Chapter 4

Two-Port Networks

CHAPTER HIGHLIGHTS

- *Classification of Networks*
- *Network Configuration*
- *Open Circuit or Impedance (Z) Parameters*
- *Y Parameters or Short-circuit Admittance Parameters*
- *Hybrid Parameters*
- *G Parameters or Inverse Hybrid Parameters*
- *Transmission or ABCD Parameters*
- *Inverse Transmission Parameters*
- *Interconnection of Networks*
- *ABCD Parameters in Terms of Z Parameters and Y – Parameters*
- *Network Graphs*

INTRODUCTION

A pair of terminals through which a current may enter or leave a network is known as a port.

The current entering one terminal leaves through the other terminals so that the net current in the port equals zero.

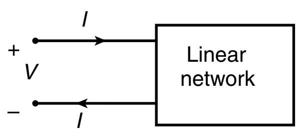


Figure 1 One-port network.

A two-port network is an electrical network with two separate ports for input and output.

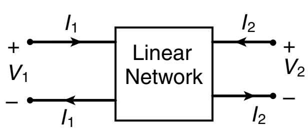


Figure 2 Two-port network.

To characterize a two-port network required that we relate the terminal quantities V_1, V_2, I_1, and I_2.

CLASSIFICATION OF NETWORKS

Linear Circuits

It is the circuit whose parameters remain constant with change in applied voltage or current ($V \alpha I$ ohm's Law).

For example,resistance, inductance, and capacitance

Non-linear Circuits

It is a circuit whose parameters changed with voltage or current. For example, diodes, transistor, etc. Non-linear circuits does not obey Ohm's Law.

Unilateral Circuits and Bilateral Circuit

When the direction of current is changed, the characteristics or properties of the circuit may change. This circuit is called unilateral circuits. For example, diode, transistor, UJT, etc. Otherwise, it is called bilateral circuit. For example, R, L, C circuits.

Active and Passive Elements

If a circuit element has the capability of enhancing the energy level of a signal passing through it, it is called an active element.

For example, transistors, op-amp, vacuum tubes, etc.

Otherwise, it is called passive elements. For example, resistors, inductors, thermistors capacitors, etc., are passive elements.

Lumped and Distributed Network

Physically separable network elements such as R, L, and C are known as lumped elements. A transmission line on a cable in the other hand is an example of distributed parameter network. They are not physically separable. If the network is fabricated with its elements in lumped form, it is called a lumped network and if it is in distributed form, it is called distributed network.

Recurrent and Non-recurrent Networks

When a large circuit consists of similar networks connected one after another, the network is called as recurrent network or cascaded network. It is also called as ladder network. Otherwise, a single network is called non-recurrent network.

Symmetrical and Asymmetrical Network

If the network looks the same from both the ports, then it is said to be symmetrical. Otherwise, it is called asymmetrical network. The following figures show the symmetrical and asymmetrical networks:

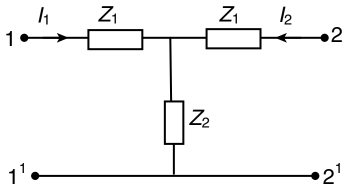


Figure 3 Symmetrical network.

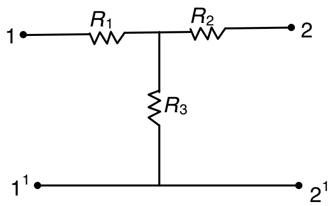


Figure 4 Asymmetrical Network.

Reciprocal and Non-reciprocal Networks

If the network obeys the reciprocity theorem, then it is called reciprocal network. Otherwise, it is called non-reciprocal network. All the passive networks are always reciprocal and all the active networks are always non-reciprocal.

Network Configuration

T-section

When a network section looks like a 'T', it is known as T-section.

Following figures are examples of T-section.

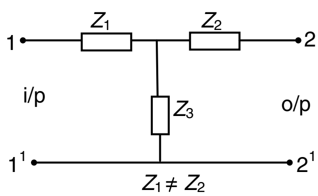


Figure 5 Unsymmetrical T-section.

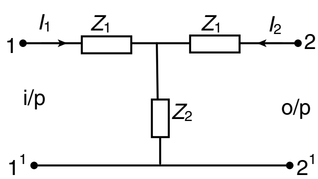


Figure 6 Symmetrical and unbalanced T-section.

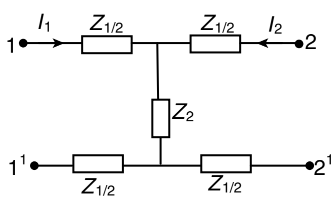


Figure 7 Balanced symmetrical T-section.

Π-section

Following are the examples of π-section.

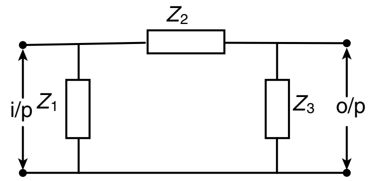


Figure 8 Unbalanced asymmetrical π-section ($Z_1 \neq Z_3$).

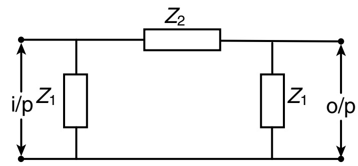


Figure 9 Unbalanced symmetrical π-section.

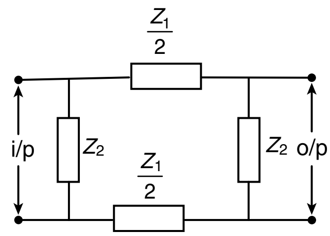
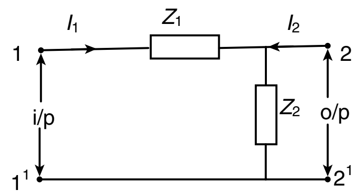


Figure 10 Balanced symmetrical π-section.

L-section

When the network section looks like '*L*', the configuration is termed as *L*-section.



Lattice Section

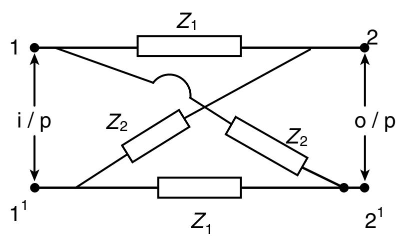
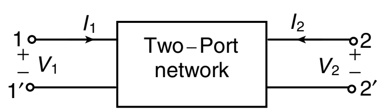


Figure 11 Symmetrical latttice section.

Two-port Networks



A two-port network has two pairs of accessible terminals; one pair represents the input and the other represents the output.

Both the currents I_1 and I_2 enter the network and the polarities of the voltages are shown in the figure. There are four variables V_1, V_2, I_1, and I_2. From these four variables, two can be taken as independent variables, and the remaining two will be dependent variables.

Open Circuit Or Impedance (Z) Parameters

Here, the two voltages V_1 and V_2 are functions of I_1 and I_2

$$V_1 = f(I_1, I_2)$$

$$V_2 = f(I_1, I_2)$$

$$[V] = [Z].[I]$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (2)$$

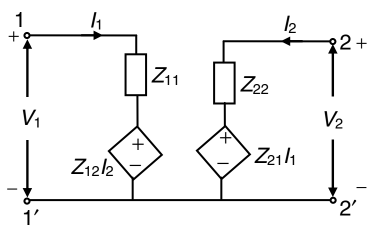
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

From Eqs (1) and (2), the network can be drawn as shown in the figure.

Equivalent Circuit of Z Parameters



Condition of Reciprocity and Symmetry

Network must be reciprocal when ratio of response at port 2 to the excitation at port 1 is same as ratio of response at port 1 to port 2, then the network is called reciprocal.

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{V_1}{I_2} \text{ at } I_1 = 0$$

The condition for reciprocal is as follows:

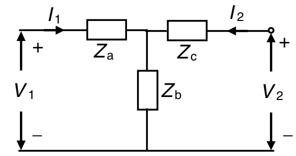
$$Z_{12} = Z_{21}.$$

The condition for symmetrical network is as follows:

$$\boxed{Z_{11} = Z_{22}}$$

Solved Examples

Example 1



Find the Z parameters for the circuit shown in figure.

Solution

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Let $I_2 = 0$

$$V_1 = I_1 (Z_a + Z_b)$$

$$Z_{11} = \frac{V_1}{I_1} = Z_a + Z_b$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$V_2 = I_1 Z_b$$

$$Z_{21} = Z_b$$

Let $I_1 = 0$

$$V_2 = I_2 (Z_c + Z_b)$$

$$Z_{22} = Z_c + Z_b$$

$$V_1 = I_2 Z_b$$

$$Z_{12} = Z_b$$

$$\Rightarrow [Z] = \begin{bmatrix} Z_a + Z_b & Z_b \\ Z_b & Z_b + Z_c \end{bmatrix}$$

Example 2

The following readings are obtained experimentally for an unknown two-port network:

	V_1	V_2	I_1	I_2
o/p open	80 V	60 V	10 A	0
i/p open	50 V	40 V	0	5 A

The Z parameters are

(A) $\begin{bmatrix} 8 & 6 \\ 10 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} 8 & 10 \\ 6 & 8 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & 10 \\ 8 & 4 \end{bmatrix}$

(D) None of these

Solution

We know $[V] = [Z]\,[I]$

If $I_2 = 0$; $Z_{11} = \frac{V_1}{I_1}$ and $Z_{21} = \frac{V_2}{I_1}$, then $Z_{11} = \frac{80}{10} = 8\ \Omega$

$$Z_{21} = \frac{60}{10} = 6\ \Omega$$

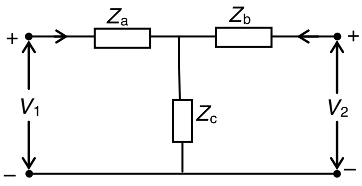
If $I_1 = 0$, $Z_{22} = \frac{V_2}{I_2}$ and $Z_{12} = \frac{V_1}{I_2}$

$$Z_{22} = \frac{40}{5} = 8\ \Omega \text{ and } Z_{12} = \frac{50}{5} = 10\ \Omega$$

$$\therefore [Z] = \begin{bmatrix} 8 & 10 \\ 6 & 8 \end{bmatrix}$$

$\therefore Z_{11} = Z_{22} = 8\ \Omega \Rightarrow$ symmetrical network
$Z_{12} \neq Z_{21} \Rightarrow$ non-reciprocal network

Example 3



If $Z_a = 2\angle 0°$, $Z_b = 5\angle -90°$, and $Z_c = 3\angle 90°$, then the abovementioned *T*-network and *Z* parameters are
(A) symmetrical and reciprocal
(B) symmetrical and non-reciprocal
(C) asymmetrical and reciprocal
(D) asymmetrical and non-reciprocal

Solution

By applying KVL, the loop equations are

$V_1 = 2I_1 + 3\angle 90°(I_1 + I_2)$

$V_1 = (2 + j3)\, I_1 + j3I_2$ (1)

$V_2 = 5\angle -90° I_2 + 3\angle 90°\,(I_1 + I_2)$

$V_2 = j3\, I_1 + (3j - 5j)\, I_2$

$V_2 = j3I_1 - 2jI_2$ (2)

From Eqs (1) and (2),

$$[Z] = \begin{bmatrix} 2 + j3 & j3 \\ j3 & -2j \end{bmatrix} = \begin{bmatrix} 3.6\angle 56° & 3\angle 90° \\ 3\angle 90° & 2\angle -90° \end{bmatrix}$$

$Z_{11} \neq Z_{22} \Rightarrow$ unsymmetrical
$Z_{12} = Z_{21} \Rightarrow$ reciprocal network

Y Parameters or Short-Circuit Admittance Parameters

In a two-port network, the input currents I_1 and I_2 can be expressed in terms of input and output voltages V_1 and V_2, respectively as $[I] = [Y]\,[V]$, where $[Y]$ is the admittance matrix.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Here, $I_1 = f(V_1, V_2)$

$I_2 = f(V_1, V_2)$

$I_1 = Y_{11}V_1 + Y_{12}V_2$ (1)

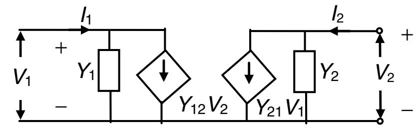
$I_2 = Y_{21}V_1 + Y_{22}V_2$ (2)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

From Eqs (1) and (2), the circuit can be drawn as shown in the figure.

Equivalent Circuit of *Y* Parameters

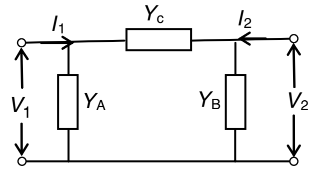


The condition for reciprocity and symmetrical are as follows:

1. If $Y_{11} = Y_{22}$, then it is called symmetrical. Otherwise, it is called asymmetrical network.
2. If $Y_{12} = Y_{21}$, then it is called reciprocal network or passive network. Otherwise, it is called non-reciprocal or active network.

Example 4

Find the *Y* parameters of the following π-circuit shown in the following figure.



Solution

Using KCL at node a,

$I_1 = V_1\, Y_A + (V_1 - V_2)\, Y_c$

$I_1 = (Y_A + Y_C)\, V_1 - (Y_C)\, V_2$ (1)

By applying KCL at node *b*

$I_2 = V_2.\, Y_B + (V_2 - V_1)\, Y_C$

$I_2 = -Y_c\, V_1 + (Y_B + Y_C)\, V_2$ (2)

From Eqs (1) and (and 2)

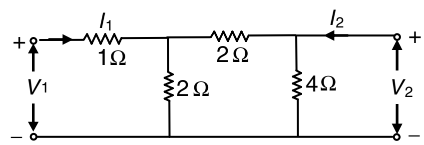
$$[Y] = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix}$$

where

$$Y = \frac{1}{R},\ Y_L = \frac{1}{SL}, \text{ and } Y_C = SC.$$

Example 5

Find the *Y* parameters for the following network.

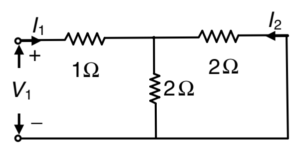


Solution

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

When $V_2 = 0$, we have



$V_1 = I_1 \times (1 + 2 \parallel 2)$

$V_1 = I_1\,(1 + 1)$

$$\frac{I_1}{V_1} = \frac{1}{2} = 0.5 \text{ mho}$$

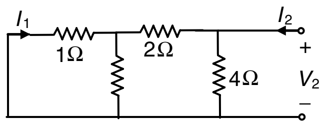
$$-I_2 = I_1 \times \frac{2}{2+2}$$

$$-I_2 = I_1 \times \frac{2}{4}$$

$$-I_2 = \frac{V_1}{2} \times \frac{1}{2}$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{-1}{4} \text{ mho}$$

When $V_1 = 0$



$$V_2 = I_2\,[(1\|2+2)\parallel 4]$$

$$V_2 = I_2\left[\left(\frac{1\times 2}{1+2}+2\right)\|4\right]$$

$$= \left[\left(\frac{2}{3}+2\right)\|4\right]$$

$$V_2 = I_2\left[\frac{\frac{8}{3}\times 4}{\frac{8}{3}+4}\right] = \frac{\frac{8}{3}}{\frac{8+12}{3}} = \frac{32}{20} = \frac{8}{5}$$

$$\frac{I_2}{V_2} = Y_{22} = \frac{5}{8} \text{ mho}$$

$$I_2' = I_2 \times \frac{4}{4+\frac{8}{3}}$$

$$I_2' = I_2 \times \frac{12}{20}$$

$$-I_1 = I_2' \times \frac{2}{2+1}$$

$$-I_1 = I_2 \times \frac{12}{20} \times \frac{2}{3}$$

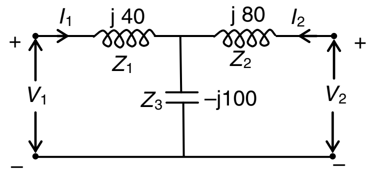
$$-I_1 = I_2 \times \frac{2}{5}$$

$$-I_1 = \frac{5}{8}V_2 \times \frac{2}{5}$$

$$\frac{I_1}{V_2} = Y_{12} = \frac{-1}{4} \text{ mho}$$

Example 6

Find Y parameters of the following network.



Solution

$$Y = [Z]^{-1}$$

$$[Z] = \begin{bmatrix} -j60 & -j100 \\ -j100 & -j20 \end{bmatrix} \Rightarrow \begin{bmatrix} j60 & j100 \\ j100 & j20 \end{bmatrix}$$

$$[Y] = [Z^{-1}]$$

$$= \frac{1}{|Z|}\{\text{adj}Z\} = \frac{1}{8{,}800}\begin{bmatrix} -j20 & j100 \\ j100 & -j60 \end{bmatrix}$$

$$= \begin{bmatrix} -j2.27\times 10^{-3}\,\Omega & j11.36\times 10^{-3}\,\Omega \\ j11.36\times 10^{-3}\,\Omega & -j6.8\times 10^{-3}\,\Omega \end{bmatrix}$$

$$Y_{11} = -j2.27\times 10^{-3}\,\Omega$$

$$Y_{12} = -j11.36\times 10^{-3}\,\Omega$$

$$Y_{21} = -j11.36\times 10^{-3}\,\Omega$$

$$Y_{22} = -j6.8\times 10^{-3}\,\Omega$$

Hybrid Parameters

The Z and Y parameters of a two-port network do not always exist. A two-port network can be represented using the h parameters. The describing equations for h parameters are as follows:

$$V_1 = h_{11}\,I_1 + h_{12}\,V_2$$

$$I_2 = h_{21}I_1 + h_{22}\,V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The value of the parameters are determined as follows:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}; \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}; \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

where h_{11} is the short-circuit input impedance; h_{12} is the open-circuit reverse voltage gain; h_{21} is the short-circuit forward current gain; and h_{22} is the open-circuit output admittance.

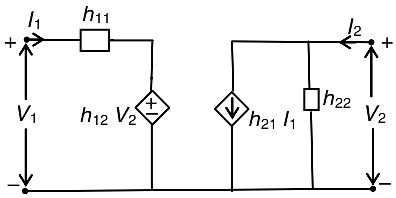


Figure 12 The h parameters equivalent network.

The condition of reciprocity is

$$\boxed{h_{12} = -h_{21}}$$

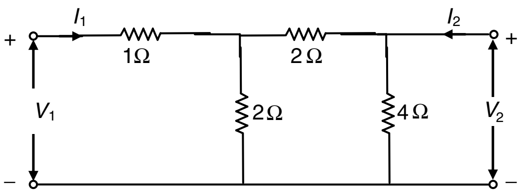
The condition of symmetry is

$$|h| = 1$$

$$\boxed{h_{11}.h_{22} - h_{12}.h_{21} = 1}$$

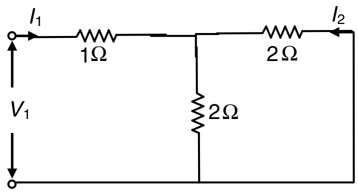
Example 7

Find the *h* parameters of the following network.



Solution

When $V_2 = 0$



$$\frac{V_1}{I_1} = h_{11} = 1 + 2 \,||\, 2$$

$$= 1 + 1 = 2\ \Omega$$

$$-I_2 = I_1 \times \frac{2}{2+2}$$

$$-I_2 = I_1 \times \frac{1}{2}$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-1}{2}$$

When $I_1 = 0$

$$V_1 = V_2 \times \frac{2}{2+2}$$

$$\frac{V_1}{V_2} = h_{12} = \frac{1}{2}$$

$$V_2 = I_2 \times 4||(2+2)$$

$$V_2 = I_2 \times 2$$

$$\frac{I_2}{V_2} = h_{22} = \frac{1}{2}$$

G Parameters or Inverse Hybrid Parameters

These are represented by

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

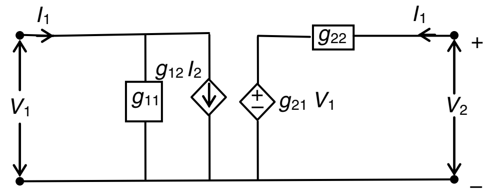


Figure 13 *g* parameter equivalent circuit.

The *g* parameters can be defined as

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2 = 0}; \quad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_i = 0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2 = 0}; \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1 = 0}$$

The condition for reciprocity is

$$g_{12} = -g_{21}$$

The condition for symmetry is

$$|g| = 1$$

$$\text{i.e.,} \quad g_{11} \cdot g_{22} - g_{12} \cdot g_{21} = 1$$

Transmission or *ABCD* Parameters

The transmission parameters express the required source variables V_1 and I_1 in terms of the existing destination variables V_2 and I_2. They are called *ABCD* or *T* parameters and are defined by

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The transmission parameters are determined as

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} \quad \text{(no units)}$$

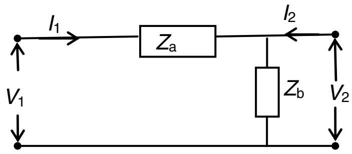
$$B = \left.\frac{-V_1}{I_2}\right|_{V_2 = 0} \quad (\Omega)$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} \quad \text{(mho)}$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2 = 0} \quad \text{(No units)}$$

Thus, the transmission parameters are called, specifically, A is the open circuit voltage ratio, B is the negative short-circuit transfer impedance, C is the open-circuit transfer admittance, and D is the negative short circuit current ratio

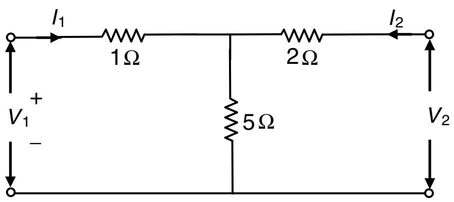
The condition for reciprocity and symmetry is $AD - BC = 1$ and $A = D$, respectively, if



abovementioned circuit T parameters defined as $[T]$ =

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\dfrac{Z_a}{Z_b} & Z_a \\ \dfrac{1}{Z_b} & 1 \end{bmatrix}$$

Example 8



Find the transmission parameters for the following circuit.

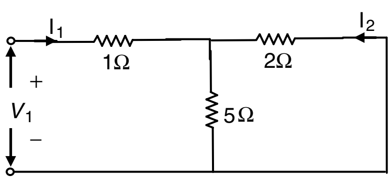
Solution

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

When $I_2 = 0$

$$V_2 = V_1 \times \frac{5}{5+1}$$
$$A = \frac{V_1}{V_2} = \frac{6}{5}$$
$$V_2 = I_1 \times 5$$
$$C = \frac{I_1}{V_2} = \frac{1}{5}$$

When $V_2 = 0$



$$-I_2 = I_1 \times \frac{5}{5+2}$$
$$V_1 = I_1 \times \left[1 + \frac{5 \times 2}{5+2}\right] = I_1\left[1 + \frac{10}{7}\right]$$

$$V_1 = \frac{-7}{5} I_2 \times \frac{17}{10}$$
$$B = \frac{-V_1}{I_2} = \frac{-119}{50}$$
$$D = \frac{-I_1}{I_2}$$
$$D = \frac{7}{5}.$$

Inverse Transmission Parameters

$$V_2 = A^1\, V_1 - B^1\, I_1$$
$$I_2 = C^1\, V_1 - D^1\, I_1$$
$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A^1 & B^1 \\ C^1 & D^1 \end{bmatrix}\begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

The inverse transmission parameters can be defined as

$$A^1 = \left.\frac{V_2}{V_1}\right|_{I_1 = 0}$$

Forward voltage ratio with sending-end open circuited.

$$C^1 = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

Transfer admittance with sending-end open circuited.

$$B^1 = \left.\frac{V_2}{-I_1}\right|_{V_1=0}$$

Transfer impedance with sending-end short circuited.

$$D = \left.\frac{I_2}{-I_1}\right|_{V_1=0}$$

$A^1 = D^1$ is symmetrical.

$A^1D^1 - B^1C^1 = 1$ is reciprocal.

The relationships between parameters are given as follows:

1. $[Y] = [Z]^{-1}$
2. $[g] = [h]^{-1}$
3. $[t] \neq [T]^{-1}$

Interconnection Of Networks

A large complex network may be divided into sub networks for the purpose of analysis and design. The sub networks are modelled as two-port networks that are interconnected to form the original network. The interconnection can be in series, in parallel, or in cascade. The interconnected network can be described by any of the six parameter sets. For example, when the networks are in series, their undivided Z parameters add up to give the Z parameter of the larger network.

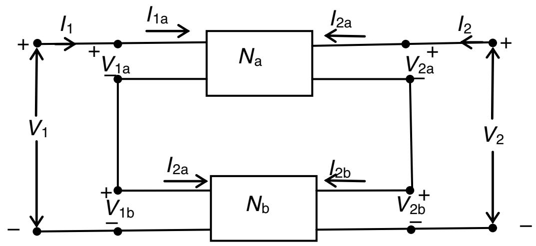


Figure 14 Series connection of two-port networks.

$$[Z] = [Z_a] + [Z_b]$$

Two-port network are in parallel when their port voltages are equal and the port currents of the large network; they are the sums of the individual port elements. The parallel connection of two two-port networks is shown in figure.

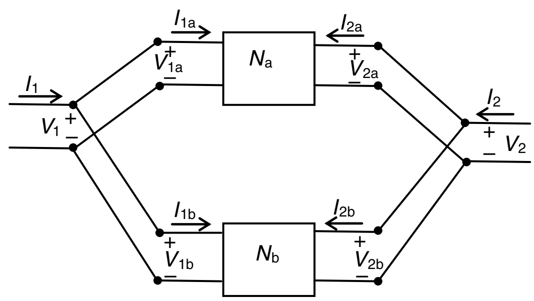


Figure 15 Parallel connection of two two-port networks.

From the abovementioned figure,

$$I_{1a} = Y_{11a}\,V_{1a} + Y_{12a}\,V_{2a}$$
$$I_{2a} = Y_{22a}\,V_{1a} + V_{22a}\cdot V_{2a}$$

and

$$I_{1b} = Y_{11b}\,V_{1b} + Y_{12b}\cdot V_{2b}$$
$$I_{2b} = Y_{21b}\,V_{1b} + V_{22b}\cdot V_{2b}$$

But from the figure

$$V_1 = V_{1a} = V_{1b}$$
$$V_2 = V_{2a} = V_{2b}$$

and

$$I_1 = I_{1a} + I_{2b}$$
$$I_2 = I_{2a} + I_{2b}$$

Thus, the Y parameters of the overall network are

$$[Y] = [Y_a] + [Y_b]$$

Two networks are said to be cascaded when the output of one is the input of the other.

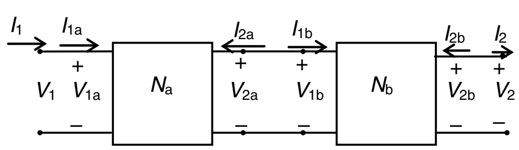
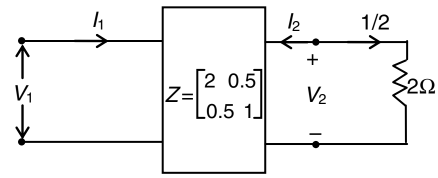


Figure 16 Cascade connection of two-port networks.

$$[T] = [T_a]\,[T_b]$$

Example 9

Determine the value of V_1.



(A) 6 V (B) 5.5 V
(C) 5.75 V (D) None of these

Solution

From the given data

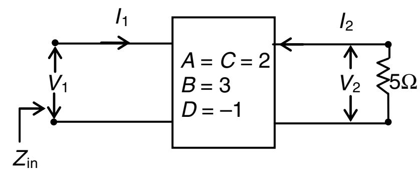
$$I_2 = -1/2 \text{ A}$$
$$V_2 = 1/2 \times 2 = 1 \text{ V}$$
$$V_1 = 2I_1 + 0.5I_2 \quad (1)$$
$$V_2 = 0.5I_1 + 1I_2 \quad (2)$$

Substitute I_2 and V_2 in Eq. (2)

$$1 = 0.5I_1 - 0.5 \Rightarrow I_1 = \frac{1.5}{0.5} = 3 \text{ A}$$
$$V_1 = 2 \times 3 + 0.5\,(-0.5)$$
$$= 6 - 0.25$$
$$= 5.75 \text{ V}$$

Example 10

Determine Z_{in}

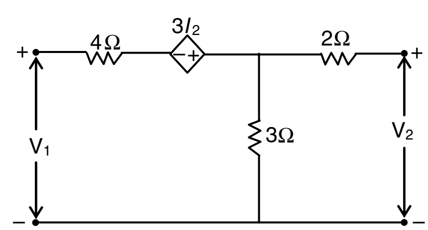


(A) −12 / 9 (B) 12 / 13
(C) 13 / 9 (D) 13 / 11

Solution

$$Z_{in} = \frac{AZ_L + B}{CZ_L + D}$$
$$= \frac{2\times5+3}{2\times5-1} = \frac{13}{9}$$

Example 11



The Z parameters of the two-port network are

(A) $\begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 3 \\ 0 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 3 \\ 7 & 0 \end{bmatrix}$

Solution

Applying KVL to the input loop

$$V_1 = 4I_1 - 3I_2 + 3(I_1 + I_2)$$

$$V_1 = 7I_1 + 0.I_2 \quad (1)$$

By applying KVL to the output loop

$$V_2 = 2I_2 + 3(I_1 + I_2)$$

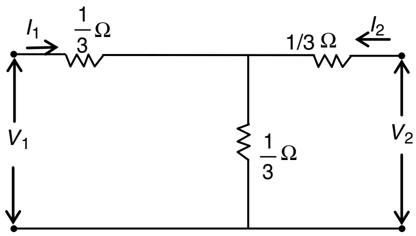
$$V_2 = 3I_1 + 5I_2 \quad (2)$$

From Eqs (1) and (2)

$$[Z] = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$$

Example 12

A Two-port network is shown in figure.



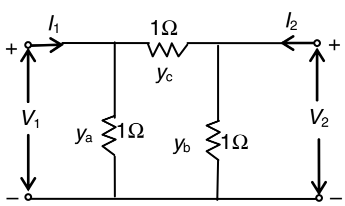
The admittance parameters Y_{11}, Y_{12}, Y_{21}, and Y_{22} are

(A) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & -2/3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -0.5 \\ -1/3 & 2 \end{bmatrix}$

Solution

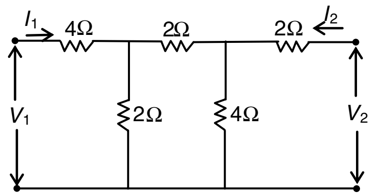
Convert star to delta connection



$$[Y] = \begin{bmatrix} y_a + y_c & -y_c \\ -y_c & -y_b + y_c \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Example 13



For the two-port network shown in the figure, the value of h_{12} is given by

(A) 0.125 (B) 0.167 (C) 0.625 (D) 0.25

Solution

h parameters can be defined by

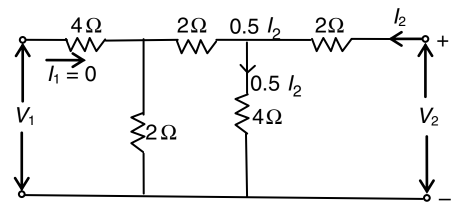
$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{12} = \frac{V_1}{V_2} \text{ at } I_1 = 0$$

Therefore, $I_1 = 0$

The circuit becomes



$$V_2 = 2I_2 + 4.\frac{I_2}{2}$$

$$V_2 = 4I_2$$

$$I_2 = \frac{V_2}{4}$$

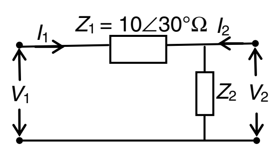
$$V_1 = \frac{I_2}{2} \times 2 = I_2$$

$$V_1 = \frac{V_2}{4}$$

$$\Rightarrow h_{12} = \frac{V_1}{V_2} = \frac{1}{4} = 0.25$$

Example 14

Two networks are connected in cascade as shown in the figure with the usual notations. The equivalent A, B, C, and D constants are obtained. Given that $C = 0.025 \angle 45°$, the value of Z_2 is

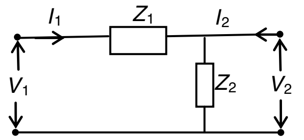


(A) $10 \angle 30°\Omega$ (B) $40 \angle -45°\Omega$

(C) $1\ \Omega$ (D) $0\ \Omega$

Solution

We know



$$[T]=\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\begin{bmatrix}1+\dfrac{Z_1}{Z_2} & Z_1\\ \dfrac{1}{Z_2} & 1\end{bmatrix}$$

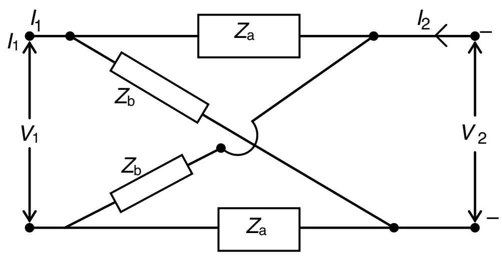
Given $C = 0.025\ \angle 45°$

Therefore, $C = \dfrac{1}{Z_2}$

$$Z_2 = \frac{1}{C} = \frac{1}{0.025}\angle -45°$$
$$= 40\ \angle -45°$$

Example 15

Find Z parameters of the two- port circuit of the following figure.



Solution

$$Z_{11} = \frac{V_1}{I_1} \text{ at } I_2 = 0$$

Z_a and Z_b having same current

Therefore, $V_1 = (Z_a + Z_b) \parallel (Z_a + Z_b)$

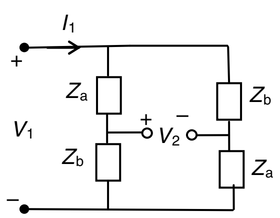
$= 1/2\ (Z_a + Z_b)\Omega$

The circuit is symmetric, and therefore,

$$Z_{11} = Z_{22} = 1/2[Z_a + Z_b]$$

Similarly,

$$Z_{21} = \frac{V_2}{I_1} \text{ at } I_2 = 0$$



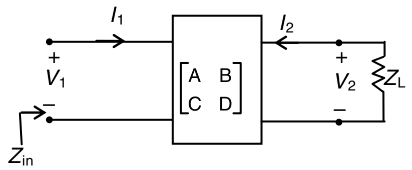
$$V_2 = Z_b\left(\frac{I_1}{2}\right) - \frac{Z_a}{2}.I_1$$

$$\frac{V_2}{I_1} = \frac{1}{2}[Z_b - Z_a]$$

Therefore, for a symmetrical lattice network

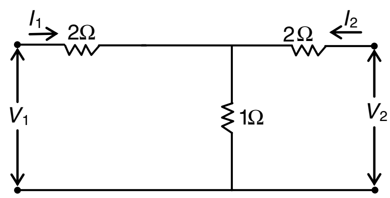
$$[Z] = \begin{bmatrix}\dfrac{Z_a+Z_b}{2} & \dfrac{Z_b-Z_a}{2}\\ \dfrac{Z_b-Z_a}{2} & \dfrac{Z_b+Z_a}{2}\end{bmatrix}$$

NOTE



Then, $Z_{in} = \dfrac{AZ_L + B}{CZ_L + D}$

Example 16

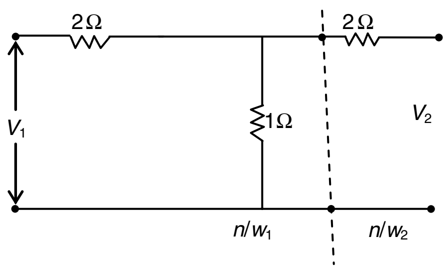


The T parameters of the network are

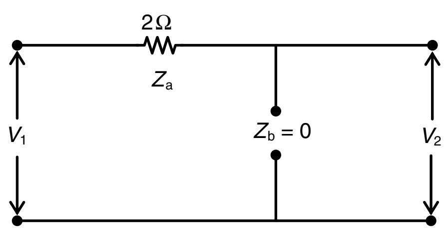
(A) $\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$ (B) $\begin{bmatrix}3 & 8\\ 1 & 3\end{bmatrix}$

(C) $\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ (D) $\begin{bmatrix}7 & 2\\ 7 & 3\end{bmatrix}$

Solution



$$[T_1] = \begin{bmatrix}1+\dfrac{2}{1} & 2\\ 1 & 1\end{bmatrix} = \begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$$



$$[T_2] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

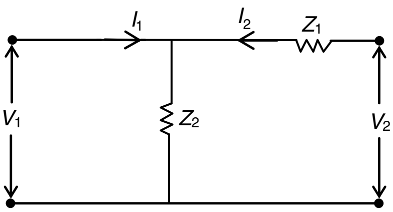
$$[T] = [T_1]\,[T_2]$$

$$= \begin{bmatrix} \rightarrow & \\ 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \downarrow \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$$

$\therefore$ $A = D$ is symmetrical and $AD - BC = 1$ is reciprocal.

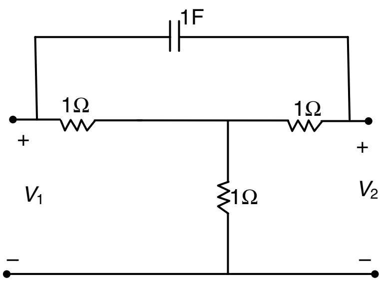
NOTE



$$[T] = \begin{bmatrix} 1 & Z_1 \\ \dfrac{1}{Z_2} & 1 + \dfrac{Z_1}{Z_2} \end{bmatrix}$$

Example 17

Determine Y parameters for the following networks



Solution

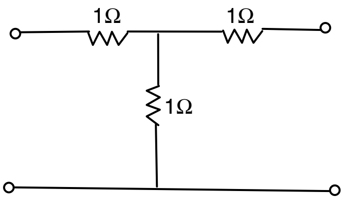


Figure 17

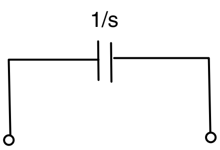


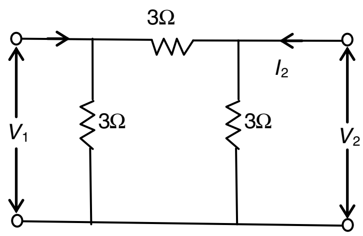
Figure 18

Figures 17 and 18 are in parallel, and so

$$[y] = [y_1] + [i_2]$$

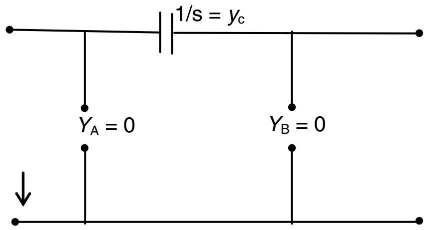
From Figure 17

Y–Δ transformation



$$[Y_1] = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix} \Omega$$

From Figure 18



$$[Y_2] = \begin{bmatrix} 1/s & -1/s \\ -1/s & 1/s \end{bmatrix}$$

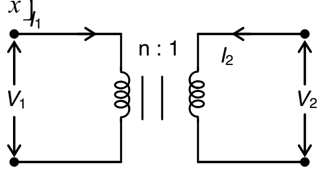
$$[Y] = \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix} + \begin{bmatrix} 1/s & -1/s \\ -1/s & 1/s \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 + 1/s & -(1/3 + 1/s) \\ -(1/3 + 1/s) & 2/3 + 1/s \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{2s+3}{3s} & \dfrac{-(s+3)}{3s} \\ \dfrac{-(s+3)}{3s} & \dfrac{2s+3}{3s} \end{bmatrix}$$

Example 18

The $ABCD$ parameters of an ideal n:1 transformer shown in figure are $\begin{bmatrix} n & 0 \\ 0 & x \end{bmatrix}$. The value of '$X$' will be



(A) n (B) $1/n$ (C) n^2 (D) $1/n^2$

Solution

For the given ideal transformer,

$$\frac{V_1}{V_2} = \frac{n}{1} = \frac{-I_2}{I_1}$$

$$V_1 = nV_2 - 0.I_2 \quad (1)$$

$$I_1 = 0.V_2 - 1/nI_2 \quad (2)$$

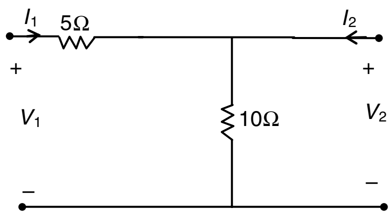
From Eqs (1) and (2)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$$

$$\Rightarrow X = 1/n$$

Example 19

The h parameters of the following circuit are



(A) $\begin{bmatrix} 5 & 1 \\ -1 & 0.1 \end{bmatrix}$ (B) $\begin{bmatrix} 0.2 & -1 \\ 1 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & -1 \\ 0.1 & 1 \end{bmatrix}$ (D) None of these

Solution

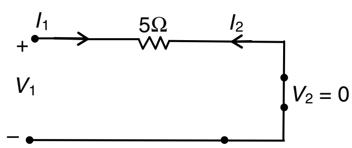
h parameters are defined by

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

From $V_2 = 0$, $h_{11} = \dfrac{V_1}{I_1}$ Ω

$$h_{21} = \frac{I_2}{I_1}$$

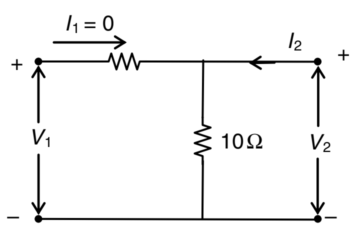


$$\frac{V_1}{I_1} = 5\Omega = h_{11}$$

$$I_1 = -I_2$$

$$\frac{I_2}{I_1} = h_{21} = -1$$

For $I_1 = 0$



$$\therefore V_1 = V_2$$

$$h_{12} = \frac{V_1}{V_2} = 1$$

$$h_{22} = \frac{I_2}{V_2}$$

$$V_2 = 10I_2$$

$$\frac{I_2}{V_2} = \frac{1}{10} = 0.1\ \Omega^{-1}$$

The relationship between Z and Y parameters is

$$Y_{11} = \frac{Z_{22}}{\Delta z};\ Y_{12} = \frac{-Z_{12}}{\Delta z}$$

$$Y_{21} = \frac{Z_{21}}{\Delta z};\ Y_{22} = \frac{+Z_{11}}{\Delta z}$$

where $\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}$

Similarly,

$$Z_{11} = \frac{Y_{22}}{\Delta y};\ Z_{12} = \frac{-Y_{12}}{\Delta y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta y};\ Z_{12} = \frac{Y_{11}}{\Delta y}$$

ABCD Parameters in Terms of Z Parameters and Y - Parameters

$$A = \frac{Z_{11}}{Z_{21}} = \frac{-Y_{22}}{Y_{21}}$$

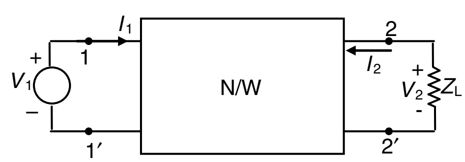
$$B = \frac{\Delta Z}{Z_{21}} = \frac{-1}{Y_{21}}$$

$$C = \frac{1}{Z_{21}} = \frac{-\Delta y}{Y_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{-Y_{11}}{Y_{21}}$$

Terminated Two-port Network

Driving point impedance at the input port of a load terminated network. Figure shows a two-port network connected to an ideal generator at the input port and to a load impedance at the output port.



$$V_2 = -I_2 Z_L$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$-I_2Z_L = Z_{21}I_1 + Z_{22}I_2$$

$$I_2 = \frac{-I_1Z_{21}}{Z_L + Z_{22}}$$

$$V_1 = Z_{11}I_1 - \frac{Z_{12}Z_{21}I_1}{Z_L + Z_{22}}$$

$$V_1 = I_1\left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_L + Z_{22}}\right]$$

$$\frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_L + Z_{22}}$$

Example 20

The Z parameter of a two-port network shown in figure are $Z_{11} = Z_{22} = 10\ \Omega$, $Z_{12} = Z_{21} = 4\ \Omega$. If the source voltage is 20 V, determine I_1, V_2, I_2, and input impedance.

Solution

$$V_1 = V_s = 20 \text{ V}$$

$$V_1 = I_1\left(Z_{11} - \frac{Z_{12}Z_{21}}{Z_L + Z_{22}}\right)$$

$$\therefore\ 20 = I_1\left(10 - \frac{4\times 4}{20+10}\right)$$

$$I_1 = 2.11 \text{ A}$$

$$I_2 = -I_1\frac{Z_{21}}{Z_L + Z_{22}}$$

$$= -2.11 \times \frac{4}{20+10} = -0.281 \text{ A}$$

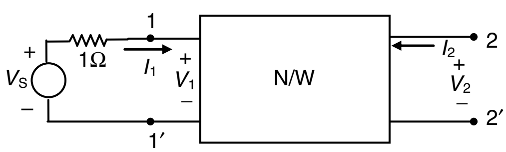
$$V_2 = -I_2 \times 20 = 0.281 \times 20 = 5.626 \text{ V}$$

$$\text{Input impedance} = \frac{V_1}{I_1} = \frac{20}{2.11}$$

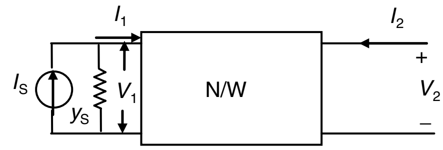
$$= 9.478\ \Omega$$

Example 21

The Y parameter of the two-port network shown in figure are $Y_{11} = Y_{22} = 6$ mho; $Y_{12} = Y_{21} = 4$ mho. Determine the driving point admittance at port 2-2′ if the source voltage is 100 V and has an impedance of 1 Ω.



Solution



$$I_1 = I_s - V_1Y_s$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$I_s - V_1Y_s = Y_{11}V_1 + Y_{12}V_2$$

$$-V_1(Y_s + Y_{11}) = Y_{12}V_2 - I_s$$

$$-V_1 = \frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}}$$

$$I_2 = -Y_{21}\left(\frac{Y_{12}V_2 - I_s}{Y_s + y_{11}}\right) + Y_{22}V_2$$

$$I_2 = -\frac{-Y_{21}Y_{12}V_2}{Y_s + Y_{11}} + Y_{22}V_2$$

$$\frac{I_2}{V_2} = \frac{Y_{22}Y_5 + Y_{22}Y_{11} - Y_{21}Y_{12}}{Y_s + Y_{11}}$$

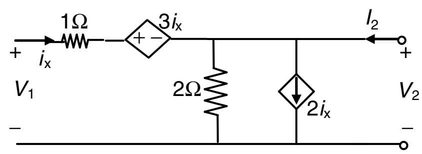
$$= \frac{6\times 1 + 6\times 6 - 4\times 4}{1+6}$$

$$= 3.714 \text{ mho}$$

Driving point impedance at port

$$2 - 2^1 = \frac{1}{3.714}\Omega$$

Example 22

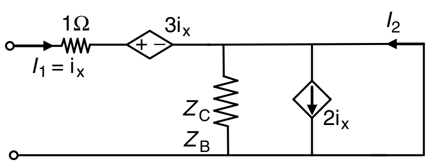


Find the h parameters for the network shown in figure

Solution

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

For $V_2 = 0$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$V_1 = I_1 \times 1 + 3I_1$$

$$\frac{V_1}{I_1} = 4 = h_{11}$$

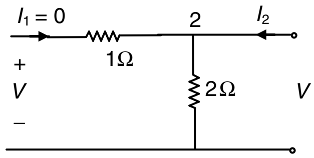
By applying KCL,

$$I_2 = 2I_1 - I_1$$

$$I_2 = I_1$$

$$\frac{I_2}{I_1} = 1 = h_{21}$$

For $I_1 = 0$



$$V_2 = V_1$$

$$\frac{V_1}{V_2} = h_{12} = 1$$

$$V_2 = I_2 \times 2$$

$$\frac{I_2}{V_2} = \frac{1}{2} = 0.5 = h_{22}$$

Network Functions

For a one-port network, the driving point impedance of the network is defined as

$$Z(s) = \frac{V(s)}{I(s)}$$

Similarly, the driving point admittance

$$Y(s) = \frac{I(s)}{V(s)}$$

For a two-port network without internal sources, the driving point impedance at port 1-1′ is

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

At port 2-2′, it is

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

Similarly $Y_{11}(s) = \dfrac{I_1(s)}{V_1(s)}$; $Y_{22}(s) = \dfrac{I_2(s)}{V_2(s)}$

$Y_{11}(s)$ and $Y_{22}(s)$ are transfer admittances. Voltage transfer ratio

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

and $G_{12}(s) = \dfrac{V_1(s)}{V_2(s)}$

Current transfer ratio

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}$$

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

Transfer impedance

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

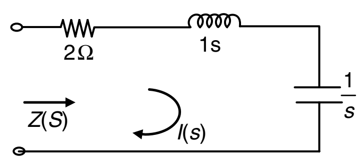
$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

Transfer admittance

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

Example 23



For the network shown in figure, obtain the driving point impedance.

Solution

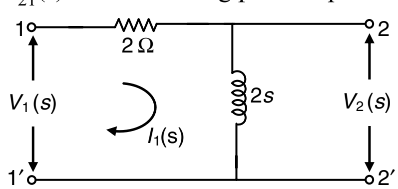
$$Z(s) = \frac{V(s)}{I(s)}$$

$$= 2 + s + \frac{1}{s}$$

$$Z(s) = \frac{2s + s^2 + 1}{s} = \frac{s^2 + 2s + 1}{s}$$

Example 24

For the network shown in figure, obtain the transfer functions $G_{21}(s)$ and $Z_{21}(s)$ and the driving point impedance $Z_{11}(s)$.



Solution

$$V_1(s) = I_1(s)\,[2 + 2s]$$

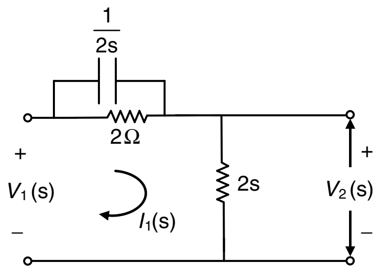
$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = 2(s + 1)$$

$$V_2(s) = I_1(s) \times 2s$$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2s}{2s + 2} = \frac{s}{s + 1}$$

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = 2s$$

Example 25



For the network shown in figure, find $G_{21}(s)$, $Z_{21}(s)$, and $Z_{11}(s)$.

Solution

$$\frac{V_1(s)}{I_1(s)} = Z_{11}(s) = \frac{2\times\frac{1}{2s}}{2+\frac{1}{2s}}+2$$

$$= \frac{2}{4s+1}+2$$

$$Z_{11}(s) = \frac{8s+4}{4s+1}$$

$$V_2(s) = \frac{V_1(s).2}{2+\frac{2}{4s+1}}$$

$$\frac{V_2(s)}{V_1(s)} = G_{21}(s) = \frac{2(4s+1)}{8s+4}$$

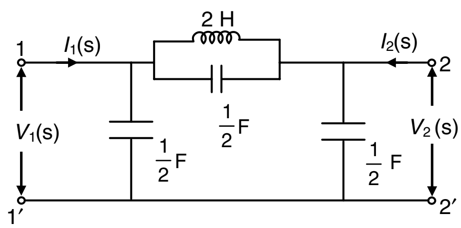
$$= \frac{8s+2}{8s+4}$$

$$V_2(\text{s}) = I_1(s).2$$

$$\frac{V_2(s)}{I_1(s)} = Z_{21}(s) = 2$$

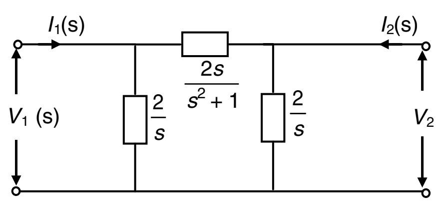
Example 26

For the network shown in figure, determine the transfer function $G_{21}(s)$ and $Z_{21}(s)$.



Solution

Transform the circuit into s-domain



$$V_2(s) = V_1(s).\ \frac{\frac{2}{s}}{\frac{2}{s}+\frac{2s}{s^2+1}}$$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{2}{s}}{2\,\frac{(s^2+1)+2s^2}{(s^2+1)s}}$$

$$= \frac{2(s^2+1)}{4s^2+2}$$

$$G_{21}(s) = \frac{s^2+1}{2s^2+1}$$

$$V_2(s) = I_1(s) = \frac{\frac{2}{s}}{\frac{4}{s}+\frac{2s}{s^2+1}}\times\frac{2}{s}$$

$$\frac{V_2(s)}{I_1(s)} = Z_{21}(s) = \frac{2(s^2+1)}{s(3s^2+2)}$$

Network Graphs

The solution of a linear network problem required for the formation of a set of equations, describing the response of the network first and then the manipulation of the co-efficient matrix so produced.

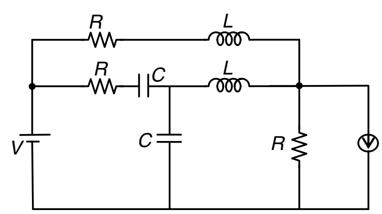
Networks' topology deals with concepts involving interconnections in the networks, rather than the actual nature of the elements.

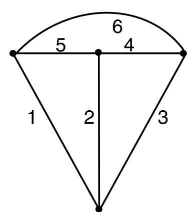
Graph

The connection of the network topology shown by replacing all physical elements by lines is called a graph.

While constructing a graph from the given network, all passive elements and the ideal voltage sources are replaced by short circuit, and all the ideal current sources are replaced by open circuit.

Let us consider the following example. A network and its related graph are shown in the below figures.



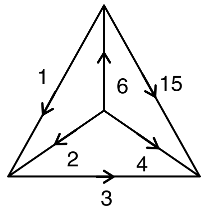


Directed Graph

A graph in which each branch is assigned a direction is called a directed or oriented graph.

Complete Graph or Standard Graph

For a standard graph between any pair of nodes, only one branch is connected for all combinations.



For example, the number of edges in a complete graph with n nodes in ${}^{n}C_2 \Rightarrow \dfrac{n(n-1)}{2} = b.$

Connected Graph

In a connected graph, all the nodes are connected by at least one branch, otherwise it is said to be unconnected. Let us consider the following example.

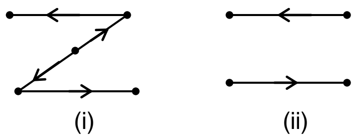
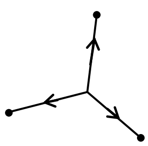


Figure 19 Connected graph.

Subgraph

It is a graph with less number of branches as compared with the original graph. Let us consider the following example.



Sub graph

Figure 20 Subgraph.

Planar and Non-planar Graphs

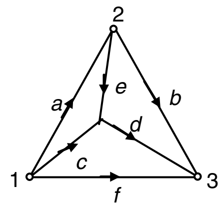
A graph is said to be planar if it can be drawn on a plane surface such that no two branches cross each other. A non-planar graph cannot be drawn on a plane surface without a crossover.

Tree and Co-tree

A tree is a connected subgraph of a network that consists of all the nodes of the original graph but no closed paths. The number of nodes in the graphs is equal to the number of nodes in the tree.

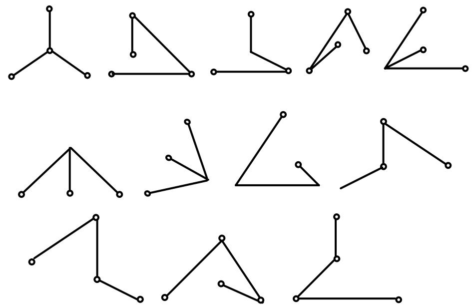
Example 27

For the given graph shown in figure, draw the number of possible trees.



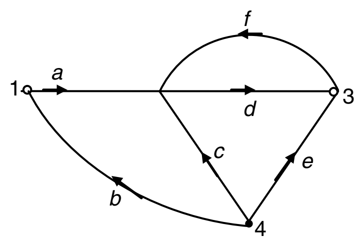
Solution

There are four nodes. The possible trees are shown in the figure.

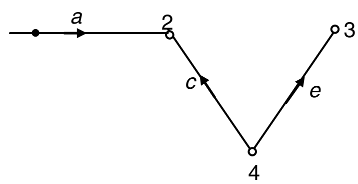


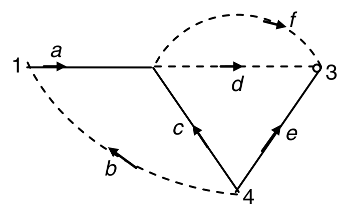
Twigs and Links

The branches of a tree are called its 'twigs'. For a given branch, the complementary set of branches of the tree is called the co-tree of the graph. The branches of co-tree are called links, that is, those elements of the connected graph that are not included in the tree links and forms a subgraph.



For the graph shown in the figure, the tree branches are 'ace', as shown in figure.

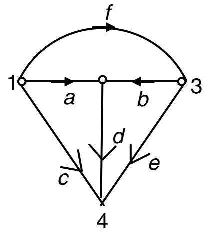




The set of branches (b, d, f) represented by dotted lines form a co-tree of the graphs. These branches are called links of this tree.

For a network with 'b' branches and 'n' nodes, the number of twigs for a selected tree is $(n - 1)$ and the number of links 'l' with respect to this tree is $b - n + 1$. The number of twigs is called the rank of the tree.

Incidence Matrix (*A*)



For the oriented graph shown in the figure, the incidence matrix is

$$A = \begin{array}{c} \downarrow \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} \text{Nodes} & & & \text{Branches}\rightarrow & & \\ a & b & c & d & e & f \\ 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix}_{n\times b}$$

In matrix A with 'n' rows and b columns, an entry a_{ij} in the ith row and jth column has the following values.

$a_{ij} = 1$, if the jth branch is incident to and oriented away from the ith node.

$a_{ij} = 1$, if the jth branch is incident and oriented towards the ith node.

$a_{ij} = 0$, if the jth branch is not incident to the ith node.

NOTE

Consider incoming branches are 'negative' sign and outgoing branches are 'positive' sign.

Example 28

Draw the graph corresponding to the given incidence matrix.

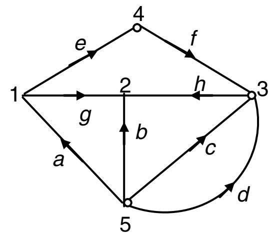
$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & +1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ +1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution

There are five rows and eight columns that indicate that there are five nodes and eight branches.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & +1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ +1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The graph can be drawn as shown in figure.

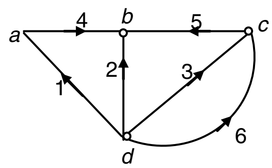


Incidence Matrix and Formulation of KCL

$$A_I I = 0$$

where A_I is the incidence matrix and I represents branch current vectors $I_1, I_2\ldots.$

Consider the graph shown in figure.



It has four nodes a, b, c, and d. Let node 'd' be taken as reference node. Let the branch currents be $i_1, i_2, \ldots, i_6$

By applying KCL at nodes a, b, and c

$$-i_1 + i_4 = 0$$

$$-i_2 + i_5 - i_4 = 0$$

$$-i_3 - i_5 - i_6 = 0$$

In matrix form, $$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1- & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_I I_b = 0 \ldots\ldots.. \text{(KCL)}$$

Relation Between Twigs and Links

The number of twigs on a tree is always one less than the number of nodes.

$$\text{twigs} = (n - 1)$$

Let n is the number of nodes. Further, if 'ℓ' represents the total number of links, while 'b' the total number of branches.

$$L = b - (n - 1)$$

$$\boxed{L = b - n + 1}$$

Tie-set Matrix

For a given tree of a graph, the addition of each link between any two nodes forms a loop called the fundamental loop.

In a loop, there exists a closed path and a circulating current, which is called the link current.

The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is called a tie set.

Consider a connected graph shown in Figure 21. It has four nodes and six branches. One of its trees is arbitrarily chosen and is shown in Figure 22.

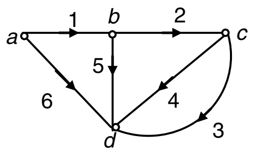


Figure 21

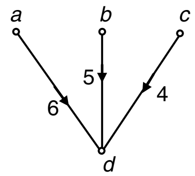


Figure 22

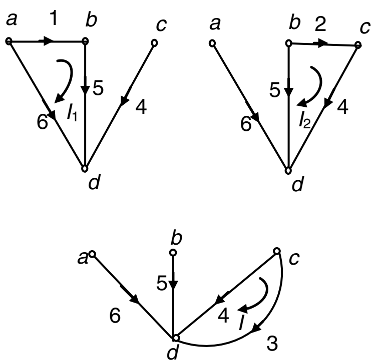
Number of nodes, $n = 4$

Number of branches, $b = 6$

Number of tree branches or twigs $= n - 1$

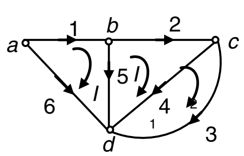
Number of link branches $I = b - (n - 1) = 3$

Let i_1, i_2 ……, i_6 be the branch currents with directions, as shown in figure ‘a’. When a link is added to the tree, a closed circuit is formed. The closed loops are as shown in the figures.



By convention, a fundamental loop is given the same orientation as its defining link, that is, the link current. I_1 coincides with the branch current i_1 direction in ab. Similarly, I_2 coincides with the branch current direction in bc and I_3 coincides with the direction of cd.

Tie-set Matrix



Consider the abovementioned figure. Kirchhoff's voltage law can be applied to the fundamental loops to get a set of linearly independent equation.

These are three fundamental loops I_1, I_2, and I_3 corresponding to the link branches 1, 2, and 3, respectively.

If V_1, V_2, …..V_6 are the branch voltages, the KVL equations for the three fundamental loops are

$$V_1 + V_5 - V_6 = 0$$

$$V_2 + V_4 - V_5 = 0$$

$$V_3 - V_4 = 0$$

The abovementioned equation can be written in matrix form:

loop ↓ 1 2 3 4   branches →

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

i.e., $BV_b = 0$ ……. (KVL)

where B is an $I \times b$ matrix called the tie-set matrix or fundamental loop matrix and V_b is a column vector of branch voltages.

Tie-set Matrix and Branch Currents

$$[I_b] = [B^T]\,[I_L]$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$[I_b] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix};\ [I_L] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The branch currents are

$$i_1 = I_1\ i_2 = I_2\ i_3 = I_3$$

$$i_4 = I_2 - I_3\ i_5 = I_1 - I_2\ i_6 = -I_1$$

Cut-set

It is a set of branches of a connected graph (G), where in the removal of all the branches of the set causes remaining graph to have two unconnected subgraphs.

Therefore, the cut-set is a minimal set of branches of the graph, and removal of which divides the graph into two subgraphs.

Consider the following example.

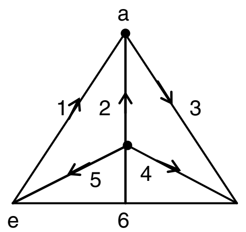
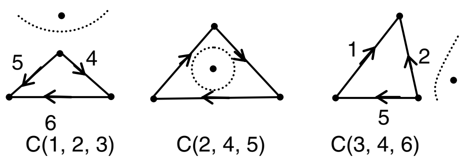


Figure 23 **Standard graph**



NOTE

$a_{ij} = \begin{cases} +1\text{: If branch } j \text{ leaves node } (i) \\ -1\text{: If branch } j \text{ enters node } (i) \\ 0\text{: If branch } j \text{ is not incident on } (i) \end{cases}$

Properties of a Tree in a Graph

1. It consists of all the nodes of the graph.
2. If the graph has N number of nodes, the tree will have $(N - 1)$ branches.
3. There will be no closed path in the tree.

NOTES

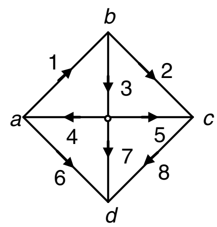
1. Rank of a graph = $(n - 1)$, where n is the number of nodes.
2. The number of trees for a given standard graph = $(n)^{n-2}$
3. Total number of KCL equations equal to $(n - 1)$.
4. Number of fundamental tie sets for a graph equal to number of links. i.e., $L = (b - n + 1)$
5. Rank of tie-set matrix = $(b - n + 1)$ = links
6. Number of nodal equations in a given graph equal to $(n - 1) \Rightarrow f$ cut-sets.
7. Number of mesh equations = f-loops = $b - n + 1$

Fundamental Cut-sets

The fundamental cut-sets are defined for a given tree of the graph. From a connected graph, first a tree is selected, and then a twig is selected. Removing this twig from the tree separates the tree into two parts. All the links that go from one part of the disconnected tree to the other, together with the twig of the selected tree will constitute a cut-set. This cut-set is called a fundamental cut-set of the graph.

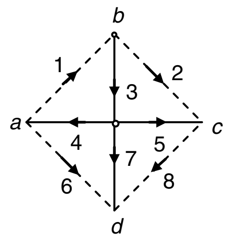
Example 29

Obtain the fundamental cut-set matrix Q_f for the network shown in figure.

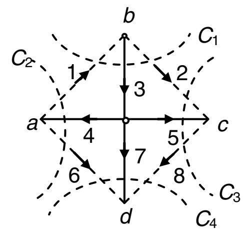


Solution

A selected tree of the graph is shown in the figure.



The twigs of the tree are {3, 4, 5, 7}. The remaining branches 1, 2, 6, and 8 are the links, corresponding to the selected tree.



The fundamental cut-set matrix is formed as fundamental.

$$\begin{array}{c} \text{Cutset} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{branches}\rightarrow \\ \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

The branch voltages in terms of twig voltages are

$$V_1 = -V_3 - V_4 = -V_{t3} - V_{t4}$$
$$V_2 = -V_3 - V_5 = +V_{t3} - V_{t5}$$
$$V_3 = V_{t3}$$
$$V_4 = V_{t4}$$
$$V_5 = V_{t5}$$
$$V_6 = V_7 - V_4 = V_{t7} - V_{t4}$$
$$V_7 = V_{t7}$$
$$V_8 = V_7 - V_5 = V_{t7} - V_{t5}$$

In matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_{t3} \\ V_{t4} \\ V_{t5} \\ V_{t7} \end{bmatrix}$$

Example 30

A standard graph consists of 55 branches, find the number of f cut-sets, tie-sets, f cut-set matrices and tie-set matrices.

Solution

Given $b = 55$

$$55 = \frac{n(n-1)}{2}$$

$$n(n-1) = 11$$

$$n = 11$$

$$L = b - n + 1 = 55 - 11 + 1 = 45$$

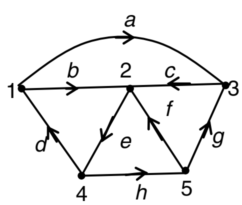
f loops or tie-sets = links = 45

f cut-set matrices = Tie-set matrices

f loop matrices = $(n)^{n-2} = (11)^9$

Example 31

Identify which of the following is not a tree of the graph shown in figure.

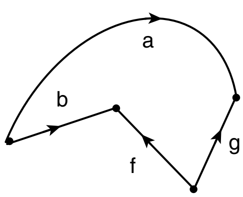


(A) $b\ c\ g\ h$ (B) $d\ e\ f\ g$
(C) $a\ b\ f\ g$ (D) $a\ e\ g\ h$

Solution

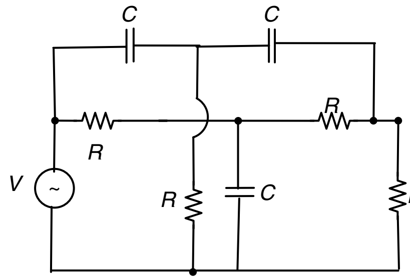
Tree is a connected graph without forming a closed path.

From the given options, (C) is not satisfied.



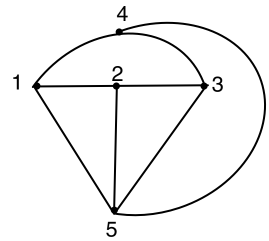
Example 32

Find the minimum number of equation required to analyse the following circuit.



(A) 3 (B) 4 (C) 6 (D) 7

Solution



Number of nodes = 5

Nodal equations = $N - 1 = 4$

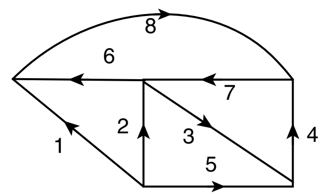
Number of mesh equations = $L = b - n + 1$

$= 8 - 5 + 1 = 4$

Minimum number of equations = min (nodal, mesh equations)

Example 33

Match List-I with List-II for the co-tree branches, 1, 2, 3, and 8 of the graph shown in the figure and select the correct answer using the following codes.



	List-I		List-II
p	Twigs	1	4, 5, 6, 7
q	Links	2	1, 2, 3, 8
r	Fundamental cut-set	3	1, 2, 3, 4
s	Fundamental loop	4	6, 7, 8

(A) $p - 1, q - 2, r - 3\ s - 4$ (B) $p - 3, q - 2, r - 1, s - 4$
(C) $p - 1, q - 4, r - 3, s - 2$ (D) $p - 3, q - 4, r - 1, s - 2$

Solution

Total number of branches = twigs + links

From the given data

Links ⇒ 1, 2, 3, 8

∴ Twigs = 4., 5, 6, 7 = $n - 1 = 4$

Given nodes = 5

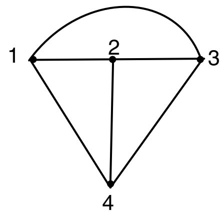
Fundamental cut-set having at a time only one tree branch

i.e., 1, 2, 3, 4
→ twig

∴ f-loops ⇒ having at a time only one link
⇒ 6, 7, 8,
→ Link

Example 34

What is the total number of trees for the following graph?



(A) 4 (B) 8
(C) 12 (D) 16

Solution

For a standard graph, total number of trees = $(n)^{n-2}$, where $n = 4$.

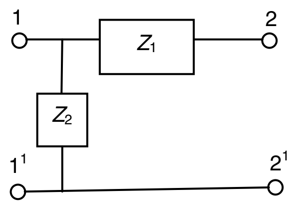
∴ Number of trees = $(4)^{4-2} = 4^2$
= 16

Exercises

Practice Problems I

Direction for questions 1 to 31: Select the correct alternative from the given choices.

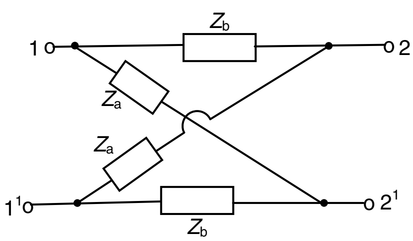
1. The z parameter of the network shown in the figure is



(A) $\begin{bmatrix} z_1+z_2 & z_1 \\ z_2 & z_1+z_2 \end{bmatrix}$ (B) $\begin{bmatrix} z_1 & z_2 \\ z_1+z_2 & z_1-z_2 \end{bmatrix}$

(C) $\begin{bmatrix} z_2 & z_2 \\ z_2 & z_1+z_2 \end{bmatrix}$ (D) $\begin{bmatrix} z_1 & z_1 \\ z_1 & z_1+z_2 \end{bmatrix}$

2. For the lattice circuit shown in figure, $Z_a = j2\ \Omega$ and $Z_b = 2\ \Omega$. The values of the open circuit impedance parameters $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ are



(A) $\begin{bmatrix} 1-j & 1+j \\ 1-j & 1+j \end{bmatrix}$ (B) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$

(C) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$ (D) $\begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$

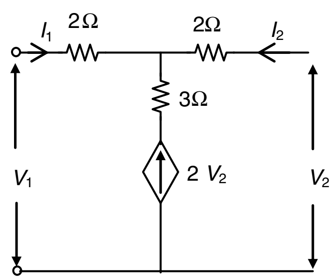
3. A two-port network is represented by $ABCD$ parameters given by $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

If port-2 is terminated by R_L, the input impedance seen at port-1 is given by ______.

(A) $\dfrac{A+BR_L}{C+DR_L}$ (B) $\dfrac{AR_L+C}{BR_L+D}$

(C) $\dfrac{DR_L+A}{BR_L+C}$ (D) $\dfrac{B+AR_L}{D+CR_L}$

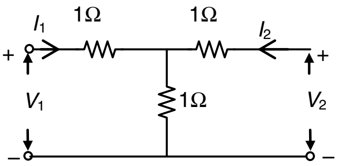
4. The admittance parameter of the network shown in the figure is ______.



(A) $\begin{bmatrix} \frac{1}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{-1}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{-3}{4} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{4} & \frac{5}{4} \\ \frac{-1}{4} & \frac{-3}{4} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & \frac{-5}{4} \\ \frac{-1}{4} & \frac{-3}{4} \end{bmatrix}$

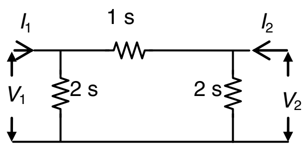
5. The transmission parameter $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ of the two-port network shown in the figure is ______.



(A) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

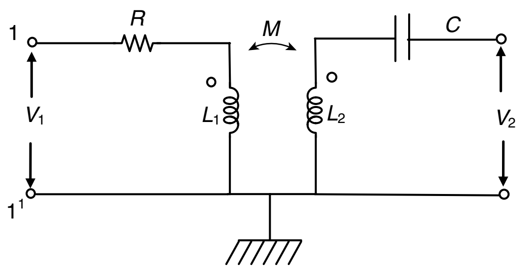
6. If two identical sections of the network shown in the figure are connected in parallel, the Y parameter of the resulting network is given by ______.



(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$

7.

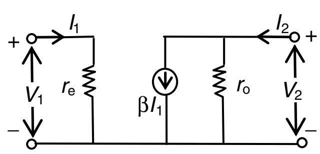


The element Z_{22} of the two-port network shown in the abovementioned figure is

(A) $R + sL_1$ (B) $\frac{1}{Cs} + sL_2$

(C) $\frac{1}{Cs} + sL_1$ (D) sL_2

8. In the following two-port network, Z_{12} and Z_{21}, respectively, are



(A) r_e and βr_o (B) 0 and $-\beta r_o$
(C) 0 and βr_o (D) r_e and $-\beta r_o$

9. A linear transformer and its T equivalent circuit are shown in figure (a) and figure (b), respectively. The values of L_a, L_b, and L_c, respectively, are ______.

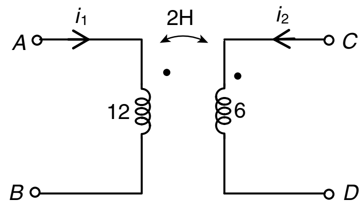


Fig (a)

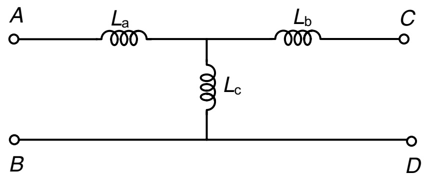
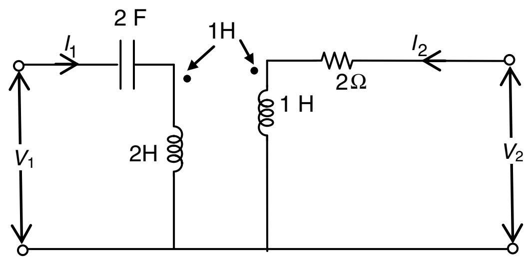


Fig (b)

(A) 10 H, 4 H, –2 H (B) 14 H, 8 H, –2 H
(C) 10 H, 4 H, 2 H (D) 14 H, 8 H, +2 H

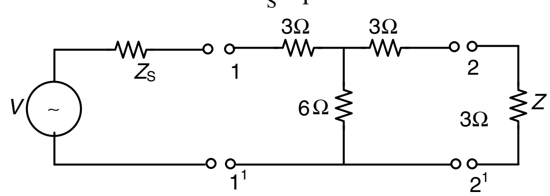
10. For the following network, the 'Z' parameter will be



(A) $\begin{bmatrix} \frac{1}{2s}+2s & 2+s \\ 2+s & s \end{bmatrix}$ (B) $\begin{bmatrix} s & \frac{1}{2s}+2s \\ 2+s & s \end{bmatrix}$

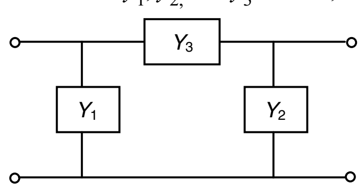
(C) $\begin{bmatrix} \frac{1}{2s}+2s & s \\ s & 2+s \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2s}-s & s \\ s & 2-s \end{bmatrix}$

11. An impedance match is desired at the 1–1[1] port of the two-port network shown in the given figure. The match will be obtained when Z_S equals



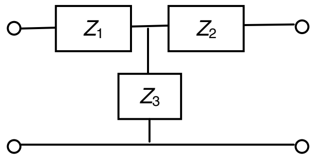
(A) 6 Ω (B) 3 Ω (B) $\frac{3}{2}$Ω (D) $\frac{2}{3}$Ω

12. The admittance parameter of the two-port network shown in the figure are $Y_{11} = 10$ ℧, $Y_{12} = Y_{21} = 6$ ℧, and $Y_{22} = 8$ ℧. The values of y_1, y_2, and y_3 will be, respectively,



(A) 2, 4, and 6 (B) 4, 2, and –6
(C) 2, 4, and –6 (D) 16, 14, and –6

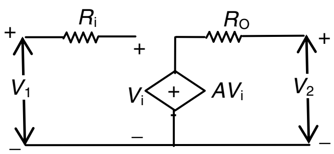
13.



To construct a high-pass filter as in the abovementioned circuit

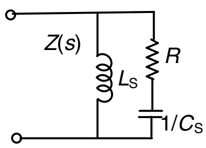
(A) z_1, z_2 are capacitors and z_3 inductor
(B) z_1 z_2 are resistors and z_3 capacitor
(C) z_1 z_2 are inductors z_3 capacitor
(D) z_1 z_2 are resistors z_3 inductor.

14. For the given equivalent circuit, find input impedance, output impedance, and output voltage.



(A) 0, α, α (B) α, R_0, Av_i
(C) R_i, R_o, Av_i (D) R_i, R_0, A

15. Find the driving point admittance of the network shown.



Given

$R = 1$ MΩ

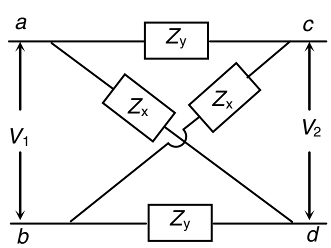
$C = 10$ μF

$L = 1{,}000$ H

(A) $\dfrac{1\times10^3}{1+10s+10^4s^2}$ (B) $\dfrac{(1+10s)10^5s}{s^2+10^3s+10^2}$

(C) $\dfrac{10s}{10^{-3}s^2+10s+1}$ (D) $\dfrac{10}{10^{-3}s^2+10s+1}$

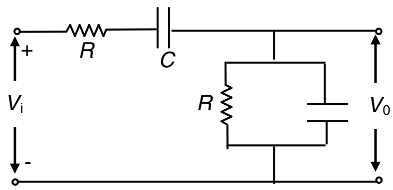
16. Following circuit shows a lattice circuit $z_x = 4\,j\Omega$ and $z_y = 4\ \Omega$, find the values of open-circuit impedance $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$



(A) $\begin{bmatrix} 2+2j & 2-2j \\ -2+2j & -2-2j \end{bmatrix}$ (B) $\begin{bmatrix} 2+2j & -2+2j \\ -2+2j & 2+2j \end{bmatrix}$

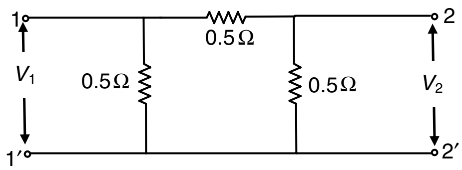
(C) $\begin{bmatrix} -2+2j & 2-2j \\ 2-2j & 2+2j \end{bmatrix}$ (D) $\begin{bmatrix} -2-2j & 2-2j \\ 2-2j & -2-2j \end{bmatrix}$

17. The *RC* circuit shown in the figure is



(A) a low-pass filter (B) a high-pass filter
(C) a band-pass filter (D) a band-reject filter

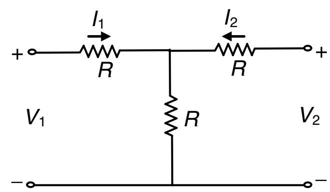
18. For the following two-port network, the short-circuit admittance parameter matrix is



(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$

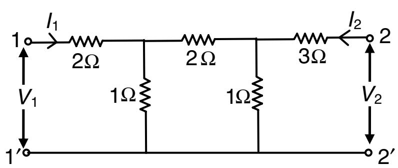
(C) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

19. A two-port network is shown in the figure. The parameter h_{21} for this network can be given by



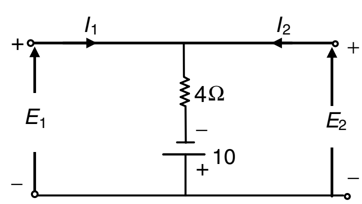
(A) –1/2 (B) $+\frac{1}{2}$ (C) –3/2 (D) +3/2

20. The impedance parameters Z_{11} and Z_{12} of the two-port network in the figure are



(A) $Z_{11} = 2.75\ \Omega$ and $Z_{12} = 0.25\ \Omega$
(B) $Z_{11} = 3\ \Omega$ and $Z_{12} = 0.5\ \Omega$
(C) $Z_{11} = 3\ \Omega$ and $Z_{12} = 0.25\ \Omega$
(D) $Z_{11} = 2.25\ \Omega$ and $Z_{12} = 0.5\ \Omega$

21. The Z parameters Z_{11} and Z_{21} for the following two-port network are



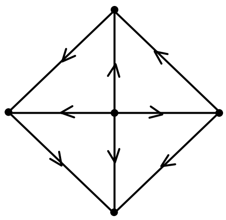
(A) $Z_{11} = \frac{-6}{11}\Omega$; $Z_{21} = \frac{16}{11}\Omega$

(B) $Z_{11} = \frac{6}{11}\Omega$; $Z_{21} = \frac{4}{11}\Omega$

(C) $Z_{11} = \frac{6}{11}\Omega$; $Z_{21} = \frac{-16}{11}\Omega$

(D) $Z_{11} = \frac{4}{11}\Omega$; $Z_{21} = \frac{4}{11}\Omega$

22. For the graph shown in the figure, the order of the tie-set matrix is



(A) 4×4 (B) 4×8 (C) 8×4 (D) 8×8

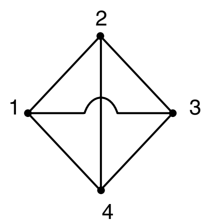
23. If V_b, Q, and V_t represent branch voltage matrix, cut-set matrix, and the twig voltage matrix, then the relationship between them is given by

(A) $V_b = Q\, V_t^T$ (B) $V_b = Q^T\, V_t^T$
(C) $V_b = V_t\, Q$ (D) $V_b = Q^T\, V_t$

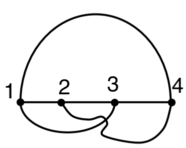
24. A planar graph has five nodes and nine branches. The number of meshes in the dual graph is

(A) 5 (B) 4
(C) 14 (D) None of these

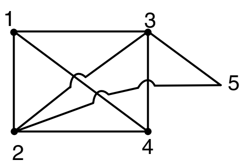
25. From the following graph, which of them is non-planar?



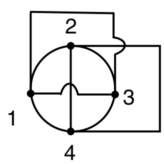
(i)



(ii)



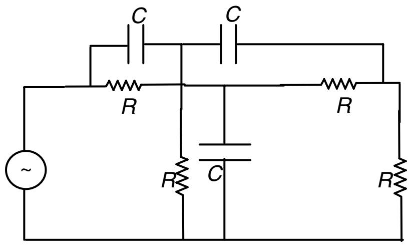
(iii)



(iv)

(A) i and ii (B) ii and iii
(C) iii only (D) iv only

26. The minimum number of equations required to analyse the following circuit is



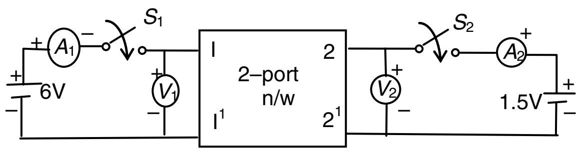
(A) 3 (B) 4 (C) 6 (D) 7

27. A two-port network shown in the following figure is excited by external DC sources. The voltages and the current are measured with voltmeters V_1 and V_2 and ammeters A_1 and A_2, as indicated.

Under following switch conditions, the readings obtained are

(i) S_1 – open, S_2 – closed
$A_1 = 0$ A, $V_1 = 4.5$ V, $V_2 = 1.5$ V, $A_2 = 1$ V

(ii) S_1 – closed, S_2 – open
$A_1 = 4$ A, $V_1 = 6$ V, $V_2 = 6$ V, $A_2 = 0$ A



The Z parameter matrix for this network is

(A) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ (B) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ (D) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

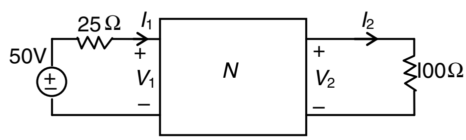
28. From the abovementioned question data, the h parameter matrix for this network is

(A) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & -1 \\ +3 & 0.67 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

29. In the following circuit, the network N is described by the following Y matrix $[Y] = \begin{bmatrix} 0.1\Omega & -0.01\Omega \\ 0.01\Omega & 0.1\Omega \end{bmatrix}$

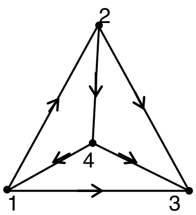
The voltage gain $\frac{V_2}{V_1}$ is

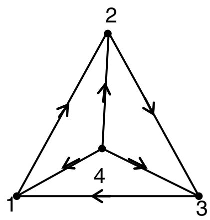


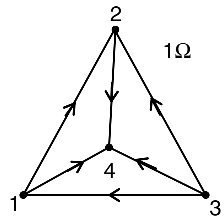
(A) 1/11 (B) –1/11 (C) –1/99 (D) 1/90

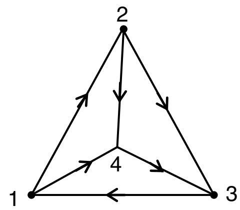
30. The incidence matrix of a graph is as follows:

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$ the graph is

(A) 

(B) 

(C) 

(D) 

31. The incidence matrix of a graph is

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The number of possible trees are

(A) 40 (B) 70 (C) 50 (D) 240

Practice Problems 2

Direction for questions 1 to 23: Select the correct alternative from the given choices.

1. Which parameters are used in the analysis of transistors?
(A) Z parameters
(B) Y parameters
(C) h parameters
(D) transmission parameters

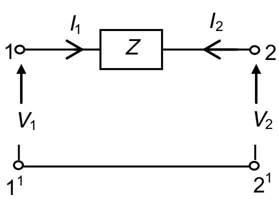
2. If a transmission line is represented by a two-port network whose parameters are A, B, C, D, then the sending- and voltage-end current are given by______.
(A) $V_S = AV_r + BI_r$, $I_S = CV_r + DI_r$
(B) $V_S = AV_r + CI_r$, $I_S = BV_r + DI_s$
(C) $V_S = AV_r - BI_r$, $I_S = CV_r - DI_r$
(D) $V_S = AV_r - CI_r$, $I_S = BV_r - DI_r$

3. A two-port network is reciprocal if and only if
(A) $Z_{11} = Z_{22}$.
(B) $Y_{12} = Y_{21.}$
(C) $BC - AD = -1$.
(D) $h_{12} = h_{21.}$

4. A two-port network is symmetrical if
(A) $z_{11} = z_{22}$
(B) $z_{11}z_{22} - z_{12}z_{21} = 1$
(C) $h_{11}h_{22} - h_{12}h_{21} = 1$
(D) Both A and C

5. A two-port network is reciprocal if
(A) $Z_{12} = Z_{21}$
(B) $A = D$
(C) $Y_{11} = Y_{22}$
(D) $BC - AD = 1$

6. For the two-port network shown in the figure, which of the following statements is true?



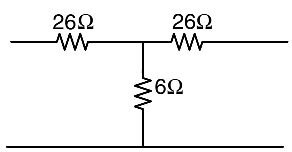
(A) It has Z parameter.
(B) It has no Z parameter.
(C) It has no Y parameter.
(D) It has no transmission parameter.

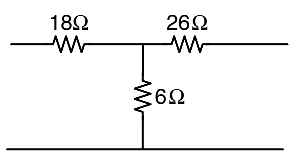
7. Two two-port networks have Z parameters

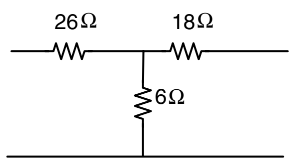
$$[Z]_x = \begin{bmatrix} Z_{11x} & Z_{12x} \\ Z_{21x} & Z_{22x} \end{bmatrix} \text{ and } [Z]_y = \begin{bmatrix} Z_{11y} & Z_{12y} \\ Z_{21y} & Z_{22y} \end{bmatrix}$$

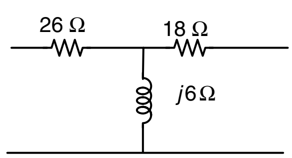
Then, the open-circuit transfer impedance of the cascaded network is
(A) $Z_{12x} + Z_{12y}$
(B) $Z_{21x} + Z_{21y}$
(C) $\dfrac{Z_{21x}\, Z_{21y}}{Z_{11x} + Z_{22y}}$
(D) $\dfrac{Z_{12x}\, Z_{12y}}{Z_{12x} + Z_{12y}}$

8. A two-port network is represented by $V_1 = 32I_1 + 6I_2$ and $V_2 = 6I_1 + 24I_2$. Which one of the following networks is represented by these equations?

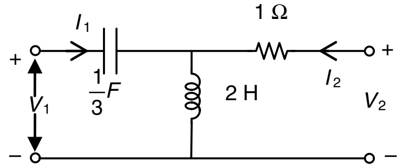
(A) 

(B) 

(C) 

(D) 

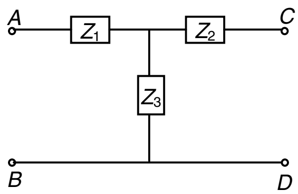
9. Z matrix for the network shown in the given figure is



(A) $\begin{bmatrix} \frac{1}{3s} & 2s \\ 2s & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2s+\frac{3}{s} & -2s \\ -2s & 2s+1 \end{bmatrix}$

(C) $\begin{bmatrix} 2s+\frac{3}{s} & 2s \\ 2s & 2s+1 \end{bmatrix}$ (D) $\begin{bmatrix} 2s & 2s+\frac{3}{s} \\ 2s+\frac{3}{s} & 2s \end{bmatrix}$

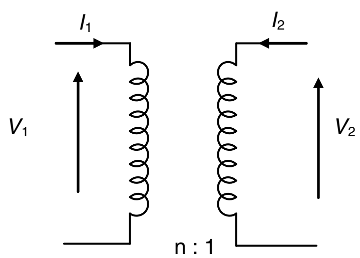
10. Find values of Z_1, Z_2, and Z_3 in the following network.



$Z_{11} = 16\ \Omega$, $Z_{12} = 10\ \Omega$, $Z_{22} = 15\ \Omega$, $Z_{21} = 10\ \Omega$

(A) $Z_1 = 10\ \Omega$, $Z_2 = 5\ \Omega$, $Z_3 = 6\ \Omega$
(B) $Z_1 = 26\ \Omega$, $Z_2 = 25\ \Omega$, $Z_3 = 10\ \Omega$
(C) $Z_1 = 6\ \Omega$, $Z_2 = 5\ \Omega$, $Z_3 = 10\ \Omega$
(D) $Z_1 = 25\ \Omega$, $Z_2 = 26\ \Omega$, $Z_3 = 10\ \Omega$

11. The *ABCD* parameters of an ideal n:1 transformer shown in the figure are $\begin{bmatrix} n & o \\ o & x \end{bmatrix}$. The value of x will be



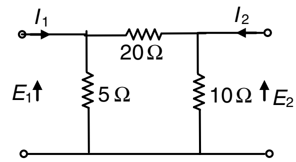
(A) n (B) $\frac{1}{n}$ (C) n^2 (D) $\frac{1}{n^2}$

12. The short-circuit admittance matrix of a two-port network is

$$\begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

The two-port network is
(A) non-reciprocal and passive
(B) non-reciprocal and active
(C) reciprocal and passive
(D) reciprocal and active

13. The admittance parameter Y_{12} in the following two-port network is

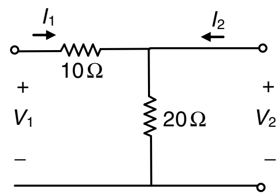


(A) −0.2 Mho (B) 0.1 Mho
(C) −0.05 Mho (D) 0.05 Mho

14. For a two-port network to be reciprocal
(A) $Z_{11} = Z_{22}$ (B) $Y_{21} = Y_{22}$
(C) $h_{21} = -h_{12}$ (D) $AD - BC = 0$

15. Which parameters are widely used in transmission line theory?
(A) *Z* parameters (B) *Y* parameters
(C) *ABCD* parameters (D) *h* parameters

16. The *h* parameters of the circuit shown in the figure are



(A) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

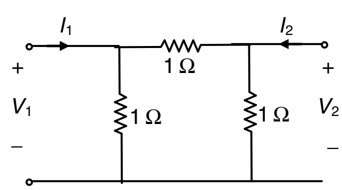
(C) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

17. The impedance matrices of two, two-port networks are given by $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 15 & 5 \\ 5 & 25 \end{bmatrix}$. If the two networks are connected in series, the impedance matrix of the combination is

(A) $\begin{bmatrix} 3 & 5 \\ 2 & 25 \end{bmatrix}$ (B) $\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 2 \\ 5 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 7 \\ 25 & 3 \end{bmatrix}$

18.



The *Y* parameters for the network is

(A) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 0.5 & 1 \\ 1 & 0.5 \end{bmatrix}$ (D) $\begin{bmatrix} 0.5 & -1 \\ -1 & 0.5 \end{bmatrix}$

19. If the graph of an electrical network has 'N' nodes and 'B' branches, the number of links 'L' is given by

(A) $N - B + 1$ (B) $B - N + 1$
(C) $N + B$ (D) $B - N$

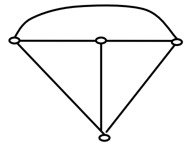
20. A connected network of $N > 2$ nodes has at the most one branch directly connecting any pair of nodes. The graph of the network

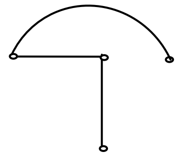
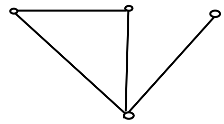
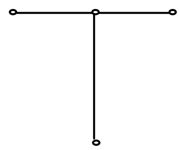
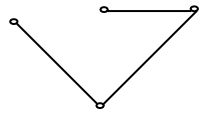
(A) must have at least 'N' branches for one or more closed paths to exist.
(B) can have an unlimited number of branches.
(C) can only have at the most N branches.
(D) can have a minimum number of branches not decided by N.

21. If B is tie-set matrix and 'I_L' is loop current matrix, then branch current matrix I_b is given by

(A) $I_b = BI_L$ (B) $I_b = I_L B^T$
(C) $I_b = B^T I_L$ (D) $I_b = B^T I_L^T$

22. Consider the following network graph. Which one of the following is not a 'tree' of this graph?



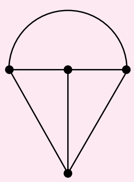
(A)

(B)

(C)

(D)


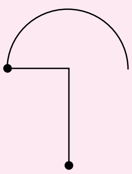
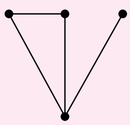
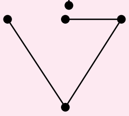
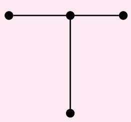
23. A network has seven nodes and five independent loops. The number of branches in the network is

(A) 13 (B) 12 (C) 11 (D) 10

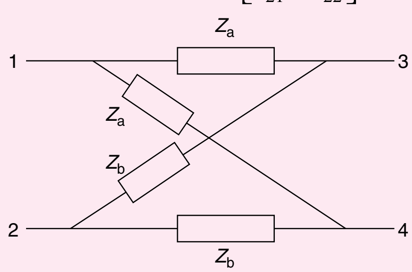
Previous Years' Questions

1. Consider the network graph shown in figure. Which one of the following is not a 'tree' of this graph? **[2004]**



(A)

(B)

(C)

(D)


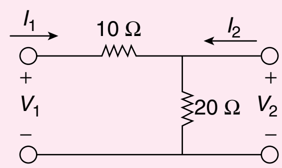
2. For the lattice circuit shown in Figure of Q.31, $Z_a = j2\ \Omega$ and $Z_b = 2\ \Omega$. The values of the open-circuit impedance parameters $Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ **[2004]**



(A) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1+j \end{bmatrix}$ (B) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$

(C) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$ (D) $\begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$

3. The h parameters of the circuit shown in figure are **[2005]**



(A) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

(C) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

4. A two-port networks is represented by $ABCD$ parameters given by **[2006]**

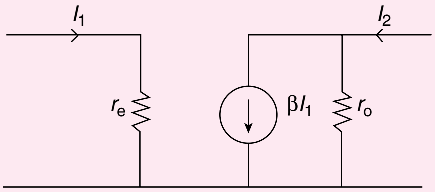
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

If port-2 terminated by R_L, the input impedance seen at port-1 is given by

(A) $\dfrac{A + BR_L}{C + DR_L}$ (B) $\dfrac{AR_L + C}{BR_L + D}$

(C) $\dfrac{DR_L + A}{BR_L + C}$ (D) $\dfrac{B + AR_L}{D + CR_L}$

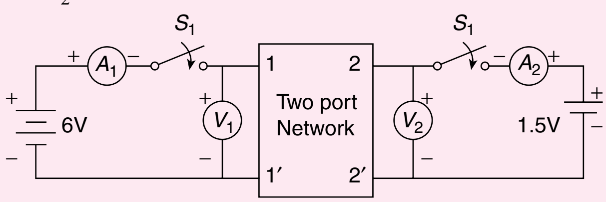
5. In the following two-port network, Z_{12} and Z_{21} are, respectively, **[2006]**



(A) r_e and βr_0 (B) 0 and $-\beta r_0$
(C) 0 and βr_0 (D) r_e and $-\beta r_0$

Direction for questions 6 and 7:
The following two-port network is excited by external DC sources. The voltages and the currents are measured with voltmeters V_1, V_2 and ammeter A_1, A_2 (all assumed to be ideal), as indicated. Under following switch conditions, the readings obtained are:

i) S_1 – open, S_2 – closed $A_1 = 0$ A, $V_1 = 4.5$ V, $V_2 = 1.5$ V, $A_2 = 1$ A
ii) S_1 – closed, S_2 – open $A_1 = 4$ A, $V_1 = 6$ V, $V_2 = 6$ V, $A_2 = 0$ A



6. The z parameter matrix for this network is **[2008]**

(A) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ (B) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$

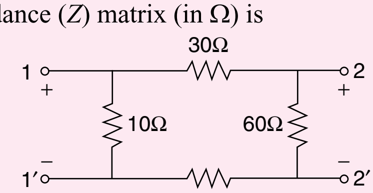
(C) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ (D) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

7. The h parameter matrix for this network is **[2008]**

(A) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & -1 \\ 3 & 0.67 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & 1 \\ -3 & 0.67 \end{bmatrix}$

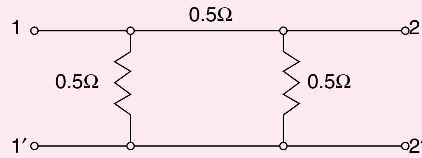
8. For the two-port network shown in the figure, the impedance (Z) matrix (in Ω) is **[2014]**



(A) $\begin{bmatrix} 6 & 24 \\ 42 & 9 \end{bmatrix}$ (B) $\begin{bmatrix} 9 & 8 \\ 8 & 24 \end{bmatrix}$

(C) $\begin{bmatrix} 9 & 6 \\ 6 & 24 \end{bmatrix}$ (D) $\begin{bmatrix} 42 & 6 \\ 6 & 60 \end{bmatrix}$

9. For the following two-port network, the short-circuit admittance parameter matrix is **[2010]**

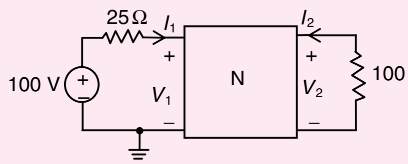


(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} S$ (B) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} S$

(C) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} S$ (D) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} S$

10. In the following circuit, the network N is described by the following Y matrix:

$Y = \begin{bmatrix} 0.1S & -0.01S \\ 0.01S & 0.1S \end{bmatrix}$. The voltage gain $\frac{V_2}{V_1}$ is **[2011]**



(A) 1/90 (B) –1/90
(C) –1/99 (D) –1/11

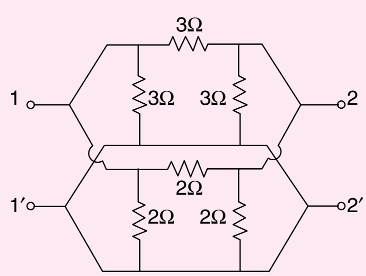
Direction for questions 11 and 12:
With 10 V DC connected at port A in the linear non-reciprocal two-port network shown in the figure, the following were observed.

(i) 1 Ω connected at port B draws a current of 3 A
(ii) 2.5 Ω connected at port B draws a current of 2 A

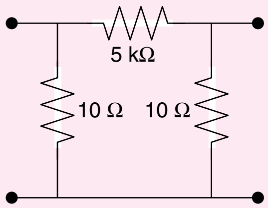
11. With 10 V DC connected at port A, the current drawn by 7 Ω connected at port B is **[2012]**
(A) 3/7 A (B) 5/7 A (C) 1 A (D) 9/7 A

12. For the same network, with 6 V DC connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V DC is connected to port A, the open-circuit voltage at port B is **[2012]**
(A) 6 V (B) 7 V (C) 8 V (D) 9 V

13. In the h parameter model of the following two-port network, the value of h_{22} (in S) is __________ **[2014]**



14. The two-port admittance matrix of the circuit shown is given by **[2015]**



(A) $\begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 15 & 5 \\ 5 & 15 \end{bmatrix}$

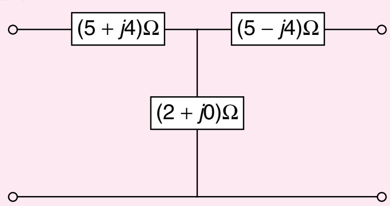
(C) $\begin{bmatrix} 3.33 & 5 \\ 5 & 3.33 \end{bmatrix}$ (D) $\begin{bmatrix} 0.3 & 0.4 \\ 0.4 & 0.3 \end{bmatrix}$

15. An LC tank circuit consists of an ideal capacitor C connected in parallel with a coil of inductance L having an internal resistance R. The resonant frequency of the tank circuit is **[2015]**

(A) $\dfrac{1}{2\pi\sqrt{LC}}$ (B) $\dfrac{1}{2\pi\sqrt{LC}}\sqrt{1-R^2\dfrac{C}{L}}$

(C) $\dfrac{1}{2\pi\sqrt{LC}}\sqrt{1-\dfrac{1}{R^2C}}$ (D) $\dfrac{1}{2\pi\sqrt{LC}}\left(1-R^2\dfrac{C}{L}\right)$

16. The ABCD parameters of the following two-port network are **[2015]**



(A) $\begin{bmatrix} 3.5+j2 & 20.5 \\ 20.5 & 3.5-j2 \end{bmatrix}$

(B) $\begin{bmatrix} 3.5+j2 & 30.5 \\ 0.5 & 3.5-j2 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & 2+j0 \\ 2+j0 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 7+j4 & 0.5 \\ 30.5 & 7-j4 \end{bmatrix}$

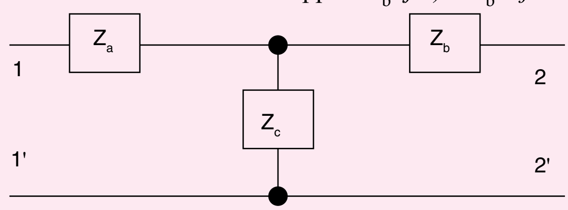
17. Consider a two port network with the transmission matrix: T = $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$. If the network is reciprocal, then **[2016]**

(A) $T^{-1} = T$
(B) $T^2 = T$
(C) Determinant (T) = 0
(D) Determinant (T) = 1

18. The z-parameter matrix for the two-port network shown is

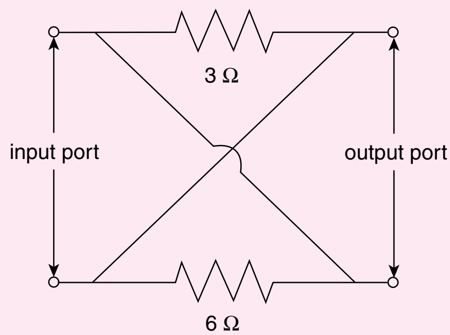
$$\begin{bmatrix} 2j\omega & j\omega \\ j\omega & 3+2j\omega \end{bmatrix}$$

Where the entries are in Ω. Suppose $z_b(j\omega) = R_b + j\omega$.



Then the value of R_b (in Ω) equals ___________. **[2016]**

19. The z – parameter matrix $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ for the two port network shown is **[2016]**



(A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 9 & -3 \\ 6 & 9 \end{bmatrix}$ (D) $\begin{bmatrix} 9 & 3 \\ 6 & 9 \end{bmatrix}$

Answer Keys

Exercises

Practice Problems 1

1. C	**2.** D	**3.** D	**4.** D	**5.** C	**6.** C	**7.** B	**8.** B	**9.** C	**10.** C
11. A	**12.** D	**13.** C	**14.** B	**15.** B	**16.** B	**17.** C	**18.** A	**19.** A	**20.** A
21. C	**22.** B	**23.** D	**24.** A	**25.** D	**26.** B	**27.** C	**28.** A	**29.** B	**30.** D
31. A									

Practice Problems 2

1. C	**2.** A	**3.** C	**4.** D	**5.** A	**6.** B	**7.** C	**8.** C	**9.** C	**10.** C
11. B	**12.** B	**13.** C	**14.** C	**15.** C	**16.** D	**17.** B	**18.** B	**19.** B	**20.** D
21. C	**22.** B	**23.** C							

Previous Years' Questions

1. B	**2.** D	**3.** D	**4.** D	**5.** B	**6.** C	**7.** A	**8.** C	**9.** A	**10.** D
11. C	**12.** B	**13.** 1.24 to 1.26		**14.** A	**15.** B	**16.** B	**17.** D	**18.** 3Ω	**19.** A