ATOMIC PHYSICS [JEE ADVANCED PREVIOUS YEAR SOLVED PAPERS]

JEE Advanced

Single Correct Answer Type

1. The shortest wavelength of X-rays emitted from an X-ray tube depends on
   a. the current in the tube
   b. the voltage applied to the tube
   c. the nature of the gas in the tube
   d. the atomic number of the target material
   (IIT-JEE 1982)

2. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be
   a. 60 **b.** 32 **c.** 4 **d.** 64
   (IIT-JEE 1983)

3. Consider the spectral line resulting from the transition $n = 2 \rightarrow n = 1$ in the atoms and ions given below. The shortest wavelength is produced by
   a. hydrogen atom **b.** deuterium atom
   c. singly ionized helium **d.** doubly ionized lithium
   (IIT-JEE 1983)

4. The X-ray beam coming from an X-ray tube will be
   a. monochromatic
   b. having all wavelengths smaller than a certain maximum wavelength
   c. having all wavelengths larger than a certain minimum wavelength
   d. having all wavelengths lying between a minimum and a maximum wavelength **(IIT-JEE 1985)**

5. The K_α X-ray emission line of tungsten occurs at $\lambda = 0.02$ nm. The energy difference between K and L levels in this atom is about
   a. 0.51 MeV **b.** 1.2 MeV
   c. 59 MeV **d.** 13.6 MeV
   (IIT-JEE 1997)

6. As per the Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is
   a. 1.51 **b.** 13.6 **c.** 40.8 **d.** 122.4
   (IIT-JEE 1997)

7. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from
   a. 0 and ∞
   b. λ_{min} to ∞; where $\lambda_{min} > 0$.
   c. 0 to λ_{max}; where $\lambda_{max} < \infty$.
   d. λ_{min} to λ_{max}; where $0 < \lambda_{min} < \infty$ **(IIT-JEE 1998)**

8. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (giving in terms of the Rydberg constant R for the hydrogen atom) equal to
   a. $9/(5R)$ **b.** $36/(5R)$ **c.** $18/(5R)$ **d.** $4/R$
   (IIT-JEE 2000)

9. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true?
   a. Its kinetic energy increases and its potential and total energies decrease.
   b. Its kinetic energy decreases, potential energy increases, and its total energy remains the same.
   c. Its kinetic and total energies decrease and its potential, energy increases.
   d. Its kinetic, potential, and total energies decrease.
   (IIT-JEE 2000)

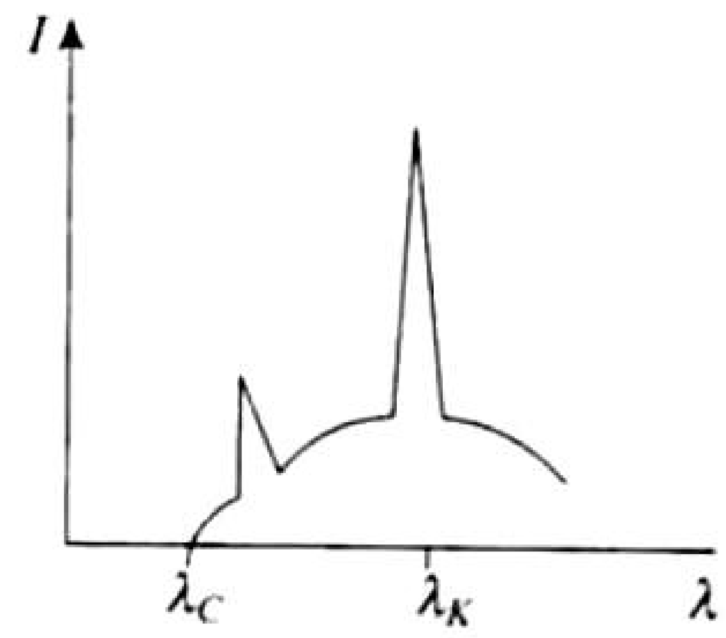
10. Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K shell electrons of tungsten have −72.5 keV energy. X-rays emitted by the tube contain only
    a. a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of −0.155 Å.
    b. continuous X-ray spectrum (Bremsstrahlung) with all wavelengths.
    c. the characteristic X-ray spectrum of tungsten.
    d. a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of −0.155 Å and the characteristic X-ray spectrum of tungsten.
    (IIT-JEE 2000)

11. The transition from the state $n = 4$ to $n = 3$ in a hydrogen-like atom results in ultra violet radiation. Infrared radiation will be obtained in the transition

a. $2 \to 1$ **b.** $3 \to 2$ **c.** $4 \to 2$ **d.** $5 \to$

(IIT-JEE 2001)

12. The intensity of X-rays from a Coolidge tube is plotted against wavelength λ as shown in the figure. The minimum wavelength found is λ_C and the wavelength of the K_α line is λ_K. As the accelerating voltage is increased,



a. $\lambda_K - \lambda_C$ increases **b.** $\lambda_K - \lambda_C$ decreases
c. λ_K increases **d.** λ_K increases

(IIT-JEE 2001)

13. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. Then, the number of electrons striking the target per second is

a. 2×10^{16} **b.** 5×10^6 **c.** 1×10^{17} **d.** 4×10^{15}

(IIT-JEE 2002)

14. A hydrogen atom and a Li^+ ion are both in second excited state. If l_H and l_{Li} are their respective electronic angular momenta and E_H and E_{Li} their respective energies, then

a. $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$
b. $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$
c. $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$
d. $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$ **(IIT-JEE 2002)**

15. The electric potential between a proton and an electron is given by $V = V_0 \ln r/r_0$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number?

a. $r_n \propto n$ **b.** $r_n \propto 1/n$ **c.** $r_n \propto n^2$ **d.** $r_n \propto 1/n^2$

(IIT-JEE 2003)

16. If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius of $_{100}Fm^{257}$ is n times the Bohr radius, then find n.

a. 100 **b.** 200 **c.** 4 **d.** 1/4

(IIT-JEE 2003)

17. K_α wavelength emitted by an atom, of atomic number $Z = 11$ is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ.

a. $Z = 6$ **b.** $Z = 4$ **c.** $Z = 11$ **d.** $Z = 44$

(IIT-JEE 2005)

18. A photon collides with a stationary hydrogen atom in the ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of a microsecond, another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?

a. One photon of energy 10.2 eV and an electron of energy 1.4 eV.
b. Two photons of energy of 1.4 eV.
c. Two photons of energy 10.2 eV.
d. One photon of energy 10.2 eV and another photon of 1.4 eV. **(IIT-JEE 2005)**

19. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

a. 802 nm **b.** 823 nm **c.** 1882 nm **d.** 1648 nm

(IIT-JEE 2007)

20. Electrons with de Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

a. $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ **b.** $\lambda_0 = \dfrac{2h}{mc}$

c. $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$ **d.** $\lambda_0 = \lambda$ **(IIT-JEE 2007)**

21. Which one of the following statement is wrong in the context of X-rays generated from an X-ray tube?

a. Wavelength of characteristic X-rays decreases when the atomic number of the target increases.
b. Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.
c. Intensity of the characteristic X-rays depends on the electric power given to the X-ray tube.
d. Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

(IIT-JEE 2008)

22. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is:

a. 1215 Å **b.** 1640 Å **c.** 2430 Å **d.** 4687 Å

(IIT-JEE 2011)

Multiple Correct Answer Type

1. In Bohr's model of the hydrogen atom,

a. the radius of the nth orbit is proportional to n^2
b. the total energy of the electron in nth orbit is inversely proportional to n
c. the angular momentum of the electron in an orbit is an integral multiple of $h/2\lambda$
d. the magnitude of potential energy of the electron in any orbit is greater than its KE **(IIT-JEE 1984)**

2. The mass number of a nucleus is

a. always less than its atomic number
b. always more than its atomic number
c. sometimes equal to its atomic number
d. sometimes more than and sometimes equal to its atomic number **(IIT-JEE 1986)**

3. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation

a. the intensity increases
b. the minimum wavelength increases
c. the intensity remains unchanged
d. the minimum wavelength decreases

(IIT-JEE 1988)

4. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are

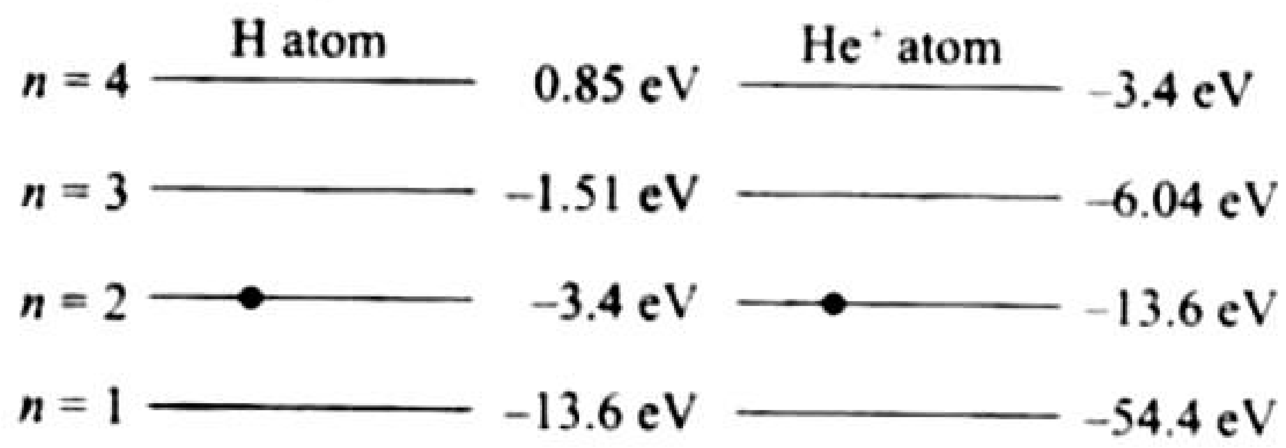
a. $n_1 = 4, n_2 = 2$ **b.** $n_1 = 8, n_2 = 2$
c. $n_1 = 8, n_2 = 1$ **d.** $n_1 = 6, n_2 = 3$

(IIT-JEE 1998)

Linked Comprehension Type

For Problems 1–3

In a mixture of H–He⁺ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.



(IIT-JEE 2008)

1. The quantum number n of the state finally populated in He^+ ions is
a. 2 **b.** 3 **c.** 4 **d.** 5

2. The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is
a. 6.5×10^{-7} m **b.** 5.6×10^{-7} m
c. 4.8×10^{-7} m **d.** 4.0×10^{-7} m

3. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He^+ ion is
a. $\frac{1}{4}$ **b.** $\frac{1}{2}$ **c.** 1 **d.** 2

For Problems 4–6

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **(IIT-JEE 2010)**

4. A diatomic molecule has moment of inertia I. By Bohr's quantization condition, its rotational energy in the nth level ($n = 0$ is not allowed) is

a. $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ **b.** $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

c. $n\left(\frac{h^2}{8\pi^2 I}\right)$ **d.** $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

5. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $4/\pi \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)
a. 2.76×10^{-46} kg m² **b.** 1.87×10^{-46} kg m²
c. 4.67×10^{-47} kg m² **d.** 1.17×10^{-47} kg m²

6. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u 5/3 × 10^{-27} kg, is close to
a. 2.4×10^{-10} m **b.** 1.9×10^{-10} m
c. 1.3×10^{-10} m **d.** 4.4×10^{-11} m

Assertion-Reasoning Type

In each of the questions, assertion (A) is given by corresponding statement of reason (R) of the statements. Mark the correct answer.

a. If both Statement I and Statement II are true, Statement II is the correct explanation of the Statement I.
b. If both Statement I and Statement II are true, Statement II is not the correct explanation of Statement I.
c. If Statement I is true but Statement II is false.
d. If Statement I is false but Statement II is true.

1. **Statement I:** If the accelerating potential in an X-ray tube is increased, the wavelength of the characteristic X-rays does not change.
Statement II: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy. **(IIT-JEE 2007)**

Fill in the Blanks Type

1. To produce characteristic X-rays using a tungsten target in an X-ray generator, the accelerating voltage should be greater than ________ volt and the energy of the characteristic radiation is ________ eV.
(The binding energy of the innermost electron in tungsten is 40 keV.) **(IIT-JEE 1983)**

2. When the number of electrons striking the anode of an X-ray tube is increased, the ________ of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased, the cut-off wavelength of the emitted X-rays ________.
(IIT-JEE 1986)

3. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen-like element is 0.32 Å. The wavelength of the K_β line emitted by the same element will be ________. **(IIT-JEE 1990)**

4. The Bohr radius of the fifth electron of phosphorous atom (atomic number = 15) acting as a dopant in silicon (relative dielectric constant = 12) is ________ Å.
(IIT-JEE 1991)

5. In the X-ray tube, electrons accelerated through a potential difference of 15000 volt strike a copper target. The speed of the emitted x-ray inside the tube is ________ m s⁻¹. **(IIT-JEE 1992)**

6. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is ______________. **(IIT-JEE 1992)**
7. The wavelength of K_α X-rays produced by an X-ray tube is 0.76 Å. The atomic number of the anode material of the tube is ______________. **(IIT-JEE 1996)**
8. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state in m $s^{-1}$is ______________. **(IIT-JEE 1997)**

Subjective Type

1. A single electron orbits around a stationary nucleus of charge $+Ze$, where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find
   a. the values of Z.
   b. the energy required to excite the electron from the third to the fourth Bohr orbit.
   c. the wavelength of electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
   d. the kinetic energy, potential energy, and the angular momentum of the electron in the first Bohr orbit.
   e. the radius of the first Bohr orbit.
   The ionization energy of the hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} m, speed of light = 3×10^{8} m s^{-1}, Planck's constant = 6.63×10^{-34} J-s. **(IIT-JEE 1981)**
2. Hydrogen atom in its ground state is excited by means of monochromic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV. **(IIT-JEE 1982)**
3. Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant. **(IIT-JEE 1983)**
4. The ionization energy of a hydrogen-like Bohr atom is 4 rydbergs.
   a. What is the wavelength of radiation emitted when the electron jumps from the first excited state to the ground state?
   b. What is the radius of the first orbit for this atom?
   Given that Bohr radius of hydrogen atom = 5×10^{-11} m and 1 rydberg = 2.2×10^{-18}J. **(IIT-JEE 1984)**
5. A doubly ionized lithium atom is hydrogen-like with atomic number 3.
   a. Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to third Bohr orbit (ionization energy of hydrogen equals 13.6 eV).
   b. How many spectral lines are observed in the emission spectrum of the above excited system? **(IIT-JEE 1985)**
6. A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of electron (called a μ-meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite.) Assuming that Bohr's model of the atom is applicable to this system: **(IIT-JEE 1988)**
   a. derive and expression for the radius of the nth Bohr orbit.
   b. find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.
   c. find the wavelength of the radiation emitted when the μ-meson jumps from the third orbit to the first orbit. (Rydberg's constant = 10967800 m)
7. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transitions to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV, some have energy more and some have less than 2.7 eV.
   a. Find the principal quantum number of the initially excited level B.
   b. Find the ionization energy for the gas atoms.
   c. Find the maximum and minimum energies of the emitted photons. **(IIT-JEE 1989)**
8. Electrons in a hydrogen-like atom ($Z = 3$) make transitions from the fourth excited state to the third excited state and from the third excited state to the second excited state. The resulting radiations are incident on a metal plate and eject photoelectrons. The stopping potential for photoelectrons ejected by shorter wavelength is 3.95 eV. Calculate the work function of the metal and stopping potential for the photoelectrons ejected by the longer wavelength. **(IIT-JEE 1990)**
9. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find
   a. the energy of the photons causing the photoelectric emission,
   b. the quantum numbers of the two levels involved in the emission of these photons,
   c. the change in the angular momentum of the electron in the hydrogen atom in the above transition, and
   d. the recoil speed of the emitting atom assuming it to be at rest before the transition.
   (Ionization potential of hydrogen is 13.6 eV) **(IIT-JEE 1992)**
10. A neutron of kinetic energy 65 eV collides inelastically with a singly ionised helium atom at rest. It is scattered at an angle 90° with respect to its original direction.
    Find the minimum allowed values of energy of neutron.
    Find the maximum allowed value of energy of He-atom? **(IIT-JEE 1993)**

11. A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV, respectively.
Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.954 eV, respectively. Determine the values of n and Z (ionization energy of hydrogen atom = 13.6 eV). **(IIT-JEE 1994)**

12. An electron in a hydrogen-like atom is in an excited state. It has a total energy of –3.4 eV. Calculate
    a. the kinetic energy, and
    b. the de Broglie wavelength of the electron.
    ($h = 6.63 \times 10^{-34}$ J-s) **(IIT-JEE 1996)**

13. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV. **(IIT-JEE 2000)**

14. A hydrogen like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between –0.85 eV and –0.544 eV (including both these values). **(IIT-JEE 2002)**
    a. Find the atomic number of the atom
    b. Calculate the smallest wavelength emitted in these transitions.
    (Take hc = 1240 eV-nm, ground state energy of hydrogen atom = –13.6 eV).

15. Frequency of a photon emitted due to transition of electron of a certain element from L to K shell is found to be 4.2×10^{18} Hz. Using Moseley's law, find the atomic number of the element, given that the Rydberg's constant $R = 1.1 \times 10^7$ m^{-1}. **(IIT-JEE 2003)**

16. Wavelengths of Balmer series lying in the range of 450 nm to 700 nm were used to eject photoelectrons from a metal surface of work function 2.2 eV. Determine the maximum kinetic energy in eV of the emitted photoelectron. Take hc = 1242 eV nm. **(IIT-JEE 2005)**

17. An electron in hydrogen-like atom makes a transition from nth orbit and emits radiation corresponding to Lyman series. If de Broglie wavelength of electron in nth orbit is equal to the wavelength of radiation emitted, find the value of n. The atomic number of atom is 11. **(IIT-JEE 2006)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. b.	**2.** a.	**3.** d.	**4.** c.	**5.** c.
6. d.	**7.** b.	**8.** c.	**9.** a.	**10.** d.
11. d.	**12.** a.	**13.** a.	**14.** b.	**15.** a.
16. d.	**17.** a.	**18.** a.	**19.** b.	**20.** a.
21. b.	**22.** b.			

Multiple Correct Answers Type

1. a., c., d. **2.** c., d. **3.** c., d. **4.** a., d.

Linked Comprehension Type

1. c. **2.** c. **3.** a. **4.** d. **5.** a. **6.** c.

Assertion–Reasoning Type

1. b.

Fill in the Blanks Type

1. 30.000 eV **2.** intensity, decrease **3.** 0.27
5. 3×10^8 m/s **4.** 3.81 Å **6.** 1
7. 41 **8.** 4.17 m s^{-1}

Subjective Type

1. (a) 5 (b) 16.53 eV (c) 36.4 Å
(d) 340 eV, –680 eV, –340 eV, $1.05 \times 10^{-34} \frac{\text{kg-m}^2}{s}$
(e) 1.06×10^{-11} m
2. six, 1.875 μm **3.** 6.57×10^{-34} Js.
4. (a) 300 Å (b) 0.2645 Å **5.** (a) 113.74 Å (b) 3
6. (a) $r_n = \frac{n^2 h^2 \varepsilon_0}{624 \pi m_e e^2}$ (b) $n \approx 25$ (c) 0.546 Å
7. (a) 2 (b) 14.4 eV (c) 13.5 eV, 0.7 eV
8. 2 eV, 0.74 V
9. (a) 2.55 eV (b) $4 \to 2$ (c) $-\frac{h}{\pi}$ (d) 0.814 m/s
10. (a) 6.36 eV, 0.32 eV, 17.84 eV, 16.33 eV
(b) 11.67×10^{15} Hz, 9.84×10^{15} Hz, 1.83×10^{15} Hz
11. 6, 3 **12.** (a) 3.4 eV (b) 6.63 Å
13. $n = 2$, $Z = 4$, = (–217.6) eV, 10.58 eV
14. (a) $Z = 3$ (b) 4052.3 mm **15.** $z = 42$
16. 0.35 eV **17.** $n \approx 25$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. b. Shortest wavelength or cut-off wavelength depends only upon the voltage applied in the Coolidge tube.

2. a. The maximum number of electrons in an orbit is $2n^2$. Since $n > 4$ is not possible, therefore the maximum number of electrons that can be in the first four orbits are

$$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(3)^2 + 2(4) = 2 + 8 + 18 + 32 = 60$$

Therefore, possible elements are 60.

3. d. We know that

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] \Rightarrow \frac{1}{\lambda} \propto z^2$$

λ is shortest when $1/\lambda$ is largest, i.e., when z is big. z is highest for lithium.

4. c. For x-ray tube, $\lambda_{min} = \frac{hc}{eV}$ – where e = change on electron.

5. c. $E = \frac{hc}{\lambda} = \left[\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.021 \times 10^{-9}}\right]$ J

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.021 \times 10^{-9} \times 1.6 \times 10^{-13}} \text{ MeV}$$

$$= 591.96 \times 10^{-4} \text{ MeV} = 59.196 \text{ keV}$$

Therefore, (c) is the correct option.

6. d. For hydrogen and hydrogen-like atoms,

$$E_n = -13.6 \frac{(z)^2}{(n)^2} \text{ eV}$$

Therefore, ground state energy of doubly ionized lithium atom ($z = 3$, $n = 1$) will be

$$E_1 = (-13.6) \frac{(3)^2}{(2)^2} = -122.4 \text{ eV}$$

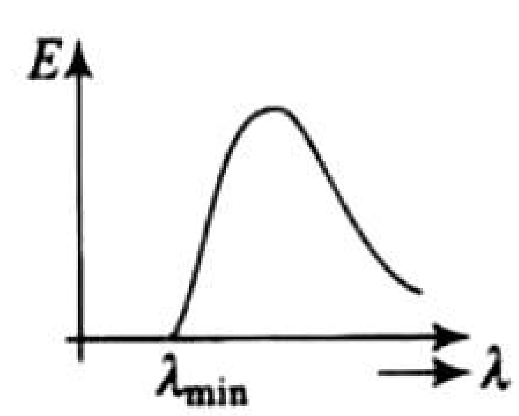
Therefore, ionization energy of an electron in ground state of doubly ionized lithium atom will be 122.4 eV.

7. b. The continuous X-ray spectrum is shown in the figure.

All wavelengths > λ_{min} are found

where $\lambda_{min} = \frac{12375}{V}$ Å

Here, V is the applied voltage.



8. c. We know that $\lambda \propto \dfrac{1}{m}$

For ordinary hydrogen atom,

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36} \quad \text{or} \quad \lambda = \frac{36}{5R}$$

With hypothetical particle, required wavelength

$$\lambda' = \frac{1}{2} \times \frac{36}{5R} = \frac{18}{5R}$$

9. a. We know that as the electron comes nearer to the nucleus, the potential energy decreases $\left(\dfrac{-KZe^2}{r} = \text{PE and} - r \text{ decreases}\right)$.

The KE will increase $\left[\because KE = \dfrac{1}{2}|PE| = \dfrac{1}{2}\dfrac{kZe^2}{r}\right]$.

The total energy decreases $\left[TE = -\dfrac{1}{2}\dfrac{kZe^2}{r}\right]$.

10. d. For continuous X-ray spectrum (Bremsstraglung),

$$\lambda_{min}(\text{in } \overset{o}{\text{A}}) = \frac{12375}{E(\text{in eV})}$$

$$\lambda_{min} = \frac{12375}{80 \times 10^3} \overset{o}{\text{A}}$$

$$\therefore \quad \lambda_{min} = 0.155 \overset{o}{\text{A}}$$

Again,

Energy of incident electron = 80 keV

Ionisation energy of K-shell electron = 72.5 keV

Since incident energy of electrons is greater than ionization energy of electrons in K-shell, the K-shell electrons will be knocked off.

Hence characteristic X-ray spectrum will be obtained.

Option (d) represents correct answer.

11. d. Method 1: Memorisation.

In Lyman series, we get energy in UV region.

In Balmer series, we get energy in visible region.

In Paschen/Brackett/Pfund series, we get energy in IR region.

Method 2:

We know that $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$ for hydrogen atom for e^- transition from $n_2 \to n_1$.

For 4 to 3, $E \propto \dfrac{1}{\lambda} = R\left(\dfrac{1}{9} - \dfrac{1}{16}\right) = \dfrac{7}{9 \times 16}R$

For 2 to 1, 3 to 2, and 4 to 2, we get more value of $1/\lambda$, i.e., more energy than $4 \to 3$.

IR radiation has less energy than UV radiation.

Therefore, the correct option is (d).

12. a. In case of coolidge tube,

$$\lambda_{min} = \frac{hc}{eV} = \lambda_c \text{ (as given here)}$$

Thus, the cut-off wavelength is inversely proportional to accelerating voltage. As V increases, λ_c decreases.

λ_k is the wavelength of K_α line which is a characteristic of an atom and does not depend on accelerating voltage of bombarding electron since λ_k always refers to a photon wavelength of transition of e^- from the target element from $2 \to 1$.

The above two facts lead to the conclusion that $\lambda_k - \lambda_c$ increases as accelerating voltage is increased.

13. a. Let n = number of electrons striking the target.

charge $q = ne$

$$\therefore \quad \text{Current} = \frac{\text{charge}}{\text{time}} = \frac{ne}{t} \therefore n = \frac{i \times t}{e}$$

$$n = \frac{(3.2 \times 10^{-3}) \times 1}{1.6 \times 10^{-19}} \text{ or } n = 2 \times 10^{16}$$

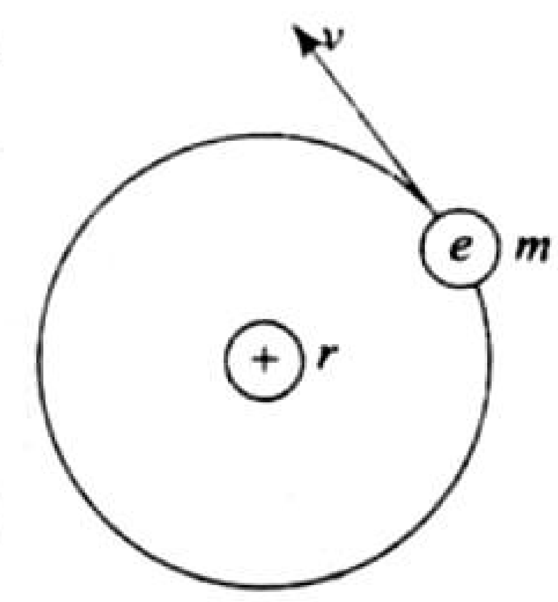
14. b. In the second excited state, $n = 3$

$$\therefore \quad l_H = l_{Li} = 3\left(\frac{h}{2\pi}\right)$$

$$Z_H = 1, Z_{Li} = 3, E \propto Z^2$$

$$\therefore \quad |E_{Li}| = 9|E_H| \text{ or } |E_H| < |E_{Li}|$$

15. a. Given potential energy between electron and proton is



$$V_0 \log_e \frac{r}{r_0}$$

$$\therefore |F| = \frac{d}{dr}\left[V_0 \log_e \frac{r}{r_0}\right] = \frac{V_0}{r_0} \times \frac{1}{r}$$

But this force acts as centripetal force

$$\therefore \quad \frac{mv^2}{r} = \frac{V_0}{rr_0} \Rightarrow mv^2 = \frac{V_0}{r_0} \quad \text{(i)}$$

$$\text{By Bohr's postulate, } mvr = \frac{nh}{2\pi} \quad \text{(ii)}$$

From (i) and (ii),

$$\frac{m^2v^2r^2}{mv^2} = \frac{n^2h^2r_0^2}{4\pi^2V_0^2} \Rightarrow r^2 = \frac{n^2h^2r_0^2}{4\pi V_0^2 m}$$

$$\Rightarrow \quad r \propto n$$

16. d. For an atom following Bohr's model, the radius is given by

$$r_m = \frac{r_0 m^2}{Z}$$

where r_0 = Bohr's radius and

m = orbit number.

For Fm, $m = 5$ (i.e. fifth orbit in which the outermost electron is present)

$$\therefore \quad r_m = \frac{r_0 5^2}{100} = 100n \text{ (given)} \quad \Rightarrow \quad n = \frac{1}{4}$$

17. a. According to Moseley's law,

$$\sqrt{f} = a(z - b) \Rightarrow f = a^2(z - b)^2$$

$$\Rightarrow \quad \frac{c}{\lambda} = a^2(z - b)^2 \quad \text{(i)}$$

For K_α line, $b = 1$

$$\text{From (i), } \frac{\lambda_2}{\lambda_1} = \frac{(z_1 - 1)^2}{(z_2 - 1)^2} \Rightarrow \frac{4\lambda}{\lambda} = \frac{(11 - 1)^2}{(z_2 - 1)^2}$$

$$\Rightarrow \quad Z_2 - 1 = \frac{10}{2} \Rightarrow z_2 = 6$$

Therefore, (a) is the correct option.

18. a. Initially, a photon of energy 10.2 eV collides inelastically with a hydrogen atom in ground state.

For hydrogen atom,

$$E_1 = -13.6\text{ eV};\ E_2 = -\frac{13.6}{4} = -3.4\text{ eV}$$

$$\therefore \quad E_2 - E_1 = 10.2\text{ eV}$$

The electron of hydrogen atom will jump to second orbit after absorbing the photon of energy 10.2 eV. The electron jumps back to its original state in less than a microsecond and releases a photon of energy 10.2 eV. Another photon of energy 15 eV strikes the hydrogen atom inelastically. This energy is sufficient to knock out the electron from the atom as ionization energy is 13.6 eV. The remaining energy of 1.4 eV is left with the electron as its kinetic energy.

19. b. The smallest frequency and largest wavelength in ultra-violet region will be for transition of electron from orbit 2 to orbit 1. Therefore,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{122 \times 10^{-9} m} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] - R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\Rightarrow \quad R = \frac{4}{3 \times 122 \times 10^{-9}}\ \text{m}^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron from ∞ to 3rd orbit.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{4}{3 \times 122 \times 10^{-9}}\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore \quad \lambda = \frac{3 \times 122 \times 9 \times 10^{-9}}{4} = 823.5\text{ nm}$$

20. a. The cut-off wavelength is given by

$$\lambda_0 = \frac{hc}{eV} \qquad \text{(i)}$$

According to de-Broglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda^2 = \frac{h^2}{2meV}$$

$$\Rightarrow \quad V = \frac{h^2}{2me\lambda^2} \qquad \text{(ii)}$$

From (i) and (ii), $\lambda_0 = \dfrac{hc \times 2me\lambda^2}{eh^2} = \dfrac{2mc\lambda^2}{h}$

21. b. Cut-off wavelength depends on the applied voltage and not on the atomic number of the target. Characteristic wavelengths depend on the atomic number of the target.

22. b.

$$\frac{1}{\lambda_{H_2}} = RZ_H^2\left[\frac{1}{4} - \frac{1}{9}\right] = R(1)^2\left[\frac{5}{36}\right]$$

$$\frac{1}{\lambda_{He}} = RZ_{He}^2\left[\frac{1}{4} - \frac{1}{16}\right] = R(4)^2\left[\frac{3}{16}\right]$$

$$\frac{\lambda_{H_2}}{\lambda_{He}} = \frac{1}{4}\left[\frac{16}{3} \times \frac{5}{36}\right] = \frac{5}{27}$$

$$\lambda_{He} = \frac{5}{27} \times 6561 = 1215\text{ Å}$$

Multiple Correct Answers Type

1. a., c., d.

a. $r_n \propto n^2$. Option (a) is correct

b. Total energy of electron T.E.

$$\text{T.E.} = \frac{-13.6Z^2}{n^2}$$

Option (b) is not correct

c. Angular momentum of electron $nh/2\pi$

Option (c) is correct

d. Potential energy of electron = $(-27.2/n^2)$ eV for hydrogen atom.

$$\therefore \quad |\text{P.E.}| = \frac{27.2}{n^2}$$

Kinetic energy of electron = $13.6/n^2$. It is always positive.

$\therefore \quad |\text{P.E.}| = 2 \times |\text{K.E.}|$

The option (d) is correct.

2. c., d.

In the case of hydrogen,

atomic number = mass number.

In other atoms, atomic number < mass number.

Therefore, (c) and (d) are the correct options.

3. c., d.

For X- ray tube, $\lambda_m(\text{in Å}) = \dfrac{12375}{V}$

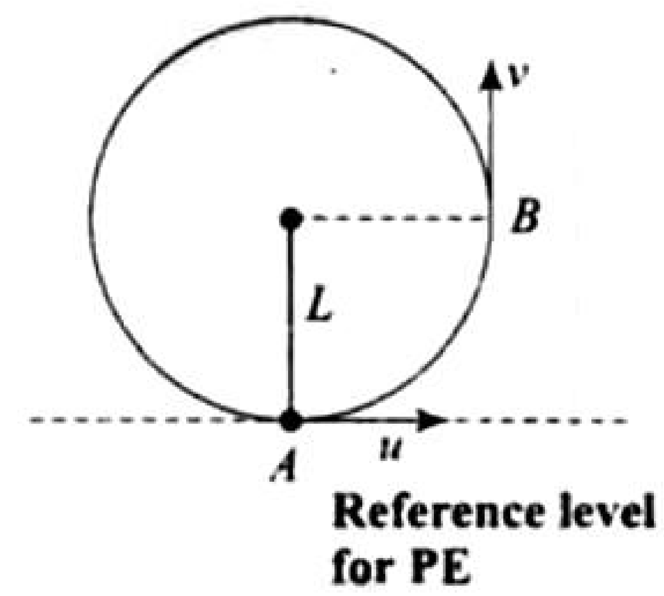
As accelerating voltage is increased, λ_m will decrease.

Number of electron bombarding the target determine the intensity (or quantity) of emitted radiation. Acceleration voltage does not change the intensity of X-ray emitted.

4. a., d.

The time period of the electron in a Bohr orbit is given by $T = 2\pi r/v$

Since for the n^{th} Bohr orbit, $mvr = n(h/2\pi)$, the time period becomes



$$T = \frac{2\pi r}{nh/(2\pi mr)} = \left(\frac{4\pi^2 m}{nh}\right)r^2$$

Since the radius of the orbit r depends on n, we replace r.

The expression of Bohr radius of a hydrogen atom is $r = n^2\left(\dfrac{h^2\varepsilon_0}{\pi me^2}\right)$

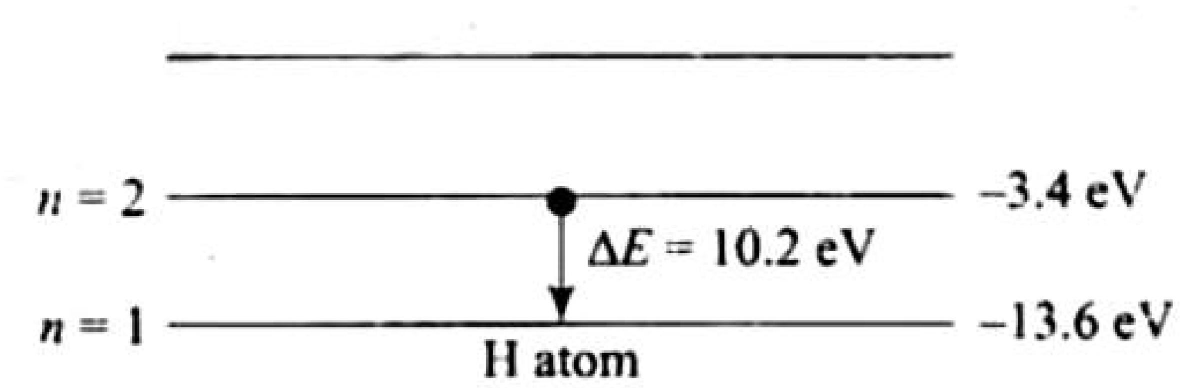
Hence, $T = \left(\dfrac{4\pi^2 m}{nh}\right)\left(\dfrac{n^4h^4\varepsilon_0^2}{\pi^2m^2e^4}\right) = n^3\left(\dfrac{4h^3\varepsilon_0^2}{me^4}\right)$

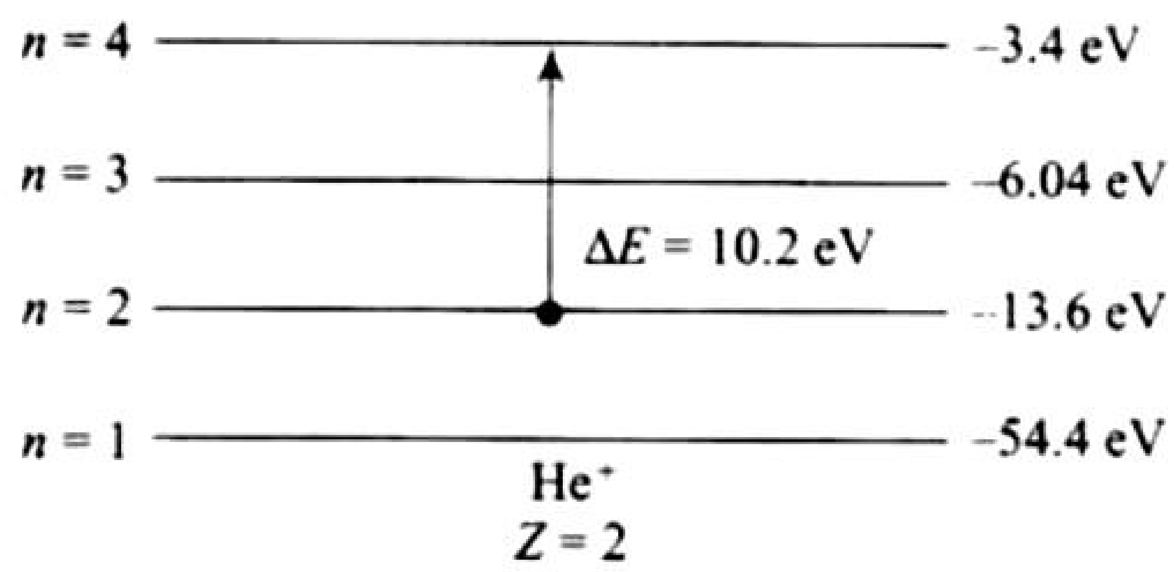
For two orbits, $\dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3$

It is given that $T_1/T_2 = 8$. Hence, $n_1/n_2 = 2$.

Linked Comprehension Type

1. c.





Energy given by H atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.

2. c. Visible light lies in the range, $\lambda_1 = 4000$ Å to $\lambda_2 = 7000$ Å. Energy of photons corresponding to these wavelengths (in eV) would be:

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV}, \quad \text{and}$$

$$E_2 = \frac{900}{11R} = 1.77 \text{ eV}$$

From energy level diagram of He^+ atom, we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04) = 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{264} \text{ Å} = 4687.5 \text{ Å} = 4.68 \times 10^{-7} \text{ m}$$

3. a. Kinetic energy, $K \propto Z^2$

$$\frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

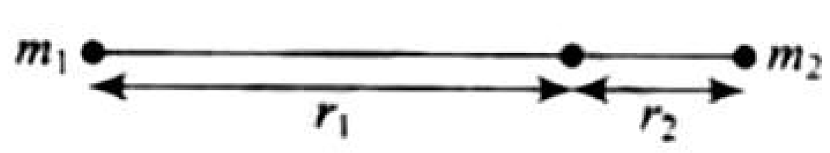
4. d. $L = \frac{nh}{2\pi}$

$$\text{K.E.} = \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I}$$

5. a. $hv = KE_{n=2} - kE_{n=1}$

$I = 1.87 \times 10^{-46}$ kg m^2

6. c. $r_1 = \frac{m_2 d}{m_1 + m_2}$ and $r_2 = \frac{m_1 d}{m_1 + m_2}$



$$I = m_1 r_1^2 + m_2 r_2^2$$

∴ $d = 1.3 \times 10^{-10}$ m

Assertion-Reasoning Type

1. b. Cut-off wavelength depends on the accelerating voltage, not on the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilized in producing X-rays. Rest 98% is lost in heat.
Therefore, option (b) is correct.

Fill in the Blanks Type

1. For minimum accelerating voltage, the electron should jump from $n = 2$ to $n = 1$ level.

For characteristic X-ray, $\frac{1}{\lambda} = R_\alpha (z-1)^2 \left[1 = \frac{1}{n^2}\right] = R_\alpha (z-1)^2$

But $E = h\frac{c}{\lambda} \Rightarrow \frac{1}{\lambda} = \frac{E}{hc}$

$$\therefore \quad \frac{E}{hc} = R_\alpha = \frac{E}{hc}$$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{\left[1 - \frac{1}{2^2}\right]}{\left[1 - \frac{1}{\alpha^2}\right]}$$

$$\Rightarrow \quad E_1 = \frac{3}{4} E_2 = \frac{3}{4} 40{,}000 \text{ eV} = 30{,}000 \text{ eV}$$

2. intensity, decrease
Cut-off wavelength is given by

$$\lambda = \left\{\frac{12375}{V(\text{in volts})}\right\} (\textit{in Å})$$

with increase in applied voltage V, speed of electrons striking the anode is increased or cut-off wavelength of the emitted X-rays decreases.
Further, with increase in number of electrons striking the anode more number of photons of X-rays will be emitted. Therefore, intensity of X-rays will increase.

3. For K_α, $\frac{1}{\lambda} = c\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\Rightarrow \quad \frac{1}{0.32} = c\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3c}{4} \qquad \text{(i)}$$

For K_β, $1 = c\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8c}{9}$ (ii)

On dividing, we get $\lambda = 0.27$

4. The fifth valance electron of phosphorous is in its third shell, i.e., $n = 3$. For phosphorous, $Z = 15$. Also, the Bohr's radius for n^{th} orbit

$$r_n = \left(\frac{n^2}{Z}\varepsilon r\right) r_0 = \frac{3^2}{15} \times 12 \times 0.529 \text{ Å} = 3.81 \text{ Å}$$

5. The speed of X-rays is always 3×10^8 ms^{-1} in vacuum. It does not depend on the potential differences through which electrons are accelerated in an X-ray tube.

6. $\text{KE} = \frac{KZe^2}{2r}$ and

Total energy, $\text{TE} = \frac{-KZe^2}{2r} \Rightarrow \frac{|KE|}{|TE|} = 1$

7. $\frac{1}{\lambda} = R(z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Since for K_α, $n_2 = 2$ and $n_1 = 1$

$$\therefore \quad \frac{1}{0.76 \times 10^{-10}} = 1.097(z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \quad z - 1 = 40 \Rightarrow z = 41$$

8. For photon emitted from hydrogen atom, the wavelength is

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{(i)}$$

But according to de Broglie concept

$$\lambda = \frac{h}{p} \Rightarrow \frac{1}{\lambda} = \frac{p}{h} \qquad \text{(ii)}$$

From (i) and (ii),

$$\frac{p}{n} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow p = Rh\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{(iii)}$$

Since the momentum of the hydrogen atom initially was zero, therefore finally the momentum of photon is equal to momentum of hydrogen atom in magnitude (By law of conservation of momentum). Let the momentum of hydrogen atom be $m_H V_H$. Then, from (iii)

$$m_H V_H = R_h\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad V_H = \frac{Rh}{m_H}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= \frac{1.097 \times 10^7 \times 6.63 \times 10^{-34}}{1.67 \times 10^{-27}}\left(\frac{1}{1^2} - \frac{1}{5^2}\right)$$

$$\Rightarrow \quad V_H = 4.178 \text{ m s}^{-1}$$

Alternatively:

Energy of photon

$$= E = E_5 - E_1 = -13.6\left[\frac{1}{5^2} - \frac{1}{2^2}\right] \text{ eV}$$

$$= 2.09 \times 10^{-18} \text{ J}$$

According to momentum conservation,

Momentum of recoil hydrogen atom = Momentum of photon

$$\therefore \quad mv = \frac{E}{c}$$

$$\Rightarrow \quad v = \frac{E}{mc} = \frac{2.09 \times 10^{-18}}{(1.67 \times 10^{-27})(3 \times 10^8)} = 4.17 \text{ m s}^{-1}$$

Subjective Type

1. The energy of the electron in nth orbit of hydrogen-like atom is given by

$$E_n = -\frac{Z^2 Rhc}{n^2} \qquad \text{(i)}$$

a. The energy required to excite the electron from second ($n = 2$) Bohr orbit to the third ($n = 3$) Bohr orbit is given by

$$\Delta E = E_3 - E_2 = -\frac{Z^2 Rhc}{3^2} - \left(-\frac{Z^2 Rhc}{3^2}\right)$$

$$= Z^2 Rhc\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36} Z^2 Rhc$$

Given, ionization energy of hydrogen atom

$= Rhc = 13.6$ eV and $\Delta E = E_3 - E_2 = 47.2$ eV

Thus, we have $47.2 \text{ eV} = \frac{5}{36} Z^2 \times 13.6$ eV

or $\quad Z^2 = \frac{36}{5} \times \frac{47.2}{13.6} = 25 \Rightarrow Z = 5$

b. The energy required to excite the atom from the third to fourth orbit is given by

$$E_4 - E_3 = -\frac{Z^2 Rhc}{4^2} - \left(-\frac{Z^2 Rhc}{3^2}\right)$$

$$= Z^2 Rhc\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 5^2 \times (13.6 \text{ eV}) \times \frac{7}{144} = 16.53 \text{ eV}$$

c. The ionization energy of atom or the energy required to remove the electron from the first Bohr orbit to ∞ is

$$E_\infty - E_1 = -\frac{Z^2 Rhc}{(\infty)^2} - \left(-\frac{Z^2 Rhc}{1^2}\right) = Z^2 Rhc$$

If λ is the wavelength of corresponding electromagnetic radiation, then

$$\frac{hc}{\lambda} = Z^2 Rhc$$

i.e., $\quad \frac{hc}{\lambda} = 5^2 \times 13.6 \text{ eV} = 340 \text{ eV}$

$$\therefore \quad \lambda = \frac{hc}{340 \text{ eV}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{340 \times 1.6 \times 10^{-19}}$$

$$= 36.056 \times 10^{-10} \text{ m} = 36.056 \text{ Å}$$

d. Kinetic energy of electron in the first Bohr's orbit,

$$E_{K_1} = -E_1 = 340 \text{ eV}$$

Potential energy of electron in the first Bohr's orbit,

$$U_1 = 2E_1 = -2 \times 340 \text{ V} = -680 \text{ eV}$$

Angular momentum of electron in the first Bohr's orbit

$$= n\frac{h}{2\pi} = 1\frac{h}{2\pi} = \frac{6.63 \times 10^{-34}}{2\pi}$$

$$= 1.0546 \times 10^{-34} \text{ J s}$$

e. Radius of the first Bohr orbit for given atom

$$= \left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right)_{n=1} = \frac{(\varepsilon_0 h^2 / \pi m e^2)}{Z}$$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z}$$

$$= \frac{5.3 \times 10^{-11}}{5} \text{m} = 1.06 \times 10^{-11} \text{ m}$$

2. The energy of an electron in nth orbit of hydrogen ($Z = 1$) is given by

$$E_n = \frac{Rhc}{n^2}$$

$\therefore$ Ionization energy $= E_\infty - E_1 = Rhc = 13.6$ eV

$$\therefore \quad E_n = -\frac{13.6}{n^2} \text{eV}$$

Therefore, energy of electron in ground state ($n = 1$),

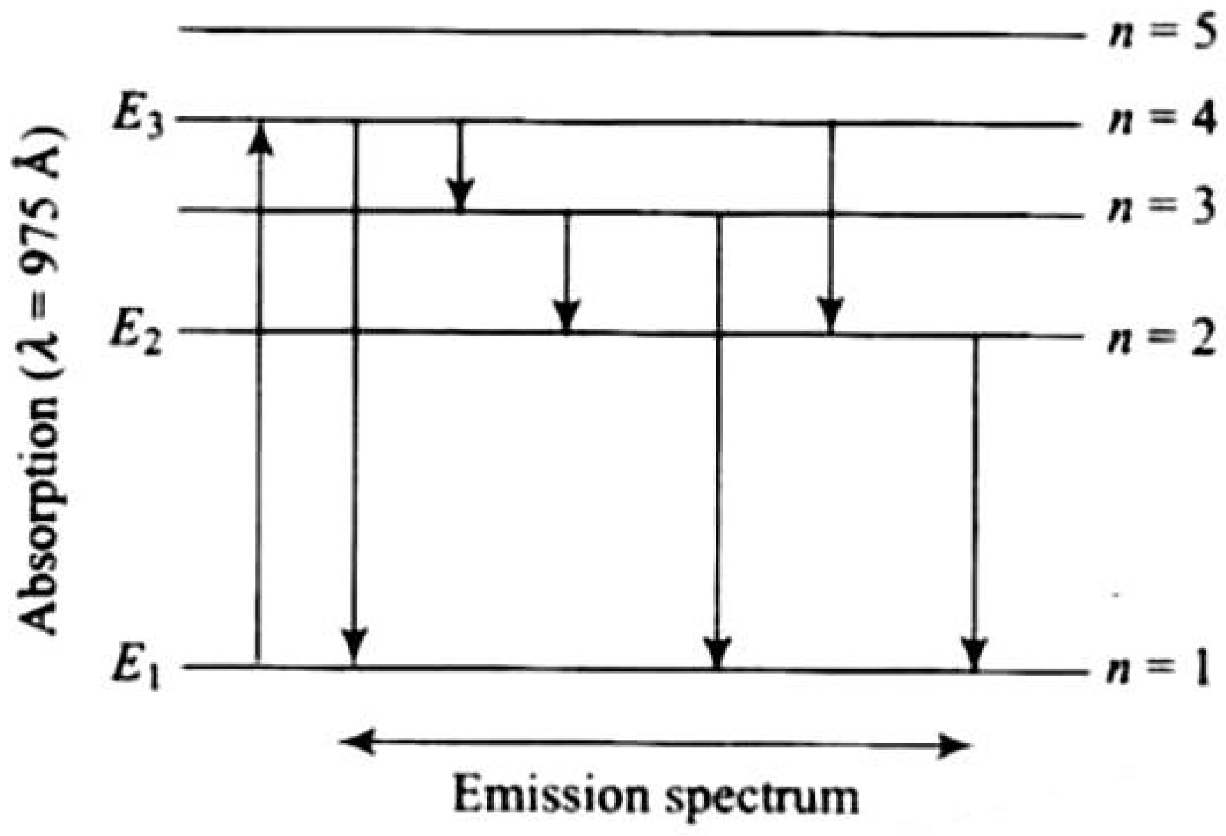
$$E_1 = -\frac{13.6}{1} = -13.6 \text{ eV}$$

Energy of electron in the first excited state ($n = 2$),

$$E_2 = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4 \text{ eV}$$

The energy of electron in the second excited state ($n = 3$),

$$E_3 = -\frac{13.6}{3^3} = \frac{13.6}{9} = -1.511 \text{ eV}$$



The energy of electron in the third excited state ($n = 4$),

$$E_4 = -\frac{13.6}{4^2} = -\frac{13.6}{16} = -0.85 \text{ eV}$$

Energy of incident photon of wavelength

$\lambda = 975 \text{ Å} = 975 \times 10^{-10}$ m

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{975\times10^{-10}} \text{ J}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{975\times10^{-10}\times1.6\times10^{-19}} \text{ eV} = 12.75 \text{ eV}$$

When the incident photon of this energy is absorbed by hydrogen atom, let its ground state electron occupy $(n-1)$th excited state or nth orbit. Then,

$$E = -\frac{Rhc}{n^2} - \left(-\frac{Rhc}{1^2}\right) = Rhc\left(1-\frac{1}{n^2}\right)$$

i.e., $12.75 \text{ eV} = 13.6 \text{ eV}\left(1-\frac{1}{n^2}\right)$

This gives $n = 4$.

That is the electron is excited to the third excited state. The emission spectrum will contain the transitions shown in the figure. The longest wavelength emitted corresponds to transition $(4 \to 3)$ for which the energy difference is minimum.

i.e., $E_{min} = E_4 - E_3 = -0.85 - (-1.511)$ eV

$= 1.511 - 0.85 = 0.661$ eV

$= 0.661 \times 1.6 \times 10^{-19}$ J

$$\therefore \quad \lambda_{max} = \frac{hc}{E_{min}} = \frac{6.63\times10^{-34}\times3\times10^{8}}{0.661\times1.6\times10^{-19}} = 18.807 \times 10^{-7} \text{ m}$$

$= 18807$ Å

3. We know the energy of a photon of wavelength λ is given by $E = \frac{hc}{\lambda}$

We can write the energies of radiations of wavelengths λ_1 and λ_2 are

$$E_1 = \frac{hc}{\lambda_1} \text{ and } E_2 = \frac{hc}{\lambda_2} \text{ – respectively}$$

$$E_1 - E_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \qquad \text{(i)}$$

We are given

$E_1 - E_2 = (4.0 - 1.8) = 2.2 \text{ eV} = 2.2 \times 1.6 \times 10^{-19}$ J,

$\lambda_1 = 700 \text{ Å} = 700 \times 10^{-10}$ m

and $\lambda_2 = 830 \text{ Å} = 830 \times 10^{-10}$ m. Also $c = 3 \times 10^8 \text{ ms}^{-1}$. Using these values of (i), we get $h = 6.57 \times 10^{-34}$ Js.

4. The energy of electron in hydrogen-like atoms in nth orbit is

$$E_n = \frac{Z^2 Rhc}{n^2}$$

We have $Rhc = 1$ rydberg.

The ionization energy

$E_\infty - E_1 = Z^2 Rhc = 4$ rydberg

$$\therefore \quad Z^2 = \frac{4 \text{ rydberg}}{Rhc} = \frac{4 \text{ rydberg}}{1 \text{ rydberg}} = 4$$

$\therefore \quad Z = 2$

a. The energy required to excite the electron from $n = 1$ to $n = 2$ is given by

$$E_2 - E_1 = -\frac{Z^2 Rhc}{2^2} - \left(-\frac{Z^2 Rhc}{1^2}\right)$$

$$= Z^2 Rhc\left(1-\frac{1}{4}\right)$$

$$= \frac{3}{4} Z^2 Rhc = \frac{3}{4} \times 4 \text{ Rydberg}$$

$= 3$ Rydberg

If λ is the wavelength of radiation emitted, then

$$\frac{hc}{\lambda} = 3 \text{ Rydberg, i.e., } \lambda = \frac{hc}{(3 \text{ Rydberg})}$$

$$\therefore \quad \lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{3\times2.2\times10^{-18}}$$

$= 301.4 \times 10^{-10} \text{ m} = 301.4$ Å

b. Radius of first Bohr orbit, $r_1 = \frac{(\varepsilon_0 h^2/\pi m e^2)}{Z}$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z} = \frac{5\times10^{-11}}{2}$$

$= 2.5 \times 10^{-11}$ m

5. The energy of electron in nth orbit of hydrogen-like atoms is

$$E_n = -\frac{Z^2 Rhc}{n^2}$$

Here, $Z = 3$

and $Rhc = 13.6$ eV

a. The energy required to excite the electron in Li^{++} from $n = 1$ to $n = 3$ is

$$E_3 - E_1 = -\frac{Z^2 Rhc}{3^2} - \left(-\frac{Z^2 Rhc}{1^2}\right)$$

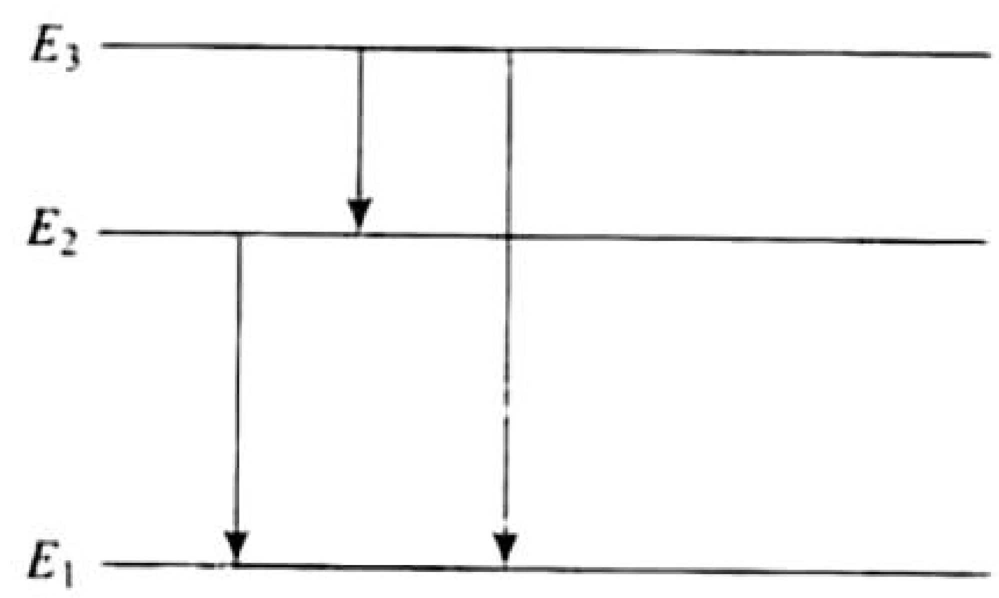
$$= Z^2 Rhc\left(1-\frac{1}{9}\right)$$

$$= \frac{8}{9} Z^2 Rhc = \frac{8}{9} \times (3)^2 \times 13.6 \text{ eV}$$

$= 8 \times 13.6 \text{ eV} = 8 \times 13.6 \times 1.6 \times 10^{-19}$ J

Therefore, the wavelength of incident radiation required for excitation is given by

$$\frac{hc}{\lambda} = E_3 - E_1$$



or $$\lambda = \frac{hc}{E_3 - E_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{8 \times 13.6 \times 1.6 \times 10^{-19}}$$

$$= 114.26 \times 10^{-10}\ \text{m} = 114.26\ \text{Å}$$

b. The possible lines in the emission spectrum of this excited system are $\left[\frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3\right]$

Three in number, represented in the above figure.

6. a. We have radius of nth orbit of a hydrogen atom as

$$r_n = \frac{n^2h^2}{4\pi^2 Kze^2 m}$$

If electron is replaced by a heavy particle of mass 208 times that of the electron, then the radius is given as

$$r_n = \frac{n^2h^2}{4\pi^2 K(3)e^2(208m)} = \frac{n^2h^2}{2496\pi^2 Kze^2 m} \qquad \text{(i)}$$

Here, we have not used reduced mass because it is given that mass of nucleus is assumed to be infinite and here we take $z = 3$.

b. The radius of first Bohr orbit is given as

$$r_1 = \frac{h^2}{4\pi^2 Ke^2 m}$$

From Eq. (i), we have

$$\frac{n^2h^2}{2496\pi^2 Ke^2 m} = \frac{h^2}{4\pi^2 Ke^2 m}$$

or $n^2 = 624$

or $n = 25$

c. Rydberg constant for hydrogen-like atom is

$$R = \frac{2\pi^2 K^2 e^4 m}{ch^3}$$

Now, when electron is replaced by μ-meson, Rydberg constant will change as

$$R' = \frac{2\pi^2 K^2 e^4 (208m)}{ch^3}$$

or $= 208R = 208 \times 1967800\ \text{m}^{-1}$

Now, the wavelength λ of the radiation emitted when μ-meson makes a transition from $n_2 = 3$ to $n = 1$ is

$$\frac{1}{\lambda} = R'Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\frac{1}{\lambda} = R' \times 9\left[\frac{1}{1^2} - \frac{1}{3^2}\right] \quad \text{or} \quad \frac{1}{\lambda} = 8R'$$

or $$\lambda = \frac{1}{8R'}$$

$$\lambda = \frac{1}{8 \times 208 \times 10967800}\ \text{m}^{-1}$$

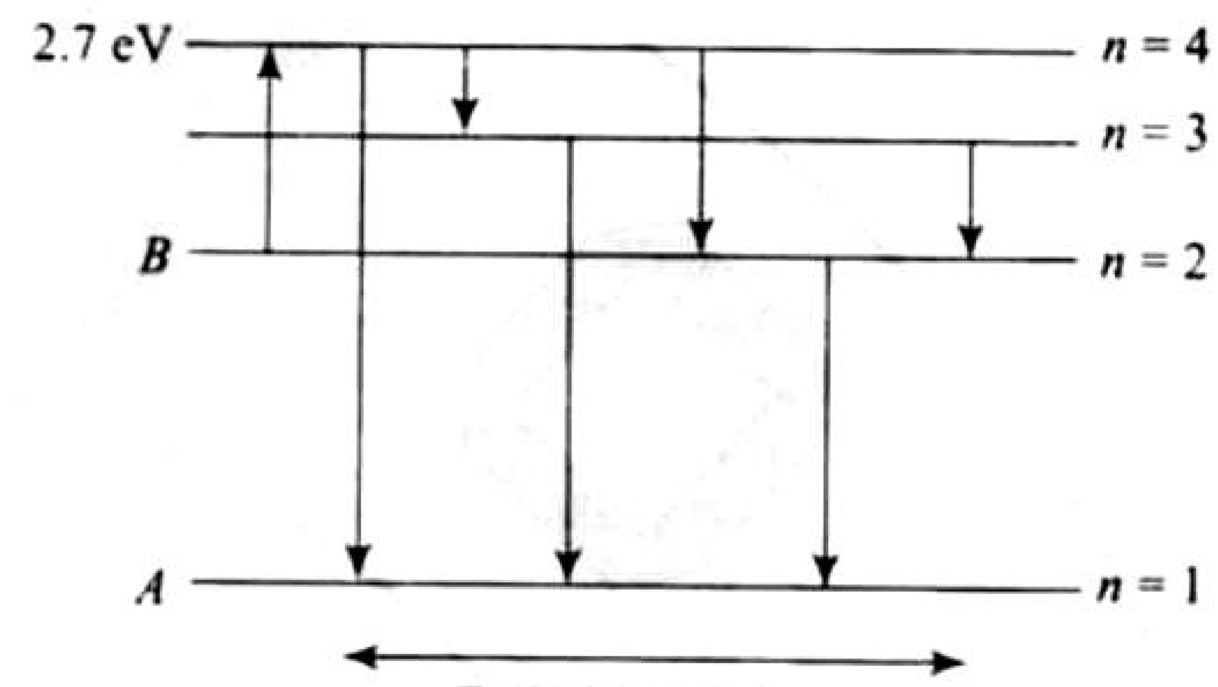
or $\lambda = 0.548$ Å

7. The energy levels of identical hydrogen-like atoms are given by

$$E_n = -\frac{B}{n^2},\ B \text{ being a constant.}$$

a. When a hydrogen-like atom absorbs energy 2.7 eV. This is only possible if transition of electron is from state B to any higher energy state. Six radiations are emitted only if the final state is $n = 4$.

The energy of emitted radiation is also more than 2.7 eV, therefore, initial state cannot be $n = 1$. As energy of emitted radiation is also less than 2.7 eV. This shows that initial excited state cannot be $n = 3$. Hence, the only possibility is that initially excited state B has $n = 2$.



That is, the principal quantum number of initially excited state B is $n = 2$.

b. Given $E_4 - E_2 = 2.7$ eV

$$\therefore \quad -\frac{B}{4^2} - \left(-\frac{B}{2^2}\right) = 2.7\ \text{eV}$$

Therefore, this gives $B = 14.4$ eV

The transition of electron should be from ground state ($n = 1$) to $n = \infty$.

Ionization energy of identical hydrogen-like gas atoms,

$$\Delta E = E_\infty - E_1 = -\frac{B}{\infty^2} - \left(-\frac{B}{1^2}\right)$$

c. Maximum energy of emitted photon is obtained when transition of electron is from $n = 4$ to $n = 1$.

$$\therefore \quad E_{max} = -\frac{B}{4^2} - \left(-\frac{B}{1^2}\right)$$

$$= B\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 14.4\left(1 - \frac{1}{16}\right)\text{eV} = 13.65\ \text{eV}$$

Minimum energy of emitted photon is obtained when electron jumps from $n = 4$ to $n = 3$.

i.e., $$E_{min} = B\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 14.4\left(\frac{1}{9} - \frac{1}{16}\right) = 0.4\ \text{eV}$$

8. Energy of a photon corresponding to transition from $n = 5$ (fourth excited state) to $n = 4$ (third excited state) is

$$h\nu = 13.6(3)^2\left[\frac{1}{4^2} - \frac{1}{5^2}\right] = 2.75\ \text{eV} \qquad \text{(i)}$$

Similarly, energy of a photon corresponding to transition from $n = 4$ to $n = 3$ is

$$h\nu = 13.6(3)^2\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = 5.95\ \text{eV} \qquad \text{(ii)}$$

From Einstein's photoelectric effect equation.

$h\nu = \phi + KE_{max}$

$KE_{max} = eV_s = h\nu - \phi$

The shorter wavelength corresponds to greater energy difference between energy levels involved in the transition. So, shorter wavelength photons are emitted for transition $n = 4$ to $n = 3$. Thus, we have

$3.95 = 5.95 - \phi \Rightarrow \phi = 2$ eV

and for longer wavelength photon, $eV_s = 2.75 - 2 = 0.75$ eV

So, stopping potential $= \left(\frac{0.75\,\text{eV}}{e}\right) = 0.75$ V

9. $E_k = 0.73$ eV, $W = 1.82$ eV

Ionization energy of H atom = 13.6 eV

a. $h\nu = W + E_k = 1.82\text{ eV} + 0.73\text{ eV} = 2.55$ eV

b. The electronic energy levels of H-atoms are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2}\text{ eV}$$

For $n = 1$, $E_1 = -13.6$ eV

For $n = 2$, $E_2 = -3.4$ eV

For $n = 3$, $E_3 = -1.51$ eV

For $n = 4$, $E_4 = -0.85$ eV

Clearly, $E_4 - E_2 = -0.85\text{ eV} - (-3.4\text{ eV}) = 2.55$ eV

i.e., Quantum numbers involved in the photon of energy 2.55 eV are 2 and 4. The transition is specified by $n_1 = 4 \rightarrow n_2 = 2$.

c. The angular momentum of electron in H atom

$$J = n\frac{h}{2\pi}$$

For $n = 4$, $J_1 = 4\frac{h}{2\pi} = \frac{2h}{\pi}$

For $n = 2$, $J_2 = 2\frac{h}{2\pi} = \frac{h}{\pi}$

Therefore, change in angular momentum,

$$\Delta J = J_1 - J_2 = \frac{2h}{\pi} - \frac{h}{\pi} = \frac{h}{\pi}$$

d. According to conservation of momentum,

$$\frac{h\nu}{c} + mv = 0$$

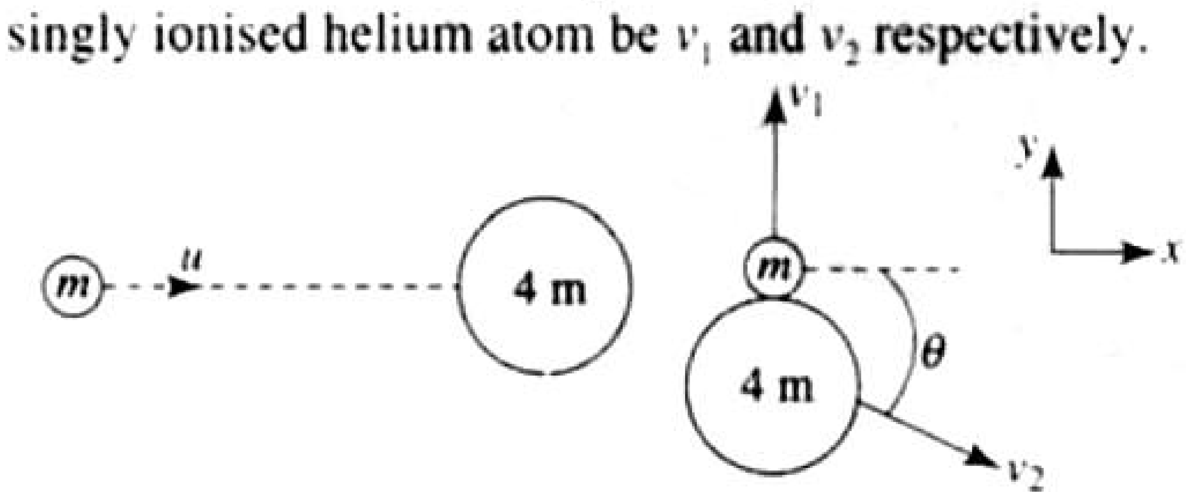
$$\therefore \quad v = -\frac{h\nu}{cm} = -\frac{2.55}{(3\times10^8)\times(1.67\times10^{-27})}$$

$$= -\frac{2.55\times1.6\times10^{-19}}{3\times10^8\times1.67\times10^{-27}}$$

$$= -0.814\text{ ms}^{-1}$$

Therefore, recoil speed of H atom = 0.814 ms^{-1}.

10. Here neutron collides inelastically with a singly ionised helium atom at rest. Let the final speeds after collision of neutron and singly ionised helium atom be v_1 and v_2 respectively.



From conservation of momentum, we have

Along x-axis: $mu = 4\,mv_2\cos\theta$ (i)

Along y-axis: $mv_1 = 4\,mv_2\sin\theta$ (ii)

On squaring and adding equations (i) and (ii), we get $u^2 + v_1^2 = 16v_2^2$

$$\frac{1}{2}mu^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}16mv_2^2 \quad \text{(iii)}$$

The initial kinetic energy of neutron = 65 eV.

Let kinetic energy of neutron and helium atom be

$$K_N = \frac{1}{2}mv_1^2,\ K_{He} = \frac{1}{2}(4m)v_2^2$$

So equation (iii) reduces to $65 + K_N = 4K_{He}$

$$4K_{He} - K_N = 65 \quad \text{(iv)}$$

a. Energy required to excited an electron from ground state of singly ionised helium atom (He$^+$) to nth energy level is $u^2 + v_1^2 = 16v_2^2$

If the neutron has sufficient energy to excite the helium atom, then from conservation of energy, the energy of neutron must equal to the sum of kinetic energy of neutron, helium atom and excitation energy. So, we have

$$65 = K_N + K_{He} + 54.4\left(1 = \frac{1}{n^2}\right)$$

$$K_{He} + K_N = 10.6 = \frac{54.4}{n^2} \quad \text{(v)}$$

On solving equations (iv) and (v), we get

$$K_{He} = \left(15.52 + \frac{10.88}{n^2}\right)\text{eV} \quad \text{(vi)}$$

$$K_N = \left(\frac{43.32}{n^2} - 4.52\right)\text{eV} \quad \text{(vii)}$$

The kinetic energy is always positive, so from equation (vii) we have

$$\frac{43.52}{n^2} > 4.52\ n < 3.1$$

So the only possible values of n are 2 and 3.

Possible values of n

2, 3

Allowed values of neutron energy

K_N

6.36 eV, 0.32 eV

Allowed values of He- atom energy

K_{He}

17.84 eV, 16.33 eV

b. When the atom deexcites,

$$v = \frac{(13.6)2^2\times1.6\times10^{-19}}{6.63\times10^{-34}}\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right]$$

$$= 13.13\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right]\times10^{15}\text{ Hz}$$

The electron excited to $n = 3$ can make three transitions:

$n = 3$ to $n = 1$, $v_1 = 11.67\times10^{15}$ Hz

$n = 3$ to $n = 2$, $v_2 = 9.84\times10^{15}$ Hz

$n = 2$ to $n = 1$, $v_3 = 1.83\times10^{15}$ Hz

11. For the transition from a higher state n to the first excited state $n_1 = 2$, the total energy released is

10.2 + 17.0 eV or 27.2 eV.

Thus, for $\Delta E = 27.2$ eV, $n_1 = 2$, and $n_2 = n$, we have

$$27.2 = 13.6\,Z^2\left[\frac{1}{4} - \frac{1}{n^2}\right] \quad \text{(i)}$$

For the eventual transition to the second excited state $n_1 = 3$, the total energy released is (4.25 + 5.95) eV or 10.2 eV. Thus,

$$10.2 = 13.6\,Z^2\left[\frac{1}{9} - \frac{1}{n^2}\right] \quad \text{(ii)}$$

Dividing Eqs. (i) and (ii), we get $\frac{27.2}{10.2} = \frac{9n^2 - 36}{4n^2 - 36}$

Solving, we get $n^2 = 36$ or $n = 6$

Substituting $n = 6$ in any of the above equations, we obtain

$Z^2 = 9$ or $Z = 3$

Thus, $n = 6$ and $Z = 3$

12. a. Energy of electron in hydrogen-like atom,

$$E_n = -\frac{Z^2 Rhc}{n^2} = -3.4 \text{ eV}$$

Kinetic energy of electron in hydrogen-like atom is equal to negative of total energy.

i.e., $E_k = -E_n = -(-3.4 \text{ eV}) = +3.4 \text{ eV}$

b. The de Broglie wavelength of electron,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{\{2\times9.1\times10^{-31}\times3.4\times1.6\times10^{-19}\}}}$$

$= 6.66$ Å

13. Let ground state energy (in eV) be E_1 Then, from the given condition $E_{2n} - E_0 = 204$ eV

We can write $\frac{E_0}{4n^2} - E_0 = 204$ eV

$$E_0\left(\frac{1}{4n^2} - 1\right) = 204 \text{ eV} \quad \text{(i)}$$

Also, $E_{2n} - E_n = 40.8$ eV

$$\frac{E_0}{4n^2} - \frac{E_0}{n^2} = 40.8 \text{ eV}$$

$$E_0\left(\frac{-3}{4n^2}\right) = 40.8 \text{ eV} \quad \text{(ii)}$$

From equations (i) and (ii)

$$\frac{\left(1 - \frac{1}{4n^2}\right)}{\frac{3}{4n^2}} = 5$$

$$1 - \frac{1}{4n^2} + \frac{15}{4n^2} \Rightarrow n = 2$$

From equation (ii), $E_0 = -\frac{4}{3}n^2(40.8)\text{eV}$

$$= -\frac{4}{3}2^2(40.8)\text{eV}$$

Which gives $E_0 = -217.6$ eV

also $E_0 = -(13.6)Z^2$

$$\Rightarrow \quad Z^2 = \frac{E_1}{-13.6} = \frac{-217.6}{-13.6} = 16$$

Hence $Z = 4$

For minimum energy

$$E_{\min} = E_{2n} - E_{2n-1}$$

$$\frac{E_0}{4n^2} - \frac{E_0}{(2n-1)^2} = \frac{E_0}{16} - \frac{E_0}{9} = -\frac{7}{144}E_0$$

$$= -\left(\frac{7}{144}\right)(-217.6)\text{ eV}$$

which gives $E_{\min} = 10.58$ eV

14. a. As total 6 lines are emitted. Therefore,

$$\frac{n(n-1)}{2} = 6 \quad \text{or} \quad n = 4$$

So, transition is taking place between m^{th} energy state and $(m+3)^{\text{th}}$ energy state.

$$E_m = -0.85 \text{ eV}$$

$$-13.6\left(\frac{z^2}{m^2}\right) = -0.85$$

$$\frac{Z}{m} = 0.25 \quad \text{(i)}$$

Similarly, $E_{m+3} = -0.544$ eV

$$-13.6\frac{Z^2}{(m+3)^2} = -0.544$$

$$\frac{Z}{(m+3)} = 0.2 \quad \text{(ii)}$$

Solving equations (i) and (ii) for z and m, we get

$m = 12$ and $Z = 3$

b. For smallest wavelength corresponds to maximum difference of energies which is obviously

$$E_{\max} = E_{m+3} - E_m$$

$$\Delta E_{\max} = -0.544 - (-0.85) = 0.306 \text{ eV}$$

$$\lambda_{\min} = \frac{hc}{\Delta E_{\max}} = \frac{1240}{0.306} = 4052.3 \text{ mm}$$

15. According to Bohr's model, the energy released during transition from n_2 to n_1 is given by

$$\Delta E = h\nu = Rhc(z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For transition from L shell to K shell

$b = 1, n_2 = 2, n_1 = 1$

$$(z-1)^2 Rhc\left[\frac{1}{1} - \frac{1}{4}\right] = h\nu$$

On putting the value of $R = 1.1\times10^7\ \text{m}^{-1}$ (given),

$c = 3\times10^8$ m/s

$\Rightarrow \quad z = 42$

16. We can write the Balmer series of hydrogen atom

$$\frac{hc}{\lambda} = (13.6\text{ eV})\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\text{For } n = 3\ \lambda_1 = \frac{hc}{(13.6\text{ eV})\{(1/2^2) - (1/3^2)\}}$$

$$= \frac{(1242\text{ eV nm})}{(13.6\text{ eV})(5/36)} = 657.5\text{ nm}$$

$$\text{For } n = 4\ \lambda_2 = \frac{hc}{(13.6\text{ eV})\{(1/2^2) - (1/4^2)\}}$$

$$= \frac{(1242\text{ eV nm})}{(13.6\text{ eV})(12/64)} = 487.1\text{ nm}$$

For $n = 5$ $\lambda_3 = \dfrac{hc}{(13.6 \text{ eV})\{(1/2^2) - (1/5^2)\}}$

$$= \frac{(1242 \text{ eV nm})}{(13.6 \text{ eV})(12/100)} = 434.9 \text{ nm}$$

To get the maximum kinetic energy of photoelectrons $\{K = (hc/\lambda) - W\}$, we will have to use the radiation of minimum wavelength in the range of 450 nm to 700 nm. Thus, for the wavelength $\lambda = 487.1$ nm, we get

$$= \frac{(1242 \text{ eV nm})}{(13.6 \text{ eV})(12/100)} = 434.9 \text{ nm}$$

$$= 2.55 \text{ eV} - 2.2 \text{ eV} = 0.35 \text{ eV}$$

17. If λ is de-Broglie wavelength, then for nth stationary orbit $2\pi r_n = n\lambda$

where r_n is radius of nth orbit $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$

$$\therefore \quad 2\pi\left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right) = n\lambda \quad \Rightarrow \quad \frac{1}{\lambda} = \frac{mZe^2}{2\varepsilon_0 h^2 n} \qquad \text{(i)}$$

For Lyman series of hydrogen like atom

$$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad \text{(ii)}$$

From Eqs. (i) and (ii), $Z^2 R\left(1 - \dfrac{1}{n^2}\right) = \dfrac{mZe^2}{2\varepsilon_0 h^2 n}$

Rydberg constant,

$$R = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

$$\therefore \quad \frac{Z^2 me^4}{8\varepsilon_0^2 ch^3}\left(1 - \frac{1}{n^2}\right) = \frac{mZe^2}{2\varepsilon_0 h^2 n}$$

$$\left(1 - \frac{1}{n^2}\right) = \frac{4\varepsilon_0 ch}{ne^2 Z}$$

$$= \frac{4 \times (8.85 \times 10^{-12}) \times (3 \times 10^8) \times (6.62 \times 10^{-34})}{n \times (1.6 \times 10^{-19})^2 \times 11} = \frac{25}{n}$$

$$\therefore \quad n^2 - 1 = 25n \quad \text{or} \quad n^2 - 25n - 1 = 0$$

$$n = \frac{25 \pm \sqrt{(-25)^2 + 4 \times 1 \times 1}}{2} \approx \frac{25 \pm \sqrt{625}}{2} \approx 25$$

As negative n is not possible, $n \approx 25$.