Government of Karnataka

MATHEMATICS

8

EIGHTH STANDARD

Part-II

विद्यया ऽमृतमश्नुते
एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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Part-II

CHAPTER

1 Cubes and Cube Roots

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1.1 Introduction

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$$1729 = 1728 + 1 = 12^3 + 1^3$$

$$1729 = 1000 + 729 = 10^3 + 9^3$$

1729 has since been known as the Hardy – Ramanujan Number, even though this feature of 1729 was known more than 300 years before Ramanujan.

Hardy – Ramanujan Number

1729 is the smallest Hardy–Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16; 9, 15), 13832 (18, 20; 2, 24), Check it with the numbers given in the brackets.

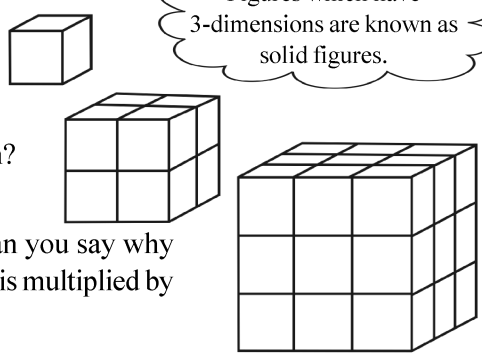
How did Ramanujan know this? Well, he loved numbers. All through his life, he experimented with numbers. He probably found numbers that were expressed as the sum of two squares and sum of two cubes also.

There are many other interesting patterns of cubes. Let us learn about cubes, cube roots and many other interesting facts related to them.

1.2 Cubes

Figures which have 3-dimensions are known as solid figures.

You know that the word 'cube' is used in geometry. A cube is a solid figure which has all its sides equal. How many cubes of side 1 cm will make a cube of side 2 cm?



How many cubes of side 1 cm will make a cube of side 3 cm?

Consider the numbers 1, 8, 27, ...

These are called **perfect cubes or cube numbers**. Can you say why they are named so? Each of them is obtained when a number is multiplied by taking it three times.

We note that $1 = 1 \times 1 \times 1 = 1^3$; $8 = 2 \times 2 \times 2 = 2^3$; $27 = 3 \times 3 \times 3 = 3^3$.
Since $5^3 = 5 \times 5 \times 5 = 125$, therefore 125 is a cube number.

Is 9 a cube number? No, as $9 = 3 \times 3$ and there is no natural number which multiplied by taking three times gives 9. We can see also that $2 \times 2 \times 2 = 8$ and $3 \times 3 \times 3 = 27$. This shows that 9 is not a perfect cube.

The following are the cubes of numbers from 1 to 10.

Table 1

Number	Cube
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 =$ _____
6	$6^3 =$ _____
7	$7^3 =$ _____
8	$8^3 =$ _____
9	$9^3 =$ _____
10	$10^3 =$ _____

The numbers 729, 1000, 1728 are also perfect cubes.

Complete it.

There are only ten perfect cubes from 1 to 1000. (Check this). How many perfect cubes are there from 1 to 100?

Observe the cubes of even numbers. Are they all even? What can you say about the cubes of odd numbers?

Following are the cubes of the numbers from 11 to 20.

Table 2

Number	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

We are even, so are our cubes

We are odd so are our cubes

Consider a few numbers having 1 as the one's digit (or unit's). Find the cube of each of them. What can you say about the one's digit of the cube of a number having 1 as the one's digit?

Similarly, explore the one's digit of cubes of numbers ending in 2, 3, 4, ... , etc.

TRY THESE

Find the one's digit of the cube of each of the following numbers.

(i) 3331 (ii) 8888 (iii) 149 (iv) 1005

(v) 1024 (vi) 77 (vii) 5022 (viii) 53

1.2.1 Some interesting patterns

1. Adding consecutive odd numbers

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$

Is it not interesting? How many consecutive odd numbers will be needed to obtain the sum as 10^3?

TRY THESE

Express the following numbers as the sum of odd numbers using the above pattern?

(a) 6^3 (b) 8^3 (c) 7^3

Consider the following pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Using the above pattern, find the value of the following.

(i) $7^3 - 6^3$ (ii) $12^3 - 11^3$ (iii) $20^3 - 19^3$ (iv) $51^3 - 50^3$

2. Cubes and their prime factors

Consider the following prime factorisation of the numbers and their cubes.

each prime factor appears three times in its cubes

Prime factorisation of a number	Prime factorisation of its cube
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 2^3 \times 3^3$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Observe that each prime factor of a number appears three times in the prime factorisation of its cube.

Do you remember that $a^m \times b^m = (a \times b)^m$

In the prime factorisation of any number, if each factor appears three times, then, is the number a perfect cube? Think about it. Is 216 a perfect cube?

By prime factorisation, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Each factor appears 3 times. $216 = 2^3 \times 3^3 = (2 \times 3)^3$

$= 6^3$ which is a perfect cube!

Is 729 a perfect cube? $729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

factors can be grouped in triples

Yes, 729 is a perfect cube.

Now let us check for 500.

Prime factorisation of 500 is $2 \times 2 \times \underline{5 \times 5 \times 5}$.

So, 500 is not a perfect cube.

There are three 5's in the product but only two 2's.

Example 1: Is 243 a perfect cube?

Solution: $243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

In the above factorisation 3×3 remains after grouping the 3's in triplets. Therefore, 243 is not a perfect cube.

TRY THESE

Which of the following are perfect cubes?

1. 400
2. 3375
3. 8000
4. 15625
5. 9000
6. 6859
7. 2025
8. 10648

1.2.2 Smallest multiple that is a perfect cube

Raj made a cuboid of plasticine. Length, breadth and height of the cuboid are 15 cm, 30 cm, 15 cm respectively.

Anu asks how many such cuboids will she need to make a perfect cube? Can you tell?

Raj said, Volume of cuboid is $15 \times 30 \times 15 = 3 \times 5 \times 2 \times 3 \times 5 \times 3 \times 5$

$= 2 \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Since there is only one 2 in the prime factorisation. So we need 2×2, i.e., 4 to make it a perfect cube. Therefore, we need 4 such cuboids to make a cube.

Example 2: Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube.

Solution: $392 = \underline{2 \times 2 \times 2} \times 7 \times 7$

The prime factor 7 does not appear in a group of three. Therefore, 392 is not a perfect cube. To make its a cube, we need one more 7. In that case

$392 \times 7 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7} = 2744$ which is a perfect cube.

Hence the smallest natural number by which 392 should be multiplied to make a perfect cube is 7.

Example 3: Is 53240 a perfect cube? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube?

Solution: $53240 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11} \times 5$

The prime factor 5 does not appear in a group of three. So, 53240 is not a perfect cube. In the factorisation 5 appears only one time. If we divide the number by 5, then the prime factorisation of the quotient will not contain 5.

So, $53240 \div 5 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5.

The perfect cube in that case is = 10648.

Example 4: Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube?

Solution: $1188 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times 11$

The primes 2 and 11 do not appear in groups of three. So, 1188 is not a perfect cube. In the factorisation of 1188 the prime 2 appears only two times and the prime 11 appears once. So, if we divide 1188 by $2 \times 2 \times 11 = 44$, then the prime factorisation of the quotient will not contain 2 and 11.

Hence the smallest natural number by which 1188 should be divided to make it a perfect cube is 44.

And the resulting perfect cube is $1188 \div 44 = 27$ $(=3^3)$.

Example 5: Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

Solution: We have, $68600 = 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 7$. In this factorisation, we find that there is no triplet of 5.

So, 68600 is not a perfect cube. To make it a perfect cube we multiply it by 5.

Thus, $68600 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$= 343000$, which is a perfect cube.

Observe that 343 is a perfect cube. From Example 5 we know that 343000 is also perfect cube.

THINK, DISCUSS AND WRITE

Check which of the following are perfect cubes. (i) 2700 (ii) 16000 (iii) 64000 (iv) 900 (v) 125000 (vi) 36000 (vii) 21600 (viii) 10,000 (ix) 27000000 (x) 1000. What pattern do you observe in these perfect cubes?

EXERCISE 1.1

1. Which of the following numbers are not perfect cubes?
   (i) 216 (ii) 128 (iii) 1000 (iv) 100
   (v) 46656
2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.
   (i) 243 (ii) 256 (iii) 72 (iv) 675
   (v) 100
3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.
   (i) 81 (ii) 128 (iii) 135 (iv) 192
   (v) 704
4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

1.3 Cube Roots

If the volume of a cube is 125 cm^3, what would be the length of its side? To get the length of the side of the cube, we need to know a number whose cube is 125.

Finding the square root, as you know, is the inverse operation of squaring. Similarly, finding the cube root is the inverse operation of finding cube.

We know that $2^3 = 8$; so we say that the cube root of 8 is 2.

We write $\sqrt[3]{8} = 2$. **The symbol $\sqrt[3]{\ }$ denotes 'cube-root.'**

Consider the following:

Statement	Inference
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$
$3^3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

Statement	Inference
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

1.3.1 Cube root through prime factorisation method

Consider 3375. We find its cube root by prime factorisation:

$$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} = 3^3 \times 5^3 = (3 \times 5)^3$$

Therefore, cube root of 3375 = $\sqrt[3]{3375} = 3 \times 5 = 15$

Similarly, to find $\sqrt[3]{74088}$, we have,

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7} = 2^3 \times 3^3 \times 7^3 = (2 \times 3 \times 7)^3$$

Therefore, $\sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

Example 6: Find the cube root of 8000.

Solution: Prime factorisation of 8000 is $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

So, $\sqrt[3]{8000} = 2 \times 2 \times 5 = 20$

Example 7: Find the cube root of 13824 by prime factorisation method.

Solution:

$$13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 2^3 \times 2^3 \times 2^3 \times 3^3.$$

Therefore, $\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

THINK, DISCUSS AND WRITE

State true or false: for any integer m, $m^2 < m^3$. Why?

1.3.2 Cube root of a cube number

If you know that the given number is a cube number then following method can be used.

Step 1 Take any cube number say 857375 and start making groups of three digits starting from the right most digit of the number.

857	375
↓	↓
second group	first group

We can estimate the cube root of a given cube number through a step by step process.

We get 375 and 857 as two groups of three digits each.

Step 2 First group, i.e., 375 will give you the one's (or unit's) digit of the required cube root.

The number 375 ends with 5. We know that 5 comes at the unit's place of a number only when it's cube root ends in 5.

So, we get 5 at the unit's place of the cube root.

Step 3 Now take another group, i.e., 857.

We know that $9^3 = 729$ and $10^3 = 1000$. Also, $729 < 857 < 1000$. We take the one's place, of the smaller number 729 as the ten's place of the required cube root. So, we get $\sqrt[3]{857375} = 95$.

Example 8: Find the cube root of 17576 through estimation.

Solution: The given number is 17576.

Step 1 Form groups of three starting from the rightmost digit of 17576.

17 576. In this case one group i.e., 576 has three digits whereas 17 has only two digits.

Step 2 Take 576.

The digit 6 is at its one's place.

We take the one's place of the required cube root as 6.

Step 3 Take the other group, i.e., 17.

Cube of 2 is 8 and cube of 3 is 27. 17 lies between 8 and 27.

The smaller number among 2 and 3 is 2.

The one's place of 2 is 2 itself. Take 2 as ten's place of the cube root of 17576.

Thus, $\sqrt[3]{17576} = 26$ (Check it!)

EXERCISE 1.2

1. Find the cube root of each of the following numbers by prime factorisation method.

(i) 64 (ii) 512 (iii) 10648 (iv) 27000

(v) 15625 (vi) 13824 (vii) 110592 (viii) 46656

(ix) 175616 (x) 91125

2. State true or false.

(i) Cube of any odd number is even.

(ii) A perfect cube does not end with two zeros.

(iii) If square of a number ends with 5, then its cube ends with 25.

(iv) There is no perfect cube which ends with 8.

(v) The cube of a two digit number may be a three digit number.

(vi) The cube of a two digit number may have seven or more digits.

(vii) The cube of a single digit number may be a single digit number.

3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768.

WHAT HAVE WE DISCUSSED?

1. Numbers like 1729, 4104, 13832, are known as Hardy – Ramanujan Numbers. They can be expressed as sum of two cubes in two different ways.

2. Numbers obtained when a number is multiplied by itself three times are known as **cube numbers**. For example 1, 8, 27, ... etc.

3. If in the prime factorisation of any number each factor appears three times, then the number is a perfect cube.

4. The symbol $\sqrt[3]{\ }$ denotes cube root. For example $\sqrt[3]{27} = 3$.

CHAPTER 2

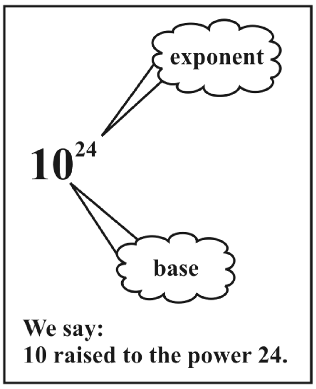
Exponents and Powers

U4Y4S5

2.1 Introduction

Do you know?

Mass of earth is 5,970,000,000,000, 000, 000, 000, 000 kg. We have already learnt in earlier class how to write such large numbers more conveniently using exponents, as, 5.97×10^{24} kg.



We read 10^{24} as 10 raised to the power 24.

We know $\quad 2^5 = 2 \times 2 \times 2 \times 2 \times 2$

and $\quad 2^m = 2 \times 2 \times 2 \times 2 \times ... \times 2 \times 2$ ... (m times)

Let us now find what is 2^{-2} is equal to?

Exponent is a negative integer.

2.2 Powers with Negative Exponents

You know that, $\quad 10^2 = 10 \times 10 = 100$

$$10^1 = 10 = \frac{100}{10}$$

$$10^0 = 1 = \frac{10}{10}$$

$$10^{-1} = ?$$

As the exponent decreases by1, the value becomes one-tenth of the previous value.

Continuing the above pattern we get, $10^{-1} = \frac{1}{10}$

Similarly $\quad 10^{-2} = \frac{1}{10} \div 10 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2}$

$$10^{-3} = \frac{1}{100} \div 10 = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} = \frac{1}{10^3}$$

What is 10^{-10} equal to?

Now consider the following.

$$3^3 = 3 \times 3 \times 3 = 27$$

$$3^2 = 3 \times 3 = 9 = \frac{27}{3}$$

$$3^1 = 3 = \frac{9}{3}$$

$$3^{\circ} = 1 = \frac{3}{3}$$

The previous number is divided by the base 3.

So looking at the above pattern, we say

$$3^{-1} = 1 \div 3 = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3} \div 3 = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^2} \div 3 = \frac{1}{3^2} \times \frac{1}{3} = \frac{1}{3^3}$$

You can now find the value of 2^{-2} in a similar manner.

We have, $10^{-2} = \frac{1}{10^2}$ or $10^2 = \frac{1}{10^{-2}}$

$10^{-3} = \frac{1}{10^3}$ or $10^3 = \frac{1}{10^{-3}}$

$3^{-2} = \frac{1}{3^2}$ or $3^2 = \frac{1}{3^{-2}}$ etc.

In general, we can say that for any non-zero integer a, $a^{-m} = \frac{1}{a^m}$, where m is a positive integer. a^{-m} is the multiplicative inverse of a^m.

TRY THESE

Find the multiplicative inverse of the following.

(i) 2^{-4} (ii) 10^{-5} (iii) 7^{-2} (iv) 5^{-3} (v) 10^{-100}

We learnt how to write numbers like 1425 in expanded form using exponents as $1 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 5 \times 10^{\circ}$.

Let us see how to express 1425.36 in expanded form in a similar way.

We have $1425.36 = 1 \times 1000 + 4 \times 100 + 2 \times 10 + 5 \times 1 + \frac{3}{10} + \frac{6}{100}$

$$= 1 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 5 \times 1 + 3 \times 10^{-1} + 6 \times 10^{-2}$$

$10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

TRY THESE

Expand the following numbers using exponents.

(i) 1025.63 (ii) 1256.249

2.3 Laws of Exponents

We have learnt that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are natural numbers. Does this law also hold if the exponents are negative? Let us explore.

(i) We know that $2^{-3} = \frac{1}{2^3}$ and $2^{-2} = \frac{1}{2^2}$

$a^{-m} = \frac{1}{a^m}$ for any non-zero integer a.

Therefore, $2^{-3} \times 2^{-2} = \frac{1}{2^3} \times \frac{1}{2^2} = \frac{1}{2^3 \times 2^2} = \frac{1}{2^{3+2}} = 2^{-5}$

–5 is the sum of two exponents – 3 and – 2

(ii) Take $(-3)^{-4} \times (-3)^{-3}$

$$(-3)^{-4} \times (-3)^{-3} = \frac{1}{(-3)^4} \times \frac{1}{(-3)^3}$$

$$= \frac{1}{(-3)^4 \times (-3)^3} = \frac{1}{(-3)^{4+3}} = (-3)^{-7}$$

$(-4) + (-3) = -7$

(iii) Now consider $5^{-2} \times 5^4$

$$5^{-2} \times 5^4 = \frac{1}{5^2} \times 5^4 = \frac{5^4}{5^2} = 5^{4-2} = 5^{(2)}$$

$(-2) + 4 = 2$

In Class VII, you have learnt that for any non-zero integer a, $\frac{a^m}{a^n} = a^{m-n}$, where m and n are natural numbers and $m > n$.

(iv) Now consider $(-5)^{-4} \times (-5)^2$

$$(-5)^{-4} \times (-5)^2 = \frac{1}{(-5)^4} \times (-5)^2 = \frac{(-5)^2}{(-5)^4} = \frac{1}{(-5)^4 \times (-5)^{-2}}$$

$$= \frac{1}{(-5)^{4-2}} = (-5)^{-(2)}$$

$(-4) + 2 = -2$

In general, we can say that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are integers.

TRY THESE

Simplify and write in exponential form.

(i) $(-2)^{-3} \times (-2)^{-4}$ (ii) $p^3 \times p^{-10}$ (iii) $3^2 \times 3^{-5} \times 3^6$

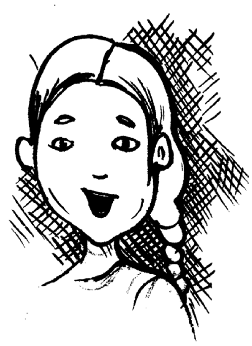
On the same lines you can verify the following laws of exponents, where a and b are non zero integers and m, n are any integers.

(i) $\frac{a^m}{a^n} = a^{m-n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m \times b^m = (ab)^m$

(iv) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ (v) $a^0 = 1$

These laws you have studied in Class VII for positive exponents only.

Let us solve some examples using the above Laws of Exponents.



Example 1: Find the value of

(i) 2^{-3} (ii) $\dfrac{1}{3^{-2}}$

Solution:

(i) $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$ (ii) $\dfrac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

Example 2: Simplify

(i) $(-4)^5 \times (-4)^{-10}$ (ii) $2^5 \div 2^{-6}$

Solution:

(i) $(-4)^5 \times (-4)^{-10} = (-4)^{(5-10)} = (-4)^{-5} = \dfrac{1}{(-4)^5}$ $\left(a^m \times a^n = a^{m+n},\ a^{-m} = \dfrac{1}{a^m}\right)$

(ii) $2^5 \div 2^{-6} = 2^{5-(-6)} = 2^{11}$ $(a^m \div a^n = a^{m-n})$

Example 3: Express 4^{-3} as a power with the base 2.

Solution: We have, $4 = 2 \times 2 = 2^2$

Therefore, $(4)^{-3} = (2 \times 2)^{-3} = (2^2)^{-3} = 2^{2 \times (-3)} = 2^{-6}$ $[(a^m)^n = a^{mn}]$

Example 4: Simplify and write the answer in the exponential form.

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$ (ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii) $\dfrac{1}{8} \times (3)^{-3}$ (iv) $(-3)^4 \times \left(\dfrac{5}{3}\right)^4$

Solution:

(i) $(2^5 \div 2^8)^5 \times 2^{-5} = (2^{5-8})^5 \times 2^{-5} = (2^{-3})^5 \times 2^{-5} = 2^{-15-5} = 2^{-20} = \dfrac{1}{2^{20}}$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = [(-4) \times 5 \times (-5)]^{-3} = [100]^{-3} = \dfrac{1}{100^3}$

[using the law $a^m \times b^m = (ab)^m$, $a^{-m} = \dfrac{1}{a^m}$]

(iii) $\dfrac{1}{8} \times (3)^{-3} = \dfrac{1}{2^3} \times (3)^{-3} = 2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \dfrac{1}{6^3}$

(iv) $(-3)^4 \times \left(\dfrac{5}{3}\right)^4 = (-1 \times 3)^4 \times \dfrac{5^4}{3^4} = (-1)^4 \times 3^4 \times \dfrac{5^4}{3^4}$

$= (-1)^4 \times 5^4 = 5^4$ $[(-1)^4 = 1]$

Example 5: Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$

Solution: $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

On both the sides powers have the same base different from 1 and – 1, so their exponents must be equal.

Therefore, $m + 6 = 7$

or $m = 7 - 6 = 1$

$a^n = 1$ only if $n = 0$. This will work for any a. For $a = 1$, $1^1 = 1^2 = 1^3 = 1^{-2} = ... = 1$ or $(1)^n = 1$ for infinitely many n.
For $a = -1$,
$(-1)^0 = (-1)^2 = (-1)^4 = (-1)^{-2} = ... = 1$ or $(-1)^p = 1$ for any even integer p.

Example 6: Find the value of $\left(\frac{2}{3}\right)^{-2}$.

Solution: $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \frac{9}{4}$

$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$

In general, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Example 7: Simplify (i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

Solution:

(i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2} = \left\{\frac{1^{-2}}{3^{-2}} - \frac{1^{-3}}{2^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$

$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2} = \{9 - 8\} \div 16 = \frac{1}{16}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}} = \frac{5^{-7}}{5^{-5}} \times \frac{8^{-5}}{8^{-7}} = 5^{(-7)-(-5)} \times 8^{(-5)-(-7)}$

$= 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25}$

EXERCISE 2.1

1. Evaluate.

   (i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

2. Simplify and express the result in power notation with positive exponent.

   (i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

   (iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

3. Find the value of.

   (i) $(3^{\circ} + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

4. Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

6. Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

7. Simplify.

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

2.4 Use of Exponents to Express Small Numbers in Standard Form

Observe the following facts.

1. The distance from the Earth to the Sun is 149,600,000,000 m.
2. The speed of light is 300,000,000 m/sec.
3. Thickness of Class VII Mathematics book is 20 mm.
4. The average diameter of a Red Blood Cell is 0.000007 mm.
5. The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
6. The distance of moon from the Earth is 384, 467, 000 m (approx).
7. The size of a plant cell is 0.00001275 m.
8. Average radius of the Sun is 695000 km.
9. Mass of propellant in a space shuttle solid rocket booster is 503600 kg.
10. Thickness of a piece of paper is 0.0016 cm.
11. Diameter of a wire on a computer chip is 0.000003 m.
12. The height of Mount Everest is 8848 m.

Observe that there are few numbers which we can read like 2 cm, 8848 m, 6,95,000 km. There are some large numbers like 150,000,000,000 m and some very small numbers like 0.000007 m.

Identify very large and very small numbers from the above facts and write them in the adjacent table:

Very large numbers	Very small numbers
150,000,000,000 m	0.000007 m
---------------	---------------
---------------	---------------
---------------	---------------
---------------	---------------

We have learnt how to express very large numbers in standard form in the previous class.

For example: $150,000,000,000 = 1.5 \times 10^{11}$

Now, let us try to express 0.000007 m in standard form.

$$0.000007 = \frac{7}{1000000} = \frac{7}{10^6} = 7 \times 10^{-6}$$

$0.000007 \text{ m} = 7 \times 10^{-6} \text{ m}$

150000000000. Decimal is moved 11 places to the left.

Similarly, consider the thickness of a piece of paper which is 0.0016 cm.

0.000007 decimal is moved 6 places to the right.

$$0.0016 = \frac{16}{10000} = \frac{1.6 \times 10}{10^4} = 1.6 \times 10 \times 10^{-4}$$

$$= 1.6 \times 10^{-3}$$

Therefore, we can say thickness of paper is 1.6×10^{-3} cm.

Again notice
0.0016 decimal is moved 3 places to the right.

TRY THESE

1. Write the following numbers in exponential form.
   (i) 0.000000564 (ii) 0.0000021 (iii) 21600000 (iv) 15240000
2. Write all the facts given in the exponential form.

2.4.1 Comparing very large and very small numbers

The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Suppose you want to compare the diameter of the Earth, with the diameter of the Sun.

Diameter of the Sun = 1.4×10^9 m

Diameter of the earth = 1.2756×10^7 m

Therefore $\frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 100}{1.2756}$ which is approximately 100

So, the diameter of the Sun is about 100 times the diameter of the earth.

Let us compare the size of a Red Blood cell which is 0.000007 m to that of a plant cell which is 0.00001275 m.

Size of Red Blood cell = 0.000007 m = 7×10^{-6} m

Size of plant cell = 0.00001275 = 1.275×10^{-5} m

Therefore, $\frac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \frac{7 \times 10^{-6-(-5)}}{1.275} = \frac{7 \times 10^{-1}}{1.275} = \frac{0.7}{1.275} = \frac{0.7}{1.3} = \frac{1}{2}$ (approx.)

So a red blood cell is half of plant cell in size.

Mass of earth is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg. What is the total mass?

Total mass = 5.97×10^{24} kg + 7.35×10^{22} kg.

$= 5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22}$

$= 597 \times 10^{22} + 7.35 \times 10^{22}$

$= (597 + 7.35) \times 10^{22}$

$= 604.35 \times 10^{22}$ kg.

When we have to add numbers in standard form, we convert them into numbers with the same exponents.

The distance between Sun and Earth is 1.496×10^{11}m and the distance between Earth and Moon is 3.84×10^8m.

During solar eclipse moon comes in between Earth and Sun.

At that time what is the distance between Moon and Sun.

Distance between Sun and Earth = 1.496×10^{11}m
Distance between Earth and Moon = 3.84×10^{8}m
Distance between Sun and Moon = $1.496 \times 10^{11} - 3.84 \times 10^{8}$
$= 1.496 \times 1000 \times 10^{8} - 3.84 \times 10^{8}$
$= (1496 - 3.84) \times 10^{8}$ m $= 1492.16 \times 10^{8}$ m

Example 8: Express the following numbers in standard form.

(i) 0.000035 (ii) 4050000

Solution: (i) $0.000035 = 3.5 \times 10^{-5}$ (ii) $4050000 = 4.05 \times 10^{6}$

Example 9: Express the following numbers in usual form.

(i) 3.52×10^{5} (ii) 7.54×10^{-4} (iii) 3×10^{-5}

Solution:

(i) $3.52 \times 10^{5} = 3.52 \times 100000 = 352000$

(ii) $7.54 \times 10^{-4} = \frac{7.54}{10^4} = \frac{7.54}{10000} = 0.000754$

(iii) $3 \times 10^{-5} = \frac{3}{10^5} = \frac{3}{100000} = 0.00003$

Again we need to convert numbers in standard form into a numbers with the same exponents.

EXERCISE 2.2

1. Express the following numbers in standard form.
   (i) 0.0000000000085 (ii) 0.00000000000942
   (iii) 6020000000000000 (iv) 0.00000000837
   (v) 31860000000
2. Express the following numbers in decimal form.
   (i) 3.02×10^{-6} (ii) 4.5×10^{4} (iii) 3×10^{-8}
   (iv) 1.0001×10^{9} (v) 5.8×10^{12} (vi) 3.61492×10^{6}
3. Express the number appearing in the following statements in standard form.
   (i) 1 micron is equal to $\frac{1}{1000000}$ m.
   (ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.
   (iii) Size of a bacteria is 0.0000005 m
   (iv) Size of a plant cell is 0.00001275 m
   (v) Thickness of a thick paper is 0.07 mm
4. In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack.

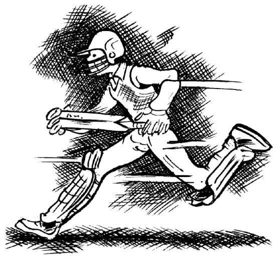
WHAT HAVE WE DISCUSSED?

1. Numbers with negative exponents obey the following laws of exponents.
   (a) $a^m \times a^n = a^{m+n}$ (b) $a^m \div a^n = a^{m-n}$ (c) $(a^m)^n = a^{mn}$
   (d) $a^m \times b^m = (ab)^m$ (e) $a^0 = 1$ (f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
2. Very small numbers can be expressed in standard form using negative exponents.

CHAPTER

3 Data Handling

3.1 Looking for Information



In your day-to-day life, you might have come across information, such as:

(a) Runs made by a batsman in the last 10 test matches.

(b) Number of wickets taken by a bowler in the last 10 ODIs.

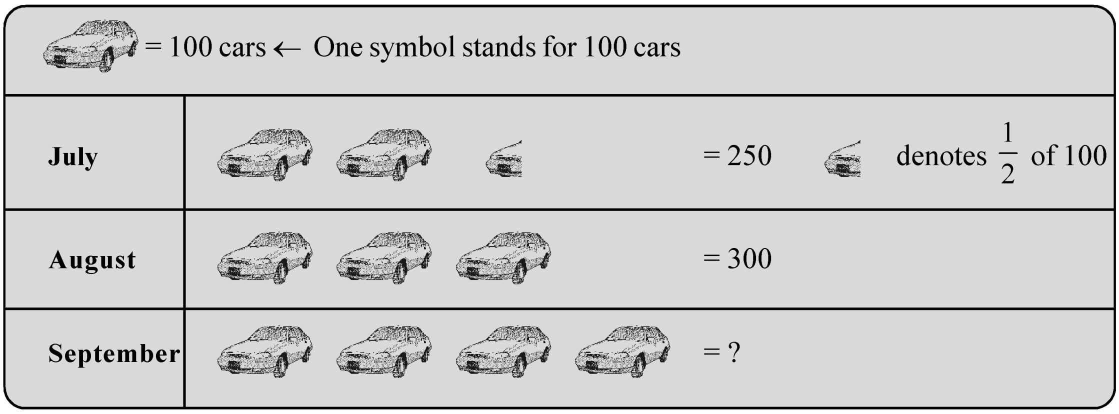
(c) Marks scored by the students of your class in the Mathematics unit test.

(d) Number of story books read by each of your friends etc.

The information collected in all such cases is called **data**. Data is usually collected in the context of a situation that we want to study. For example, a teacher may like to know the average height of students in her class. To find this, she will write the heights of all the students in her class, organise the data in a systematic manner and then interpret it accordingly.

Sometimes, data is represented **graphically** to give a clear idea of what it represents. Do you remember the different types of graphs which we have learnt in earlier classes?

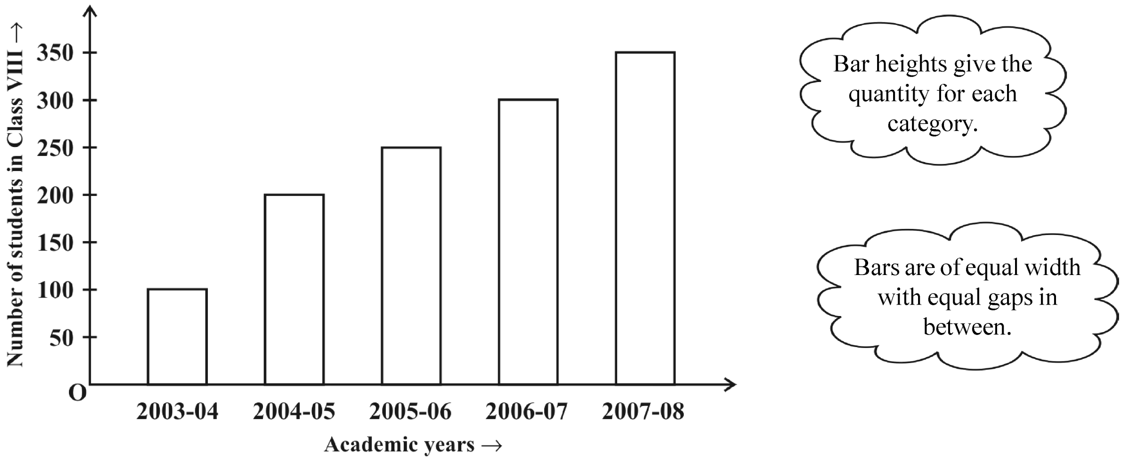
1. A Pictograph: Pictorial representation of data using symbols.



(i) How many cars were produced in the month of July?

(ii) In which month were maximum number of cars produced?

2. **A bar graph:** A display of information using bars of uniform width, their heights being proportional to the respective values.



(i) What is the information given by the bar graph?

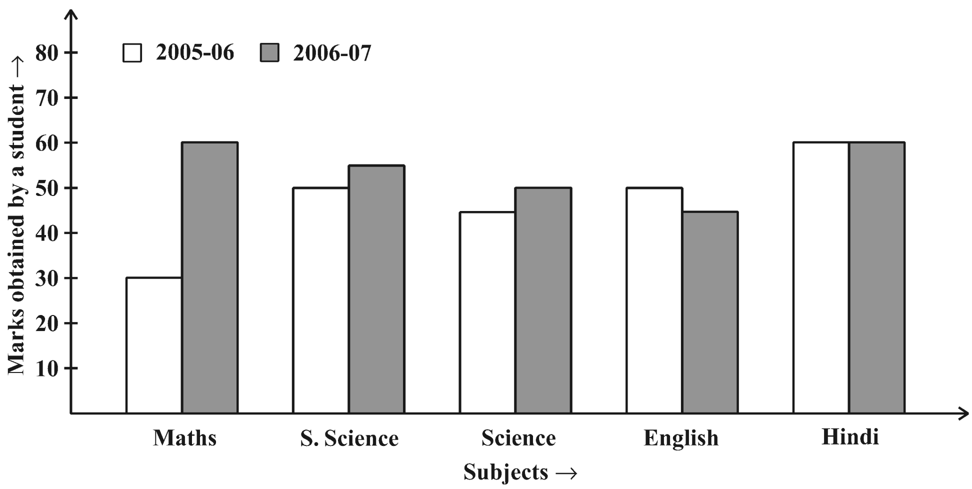
(ii) In which year is the increase in the number of students maximum?

(iii) In which year is the number of students maximum?

(iv) State whether true or false:

'The number of students during 2005-06 is twice that of 2003-04.'

3. **Double Bar Graph:** A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data.



(i) What is the information given by the double bar graph?

(ii) In which subject has the performance improved the most?

(iii) In which subject has the performance deteriorated?

(iv) In which subject is the performance at par?

THINK, DISCUSS AND WRITE

If we change the position of any of the bars of a bar graph, would it change the information being conveyed? Why?

TRY THESE

Draw an appropriate graph to represent the given information.

1.

Month	July	August	September	October	November	December
Number of watches sold	1000	1500	1500	2000	2500	1500

2.

Children who prefer	School A	School B	School C
Walking	40	55	15
Cycling	45	25	35

3. Percentage wins in ODI by 8 top cricket teams.

Teams	From Champions Trophy to World Cup-06	Last 10 ODI in 07
South Africa	75%	78%
Australia	61%	40%
Sri Lanka	54%	38%
New Zealand	47%	50%
England	46%	50%
Pakistan	45%	44%
West Indies	44%	30%
India	43%	56%

3.2 Organising Data

Usually, data available to us is in an unorganised form called **raw data**. To draw meaningful inferences, we need to organise the data systematically. For example, a group of students was asked for their favourite subject. The results were as listed below:

Art, Mathematics, Science, English, Mathematics, Art, English, Mathematics, English, Art, Science, Art, Science, Science, Mathematics, Art, English, Art, Science, Mathematics, Science, Art.

Which is the most liked subject and the one least liked?

It is not easy to answer the question looking at the choices written haphazardly. We arrange the data in Table 3.1 using tally marks.

Table 3.1

Subject	Tally Marks	Number of Students
Art	卌 ||	7
Mathematics	卌	5
Science	卌 |	6
English	||||	4

The number of tallies before each subject gives the number of students who like that particular subject.

This is known as the **frequency** of that subject.

Frequency gives the number of times that a particular entry occurs.

From Table 3.1, Frequency of students who like English is 4

Frequency of students who like Mathematics is 5

The table made is known as **frequency distribution table** as it gives the number of times an entry occurs.

TRY THESE

1. A group of students were asked to say which animal they would like most to have as a pet. The results are given below:
   dog, cat, cat, fish, cat, rabbit, dog, cat, rabbit, dog, cat, dog, dog, dog, cat, cow, fish, rabbit, dog, cat, dog, cat, cat, dog, rabbit, cat, fish, dog.
   Make a frequency distribution table for the same.

3.3 Grouping Data

The data regarding choice of subjects showed the occurrence of each of the entries several times. For example, Art is liked by 7 students, Mathematics is liked by 5 students and so on (Table 5.1). This information can be displayed graphically using a pictograph or a bargraph. Sometimes, however, we have to deal with a large data. For example, consider the following marks (out of 50) obtained in Mathematics by 60 students of Class VIII:

21, 10, 30, 22, 33, 5, 37, 12, 25, 42, 15, 39, 26, 32, 18, 27, 28, 19, 29, 35, 31, 24, 36, 18, 20, 38, 22, 44, 16, 24, 10, 27, 39, 28, 49, 29, 32, 23, 31, 21, 34, 22, 23, 36, 24, 36, 33, 47, 48, 50, 39, 20, 7, 16, 36, 45, 47, 30, 22, 17.

If we make a frequency distribution table for each observation, then the table would be too long, so, for convenience, we make groups of observations say, 0-10, 10-20 and so on, and obtain a frequency distribution of the number of observations falling in each group. Thus, the frequency distribution table for the above data can be.

Table 3.2

Groups	Tally Marks	Frequency
0-10	||	2
10-20	𝍸 𝍸	10
20-30	𝍸 𝍸 𝍸 𝍸 |	21
30-40	𝍸 𝍸 𝍸 ||||	19
40-50	𝍸 ||	7
50-60	|	1
	Total	**60**

Data presented in this manner is said to be **grouped** and the distribution obtained is called **grouped frequency distribution**. It helps us to draw meaningful inferences like –

(1) Most of the students have scored between 20 and 40.

(2) Eight students have scored more than 40 marks out of 50 and so on.

Each of the groups 0-10, 10-20, 20-30, etc., is called a **Class Interval** (or briefly a class).

Observe that 10 occurs in both the classes, i.e., 0-10 as well as 10-20. Similarly, 20 occurs in classes 10-20 and 20-30. But it is not possible that an observation (say 10 or 20) can belong simultaneously to two classes. To avoid this, we adopt the convention that the common observation will belong to the higher class, i.e., 10 belongs to the class interval 10-20 (and not to 0-10). Similarly, 20 belongs to 20-30 (and not to 10-20). In the class interval, 10-20, 10 is called the **lower class limit** and 20 is called the **upper class limit**. Similarly, in the class interval 20-30, 20 is the lower class limit and 30 is the upper class limit. Observe that the difference between the upper class limit and lower class limit for each of the class intervals 0-10, 10-20, 20-30 etc., is equal, (10 in this case). This difference between the upper class limit and lower class limit is called the **width** or **size** of the class interval.

TRY THESE

1. Study the following frequency distribution table and answer the questions given below.

Frequency Distribution of Daily Income of 550 workers of a factory

Table 3.3

Class Interval (Daily Income in ₹)	Frequency (Number of workers)
100-125	45
125-150	25

150-175	55
175-200	125
200-225	140
225-250	55
250-275	35
275-300	50
300-325	20
Total	**550**

(i) What is the size of the class intervals?
(ii) Which class has the highest frequency?
(iii) Which class has the lowest frequency?
(iv) What is the upper limit of the class interval 250-275?
(v) Which two classes have the same frequency?

2. Construct a frequency distribution table for the data on weights (in kg) of 20 students of a class using intervals 30-35, 35-40 and so on.

40, 38, 33, 48, 60, 53, 31, 46, 34, 36, 49, 41, 55, 49, 65, 42, 44, 47, 38, 39.

3.3.1 Bars with a difference

Let us again consider the grouped frequency distribution of the marks obtained by 60 students in Mathematics test. (Table 3.4)

Table 3.4

Class Interval	Frequency
0-10	2
10-20	10
20-30	21
30-40	19
40-50	7
50-60	1
Total	**60**

This is displayed graphically as in the adjoining graph (Fig 3.1).

Is this graph in any way different from the bar graphs which you have drawn in Class VII? Observe that, here we have represented the groups of observations (i.e., class intervals)

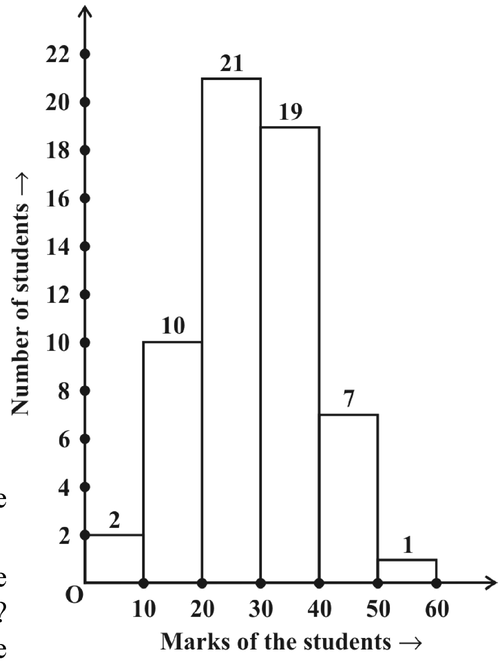


Fig 3.1

on the horizontal axis. The **height** of the bars show the **frequency** of the class-interval. Also, there is no gap between the bars as there is no gap between the class-intervals.

The graphical representation of data in this manner is called a **histogram**.
The following graph is another histogram (Fig 3.2).

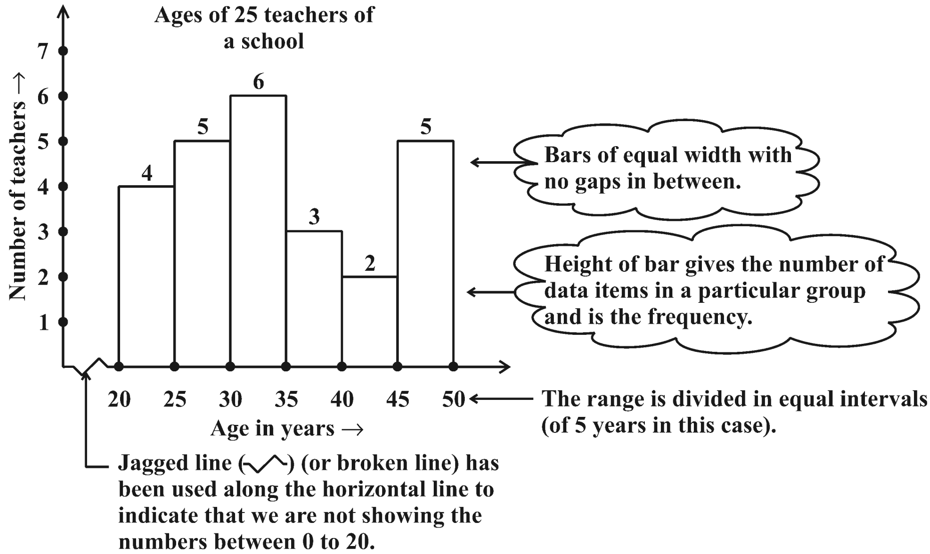


Fig 3.2

From the bars of this histogram, we can answer the following questions:

(i) How many teachers are of age 45 years or more but less than 50 years?

(ii) How many teachers are of age less than 35 years?

TRY THESE

1. Observe the histogram (Fig 3.3) and answer the questions given below.

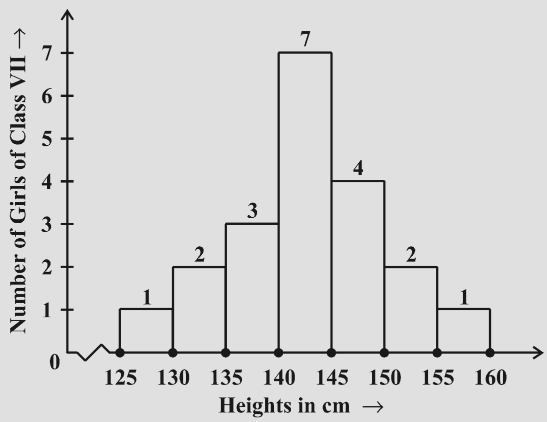


Fig 3.3

(i) What information is being given by the histogram?

(ii) Which group contains maximum girls?

(iii) How many girls have a height of 145 cms and more?
(iv) If we divide the girls into the following three categories, how many would there be in each?

150 cm and more	— Group A
140 cm to less than 150 cm	— Group B
Less than 140 cm	— Group C

EXERCISE 3.1

1. For which of these would you use a histogram to show the data?
   (a) The number of letters for different areas in a postman's bag.
   (b) The height of competitors in an athletics meet.
   (c) The number of cassettes produced by 5 companies.
   (d) The number of passengers boarding trains from 7:00 a.m. to 7:00 p.m. at a station.

   Give reasons for each.

2. The shoppers who come to a departmental store are marked as: man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning:

   W W W G B W W M G G M M W W W W G B M W B G G M W W M M W W
   W M W B W G M W W W W G W M M W W M W G W M G W M M B G G W

   Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

3. The weekly wages (in ₹) of 30 workers in a factory are.

   830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840

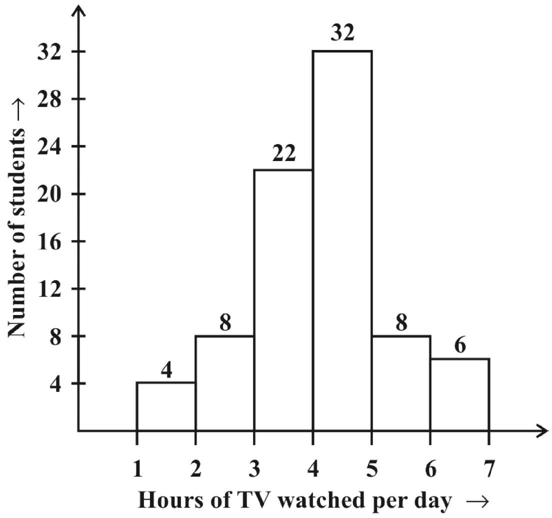
   Using tally marks make a frequency table with intervals as 800–810, 810–820 and so on.

4. Draw a histogram for the frequency table made for the data in Question 3, and answer the following questions.
   (i) Which group has the maximum number of workers?
   (ii) How many workers earn ₹ 850 and more?
   (iii) How many workers earn less than ₹ 850?

5. The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

   Answer the following.
   (i) For how many hours did the maximum number of students watch TV?
   (ii) How many students watched TV for less than 4 hours?

(iii) How many students spent more than 5 hours in watching TV?



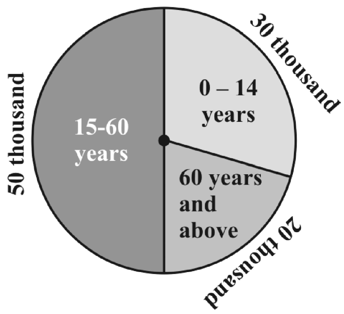
3.4 Circle Graph or Pie Chart

Have you ever come across data represented in circular form as shown (Fig 3.4)?

The time spent by a child during a day

(i)

Age groups of people in a town



(ii)

Fig 3.4

These are called **circle graphs**. A circle graph shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents.

For example, in the above graph, the proportion of the sector for hours spent in sleeping

$$= \frac{\text{number of sleeping hours}}{\text{whole day}} = \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{1}{3}$$

So, this sector is drawn as $\frac{1}{3}$rd part of the circle. Similarly, the proportion of the sector

for hours spent in school $= \frac{\text{number of school hours}}{\text{whole day}} = \frac{6 \text{ hours}}{24 \text{ hours}} = \frac{1}{4}$

So this sector is drawn $\frac{1}{4}$th of the circle. Similarly, the size of other sectors can be found.

Add up the fractions for all the activities. Do you get the total as one?

A circle graph is also called a **pie chart**.

TRY THESE

1. Each of the following pie charts (Fig 3.5) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.

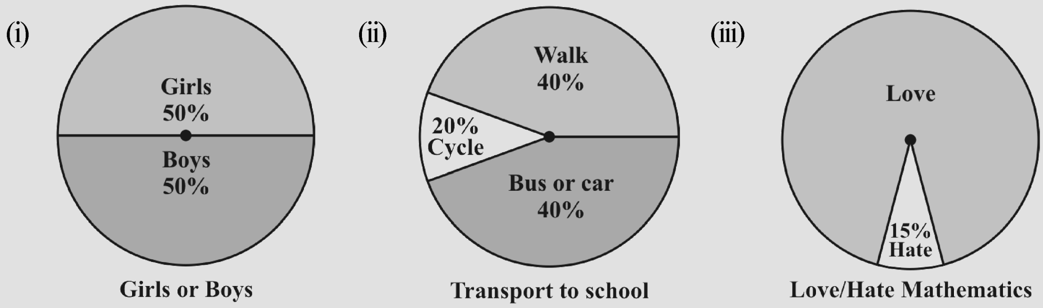
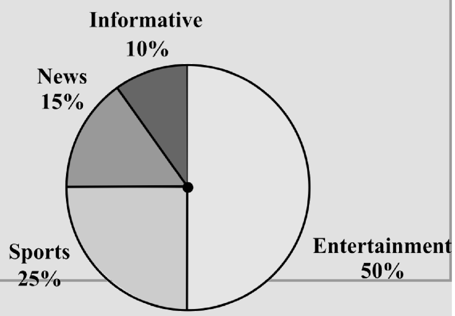


Fig 3.5

2. Answer the following questions based on the pie chart given (Fig 3.6).
   (i) Which type of programmes are viewed the most?
   (ii) Which two types of programmes have number of viewers equal to those watching sports channels?



Viewers watching different types of channels on T.V.

Fig 3.6

3.4.1 Drawing pie charts

The favourite flavours of ice-creams for students of a school is given in percentages as follows.

Flavours	Percentage of students Preferring the flavours
Chocolate	50%
Vanilla	25%
Other flavours	25%

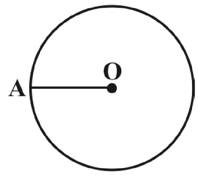
Let us represent this data in a pie chart.

The total angle at the centre of a circle is 360°. The central angle of the sectors will be a fraction of 360°. We make a table to find the central angle of the sectors (Table 3.5).

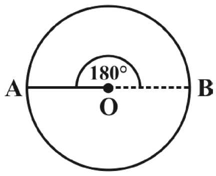
Table 3.5

Flavours	Students in per cent preferring the flavours	In fractions	Fraction of 360°
Chocolate	50%	$\frac{50}{100}=\frac{1}{2}$	$\frac{1}{2}$ of 360° = 180°
Vanilla	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°
Other flavours	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°

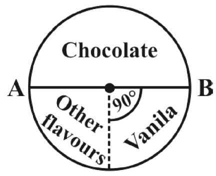
1. Draw a circle with any convenient radius. Mark its centre (O) and a radius (OA).



2. The angle of the sector for chocolate is 180°. Use the protractor to draw ∠AOB = 180°.



3. Continue marking the remaining sectors.



Example 1: Adjoining pie chart (Fig 3.7) gives the expenditure (in percentage) on various items and savings of a family during a month.

(i) On which item, the expenditure was maximum?

(ii) Expenditure on which item is equal to the total savings of the family?

(iii) If the monthly savings of the family is ₹ 3000, what is the monthly expenditure on clothes?

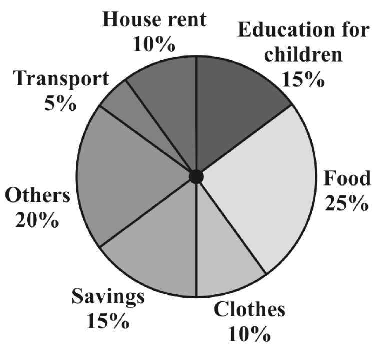


Fig 3.7

Solution:

(i) Expenditure is maximum on food.

(ii) Expenditure on Education of children is the same (i.e., 15%) as the savings of the family.

(iii) 15% represents ₹ 3000

Therefore, 10% represents ₹ $\frac{3000}{15} \times 10$ = ₹ 2000

Example 2: On a particular day, the sales (in rupees) of different items of a baker's shop are given below.

ordinary bread	: 320
fruit bread	: 80
cakes and pastries	: 160
biscuits	: 120
others	: 40
Total	**: 720**

Draw a pie chart for this data.

Solution: We find the central angle of each sector. Here the total sale = ₹ 720. We thus have this table.

Item	Sales (in ₹)	In Fraction	Central Angle
Ordinary Bread	320	$\frac{320}{720} = \frac{4}{9}$	$\frac{4}{9} \times 360° = 160°$
Biscuits	120	$\frac{120}{720} = \frac{1}{6}$	$\frac{1}{6} \times 360° = 60°$
Cakes and pastries	160	$\frac{160}{720} = \frac{2}{9}$	$\frac{2}{9} \times 360° = 80°$
Fruit Bread	80	$\frac{80}{720} = \frac{1}{9}$	$\frac{1}{9} \times 360° = 40°$
Others	40	$\frac{40}{720} = \frac{1}{18}$	$\frac{1}{18} \times 360° = 20°$

Now, we make the pie chart (Fig 3.8):

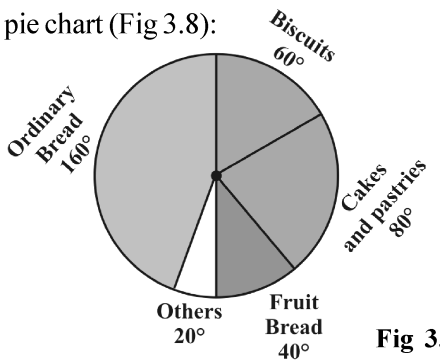


Fig 3.8

TRY THESE

Draw a pie chart of the data given below.

The time spent by a child during a day.

Sleep — 8 hours
School — 6 hours
Home work — 4 hours
Play — 4 hours
Others — 2 hours

THINK, DISCUSS AND WRITE

Which form of graph would be appropriate to display the following data.

1. **Production of food grains of a state.**

Year	2001	2002	2003	2004	2005	2006
Production (in lakh tons)	60	50	70	55	80	85

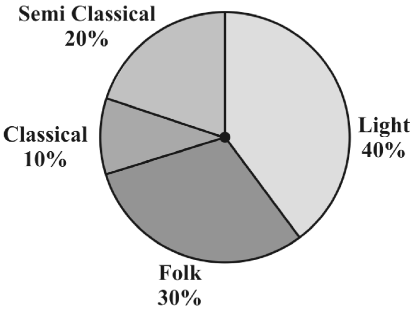
2. **Choice of food for a group of people.**

Favourite food	**Number of people**
North Indian	30
South Indian	40
Chinese	25
Others	25
Total	**120**

3. **The daily income of a group of a factory workers.**

Daily Income (in Rupees)	**Number of workers (in a factory)**
75-100	45
100-125	35
125-150	55
150-175	30
175-200	50
200-225	125
225-250	140
Total	**480**

EXERCISE 3.2



1. A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey.
From this pie chart answer the following:
- (i) If 20 people liked classical music, how many young people were surveyed?
- (ii) Which type of music is liked by the maximum number of people?
- (iii) If a cassette company were to make 1000 CD's, how many of each type would they make?

2. A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.
- (i) Which season got the most votes?
- (ii) Find the central angle of each sector.
- (iii) Draw a pie chart to show this information.

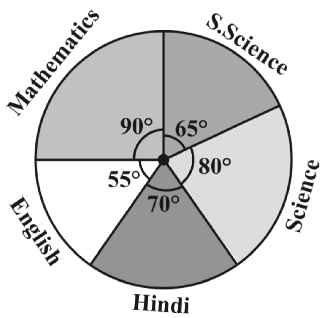
Season	No. of votes
Summer 	90
Rainy 	120
Winter 	150

3. Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	**36**

Find the proportion of each sector. For example, Blue is $\frac{18}{36} = \frac{1}{2}$; Green is $\frac{9}{36} = \frac{1}{4}$ and so on. Use this to find the corresponding angles.

4. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.
- (i) In which subject did the student score 105 marks?
  (**Hint:** for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)
- (ii) How many more marks were obtained by the student in Mathematics than in Hindi?
- (iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.
  (**Hint:** Just study the central angles).



5. The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

3.5 Chance and Probability

Sometimes it happens that during rainy season, you carry a raincoat every day and it does not rain for many days. However, by chance, one day you forget to take the raincoat and it rains heavily on that day.

Sometimes it so happens that a student prepares 4 chapters out of 5, very well for a test. But a major question is asked from the chapter that she left unprepared.

Everyone knows that a particular train runs in time but the day you reach well in time it is late!

You face a lot of situations such as these where you take a chance and it does not go the way you want it to. Can you give some more examples? These are examples where the chances of a certain thing happening or not happening are not equal. The chances of the train being in time or being late are not the same. When you buy a ticket which is wait listed, you do take a chance. You hope that it might get confirmed by the time you travel.

We however, consider here certain experiments whose results have an equal chance of occurring.

3.5.1 Getting a result

You might have seen that before a cricket match starts, captains of the two teams go out to toss a coin to decide which team will bat first.

What are the possible results you get when a coin is tossed? Of course, Head or Tail.

Imagine that you are the captain of one team and your friend is the captain of the other team. You toss a coin and ask your friend to make the call. Can you control the result of the toss? Can you get a head if you want one? Or a tail if you want that? No, that is not possible. Such an experiment is called a **random experiment**. Head or Tail are the two **outcomes** of this experiment.

TRY THESE

1. If you try to start a scooter, what are the possible outcomes?
2. When a die is thrown, what are the six possible outcomes?

3. When you spin the wheel shown, what are the possible outcomes? (Fig 3.9) List them.
   (Outcome here means the sector at which the pointer stops).

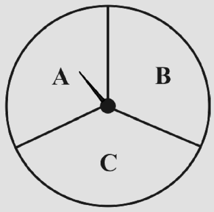


Fig 3.9

Fig 3.10

4. You have a bag with five identical balls of different colours and you are to pull out (draw) a ball without looking at it; list the outcomes you would get (Fig 3.10).

THINK, DISCUSS AND WRITE

In throwing a die:

- Does the first player have a greater chance of getting a six?
- Would the player who played after him have a lesser chance of getting a six?
- Suppose the second player got a six. Does it mean that the third player would not have a chance of getting a six?

3.5.2 Equally likely outcomes:

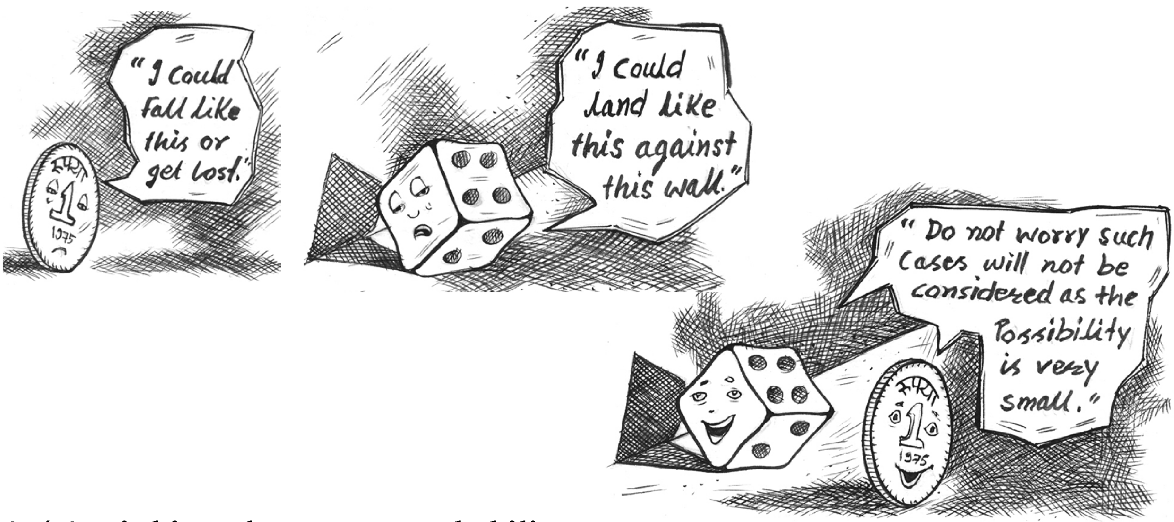
A coin is tossed several times and the number of times we get head or tail is noted. Let us look at the result sheet where we keep on increasing the tosses:

Number of tosses	Tally marks (H)	Number of heads	Tally mark (T)	Number of tails
50	卌 卌 卌 卌 卌 ||	27	卌 卌 卌 卌 |||	23
60	卌 卌 卌 卌 卌 |||	28	卌 卌 卌 卌 卌 卌 ||	32
70	...	33	...	37
80	...	38	...	42
90	...	44	...	46
100	...	48	...	52

Observe that as you increase the number of tosses more and more, the number of heads and the number of tails come closer and closer to each other.

This could also be done with a die, when tossed a large number of times. Number of each of the six outcomes become almost equal to each other.

In such cases, we may say that the different outcomes of the experiment are equally likely. This means that each of the outcomes has the same chance of occurring.



3.5.3 Linking chances to probability

Consider the experiment of tossing a coin once. What are the outcomes? There are only two outcomes – Head or Tail. Both the outcomes are equally likely. Likelihood of getting a head is one out of two outcomes, i.e., $\frac{1}{2}$. In other words, we say that the probability of getting a head = $\frac{1}{2}$. What is the probability of getting a tail?

Now take the example of throwing a die marked with 1, 2, 3, 4, 5, 6 on its faces (one number on one face). If you throw it once, what are the outcomes?

The outcomes are: 1, 2, 3, 4, 5, 6. Thus, there are six equally likely outcomes.

What is the probability of getting the outcome '2'?

It is $\frac{1}{6}$ ← Number of outcomes giving 2 / ← Number of equally likely outcomes.

What is the probability of getting the number 5? What is the probability of getting the number 7? What is the probability of getting a number 1 through 6?

3.5.4 Outcomes as events

Each outcome of an experiment or a collection of outcomes make an **event**.

For example in the experiment of tossing a coin, getting a Head is an event and getting a Tail is also an event.

In case of throwing a die, getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

Is getting an even number an event? Since an even number could be 2, 4 or 6, getting an even number is also an event. What will be the probability of getting an even number?

It is $\frac{3}{6}$ ← Number of outcomes that make the event
← Total number of outcomes of the experiment.

Example 3: A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball? Is it more or less than getting a yellow ball?

Solution: There are in all (4 + 2) = 6 outcomes of the event. Getting a red ball consists of 4 outcomes. (Why?)

Therefore, the probability of getting a red ball is $\frac{4}{6} = \frac{2}{3}$. In the same way the probability of getting a yellow ball $= \frac{2}{6} = \frac{1}{3}$ (Why?). Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

TRY THESE

Suppose you spin the wheel

1. (i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel (Fig 3.11).
   (ii) Find the probability of getting a green sector.
   (iii) Find the probability of not getting a green sector.

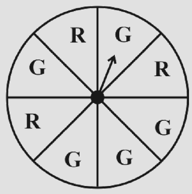


Fig 3.11

3.5.5 Chance and probability related to real life

We talked about the chance that it rains just on the day when we do not carry a rain coat.

What could you say about the chance in terms of probability? Could it be one in 10 days during a rainy season? The probability that it rains is then $\frac{1}{10}$. The probability that it does not rain $= \frac{9}{10}$. (Assuming raining or not raining on a day are equally likely)

The use of probability is made in various cases in real life.

1. To find characteristics of a large group by using a small part of the group.

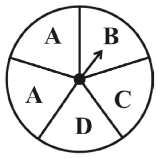
   For example, during elections 'an exit poll' is taken. This involves asking the people whom they have voted for, when they come out after voting at the centres which are chosen off hand and distributed over the whole area. This gives an idea of chance of winning of each candidate and predictions are made based on it accordingly.

2. Metrological Department predicts weather by observing trends from the data over many years in the past.

EXERCISE 3.3

1. List the outcomes you can see in these experiments.

(a) Spinning a wheel



(b) Tossing two coins together

2. When a die is thrown, list the outcomes of an event of getting

(i) (a) a prime number (b) not a prime number.

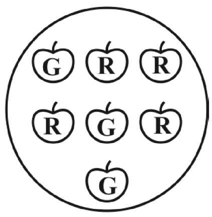
(ii) (a) a number greater than 5 (b) a number not greater than 5.

3. Find the.

(a) Probability of the pointer stopping on D in (Question 1-(a))?

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards?

(c) Probability of getting a red apple. (See figure below)



4. Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of .

(i) getting a number 6?

(ii) getting a number less than 6?

(iii) getting a number greater than 6?

(iv) getting a 1-digit number?

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?

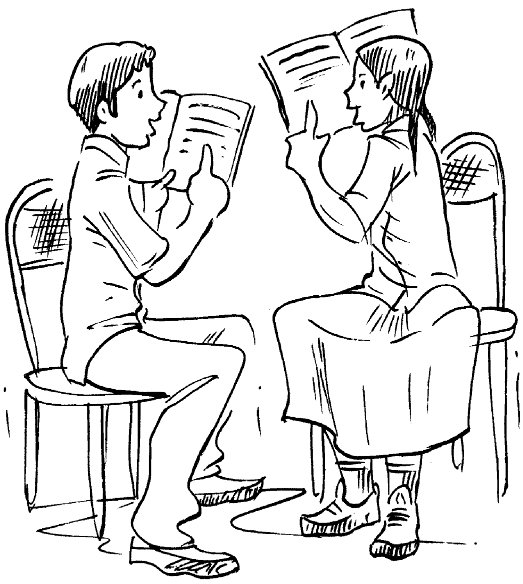
6. Find the probabilities of the events given in Question 2.

WHAT HAVE WE DISCUSSED?

1. Data mostly available to us in an unorganised form is called **raw data**.

2. In order to draw meaningful inferences from any data, we need to organise the data systematically.

3. **Frequency** gives the number of times that a particular entry occurs.
4. Raw data can be 'grouped' and presented systematically through 'grouped frequency distribution'.
5. Grouped data can be presented using **histogram**. Histogram is a type of bar diagram, where the class intervals are shown on the horizontal axis and the heights of the bars show the frequency of the class interval. Also, there is no gap between the bars as there is no gap between the class intervals.
6. Data can also presented using **circle graph** or **pie chart**. A circle graph shows the relationship between a whole and its part.
7. There are certain experiments whose outcomes have an equal chance of occurring.
8. A **random experiment** is one whose outcome cannot be predicted exactly in advance.
9. Outcomes of an experiment are **equally likely** if each has the same chance of occurring.
10. **Probability of an event** = $\frac{\text{Number of outcomes that make an event}}{\text{Total number of outcomes of the experiment}}$, when the outcomes are equally likely.
11. One or more outcomes of an experiment make an **event**.
12. Chances and probability are related to real life.



CHAPTER

4

Direct and Inverse Proportions

G9W3D7

4.1 Introduction

Mohan prepares tea for himself and his sister. He uses 300 mL of water, 2 spoons of sugar, 1 spoon of tea leaves and 50 mL of milk. How much quantity of each item will he need, if he has to make tea for five persons?

If two students take 20 minutes to arrange chairs for an assembly, then how much time would five students take to do the same job?

We come across many such situations in our day-to-day life, where we need to see variation in one quantity bringing in variation in the other quantity.

For example:

(i) If the number of articles purchased increases, the total cost also increases.

(ii) More the money deposited in a bank, more is the interest earned.

(iii) As the speed of a vehicle increases, the time taken to cover the same distance decreases.

(iv) For a given job, more the number of workers, less will be the time taken to complete the work.

Observe that change in one quantity leads to change in the other quantity.

Write five more such situations where change in one quantity leads to change in another quantity.

How do we find out the quantity of each item needed by Mohan? Or, the time five students take to complete the job?

To answer such questions, we now study some concepts of variation.

4.2 Direct Proportion

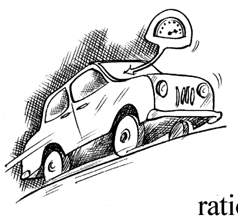
If the cost of 1 kg of sugar is ₹ 36, then what would be the cost of 3 kg sugar? It is ₹ 108.

Similarly, we can find the cost of 5 kg or 8 kg of sugar. Study the following table.

Weight of sugar (in kg)	1	3	5	6	8	10
Cost (in Rs)	36	108	180	...	...	...

Observe that as weight of sugar increases, cost also increases in such a manner that their ratio remains constant.

Take one more example. Suppose a car uses 4 litres of petrol to travel a distance of 60 km. How far will it travel using 12 litres? The answer is 180 km. How did we calculate it? Since petrol consumed in the second instance is 12 litres, i.e., three times of 4 litres, the distance travelled will also be three times of 60 km. In other words, when the petrol consumption becomes three-fold, the distance travelled is also three fold the previous one. Let the consumption of petrol be x litres and the corresponding distance travelled be y km . Now, complete the following table:



Petrol in litres (x)	4	8	12	15	20	25
Distance in km (y)	60	...	180	...	...	...

We find that as the value of x increases, value of y also increases in such a way that the ratio $\frac{x}{y}$ does not change; it remains constant (say k). In this case, it is $\frac{1}{15}$ (check it!).

We say that **x and y are in direct proportion, if $\frac{x}{y} = k$ or $x = ky$.**

In this example, $\frac{4}{60} = \frac{12}{180}$, where 4 and 12 are the quantities of petrol consumed in litres (x) and 60 and 180 are the distances (y) in km. So when x and y are in **direct proportion**, we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$. [$y_1$, y_2 are values of y corresponding to the values x_1, x_2 of x respectively]

The consumption of petrol and the distance travelled by a car is a case of direct proportion. Similarly, the total amount spent and the number of articles purchased is also an example of direct proportion.