans(c)

(4/76) $\frac{1+aTs}{1+Ts} = \frac{1+T_1 s}{1+\alpha T_1 s}$

$T_1 = aT$; $\alpha T_1 = T$

$\alpha \times aT = T$; $\alpha = \frac{1}{a}$

$$\phi_m = \sin^{-1}\left(\frac{1-1/a}{1+1/a}\right) = \sin^{-1}\left(\frac{a-1}{a+1}\right)$$

(6/76) $\frac{K(1+0.3S)}{(1+0.17S)} = \frac{1+TS}{1+\alpha TS}$

$T=0.3$; $\alpha T=0.17$

$\alpha \times 0.3 = 0.17$

$\alpha = 0.56$

$T = R_1 C$

$R_1 \times 10^{-6} = 0.3$

$\boxed{R_1 = 300k\Omega}$

$\alpha = \frac{R_2}{R_1+R_2}$

$0.56 = \frac{R_2}{300+R_2}$

$\boxed{R_2 = 400k\Omega}$

(10/77) $\frac{1+2S}{1+0.2S} = \frac{1+TS}{1+KTS}$

$T=2$; $KT=0.2$

$K\times 2 = 0.2$, $K=0.1$

(Lead Controller)

(11/77) (b.) $\frac{S+9.9}{S+3} \Rightarrow$ lag.

(d) $\frac{S+6}{S} = 1+\frac{6}{S} \rightarrow$ PI

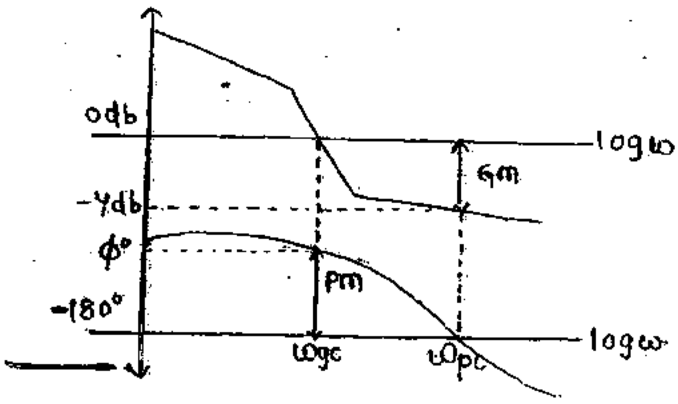
$K_p + \frac{K_I}{3}$

(c.) Can't be ans.

~~(a)~~ To improved transient state lead compensator must be used

(5/76) (d) (7/77) d (10/77) (c) (12/77) (a)

* To Find Gm & Pm From BODE plots →

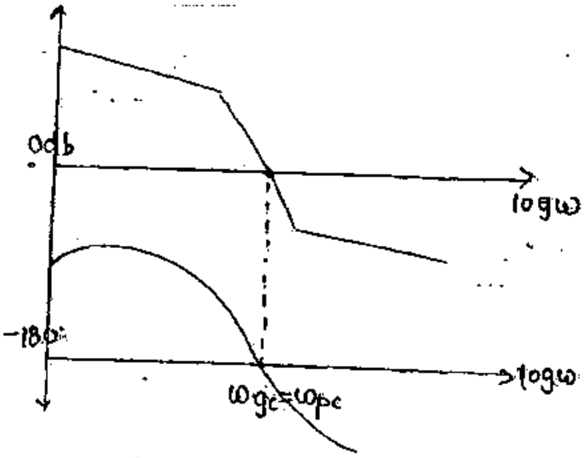


Stable

$Pm = \phi - (-180°) = 180° + \phi = +ve$

$Gm = 0 - (-y) = +y\,db = +ve$

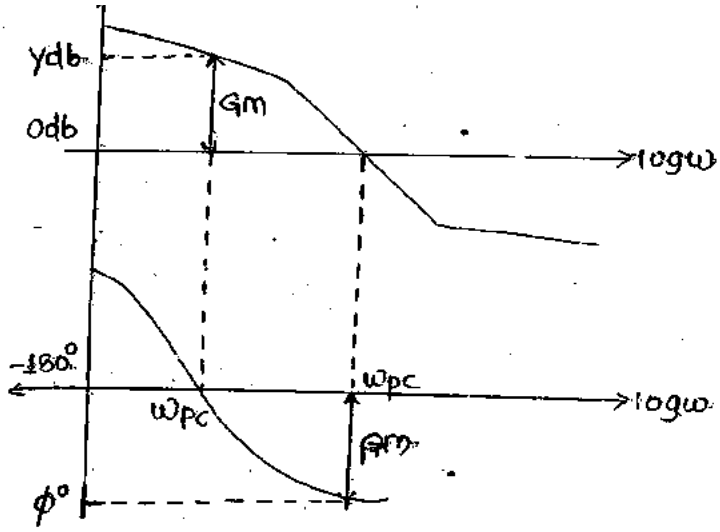
$\boxed{\omega_{gc} < \omega_{pc}}$



$Gm = Pm = 0$

Marginaly stable

$\boxed{\omega_{gc} = \omega_{pc}}$



$pm = 180° + \phi = -ve$

$Gm = 0 - y = -ydb = -ve$

$\boxed{\omega_{gc} > \omega_{pc}}$

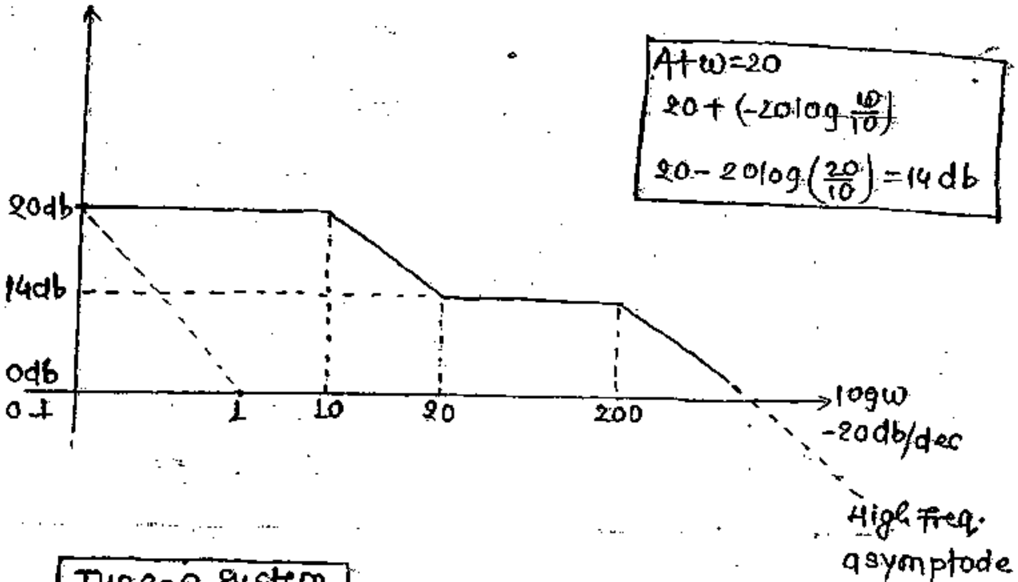
Unstable

Que.→ $G(s) = \dfrac{10^3(s+20)}{(s+10)(s+200)}$

Sol^n→ $G(s) = \dfrac{\frac{10^3}{20}(20)\left(\frac{s}{20}+1\right)}{10 \times 200\left(\frac{s}{10}+1\right)\left(\frac{s}{200}+1\right)}$

$$= \frac{10\left(1 + j\omega/20\right)}{(1+j\omega/10)(1+j\omega/200)}$$

	Factors	CF	magnitude
(1.)	$k=10$	-	$20\log 10 = 20db$
(2.)	$(j\omega)^{\pm n}$	-	Nil
		0	
(3.)	$\dfrac{1}{1+\frac{j\omega}{10}}$	10	$-20\log\left(\frac{\omega}{10}\right)$
		-20	(-20db/dec)
(4.)	$1+\left(\frac{j\omega}{20}\right)$	20	$+20\log\left(\frac{\omega}{20}\right)$
		0	(+20db/dec)
(5.)	$\dfrac{1}{1+\frac{j\omega}{200}}$	200	$-20\log(\omega/200)$
		-20	(-20db/dec)



At ω=20

$20 + (-20\log \frac{\omega}{10})$

$20 - 20\log\left(\frac{20}{10}\right) = 14\,db$

Type-0 system

$K_v = K_A = 0$

$20\log \cancel{K_p =} 20$

$K_p = \log^{-1}(1) = 10$

Que→ $G(s) = \dfrac{20(s+1)}{s(s+2)(s+5)}$

Soln→ $G(s) = \dfrac{20(1+s)}{10\,s\,(1+\frac{s}{2})(1+\frac{s}{5})}$

$G(j\omega) = \dfrac{2(1+j\omega)}{(j\omega)(1+0.5j\omega)(1+0.2j\omega)}$

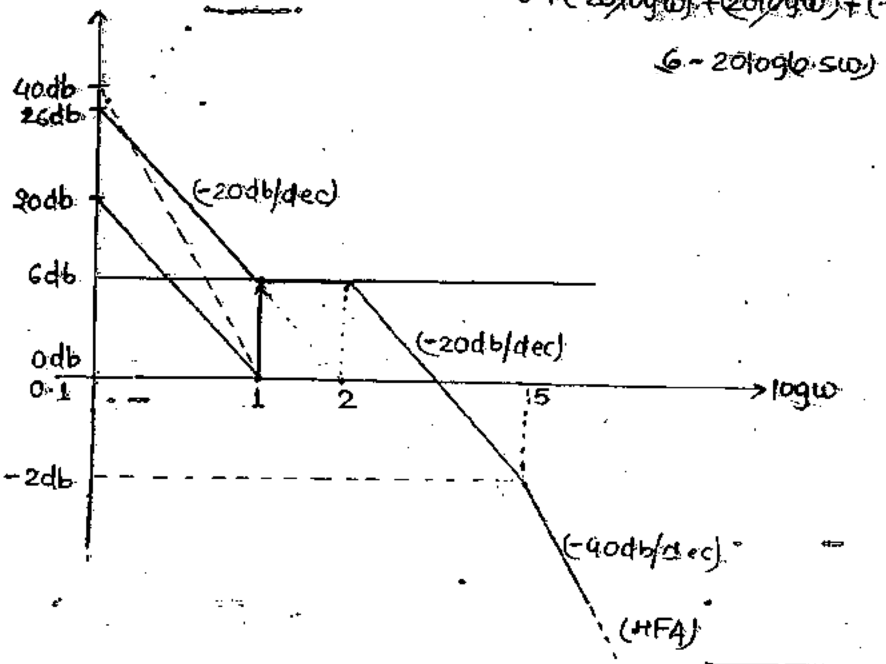
Factor	CF	magnitude
(i) $K=2$	-	$20\log 2 = 6db$
(ii) $\frac{1}{j\omega}$	-	$-20\log\omega$ (−20db/dec)
	-20	
(iii) $1+j\omega$	1	$+20\log\omega$ (+20db/dec)
	0	
(iv) $\frac{1}{1+0.5j\omega}$	2	$-20\log 0.5\omega$ (−20db/dec)
	-20	
(v) $\frac{1}{1+0.2j\omega}$	5	$-20\log 0.2\omega$ (−20db/dec)
	-40	

at $\omega = 5$

$6 + (\cancel{-20\log\omega}) + (\cancel{20\log\omega}) + (-20\log 0.5\omega)$

$6 - 20\log(0.5\omega) = 6 - 20\log(0.5\times 5)$

$= -2db$



y m x c $\boxed{C = 20\log k}$

$y = -20\log\omega + 6$

At $\omega = 0.1$

$y = -20\log 0.1 + 6 = 20 + 6 = 26db$

Type-1 system
$k_p = \infty$, $k_A = 0$
$k_v = \omega = 2$
$20\log k - 20\log\omega = 0$
$k = k_v = \omega$

Conv que (1) → (74)

$$G(s) = \frac{3(s+1)(s+700)}{s^2(s^2+18s+400)}$$

Sol^n → quadratic factors must be considered as quadratic factor.

$$G(s) = \frac{3(1+s)\left(1+\frac{s}{700}\right)700}{s^2\times 400\left[1+\frac{18s}{400}+\frac{s^2}{400}\right]}$$

$$G(j\omega) = \frac{3(1+j\omega)\left(1+\frac{j\omega}{700}\right)700}{-\omega^2\times 400\left[1-\frac{\omega^2}{400}+\frac{j18\omega}{400}\right]}$$

$$\underline{|G(j\omega)}\Big|_{\omega<20} = \frac{(0^\circ)(\tan^{-1}\omega)\left(\tan^{-1}\frac{\omega}{700}\right)}{(180^\circ)\left[\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]}$$

$$\underline{|G(j\omega)}\Big|_{\omega>20} = \frac{(0^\circ)(\tan^{-1}\omega)\left(\tan^{-1}\frac{\omega}{700}\right)}{(180^\circ)\left[180^\circ-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]}$$

Ex:- Cal^n of phase angle for quadratic term

$\omega =$

$-180 + \left[-\tan^{-1}\left(\frac{18\omega}{400-\omega^2}\right)\right]$

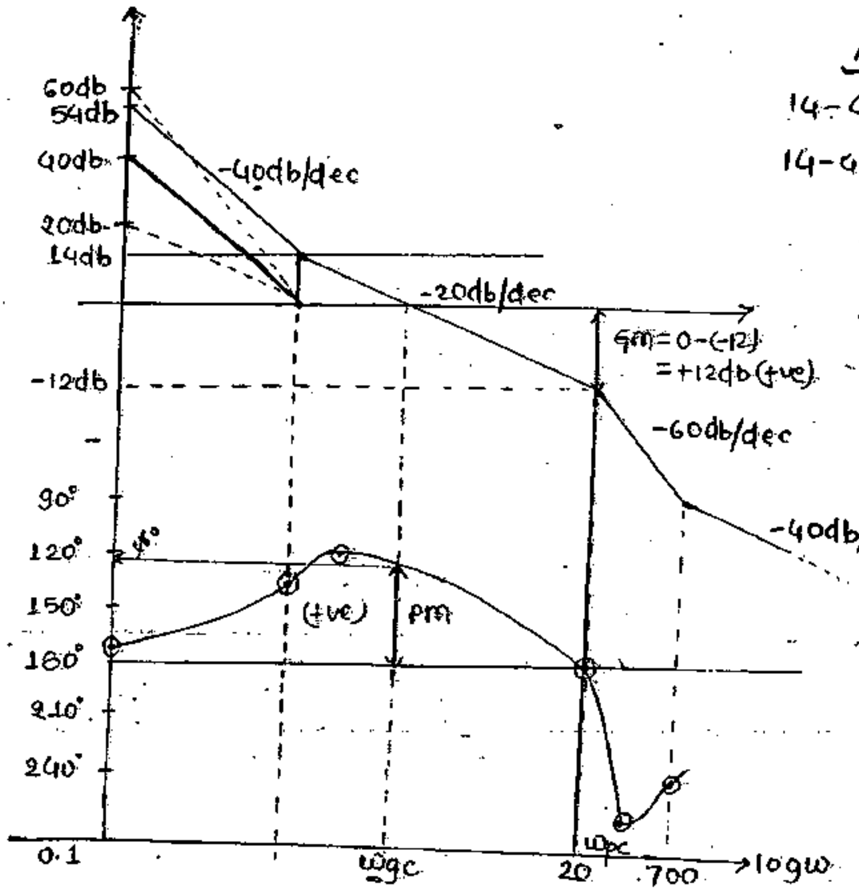
$(-180^\circ + 10.6^\circ) = -169.4^\circ$

Phase angle table→

ω	$-180°$ +	$\tan^{-1}\omega$ +	$\tan^{-1}\frac{\omega}{700}$ +	$\tan^{-1}\frac{-18\omega}{400-\omega^2}$	ϕ_R
0.1	$-180°$	$5.7°$	$0.008°$	$-0.25°$	$-174°$
1	$-180°$	$45°$	$0.08°$	$-2.5°$	$-137°$
5	$-180°$	$78.6°$	$0.4°$	$-13.5°$	$-114°$
20	$-180°$	$87°$	$1.6°$	$-90°$	$-181°$
100	$-180°$	$89.4°$	$8°$	$+10.6-180° = -169.4°$	$-251°$
700	$-180°$	$89.9°$	$45°$	$+1.5-180° = -178.5°$	$-223°$

magnitude table→

	factor	CF	magnitude
(1)	$k = 5.2$	–	$20\log 5.2 = 14\,db$
(2)	$\frac{1}{(j\omega)^2}$	–	$-40\log\omega$ $(-40\,db/dec)$
		-40	
(3)	$(1+j\omega)$	1	$20\log\omega$ $(+20\,db/dec)$
		-20	
(4)	$\frac{1}{1-\frac{\omega^2}{400}+j\frac{18\omega}{400}}$	20	$-40\log\left(\frac{\omega}{20}\right)$ $(-40\,db/dec)$
		-60	
(5)	$1+\frac{j\omega}{700}$	700	$20\log\left(\frac{\omega}{700}\right)$ $(+20\,db/dec)$
		-40	



At $\omega = 20$

$14 - 40\log\omega + 20\log\omega$

$14 - 40\log 20 + 20\log 20 = -12db$

At $\omega = 0.1$

$Y = -40\log 0.1 + 14$

$Y = 40 + 14 = 54db$

Type-2 system

$K_p = K_v = \infty$

$K_A = \omega^2$

$20\log K - 20\log\omega^2 = 0$

$K = K_A = \omega^2$

$\omega_{pc} > \omega_{gc}$

stable system

$Gm = Pm = +ve$

13/71

$$G(s) = \frac{K(s+10)(s+20)}{s^3(s+100)(s+200)}$$

Solⁿ→

$$G(s) = \frac{0.01K(1+0.1s)(1+0.05s)}{s^3\left(1+\frac{s}{100}\right)\left(1+\frac{s}{200}\right)}$$

$$G(j\omega) = \frac{\frac{K}{100}\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{20}\right)}{(j\omega)^3\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{200}\right)}$$

$$\angle G(j\omega) = \frac{(0^\circ)\left(\tan^{-1}\frac{\omega}{10}\right)\left(\tan^{-1}\frac{\omega}{20}\right)}{270^\circ\left(\tan^{-1}\frac{\omega}{100}\right)\left(\tan^{-1}\frac{\omega}{200}\right)}$$

$$\phi_R = -270^\circ + \tan^{-1}\frac{\omega}{10} + \tan^{-1}\frac{\omega}{20} - \tan^{-1}\frac{\omega}{100} - \tan^{-1}\frac{\omega}{200}$$

ω	0.1	1	10	20	50	100	200
ϕ_R	$-269°$	$-262°$	$-207°$	$-179°$	$-163°$	$-179°$	$-207°$

<u>mag table →</u>

	Factor	CF	magnitude
(1.)	$k' = \frac{k}{100}$	–	$20\log k'$
(2.)	$\frac{1}{(j\omega)^3}$	–	$-60\log \omega$ (-60 db/dec)
(3.)	$1+\frac{j\omega}{10}$	10	$20\log\frac{\omega}{10}$ ($+20$ db/dec)
(4.)	$1+\frac{j\omega}{20}$	20	$20\log \omega/20$ ($+20$ db/dec)
(5.)	$\frac{1}{1+\frac{j\omega}{100}}$	100	$-20\log\frac{\omega}{100}$ (-20 db/dec)
(6.)	$\frac{1}{1+\frac{j\omega}{200}}$	200	$-20\log\frac{\omega}{200}$ (-20 db/dec)

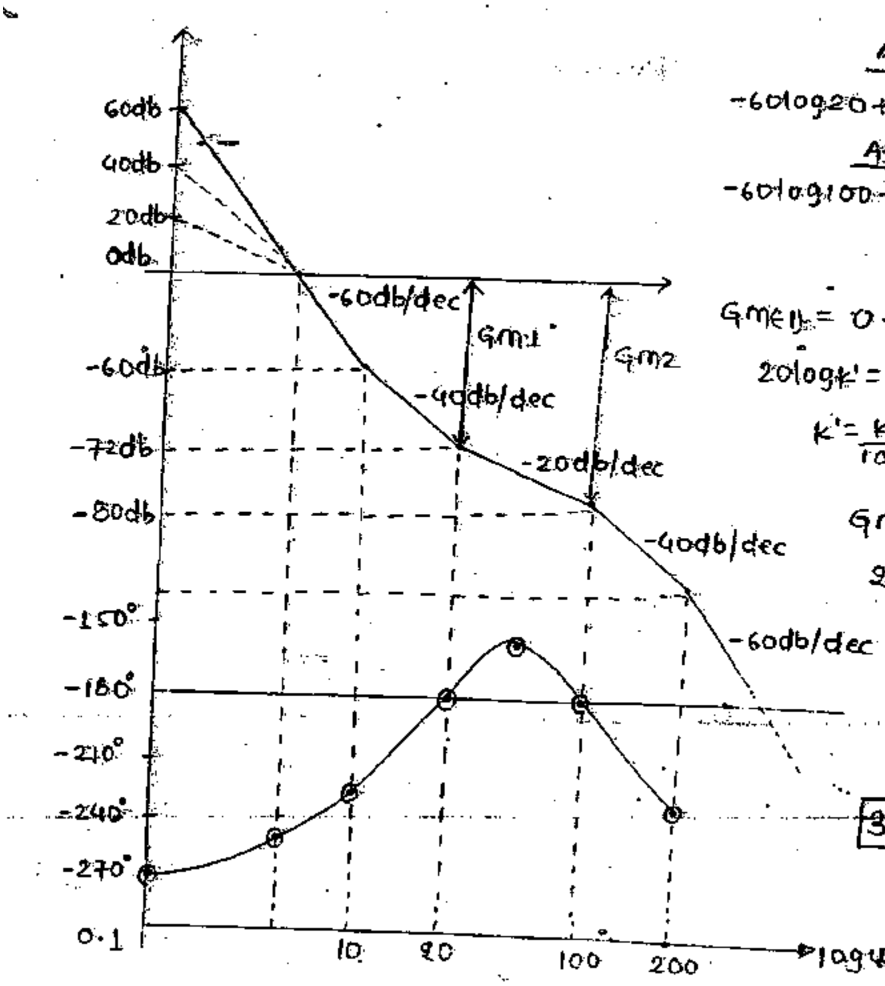
-60

-40

-20

-40

-60



<u>At ω=20</u>

$-60\log 20 + 20\log 2 = -72$ db

<u>At ω=100</u>

$-60\log 100 + 20\log 10 + 20\log 5 = -86$ db

$Gm(1) = 0 - (-72) = 72$ db

$20\log k' = 72 \Rightarrow k' = 3981$

$k' = \frac{k}{100} = 3981$; $k = 398100$

$Gm(2) = 0 - (-86) = 86$ db

$20\log k' = 86 \Rightarrow k' = 19952$

$k' = \frac{k}{100} = 19952$

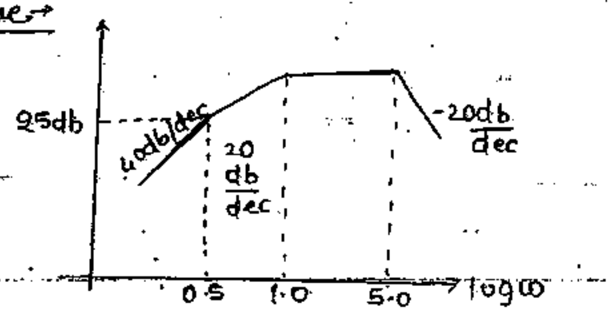
$k = 1995200$

$\boxed{398100 < k < 1995200}$

Inverse Bode Plot

* Observe the starting slope. This will give the information of poles (or) zeros at origin.
* For the starting slope write the eqn $y = mx + c$.
  $c = 20\log K$
* At every corner freq. observe the change in slopes. This will give the information of 1st (or) higher order factors.

Que→



Soln→

$$G(s) = \frac{70s^2}{(1+2s)(1+s)\left(1+\frac{s}{5}\right)}$$

At $\omega = 0.5$

$y = mx + c$

$25 = 40\log 0.5 + c$

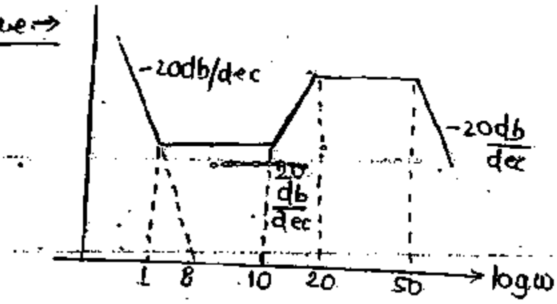
$c = 37$

$20\log K = 37$

$K = \log^{-1}\left(\frac{37}{20}\right)$

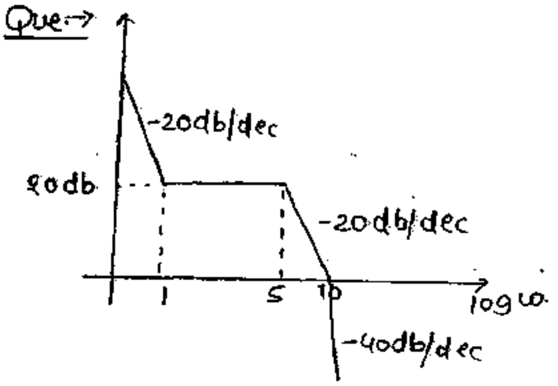
$K = 70$

Que→



Soln→ $$G(s) = \frac{8(1+s)\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{20}\right)\left(1+\frac{s}{50}\right)}$$

Que→



Soln→ $$G(s) = \frac{10(1+s)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)}$$

At $\omega = 1$

$y = mx + c$

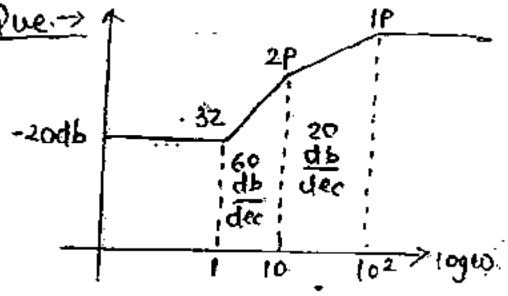
$20 = \log 1 + c$

$c = 20$

$20\log K = 20$

$K = \log^{-1}(1) = 0$

Que→



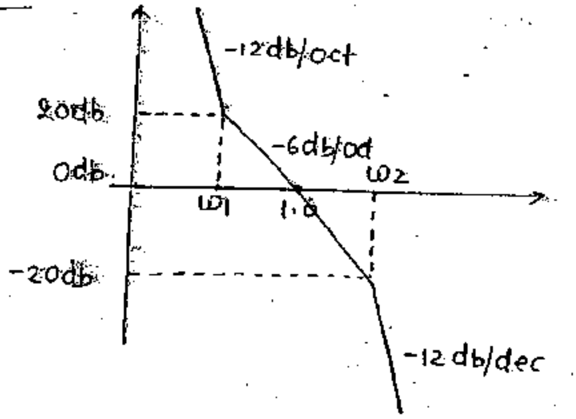
Soln→ $$G(s) = \frac{0.1(1+s)^3}{\left(1+\frac{s}{10}\right)^2\left(1+\frac{s}{100}\right)}$$

$20\log K = -20$

$K = \log^{-1}(-1)$

$K = 0.1$

Que →



Sol^n →

decade scale

$\omega_2 = 10\omega_1$

db value magnitude $= \pm 20 \times n \log \omega$

slope (m) $= \pm 20 \times n \log_{10} 10$

$= \pm 20 \times n$ db/dec

Octave scale

$\omega_2 = 2\omega_1$

db value magnitude $= \pm 20 \times n \log \omega$

slope (m) $= \pm 20 \times n \log 2$

$= \pm 6 \times n$ db/oct

$$\boxed{\pm 6 \times n \frac{db}{oct} \simeq \pm 20 \times n \frac{db}{dec}}$$

2nd line →

At $\omega = 1$

$y = mx + c$

$0 = -20 \log 1 + c$

$c = 0$

At $\omega = \omega_1$

$y = mx + c$

$20 = -20 \log \omega_1 + 0$

$\omega_1 = \log^{-1}(-1) = 0.1$ r/s

At $\omega = \omega_2$

$y = mx + c$

$-20 = -20 \log \omega_2 + 0$

$\omega_2 = \log^{-1}(1) = 10$ r/s

To Find k →

1st line

At $\omega = \omega_1 = 0.1$

$y = mx + c$

$20 = -40 \log 0.1 + c$

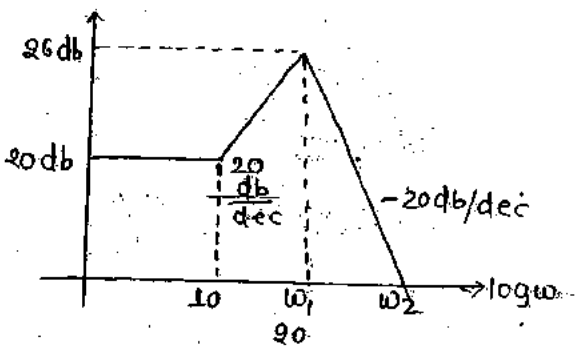
$c = -20$

$20 \log k = -20$

$\boxed{k = 0.1}$

$$\boxed{G(s) = \frac{0.1(1+10s)}{s^2(1+0.1s)}}$$

Que.→



Sol^n→

To find ω_1

2nd line

At $\omega=10$

$y=mx+c$

$20=20\log 10+c$

$c=0$

At $\omega=\omega_1$

$y=mx+c$

$26=20\log\omega_1+0$

$\omega_1=\log^{-1}\left(\frac{26}{20}\right)=20\,r/s$

To find ω_2

3rd line

At $\omega=\omega_1=20$

$y=mx+c$

$26=-20\log 20+c$

$c=52$

At $\omega=\omega_2$

$y=mx+c$

$0=-20\log\omega_2+52$

$-52=-20\log\omega_2$

$\omega_2=\log^{-1}\left(\frac{52}{20}\right)=400\,r/s$

To find K

$20\log K=20$

$K=\log^{-1}(1)$

$=10$

$$\boxed{G(s)=\frac{10\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{20}\right)^2}}$$

M & N circles

* Nicol charts →

* The Nicol chart consist of magnitudes & phase angles of a closed loop system represented as a family of circles known as m & N circles.
* This chart gives information about closed loop freq. responce.
*

* M-circles →

Let the CLTF

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Let $G(s) = x + jy$

$$\frac{C(s)}{R(s)} = \frac{x+jy}{1+x+jy}$$

The magnitude (m)

$$m = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

$$m^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$m^2x^2-x^2+m^2y^2-y^2+2xm^2+m^2=0$$

$$(m^2-1)x^2+y^2(m^2-1)+2xm^2+m^2=0 \text{ ----- (i)}$$

In eqn (i) $m=1$

$2x+1=0$, It reprents a straight line passing through $-\frac{1}{2}, 0$

If $m \neq 1$ eqn (i) represents a family of circles.

$$x^2+y^2+2x\frac{m^2}{m^2-1}+\frac{m^2}{m^2-1}=0 \text{ ---- (ii)}$$

$$x^2+y^2+2x\frac{m^2}{m^2-1}+\frac{m^2}{m^2-1}+\frac{m^2}{(m^2-1)^2}=\frac{m^2}{(m^2-1)^2}$$

$$x^2+2x\frac{m^2}{m^2-1}+\frac{m^4}{(m^2-1)^2}+y^2=\frac{m^2}{(m^2-1)^2}$$

$$\left[x+\frac{m^2}{m^2-1}\right]^2+y^2=\frac{m^2}{(m^2-1)^2}$$

Center: $\frac{-m^2}{m^2-1}, 0$ Radius $= \frac{m}{m^2-1}$

(4.) N-circles →

Let α = phase angles of CL system

$N = \tan\alpha$ represents a family of circles

$$\alpha = \tan^{-1}\frac{y}{x} - \tan^{-1}\left(\frac{y}{1-x}\right)$$

$$N = \tan\alpha = \tan\left[\tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}\right]$$

$$N = \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y^2}{x(1+x)}}$$

$$x^2 + x + y^2 - \frac{y}{N} = 0 \quad \text{----- (i)}$$

adding term $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on both side.

$$x^2 + x + \frac{1}{4} + y^2 - \frac{y}{N} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

Center; $= -\frac{1}{2}, \frac{1}{2N}$ Radius $= \sqrt{\frac{1}{4}+\left(\frac{1}{2N}\right)^2}$

All N-circles intersects the real axis b/w -1 & origin only.

(23/74) (b) 3 is wrong because geometric rules are not applicable for log-scale.

(24/74)

$$x^2+y^2+2x\frac{m^2}{m^2-1}+\frac{m^2}{m^2-1}=0 \quad ----(i)$$

$$x^2+y^2+2.25x+1.125=0$$

$$\frac{m^2}{m^2-1}=1.125$$

$$\boxed{m=3}$$

(25/74) (a)