

5. Circle

- A tangent to a curve at a point is the limiting position of a line joining two points on the curve in the neighbourhood of the given point (at which tangent has to be drawn), as both the points tend to the given point.
- Equation of tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at point x_1, y_1 is $xx_1+yy_1+gx+x_1+fy+y_1+c=0$.
- Equation of tangent to the circle $x^2+y^2=a^2$ at point x_1, y_1 is $xx_1+yy_1=a^2$.
- Equation of tangent to the circle $x^2+y^2=a^2$ at point $a\cos\theta, a\sin\theta$ is $x\cos\theta+y\sin\theta=a$.
- Conditions for which the line $y=mx+c$ is a tangent to the circle $x^2+y^2=a^2$ is $c^2=a^2(1+m^2)$:
- Point of contact is $-a^2mc/a^2c$
- Equation of tangent to the circle $x^2+y^2=a^2$ is $y=mx\pm a\sqrt{1+m^2}$.
- The length of the tangent drawn from an external point x_1, y_1 to the circle $x^2+y^2+2gx+2fy+c=0$ is $\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$.
- The length of the tangent drawn from an external point x_1, y_1 to the circle $x^2+y^2=a^2$ is $\sqrt{x_1^2+y_1^2-a^2}$.
- The equation of a pair of tangents drawn from an external point x_1, y_1 to the circle $x^2+y^2+2gx+2fy+c=0$ is $x^2+y^2+2gx+2fy+cx_1^2+y_1^2+2gx_1+2fy_1+c=0$
Or $SS_1=T^2$ where, $S=x^2+y^2+2gx+2fy+c$, $S_1=x_1^2+y_1^2+2gx_1+2fy_1+c$ and $T=xx_1+yy_1+gx+x_1+fy+y_1+c$
- The equation of a pair of tangents drawn from an external point x_1, y_1 to the circle $x^2+y^2=a^2$ is $x^2+y^2-a^2-x_1^2-y_1^2+a^2=xx_1+yy_1-a^2$
Or $SS_1=T^2$, where, $S=x^2+y^2-a^2$, $S_1=x_1^2+y_1^2-a^2$ and $T=xx_1+yy_1-a^2$
- The normal to a circle at a point is defined as the straight line passing through the point of contact of the tangent to the circle and perpendicular to the tangent. Every normal to the circle always passes through the centre of the circle.
- The equation of the normal to the circle $x^2+y^2+2gx+2fy+c=0$ at point $P(x_1, y_1)$ is $x-x_1/x_1+g=y-y_1/y_1+f$.
- The equation of the normal to the circle $x^2+y^2=a^2$ at point $P(x_1, y_1)$ is $x-x_1/x_1+0=y-y_1/y_1+0 \Rightarrow xx_1=yy_1$.