Quadratic Equations

Tip 1

- Quadratic Equations is one of the important topics for CAT
- The theory involved in this topic is very simple and students should be comfortable with the some basic formulas and concepts.
- The techniques like option elimination, value assumption can help to solve questions from this topic quickly.
- This pdf covers all the important formulas and concepts related to Quadratic Equations.

Tip 2

General Quadratic equation will be in the form of $ax^2+bx+c = 0$

The values of ‘x’ satisfying the equation are called roots of the equation.

- The value of roots, p and q = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The above formula is known as the Shreedhara Acharya's Formula, after the ancient **Indian Mathematician** who derived it.

- Sum of the roots = p+q = $\frac{-b}{a}$
- Product of the roots = p*q = $\frac{c}{a}$
- If 'c' and 'a' are equal then the roots are reciprocal to each other
- If b = 0, then the roots are equal and are opposite in sign.

Tip 3

Let D denote the discriminant, $D = b^2 - 4ac$. Depending on the sign and value of D, nature of the roots would be as follows:

- $D < 0$ and $|D|$ is not a perfect square:

Roots will be in the form of p+iq and p-iq where p and q are the real and imaginary parts of the complex roots. p is rational and q is irrational.

- $D < 0$ and $|D|$ is a perfect square:

Roots will be in the form of p+iq and p-iq where p and q are both rational.

- $D = 0$

Roots are real and equal. $X = -b/2a$

Tip 4

- D > 0 and D is not a perfect square:

Roots are conjugate surds

- D > 0 and D is a perfect square:

Roots are real, rational and unequal

Tip 5

Signs of the roots: Let P be product of roots and S be their sum

- $P > 0, S > 0$: Both roots are positive
- $P > 0, S < 0$: Both roots are negative
- $P < 0, S > 0$: Numerical smaller root is negative and the other root is positive
- $P < 0, S < 0$: Numerical larger root is negative and the other root is positive

Tip 6

- Minimum and maximum values of $ax^2+bx+c = 0$
- If a > 0: minimum value = $\frac{4ac-b^2}{4a}$ and occurs at x = -b/2a
- If a < 0: maximum value = $\frac{4ac-b^2}{4a}$ and occurs at x = -b/2a

Tip 7

If $A_nX^n + A_{n-1}X^{n-1} + ... + A_1X + A_0 = 0$, then

- Sum of the roots = $-A_{n-1}/A_n$
- Sum of roots taken two at a time = A_{n-2}/A_n
- Sum of roots taken three at a time = $-A_{n-3}/A_n$ and so on
- Product of the roots = $[(-1)^n A_0] /A_n$

Tip 8

Finding a quadratic equation:

- If roots are given: $(x-a)(x-b)=0 \Rightarrow x^2-(a+b)x+ab = 0$
- If sum s and product p of roots are given: $x^2-sx+p = 0$
- If roots are reciprocals of roots of equation $ax^2+bx+c = 0$, then equation is $cx^2+bx+a = 0$
- If roots are k more than roots of $ax^2+bx+c = 0$ then equation is $a(x-k)^2+b(x-k)+c = 0$
- If roots are k times roots of $ax^2+bx+c = 0$ then equation is $a(x/k)^2+b(x/k)+c = 0$

Tip 9

- Descartes Rules: A polynomial equation with n sign changes can have a maximum of n positive roots. To find the maximum possible number of negative roots, find the number of positive roots of f(-x).

- An equation where highest power is odd must have at least one real root