CBSE Class 10 Mathematics Standard
Sample Paper - 07 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. State whether $\frac{427}{625}$ have terminating decimal expansion or non-terminating repeating decimal expansion.

   OR

   In what form of decimals can irrational numbers be represented?

2. State whether the quadratic equation $(x + 4)^2 - 8x = 0$ has two distinct real roots. Justify your answer.
3. Write the value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky - 3 = 0$ has

no solution.

4. Write the name of the common point of the tangent to a circle and the circle.
5. Find $a_{30} - a_{20}$ for the AP a, a + d, a + 2d, a + 3d,...

OR

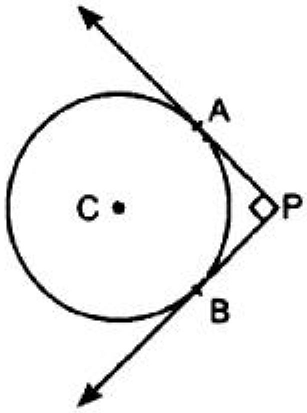
If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then find the value of a.

6. If sum of first n terms of an AP is $2n^2 + 5n$. Then find S_{20}.
7. State whether $x^2 + 6x - 4 = 0$ is a quadratic equation or not?

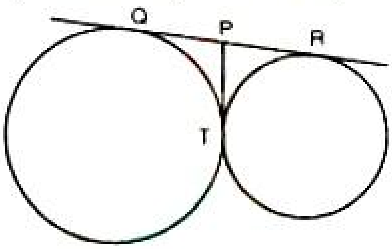
OR

State whether $2x^2 - 7x = 0$ is a quadratic equation or not?

8. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, find the length of each tangent.



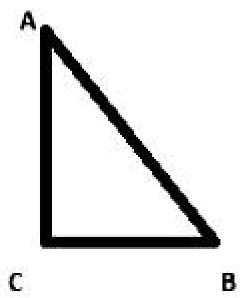
9. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If QP = 3.8 cm, then find length of QR.



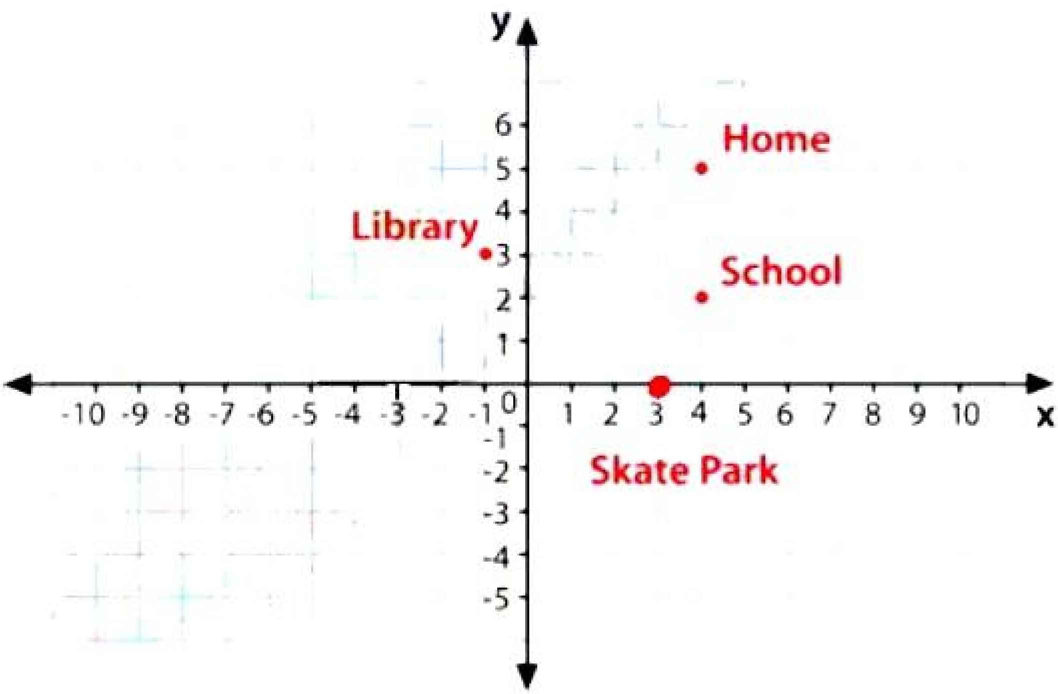
OR

How many common tangents can be drawn to two circles touching internally?

10. ABC is an isosceles right triangle right-angled at C. Prove that $AB^2 = 2AC^2$



11. The n^{th} term of an A.P. is (5n - 2). Find its 19th term.
12. Find the value of x, if $\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$.
13. Prove that: $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$
14. A rectangular sheet of paper 40cm $\times$ 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder.
15. The sum of first n terms of an A.P. is $5n - n^2$. Find the n^{th} term of the A.P.
16. A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is the ace of spades.
17. Two brothers Ramesh and Pulkit were at home and have to reach School. Ramesh went to Library first to return a book and then reaches School directly whereas Pulkit went to Skate Park first to meet his friend and then reaches School directly.



i. How far is School from their Home?
   a. 5 m
   b. 3 m
   c. 2 m

d. 4 m

ii. What is the extra distance travelled by Ramesh in reaching his School?

a. 4.48 metres

b. 6.48 metres

c. 7.48 metres

d. 8.48 metres

iii. What is the extra distance travelled by Pulkit in reaching his School? (All distances are measured in metres as straight lines)

a. 6.33 metres

b. 7.33 metres

c. 5.33 metres

d. 4.33 metres

iv. The location of the library is:

a. (-1, 3)

b. (1, 3)

c. (3, 1)

d. (3, -1)

v. The location of the Home is:

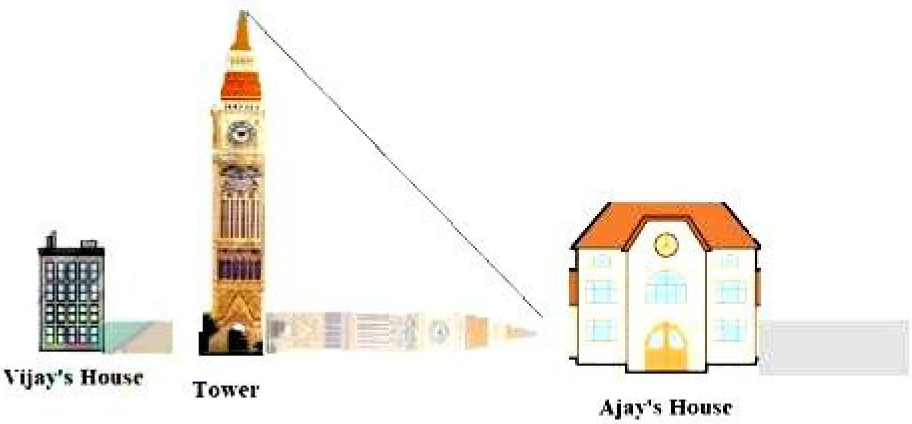
a. (4, 2)

b. (1, 3)

c. (4, 5)

d. (5, 4)

18.



Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house is 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground. At the same time, the house of Ajay casts 20 m shadow on the ground.

i. What is the height of the tower?
   a. 20 m
   b. 50 m
   c. 100 m
   d. 200 m

ii. What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m?
   a. 75 m
   b. 50 m
   c. 45 m
   d. 60 m

iii. What is the height of Ajay's house?
   a. 30 m
   b. 40 m
   c. 50 m
   d. 20 m

iv. When the tower cast shadow of 40 m, Same time what will be the length of the shadow of Ajay's house?
   a. 16 m
   b. 32 m
   c. 20 m
   d. 8 m

v. When the tower cast shadow of 40 m, Same time what will be the length of the shadow of Vijay's house?
   a. 15 m
   b. 32 m
   c. 16 m
   d. 8 m

19. The agewise participation of students of a school in the International Yoga day Celebration that was held in Central City Ground Patna is shown in the following distribution. By Analysing the data given below answer the questions that follow:

Age(in years)	5 - 7	7 - 9	9 - 11	11 - 13	13 - 15	15 - 17	17 - 19
Number of students	x	15	18	30	50	48	x

Find the Following when the sum of frequencies is 181.

i. The mode of the data is:
   a. 17.81
   b. 11.81
   c. 18.41
   d. 14.81

ii. The value of missing frequency(x) is:
   a. 12
   b. 10
   c. 13
   d. 14

iii. The modal class is:
   a. 13 - 15
   b. 11 - 13
   c. 15 - 17
   d. 17 - 19

iv. The upper limit of the modal class is:
   a. 17
   b. 19
   c. 15
   d. 13

v. The construction of the cumulative frequency table is useful in determining the:
   a. Mean

b. Median

c. Mode

d. All of the above

20. 

A mathematics teacher took her grade X students to the Taj Mahal. It was an educational trip. She was interested in history also. On reaching there she told them about the history and facts about the seventh wonder. She also told them that the structure of the monument is a combination of several solid figures. There are 4 pillars that are cylindrical in shape. A big dome in the center and 2 more small domes on both sides of the big dome on its side. The domes are hemispherical. The pillars also have domes on them.

i. How much cloth material will be required to cover a big dome of a diameter of 7m?

a. 77 m^2

b. 78 m^2

c. 79 m^2

d. 80 m^2

ii. Write the formula to calculate the volume of the pillar.

a. $\pi r^2 h + \pi r^3$

b. $\pi r^2 h + \frac{2}{3}\pi r^2 l$

c. $\pi r l + \frac{2}{3}\pi r^3$

d. $\pi r^2 h + \frac{2}{3}\pi r^3$

iii. How much is the volume of the hemisphere if the radius of the base is 3 m?

a. 65.57 m^3

b. 75.77 m^3

c. 56.57 m^3

d. 85.57 m^3

iv. Find the curved surface area of 4 pillars if the height of pillars is 7.5 m and the radius of the base is 2.5 m.

a. 768.56 m^2

b. 658.56 m^2

c. 766.56 m^2

d. 628.57 m^2

v. What is the ratio of the sum of volumes of two-cylinder of radius 1 cm and height 2 cm each to the volume of a sphere of radius 3 cm?

a. 2:3

b. 3:2

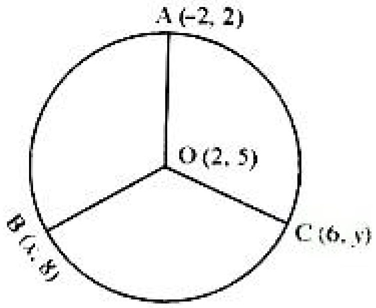
c. 1:1

d. 1:2

Part-B

21. Prove that $5-\sqrt{3}$ is an irrational number.

22. (-2, 2) (x, 8) and(6, y) are three concyclic points whose centre is (2, 5). Find the possible values of x and Y.



OR

Show that the points A (a, a), B (-a, -a) and C $(-a\sqrt{3}, a\sqrt{3})$ form an equilateral triangle.

23. Find the zeroes of a quadratic polynomial given as $3x^2 - x - 4$ and also verify the relationship between the zeroes and the coefficients.

24. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60°.

25. Prove the trigonometric identity: $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$

OR

Prove that $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

26. Two circles touch each other externally at C. AB and CD are two common tangents. If D lies on AB such that CD = 6 cm, then find AB.

27. If $\frac{241}{4000} = \frac{241}{2^m 5^n}$ find the values of m and n where m and n are non-negative integers. Hence, write its decimal expansion without actual division.

28. Ashu is x years old while his mother Mrs Veena is x^2 years old. Five years hence Mrs Veena will be three times old as Ashu. Find their present ages.

OR

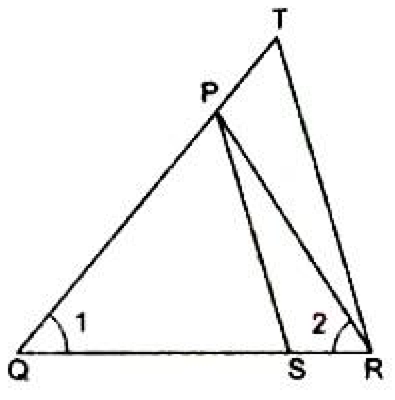
A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

29. Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients.

30. In a trapezium ABCD, diagonals AC and BD intersect at O. If AB = 3CD, then find ratio of areas of triangles COD and AOB.

OR

In Fig. if $\frac{QT}{PR} = \frac{QR}{QS}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$.



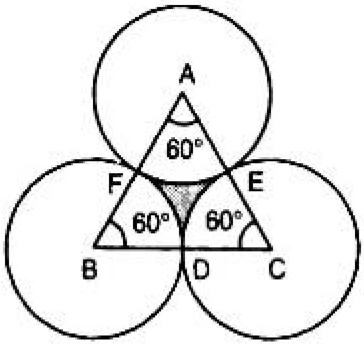
31. From all the two-digit numbers, a number is chosen at random. Find the probability that the chosen number is a multiple of 7.

32. The angles of elevation of the top of the tower from two points P and Q at distances of a and b respectively from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$.

33. A frequency distribution of the life times of 400 T.V. picture tubes tested in a tube company is given below. Find the average life of tube.

Life time (in hrs)	Frequency	Life time (in hrs)	Frequency
300-399	14	800-899	62
400-499	46	900-999	48
500-599	58	1000-1099	22
600-699	76	1100-1199	6
700-799	68		

34. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles. [Take $\sqrt{3}$ = 1.73.]



35. Draw the graphs of the following equations on the same graph paper:

$2x + 3y = 12$

$x - y = 1$

Find the coordinates of the vertices of the triangle formed by the two straight lines and the y-axis.

36. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant.

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Solution

Part-A

1. $\frac{427}{625} = \frac{427}{5^4} = \frac{427}{2^0 \cdot 5^4}$

   Here, the denominator is $2^0 \times 5^4$ which is of the form $2^m \times 5^n$, where m and n are non-negative integers.

   Hence, $\frac{427}{625}$ has terminating decimal expansion.

   OR

   Irrational numbers can be represented as non-terminating and non-repeating decimal expansions

2. Given $(x + 4)^2 - 8x = 0$

   $\Rightarrow x^2 + 8x + 16 - 8x = 0$

   $\therefore x^2 + 16 = 0$

   $\therefore D = b^2 - 4ac = (0) - 4(1)(16) < 0$

   Hence, the equation $(x + 4)^2 - 8x = 0$ has no real roots.

3. Given system of equations $x + y - 4 = 0$ and $2x + ky - 3 = 0$. With $a_1 = 1, b_1 = 1, c_1 = -4$ and $a_2 = 2, b_2 = k, c_2 = -3$

   For no solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

   then, $\frac{1}{2} = \frac{1}{k} = \frac{-4}{-3}$ ...... (1)

   then $\frac{1}{2} = \frac{1}{k}$

   $\Rightarrow k = 2$

   Substitute k = 2 in (1) we get $\frac{1}{k} = \frac{1}{2} \neq \frac{4}{3}$

   Hence, $k \neq \frac{3}{4}$ for no solution

4. The common point of a tangent to a circle and the circle is called a **point of contact.**

5. $a1 = a,\ d = (a + d) - a = d$

$a_{30} - a_{20} = (a + 29d) - (a + 19d) = 10d$

OR

Given $\frac{4}{5}$, a, 2 are in AP

$\Rightarrow$ common difference d = a - $\frac{4}{5}$ = 2 - a

$\Rightarrow$ 5a - 4 = 10 - 5a

$\Rightarrow$ 10a = 14

$\Rightarrow$ a = $\frac{7}{5}$

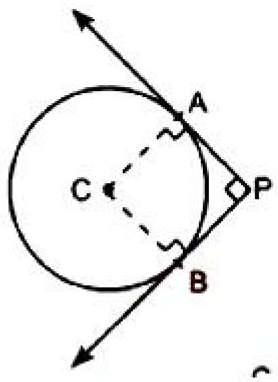
6. $S_n = 2n^2 + 5n$
   $S_{20} = 2(20)^2 + 5(20)$
   $= 2(400) + 100 = 900.$

7. A polynomial equation is a quadratic equation if it is of the form $ax^2 + bx + c = 0$ such that $a \neq 0$
   for $x^2 + 6x - 4 = 0$, it is a quadratic equation.

OR

$2x^2 - 7x$ is a quadratic equation because the highest degree of x is 2.

8. PA and PB are two tangents drawn from an external point P to a circle.



CA $\perp$ AP

CB $\perp$ BP

PA $\perp$ PB

$\therefore$ BPAC is a square.

$\Rightarrow$ AP = PB = BC = 4 cm

9. QP = 3.8

QP = PT (Length of tangents from the same external point are equal)

Therefore, PT = 3.8 cm

Also, PR = PT = 3.8 cm

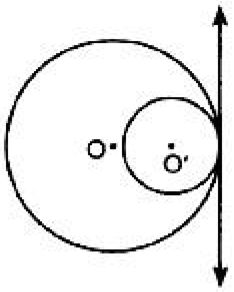
Now, QR = QP + PR

QR = 3.8 + 3.8 = 7.6 cm.

OR

1 common tangent can be drawn to two circles touching internally

Figure:



10. In right-angled $\triangle$ABC, right angled at C

$\Rightarrow AB^2 = AC^2 + BC^2$ [By pythagoras theorem]

$\Rightarrow AC^2 + AC^2 = 2AC^2$ [$\because$ BC = AC (given)]

$\Rightarrow AB^2 = 2AC^2$

11. $T_n = (5n - 2)$ (given)

$\Rightarrow T_1 = [(5 \times 1) - 2] = 3$ and $T_2 = [(5 \times 2) - 2] = 8$

Thus, we have19th term = a + (19 -1) d, where a = 3 and d = 5

= (3 + 18 × 5) = 93

12. Given ,

$\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \sqrt{3}\tan 2x = 1$

$\Rightarrow \tan 2x = \frac{1}{\sqrt{3}}$

$\Rightarrow$ tan 2x = tan30 ($Since, \tan 30^\circ = \sqrt{\frac{1}{3}}$)

$\Rightarrow 2x = 30^{\circ}$

$\Rightarrow x = 15^{\circ}$

13. We know that $\sin^2\theta + \cos^2\theta = 1$

    Therefore, $(\sin^2\theta + \cos^2\theta)^3 = 1$

    or, $(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) = 1$

    or, $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

14. Here, h = 40 cm

    When the sheet is rolled 22cm becomes the circumference of the base circles.

    Circumference = 22 cm

    $2\pi r = 22cm$

    or, $r = \frac{22\times7}{2\times22}$

    or, $r = \frac{7}{2}$

    = 3.5 cm

    Therefore, the radius of the cylinder is 3.5 cm

15. Let the sum of first n terms of A.P. = S_n

    Given, Sn = $5n - n^2$

    Now, n^{th} term of A.P. = $S_n - S_{n-1}$

    or, $a_n = (5n - n^2) - [5(n-1) - (n-1)2]$

    $= 5n - n^2 - [5n - 5 - (n^2 + 1 - 2n)]$

    $= 5n - n^2 - (5n - 5 - n^2 - 1 + 2n)$

    $= 5n - n^2 - 5n + 5 + n^2 + 1 - 2n$

    $= 5n - n^2 - 7n + 6 + n^2$

    $= -2n + 6$

    or, $a_n = -2(n-3)$

    $\therefore n^{th}$ term $= -2(n-3)$

16. Given: A card is drawn at random from a pack of 52 cards

    TO FIND: Probability of the following

    Total number of cards = 52

    Total number of the ace of the spade is 1

    We know that PROBABILITY $= \frac{\text{Number of favourable event}}{\text{Total number of event}}$

    Hence the probability of getting an ace of spade = $\frac{1}{52}$

17. Let Home represented by point H(4, 5), Library by point L(-1, 3), Skate Park by point P(3, 0) and School by S(4, 2).

i. (b) Distance between Home and School, HS = $\sqrt{(4-4)^2+(2-5)^2}$ = 3 metres

ii. (c) Now, HL = $\sqrt{(-1-4)^2+(3-5)^2}=\sqrt{25+4}=\sqrt{29}$

LS = $\sqrt{[4-(-1)]^2+(2-3)^2}=\sqrt{25+1}=\sqrt{26}$

Thus, HL + LS = $\sqrt{29}+\sqrt{26}$ = 10.48 metres

So, extra distance covered by Ramesh is = HL + LS - HS = 10.48 - 3 = 7.48 metres

iii. (d) Now, HP = $\sqrt{(3-4)^2+(0-5)^2}=\sqrt{1+25}=\sqrt{26}$

PS = $\sqrt{[4-3]^2+(2-0)^2}=\sqrt{1+4}=\sqrt{5}$

Thus, HP + PS = $\sqrt{26}+\sqrt{5}$ = 7.33 metres

So, extra distance covered by Pulkit is = HP + PS - HS = 7.33 - 3 = 4.33 metres

iv. (a) (-1, 3)

v. (c) (4, 5)

18. i. (c) 100 m

ii. (d) 60 m

iii. (b) 40 m

iv. (a) 16 m

v. (d) 8 m

19. Sum of the frequencies = 181

$\Rightarrow$ x + 15 + 18 + 30 + 50 + 48 + x = 181

$\Rightarrow$ 2x + 161 = 181

$\Rightarrow$ x = 10

Thus, the missing frequencies are 10 and 10.

Clearly, the modal class is 13 - 15, as it has the maximum frequency.

$\therefore l=13$, h = 2, f_1 = 50, f_0 = 30, f_2 = 48

Mode, M_o = $l+\left\{h\times\frac{(f_1-f_0)}{(2f_1-f_0-f_2)}\right\}$

$=13+2\left\{\frac{50-30}{2(50)-30-48}\right\}$

$=13+2\times\frac{20}{22}$

= 13 + 1.81 = 14.81

i. (d) 14.81

ii. (b) 10

iii. (a) 13 - 15

iv. (c) 15

v. (b) Median

20. i. (a) 77 m^2

ii. (d) $\pi r^2 h + \frac{2}{3}\pi r^3$

iii. (c) 56.57 m^3

iv. (d) 628.57 m^2

v. (c) 1:1

Part-B

21. We will solve it by contradiction method i.e., assume that $5-\sqrt{3}$ is rational.

Then, there exist co-prime positive integers a and b such that

$5-\sqrt{3}=\frac{a}{b}$
$\Rightarrow 5-\frac{a}{b}=\sqrt{3}$
$\Rightarrow \frac{5b-a}{b}=\sqrt{3}$
$\Rightarrow \frac{5b-a}{b}=\sqrt{3}$

Since a, b are integers, therefore $\sqrt{3}$ is a rational number which is a contradiction.

So, our assumption is incorrect.

Hence, $5-\sqrt{3}$ is an irrational number.

22. Let O(2, 5) be the centre of the circle and A(-2, 2), B(x, 8) and C(6, y) be points on the circumference.

Here, OA = OB = OC = radius of circle

$\therefore OA^2 = OB^2 = OC^2$

Now $OB^2 = OA^2$

$\Rightarrow (x - 2)^2 + (8 - 5)^2 = (2 + 2)^2 + (5 - 2)^2$

$\Rightarrow x^2 + 4 - 4x + 3^2 = 4^2 + 3^2$

$\Rightarrow x^2 - 4x + 4 + 9 = 16 + 9$

$\Rightarrow x^2 - 4x + 4 - 16 = 0$

$\Rightarrow x^2 - 4x - 12 = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x - 6) + 2(x - 6) = 0$

$\Rightarrow$ (x - 6) (x + 2) = 0

$\Rightarrow$ x - 6 = 0 or x + 2 = 0

$\therefore$ x = 6 or x = -2

Similarly $OC^2 = OA^2$

$\Rightarrow (6 - 2)^2 + (y - 5)^2 = (2 + 2)^2 + (5 - 2)^2$

$\Rightarrow 4^2 + y^2 + 25 - 10y = 4^2 + 3^2$

$\Rightarrow 16 + y^2 + 25 - 10y = 16 + 9$

$\Rightarrow y^2 - 10y + 25 - 9 = 0$

$\Rightarrow y^2 - 10y + 16 = 0$

$\Rightarrow y^2 - 8y - 2y + 16 = 0$

$\Rightarrow$ y(y - 8) -2(y - 8) = 0

$\Rightarrow$ (y - 8) (y - 2) = 0

$\Rightarrow$ y - 8 = 0 or y - 2 = 0

$\therefore$ y = 8 or y = 2

OR

AB = $\sqrt{(-a-a)^2+(-a-a)^2} = \sqrt{(-2a)^2+(-2a)^2}$
$= \sqrt{4a^2+4a^2} = \sqrt{8a^2} = 2\sqrt{2}a$
BC = $\sqrt{(-a\sqrt{3}+a)^2+(a\sqrt{3}+a)^2} = \sqrt{3a^2+a^2-2\sqrt{3}a^2+3a^2+a^2+2\sqrt{3}a^2}$
$= \sqrt{8a^2} = 2\sqrt{2}a$
AC = $\sqrt{(-a\sqrt{3}-a)^2+(a\sqrt{3}-a)^2} = \sqrt{3a^2+a^2+2\sqrt{3}a^2+3a^2+a^2-2\sqrt{3}a^2}$
$= \sqrt{8a^2} = 2\sqrt{2}a$

Since AB = BC = AC

$\therefore \triangle$ABC is equilateral.

23. The quadratic equation is given as: $3x^2 - x - 4$

(Now we will factorize 1 in such a way that the product of factors is equal to -12 and the sum is equal to 1)

$= 3x^2 - 4x + 3x - 4$

= x(3x - 4) + 1(3x - 4)= (3x – 4)(x + 1)

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1

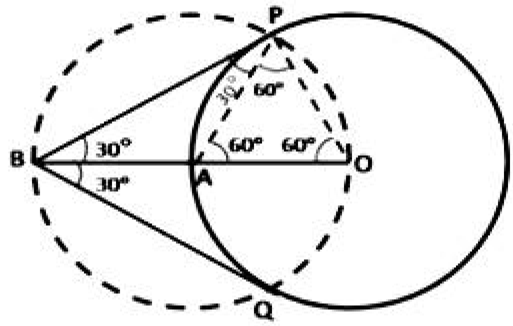
Sum of zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes = $\frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence verified.

24. Steps of construction:

   1. Take a point O on the plane of the paper and draw a circle of radius OA = 4cm.
   2. Produce OA to B such that OA = AB = 4cm
   3. Draw a circle with centre at A and radius AB.
   4. Suppose it cuts the circle drawn in step (i) at P and Q.
   5. Join BP and BQ to get the desired tangets.



Justification:

In $\triangle$OAP, OA = OP = 4 cm.. (radii of the same circle)

Also, AO = 4cm ..(Radius of the circle with centre A)

$\triangle$OAP is equilateral.

$\angle$PAO = 60°

$\therefore$ $\angle$BAP = 120°

In BAP, we have BA = AP and $\angle$BAP = 120°

$\therefore$ $\angle$ABP = $\angle$APB = 30°

Similary, we can get $\angle$ABQ = 30°

$\therefore$ $\angle$PBQ = 60°

25. We have,

$$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad \text{LHS} = \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$\Rightarrow \quad \text{LHS} = \frac{(\sin^2\theta+\cos^2\theta)+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$

$\Rightarrow \quad \text{LHS} = \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \quad \text{LHS} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$

OR

$\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

$\text{L.HS.} = \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

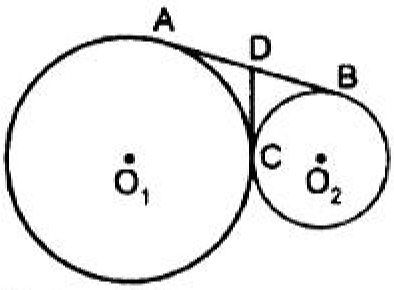
$= \frac{\sin^2 A\cos^2 B - \cos^2 A\sin^2 B}{\cos^2 A\cos^2 B} = \frac{\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B}{\cos^2 A\cos^2 B}$

$[\because \sin^2 A + \cos^2 A 1 \text{ or } \cos^2 A = 1 - \sin^2 A]$

$= \frac{\sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B}{\cos^2 A\cos^2 B}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

$\therefore \text{L. HS.} = \text{R. HS}$

26. 

$AD = CD$[Tangents drawn from an exterior point are equal]

$\therefore AD = 6cm$

Also $DB = CD$[Tangents drawn from an exterior point are equal]

$\therefore DB = 6cm$

$\Rightarrow AB = 6 + 6 = 12cm.$

27. According to question,

$\frac{241}{4000} = \frac{241}{2^m 5^n}$

$\Rightarrow \frac{241}{2^5\times 5^3} = \frac{241}{2^m 5^n}$

$\Rightarrow m = 5, n = 3$

Now, $\frac{241}{4000} = \frac{241}{2^5\times 5^3}$

$= \frac{241\times 5^2}{2^5\times 5^3\times 5^2}$ (by multiplying and dividing by 5^2)

$= \frac{6025}{(2\times5)^5}$

$= \frac{6025}{(10)^5}$

= 0.06025

28. We have,

Ashu's present age = x years,

Mrs Veena's present age = x^2 years

Five years hence, we have

Ashu's age = (x + 5) years

Mrs Veena's age = $(x^2 + 5)$ years

It is given that five years hence Mrs Veena will be three times old as Ashu.

$\therefore x^2 + 5 = 3(x + 5)$

$\Rightarrow x^2 + 5 = 3x + 15$

$\Rightarrow x^2 - 3x + 5 - 15 = 0$

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x = 5$ [$\because$ Age can never negative $\therefore x + 2 \neq 0$]

Hence, Mrs Veena's present age = $x^2 = (5)^2 = 25$ years

And, Ashu's present age = 5 years.

OR

Let the present age of girl's sister be 'x' years.

So, girl's present age = 2x years.

After 4 years;

Girl's age =(2x+4) years.

Sister's age =(x+4)years.

According to the question ;

$(x + 4)(2x + 4) = 160$ ($\because$ product of their ages 4 years hence is 160

$\Rightarrow 2x^2 + 12x - 144 = 0 \Rightarrow x^2 + 6x - 72 = 0$

$\Rightarrow x^2 + 12x - 6x - 72 = 0 \Rightarrow x(x + 12) - 6(x + 12) = 0$

$\Rightarrow (x + 12)(x - 6) = 0 \Rightarrow x + 12 = 0$ or $x - 6 = 0$

$\Rightarrow x = -12$ or $x = 6$

$\Rightarrow x = 6$ [$\because$ age cannot be negative]

Hence, sister's present age =x= 6 years and girl's present age = 2x = 12 years

29. Here, $p(x) = 3x^2 - 2$.

Now $p(x) = 0$

$\Rightarrow \quad 3x^2 - 2 = 0$

$\Rightarrow 3x^2 = 2$

$\Rightarrow \quad x^2 = \frac{2}{3}$

$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$

Therefore, zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.

If $p(x) = 3x^2 - 2$, then a = 3, b = 0 and c = -2

Now, sum of zeroes = $\sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right) = 0$ .... (i)

Also, $\frac{-b}{a} = \frac{-0}{3} = 0$ .... (ii)

From (i) and (ii)

Sum of zeroes= $\frac{-b}{a}$

and product of zeroes = $\sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3}$ .... (iii)

Also, $\frac{c}{a} = \frac{-2}{3}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

30. In ΔAOB and ΔCOD

$\angle AOB = \angle COD$

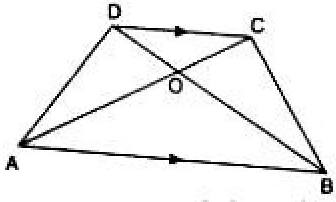
$\angle ABO = \angle CDO$

Hence $\triangle AOB \sim \triangle COD$

$\frac{\text{ar}(\Delta COD)}{\text{ar}(\Delta AOB)} = \frac{CD^2}{AB^2}$

$= \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9}$

ratio = 1 : 9



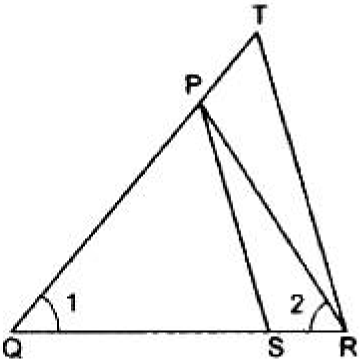
OR

According to the question, we are given that,

$\frac{QT}{PR} = \frac{QR}{QS}$ [Given]

$\Rightarrow \quad \frac{QT}{QR} = \frac{PR}{QS}$ ...(i)

We also have,



$\angle 1 = \angle 2$ [Given]

$\Rightarrow$ PR = PQ [Sides opposite to equal angles are equal] ... (ii)

From (i) and (ii), we get,

$\frac{QT}{QR} = \frac{PQ}{QS}$

$\Rightarrow \quad \frac{PQ}{QT} = \frac{QS}{QR}$ ...(iii)

Thus, in triangles PQS and TQR, we have

$\frac{PQ}{QT} = \frac{QS}{QR}$ and $\angle PQS = \angle TQR = \angle Q$

So, by SAS-criterion of similarity, we obtain $\Delta PQS \sim \triangle TQR$.

31. Two digits numbers are from 10 to 99.

Total no. of all possible outcomes = 99 - 10 +1 = 90

Two-digit numbers which are multiples of 7 are 14,21,28 98

No. of favourable outcomes = 13

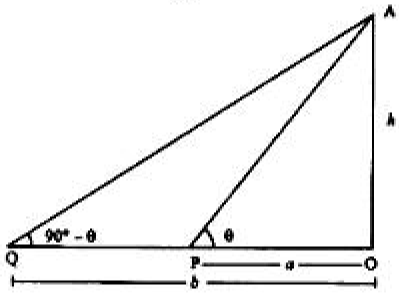
P(getting a number multiple of 7) = $\frac{13}{90}$

32. Let OQ be a horizontal line. Let, A be the top of the tower OA.

From P and Q, θ and $(90^o - \theta)$ respectively be the angles of elevation of the top of the tower A.

It is given that OP = a and OQ = b.

Let h be height of the tower OA, i.e. AO = h.



In the right triangles AOP, right angles at O,

$\tan\theta = \frac{h}{a}$ ....(i)

In right triangle AOQ, right angles at O, we have

$\tan(90^\circ - \theta) = \frac{h}{b} \Rightarrow \cot\theta = \frac{h}{b}$

$\Rightarrow \tan\theta = \frac{b}{h}$ ....(ii)

From (i) and (ii), we get $\frac{h}{a} = \frac{b}{h} \Rightarrow h^2 = ab$

$\therefore h = \sqrt{ab}$ proved.

33.

Life time (in hrs)	**Frequency f_i**	**Mid-Values x_i**	**$d_i = x_i - A = x_i - 749.5$**	$u_i = \frac{x_i - A}{h}$ $u_i = \frac{x_i - 749.5}{100}$	**$f_i u_i$**
300-399	14	349.5	-400	-4	-56
400-499	46	449.5	-300	-3	-138
500-599	58	549.5	-200	-2	-116
600-699	76	649.5	-100	-1	-76
700-799	68	749.5	0	0	0
800-899	62	849.5	100	1	62
900-999	48	949.5	200	2	96
1000-1099	22	1049.5	300	3	66
1100-1199	6	1149.5	400	4	24
	$N = \Sigma f_i = 400$				$\Sigma f_i u_i = -138$

Let the assumed mean be A=749.5.

We have, N=400, A=749.5, h=100 and $\Sigma f_i u_i = -138$

$mean = \overline{x} = A + h\left\{\frac{1}{N}\Sigma f_i u_i\right\}$

$\Rightarrow \overline{x} = 749.5 + 100 \times \left(\frac{-138}{400}\right) = 749.5 - \frac{138}{4} = 749.5 - 34.5 = 715$

34. Area of equilateral triangle ABC

$= 49\sqrt{3}\text{cm}^2$

Let a be its side

$\therefore \frac{\sqrt{3}}{4}a^2$

$= 49\sqrt{3}$

or $a^2 = 49 \times 4$

$\therefore a = 7 \times 2$

$\Rightarrow a = 14\text{cm}$

Area of sector BDF = $\pi r^2 \times \frac{\theta}{360^\circ}$

$= \frac{22}{7} \times 7 \times 7 \times \frac{60}{360}\text{cm}$

$= \frac{11 \times 7}{3}\text{cm}^2$

$= \frac{77}{3}\text{cm}^2$

Area of sector BDF

= Area of sector CDE

= Area of sector AEF

Sum of area of all the sectors

$= \frac{77}{3} \times 3\text{cm}^2$

= 77cm^2

$\therefore$ Shaded area = Area of $\triangle$ABC - Sum of the area of all sectors

$= 49\sqrt{3} - 77\text{cm}^2$

= $(84.77 - 77.00)\text{cm}^2$

= 7.77 cm^2

35. $2x + 3y = 12$

$x = \frac{12-3y}{2}$

When y = 0, then x = 6

When y = 2, then x = 3

x	6	3
y	0	2

We have,

$x - y = 1$

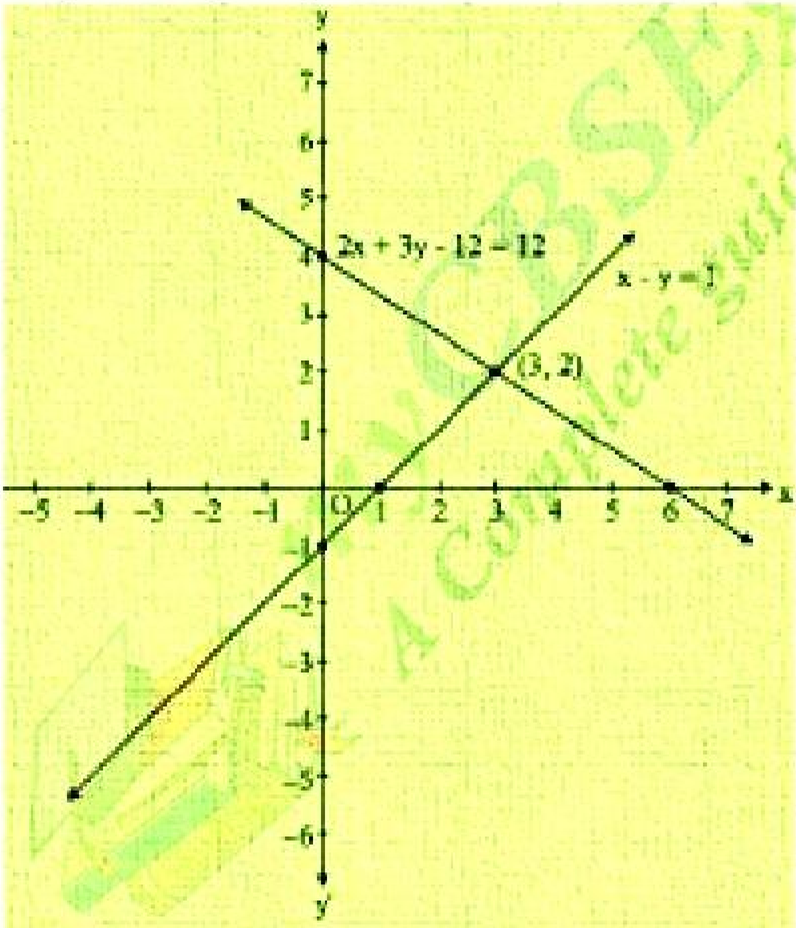
$x = 1 + y$

When y = 0, then x = 1

When y = -1, then x = 0

x	1	0
y	0	-1

Graph of the given system is:



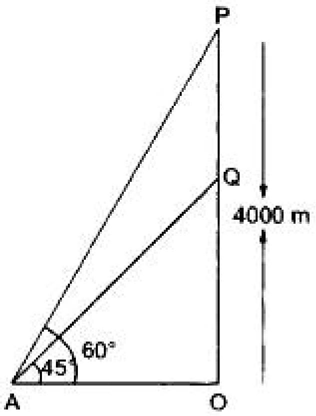
Clearly, the two lines intersect at $A(3, 2)$

We also observe that the lines meet y-axis B(0, -1) and C(0, 4)

Hence the vertices of the required triangle are $A(3, 2), B(0, -1)\ and\ C(0, 4)$.

36. Let P and Q be the positions of two aeroplanes when Q is vertically below P and OP = 4000 m. Let the angles of elevation of P and Q at a point A on the ground be 60° and 45° respectively.

In triangles AOP and AOQ, we have



$$\tan 60^\circ = \frac{OP}{OA} \text{ and } \tan 45^\circ = \frac{OQ}{OA}$$
$$\Rightarrow \sqrt{3} = \frac{4000}{OA} \text{ and } 1 = \frac{OQ}{OA}$$
$$\Rightarrow OA = \frac{4000}{\sqrt{3}} \text{ and } OQ = OA$$
$$\Rightarrow OQ = \frac{4000}{\sqrt{3}} \text{m}$$

∴ Vertical distance PQ between the aeroplanes is given by

PQ = OP - OQ

$$\Rightarrow PQ = \left(4000 - \frac{4000}{\sqrt{3}}\right) \text{m}.$$
$$= 4000\frac{(\sqrt{3}-1)}{\sqrt{3}}\text{m} = 1690.53\text{m}$$