CBSE Class 10 Mathematics Basic

Sample Paper - 05 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. If $\frac{p}{q}$ is a rational number ($q \neq 0$), what is condition of q so that the decimal expansion of $\frac{p}{q}$ is terminating?

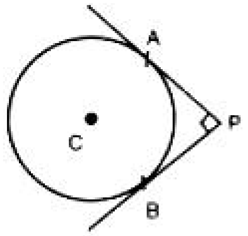
OR

Find the HCF and LCM of 11008 and 7344 using fundamental theorem of arithmetic.

2. Find the roots of the following quadratic equation by factorisation: $x^2 - 9x + 20 = 0$.
3. For what value of k the following pair of linear equations has unique solution?
$7x + 8y = k$

$9x - 4y = 12$

4. In fig., PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA ⊥ PB, then find the length of each tangent.



5. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$ an A.P.? Give reason.

OR

Find $a_{30} - a_{20}$ for the AP in -9, -14, -19, -24 ....

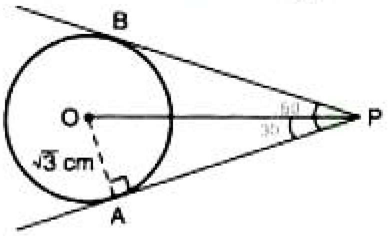
6. Find 10^{th} term of the A.P. -40, -15, 10, 35, ...

7. Write the set of values of 'a' for which the equation $x^2 + ax - 1 = 0$ has real roots.

OR

State whether $3x^2 - 4x + 2 = 2x^2 - 2x + 4$ is a quadratic equation or not?

8. Two tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm then find the length of each tangent.

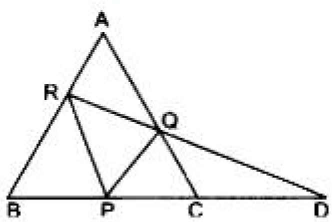


9. At which point a tangent is perpendicular to the radius?

OR

What term will you use for a line which intersect a circle at two distinct points?

10. In the given figure PQ | | BA; PR | | CA.



If PD = 12 cm, Find BD × CD.

11. For an AP, if $a_{18} - a_{14} = 32$ then find the common difference d.
12. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A - 1$
13. Prove that : -

    $$\left(1 - \sin^2\theta\right)\sec^2\theta = 1$$
14. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.
15. If k, (2k-1) and (2k+ 1) are the three successive terms of an AP, find the value of k.
16. A single letter is selected at random from the word "PROBABILITY". Find the probability that it is vowel.
17. **2-DIMENSINAL PLANE/ CARTESIAN PLANE**

    Using Cartesian Coordinates we mark a point on a graph by **how far along** and **how far up** it is.

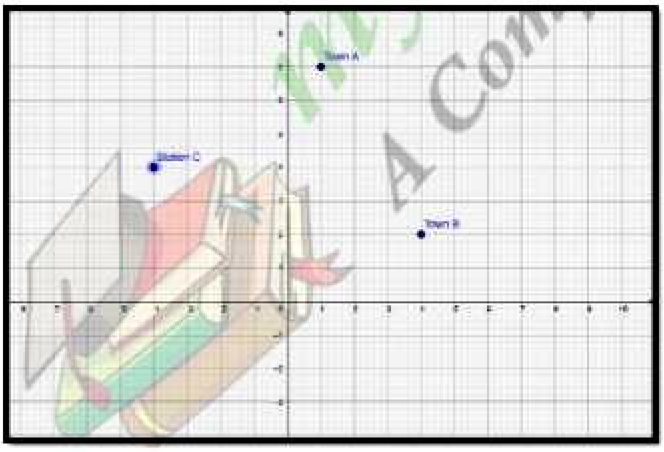
    The left-right (**horizontal**) direction is commonly called X-axis.

    The up-down (**vertical**) direction is commonly called Y-axis.

    When we include negative values, the x and y axes divide the space up into 4 pieces.

    Read the information given above and below and answer the questions that follow:

    Two friends Seema and Aditya work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



i. Who will travel more distance to reach to their hometown?
   a. Seema
   b. Aditya
   c. Both travel the same distance

d. None of these

ii. Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the point represented by Town A and Town B. Then the coordinates of the point represented by the point D are:

a. $\left(\frac{2}{5}, \frac{9}{2}\right)$

b. $\left(\frac{5}{2}, \frac{2}{9}\right)$

c. $\left(\frac{9}{2}, \frac{5}{2}\right)$

d. $\left(\frac{5}{2}, \frac{9}{2}\right)$

iii. The area of the triangle formed by joining the points represented by A, B and C is:

a. 17 sq. units

b. 27 sq. units

c. 7 sq. units

d. 15 sq. units

iv. The location of the station is given by:

a. (4, -4)

b. (-4, 4)

c. (-2, 4)

d. (4, 2)

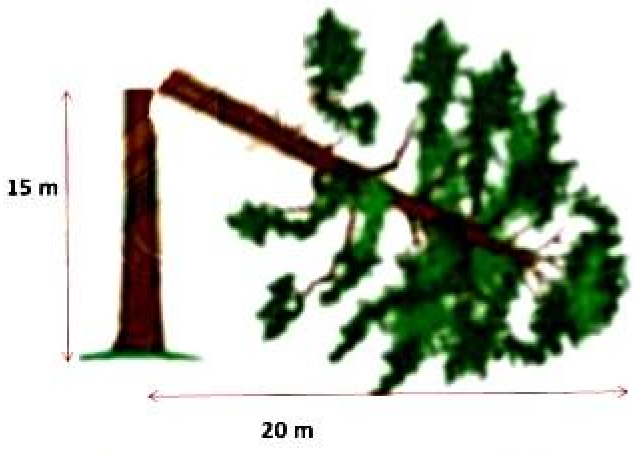
v. The location of the Town B is given by:

a. (4, -4)

b. (1, 7)

c. (2, 4)

d. (4, 2)

18. 

Suresh is having a garden near Delhi. In the garden, there are different types of trees and flower plants. One day due to heavy rain and storm one of the trees got broken as shown in the figure.

The height of the unbroken part is 15 m and the broken part of the tree has fallen at 20 m away from the base of the tree.

Using the Pythagoras answer the following questions:

i. What is the length of the broken part?
   a. 15 m
   b. 20 m
   c. 25 m
   d. 30 m

ii. What was the height of the full tree?
   a. 40 m
   b. 50 m
   c. 35 m
   d. 30 m

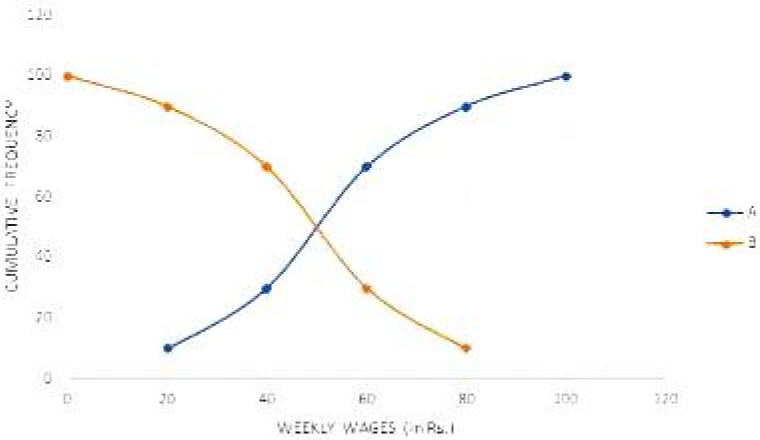
iii. In the formed right-angle triangle what is the length of the hypotenuse?
   a. 15 m
   b. 20 m
   c. 25 m
   d. 30m

iv. What is the area of the formed right angle triangle?
   a. 100 m^2
   b. 200 m^2
   c. 60 m^2
   d. 150 m^2

v. What is the perimeter of the formed triangle?
   a. 60 m
   b. 50 m
   c. 45 m
   d. 100 m

19. A Mall is constructing in the city, Jaipur. 100 workers are working in the Mall. The data of the distribution of weekly wages of 100 workers are recorded and the following graph is

made:



Based on the above graph, answer the following questions:

i. Identify less than type ogive from the given graph.
   a. A
   b. point of intersection of A and B
   c. B
   d. none of these

ii. Find the Median Wages.
   a. Rs.60
   b. Rs.150
   c. Rs.50
   d. Rs.55

iii. Find Mode of the data if Mean Wages is Rs. 50
   a. Rs.52
   b. Rs.60
   c. Rs.55
   d. Rs.50

iv. The construction of the cumulative frequency table is useful in determining the:
   a. Median
   b. Mean
   c. Mode
   d. All of the above

v. The intersection of the Ogive graph(abscissa) represents which of the following:
   a. Mean
   b. Median

c. Mode

d. All of these

20. To make the teaching, learning process easier, creative, and innovative, A teacher brings clay in the classroom to teach the topic mensuration. She thought this method of teaching is more interesting, leave a long-lasting impact She forms a cylinder of radius 6 cm and height 8 cm with the clay, then she moulds the cylinder into a sphere and asks some question to students [use π = 3.14]

i. The radius of the sphere so form:

a. 6 cm

b. 7 cm

c. 4 cm

d. 8 cm

ii. The volume of the sphere so formed:

a. 902.32 cm^3

b. 899.34 cm^3

c. 904.32 cm^3

d. 999.33 cm^3

iii. What is the ratio of the volume of a sphere to the volume of a cylinder?

a. 1:2

b. 2:1

c. 1:1

d. 3:1

iv. The total surface area of the cylinder is:

a. $525.57\ cm^2$

b. $557.55\ cm^2$

c. $534.32\ cm^2$

d. $527.52\ cm^2$

v. During the conversion of a solid from one shape to another the volume of the new shape will:

a. increase

b. decrease

c. remain unaltered

d. be double

Part-B

21. The HCF of two numbers is 27 and their LCM is 162. If one of the number is 81, find the other.

22. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

OR

Find the point on y-axis which is equidistant from (–5, –2) and (3, 2).

23. A teacher told 8 students to write a polynomial on the blackboard. Students wrote the following polynomials:

(i) $x^2 + 9$	(v) $x^3 + 5x + 2x + 6$
(ii) $x^3 + 2x^2 + x + 5$	(vi) $5x + 6$
(iii) $2x^4 + 3x^3 + 2x + 7$	(vii) $x^4 + x^3 - 5x^2 + 3x + 8$
(iv) $x^2 + 5x + 6$	(viii) $x^2 - 7x + 12$

i. How many students wrote quadratic polynomials?

ii. If α and β are zeros of the polynomial $x^2 + 5x + 6$, then what is the value of α + β?

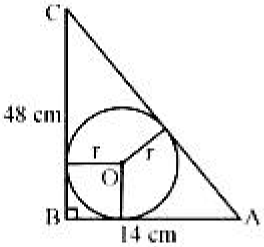
24. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.

25. Prove that: $\frac{\cos^2\theta}{\sin\theta} - cosec\theta + \sin\theta = 0$

OR

Given that sin θ + 2 cos θ = 1, then prove that 2 sin θ - cosθ = 2.

26. In the given figure, ABC is a triangle in which $\angle B = 90^o$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius r of in-circle.



27. Prove that $6+\sqrt{2}$ is irrational.

28. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{5}{14}$ Find the numbers.

OR

Solve the quadratic equation by factorization:

$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x \neq 1,-5$

29. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

30. Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

OR

Find the length of altitude AD of an isosceles $\triangle$ABC in which AB = AC = 2a units and BC = a units.

31. i. Two dice, one blue and one grey, are thrown at the same time. Complete the following table:

Event: Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

ii. A student argues that there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

32. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant

height of $3600\sqrt{3}$ m, find the speed of the jet plane.

33. The arithmetic mean of the following frequency distribution is 53. Find the missing frequency p :

Class	0 - 20	20 -40	40 -60	60 -80	80 -100
Frequency	12	15	32	p	13

34. The lengths of the sides of a triangle are in the ratio 3:4:5, and its perimeter is 144 cm. Find (i) the area of the triangle, and (ii) the height corresponding to the longest side.

35. Solve for x and y: $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4}, \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

36. At the foot of a mountain the elevation of its summit is 45°; after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.

CBSE Class 10 Mathematics Basic

Sample Paper - 05 (2020-21)

Solution

Part-A

1. The rational number of the form , $\frac{p}{q}$ will have a terminating decimal expansion,

   If q is power of 10

   or q is power of 2×5

   or q is of form $2^n \times 5^m$

   Any rational number $\frac{p}{q}$ will have terminating decimal expansion,if q is of the form $2^n \times 5^m$

   where n and m are positive integers.

   OR

   11008 =$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 43$

   = $2^8 \times 43$

   7344 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 17$

   = $2^4 \times 3^3 \times 17$

   HCF = 2^4 = 16

   LCM = $2^8 \times 43 \times 3^3 \times 17$= 5052672

2. Given equation is $x^2 - 9x + 20 = 0$

   $\Rightarrow x^2 - 5x - 4x + 20 = 0$

   $\Rightarrow x(x - 5) - 4(x - 5) = 0$

   $\Rightarrow (x - 5)(x - 4) = 0$

   $\Rightarrow$ either $x - 5 = 0$ or $x - 4 = 0$

   $\Rightarrow x = 5$ or $x = 4$

   $\therefore$ x = 4 or 5 are the roots of the given quadratic equation.

3. we have, $7x + 8y = k \Rightarrow 7x + 8y - k = 0$

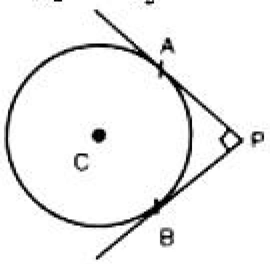
   Also, $9x - 4y = 12 \Rightarrow 9x - 4y - 12 = 0$

   Here, $a_1 = 7, b_1 = 8, c_1 = -k,$

   $a_2 = 9, b_2 = -4, c_2 = -12$

Now, $\frac{a_1}{a_2} = \frac{7}{9}, \frac{b_1}{b_2} = \frac{8}{-4}$ = -2, $\frac{c_1}{c_2} = \frac{k}{12}$

$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

4. 

Construction: Join AC and BC

Now, $AC \perp AP\ and\ CB \perp BP$

$\angle APB = 90^0$

Therefore, CAPB will be a square

CA = AP = PB = BC = 4 cm

$\therefore$ Length of tangent = 4 cm.

5. Common difference,

$d_1 = \sqrt{6} - \sqrt{3}$

$= \sqrt{3}(\sqrt{2} - 1)$

$d_2 = \sqrt{9} - \sqrt{6}$

$= \sqrt{3 \times 3} - \sqrt{2 \times 3}$

$= 3 - \sqrt{6}$

$d_3 = \sqrt{12} - \sqrt{9}$

$= \sqrt{4 \times 3} - \sqrt{9}$

$= 2\sqrt{3} - 3$

As common difference does not equal.

Hence, The given series is not in A.P.

OR

a = -9

d = -14 - (-9) = -14 + 9 = -5

$a_{30} - a_{20}$ = (a + 29d) - (a + 19d)

$= 10d = 10 \times (-5) = -50$

6. A.P = -40, -15, 10, 35......

First term(a) = -40

Common difference(d) = (-15) - (-40) = 25

We have,

n^{th} term(a_n) = a + (n - 1)d

$\Rightarrow a_{10} = -40 + (10 - 1) \times 25$

= - 40 + 9 $\times$ 25

= -40 + 225

= 185

7. Given, $x^2 + ax - 1 = 0$

   $D = b^2 - 4ac$

   $D = (a)^2 - 4(1)(-1) = a^2 + 4$

   We know $a^2 \geq 0$, for all the real values of a

   $\Rightarrow a^2 + 4 \geq 0$ (as 4 > 0)

   $\Rightarrow D \geq 0$

   Therefore, the equation has real roots.

OR

We have,

$3x^2 - 4x + 2 = 2x^2 - 2x + 4$

$\Rightarrow x^2 - 2x - 2 = 0$

Clearly, it is of the form of $ax^2 + bx + c = 0$. So, the given equation is a quadratic equation.

8. $\tan 30^0 = \frac{OA}{AP}$

   $\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$

   $\Rightarrow AP = 3$

   $\therefore AP = BP = 3cm$

9. A line which intersects a circle at any one point is called the tangent. The tangent at any point of a circle is perpendicular to the radius through the all point of contact.

OR

A line that interests a circle at two points in a circle is called a Secant.

10. In $\triangle$BRD,

    BR || PQ

    Therefore,by basic proportionality theorm,

$\frac{\mathrm{BD}}{\mathrm{PD}}=\frac{\mathrm{RD}}{\mathrm{QD}}$ ..(i)

In $\triangle$ RDP, PR || QC (given)

Therefore,by basic proportionality theorm,

$\frac{\mathrm{RD}}{\mathrm{QD}}=\frac{\mathrm{PD}}{\mathrm{CD}}$ ..(ii)

from (i) and (ii)

$\frac{\mathrm{PD}}{\mathrm{CD}}=\frac{\mathrm{BD}}{\mathrm{PD}}$

$\Rightarrow \quad BD \times CD = PD \times PD = 12 \times 12$ = 144 cm^2

11. We know, n^{th} term of an AP is given by a_n = a + (n - 1)d, where a is the first term and d is the common difference

Given, $a_{18} - a_{14}$ = 32

$\Rightarrow (a+(18-1)d)-(a+(14-1)d)=32$

$\Rightarrow (a+17d)-(a+13d)=32$

$\Rightarrow 4d=32$

$\Rightarrow d=8$

12. $\sin \mathrm{A}=\frac{\sqrt{3}}{2}$ sin A = sin 60°

$\Rightarrow$ A = 60°

Now, cot A = cot 60° = $\frac{1}{\sqrt{3}}$

$2\cot^2$ A - 1 = 2 $\cot^2$ 60° - 1

= $2(\frac{1}{\sqrt{3}})^2$ - 1

= $\frac{2}{3}$ - 1

= $\frac{-1}{3}$

13. LHS = $\left(1-\sin^2\theta\right)\sec^2\theta$

= $\cos^2\theta\sec^2\theta$ [$\because$1 - $\sin^2\theta$ = $\cos^2\theta$]

$=\cos^2\theta\left(\frac{1}{\cos^2\theta}\right)$ [Since,SecA = (1/CosA)]

= 1 = RHS,

Hence Proved.

14. Let 'r' be the radius of the cone, cylinder and hemisphere respectively and ' h ' be the height of the cone, cylinder and hemisphere respectively then h = r

V_1 = Volume of cylinder = $\pi r^2 h$

V_2 = Volume of a cone = $\frac{1}{3}\pi r^2 h$

V_3= Volume of a hemisphere = $\frac{2}{3}\pi r^3$

or, V_1: V_2 : $V_3 = \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$

$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^2 \times h \ (\because h = r)$

$= 1 : \frac{1}{3} : \frac{2}{3}$

= 3: 1 : 2

15. So, (2k - 1) - k = (2k+1) - (2k-1)

k - 1 = 2

k = 2 + 1

k = 3

16. Total number of outcomes = 11

There are four vowels namely O, A, I, I

So, favourable outcomes = 4

$Probability = \frac{Number\ of\ \ favorable\ outcome}{Total\ number\ of\ outcome}$

Hence, P (getting a vowel) = $\frac{4}{11}$

17. i. (b) A(1, 7), B(4, 2), C(-4, 4)

Distance travelled by Seema, AC = $\sqrt{[-4-1]^2 + [4-7]^2} = \sqrt{34}$ units

Distance travelled by Aditya, BC = $\sqrt{[-4-4]^2 + [4-2]^2} = \sqrt{68}$ units

$\therefore$ Aditya travels more distance

ii. (d) By using mid-point formula,

Coordinates of D are $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

iii. $\text{ar}(\Delta\text{ABC}) = \frac{1}{2}$ [1(2 - 4) + 4(4 - 7) - 4(7 - 2)]

= 17 sq. units

iv. (b) (-4, 4)

v. (d) (4, 2)

18. i. (c) 25 m

ii. (a) 40 m

iii. (c) 25 m

iv. (d) 150 m^2

v. (a) 60 m

19. i. (a) Curve A - Less than type ogive and Curve B - More than type ogive

ii. (c) Median Wages = 50 Rs.

iii. (d) Mode = 3 Median - 2 Mean = 3(50) - 2(50) = 50 Rs.

As, Mean = Median = Mode, so it is a symmetrical distribution

iv. (a) Median

v. (b) Median

20. i. (a) 6 cm

ii. (c) 904.32 cm^3

iii. (c) 1:1

iv. (d) 527.52 cm^2

v. (c) Remain unaltered

Part-B

21. It is given that the HCF of two numbers is 27 and their LCM is 162.

Let the numbers be "a "and 81.

HCF × LCM = product of the two numbers

$\Rightarrow 27 \times 162 = 81 \times a$

$\Rightarrow a = 54$

So, the other number is 54.

22. Let the coordinates of the third vertex be (x, y), Then

Coordinates of centroid of triangle are

$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$

We have centroid is at origin (0, 0)

$\therefore \frac{x+1+3}{3} = 0$ and $\frac{y+2+5}{3} = 0$

⇒ x + 4 = 0 ⇒ y + 7 = 0

⇒ x = -4 ⇒ y = -7

Hence, the coordinates of the third vertex are (-4, -7).

OR

The point P on y-axis will have its abscissa = 0

Let the coordinates of point P be (0, y)

Let the given points be A(–5, –2) and B(3, 2)

Then PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (-5-0)^2 + (-2-y)^2 = (3-0)^2 + (2-y)^2$

$\Rightarrow (-5)^2 + (-2 - y)^2 = (3)^2 + (2 - y)^2$

$\Rightarrow 25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$

$\Rightarrow 4y + 4y = 9 - 25$

$\Rightarrow 8y = -16$

$\Rightarrow y = \frac{-16}{8} = -2$

$\therefore$ The point equidistant from given points is (0, – 2)

23. i. We observe that polynomials $x^2 + 9$, $x^2 + 5x + 6$ and $x^2 - 7x + 12$ are in the form of $ax^2 + bx + c$, which is the standard form of quadratic polynomial. Hence, 3 quadratic polynomials were written.

ii. α and β are zeros of polynomial $x^2 + 5x + 6$.

We know that for any quadratic polynomial:

Sum of zeros $= -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$

So, $\alpha + \beta = -\frac{5}{1} = -5$

24. The steps of construction are as follows:

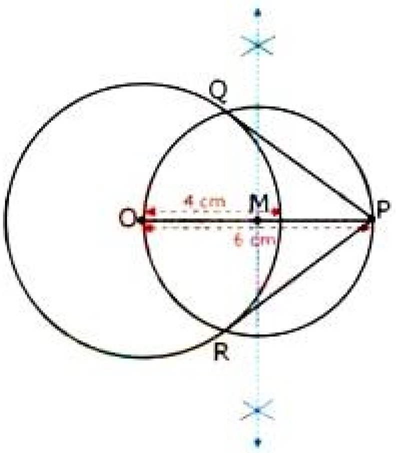
i. Taking any point O as centre, draw a circle of 4 cm radius. Locate a point P which is 6 cm away from O. Join OP.

ii. Bisect OP. Let M be the mid-point of PO.

iii. Taking M as centre and MO as radius, draw a circle.

iv. Let this circle intersect the previous circle at point Q and R.

v. Join PQ and PR. PQ and PR are the required tangents.



25. LHS $= \frac{\cos^2\theta}{\sin\theta} - \cos ec\,\theta + \sin\theta$

$= \frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta} + \sin\theta \left[\because \cos ec\theta = \frac{1}{\sin\theta}\right]$

$= \frac{\cos^2\theta - 1 + \sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta - 1}{\sin\theta}$

$= \frac{1-1}{\sin\theta}\ [\because \cos^2\theta + \sin^2\theta = 1]$

$= \frac{0}{\sin\theta}$

= 0

= RHS

Hence proved

OR

Given, $\sin\theta + 2\cos\theta = 1$

On squaring both sides, we get

$(sin\theta + 2cos\theta)^2 = 1$

$\Rightarrow sin^2\theta + 4cos^2\theta + 4sin\theta cos\theta = 1$

$\Rightarrow 1 - \cos^2\theta + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1\ [\because sin^2\theta + cos^2\theta = 1]$

$\Rightarrow 1 - \cos^2\theta + 4 - 4\sin^2\theta + 4\sin\theta\cos\theta = 1$

$\Rightarrow -\cos^2\theta - 4\sin^2\theta + 4\sin\theta\cos\theta = -4$

$\Rightarrow -(cos^2\theta + 4sin^2\theta - 4sin\theta cos\theta) = -4$

$\Rightarrow \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$

$\Rightarrow (\cos\theta)^2 + (2\sin\theta)^2 - 2(\cos\theta)(2\sin\theta) = 4$

$\Rightarrow (2\sin\theta - \cos\theta)^2 = 2^2$

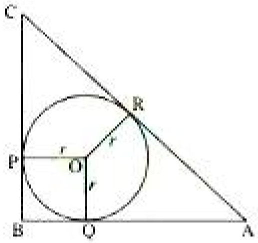
$\Rightarrow 2\sin\theta - \cos\theta = 2$

Hence proved.

26. Here,

AC = $\sqrt{AB^2 + BC^2}$

$= \sqrt{14^2 + 48^2} = \sqrt{2500} = 50$ cm



$\angle OQB = 90^o$

$\Rightarrow$ OPBQ is a square

$\Rightarrow$ BQ = r

QA = 14 - r = AR (tangents from a external point are equal in length)

Again PB = r,

PC = 48 - r

⇒ RC = 48 - r (tangents from a external point are equal in length)

AR + RC = AC

⇒ 14 - r + 48 - r = 50

⇒ r = 6 cm.

27. We will prove $6+\sqrt{2}$ irrational by contradiction.

Let us suppose that ($6+\sqrt{2}$) is rational.

It means that we have co-prime integers a and b ($b \neq 0$)

Such that

$6+\sqrt{2}=\frac{a}{b}$

$\Rightarrow \sqrt{2}=\frac{a}{b}-6$

$\Rightarrow \sqrt{2}=\frac{a-6b}{b}$ .... (1)

a and b are integers.

It means L.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. ($6+\sqrt{2}$) cannot be rational.

Hence, ($6+\sqrt{2}$) is irrational.

28. Let the required numbers be x and (x - 5).

Then, according to question we have:

$\frac{1}{x-5}-\frac{1}{x}=\frac{5}{14}$

$\Rightarrow \frac{x-x+5}{x(x-5)}=\frac{5}{14}$

⇒ 70 = $5x^2$ - 25x

⇒ $5x^2$ - 25x - 70 = 0

⇒ x^2 - 5x - 14 = 0

By factorization method, we have:

x^2 - 7x + 2x - 14 = 0

⇒ x(x - 7) + 2(x - 7) = 0

⇒ (x + 2) (x - 7) = 0

Therefore, either (x + 2) = 0 or (x - 7) = 0

⇒ x = - 2 or x = 7

Since x is a natural number, x $\neq$ -2.

$\Rightarrow$ x = 7 and x - 5 = 7 - 5 = 2

Hence, the required numbers are 7 and 2.

OR

The given equation is

$\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}$

$\Rightarrow \frac{x+5-x+1}{(x-1)(x+5)} = \frac{6}{7}$

$\Rightarrow \frac{6}{x^2+5x-x-5} = \frac{6}{7}$ (Cancel 6 on both sides)

$\Rightarrow \frac{1}{x^2+4x-5} = \frac{1}{7}$

$\Rightarrow 7 = x^2 + 4x - 5$ (by cross multiplication)

$\Rightarrow x^2 + 4x - 12 = 0$

Now by middle term splitting method ,

$x^2 + 6x - 2x - 12 = 0$

$\Rightarrow x(x + 6) -2(x + 6) = 0$

$\Rightarrow (x + 6)(x - 2) = 0$

$\Rightarrow x + 6 = 0$ or $x - 2 = 0$

$\Rightarrow x = -6$ or $x = 2$

Therefore, the two values of x are -6 and 2

29. Assume f(x) = $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

If $\sqrt{2}$ is the zero of f(x), then $(x - \sqrt{2})$ will be a factor of f(x). So, by remainder theorem when f(x) is divided by $(x - \sqrt{2})$, the quotient comes out to be quadratic.

Now we divide $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$ by $(x - \sqrt{2})$.

$$
\begin{array}{r|l}
 & 6x^2 + 7\sqrt{2}x + 4 \\
\hline
x - \sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & \underset{-}{6x^3} \underset{+}{-} 6\sqrt{2}x^2 \\
\hline
 & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} \\
 & \underset{-}{7\sqrt{2}x^2} \underset{+}{-} 14x \\
\hline
 & 4x - 4\sqrt{2} \\
 & \underset{-}{4x} \underset{+}{-} 4\sqrt{2} \\
\hline
 & 0
\end{array}
$$

$\therefore\ f(x) = (x - \sqrt{2})(6x^2 + 7\sqrt{2}x + 4)$ (By Euclid's division algorithm)

$= (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4)$ (By factorization method)

For zeroes of f(x), put f(x) = 0

$\therefore (x - \sqrt{2})(6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4) = 0$

$\Rightarrow \ (x - \sqrt{2})[2x(3x + 2\sqrt{2}) + \sqrt{2}(3x + 2\sqrt{2})] = 0$

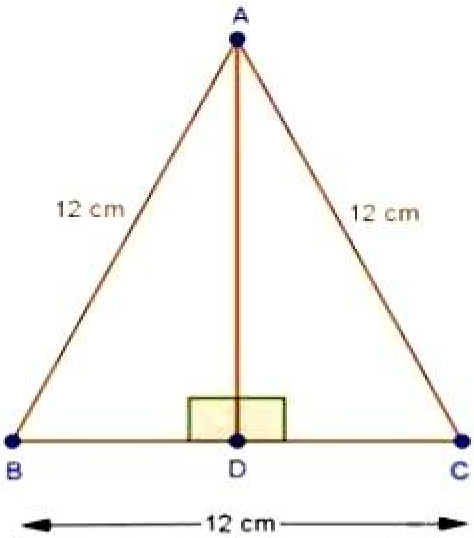
$\Rightarrow (x - \sqrt{2})(3x + 2\sqrt{2})(2x + \sqrt{2}) = 0$

$\Rightarrow x - \sqrt{2} = 0$ or $3x + 2\sqrt{2} = 0$ or $2x + \sqrt{2} = 0$

$\Rightarrow x = \sqrt{2}$ or $x = \frac{-2\sqrt{2}}{3}$ or $x = \frac{-\sqrt{2}}{2}$

So, other two roots are $= \frac{-2\sqrt{2}}{3}$ and $\frac{-\sqrt{2}}{2}$.

30. We have,



ΔABC is an equilateral Δ with side 12 cm.

Draw $AD \perp BC$

In ΔABD and ΔACD

$\angle ADB = \angle ADC$ [Each 90°]

AB = AC [Each 12 cm]

AD = AD [Common]

Then, $\Delta ABD \cong \Delta ACD$ [By RHS condition]

$\therefore$ BD = CD [Corresponding Parts of Congruent Triangles are equal]

We know that

BC = BD + CD

$\Rightarrow$ 12 = BD + BD

$\Rightarrow$ 12 = 2BD

$\Rightarrow$ BD = 6 cm

In ΔADB, by Pythagoras Theorem

$AD^2 + BD^2 = AB^2$

$\Rightarrow AD^2 + 6^2 = 12^2$

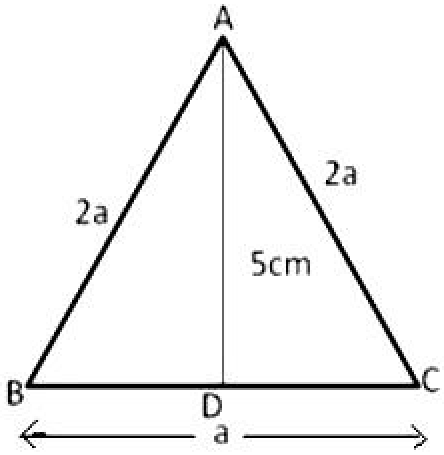
$\Rightarrow AD^2 = 144 - 36 = 108$

$\Rightarrow AD = \sqrt{108} = 10.39$ cm

OR

Given: $\triangle ABC$ in which AB=AC=2a units and BC= a units

Const: Draw $AD \perp BC$ then D is the midpoint of BC.



In $\triangle ABC$

BC = a

and BD= $\frac{BC}{2} = \frac{a}{2}$

In $\triangle ADB$,

$(AB)^2 = AD^2 + BD^2$

$AD^2 = (AB^2 - BD^2)$

$AD^2 = [(2a)^2 - (\frac{a}{2})^2]$

$AD^2 = \left[4a^2 - \frac{a^2}{4}\right] = \frac{15a^2}{4}$

$\Rightarrow AD = \frac{a\sqrt{15}}{2}$ units.

31. i. It can be observed that,

To get the sum as 2, possible outcomes = (1, 1)

To get the sum as 3, possible outcomes = (2, 1) and (1, 2)

To get the sum as 4, possible outcomes = (3, 1), (1, 3), (2, 2)

To get the sum as 5, possible outcomes = (4, 1), (1, 4), (2, 3), (3, 2)

To get the sum as 6, possible outcomes = (5, 1), (1, 5), (2, 4), (4, 2), (3, 3)

To get the sum as 7, possible outcomes = (6, 1), (1, 6), (2, 5), (5, 2), (3, 4), (4, 3)

To get the sum as 8, possible outcomes = (6, 2), (2, 6), (3, 5), (5, 3), (4, 4)

To get the sum as 9, possible outcomes = (3, 6), (6, 3), (4, 5), (5, 4)

To get the sum as 10, possible outcomes = (4, 6), (6, 4), (5, 5)

To get the sum as 11, possible outcomes = (5, 6), (6, 5)

To get the sum as 12, possible outcomes = (6, 6)

Event	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

ii. The probability of each of these sums will not be $\frac{1}{11}$ as their sums are not equally likely.

32. Let P and Q be the two positions of the plane and let A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from point A are 60° and 30° respectively.

$\therefore \angle PAB = 60^o, \angle QAB = 30^o$. It is also given that PB = 3600 $\sqrt{3}$ metres

In $\triangle$ ABP, we have

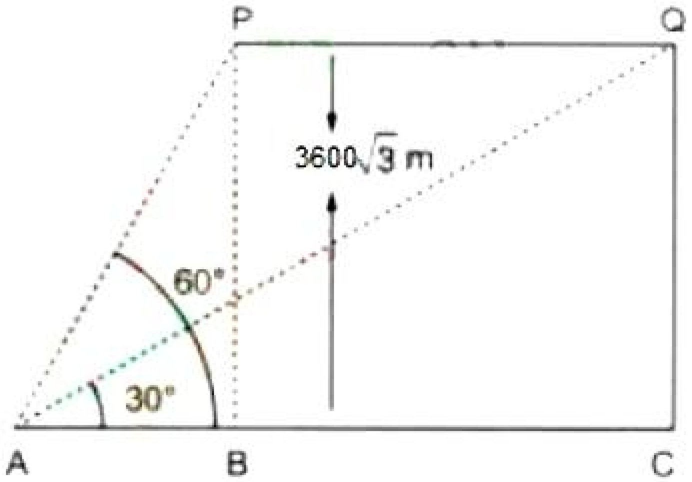
$\tan 60^o = \frac{BP}{AB}$

$\Rightarrow \sqrt{3} = \frac{3600\sqrt{3}}{AB}$

$\Rightarrow$ AB = 3600 m

In $\triangle$ ACQ, we have

$\tan 30^o = \frac{CQ}{AC}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$

$\Rightarrow$ AC = 3600 $\times$ 3 = 10800 m

$\therefore$ PQ = BC = AC - AB = 10800 - 3600 = 7200 m

Thus, the plane travels 7200 m in 30 seconds.

Hence, Speed of plane = $\frac{7200}{30}$ = 240 m / sec = $\frac{240}{1000} \times 60 \times 60$ = 864 km / hr

33.

Class	x_i (Class marks)	f_i	f_ix_i
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	p	70p
80-100	90	13	1170
	Total	$\Sigma f_i = 72 + p$	$\Sigma f_ix_i = 3340 + 70p$

$\therefore$ Mean, $\overline{x} = \frac{\Sigma f_ix_i}{\Sigma f_i}$

$\Rightarrow 53 = \frac{3340+70p}{72+p}$

$\Rightarrow 3340 + 70p = 53(72 + p)$

$\Rightarrow 3340 + 70p = 3816 + 53p$

$\Rightarrow 70p - 53p = 3816 - 3340$

$\Rightarrow 17p = 476$

$\Rightarrow$ p = $\frac{476}{17}$

$\therefore p = 28$

34. It is given that lengths of the sides of the triangle are in the ratio of 3 : 4 : 5 and also perimeter of a given triangle is 144 cm therefore on dividing 144 cm in the ratio 3:4:5, we get

$a = \left(144 \times \frac{3}{12}\right) \text{cm} = 36\text{cm}, b = \left(144 \times \frac{4}{12}\right) \text{cm} = 48\text{cm}$

and $c = \left(144 \times \frac{5}{12}\right) \text{cm} = 60\text{cm}$

$\therefore$ s = $\frac{1}{2}$ (36 + 48 + 60) cm = 72 cm.

(s - a) = (72 - 36) cm = 36 cm,

(s - b) = (72- 48) cm = 24 cm

and (s - c) = (72 - 60) cm = 12 cm.

i. Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{72 \times 36 \times 24 \times 12}\text{cm}^2$ = 72×12 cm^2 = 864 cm^2.

ii. Let base = 60 cm and the corresponding height = h cm.

Then, area of the triangle = $= \left(\frac{1}{2} \times 60 \times h\right) \text{cm}^2 = (30h)\text{cm}^2.$

$\therefore \quad 30h = 864 \Rightarrow h = \frac{864}{30} = 28.8.$

Longest side = 60 cm, corresponding height = 28.8 cm.

35. The given equations are

$\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8},$

$\Rightarrow \frac{1}{(3x+y)} - \frac{1}{(3x-y)} = \frac{-1}{4}$

Putting $\frac{1}{3x+y}$ =u and $\frac{1}{3x-y}$ = v , the given equations are

u + v = $\frac{3}{4}$ ....(i)

u - v = $-\frac{1}{4}$ ....(ii)

Adding (i) and (ii), we get

$\Rightarrow 2u = \frac{1}{2}$

$\Rightarrow u = \frac{1}{4}$

Substituting u = $\frac{1}{4}$ in (i), we get v = $\frac{1}{2}$

$\Rightarrow \frac{1}{3x+y} = \frac{1}{4}$ and $\frac{1}{3x-y} = \frac{1}{2}$

⇒ 3x + y = 4 ....(iii)

and 3x - y = 2.........(iv)

Adding (iii) and (iv),we get

3x + y + 3x - y = 4 + 2

⇒ 6x = 6

⇒x = 1

Substituting x = 1 in (iii), we get

3(1) + y = 4

⇒ 3 + y = 4

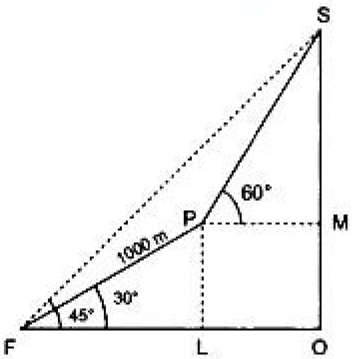
⇒ y = 1

Hence, the solution is x = 1 and y = 1

36. Given that at the foot of a mountain the elevation of its summit is 45°; after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. We have to find the height of the mountain.

Let F be the foot and S be the summit of the mountain FOS. Then $\angle OFS = 45°$and therefore, $\angle OSF = 45°.$Consequently, OF = OS = h km (say). Let FP = 1000 m = 1 km be

the slope so that $\angle OFP = 30^{\circ}$.Draw PM $\perp$OF. join PS. It is given that $\angle MPS = 60^{\circ}$.
In $\triangle FPL$, we have



$\sin 30^{\circ} = \frac{PL}{PF}$

$\Rightarrow \quad PL = PF \sin 30^{\circ} = \left(1 + \frac{1}{2}\right) \text{km} = \frac{1}{2} \text{km}$

$\therefore \quad OM = PL = \frac{1}{2} \text{km}$

$\Rightarrow \quad MS = OS - OM = \left(h - \frac{1}{2}\right) \text{km}$ ...(i)

Also, $\cos 30^{\circ} = \frac{FL}{PF}$

$\Rightarrow \quad FL = PF \cos 30^{\circ} = \left(1 \times \frac{\sqrt{3}}{2}\right) \text{km} = \frac{\sqrt{3}}{2} \text{km}$

Now, h = OS = OF = OL + LF

$\Rightarrow \quad h = OL + \frac{\sqrt{3}}{2}$

$\Rightarrow \quad OL = \left(h - \frac{\sqrt{3}}{2}\right) \text{km}$

$\Rightarrow \quad PM = \left(h - \frac{\sqrt{3}}{2}\right) \text{km}$ ...(ii)

In $\triangle SPM$, we have

$\tan 60^{\circ} = \frac{SM}{PM}$

$\Rightarrow$ SM = PM . tan60°

$\Rightarrow \left(h - \frac{1}{2}\right) = \left(h - \frac{\sqrt{3}}{2}\right) \sqrt{3}$

$\Rightarrow \quad h - \frac{1}{2} = h\sqrt{3} - \frac{3}{2}$

$\Rightarrow \quad \sqrt{3}h - h = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow \quad h(\sqrt{3} - 1) = 1$

$\Rightarrow \quad h = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{2} = \frac{2.732}{2} = 1.366 \text{km}$

Hence, the height of the mountain is 1.366 km.