Chapter 4. Expansions

Ex 4.1

Answer 1.

(i) $(a + 4)(a + 7) = a^2 + 4a + 7a + 28 = a^2 + 11a + 28$

(Using identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$)

(ii) $(m + 8)(m - 7) = m^2 + 8m - 7m - 56 = m^2 + m - 56$

(Using identity: $(x + a)(x - b) = x^2 + (a - b)x - ab$)

(iii) $(x - 5)(x - 4) = x^2 - 5x - 4x + 20 = x^2 - 9x + 20$

(Using identity: $(x - a)(x - b) = x^2 - (a + b)x + ab$)

(iv) $(3x + 4)(2x - 1) = 6x^2 - 3x + 8x - 4 = 6x^2 + 5x - 4$

(Using identity: $(x + a)(x - b) = x^2 + (a - b)x - ab$)

(V)

$(2x - 5)(2x + 5)(2x - 3)$

$= (4x^2 - 25)(2x - 3) = 8x^3 - 12x^2 - 50x + 75$

(Using identity: $(x - a)(x + b) = x^2 - (a - b)x - ab$)

Answer 2.

a. Using $(x + y)^2 = x^2 + 2xy + y^2$, we get

$$(a + 3b)^2 = a^2 + 2(a)(3b) + (3b)^2 = a^2 + 6ab + 9b^2$$

b. $$(2p - 3q)^2 = (2p)^2 - 2(2p)(3q) + (3q)^2 = 4p^2 - 12pq + 9q^2$$

c. $$\left(2a + \frac{1}{2a}\right)^2 = (2a)^2 + 2(2a)\left(\frac{1}{2a}\right) + (2a)^2 = 4a^2 + 2 + \frac{1}{4a^2}$$

d. Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$$(x - 3y - 2z)^2 = x^2 + (3y)^2 + (2z)^2 + 2(x)(-3y) + 2(-3y)(-2z) + 2(x)(-2z) = x^2 + 9y^2 + 4z^2 - 6xy + 12yz - 4xz$$

Answer 3.

a. Using $(x+y)^2 = x^2 + 2xy + y^2$, we get

$$(9m-2n)^2 = (9m)^2 + 2(9m)(-2n) + (-2n)^2$$
$$= 81m^2 - 36mn + 4n^2$$

b. $(3p-4q)^2 = (3p)^2 - 2(3p)(4q) + (4q)^2$
$$= 9p^2 - 12pq + 16q^2$$

c. $\left(\frac{7x}{9y} - \frac{9y}{7x}\right)^2 = \left(\frac{7x}{9y}\right)^2 + 2\left(\frac{7x}{9y}\right)\left(\frac{9y}{7x}\right) + \left(\frac{9y}{7x}\right)^2$
$$= \frac{49x^2}{81y^2} + 2 + \frac{81y^2}{49x^2}$$

d. Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$$(2a+3b-4c)^2 = (2a)^2 + (3b)^2 + (4c)^2 + 2(2a)(3b) + 2(3b)(-4c) + 2(2a)(-4c)$$
$$= 4a^2 + 9b^2 + 16c^2 + 12ab - 24bc - 8ac$$

Answer 4.

(i) $(5x-9)(5x+9) = (5x)^2 - (9)^2 = 25x^2 - 81$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

(ii) $(2x+3y)(2x-3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

(iii) $(a+b-c)(a-b+c) = (a+b-c)[a-(b-c)]$

$$= (a)^2 - (b-c)^2$$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

$$= a^2 - (b^2 + c^2 - 2bc)$$
$$= a^2 - b^2 - c^2 + 2bc$$

(iv) $(x+y-3)(x+y+3) = (x+y)^2 - (3)^2$

$$= x^2 + y^2 + 2xy - 9$$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

(v) $(1 + a)(1 - a)(1 + a^2) = [(1)^2 - (a)^2](1 + a^2)$

$= (1 - a^2)(1 + a^2)$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

$= (1)^2 - (a^2)^2$

$= 1 - a^4$

(vi) $\left(a + \frac{2}{a} - 1\right)\left(a - \frac{2}{a} - 1\right) = (a - 1)^2 - \left(\frac{2}{a}\right)^2$

$= a^2 + 1 - 2a - \frac{4}{a^2}$

(Using identity: $(a+b)(a-b) = a^2 - b^2$)

Answer 5.

a. Using $(x + y)^2 = x^2 + 2xy + y^2$, we get

$(95)^2 = (100 - 5)^2$

$= (100)^2 - 2(100)(5) + (5)^2$

$= 10000 - 1000 + 25$

$= 9025$

b. $(103)^2 = (100 + 3)^2$

$= (100)^2 + 2(100)(3) + (3)^2$

$= 10000 + 600 + 9$

$= 10609$

c. $(999)^2 = (1000 - 1)^2$

$= (1000)^2 - 2(1000)(1) + (1)^2$

$= 1000000 - 2000 + 1$

$= 998001$

d. $(1005)^2 = (1000 + 5)^2$

$= (1000)^2 + 2(1000)(5) + (5)^2$

$= 1000000 + 10000 + 25$

$= 1010025$

Answer 6.

a. $399 \times 401 = (400-1)\times(400+1)$

$= (400)^2 - (1)^2$

$= 160000 - 1$

$= 159999$

b. $999 \times 1001 = (1000-1)\times(1000+1)$

$= (1000)^2 - (1)^2$

$= 1000000 - 1$

$= 999999$

c. $4.9 \times 5.1 = (5-0.1)\times(5+0.1)$

$= (5)^2 - (0.1)^2$

$= 25 - 0.01$

$= 24.99$

d. $15.9 \times 16.1 = (16-0.1)\times(16+0.1)$

$= (16)^2 - (0.1)^2$

$= 256 - 0.01$

$= 255.99$

Answer 7.

$a - b = 10, ab = 11$

We know that:

$(a - b)^2 = a^2 - 2ab + b^2$

$\Rightarrow (10)^2 = a^2 + b^2 - 2 \times 11$

$\Rightarrow 100 = a^2 + b^2 - 22$

$\Rightarrow a^2 + b^2 = 100 + 22 = 122$

Using $(a + b)^2 = a^2 + b^2 + 2ab$, we get

$(a + b)^2 = 122 + 2(11) = 122 + 22 = 144$

$\Rightarrow (a + b) = \sqrt{144} = \pm 12$

Answer 8.

$x + y = 9,\ xy = 20$

(i) We know $(a + b) = a^2 + 2ab + b^2$

$\therefore (x + y)^2 = 81\ x^2 + y^2 + 2xy$

$\Rightarrow x^2 + y^2 = 81 - 2(120) = 41$

We also know $(a - b)^2 = a^2 - 2ab + b^2$

$\Rightarrow (x - y)^2 = x^2 - 2xy + y^2$

$\Rightarrow (x - y)^2 = 41 - 2(20) = 1$

$\Rightarrow x - y = \pm 1$

(ii) We know $(x - y)(x + y) = x^2 - y^2$

$\Rightarrow x^2 - y^2 = (\pm 1)(9) = \pm 9$

Answer 9.

(i) $\left(a + \frac{1}{a}\right)^2 = \left(a^2\right) + 2(a)\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$\Rightarrow 36 = a^2 + \frac{1}{a^2} + 2$

$\Rightarrow a^2 + \frac{1}{a^2} = 34$

$$\begin{aligned}\left(a - \frac{1}{a}\right)^2 &= (a)^2 - 2(a)\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2 \\ &= a^2 + \frac{1}{a^2} - 2 \\ &= 34 - 2 = 32\end{aligned}$$

$\Rightarrow a - \frac{1}{a} = \pm\sqrt{32} = \pm 4\sqrt{2}$

(ii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$= (6)\left(\pm 4\sqrt{2}\right) = \pm 24\sqrt{2}$

Answer 10.

$$a - \frac{1}{a} = 10$$

(i) $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2(a)\left(\frac{1}{a}\right)$

$$\Rightarrow (10)^2 = a^2 + \frac{1}{a^2} - 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 102$$

Now, $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2(a)\left(\frac{1}{a}\right)$

$$= 102 + 2 = 104$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{104} = \pm 2\sqrt{26}$$

(ii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$$= \left(\pm 2\sqrt{26}\right)(10)$$

$$= \pm 20\sqrt{26}$$

Answer 11.

(i) $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)$

$$\Rightarrow (3)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

(ii) squaring both sides of the equation $\left(x^2 + \frac{1}{x^2}\right) = 7$, we get:

$$x^4 + \frac{1}{x^4} + 2 = 49$$

$$x^4 + \frac{1}{x^4} = 47$$

Answer 12.

(i) $(p + q)^2 = (8)^2$

$p^2 + q^2 + 2pq = 64 \quad \text{...(i)}$

$(p - q)^2 = (4)^2$

$p^2 + q^2 - 2pq = 16$

$p^2 + q^2 = 16 + 2pq \quad \text{...(ii)}$

Using (ii) in (i), we get:

$16 + 2pq + 2pq = 64$

$\Rightarrow 4pq = 64 - 16 = 48$

$\Rightarrow pq = 12$

(ii) Putting pq = 12 in (i) we get:

$p^2 + q^2 = 64 - 2(12) = 64 - 24 = 40$

Answer 13.

Given $m - n = 0.9$ and $mn = 0.36$

a. $(m-n)^2 = m^2 - 2mn + n^2$

$\Rightarrow (0.9)^2 = m^2 - 2mn + n^2$

$\Rightarrow 0.81 = m^2 + n^2 - 2(0.36)$

$\Rightarrow 0.81 = m^2 + n^2 - 0.72$

$\Rightarrow m^2 + n^2 = 1.53$

So, $(m+n)^2 = m^2 + 2mn + n^2$

$\Rightarrow (m+n)^2 = m^2 + n^2 + 2mn$

$\Rightarrow (m+n)^2 = 1.53 + 2(0.36)$

$\Rightarrow (m+n)^2 = 2.25$

$\Rightarrow m + n = \pm 1.5$

b. $m^2 - n^2 = (m+n)(m-n)$

$= (\pm 1.5)(0.9)$

$= \pm 1.35$

Answer 14.

(i) $(x + y)^2 = (1)^2$

$$\Rightarrow x^2 + y^2 + 2xy = 1$$
$$\Rightarrow x^2 + y^2 = 1 - 2(-12) = 1 + 24 = 25$$

Now, $(x - y)^2 = x^2 + y^2 - 2xy$

$$= 25 - 2(-12)$$

$$= 25 + 24$$

$$= 49$$

$$\Rightarrow x - y = \pm 7$$

(ii) $x^2 - y^2 = (x + y)(x - y)$

$$= (1)(\pm 7) = \pm 7$$

Answer 15.

(i) Dividing the given equation by a , we get:

$$\frac{a^2}{a} - \frac{7a}{a} + \frac{1}{a} = 0, a - 7 + \frac{1}{a} = 0$$

$$\Rightarrow a + \frac{1}{a} = 7$$

(ii) $a + \frac{1}{a} = 7$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 49$$
$$\Rightarrow a^2 + \frac{1}{a^2} = 49 - 2 = 47$$

Answer 16.

(i) Dividing the given equation by a we get

$$a - 3 - \frac{1}{a} = 0$$

$$\Rightarrow a - \frac{1}{a} = 3$$

(ii) $a - \frac{1}{a} = 3$

Squaring both sides ,we get

$$\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 11$$

Now,

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2 = 11+2=13$$

$$\Rightarrow a + \frac{1}{a} = \pm\sqrt{13}$$

(iii) $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right)$

$$= \left(\pm\sqrt{13}\right)(3)$$
$$= \pm 3\sqrt{13}$$

Answer 17.

Given $2x + 3y = 10$ and $xy = 5$

$4x^2 + 9y^2 = \left(2x^2\right) + (3y)^2$

$= (2x + 3y)^2 - 2(2x)(3y)$

......$[\because (a+b)^2 = a^2 + b^2 + 2ab,\ so,\ a^2 + b^2 = (a+b)^2 - 2ab]$

$= (10)^2 - 12(5)$

$= 100 - 60$

$= 40$

Answer 18.

$(x + y + z)^2 = (12)^2$

$\Rightarrow x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 144$

$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 144$

$\Rightarrow x^2 + y^2 + z^2 + 2(27) = 144$

$\Rightarrow x^2 + y^2 + z^2 = 144 - 54 = 90$

Answer 19.

$(a + b + c)^2 = (9)^2$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 81$

$\Rightarrow 41 + 2(ab + bc + ca) = 81$

$\Rightarrow 2(ab + bc + ca) = 81 - 41 = 40$

$\Rightarrow ab + bc + ca = 20$

Answer 20.

$(p + q + r)^2 = p^2 + q^2 + r^2 + 2pq + 2qr + 2pr$

$= 8^2 + 2(18)$

$= 64 + 36$

$= 100$

$\Rightarrow p + q + r = \sqrt{100} = \pm 10$

Answer 21.

Given $x + y + z = p$ and $xy + yz + zx = q$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\Rightarrow x^2 + y^2 + z^2 = (x + y + z)^2 - 2xy + 2yz + 2zx$

$\Rightarrow x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx)$

$\Rightarrow x^2 + y^2 + z^2 = (p)^2 - 2(q)$

$\Rightarrow x^2 + y^2 + z^2 = p^2 - 2q$

Ex 4.2

Answer 1.

(i) Using $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

$(2a - 5b) = (2a)^3 - (5b)^3 - 3(2a)(5b)(2a - 5b)$

$= 8a^3 - 125b^3 - 30ab\,(2a - 5b)$

$= 8a^3 - 125b^3 - 60a^2b + 150ab^2$

(ii) Using $(a + b)^3 = a^3 + b^3 + 3ab + (a + b)$

$(4x + 7y)^3 = (4x)^3 + (7y)^3 + 3(4x)(7y)(4x + 7y)$

$= 64x^3 + 343y^3 + 84xy\,(4x + 7y)$

$= 64x^3 + 343y^3 + 336x^2y + 588xy^2$

(iii) $\left(3a + \frac{1}{3a}\right)^3 = (3a)^3 + \left(\frac{1}{3a}\right)^3 + 3(3a)\left(\frac{1}{3a}\right)\left(3a + \frac{1}{3a}\right)$

$= 27a^3 + \frac{1}{27a^3} + 9a + \frac{1}{a}$

(iv) $\left(4p - \frac{1}{p}\right)^3 = (4p)^3 - \left(\frac{1}{p}\right)^3 - 3(4p)\left(\frac{1}{p}\right)\left(4p - \frac{1}{p}\right)$

$= 64p^3 - \frac{1}{p^3} - 48p + \frac{12}{p}$

(v) $\left(\frac{2m}{3n} + \frac{3n}{2m}\right)^3 = \left(\frac{2m}{3n}\right)^3 + \left(\frac{3n}{2m}\right)^3 + 3\left(\frac{2m}{3n}\right)\left(\frac{3n}{2m}\right)\left(\frac{2m}{3n} + \frac{3n}{2m}\right)$

$= \frac{8m^3}{27n^3} + \frac{27n^3}{8m^3} + \frac{2m}{n} + \frac{9n}{2m}$

(v)

Using $(a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2a + 3c^2a + 6abc$

$\left(a - \frac{1}{a} + b\right)^3$

$= a^3 + \left(-\frac{1}{a}\right)^3 + b^3 + 3a^2\left(-\frac{1}{a}\right) + 3a^2 + 3\left(-\frac{1}{a}\right)^2 b + 3\left(-\frac{1}{a}\right)^2 a + 3b^2a + 3b^2\left(-\frac{1}{a}\right) + 6a\left(-\frac{1}{a}\right)b$

$= a^3 - \frac{1}{a^3} + b^3 - 3a + 3a^2b + \frac{3b}{a^2} + \frac{3}{a} + 3b^2a - \frac{3b^2}{a} - 6b$

Answer 2.

$$5x + \frac{1}{5x} = 7$$

Using $\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$, we get :

$$\left(5x + \frac{1}{5x}\right)^3 = (5x)^3 + \left(\frac{1}{5x}\right)^3 + 3\left(5x + \frac{1}{5x}\right)$$

$$\Rightarrow 343 = 125x^2 + \frac{1}{125x^3} + 3(7)$$

$$\Rightarrow 125x^3 + \frac{1}{125x^3} = 343 - 21 = 322$$

Answer 3.

$$3x - \frac{1}{3x} = 9$$

Using $\left(a - \frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right)$, we get :

$$\left(3x - \frac{1}{3x}\right)^3 = (3x)^3 - \left(\frac{1}{3x}\right)^3 - 3\left(3x - \frac{1}{3x}\right)$$

$$\Rightarrow 729 = 27x^3 - \frac{1}{27x^3} - 3(9)$$

$$\Rightarrow 27x^3 - \frac{1}{27x^3} = 729 + 27 = 756$$

Answer 4.

$$x + \frac{1}{x} = 5 \qquad ...(1)$$

Squaring both sides of (1),

$$\left(x + \frac{1}{x}\right)^2 = (5)^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 25 - 2 = 23 \qquad ...(2)$$

Cubing both sides of (1),

$$\left(x + \frac{1}{x}\right)^3 = 95^3$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 125$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\ (5) = 125$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 15 = 110$$

Squaring both sides of (2),

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (23)^2$$

$$\Rightarrow x^4 + \frac{1}{x^4} + \quad = 529$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 529 - 2 = 527$$

Answer 5.

$$a - \frac{1}{a} = 7 \quad ...(1)$$

Squaring both sides of (1),

$$\left(a - \frac{1}{a}\right)^2 = (7)^2$$

$$\Rightarrow a^2 + \frac{1}{a^2} - 2 = 49$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 49 + 2 = 51$$

Now, $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$

$$= 51 + 2 = 53$$

$$\Rightarrow a + \frac{1}{a} = \pm\sqrt{53}$$

Now $a^2 - \frac{1}{a^2} = \left(a + \frac{1}{a}\right)\left(a - \frac{1}{a}\right) = \left(\pm\sqrt{53}\right)(7) = \pm 7\sqrt{53}$

Answer 6A.

Using $(a+b)^2 = a^2 + 2ab + b^2$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 14 + 2$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 16$$

$$\Rightarrow a + \frac{1}{a} = \pm 4$$

Answer 6B.

Using $(a+b)^2 = a^2 + 2ab + b^2$

$$\left(a+\frac{1}{a}\right)^2 = a^2 + 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 14 + 2$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = 16$$

$$\Rightarrow a + \frac{1}{a} = \pm 4$$

$$a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)\left(a^2 + \frac{1}{a^2} - 1\right) \quad .....\left[\text{Using } a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)\right]$$

$$= (\pm 4)(14-1)$$

$$= (\pm 4)(13)$$

$$= \pm 52$$

Answer 7.

$$m^2 + \frac{1}{m^2} = 51$$

We know that

$$\left(m - \frac{1}{m}\right)^2 = m^2 + \frac{1}{m^2} - 2$$

$$\Rightarrow \left(m - \frac{1}{m}\right)^2 = 51 - 2$$

$$\Rightarrow \left(m - \frac{1}{m}\right)^2 = 49 = 7^2$$

$$\Rightarrow m - \frac{1}{m} = 7$$

$$\Rightarrow \left(m - \frac{1}{m}\right)^3 = 7^3$$

$$\Rightarrow m^3 - \frac{1}{m^3} - 3\left(m - \frac{1}{m}\right) = 343$$

$$\Rightarrow m^3 - \frac{1}{m^3} - 3 \times 7 = 343$$

$$\Rightarrow m^3 - \frac{1}{m^3} = 343 + 21 = 364$$

Answer 8.

$$9a^2 + \frac{1}{9a^2} = 23$$

Using $\left(3a + \frac{1}{3a}\right)^2 = (3a)^2 + \left(\frac{1}{3a}\right)^2 + 2(3a)\left(\frac{1}{3a}\right)$

$$\Rightarrow \left(3a + \frac{1}{3a}\right)^2 = 9a^2 + \frac{1}{9a^2} + 2$$
$$= 23 + 2 = 25$$
$$\Rightarrow 3a + \frac{1}{3a} = 5$$

Cubing both sides, we get:

$$(3a)^3 + \left(\frac{1}{3a}\right)^3 + 3(3a)\left(\frac{1}{3a}\right)\left(3a + \frac{1}{3a}\right) = (5)^3$$
$$\Rightarrow 27a^3 + \frac{1}{27a^3} + 3(5) = 125$$
$$\Rightarrow 27a^3 + \frac{1}{27a^3} = 125 - 15 = 110$$

Answer 9.

$$x^2 + \frac{1}{x^2} = 18$$

(i) Using $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 18 - 2 = 16$$
$$\Rightarrow x - \frac{1}{x} = 4$$

(ii) $\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

$$\Rightarrow 64 = x^3 - \frac{1}{x^3} - 3(4)$$
$$\Rightarrow x^3 - \frac{1}{x^3} = 64 + 12 = 76$$

Answer 10.

(i) $\left(p+\frac{1}{p}\right)^2 = p^2+\frac{1}{p^2}+2$

$\Rightarrow 36 = p^2+\frac{1}{p^2}+2$

$\Rightarrow p^2+\frac{1}{p^2} = 36-2 = 34$

(ii) $\left(p^2+\frac{1}{p^2}\right)^2 = p^4+\frac{1}{p^4}+2$

$\Rightarrow (34)^2 = p^4+\frac{1}{p^4}+2$

$\Rightarrow p^4+\frac{1}{p^4} = 1158-2 = 1154$

(iii) $\left(p+\frac{1}{p}\right)^3 = p^3+\frac{1}{p^3}+3\left(p+\frac{1}{p}\right)$

$\Rightarrow 216 = p^3+\frac{1}{p^3}+3(6)$

$\Rightarrow p^3+\frac{1}{p^3} = 216-18 = 198$

Answer 11.

(i) $\left(r-\frac{1}{r}\right)^2 = r^2+\frac{1}{r^2}-2$

$\Rightarrow (4)^2 = r^2+\frac{1}{r^2}-2$

$\Rightarrow r^2+\frac{1}{r^2} = 16+2 = 18$

(ii) $\left(r^2+\frac{1}{r^2}\right)^2 = r^4+\frac{1}{\sqrt{4}}+2$

$\Rightarrow (18)^2 = r^4+\frac{1}{r^4}+2$

$\Rightarrow r^4+\frac{1}{r^4} = 324-2 = 322$

(iii) $\left(r-\frac{1}{r}\right)^3 = r^3-\frac{1}{r^3}-3\left(r-\frac{1}{r}\right)$

$\Rightarrow (4)^3 = r^3-\frac{1}{r^3}-3(4)$

$\Rightarrow r^3-\frac{1}{r^3} = 64+12 = 76$

Answer 12.

$$a + \frac{1}{a} = 2$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow (2)^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 4 - 2 = 2$$

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow (2)^3 = a^3 + \frac{1}{a^3} + 3(2)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 8 - 6 = 2$$

$$\left(a^2 + \frac{1}{a^2}\right)^2 = a^4 + \frac{1}{a^4} + 2$$

$$\Rightarrow (2a)^2 = a^4 + \frac{1}{a^4} + 2$$

$$\Rightarrow a^4 + \frac{1}{a^4} = 4 - 2 = 2$$

Thus, $a^2 + \frac{1}{a^2} = a^3 + \frac{1}{a^3} = a^4 + \frac{1}{a^4}$

Answer 13.

$$x + \frac{1}{x} = p,\ x - \frac{1}{x} = q$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow p^2 = x^2 + \frac{1}{x^2} + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = p^2 - 2 \qquad ...(1)$$

Also, $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$$\Rightarrow q^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = q^2 + 2 \qquad ...(2)$$

Equating the value of $x^2 + \frac{1}{x^2}$ from and (2), we get:

$p^2 - 2 = q^2 + 2$

$\Rightarrow p^2 - q^2 = 4$

Answer 14.

$$a + \frac{1}{a} = p$$

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow p^3 = a^3 + \frac{1}{a^3} + 3(p)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = p^3 - 3p = p\left(p^2 - 3\right)$$

Answer 15.

$$\left(a + \frac{1}{a}\right)^2 = 3$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{3}$$

$$\text{Now, } \left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow \left(\sqrt{3}\right)^3 = a^3 + \frac{1}{a^3} + 3\left(\sqrt{3}\right)$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

Answer 16.

$a + b + c = 0 \qquad \text{...(i)}$

$\Rightarrow (a + b) + c = 0$

cubing both sides

$\Rightarrow (a + b)^3 + c^3 + 3(a + b)(c)(a + b + c) = 0$

$\Rightarrow a^3 + b^3 + 3ab(a + b) + c^3 + 0 = 0$

$\Rightarrow a^3 + b^3 + c^3 + 3ab(a + b) = 0 \qquad \text{...(2)}$

Using (i), we get, $a + b = -c$ From (2),

$a^3 + b^3 + c^3 + 3ab(-c) = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Answer 17.

$a + 2b + c = 0 \qquad \ldots(i)$

$$\Rightarrow (a + 2b) + c = 0$$
$$\Rightarrow (a + 2b)^3 + c^3 + 3(a + 2b)\ c\ (a + 2b + c) = 0$$
$$\Rightarrow a^3 + 8b^2 + 6ab\ (a + 2b) + c^3 + 0 = 0$$
$$\Rightarrow a^3 + 8b^3 + c^3 + 6ab\ (a + 2b) = 0 \qquad \ldots(2)$$

Using (1), we get $a + 2b = -c$

From (2),

$a^3 + 8b^3 + 6ab\ (-c) = 0$

$$\Rightarrow a^3 + 8b^3 + c^3 = 6abc$$

Answer 18.

$x^3 + y^3 = 9,\ x + y = 3$

$$(x + y)^3 = x^3 + y^3 + 3xy\ (x + y)$$
$$\Rightarrow (3)^3 = 9 + 3xy\ (3)$$
$$\Rightarrow 27 = 9 + 9xy$$
$$\Rightarrow 9xy = 27 - 9 = 18$$
$$\Rightarrow xy = 2$$

Answer 19.

Using $(a+b)^2 = a^2 + 2ab + b^2$

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$\Rightarrow a^2 + b^2 = (5)^2 - 2(2)$$

$$\Rightarrow a^2 + b^2 = (5)^2 - 2(2)$$

$$\Rightarrow a^2 + b^2 = 25 - 4$$

$$\Rightarrow a^2 + b^2 = 21$$

$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$= (5)(21 - 2)$$

$$= (5)(19)$$

$$= 95$$

Answer 20.

$p - q = -1, \quad pq = -12$

$$(p-q)^3 = p^3 - q^3 - 3pq\,(p-q)$$
$$\Rightarrow (-1)^3 = p^3 - q^3 - 3(-12)(-1)$$
$$\Rightarrow p^3 - q^3 = -14 + 36 = 35$$

Answer 21.

$m - n = -2, m^3 - n^3 = -26$

$$(m-n)^3 = m^3 - n^3 - 3mn\,(m-n)$$
$$\Rightarrow (-2)^3 = -26 - 3mn\,(-2)$$
$$\Rightarrow 6mn = -8 + 26 = 18$$
$$\Rightarrow mn = 3$$

Answer 22.

$2a - 3b = 10$

$$(2a-3b)^3 = (2a)^3 - (3b)^3 - 2(2a)(3b)(2a-3b)$$
$$\Rightarrow 1000 = 8a^3 - 27b^3 - 12(16)(10)$$
$$\Rightarrow 8a^3 - 27b^3 = 1000 + 1920 = 2920$$

Answer 23.

Given $x + 2y = 5$

$$(x+2y)^3 = 5^3$$
$$\Rightarrow (x)^3 + (2y)^3 + 3(x)(2y)(x+2y) = 5^3 \quad \ldots\left[\text{Using } (a+b)^3 = (a)^3 + (b)^3 + 3ab\,(a+b)\right]$$
$$\Rightarrow (x)^3 + (2y)^3 + 6xy(x+2y) = 125$$
$$\Rightarrow (x)^3 + (2y)^3 + 6xy(5) = 125$$
$$\Rightarrow x^3 + 8y^3 + 30xy = 125$$

Answer 24A.

$$(4x+5y)^2+(4x-5y)^2$$
$$=(4x)^2+(5y)^2+2(4x)(5y)+(4x)^2+(5y)^2-2(4x)(5y)$$
$$=16x^2+25y^2+40xy+16x^2+25y^2-40xy$$
$$=32x^2+50y^2$$

Answer 24B.

$$(7a+5b)^2-(7a-5b)^2$$
$$=(7a)^2+(5b)^2+2(7a)(5b)-\left[(7a)^2+(5b)^2-2(7a)(5b)\right]$$
$$=49a^2+25b^2+70ab-\left[49a^2+25b^2-70ab\right]$$
$$=70a+70ab$$
$$=140ab$$

Answer 24C.

$$(a+b)^3+(a-b)^3$$
$$=a^3+b^3+3ab(a+b)+a^3-3ab(a-b)-b^3$$
$$=a^3+b^3+3a^2b+3ab^2+a^3-3a^2b+3ab^2-b^3$$
$$=2a^3+6ab^2$$

Answer 24D.

$$\left(a-\frac{1}{a}\right)^2+\left(a+\frac{1}{a}\right)^2$$
$$=(a)^2+\left(\frac{1}{a}\right)^2-2(a)\left(\frac{1}{a}\right)+(a)^2+\left(\frac{1}{a}\right)^2+2(a)\left(\frac{1}{a}\right)$$
$$=a^2+\frac{1}{a^2}-2+a^2+\frac{1}{a^2}+2$$
$$=2a^2+\frac{2}{a^2}$$

Answer 24E.

$$(x+y-z)^2+(x-y+z)^2$$
$$= x^2+y^2+z^2+2(x)(y)+2(y)(-z)+2(x)(-z)+x^2+y^2+z^2+2(x)(-y)+2(-y)(z)+2(x)(z)$$
$$= x^2+y^2+z^2+2xy-2yz-2xz+x^2+y^2+z^2-2xy-2yz+2xz$$
$$= 2x^2+2y^2+2z^2-4yz$$

Answer 24F.

$$\left(a+\frac{1}{a}\right)^3-\left(a-\frac{1}{a}\right)^3$$
$$= (a)^3+\left(\frac{1}{a}\right)^3+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)-\left[(a)^3-\left(\frac{1}{a}\right)^3-3(a)\left(\frac{1}{a}\right)\left(a-\frac{1}{a}\right)\right]$$
$$= a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)-\left[a^3-\frac{1}{a^3}-3\left(a-\frac{1}{a}\right)\right]$$
$$= a^3+\frac{1}{a^3}+3a+\frac{3}{a}-a^3+\frac{1}{a^3}+3a-\frac{3}{a}$$
$$= \frac{2}{a^3}+6a$$

Answer 24G.

$$(2x+y)\left(4x^2-2xy+y^2\right)$$
$$= 2x\left(4x^2-2xy+y^2\right)+y\left(4x^2-2xy+y^2\right)$$
$$= 8x^3-4x^2y+2xy^2+4x^2y-2xy^2+y^3$$
$$= 8x^3+y^3$$

Answer 24H.

$$\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right)$$
$$= x\left(x^2+1+\frac{1}{x^2}\right)-\frac{1}{x}\left(x^2+1+\frac{1}{x^2}\right)$$
$$= x^3+x+\frac{1}{x}-x-\frac{1}{x}-\frac{1}{x^3}$$
$$= x^3-\frac{1}{x^3}$$

Answer 24I.

$$(x+2y+3z)\left(x^2+4y^2+9z^2-2xy-6yz-3zx\right)$$
$$= x\left(x^2+4y^2+9z^2-2xy-6yz-3zx\right)+2y\left(x^2+4y^2+9z^2-2xy-6yz-3zx\right)$$
$$+3z\left(x^2+4y^2+9z^2-2xy-6yz-3zx\right)$$
$$= x^3+4xy^2+9xz^2-2x^2y-6xyz-3zx^2+2x^2y+8y^3+18yz^2-4xy^2-12y^2z-6xyz$$
$$+3x^2z+12y^2z+27z^3-6xyz-18yz^2-9xz^2$$
$$= x^3+8y^3+27z^3-18xyz$$

Answer 24J.

$$(1+x)(1-x)\left(1-x+x^2\right)\left(1+x+x^2\right)$$
$$= (1+x)(1-x)\left(x^2+1-x\right)\left(x^2+1+x\right)$$
$$= \left(1^2-x^2\right)\left[\left(x^2+1\right)^2-x^2\right] \qquad .....\left(\text{Using } a^2-b^2=(a+b)(a-b)\right)$$
$$= \left(1-x^2\right)\left[x^4+2x^2+1-x^2\right]$$
$$= \left(1-x^2\right)\left(x^4+x^2+1\right)$$
$$= 1\left(x^4+x^2+1\right)-x^2\left(x^4+x^2+1\right)$$
$$= x^4+x^2+1-x^6-x^4-x^2$$
$$= 1-x^6$$

Answer 24K.

$$(3a+2b-c)\left(9a^2+4b^2+c^2-6ab+2bc+3ca\right)$$
$$= 3a\left(9a^2+4b^2+c^2-6ab+2bc+3ca\right)+2b\left(9a^2+4b^2+c^2-6ab+2bc+3ca\right)$$
$$-c\left(9a^2+4b^2+c^2-6ab+2bc+3ca\right)$$
$$= 27a^3+12ab^2+3ac^2-18a^2b+6abc+9a^2c+18a^2b+8b^3+2bc^2-12ab^2+4b^2c+6abc$$
$$-9a^2c-4b^2c-c^3+6abc-2bc^2-3ac^2$$
$$= 27a^3+8b^3-c^3+18abc$$

Answer 24L.

$$(3x+5y+2z)(3x-5y+2z)$$
$$= (3x+2z+5y)(3x+2z-5y)$$
$$= (3x+2z)^2-(5y)^2$$
$$= 9x^2+2(3x)(2z)+4z^2-25y^2$$
$$= 9x^2-25y^2+4z^2+12xz$$

Answer 24M.

$$(2x-4y+7)(2x+4y+7)$$
$$=(2x+7-4y)(2x+7+4y)$$
$$=(2x+7)^2-(4y)^2$$
$$=4x^2+2(2x)(7)+7^2-16y^2$$
$$=4x^2-16y^2+28x+49$$

Answer 24N.

$$(3a-7b+3)(3a-7b+5)$$
$$=3a(3a-7b+5)-7b(3a-7b+5)+3(3a-7b+5)$$
$$=9a^2-21ab+15a-21ab+49b^2-35b+9a-21b+15$$
$$=9a^2-42ab+24a+49b^2-56b+15$$

Answer 24O.

$$(4m-5n-8)(4m-5n+5)$$
$$=4m(4m-5n+5)-5n(4m-5n+5)-8(4m-5n+5)$$
$$=16m^2-20mn+20m-20mn+25n^2-25n-32m+40n-40$$
$$=16m^2+25n^2-40mn-12m+15n-40$$

Answer 25.

(i) $(3.29)^3+(6.71)^3$

$$=(3.29+6.71)^3-3(3.29)(6.71)(3.029+6.71)$$
$$=(10)^3-3(3.29)(6.71)(10)$$
$$=1000-30(5-1.71)(5+1.71)$$
$$=1000-30(5)^2-(1.71)^2$$
$$=1000-30(25-2.9241)$$
$$=1000-30\times 22.0759$$
$$=1000-662.277$$
$$=337.723$$

(ii) $(5.45)^3+(3.55)^3$

$$=(5.45+3.55)^3-3(5.45)(3.55)(5.45+3.55)$$
$$=(9)^3-3(4+1.45)(4-1.45)(9)$$
$$=81-3\left(16-(1.45)^2\right)(9)$$
$$=81-27(16-2.1025)$$
$$=81-27\times 13.8975$$
$$=81-522.3825=206.6175$$

(iii) $(8.12)^3 - (3.12)^3$

$$= (8.12 - 3.12)^3 + 3(8.12)(3.12)(8.12 - 3.12)$$
$$= 5^3 + 3(8.12)(3.12) \times 5$$
$$= 125 + 15 \times (8.12)(3.12)$$
$$= 125 + 15 \times 25.3344$$
$$= 125 + 380.016 = 505.016$$

(iv) 7.16 x 7.16 +2.16 x 7.16 + 2.16 x 2.16

$$= (7.16)^2 + (2.16)(7.16) + (2.16)^2$$
$$= (7.16)^2 + (2.16)(7.16) + (2.16)^2 + (2.16)(7.16) - (2.16)(7.16)$$
$$= (7.16)^2 + 2(2.16)(7.16) + (2.16)^2 - (2.16)(7.16)$$
$$= (7.16 + 2.16)^2 - (2.16)(7.16)$$
$$= (9.32)^2 - 15.4656$$
$$= 86.8624 - 15.4656 = 71.3968$$

(v) 1.81 x 1.81 - 1.81 x 2.19 + 2.19 x 2.19

Sol: 1.81 x 1.81 - 1.81 x 2.19 + 2.19 x 2.19

$$= (1.81)^2 - (1.81 \times 2.19) + (2.19)^2$$
$$= (1.81)^2 - (1.81 \times 2.19) + (2.19)^2 - (1.81 \times 2.19) + (1.81 \times 2.19)$$
$$= (1.81)^2 - 2(1.81 \times 2.19) + (2.19)^2 + (1.81 \times 2.19)$$
$$= (1.81 - 2.19)^2 + (2.00 - 0.19)(2.00 + 0.19)$$
$$= (0.38)^2 + \left(4 - (0.19)^2\right)$$
$$= 0.1444 + (4 - 0.0361)$$
$$= 0.1444 + 3.9639 = 4.1083$$