NUMBER SYSTEMS

1

INTRODUCTION

The chapter on Number Systems is amongst the most important chapters in the entire syllabus of Quantitative Aptitude for the CAT examination (and also for other parallel MBA entrance exams). Students are advised to go through this chapter with utmost care; understanding each concept and questiontype on this topic. The CAT has consistently contained anything between 20–40 percent of the marks based on questions taken from this chapter. Naturally, this chapter becomes one of the most crucial as far as your quest to reach close to the qualification score in the section of Quantitative Aptitude is concerned.

Hence, going through this chapter and its concepts properly is imperative for you. It would be a good idea to first go through the basic definitions of all types of numbers. Also, closely follow the solved examples based on various concepts discussed in the chapter. Also, the approach and attitude while solving questions on this chapter is to try to maximize your learning experience out of every question. Hence, do not just try to solve the questions but also try to think of alternative processes in order to solve the same question. Refer to hints or solutions only as a last resort.

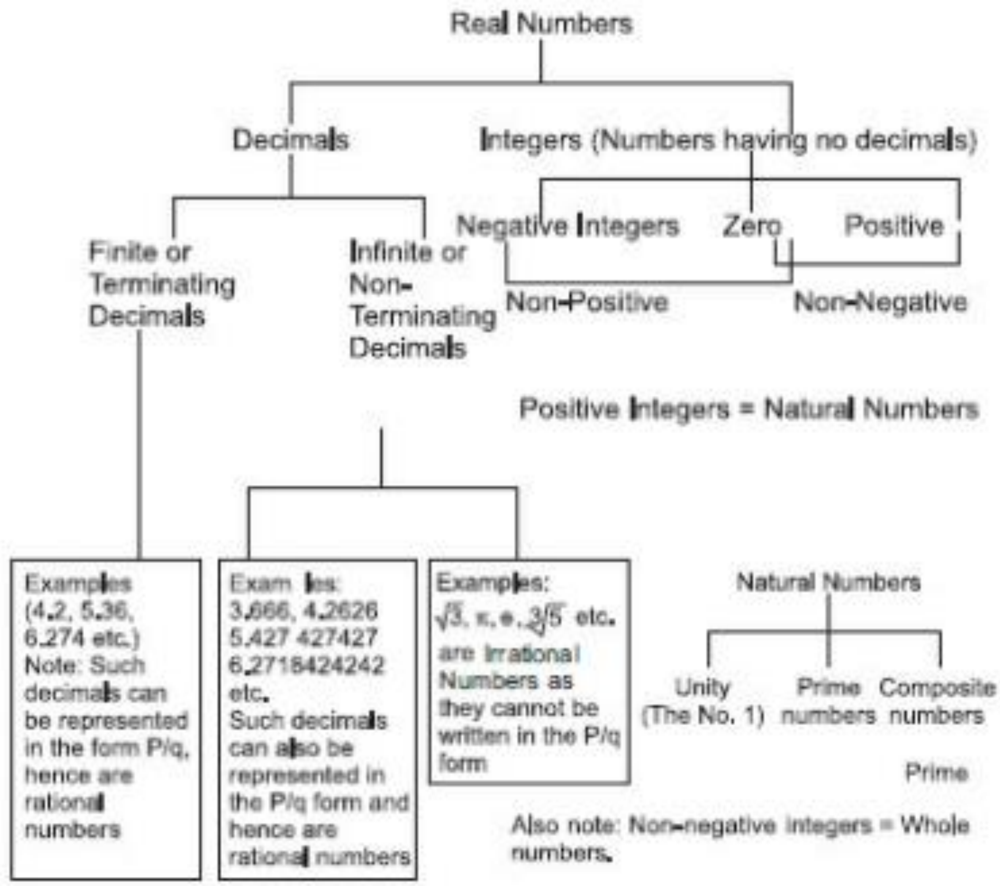
To start off, the following pictorial representation of the types of numbers will help you improve your quality of comprehension of different types of numbers.

DEFINITION

Natural Numbers These are the numbers (1, 2, 3, etc.) that are used for counting. In other words, all positive integers are natural numbers.

There are infinite natural numbers and the number 1 is the least natural number.

Examples of natural numbers: 1, 2, 4, 8, 32, 23, 4321 and so on.



The following numbers are examples of numbers that are not natural: –2, –31, 2.38, 0 and so on.

Based on divisibility, there could be two types of natural numbers: *Prime* and *Composite*.

Prime Numbers A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.

Note: Unity (i.e. 1) is not a prime number.

Some Properties of Prime Numbers

- The lowest prime number is 2.
- Number 2 is also the only even prime number.
- The lowest odd prime number is 3.
- The remainder when a prime number $p \geq 5$ is divided by 6 is 1 or 5. However, if a number on being divided by 6 gives a remainder of 1 or 5, the number need not be prime. Thus, this can be referred to as a necessary but not sufficient condition. In mathematical and algebraic writing, this logic is represented as: Prime numbers after 3, would always be of the form $6n+1$ or $6n+5$. It can also be referred to as: Prime numbers after 3 would always be of the form: $6n \pm 1$. Its
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 24 is 1.
- For prime numbers $p > 3$, $p^2 - 1$ is divisible by 24.
- Prime numbers between 1 to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
- Prime numbers between 100 to 200 are: 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.
- If a and b are any two odd primes then $a^2 - b^2$ is composite. Also, $a^2 + b^2$ is composite.

- The remainder of the division of the square of a prime number $p \geq 5$ divided by 12 is 1.

SHORT-CUT PROCESS

To Check Whether a Number is Prime or Not

To check whether a number N is prime, adopt the following process.

(a) Take the square root of the number.

(b) Round of the square root to the immediately higher integer. Call this number z. For example if you have to check for 181, its square root will be 13. Hence, the value of z, in this case will be 14.

(c) Check for divisibility of the number N by all prime numbers below z. If there is no prime number below the value of z which divides N then the number N will be prime.

To illustrate :-

The value of $\sqrt{239}$ lies between 15 to 16. Hence, take the value of z as 16.

Prime numbers less than or equal to 16 are 2, 3, 5, 7, 11 and 13, 239 is not divisible by any of these. Hence you can conclude that 239 is a prime number.

A Brief Look into Why this Works?

Suppose you are asked to find the factors of the number 40.

An untrained mind will find the factors as : 1, 2, 4, 5, 8, 10, 20 and 40.

The same task will be performed by a trained mind as follows:

1 × 40

2 × 20

4 × 10

and 5 × 8

i.e., The discovery of one factor will automatically yield the other factor. In other words, factors will appear in terms of what can be called as factor pairs. The locating of one factor, will automatically pinpoint the other one for you. Thus, in the example above, when you find 5 as a factor of 40, you will automatically get 8 too as a factor.

Now take a look again at the pairs in the example above. If you compare the values in each pair with the square root of 40 (i.e. 6,) you will find that for each pair the number in the left column is lower than the square root of 40, while the number in the right column is higher than the square root of 40.

This is a property for all numbers and is always true.

Hence, we can now phrase this as: Whenever you have to find the factors of any number *N*, you will get the factors in pairs (i.e. factor pairs). Further, the factor pairs will be such that in each pair of factors, one of the factors will be lower than the square root of *N* while the other will be higher than the square root of *N*.

As a result of this fact, one need not make any effort to find the factors of a number above the square root of the number. These come automatically. All you need to do is to find the factors below the square root of the number.

Extending this logic, we can say that if we are not able to find a factor of a number up to the value of its square root, we will not be able to find any factor above the square root and the number under consideration will be a prime number. This is the reason why when we need to check whether a number is prime, we have to check for factors only below the square root.

But, we have said that you need to check for divisibility only with the prime numbers below (and including) the square root of the number.

Let us look at an example to understand why you need to look only at prime numbers below the square root.

Uptil now, we have deduced that in order to check whether a number is prime, we just need to do a factor search below (and including) the square root.

Thus, for example, in order to find whether 181 is a prime number, we need to check for it's divisibility with the numbers = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.

The first thing you will realise, when you first look at the list above is that all even numbers will get eliminated automatically (since no even number can divide an odd number and of course you will check a number for being prime only if it is odd!)

This will leave you with the numbers 3, 5, 7, 9, 11 and 13 to check 181.

Why do we not need to check with composite numbers below the square root? This will again be understood best if explained in the context of the example above. The only composite number in the list above is 9. You do not need to check with 9, because when you checked *N* for divisibility with 3 you would get either of two cases:

Case 1: If N is divisible by 3: In such a case, *N* will automatically become non-prime and you can stop your checking. Hence, you will not need to check for the divisibility of the number by 9.

Case 2: N is not divisible by 3: If *N* is not divisible by 3, it is obvious that it will not be divisible by 9. Hence, you will not need to check for the divisibility of the number by 9.

Thus, in either case, checking for divisibility by a composite number (9 in this case) is an unnecessary step. This will be true for all composite numbers.

Hence, when we have to check whether a number *N* is prime or not, we need to only check for its divisibility by prime factors below the square root of *N*.

Finding Prime Numbers: The Short Cut

Using the logic that we have to look at only the prime numbers below the square root in order to check whether a number is prime, we can actually cut short the time for finding whether a number is prime drastically.

Before I start to explain this, you should perhaps realise that in an examination like the CAT, or any other aptitude test for that matter whenever you would need to be checking for whether a number is prime or not, you would typically be checking 2 digit or maximum 3 digit numbers in the range of 100 to 200.

Also, one would never really need to check with the prime number 5, because divisibility by 5 would automatically be visible and thus, there is no danger of anyone ever declaring a number like 35 to be prime. Hence, in the list of prime numbers below the square root, we would never include 5 as a number to check with.

Checking Whether a Number is Prime (For Numbers below 49)

The only number you would need to check for divisibility with is the number 3. Thus, 47 is prime because it is not divisible by 3.

Checking Whether a Number is Prime(For Numbers above 49 and below 121)

Naturally, you would need to check this with 3 and 7. But if you remember that 77, 91 and 119 are not prime, you would be able to spot the prime numbers below 121 by just checking for divisibility with the number 3.

Why? Well, the odd numbers between 49 and 121 which are divisible by 7 are 63, 77, 91, 105 and 119. Out of these, perhaps 91 and 119 are the only numbers that you can mistakenly declare as prime. The numbers 77 and 105 are so obviously not-prime that you would never be in danger of declaring them prime.

Thus, for numbers between 49 and 121, you can find whether a number is prime or not by just dividing by 3 and checking for its divisibility.

For example:

61 is prime because it is not divisible by 3 and it is neither 91 nor 119.

Checking Whether a Number is Prime(For Numbers above 121 and below 169)

Naturally you would need to check this with 3, 7 and 11. But if you remember that 133,143 and 161 are not prime, you would be able to spot the prime numbers between 121 to 169 by just checking for divisibility with the number 3.

Why? The logic is same as explained above. The odd numbers between 121 and 169 which are divisible by either 7 or 11 are 133, 143, 147, 161 and 165. Out of these 133, 143 and 161 are the only numbers that you can mistakenly declare as prime if you do not check for 7 or 11. The number 147 would be found not to be prime when you check its divisibility by 3 while the number 165 you would never need to check for, for obvious reasons.

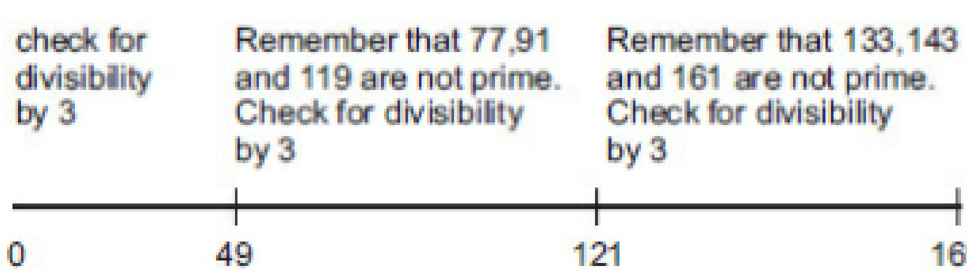
Thus, for numbers between 121 and 169 you can find whether a number is prime or not by just dividing by 3 and checking for its divisibility.

For example:

149, is prime because it is not divisible by 3 and it is neither 133, 143 nor 161.

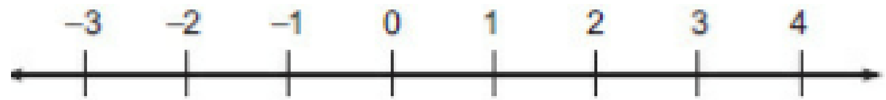
Thus, we have been able to go all the way till 169 with just checking for divisibility with the number 3.

This logic can be represented on the number line as follows:



Integers A set which consists of natural numbers, negative integers (–1, –2, –3…–n…) and zero is known as the set of integers. The numbers belonging to this set are known as integers.

Integers can be visualised on the number line:



Note: Positive integers are the same thing as natural numbers.

The moment you define integers, you automatically define **decimals.**

Decimals

A *decimal* number is a number with a decimal point in it, like these: 1.5, 3.21, 4.173, 5.1, etc.

The number to the left of the decimal is an ordinary whole number. The first number to the right of the decimal is the number of tenths (1/10's). The second is the number of hundredths (1/100's) and so on. So, for the number 5.1, this is a shorthand way of writing the mixed number $5\frac{1}{10}$. The number 3.27 is the same as 3 + 2/10 + 7/100.

A word on where decimals originate from

Consider the situation where there are 5 children and you have to distribute 10 chocolates between them in such a way that all the chocolates should be distributed and each child should get an equal number of chocolates. How would you do it? Well, simple—divide 10 by 5 to get 2 chocolates per child.

Now consider what if you had to do the same thing with 9 chocolates amongst 5 children. In such a case, you would not be able to give an integral number of chocolates to each person. You would give 1 chocolate each to all the 5 and the 'remainder' 4 would have to be divided into 5 parts. 4 out of 5 would give rise to the decimal 0.8 and hence you would give 1.8 chocolates to each child. That is how the concept of decimals enters mathematics in the first place.

Taking this concept further, you can realise that the decimal value of any fraction essentially emerges out of the remainder when the numerator of the fraction is divided by the denominator. Also, since we know that each divisor has a few defined remainders possible, there would be a limited set of decimals that each denominator gives rise to.

Thus, for example the divisor 4 gives rise to only 4 remainders (viz. 0,1,2 and 3) and hence it would give rise to exactly 4 decimal values when it divides any integer. These values are:

0 (when the remainder is 0)

0.25 (when the remainder is 1)

0.50 (when the remainder is 2)

0.75 (when the remainder is 3)

There would be similar connotations for all integral divisors—although the key is to know the decimals that the following divisors give you:

Primary list:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16 (A detailed table for the decimal values for these numbers as divisors is provided in the chapter on Percentages in this book)

Secondary list:

18, 20, 24, 25, 30, 40, 50, 60, 80, 90, 120

A detailed discussion of some of these decimals is given in the chapter on Percentages in this book.

Composite Numbers It is a natural number that has at least one divisor different from unity and itself.

Every composite number *n* can be factored into its prime factors. (This is sometimes called the canonical form of a number.)

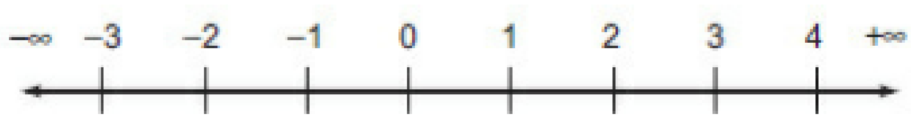
In mathematical terms: $n = p_1^m . p_2^n \ldots p_k^s$, where $p_1, p_2 \ldots p_k$ are prime numbers called factors and m, n…k are natural numbers.

Thus, $24 = 2^3 \cdot 3, 84 = 7 \cdot 3 \cdot 2^2$, etc.

This representation of a composite number is known as the standard form of a composite number. It is an extremely useful form of seeing a composite number as we shall see.

Whole Numbers The set of numbers that includes all natural numbers and the number zero are called whole numbers. Whole numbers are also called as non-negative integers.

Concept of the Number Line The number line is a straight line between negative infinity on the left to positive infinity to the right.



The distance between any two points on the number line is obtained by subtracting the lower value from the higher value. Alternately, we can also start with the lower number and find the required addition to reach the higher number. This is an important method to replace your need to follow the standard process for subtraction. Instead of using the carryover method we learnt in school, whenever you face a A-B subtraction, try to add an appropriate number to B, to get to A. You can think about this on the number line. For instance, if you wanted to subtract 28 from 82, think of what you would add to 28 to get to 82. Thus, 28 + 4 = 32 and 32 + 50 = 82. Hence, 54 would be the answer if you were to subtract 28 from 82.

Likewise, for subtracting 287 from 711, think like this: 287 + 13 = 300 and 300 + 411 = 711. Hence, the answer of 711 – 287 = 411 + 13 = 424.

Real Numbers All numbers that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

The following operations of addition, subtraction, multiplication and division are valid for both whole numbers and real numbers: [For any real or whole numbers a, b and c].

(a) Commutative property of addition: $a + b = b + a$

(b) Associative property of addition: $(a + b) + c = a + (b + c)$

(c) Commutative property of multiplication: $a \cdot b = b \cdot a$

(d) Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

(e) Distributive property of multiplication with respect to addition: $(a + b) c = ac + bc$

(f) Subtraction and division are defined as the inverse operations to addition and multiplication respectively

Thus, if a + b = c, then $c - b$ = a and if $q = a/b$ then $b \cdot q$ = a (where b π 0).

Division by zero is not possible since there is no number q for which $b \cdot q$ equals a, non zero number a if b is zero.

Rational Numbers A rational number is defined as a number of the form a/b where a and b are integers and b π 0.

The set of rational numbers encloses the set of integers and fractions. The rules given above for addition, subtraction, multiplication and division also apply on rational numbers.

Rational numbers that are not integral will have decimal values. These values can be of two types:

(a) **Terminating (or finite) decimal fractions:** For example, 17/4 = 4.25, 21/5 = 4.2 and so forth.

(b) **Non-terminating decimal fractions:** Amongst non-terminating decimal fractions there are two types of decimal values:

(i) *Non-terminating periodic fractions:* These are non- terminating decimal fractions of the type $x \cdot a_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_n$. For example $\frac{16}{3}$ = 5.3333, 15.23232323, 14.287628762 876 … and so on.

(ii) *Non-terminating non-periodic fractions:* These are of the form $x \cdot b_1b_2b_3b_4 \ldots b_nc_1c_2c_3 \ldots c_n$. For example: 5.2731687143725186....

Of the above categories, terminating decimal and non-terminating periodic decimal fractions belong to the set of rational numbers.

Every positive irrational number has a negative irrational number corresponding to it.

All operations of addition, subtraction, multiplication and division applicable to rational numbers are also applicable to irrational numbers.

As briefly stated in the Back to School section, whenever an expression contains a rational and an irrational number together, the two have to be carried together till the end. In other words, an irrational number once it appears in the solution of a question will continue to appear till the end of the question. This concept is particularly useful in Geometry. For example: If you are asked to find the ratio of the area of a circle to that of an equilateral triangle, you can expect to see a $\pi/\sqrt{3}$ in the answer. This is because the area of a circle will always have a π component in it, while that of an equilateral triangle will always have $\sqrt{3}$.

You should realise that once an irrational number appears in the solution of a question, it can only disappear if it is multiplied or divided by the same irrational number.

CONCEPT OF GCD (GREATEST COMMON DIVISOR OR HIGHEST COMMON FACTOR)

Consider two natural numbers n_1 and n_2.

If the numbers n_1 and n_2 are exactly divisible by the same number x, then x is a common divisor of n_1 and n_2.

The highest of all the common divisors of n_1 and n_2 is called as the GCD or the HCF. This is denoted as GCD (n_1, n_2).

Rules for Finding the GCD of Two Numbers n_1 and n_2

(a) Find the standard form of the numbers n_1 and n_2.

(b) Write out all prime factors that are common to the standard forms of the numbers n_1 and n_2.

(c) Raise each of the common prime factors listed above to the lesser of the powers in which it appears in the standard forms of the numbers n_1 and n_2.

(d) The product of the results of the previous step will be the GCD of n_1 and n_2.

Illustration: Find the GCD of 150, 210, 375.

Step 1: Writing down the standard form of numbers

$$150 = 5 \times 5 \times 3 \times 2$$

$$210 = 5 \times 2 \times 7 \times 3$$

$$375 = 5 \times 5 \times 5 \times 3$$

Step 2: Writing prime factors common to all the three numbers is $5_1 \times 3_1$

Step 3: This will give the same result, i.e. $5_1 \times 3_1$

Step 4: Hence, the HCF will be $5 \times 3 = 15$

For practice, find the HCF of the following:

(a) 78, 39, 195

(b) 440, 140, 390

(c) 198, 121, 1331

SHORTCUT FOR FINDING THE HCF

The above 'school' process of finding the HCF (or the GCD) of a set of numbers is, however, extremely cumbersome and time-taking. Let us take a look at a much faster way of finding the HCF of a set of numbers.

Suppose you were required to find the HCF of 39,78 and 195.

Logic The HCF of these numbers would necessarily have to be a factor (divisor) of the difference between any pair of numbers from the above 3, i.e. the HCF has to be a factor of (78 – 39 = 39) as well as of (195 – 39 = 156) and (195 – 78 = 117). Why?

Well the logic is simple if you were to consider the tables of numbers on the number line.

For any two numbers on the number line, a common divisor would be one which divides both. However, for any number to be able to divide both the numbers, it can only do so if it is a factor of the difference between the two numbers. Got it?

Take an example:

Let us say, we take the numbers 68 and 119. The difference between them being 51, it is not possible for any number outside the factor list of 51 to divide both 68 and 119. Thus, for example a number like 4, which divides 68 can never divide any number which is 51 away from 68– because 4 is not a factor of 51.

Only factors of 51, i.e. 51,17,3 and 1 'could' divide both these numbers simultaneously.

Hence, getting back to the HCF problem we were trying to tackle—take the difference between any two numbers of the set—of course, if you want to reduce your calculations in the situation, take the difference between the two closest numbers. In this case, that would be the difference between 78 and 39 = 39.

'You can of course realise that in case you see a prime number difference between two of the numbers, you would prefer the prime number difference even if it is not the smallest difference—as in such a case, either the prime number itself would be the HCF of all the numbers or in case it does not divide even one of the numbers, 1 would become the HCF. For example: If you had to find the HCF of 122,144, 203 and 253, the difference between 144 and 203 can be seen to be 59 and it is evident that 59 is not the HCF here and hence the HCF of the given numbers would be 1.'

The HCF has then to be a factor of this number. In order to find the factors quickly, remember to use the fact we learnt in the back to school section of this part of the book—that whenever we have to find the list of factors/divisors for any number we have to search the factors below the square root and the factors above the square root would be automatically visible).

A factor search of the number 39 yields the following factors:

$$1 \times 39$$

$$3 \times 13$$

Hence, one of these 4 numbers has to be the HCF of the numbers 39,78 and 195. Since we are trying to locate the **highest** common factor—we would begin our search from the highest number (viz:39).

Check for divisibility by 39 Any one number out of 39 and 78 and also check the number 195 for divisibility by 39. You would find all the three numbers are divisible by 39 and hence 39 can be safely taken to be the correct answer for the HCF of 39,78 and 195.

Suppose the numbers were:

39, 78 and 182.

The HCF would still be a factor of 78 – 39 = 39. The probable candidates for the HCF's value would still remain 1,3,13 and 39.

When you check for divisibility of all these numbers by 39, you would realise that 182 is not divisible and hence 39 would not be the HCF in this case.

The next check would be with the number 13. It can be seen that 13 divides 39 (hence would automatically divide 78—no need to check that) and also divides 182. Hence, 13 would be the required HCF of the three numbers.

Typical questions where HCF is used directly

Example 1: The sides of a hexagonal field are 216, 423, 1215, 1422, 2169 and 2223 metres. Find the greatest length of tape that would be able to exactly measure each of these sides without having to use fractions/parts of the tape.

Solution: In this question, we are required to identify the HCF of the numbers 216,423,1215, 1422, 2169 and 2223.

In order to do that, we first find the smallest difference between any two of these numbers. It can be seen that the difference between 2223-2169 = 54. Thus, the required HCF would be a factor of the number 54.

The factors of 54 are:

$$1 \times 54$$

$$2 \times 27$$

3 × 18

6 × 9

One of these 8 numbers has to be the HCF of the 6 numbers. Number 54 cannot be the HCF because the numbers 423 and 2223 being odd numbers would not be divisible by any even number. Thus, we do not need to check any even numbers in the list.

Number 27 does not divide 423 and hence, cannot be the HCF. Number 18 can be skipped as it is even.

Checking for 9:

9 divides 216,423,1215,1422 and 2169. Hence, it would become the HCF. (**Note:** we do not need to check 2223, once we know that 2169 is divisible by 9)

Example 2: A nursery has 363,429 and 693 plants respectively of 3 distinct varieties. It is desired to place these plants in straight rows of plants of 1 variety only so that the number of rows required is the minimum. What is the size of each row and how many rows would be required?

Solution: The size of each row would be the HCF of 363, 429 and 693. Difference between 363 and 429 = 66. Factors of 66 are 66, 33, 22, 11, 6, 3, 2, 1.

Number 66 need not be checked as it is even and 363 is odd. Number 33 divides 363, hence would automatically divide 429 and also divides 693. Hence, 33 is the correct answer for the size of each row.

For how many rows would be required, we need to follow the following process:

Minimum number of rows required = 363/33 + 429/33 + 693/33 = 11 + 13 + 21 = 45 rows.

CONCEPT OF LCM (LEAST COMMON MULTIPLE)

Let n_1 and n_2 be two natural numbers distinct from each other. The smallest natural number n that is exactly divisible by n_1 and n_2 is called the Least Common Multiple (LCM) of n_1 and n_2 and is designated as LCM (n_1, n_2).

RULE FOR FINDING THE LCM OF TWO NUMBERS n_1 and n_2

(a) Find the standard form of the numbers n_1 and n_2.

(b) Write out all the prime factors, which are contained in the standard forms of either of the numbers.

(c) Raise each of the prime factors listed above to the highest of the powers in which it appears in the standard forms of the numbers n_1 and n_2.

(d) The product of results of the previous step will be the LCM of n_1 and n_2.

Illustration: Find the LCM of 150, 210, 375.

Step 1: Writing down the standard form of numbers

$$150 = 5 \times 5 \times 3 \times 2$$

$$210 = 5 \times 2 \times 7 \times 3$$

$$375 = 5 \times 5 \times 5 \times 3$$

Step 2: Write down all the prime factors: that appear at least once in any of the numbers: 5, 3, 2, 7.

Step 3: Raise each of the prime factors to their highest available power (considering each to the numbers).

The LCM = $2^1 \times 3^1 \times 5^3 \times 7^1 = 5250$

Important Rule:

GCD (n_1, n_2). LCM $(n_1, n_2) = n_1 \cdot n_2$

i.e. the product of the HCF and the LCM equals the product of the numbers.

Note: This rule is applicable only for two numbers

SHORT CUT FOR FINDING THE LCM

The LCM (least common multiple) again has a much faster way of doing it than what we learnt in school.

The process has to do with the use of co-prime numbers.

Before we look at the process, let us take a fresh look at what co-prime numbers are.

Co-prime numbers are any two numbers which have an HCF of 1, i.e. when two numbers have no common prime factor apart from the number 1, they are called co-prime or relatively prime to each other.

Some Rules for Co-primes

Two Numbers being Co-prime

(i) Two consecutive natural numbers are always co-prime (Example 5, 6; 82, 83; 749, 750 and so on)

(ii) Two consecutive odd numbers are always co-prime (Examples: 7, 9; 51, 53; 513, 515 and so on)

(iii) Two prime numbers are always co-prime (Examples: 13, 17; 53, 71 and so on)

(iv) One prime number and another composite number (such that the composite number is not a multiple of the prime number) are always co-prime (Examples: 17, 38; 23, 49 and so on, but note that 17 and 51 are not co-prime)

(v) Two odd numbers with a difference that is equal to any power of 2. (Examples: 17, 21; 33, 97; 21, 85; 33, 65)

(vi) Two numbers that have a difference of 3, but are themselves not multiples of 3 (Examples: 32, 35; 43, 46; 71, 74 and so on)

Three or more numbers being co-prime with each other means that all possible pairs of the numbers would be co-prime with each other.

Thus, 47, 49, 51 and 52 are co-prime since each of the 6 pairs (47, 49); (47, 51); (47, 52); (49, 51); (49, 52) and (51, 52) are co-prime.

Rules for Spotting Three Co-prime Numbers

(i) Three consecutive odd numbers are always co-prime (Examples: 15, 17, 19; 51, 53, 55 and so on)

(ii) Three consecutive natural numbers with the first one being odd (Examples: 15, 16, 17; 21, 22, 23; 41, 42, 43 and so on). (**Note:** 22, 23, 24 are not co-prime.)

(iii) Two consecutive natural numbers along with the next odd number such that the first number. is even (Examples: 22, 23, 25; 52, 53, 55; 68, 69, 71 and so on). Please note that this would not be true in case the first and the last number would be a multiple of 3.

(iv) Three prime numbers (Examples: 17, 23, 29; 13, 31, 43 and so on)

(v) Two prime numbers and one composite number such that the composite number is not a multiple of either of the primes (Examples: 23, 31 and 42; 17, 23 and 28; 13, 23 and 27 and so on)

(vi) Three odd numbers for which the pairwise difference for each of the three pairs of numbers taken two at a time is a power of 2 (Examples: 21, 25, 29; 55, 63, 71 and so on)

So what do co-prime numbers have to do with LCMs?

By using the logic of co-prime numbers, you can actually bypass the need to take out the prime factors of the set of numbers for which you are trying to find the LCM. How?

The following process will make it clear:

Let us say that you were trying to find the LCM of 9, 10, 12 and 15.

The LCM can be directly written as: 9 × 10 × 2. The thinking that gives you the value of the LCM is as follows:

Step 1: If you can see a set of 2 or more co-prime numbers in the set of numbers for which you are finding the LCM– write them down by multiplying them.

So in the above situation, since we can see that 9 and 10 are co-prime to each other, we can start off writing the LCM by writing 9 × 10 as the first step.

Step 2: For each of the other numbers, consider what part of them have already been taken into the answer and what part remains outside the answer. In case, you see any part of the other numbers such that it is not a part of the value of the LCM you are writing—such a part would need to be taken into the answer of the LCM.

The process will be clear once you see what we do (and how we think) with the remaining 2 numbers in the above problem.

At this point, when we have written down 9 × 10 we already have taken into account the numbers 9 and 10 leaving us to account for 12 and 15.

Thought about 12: 12 is 2 × 2 × 3

9 × 10 already has a 3 and 2 in its prime factors. However, the number 12 has two 2's. This means that one of the two 2's of the number 12 is still not accounted for in our answer. Hence, we need to modify the LCM by multiplying the existing 9 × 10 by a 2. With this change, the LCM now becomes:

$$9 \times 10 \times 2$$

Thought about 15: 15 is 5 × 3

9 × 10 × 2 already has a 5 and a 3. Hence, there is no need to add anything to the existing answer.

Thus, 9 × 10 × 2 would become the correct answer for the LCM of the numbers 9, 10, 12 and 15.

What if the numbers were: 9, 10, 12 and 25

Step 1: 9 and 10 are co-prime

Hence, the starting value is 9 × 10

Thought about 12: 12 is 2 × 2 × 3

9 × 10 already has a 3 and one 2 in it prime factors. However, the number 12 has two 2's. This means that one of the two 2's of the number 12 is still not accounted for in our answer. Hence, we need to modify the LCM by multiplying the existing 9 × 10 by a 2. With this change, the LCM now becomes:

$$9 \times 10 \times 2$$

Thought about 25: 25 is 5 × 5

Thought about 12: 12 is 2 × 2 × 3

9 × 10 already has a 3 and one 2 in it prime factors. However, the number 12 has two 2's. This means that one of the two 2's of the number 12 is still not accounted for in our answer. Hence, we need to modify the LCM by multiplying the existing 9 × 10 by a 2. With this change, the LCM now becomes:

$$9 \times 10 \times 2$$

Thought about 25: 25 is 5 × 5

9 × 10 × 2 has only one 5. Hence, we need to add another 5 to the answer.

Thus, 9 × 10 × 2 × 5 would become the correct answer for the LCM of the numbers 9, 10, 12 and 25.

Rule for Finding out HCF and LCM of Fractions

(a) HCF of two or more fractions is given by:

$$\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

(b) LCM of two or more fractions is given by:

$$\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

Note: Make sure that you reduce the fractions to their lowest forms before you use these formulae, otherwise you may get the wrong answer.

You would be able to see most of the standard questions on LCMs in the practice exercise on HCF and LCM given below.

PRACTICE EXERCISE 1

(Typical questions asked in Exams)

1. Find the common factors for the numbers

   (a) 24 and 64

   (b) 42, 294 and 882

   (c) 60, 120 and 220

2. Find the HCF of

   (a) 420 and 1782

   (b) 36 and 48

   (c) 54, 72, 198

   (d) 62, 186 and 279

3. Find the LCM of

   (a) 13, 23 and 48

   (b) 24, 36, 44 and 62

   (c) 22, 33, 45, and 72

   (d) 13, 17, 21 and 33

4. Find the series of common multiples of (a) 54 and 36,

   (b) 33, 45 and 60

   [*Hint:* Find the LCM and then create an arithmetic progression with the first term as the LCM and the common difference also as the LCM.]

5. The LCM of two numbers is 936. If their HCF is 4 and one of the numbers is 72, the other is:

(a) 42

(b) 52

(c) 62

(d) None of these

[Answer: (b). Use HCF × LCM = product of numbers.]

6. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at noon, at what time will they beep again for the first time?

(a) 12:10 p.m.

(b) 12:12 p.m.

(c) 12:11 p.m.

(d) None of these

[Answer: (d). The LCM of 50 and 48 being 1200, the two clocks will ring again after 1200 seconds.]

7. Four Bells toll together at 9:00 a.m. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

(a) 3

(b) 4

(c) 5

(d) 6

[Answer: (c). The LCM of 7, 8, 11 and 12 is 1848. Hence, the answer will be obtained by the quotient of the ratio (10800)/(1848) → 5.]

8. On Ashok Marg, three consecutive traffic lights change after 36, 42 and 72 seconds, respectively. If the lights are first switched on at 9:00 a.m. sharp, at what time will they change simultaneously?

(a) 9 : 08 : 04

(b) 9 : 08 : 24

(c) 9 : 08 : 44

(d) None of these

[Answer (b). The LCM of 36,42 and 72 is 504. Hence, the lights will change simultaneously after 8 minutes and 24 seconds.]

9. The HCF of 2472, 1284 and a third number 'N' is 12. If their LCM is 23 × 32 × 51 × 103 × 107, then the number 'N' could be

(a) 22 × 32 × 71

(b) 22 × 33 × 103

(c) 22 × 32 × 51

(d) None of these

[Answer: (c)]

10. Two equilateral triangles have the sides of lengths 34 and 85, respectively.

(a) The greatest length of tape that can measure both of them exactly is

[Answer: HCF of 34 and 85 is 17.]

(b) How many such equal parts can be measured?

[Answer: $\frac{34}{17} \times 3 + \frac{85}{17} \times 3 = 2 \times 3 + 5 \times 3 = 21$]

11. Two numbers are in the ratio 17:13. If their HCF is 15, what are the numbers?

(Answer: 17 × 15 and 13 × 15 i.e. 255 and 195 respectively.) (**Note:** This can be done when the numbers are co-prime.)

12. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The number of rows (minimum) that are required are

(a) 2

(b) 3

(c) 10

(d) 11

[Answer: (c) 44/22 + 66/22 + 110/22 (Since 22 is the HCF)]

13. Three runners running around a circular track can complete one revolution in 2, 4 and 5.5 hours, respectively. When will they meet at the starting point?

(a) 22

(b) 33

(c) 11

(d) 44

[Answer: (d). The LCM of 2, 4 and 11/2. This will give you 44 as the answer].

14. The HCF and LCM of two numbers are 20 and 120, respectively. When the first number is divided by 2, the quotient is 20. The other number is

[Answer: $20 \times 120 = 40 \times n$. Hence, $n = 60$.]

15. The greatest number which will divide: 4003, 4126 and 4249, is

(a) 43

(b) 41

(c) 45

(d) None of these

The answer will be the HCF of the three numbers. (1 in this case)

16. Which of the following represents the largest 4 digit number which can be added to 7249 in order to make the derived number divisible by each of 12, 14, 21, 33, and 54?

(a) 9123

(b) 9383

(c) 8727

(d) None of these

[Answer: The LCM of the numbers 12, 14, 21, 33 and 54 is 8316. Hence, in order for the condition to be satisfied we need to get the number as:

$$7249 + n = 8316 \times 2$$

Hence, $n = 9383$]

17. Find the greatest number of 5 digits, that will give us a remainder of 5, when divided by 8 and 9, respectively.

(a) 99931

(b) 99941

(c) 99725

(d) None of these

[Answer: The LCM of 8 and 9 is 72. The largest 5 digit multiple of 72 is 99936. Hence, the required answer is 99941.]

18. The least perfect square number which is divisible by 3, 4, 6, 8, 10 and 11 is

[Answer: The number should have at least one 3, three 2's, one 5 and one 11 for it to be divisible by 3, 4, 6, 8, 10 and 11.]

Further, each of the prime factors should be having an even power in order to be a perfect square. Thus, the correct answer will be: 3 × 3 × 2 × 2 × 2 × 2 × 5 × 5 × 11 × 11

19. Find the greatest number of four digits which when divided by 10, 11, 15 and 22 leaves 3, 4, 8 and 15 as remainders, respectively.

(a) 9907

(b) 9903

(c) 9893

(d) None of these

[Answer: First find the greatest 4 digit multiple of the LCM of 10, 11, 15 and 22. (In this case it is 9900). Then, subtract 7 from it to give the answer.]

20. Find the HCF of $(3^{125} – 1)$ and $(3^{35} – 1)$.

[Answer: The solution of this question is based on the rule that:

The HCF of $(a^m – 1)$ and $(a^n – 1)$ is given by$(a^{\text{HCF of m, n}} – 1)$

Thus, in this question the answer is: (35 – 1). Since 5 is the HCF of 35 and 125.]

PRACTICE EXERCISE 2

21. What will be the least possible number of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?

(a) 7

(b) 8

(c) 22

(d) None of these

22. Find the greatest number, which will divide 215, 167 and 135 so as to leave the same remainder in each case.

(a) 64

(b) 32

(c) 24

(d) 16

23. Find the L.C.M of 2.5, 0.5 and 0.175.

(a) 2.5

(b) 5

(c) 7.5

(d) 17.5

24. The L.C.M of 4.5; 0.009; and 0.18 = ?

(a) 4.5

(b) 45

(c) 0.225

(d) 2.25

25. The L.C.M of two numbers is 1890 and their H.C.F is 30. If one of them is 270, the other will be

(a) 210

(b) 220

(c) 310

(d) 320

26. What is the smallest number which when increased by 6 is divisible by 36, 63 and 108?

(a) 750

(b) 752

(c) 754

(d) 756

27. The smallest square number, which is exactly divisible by 2, 3, 4, – 9, 6, 18, 30 and 60, is

(a) 900

(b) 1600

(c) 3600

(d) None of these

28. The H.C.F of two numbers is 11, and their L.C.M is 616. If one of the numbers is 88, find the other.

(a) 77

(b) 87

(c) 97

(d) None of these

29. What is the greatest possible rate at which a man can walk 51 km and 85 km in an exact number of minutes?

(a) 11 km/min

(b) 13 km/min

(c) 17 km/min

30. The HCF and LCM of two numbers are 12 and 144 respectively. If one of the numbers is 36, the other number is

(a) 4

(b) 48

(c) 72

(d) 432

31. HCF of (9000, X) = 60; where X is a natural number less than 9000. Find the number of possible values of X.

(a) 24

(b) 40

(c) 30

(d) 36

32. The sum of two numbers is 2800 and 7 is their HCF, how many pairs of such numbers are possible?

(a) 80

(b) 160

(c) 60

(d) 100

33. There are three positive integers *a*, *b* and *c* such that HCF of each possible pair is 11 and product of all three numbers is 13310. How many sets of $\{a, b, c\}$ is possible?

(a) 2

(b) 6

(c) 3

(d) 9

34. The HCF and LCM of two numbers are 75 and 920 respectively. If one of the numbers is 600, find the other number.

(a) 115

(b) 230

(c) 23

(d) Data inconsistent

35. If the LCM of 1, 2, 3, ..., 120 is n, find the LCM of 1, 2, 3, ..., 125.

(a) n

(b) $5n$

(c) $11n$

(d) $55n$

36. The sum of two numbers is 112 and the sum and the difference of their LCM and HCF are 208 and 176 respectively. Then the smaller number is

(a) 92

(b) 24

(c) 48

(d) 64

ANSWER KEY

21. (c)

22. (d)

23. (d)

24. (a)

25. (a)

26. (a)

27. (a)

28. (a)

29. (c)

30. (b)

31. (b)
32. (a)
33. (b)
34. (d)
35. (d)
36. (c)

Solutions and Shortcuts

Practice Exercise 1

4. (a) The first common multiple is also the LCM. The LCM of 36 and 54 would be 108. The next common multiple would be 216, 324 and so on. Thus, the required series would be 108, 216, 324, 432, 540, 648....

   (b) The LCM of 33, 45 and 60 = 60 × 3 × 11 = 1980. Thus, the required series is: 1980, 3960, 5940...

5. LCM × HCF = 936 × 4 = $N_1 \times N_2$ →

   936 × 4 = 72 × N_2 → N_2 = 13 × 4 = 52. Option (b) is correct.

6. The first time the alarm clocks would ring together would be after a time that is equal to the LCM of 50 and 48. The LCM of 50 and 48 is 50 × 24 = 1200. Hence, the first time they would ring together after noon would be exactly 1200 seconds or 20 minutes later. Option (d) is correct.

7. The LCM of 7, 8, 11 and 12 is given by 12 × 11 × 2 × 7 = 264 × 7 = 1848. 1848 seconds is 30 minutes 48 seconds. Hence, the 4 bells would toll together every 30 minutes 48 seconds.

   The number of times they would toll together in the next 3 hours would be given by the quotient of the division:

   3 × 60 × 60 /1848 → 5 times

Alternately, by thinking of 1848 seconds as 30 minutes 48 seconds you can also solve the same question by thinking as follows:

9. 2472 = $2^3 \times 103^1 \times 3$; 1284 = $2^2 \times 107 \times 3$. Since the HCF is 12, the number must have a component of $2^2 \times 3^1$ at the very least in it. Also, since the LCM is $2^3 \times 3^2 \times 5^1 \times 103 \times 107$ we can see that the minimum requirement in the required number has to be $3^2 \times 5^1$. Combining these two requirements, we get that the number should have $2^2 \times 3^2 \times 5^1$ at the minimum and the power of 2 could also be 2^3 while we could also have either one of 103^1 and/or 107^1 as a part of the required number.

Thus, for instance, the number could also be $2^3 \times 3^2 \times 5^1 \times 103^1 \times 107^1$. The question has asked us- what ***'could'*** the number be?

Option (c) gives us a possible value of the number and is hence the correct answer.

Practice Exercise 2

21. The least possible number of planks would occur when we divide each plank into a length equal to the HCF of 42, 49 and 63. The HCF of these numbers is clearly 7- and this should be the size of each plank. Number of planks in this case would be: 42/7 + 49/7 + 63/7 = 6 + 7 + 9 = 22 planks. Hence, Option (c) is correct.

22. The difference between 135 and 167 is 32, while the difference between 167 and 215 is 48. The answer to this question would be the HCF of these differences. Hence, HCF of 32 and 48 = 16. Hence, Option (d) is correct.

23. The numbers are 5/2, ½ and 175/1000 = 7/40. The LCM of three fractions is given by the formula:

LCM of numerators/HCF of denominators = (LCM of 5, 1 and 7)/(HCF of 2 and 40) = 35/2 = 17.5

24. Use the same process as for question 23 for the numbers 9/2; 9/1000 and 9/50.

(LCM of 9,9,9)/(HCF of 2,1000 & 50) = 9/2= 4.5

25. $1890 \times 30 = 270 \times N_2 \rightarrow N_2 = 210$. Hence, option (a) is correct.

26. The LCM of 36, 63 and 108 is 756. Hence, the required number is 750. Option (a) is correct.

27. The LCM of the given numbers is 180. Hence, all multiples of 180 would be divisible by all of these numbers. Checking the series 180, 360, 540, 720, 900 we can see that 900 is the first perfect square in the list. Option (a) is correct.

28. Using the property HCF × LCM = product of the numbers, we get:

$616 \times 11 = 88 \times N_2 \rightarrow N_2 = 77$. Option (a) is correct.

29. The answer would be given by the HCF of 51 and 85 – which is 17. Hence, Option (c) is correct.

30. Using the property HCF × LCM = product of the numbers, we get:

$12 \times 144 = 36 \times N_2 \rightarrow N_2 = 48$. Option (b) is correct.

31. The numbers would be of the form $60 \times a$ and $60 \times b$, where 'a' and 'b' are co-prime numbers. (**Note:** This is a general property of HCFs, where if H is the HCF of 3 numbers x, y, z, then the numbers can be denoted as $H \times a$, $H \times b$ and $H \times c$, where a, b and c are co-prime numbers. In this question, since the first number is given to us as 900, the value of a can be calculated as: $60 \times a = 9000$. Hence, $a = 150$. We need to find values of b that are co-prime to 150 – and are below 150. To find this, we need to look for numbers below 150, that can be constructed using prime numbers other than 2,3, and 5 (since 150 has 2's, 3's and 5's in it). There are 22 prime numbers below 100, other than 2,3 and 5. Between 100 and 150, there are an additional 10 prime numbers. Besides this, numbers like 7^2, 11^2, 7×11, 7×13, 7×17, 7×19, 11×13. This makes a total of $22 + 10 + 7 = 39$ numbers. Besides, this we also need to visualise that if we take the value of b as 1, then again we would get the HCF of 60 and 9000 as 60. Thus, there are a total of 40 such numbers below 9000, that would have an HCF of 60 along with the number 9000. Option (b) is correct.

32. In this question, we can again think of the numbers are $7 \times a$ and $7 \times b$, where a and b should be co-prime. Since the sum of the numbers is 2800, we have $7 \times a + 7 \times b = 2800$, which means that $a + b = 400$. Thus, a and b should fulfill two conditions – they should add up to 400 and they should be co-prime. Since, $400 = 2^4 \times 5^2$, all numbers below 400 that are not multiples of 2 or 5, would be co-prime. If we take the value of $a = 1, 3, 7$ or 9; the value of b would come out as 399, 397, 393 and 391. Going forward

between every set of numbers from 11 to 20, 21 to 30 and so on, we will find 4 such numbers that would not be a multiple of 2 or 5. Thus, there would be a total of 20 × 4 = 80 such pairs of numbers, whose HCF is 7 and sum is 2800. Option (a) is correct.

33. The numbers can be taken as $11x$, $11y$ and $11z$, where x, y, z are co-prime. The product of the three numbers would be $11 \times 11 \times 11 \times xyz = 1331 \times xyz$. For this to be equal to 13310, the product xyz needs to be 10. There is only one way of making a product of 10 using three distinct numbers – viz: 1 × 2 × 5. However, these three values can be used interchangeably between x, y and z in 6 ways (3! = 6). Hence, there are 6 sets of $\{a, b, c\}$ possible. Option (b) is correct.

34. Normally, students would solve this question as $75 \times 920 = 600 \times n \rightarrow n = 115$. But this is not correct in this situation. The reason for the same is that it is not possible to have the LCM of two numbers such that it is not the product of their HCF. The LCM would always necessarily be a product of the HCF. Hence, the correct answer for this question is Data inconsistent. Hence, option (d) is correct.

35. Going forward from 121 to 125, the number 121 being 11^2 would alter the LCM to $11 \times n$ (since, the LCM is got by multiplying all the prime factors using their highest powers.). Also, 125 being 5^3, would necessitate a multiplication by 5 for the LCM. Thus, the correct LCM would be $11 \times 5 \times n = 55 \times n$. Hence, option (d) is correct.

36. LCM + HCF = 208 and LCM – HCF = 176 → LCM = 192 and HCF = 16. Thus, the two numbers are 16*a* and 16*b* with *a* and *b* as co-prime. Also, since 16*a* + 16*b* = 112, we get that *a* + *b* = 7. Trial and error with different values of *a* and *b*, gives us that the numbers would be 48 and 64 (using *a* = 3 and *b* = 4). Hence, the correct answer is option (c).

DIVISIBILITY

A number *x* is said to be divisible by another number 'y' if it is completely divisible by *y* (i.e. it should leave no remainder).

In general, it can be said that any integer N, when divided by a natural number D, there exists a unique pair of numbers Q and R which are called the quotient and remainder respectively.

Thus, $N = QN + R$.

Where *Q* is an integer and *D* is a natural number or zero and $0 \le R < N$ (i.e. remainder has to be a whole number less than *N*.)

If the remainder is zero, we say that the number *N* is divisible by *D*.

When $R > 0$, we say that the number *N* is divisible by *D* with a remainder.

Thus, 25/8 can be written as: 25 = 3 × 8 + 1 (3 is the quotient and 1 is the remainder)

While, –25/7 will be written as –25 = 7 × (–4) + 3 (–4 is the quotient and 3 is the remainder)

Note: An integer $b > 0$ is said to divide an integer a if there exists another integer *c* such that:

$$a = bc$$

It is important to explain at this point a couple of concepts with respect to the situation, when we divide a negative number by a natural number N.

Suppose, we divide – 32 by 7. Contrary to what you might expect, the remainder in this case is + 3 (and not – 4). This is because the remainder is always non negative.

Thus, –32/7 gives quotient as – 5 and remainder as + 3.

The relationship between the remainder and the decimal:

1. Suppose we divide 42 by 5. The result has a quotient of 8 and remainder of 2.

   But 42/5 = 8.4. As you can see, the answer has an integer part and a decimal part. The integer part being 8 (equals the quotient), the decimal part is 0.4 (and is given by 2/5).

   Since, we have also seen that for any divisor N, the set of remainders obeys the inequality $0 \le R < N$, we should realise that any divisor N, will yield exactly N possible remainders. (For example, if the divisor is 3, we have 3 possible remainders 0, 1 and 2. Further, when 3 is the divisor, we can have only 3 possible decimal values 0.00, 0.333 and 0.666 corresponding to remainders of 0, 1 or 2. Remember this concept when you study the fraction to percentage conversion table in the chapter on percentages.)
2. In the case of –42 being divided by 5, the value is– 8.4. In this case, the interpretation should be thus:

The integer part is – 9 (which is also the quotient of this division) and the decimal part is 0.6 (corresponding to 3/5). Notice that since the remainder cannot be negative, the decimal too cannot be negative.

Theorems of Divisibility

(a) If a is divisible by b, then ac is also divisible by b.

(b) If a is divisible by b and b is divisible by c, then a is divisible by c.

(c) If a and b are natural numbers such that a is divisible by b and b is divisible by a, then $a = b$.

(d) If n is divisible by d and m is divisible by d then $(m + n)$ and $(m – n)$ are both divisible by d. This has an important implication. Suppose 28 and 742 are both divisible by 7. Then (742 + 28) as well as (742 – 28) are divisible by 7 (and in fact so is + 28 – 742).

(e) If a is divisible by b and c is divisible by d then ac is divisible by bd.

(f) The highest power of a prime number p, which divides $n!$ exactly is given by

$$[n/p] + [n/p^2] + [n/p^3] + \ldots$$

where $[x]$ denotes the greatest integer less than or equal to x.

As we have already seen earlier—

Any composite number can be written down as a product of its prime factors (also called standard form).

Thus, for example the number 1240 can be written as $2^3 \times 31^1 \times 5^1$.

The standard form of any number has a huge amount of information stored in it. The best way to understand the information stored in the standard form of a number is to look at concrete examples. As a reader must you to understand each of the processes defined below and use them to solve similar questions given in the exercise that follows and beyond:

1. **Using the standard form of a number to find the sum and the number of factors of the number:**

(a) Sum of factors of a number:

Suppose, we have to find the sum of factors and the number of factors of 240.

$$240 = 2^4 \times 3^1 \times 5^1$$

The sum of factors will be given by:

$$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1)$$

$$= 31 \times 4 \times 6 = 744$$

Note: This is a standard process, wherein you create the same number of brackets as the number of distinct prime factors the number contains and then each bracket is filled with the sum of all the powers of the respective prime number starting from 0 to the highest power of that prime number contained in the standard form.

Thus, for 240, we create 3 brackets-one each for 2, 3 and 5. Further in the bracket corresponding to 2, we write $(2^0 + 2^1 + 2^2 + 2^3 + 2^4)$.

Hence, for example for the number $40 = 2^3 \times 5^1$, the sum of factors will be given by: $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ {2 brackets since 40 has 2 distinct prime factors 2 and 5}

(b) Number of factors of the number:

Let us explore the sum of factors of 40 in a different context.

$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$

$= 2^0 \times 5^0 + 2^0 \times 5^1 + 2^1 \times 5^0 + 2^1 \times 5^1 + 2^2 \times 5^0 + 2^2 \times 5^1 + 2^3 \times 5^0 + 2^3 \times 5^1$

$= 1 + 5 + 2 + 10 + 4 + 20 + 8 + 40 = 90$

A clear look at the numbers above will make you realise that it is nothing but the addition of the factors of 40.

Hence, we realise that the number of terms in the expansion of $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ will give us the number of factors of 40. Hence, 40 has $4 \times 2 = 8$ factors.

Note: The moment you realise that $40 = 2^3 \times 5^1$ the answer for the number of factors can be obtained by $(3 + 1)(1 + 1) = 8$

2. **Sum and number of even and odd factors of a number:**

Suppose, you are trying to find out the number of factors of a number represented in the standard form by: $2^3 \times 3^4 \times 5^2 \times 7^3$

As you are already aware, the answer to the question is (3 + 1) (4 + 1) (2 + 1) (3 + 1) and is based on the logic that the number of terms will be the same as the number of terms in the expansion: $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$.

Now, suppose you have to find out the sum of the even factors of this number. The only change you need to do in this respect will be evident below. The answer will be given by:

$(2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

Note: We have eliminated 2^0 from the original answer. By eliminating 2^0 from the expression for the sum of all factors, you are ensuring that you have only even numbers in the expansion of the expression.

Consequently, the number of even factors will be given by: (3) (4 + 1) (2 + 1) (3 + 1)

i.e. since 2^0 is eliminated, we do not add 1 in the bracket corresponding to 2.

Let us now try to expand our thinking to try to think about the number of odd factors for a number.

In this case, we just have to do the opposite of what we did for even numbers. The following step will make it clear:

Odd factors of the number whose standard form is : $2^3 \times 3^4 \times 5^2 \times 7^3$

Sum of odd factors = $(2^0)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

i.e. ignore all powers of 2. The result of the expansion of the above expression will be the complete set of odd factors of the number. Consequently, the number of odd factors for the number will be given by the number of terms in the expansion of the above expression.

Thus, the number of odd factors for the number $2^3 \times 3^4 \times 5^2 \times 7^3 = 1 \times (4 + 1)(2 + 1)(3 + 1)$.

3. Sum and number of factors satisfying other conditions for any composite number:

These are best explained through examples:

(i) Find the sum and the number of factors of 1200 such that the factors are divisible by 15.

Solution : $1200 = 2^4 \times 5^2 \times 3^1$.

For a factor to be divisible by 15 it should compulsorily have 3^1 and 5^1 in it. Thus, sum of factors divisible by 15 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (5^1 + 5^2) \times (3^1)$ and consequently the number of factors will be given by $5 \times 2 \times 1 = 10$.

(What we have done is ensured that in every individual term of the expansion, there is a minimum of $3^1 \times 5^1$. This is done by removing powers of 3 and 5 which are below 1.)

Task for the student: Physically verify the answers to the question above and try to convert the logic into a mental algorithm.

<u>NOTE FROM THE AUTHOR:</u> The need for thought algorithms:

I have often observed that the key difference between understanding a concept and actually applying it under examination pressure, is the presence or absence of a mental thought algorithm which clarifies the concept to you in your mind. The thought algorithm is a personal representation of a concept—and any concept that you read/understand in this book (or elsewhere) will remain an external concept till it remains in someone else's words. The moment the thought becomes internalised, the concept becomes yours to apply and use.

PRACTICE EXERCISE ON FACTORS

For the number 2450, find

1. The sum and number of all factors
2. The sum and number of even factors
3. The sum and number of odd factors
4. The sum and number of factors divisible by 5
5. The sum and number of factors divisible by 35
6. The sum and number of factors divisible by 245

For the number 7200, find

7. The sum and number of all factors
8. The sum and number of even factors
9. The sum and number of odd factors
10. The sum and number of factors divisible by 25
11. The sum and number of factors divisible by 40

12. The sum and number of factors divisible by 150

13. The sum and number of factors not divisible by 75

14. The sum and number of factors not divisible by 24

15. Find the number of divisors of 1728.

(a) 18

(b) 30

(c) 28

(d) 20

16. Find the number of divisors of 1080 excluding the divisors, which are perfect squares.

(a) 28

(b) 29

(c) 30

(d) 31

17. Find the number of divisors of 544 excluding 1 and 544.

(a) 12

(b) 18

(c) 11

(d) 10

18. Find the number of divisors of 544 which are greater than 3.

(a) 15

(b) 10

(c) 12

(d) None of these

19. Find the sum of divisors of 544 excluding 1 and 544.

(a) 1089

(b) 545

(c) 589

(d) 1134

20. Find the sum of divisors of 544 which are perfect squares.

(a) 32

(b) 64

(c) 42

(d) 21

21. Let d_1, d_2, ... d_k be all the factors of a positive integer n, in ascending order including 1 and n. Suppose $d_1 + d_2 + ... d_k = 72$.Then the value of $1/d_1 + 1/d_2 + ... + 1/d_k$ is

(a) $n2/72$

(b) $n/72$

(c) $72/n$

(d) $36/n$

22. How many factors of $4_5 \times 5_4 \times 6_3$ are perfect squares but not perfect cubes?

(a) 6

(b) 24

(c) 30

(d) 39

23. How many of the following statements are true?

(i) Only squares of prime numbers have three factors

(ii) If a number has 4 factors, the number must be a perfect cube

(iii) If a number has 5 factors, the number must be a fourth power of a natural number

(a) None of these

(b) One

(c) Two

(d) All three

24. n is the smallest positive integer such that $n/2$ is a perfect square, $n/3$ is a perfect cube and $n/5$ is a perfect fifth power. If n is represented in its factorised form as $p_a \times q_b \times r_c \times$ ....., where p, q, r, ... are all distinct prime numbers, then find the minimum sum $a + b + c +$ .... .

(a) 31

(b) 37

(c) 54

(d) 56

25. n is the smallest positive integer such that $2n$ is a perfect square, $3n$ is a perfect cube and $5n$ is a perfect fifth power. If n is represented in its factorised form as $p^a \times q^b \times r^c \times$ ......, where $p, q, r, \ldots$ are all distinct prime numbers, then find the minimum sum $a + b + c + \ldots$.

(a) 31

(b) 37

(c) 54

(d) 59

26. N^2 has 13 factors smaller than N. Find the maximum number of factors of N^3.

(a) 40

(b) 52

(c) 64

(d) None of these

ANSWER KEY

15. (c)

16. (a)

17. (d)

18. (b)

19. (c)

20. (d)

21. (c)

22. (d)

23. (c)

24. (a)

25. (d)

26. (c)

Solutions and Shortcuts

Solutions for Questions 1 to 6:

$2450 = 50 \times 49 = 2^1 \times 5^2 \times 7^2$

1. Sum and number of all factors:

   Sum of factors = $(2^0 + 2^1)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of factors = $2 \times 3 \times 3 = 18$

2. Sum of all even factors:

   $(2^1)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of even factors = $1 \times 3 \times 3 = 9$

3. Sum of all odd factors:

   $(2^0)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2)$

   Number of odd factors = $1 \times 3 \times 3 = 9$

4. Sum of factors divisible by 5:

$(2^0 + 2^1)(5^1 + 5^2)(7^0 + 7^1 + 7^2)$

Number of factors divisible by 5 = 2 × 2 × 3 = 12

5. Sum of factors divisible by 35:

$(2^0 + 2^1)(5^1 + 5^2)(7^1 + 7^2)$

Number of factors divisible by 35 = 2 × 2 × 2 = 8

6. Sum of all factors divisible by 245:

$(2^0 + 2^1)(5^1 + 5^2)(7^2)$

Number of factors divisible by 245 = 2 × 2 × 1 = 4

Solutions for Questions 7 to 14:

$7200 = 72 \times 100 = 12 \times 6 \times 100 = 2^5 \times 3^2 \times 5^2$

7. **Sum and number of all factors:**

Sum of factors = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)$

$(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$

Number of factors = 6 × 3 × 3 = 54

8. **Sum and number of even factors:**

Sum of even factors = $(2^1 + 2^2 + 2^3 + 2^4 + 2^5)$

$(3^0 + 3^1 + 3^2)(5^0 + 5^1 + 5^2)$

Number of even factors = 5 × 3 × 3 = 45

9. **Sum and number of odd factors:**

Sum of odd factors = $(2_0)(3_0 + 3_1 + 3_2)(5_0 + 5_1 + 5_2)$

Number of odd factors = $1 \times 3 \times 3 = 9$

10. **Sum and number of factors divisible by 25:**

Sum of factors divisible by 2_5= $(2_0 + 2_1 + 2_2 + 2_3 + 2_4 + 2_5)(3_0 + 3_1 + 3_2)(5_2)$

Number of factors divisible by 2_5 = $6 \times 3 \times 1 = 1_8$

11. **Sum and number of factors divisible by 40:**

Sum of factors divisible by 4_0 = $(2_3 + 2_4 + 2_5)(3_0 + 3_1 + 3_2)(5_1 + 5_2)$

Number of factors = $3 \times 3 \times 2 = 1_8$

12. **Sum and number of factors divisible by 150:**

Sum of factors divisible by 150 = $(2_1 + 2_2 + 2_3 + 2_4 + 2_5)(3_1 + 3_2)(5_2)$

Number of factors divisible by 150 = $5 \times 2 \times 1 = 10$

13. **Sum and number of factors not divisible by 75:**

Sum of factors not divisible by 75 = Sum of all factors – Sum of factors divisible by 75 =

$(2_0 + 2_1 + 2_2 + 2_3 + 2_4 + 2_5)(3_0 + 3_1 + 3_2)(5_0 + 5_1 + 5_2) – (2_0 + 2_1 + 2_2 + 2_3 + 2_4 + 2_5)(3_1 + 3_2)(5_2)$

Number of factors not divisible by 75 = Number of factors of 7200 – Number of factors of 7200 which are divisible by 75 = $6 \times 3 \times 3 – 6 \times 2 \times 1 = 54 – 12 = 42$

14. **Sum and number of factors not divisible by 24:**

Sum of factors not divisible by 24 = Sum of all factors – Sum of factors divisible by 24 =

$(2_0 + 2_1 + 2_2 + 2_3 + 2_4 + 2_5)(3_0 + 3_1 + 3_2)(5_0 + 5_1 + 5_2) – (2_3 + 2_4 + 2_5)(3_1 + 3_2)(5_0 + 5_1 + 5_2)$

Number of factors not divisible by 24 = Number of factors of 7200 – Number of factors of 7200 which are divisible by 24 = 6 × 3 × 3 – 3 × 2 × 3 = 54 – 18 = 36

15. **Number of divisors of 1728**

1728 = 4 × 432 = 16 × 108 = 64 × 27 = $2_6 \times 3_3$

Number of factors = 7 × 4 = 28. Option (c) is correct.

16. 1080 = 108 × 10 = 27 × 4 × 10 = $3_3 \times 2_3 \times 5_1$

Number of factors = 4 × 4 × 2 = 32

In order to see the number of factors of 1080 which are perfect squares we need to visualise the structure for writing down the sum of perfect square factors of 1080.

This would be given by:

Sum of all perfect square factors of 1080 = $(2_0 + 2_2)(3_0 + 3_2)(5_0)$.

From the above structure, it is clear that the number of perfect square factors is going to be 2 × 2 × 1 = 4.

Thus, the number of factors of 1080 which are not perfect squares are equal to 32 – 4 = 28. Option **(a)** is correct.

17. $544 = 17^1 \times 2^5$. Hence, the total number of factors of 544 is 2 × 6 = 12. But we have to count factors excluding 1 and 544. Thus, we need to remove 2 factors from this. The required answer is 12 – 2 =10. Option **(d)** is correct.

18. Using the fact that 544 has a total of 12 factors and the numbers 1 and 2 are the two factors which are lower than 3, we would get a total of 10 factors greater than 3. Option **(b)** is correct.

19. The required answer would be given by: Sum of all factors of 544 – 1 – 544 $= (2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5)(17^0 + 17^1) - 545 = 63 \times 18 - 545 = 589$. Option **(c)** is correct.

20. Sum of divisors of 544 which are perfect square is:

$(2^0 + 2^2 + 2^4)(17^0) = 21$. Option **(d)** is correct.

21. Since the sum of factors of n are given as 72, we can visualise that n would be a number like $2^1 \times 3^1 \times 5^1 = 30$.

(**Note:** For this visualize 72 as 3 × 4 × 6 and then visualise 3 × 4 × 6 as $(2^0 + 2^1) \times (3^0 + 3^1) \times (5^0 + 5^1)$] The factors of 30 are: 1, 2, 3, 5, 6, 10, 15 and 30. The value of:

$1/d_1 + 1/d_2 + ... + 1/d_k = 72/30$. Hence, option **(c)** is correct.

22. The number is: $2^{13} \times 3^3 \times 5^4$. Sum of the factors that are perfect squares would be given by the expression: $(2^0 + 2^2 + 2^4 + \dots 2^{12}) \times (3^0 + 3^2) \times (5^0 + 5^2 + 5^4)$. For the number of factors, we would need to look at the number of terms in the expansion of this expression. There are: 7 × 2 × 3 = 42 terms in this expansion. Hence, the given number has 42 factors that are

perfect squares. Out of these, three factors viz: $2^0 \times 3^0 \times 5^0$; $2^6 \times 3^0 \times 5^0$ and $2^{12} \times 3^0 \times 5^0$ are perfect cubes too. Hence, the required answer is 42 – 3 = 39. Option (d) is correct.

23. The first statement would always be true – as we cannot make 3 by multiplying any two numbers. Thus, only numbers like 2^2, 3^2, 5^2, 7^2,11^2 would give us 3 factors. The second statement need not be true, since we can easily visualise that numbers like: $2^1 \times 3^1$ would also give us 4 factors. The third statement would again be true since only numbers like 2^4, 3^4 11^4 would give us 5 factors. Hence, the correct answer would be **(c)**.

24. First visualise the number in its general form as: $2^{2m+1} \times 3^{3x+1} \times 5^{5y+1}$ with $2m + 1$, simultaneously being a multiple of 3 and 5, $3x + 1$ being a multiple of 2 as well as a multiple of 5 and $5y + 1$ being a multiple of 2 and 3. Thus, the minimum values with these constraints would be: $2^{15} \times 3^{10} \times 5^6$. In such a case, the number on division by 2, becomes a perfect square, on division by 3 becomes a perfect cube and on division by 5 becomes a perfect fifth power. In this case, the sum of the powers on 2, 3 and 5 are 15 + 10 + 6 = 31. Option (a) is correct.

25. First visualise the number in its general form as: $2^{2m-1} \times 3^{3x-1} \times 5^{5y-1}$ with $2m - 1$, simultaneously being a multiple of 3 and 5, $3x - 1$ being a multiple of 2 as well as a multiple of 5 and $5y - 1$ being a multiple of 2 and 3. Thus, the minimum values with these constraints would be: $2^{15} \times 3^{20} \times 5^{24}$. In such a case, the number on multiplication by 2 becomes a perfect square, on multiplication by 3 becomes a perfect cube and on multiplication by 5 becomes a perfect fifth power. In this case, the sum of the powers on 2, 3 and 5 are 15 + 20 + 24 = 59. Option (d) is correct.

26. From the given information, we can see that N^2 would have 13 pairs of factors apart from its square root N. Thus, N^2 has 27 factors. 27 factors for a number would occur if the number N^2 is of the form: $p_1^8 \times p_2^2$ or $p_1^2 \times p_2^2 \times p_3^2$. In the first case: N would be: $p_1^4 \times p_2^1$. Thus, N^3 would be $p_1^{12} \times p_2^3$. This number would have 13 × 4 = 52 factors. In the second case: N would be: $p_1^1 \times p_2^1 \times p_3^1$ and consequently N^3 would be: $p_1^3 \times p_2^3 \times p_3^3$. In this case, the number of factors would be 4 × 4 × 4 = 64 factors. Option (c) is correct.

(**Note:** p^{26} can also be the form of the number N^2. In such a case, N = p^{13} and $N^3 = P^{39}$. This would have only 40 factors.)

NUMBER OF ZEROES IN AN EXPRESSION

Suppose you have to find the number of zeroes in a product: 24 × 32 × 17 × 23 × 19 = $(2^3 \times 3^1) \times (2^5) \times 17^1 \times 23 \times 19$

As you can notice, this product will have no zeroes because it has no 5 in it.

However, if you have an expression like: 8 × 15 × 23 × 17 × 25 × 22

The above expression can be rewritten in the standard form as:

$$2^3 \times 3^1 \times 5^1 \times 23 \times 17 \times 5^2 \times 2^1 \times 11^1$$

Zeroes are formed by a combination of 2 × 5. Hence, the number of zeroes will depend on the number of pairs of 2's and 5's that can be formed.

In the above product, there are four twos and three fives. Hence, we shall be able to form only three pairs of (2 × 5). Hence, there will be 3 zeroes in the product.

Finding the Number of Zeroes in aFactorial Value

Suppose you had to find the number of zeroes in 6!

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (3 \times 2) \times (5) \times (2 \times 2) \times (3) \times (2) \times (1)$

The above expression will have only one pair of 5×2, since there is only one 5 and an abundance of 2's.

It is clear that in any factorial value, the number of 5's will always be lesser than the number of 2's. Hence, all we need to do is to count the number of 5's. The process for this is explained in Solved Examples 1.1 to 1.3.

EXERCISE FOR SELF-PRACTICE

Find the number of zeroes in the following cases:

1. 47!
2. 58!
3. $13 \times 15 \times 22 \times 125 \times 44 \times 35 \times 11$
4. $12 \times 15 \times 5 \times 24 \times 13 \times 17$
5. 173!
6. $144! \times 5 \times 15 \times 22 \times 11 \times 44 \times 135$
7. 148!
8. 1093!
9. 1132!
10. $1142! \times 348! \times 17!$

Solutions and Shortcuts

1. 47/5 → Quotient 9. 9/5 → Quotient → 1. 9 + 1 = 10 zeroes.
2. 58/5 → Quotient 11. 11/5 → Quotient → 2. 11 + 2 = 13 zeroes.
3. The given expression has five 5's and three 2's. Thus, there would be three zeroes in the expression.
4. The given expression has two 5's and five 2's. Thus, there would be two zeroes in the expression.
5. 173/5 → Quotient 34. 34/5 → Quotient 6. 6/5 → Quotient 1. 34 + 6 + 1 = 41 zeroes.
6. 144! Would have 28 + 5 + 1 = 34 zeroes and the remaining part of the expression would have three zeroes. A total of 34 + 3 = 37 zeroes.
7. 148/5 → Quotient 29. 29/5 → Quotient 5. 5/5 → Quotient 1. 29 + 5 + 1 = 35 zeroes.
8. 1093/5 → Quotient 218. 218/5 → Quotient 43. 43/5 → Quotient 8. 8/5 → Quotient 1. 218 + 43 + 8 + 1 = 270 zeroes.
9. 1132/5 → Quotient 226. 226/5 → Quotient 45. 45/5 → Quotient 9. 9/5 → Quotient 1. 226 + 45 + 9 + 1 = 281 zeroes.
10. 1142/5 → Quotient 228. 228/5 → Quotient 45. 45/5 → Quotient 9. 9/5 → Quotient 1. 228 + 45 + 9 + 1 = 283 zeroes.

    348/5 → Quotient 69. 69/5 → Quotient 13. 13/5 → Quotient 2. 69 + 13 + 2 = 84 zeroes.

    17/5 → Quotient 3 → 3 zeroes.

Thus, the total number of zeroes in the expression is: 283 + 84 + 3 = 370 zeroes.

A special implication: Suppose you were to find the number of zeroes in the value of the following factorial values:

$$45!, 46!, 47!, 48!, 49!$$

What do you notice? The number of zeroes in each of the cases will be equal to 10. Why does this happen? It is not difficult to understand that the number of fives in any of these factorials is equal to 10. The number of zeroes will only change at 50! (It will become 12).

In fact, this will be true for all factorial values between two consecutive products of 5.

Thus, 50!, 51!, 52!, 53! and 54! will have 12 zeroes (since they all have 12 fives).

Similarly, 55!, 56!, 57!, 58! and 59! will each have 13 zeroes.

Apart from this fact, did you notice another thing? That while there are 10 zeroes in 49!, there are directly 12 zeroes in 50!. This means that there is no value of a factorial which will give 11 zeroes. This occurs because to get 50!, we multiply the value of 49! by 50. When you do so, the result is that we introduce two 5's in the product. Hence, the number of zeroes jumps by two (since we never had any paucity of twos.)

Note: At 124! you will get 24 + 4 ⇒ 28 zeroes

At 125!, you will get 25 + 5 + 1 = 31 zeroes (A jump of 3 zeroes.)

EXERCISE FOR SELF-PRACTICE

1. $n!$ has 23 zeroes. What is the maximum possible value of n?

2. $n!$ has 13 zeroes. The highest and least values of n are

3. Find the number of zeroes in the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times .... \times 49^{49}$

4. Find the number of zeroes in

   $100^1 \times 99^2 \times 98^3 \times 97^4 \times .... \times 1^{100}$

5. Find the number of zeroes in

   $1^1! \times 2^2! \times 3^3! \times 4^4! \times 5^5! \times ....10^{10}!$

6. Find the number of zeroes in the value of

   $2^2 \times 5^4 \times 4^6 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$

7. What is the number of zeroes in the following?

   (a) 3200 + 1000 + 40000 + 32000 + 15000000

   (b) 3200 × 1000 × 40000 × 32000 × 16000000

Solutions and Shortcuts

1. This can never happen because at 99!, number of zeroes is 22 and at 100!, the number of zeroes is 24.

2. 59 and 55, respectively.

3. The fives will be less than the twos. Hence, we need to count only the fives.

   Thus : $5^5 \times 10^{10} \times 15^{15} \times 20^{20} \times 25^{25} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$

   gives us: 5 + 10 + 15 + 20 + 25 + 25 + 30 + 35 + 40 + 45 fives. Thus, the product has 250 zeroes.

4. Again the key here is to count the number of fives. This can get done by:

   $100_1 \times 95_6 \times 90_{11} \times 85_{16} \times 80_{21} \times 75_{26} \times \ldots\ldots 5_{96}$

   (1 + 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + …….. + 96) + (1 + 26 + 51 + 76)

   = 20 × 48.5 + 4 × 38.5 (Using sum of A.P. ex-plained in the next chapter.)

   = 970 + 154 = 1124

5. The answer will be the number of 5's. Hence, it will be 5! + 10!

6. The number of fives is again lesser than the number of twos.

   The number of 5's will be given by the power of 5 in the product:

   $$5_4 \times 10_8 \times 15_{12} \times 20_{16} \times 10_{18} \times 25_{20}$$

   $$= 4 + 8 + 12 + 16 + 18 + 40 = 98$$

7. (a) The number of zeroes in the sum will be two, since:

3200
1000
40000
32000
15000000
15076200

   Thus, in such cases the number of zeroes will be the least number of zeroes amongst the numbers.

   Exception: 3200 + 1800 = 5000 (three zeroes, not two).

   (b) The number of zeroes will be:

2 + 3 + 4 + 3 + 6 = 18

An extension of the process for finding the number of zeroes.

Consider the following questions:

1. Find the highest power of 5 which is contained in the value of 127!
2. When 127! is divided by 5*n* the result is an integer. Find the highest possible value for *n*.
3. Find the number of zeroes in 127!

In each of the above cases, the value of the answer will be given by:

[127/5] + [127/25] + [127/125]

= 25 + 5 + 1 = 31

This process can be extended to questions related to other prime numbers. For example:

Find the highest power of:

1. 3 which completely divides 38!

   Solution: [38/3] + [38/32] + [38/33] = 12 + 4 + 1 = 17

2. 7 which is contained in 57!

   [57/7] + [57/72] = 8 + 1 = 9

This process changes when the divisor is not a prime number. You are first advised to go through worked out problems 1.4, 1.5, 1.6 and 1.19.

Now try to solve the following exercise:

1. Find the highest power of 7 which divides 81!
2. Find the highest power of 42 which divides 122!
3. Find the highest power of 84 which divides 342!
4. Find the highest power of 175 which divides 344!
5. Find the highest power of 360 which divides 520!

Solutions and Shortcuts

1. 81/7 → Quotient 11. 11/7 → Quotient 1. Highest power of 7 in 81! = 11 + 1 = 12.

2. In order to check for the highest power of 42, we need to realise that 42 is 2 × 3 × 7. In 122! the least power between 2,3 and 7 would obviously be for 7. Thus, we need to find the number of 7's in 122! (or in other words— the highest power of 7 in 122!).

   This can be done by:

   122/7 → Quotient 17. 17/7 → Quotient 2. Highest power of 7 in 122! = 17 + 2 = 19.

3. 84 = 2 × 2 × 3 × 7. This means we need to think of which amongst 22, 3 and 7 would appear the least number of times in 342! It is evident that there would be more 22s and 3's than 7's in any factorial value (Because if you write any factorial 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14 × 15…you can clearly see that before a 7 or it's multiple appears in the multiplication, there are at least two 2's and one 3 which appear beforehand.)

   Hence, in order to solve this question we just need to find the power of 7 in 342!

This can be done as:

342/7 → Quotient 48. 48/7 → Quotient 6. 6/7 → Quotient 0. Highest power of 7 in 342! = 48 + 6 = 54.

4. 175 = 5 × 5 × 7. This means we need to think of which amongst 52 and 7 would appear the least number of times in 175! In this case, it is not immediately evident that whether there would be more 5's or more 7's, in any factorial value (Because if you write any factorial 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12 × 13 × 14 × 15…you can clearly see that although the 5's appear more frequently than the 7's it is not evident that we would have at least two fives before the 7 appears.) Hence, in this question we would need to check for both the number of 5's and the number of 7's.

Number of 7's in 344!

344/7 → Quotient 49. 49/7 → Quotient 7. 7/7 → Quotient 1. Highest power of 7 in 344! = 49 + 7 + 1 = 57.

In order to find the number of 52s in 344!, we first need to find the number of 5's in 344!

344/5 → Quotient 68. 68/5 → Quotient 13. 13/5 → Quotient 2. Number of 5's in 344! = 68 + 13 + 2 = 83.

83 fives would obviously mean [83/2] = 41

Thus, there are 41 5 squares and 57 7's in 344!

Since, the number of 5 squares are lower, they would determine the highest power of 175 that would divide 344!

The answer is 41.

5. 360 = 5 × 2 × 2 × 2 × 3 × 3. This means we need to think of which amongst 23, 32 and 5 would appear the least number of times in 520! In this case, it is not immediately evident which of these three would appear least number of times. Hence, in this question we would need to check for all three – 23s, 32s and 5s.

Number of 5's in 520!

520/5 → Quotient 104. 104/5 → Quotient 20. 20/5 → Quotient 4. Highest power of 5 in 520! = 104 + 20 + 4 = 128.

In order to find the number of 32s in 520! we first need to find the number of 3's in 520!

520/3 → Quotient 173. 173/3 → Quotient 57. 57/3 → Quotient 19. 19/3 → Quotient 6. 6/3 → Quotient 2. 2/3 → Quotient 0. Number of 3's in 520! = 173 + 57 + 19 + 6 + 2 = 257.

257 threes would obviously mean [257/2] = 128 32s.

In order to find the number of 23s in 520!, we first need to find the number of 2's in 520!

520/2 → Quotient 260. 260/2 → Quotient 130. 130/2 → Quotient 65. 65/2 → Quotient 32. 32/2 → Quotient 16. 16/2 → Quotient 8. 8/2 → Quotient 4. 4/2 → Quotient 2. 2/2 → Quotient 1. 1/2 → Quotient 0.

Number of 2's in 520! = 260 + 130 + 65 + 32 + 16 + 8 + 4 + 2 + 1 = 518. 518 twos would obviously mean [518/3] = 172 23s.

Thus, there are 128 32s, 128 5's and 172 23's in 520!

The highest power of 360 that would divide 520! would be the least of 128, 128 and 172.

The answer is 128.

EXERCISE FOR SELF-PRACTICE

1. Find the maximum value of n such that 157! is perfectly divisible by 10^n.

   (a) 37

   (b) 38

   (c) 16

   (d) 31

2. Find the maximum value of n such that 157! is perfectly divisible by 12^n.

   (a) 77

   (b) 76

   (c) 75

   (d) 78

3. Find the maximum value of n such that 157! is perfectly divisible by 18^n.

   (a) 37

   (b) 38

   (c) 39

(c) 16

(d) 31

2. Find the maximum value of n such that 157! is perfectly divisible by 12n.

(a) 77

(b) 76

(c) 75

(d) 78

3. Find the maximum value of n such that 157! is perfectly divisible by 18n.

(a) 37

(b) 38

(c) 39

(d) 40

4. Find the maximum value of n such that 50! is perfectly divisible by 2520n.

(a) 6

(b) 8

(c) 7

(d) None of these

5. Find the maximum value of n such that 50! is perfectly divisible by 12600n.

(a) 7

(b) 6

(c) 8

(d) None of these

6. Find the maximum value of n such that 77! is perfectly divisible by 720n.

(a) 35

(b) 18

(c) 17

(d) 36

7. Find the maximum value of n such that

42 × 57 × 92 × 91 × 52 × 62 × 63 × 64 × 65 × 66 × 67 is perfectly divisible by 42n.

(a) 4

(b) 3

(c) 5

(d) 6

8. Find the maximum value of n such that

570 × 60 × 30 × 90 × 100 × 500 × 700 × 343 × 720 × 81 is perfectly divisible by 30n.

(a) 12

(b) 11

(c) 14

(d) 13

9. Find the maximum value of n such that

77 × 42 × 37 × 57 × 30 × 90 × 70 × 2400 × 2402 × 243 × 343 is perfectly divisible by 21n.

(a) 9

(b) 11

(c) 10

(d) 6

Find the number of consecutive zeroes at the end of the following numbers.

10. 72!

(a) 17

(b) 9

(c) 8

(d) 16

11. 77! × 42!

(a) 24

(b) 9

(c) 27

(d) 18

12. 100! + 200!

(a) 73

(b) 24

(c) 11

(d) 22

13. 57 × 60 × 30 × 15625 × 4096 × 625 × 875 × 975

(a) 6

(b) 16

(c) 17

(d) 15

14. 1! × 2! × 3! × 4! × 5! × - - - - - - - × 50!

(a) 235

(b) 12

(c) 262

(d) 105

15. $1_1 \times 2_2 \times 3_3 \times 4_4 \times 5_5 \times 6_6 \times 7_7 \times 8_8 \times 9_9 \times 10_{10}$.

(a) 25

(b) 15

(c) 10

(d) 20

16. 100! × 200!

(a) 49

(b) 73

(c) 132

(d) 33

ANSWER KEY

1. (b)
2. (c)
3. (a)
4. (b)
5. (b)
6. (c)
7. (b)
8. (b)
9. (d)
10. (d)
11. (c)
12. (b)
13. (d)
14. (c)
15. (b)
16. (b)

Solutions and Shortcuts

1. [157/5] = 31. [31/5] = 6. [6/5] = 1. 31 + 6 + 1 = 38. Option (b) is correct.

2. Number of 2's in 157! = [157/2] + [157/4] + [157/8]… + [157/128] = 78 + 39 + 19 + 9 + 4 + 2 + 1 = 152. Hence, the number of 22s would be [152/2] = 76.

   Number of 3's in 157! = 52 + 17 + 5 +1 = 75.

   The answer would be given by the lower of these values. Hence, 75 Option (c) is correct.

3. From the above solution:

   Number of 2's in 157! = 152

   Number of 32s in 157! = [75/2] = 37.

   Hence, Option **(a)** is correct.

4. 2520 = 7 × 3^2 × 2^3 × 5

   The value of *n* would be given by the value of the number of 7*s* in 50!

   This value is equal to [50/7] + [50/49] = 7 + 1 = 8

   Option (b) is correct.

5. 12600 = 7 × 32 × 2^3 × 5^2

   The value of 'n' would depend on which of number of 7's and number of 52s is lower in 50!.

   Number of 7's in 50! = 8. (**Note:** If we check for 7's we do not need to check for 3^2s as there would be at least two 3's before a 7 comes in every factorial's value. Similarly, there would always be at least three 2's before a 7 comes in any factorial's value. Thus, the number of 32s and the number of 23s can never be lower than the number of 7's in any factorial's value.)

Number of 5s in 50! = 10 + 2 = 12. Hence, the number of 52s in 50! = [12/2] = 6.

6 will be the answer as the number of 52s is lower than the number of 7's.

Option (b) is correct.

6. $720 = 2^4 \times 5^1 \times 3^2$

   In 77! there would be 38 + 19 + 9 + 4 + 2 + 1 = 73 twos → hence [73/4] = 18 2^4s

   In 77! there would be 25 + 8 + 2 = 35 threes → hence [35/2] = 17 3^2s

   In 77! there would be 15 + 3 = 18 fives

   Since 17 is the least of these values, Option (c) is correct.

7. In the expression given, there are three 7's and more than three 2's and 3's. Thus, Option (b) is correct.

8. Checking for the number of 2's, 3's and 5's in the given expression, you can see that the minimum is for the number of 3's (there are 11 of them while there are 12 5's and more than 11 2's) Hence, Option (b) is correct.

9. The number of 7's in the number is 6, while there are more than six 3's. Hence, Option (d) is correct.

10. The number of zeroes would depend on the number of 5's in the value of the factorial.

72! → 14 + 2 = 16. Option (d) is correct.

11. The number of zeroes would depend on the number of 5's in the value of the factorial.

77! × 42! → 15 + 3 = 18 (for 77!) and 8 + 1 = 9 (for 42!).

Thus, the total number of zeroes in the given expression would be 18 + 9 = 27. Option (c) is correct.

12. The number of zeroes would depend on the number of 5's in the value of the factorial.

100! would end in 20 + 4 = 24 zeroes

200! Would end in 40 + 8 + 1 = 49 zeroes.

When you add the two numbers (one with 24 zeroes and the other with 49 zeroes at its end), the resultant total would end in 24 zeroes. Option (b) is correct.

13. The given expression has fifteen 2's and seventeen 5's. The number of zeroes would be 15 as the number of 2's is lower in this case. Option (d) is correct.

14. 1! to 4! would have no zeroes while 5! to 9! all the values would have 1 zero. Thus, a total of 5 zeroes till 9!. Going further 10! to 14! would have two zeroes each — so a total of 10 zeroes would come out of the product of 10! × 11! × 12! × 13! × 14!.

Continuing this line of thought further we get:

Number of zeroes between 15! × 16!... × 19! = 3 + 3 + 3 + 3 + 3 = 3 × 5 = 15

Number of zeroes between 20! × 21!... × 24! = 4 × 5 = 20

Number of zeroes between 25! × 26!... × 29! = 6 × 5 = 30

Number of zeroes between 30! × 31!... × 34! = 7 × 5 = 35

Number of zeroes between 35! × 36!... × 39! = 8 × 5 = 40

Number of zeroes between 40! × 41!... × 44! = 9 × 5 = 45

Number of zeroes between 45! × 46!... × 49! = 10 × 5 = 50

Number of zeroes for 50! = 12

Thus, the total number of zeroes for the expression 1! × 2! × 3! …. × 50! = 5 + 10 +15 + 20 + 30 + 35 + 40 + 45 + 50 + 12 = 262 zeroes. Option (c) is correct.

15. The number of 5's is 15 while the number of 2's is much more. Option (b) is correct.

16. The number of zeroes would depend on the number of 5's in the value of the factorial.

100! would end in 20 + 4 = 24 zeroes

200! would end in 40 + 8 + 1 = 49 zeroes.

When you multiply the two numbers (one with 24 zeroes and the other with 49 zeroes at its end), the resultant total would end in 24 + 49 = 73 zeroes. Option (b) is correct.

Co-Prime or Relatively Prime Numbers Two or more numbers that do not have a common factor are known as co-prime or relatively prime. In other words, these numbers have a highest common factor of unity.

If two numbers m and n are relatively prime and the natural number x is divisible by both m and n independently then the number x is also divisible by mn.

Key Concept 1: The spotting of two numbers as co-prime has a very important implication in the context of the two numbers being in the denominators of fractions.

The concept is again best understood through an example:

Suppose, you are doing an operation of the following format — $M/8 + N/9$ where M and N are integers.

What are the chances of the result being an integer, if M is not divisible by 8 and N is not divisible by 9? A little bit of thought will make you realise that the chances are zero. The reason for this is that 8 and 9 are co-prime and the decimals of co-prime numbers never match each other.

Note: This will not be the case in the case of: $M/3 + N/27$

In this case, even if 3 and 27 are not dividing M and N respectively, there is a possibility of the values of M and N being such that you have an integral answer.

For instance: $5/3 + 36/27 = 81/27 = 3$

The result will never be integral if the two denominators are co-prime.

Note: This holds true even for expressions of the nature $A/7 - B/6$, etc.

This has huge implications for problem solving especially in the case of solving linear equations related to word based problems. Students are advised to try to use these throughout Blocks I, II and III of this book.

Example: Find all five-digit numbers of the form $34x5y$ that are divisible by 36.

Solution: 36 is a product of two co-primes 4 and 9. Hence, if $34x5y$ is divisible by 4 and 9, it will also be divisible by 36. Hence, for divisibility by 4, we have that the value of y can be 2 or 6. Also, if y is 2 the number becomes $34x52$. For this to be divisible by 9, the addition of $3 + 4 + x + 5 + 2$ should be divisible by 9. For this, x can be 4.

Hence, the number 34452 is divisible by 36.

Also for $y = 6$, the number $34x56$ will be divisible by 36 when the addition of the digits is divisible by 9. This will happen when x is 0 or 9. Hence, the numbers 34056 and 34956 will be divisible by 36.

Key Concept 2: The prime factors of the numbers, is another way to check whether the numbers under consideration are co-prime or not. For two or more co-prime numbers, their prime factors would have no commonality between themselves. Thus, if we have A and B co-prime, and $A = 2n \times 3m$, then it necessarily follows that in the prime factors of B, there would be no 2 or 3. In other words, B would be created out of prime numbers like 5,7,11 and so on, but cannot have a 2 or 3 in it.

EXERCISE FOR SELF-PRACTICE

1. Find all numbers of the form $56x3y$ that are divisible by 36.
2. Find all numbers of the form $72xy$ that are divisible by 45.
3. Find all numbers of the form $135xy$ that are divisible by 45.
4. Find all numbers of the form $517xy$ that are divisible by 89.

Divisibility Rules

Divisibility by 2 or 5: A number is divisible by 2 or 5 if the last digit is divisible by 2 or 5.

Divisibility by 3 or 9: all such numbers, the sum of whose digits are divisible by 3 (or 9) are divisible by 3 (or 9).

Divisibility by 4: A number is divisible by 4 if the last 2 digits are divisible by 4.

Divisibility by 6: A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 8: A number is divisible by 8 if the last 3 digits of the number are divisible by 8.

Divisibility by 11: A number is divisible by 11, if the difference of the sum of the digits in the odd places and the sum of the digits in the even places is zero or is divisible by 11.

Divisibility by 12: All numbers divisible by 3 and 4 are divisible by 12.

Divisibility by 7, 11 or 13: The integer n is divisible by 7, 11 or 13 if and only if the difference of the number of its thousands and the remainder of its division by 1000 is divisible by 7, 11 or 13.

For example: 473312 is divisible by 7 since the difference between 473 – 312 = 161 is divisible by 7.

Even Numbers: All integers that are divisible by 2 are even numbers. They are also denoted by $2n$s.

Example: 2, 4, 6, 12, 122, –2, –4, –12.

Also note that zero is an even number.

2 is the lowest positive even number.

Odd Numbers: All integers that are not divisible by 2 are odd numbers. Odd numbers leave a remainder of 1 on being divided by 2. They are denoted by $2n + 1$ or $2n - 1$.

Lowest positive odd number is 1.

Example: –1, –3, –7, –35, 3, 11, etc.

Complex Numbers: The arithmetic combination of real numbers and imaginary numbers are called complex numbers.

Alternately: All numbers of the form $a + ib$, where $i = \frac{12\times12+12\times11+5\times12+5\times11}{12}$ are called complex numbers.

Twin Primes: A pair of prime numbers are said to be twin prime when they differ by 2.

Example: 3 and 5 are twin primes, so also are 11 and 13.

Perfect Numbers: A number n is said to be a perfect number if the sum of all the divisors of n (including n) is equal to $2n$.

Example: 6 = 1 × 2 × 3 sum of the divisors = 1 + 2 + 3 + 6 = 12 = 2 × 6

28 = 1, 2, 4, 7, 14, 28, = 56 = 2 × 28

Task for student: Find all perfect numbers below 1000.

Mixed Numbers: A number that has both an integral and a fractional part is known as a mixed number.

Triangular Numbers: A number which can be represented as the sum of consecutive natural numbers starting with 1 are called as triangular numbers.

e.g.: 1 + 2 + 3 + 4 = 10. So, 10 is a triangular number.

Certain Rules

1. Of n consecutive whole numbers $a, a + 1 \ldots a + n - 1$, one and only one is divisible by n.

2. Mixed numbers: A number that has both the integral and fractional parts is known as mixed number.

3. if a number n can be represented as the product of two numbers p and q, that is, $n = p \cdot q$, then we say that the number n is divisible by p and by q and each of the numbers p and q is a divisor of the number n. Also, each factor of p and q would be a divisor of n.

4. Any number n can be represented in the decimal system of numbers as

   $N = a_k \times 10_k + a_{k-1} \times 10_{k-1} + \ldots + a_i \times 10 + a_0$

   Example: 2738 can be written as: $2 \times 10_3 + 7 \times 10_2 + 3 \times 10_1 + 8 \times 10_0$

5. 3_n will always have an even number of tens. (Example: 2 in 27,8 in 81,24 in 243,72 in 729 and so on.)

6. A sum of 5 consecutive whole numbers will always be divisible by 5.

7. The difference between 2 two digit numbers:

   $(xy) - (yx)$ will be divisible by 9

8. The square of an odd number when divided by 8 will always leave a remainder of 1.

9. The product of 3 consecutive natural numbers is divisible by 6.

10. The product of 3 consecutive natural numbers is the first of which is even is divisible by 24.

11. Products:

    Odd × odd = odd

Odd × even = even

Even × even = even

12. All numbers not divisible by 3 have the property that their square will have a remainder of 1 when divided by 3.

13. $(a^2 + b^2)/(b^2 + c^2) = (a^2/b^2)$ if $a/b = b/c$.

14. The product of any r consecutive integers (numbers) is divisible by $r!$

15. If m and n are two integers then $(m + n)!$ is divisible by $m!n!$

16. Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9. (for any number of digits)

17. Any number written in the form $10^n - 1$ is divisible by 3 and 9.

18. If a numerical expression contains no parentheses, first the operations of the third stage (involution or raising a number to a power) are performed, then the operations of the second stage (multiplication and division) and, finally, the operations of the first stage (addition and subtraction) are performed. In this case, the operations of one and the same stage are performed in the sequence indicated by the notation.

    If an expression contains parentheses, then the operation indicated in the parentheses are to be performed first and then all the remaining operations. In this case, operations of the numbers in parentheses as well as standing without parentheses are performed in the order indicated above.

If a fractional expression is evaluated, then the operations indicated in the numerator and denominator of the function are performed and the first result is divided by the second.

19. $(a)^n/(a + 1)$ leaves a remainder of

    a if n is odd

    1 if n is even

20. $(a + 1)^n/a$ will always give a remainder of 1.

21. For any natural number n, n^5 has the same units digit as n has.

22. For any natural number: $n^3 - n$ is divisible by 6.

23. The expression $\frac{1\times2\times3\times4\times\cdots(n-1)}{n}$ gives a remainder of $(n - 1)$ if n is prime.

    In case n is composite, the remainder would be 0.

REMAINDER THEOREM

Consider the following question:

$$17 \times 23$$

Suppose you have to find the remainder of this expression when divided by 12.

We can write this as:

$$17 \times 23 = (12 + 5) \times (12 + 11)$$

Which when expanded gives us:

$$12 \times 12 + 12 \times 11 + 5 \times 12 + 5 \times 11$$

You will realise that, when this expression is divided by 12, the remainder will only depend on the last term above:

Thus, $\frac{12\times12+12\times11+5\times12+5\times11}{12}$ gives the same remainder as $\frac{5\times11}{12}$

Hence, 7.

This is the remainder when 17 × 23 is divided by 12.

Learning Point: In order to find the remainder of 17 × 23 when divided by 12, you need to look at the individual remainders of 17 and 23 when divided by 12. The respective remainders (5 and 11) will give you the remainder of the original expression when divided by 12.

Mathematically, this can be written as:

The remainder of the expression $[A \times B \times C + D \times E]/M$, will be the same as the remainder of the expression $[A_R \times B_R \times C_R + D_R \times E4_R]/M$.

Where A_R is the remainder when A is divided by M,

B_R is the remainder when B is divided by M,

C_R is the remainder when C is divided by M,

D_R is the remainder when D is divided by M and

E_R is the remainder when E is divided by M.

We call this transformation as the remainder theorem transformation and denote it by the sign $\xrightarrow{R}$

Thus, the remainder of

1421 × 1423 × 1425 when divided by 12 can be given as:

$$\frac{1421\times1423\times1425}{12} \xrightarrow{R} \frac{5\times7\times9}{12} = \frac{35\times9}{12}$$

$$\xrightarrow{R} \frac{11\times9}{12}$$

$\xrightarrow{R}$ gives us a remainder of 3.

In the above question, we have used a series of remainder theorem transformations (denoted by $\xrightarrow{R}$) and equality transformations to transform a difficult looking expression into a simple expression.

Try to solve the following questions on remainder theorem:

Find the remainder in each of the following cases:

1. 17 × 23 × 126 × 38 divided by 8.
2. 243 × 245 × 247 × 249 × 251 divided by 12.
3. $\frac{173 \times 261}{13} + \frac{248 \times 249 \times 250}{15}$
4. $\frac{1021 \times 2021 \times 3021}{14}$
5. $\frac{37 \times 43 \times 51}{7} + \frac{137 \times 143 \times 151}{9}$

USING NEGATIVE REMAINDERS

Consider the following question:

Find the remainder when: 14 × 15 is divided by 8.

The obvious approach in this case would be

$$\frac{14 \times 15}{8} \xrightarrow{R} \frac{6 \times 7}{8} = \frac{42}{8} \xrightarrow{R} 2$$

However there is another option by which you can solve the same question:

When 14 is divided by 8, the remainder is normally seen as + 6. However, there might be times when using the negative value of the remainder might give us more convenience. Which is why you should know the following process:

Concept Note: Remainders by definition are always non-negative. Hence, even when we divide a number like – 27 by 5 we say that the remainder is 3 (and not – 2). However, looking at the negative value of the remainder—it has its own advantages in Mathematics as it results in reducing calculations.

Thus, when a number like 13 is divided by 8, the remainder being 5, the negative remainder is – 3.

Note: It is in this context that we mention numbers like 13, 21, 29, etc., as $8n + 5$ or $8n - 3$ numbers.

Thus, $\frac{14 \times 15}{8}$ will give us $\frac{-2 \times -1}{8} R \rightarrow 2$.

Consider the advantage this process will give you in the following question:

$$\frac{51 \times 52}{53} \xrightarrow{R} \frac{-2 \times -1}{53} \xrightarrow{R} 2$$

(The alternative will involve long calculations. Hence, the principle is that you should use negative remainders wherever you can. They can make life much simpler!!!)

What if the Answer Comes Out Negative

For instance, $\frac{62 \times 63 \times 64}{66} \xrightarrow{R} \frac{-4 \times -3 \times -2}{66} R \rightarrow \frac{-24}{66}$

But, we know that a remainder of –24, equals a remainder of 42 when divided by 66. Hence, the answer is 42.

Of course, nothing stops you from using positive and negative remainders at the same time in order to solve the same question —

Thus, $\frac{17 \times 19}{9} \xrightarrow{R} \frac{(-1) \times (1)}{9}$ $R \to -1$ $R \to 8$

Dealing with Large Powers There are two tools which are effective in order to deal with large powers —

(a) If you can express the expression in the form $\frac{(ax+1)^n}{a}$, the remainder will become 1 directly. In such a case, no matter how large the value of the power n is, the remainder is 1.

For instance, $\frac{(37^{12635})}{9} \xrightarrow{R} \frac{(1^{12635})}{9} \xrightarrow{R}$ 1. In such a case, the value of the power does not matter.

(b) $\frac{(ax-1)^n}{a}$. In such a case using -1 as the remainder, it will be evident that the remainder will be $+1$ if n is even and it will be -1 (Hence $a - 1$) when n is odd.

e.g.: $\frac{31^{127}}{8} \xrightarrow{R} \frac{(-1)^{127}}{8} \xrightarrow{R} \frac{(-1)}{8} \xrightarrow{R} 7$

ANOTHER IMPORTANT POINT

Suppose you were asked to find the remainder of 14 divided by 4. It is clearly visible that the answer should be 2.

But consider the following process:

$14/4 = 7/2 \xrightarrow{R} 1$ (The answer has changed!!)

What has happened?

We have transformed 14/4 into 7/2 by dividing the numerator and the denominator by 2. The result is that the original remainder 2 is also divided by 2 giving us 1 as the remainder. In order to take care of this problem, we need to reverse the effect of the division of the remainder by 2. This is done by multiplying the final remainder by 2 to get the correct answer.

Note: In any question on remainder theorem, you should try to cancel out parts of the numerator and denominator as much as you can, since it directly reduces the calculations required.

APPLICATION OF REMAINDER THEOREM

Finding the last two digits of an expression:

Suppose you had to find the last 2 digits of the expression:

$$22 \times 31 \times 44 \times 27 \times 37 \times 43$$

The remainder the above expression will give when it is divided by 100 is the answer to the above question.

Hence, to answer the question above, find the remainder of the expression when it is divided by 100.

Solution:

$$\frac{22 \times 31 \times 44 \times 27 \times 37 \times 43}{100}$$

$$= \frac{22 \times 31 \times 11 \times 27 \times 37 \times 43}{25} \text{ (on dividing by 4)}$$

$$\xrightarrow{R} \frac{22 \times 6 \times 11 \times 2 \times 12 \times 18}{25} = \frac{132 \times 22 \times 216}{25}$$

$$\xrightarrow{R} \frac{7 \times 22 \times 16}{25}$$

$$= \frac{154 \times 16}{25} \xrightarrow{R} \frac{4 \times 16}{25} \xrightarrow{R} 14$$

Thus, the remainder being 14, (after division by 4). The actual remainder should be 56.

[Do not forget to multiply by 4 !!]

Hence, the last 2 digits of the answer will be 56.

Using negative remainders here would have helped further.

Note: Similarly finding the last three digits of an expression means finding the remainder when the expression is divided by 1000.

THE PRIME NUMBER DIVISOR RULE

This rule states that: If 'P' is a prime number then:

The remainder of the expression $\frac{A^{P-1}}{P}$ is 1. (Provided A is not a multiple of P)

For example: The remainder of $\frac{24^{82}}{83} = 1$

SPLITTING THE DENOMINATOR INTO CO-PRIME NUMBERS:

This is also sometimes referred to as the 'Chinese Remainder Theorem'. It is useful when you have to find the remainder when there is a large denominator, and no other short cuts are working. It is best explained through an example.

Suppose you were trying to find the remainder of $\frac{107^{1444}}{136}$ You can split the denominator into two co-prime numbers as 17 and 8.

First find the remainder of $\frac{107^{1444}}{17} \rightarrow \frac{5^{1444}}{17} =$

$\frac{5^{16n} \times 5^4}{17} \rightarrow \frac{1 \times 5^4}{17} \rightarrow \text{Remainder} = 13$. This means that 107^{1444} is a $17n+13$ number.

Next, find the remainder of $\frac{107^{1444}}{8} \rightarrow \frac{3^{1444}}{8} = \frac{3^{2n}}{8} \rightarrow$. This means that 107^{1444} is a $8n + 1$ number.

The next step is to find a number below 136 that is both a $17n + 13$ as well as an $8n + 1$ number. That number would be the answer.

The list of $17n + 13$ numbers below 136 is: 13, 30, 47, 64, 81, 98, 115 and 132. 81 can be seen to be an $8n + 1$ number too.

Thus, the correct answer is 81.

EXERCISE FOR SELF-PRACTICE

1. Find the remainder when 73 + 75 + 78 + 57 + 197 is divided by 34.

   (a) 32

   (b) 4

   (c) 15

   (d) 28

2. Find the remainder when 73 × 75 × 78 × 57 × 197 is divided by 34.

   (a) 22

   (b) 30

   (c) 15

   (d) 28

3. Find the remainder when 73 × 75 × 78 × 57 × 197 × 37 is divided by 34.

   (a) 32

   (b) 30

(c) 15

(d) 28

4. Find the remainder when 43^{197} is divided by 7.

(a) 2

(b) 4

(c) 6

(d) 1

5. Find the remainder when 51^{203} is divided by 7.

(a) 4

(b) 2

(c) 1

(d) 6

6. Find the remainder when 59^{28} is divided by 7.

(a) 2

(b) 4

(c) 6

(d) 1

7. Find the remainder when 67^{99} is divided by 7.

(a) 2

(b) 4

(c) 6

(d) 1

8. Find the remainder when 75^{80} is divided by 7.

(a) 4

(b) 3

(c) 2

(d) 6

9. Find the remainder when 41^{77} is divided by 7.

(a) 2

(b) 1

(c) 6

(d) 4

10. Find the remainder when 21^{875} is divided by 17.

(a) 8

(b) 13

(c) 16

(d) 9

11. Find the remainder when 54^{124} is divided by 17.

(a) 4

(b) 5

(c) 13

(d) 15

12. Find the remainder when 83^{261} is divided by 17.

(a) 13

(b) 9

(c) 8

(d) 2

13. Find the remainder when 25^{102} is divided by 17.

(a) 13

(b) 15

(c) 4

(d) 2

ANSWER KEY

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1. (b)
2. (a)
3. (a)
4. (d)
5. (a)
6. (b)
7. (d)

8. (a)

9. (c)

10. (b)

11. (a)

12. (d)

13. (c)

Solutions and Shortcuts

1. The remainder would be given by: (5 + 7 + 10 + 23 + 27)/34 = 72/34 → remainder = 4. Option (b) is correct.

2. The remainder would be given by: (5 × 7 × 10 × 23 × 27)/34 → 35 × 230 × 27/ 34 → 1 × 26 × 27/34 = 702/34 → remainder = 22. Option (a) is correct.

3. The remainder would be given by: (5 × 7 × 10 × 23 × 27 × 3)/34 → 35 × 230 × 27 × 3/34 → 1 × 26 × 81/34 → 26 × 13/34 = 338/34 → remainder = 32. Option (a) is correct.

4. $43^{197}/7 \rightarrow 1^{197}/7$ → remainder = 1. Option (d) is correct.

5. $51^{203}/7 \rightarrow 2^{203}/7 = (2^3)^{67} \times 2^2/7 = 8^{67} \times 4/7$ → remainder = 4. Option (a) is correct.

6. $59^{28}/7 \rightarrow 3^{28}/7 = (3^6)^4 \times 3^4/7 = 729^6 \times 81/7$ → remainder = 4. Option (b) is correct.

7. $67^{99}/7 \rightarrow 4^{99}/7 = (4^3)^{33}/7 = 64^{33}/7$ → remainder = 1. Option (d) is correct.

8. $75^{80}/7 \rightarrow 5^{80}/7 = (5^6)^{13} \times 5^2/7 \rightarrow 1^{13} \times 25/7$ → remainder = 4. Option (a) is correct.

9. $41^{77}/7 \rightarrow 6^{77}/7 \rightarrow$ remainder = 6 (as the expression is in the form $a^n/(a + 1)$. Option (c) is correct.

10. $21^{875}/17 \rightarrow 4^{875}/17 = (4^4)^n \times 4^3/17 = 256^n \times 64/17 \rightarrow 1^n \times 13/17 \rightarrow$ remainder = 13. Option (b) is correct.

11. $54^{124}/17 \rightarrow 3^{124}/17$. At this point, like in each of the other questions solved above, we need to plan the power of 3 which would give us a convenient remainder of either 1 or –1. As we start to look for remainders that powers of 3 would have when divided by 17, we get that at the power 3^6 the remainder is 15. If we convert this to –2, we will get that at the fourth power of 3^6, we should get a 16/17 situation (as $-2 \times -2 \times -2 \times -2 = 16$). This means that at a power of 3^{24}, we are getting a remainder of 16 or –1. Naturally then if we double the power to 3^{48}, the remainder would be 1.

With this thinking, we can restart solving the problem:

$3^{124}/17 = 3^{48} \times 3^{48} \times 3^{24} \times 3^4/17 \rightarrow 1 \times 1 \times 16 \times 81/17 \rightarrow 16 \times 13/17$
$= 208/17 \rightarrow$ remainder = 4. Option (a) is correct.

(**Note:** That if we are dividing a number by 17 and if we see the remainder as 15, we can logically say that the remainder is –2 — even though negative remainders are not allowed in Mathematics)

12. Using the logic developed in Question 11 above, we have $83^{261}/17 \rightarrow 15^{261}/17 \rightarrow$

$(-2)^{261}/17 \rightarrow (-2^4)^{65} \times (-2)/17 \rightarrow 16^{65} \times (-2)/17 \rightarrow (-1) \times (-2)/17 \rightarrow$ remainder = 2. Option (d) is correct.

13. $25^{102}/17 \rightarrow 8^{102}/17 = 2^{306}/17 = (2^4)^{76} \times 2^2/17 \rightarrow 16^{76} \times 4/17 \rightarrow 1 \times 4/17 \rightarrow$ remainder = 4. Option (c) is correct.

BASE SYSTEM

All the work that we carry out with numbers is called as the decimal system. In other words, we work in the decimal system. Why is it called decimal? It is because there are 10 digits in the system 0–9.

However, depending on the number of digits contained in the base system other number systems are also possible. Thus, a number system with base 2 is called the binary number system and will have only two digits 0 and 1. Some of the most commonly used systems are: Binary (base 2), Octal (base 8), Hexadecimal (base 16).

Binary system has 2 digits : 0, 1. Octal has 8 digits : – 0, 1, 2, 3, ... 7.

Hexadecimal has 16 digits – 0, 1, 2, ... 9, *A*, *B*, *C*, *D*, *E*, *F*.

Where *A* has a value 10, *B* = 11 and so on.

Before coming to the questions asked under this category, let us first look at a few issues with regard to converting numbers between different base systems.

1. Conversion from any base system into decimal:

Suppose you have to write the decimal equivalent of the base 8 number 146_8.

In such a case, follow the following structure for conversion:

$$146_8 = 1 \times 8^2 + 4 \times 8^1 + 6 \times 8^0$$

$$= 64 + 32 + 6 = 102$$

Note: If you remember the process, for writing the value of any random number, say 146, in our decimal system (base 10), we use: $1 \times 10^2 + 4 \times 10^1 + 6 \times 10^0$. All you need to change, in case you are trying to write the value of the number in base 8, is that you replace 10 with 8 in every power.

Try to write the decimal equivalents of the following numbers:

143_5, 143_6, 143_7, 143_8, 143_9

1256_7, 1256_8, 1256_9

2. Conversion of a number in decimals into any base:

Suppose you have to find out the value of the decimal number 347 in base 6. The following process is to be adopted:

Step 1: Find the highest power of the base (6 in this case) that is contained in 347. In this case, you will realise that the value of $6^3 = 216$ is contained in 347, while the value of $6^4 = 1296$ is not contained in 347. Hence, we realise that the highest power of 6 contained in 347 is 3. This should make you realise that the number has to be constructed by using the powers 6^0, 6^1, 6^2, 6^3 respectively. Hence, a 4-digit number.

Structure of number: – – – –

Step 2: We now need to investigate, how many times each of the powers of 6 is contained in 347. For this, we first start with the highest power as found above. Thus we can see that 6^3 (216) is contained in 347 once. Hence, our number now becomes:

1 – – –

That is, we now know that the first digit of the number is 1. Besides, when we have written the number 1 in this place, we have accounted for a value of 216 out of 347. This leaves us with 131 to account for.

We now need to look for the number of times 6^2 is contained in 131. We can easily see that $6^2 = 36$ is contained in 131 three times. Thus, we write 3 as the next digit of our number which will now look like:

1 3 3 –

In other words, we now know that the first three digits of the number are 133. Besides, when we have written the number 3 in this place, we have accounted for a value of 18 out of the 23 which was left to be accounted for. This leaves us with 23 – 18 = 5 to account for.

The last digit of the number corresponds to $6^0 = 1$. In order to make a value of 5 in this place, we will obviously need to use this power of 6, 5 times, thus, giving us the final digit as 5. Hence, our number is:

1 3 3 5

A few points you, should know about base systems:

(i) In single digits there is no difference between the value of the number—whichever base we take. For example, the equality $5_6 = 5_7 = 5_8 = 5_9 = 5_{10}$.

(ii) Suppose you have a number in base x. When you convert this number into its decimal value, the value should be such that when it is divided by x, the remainder should be equal to the units digit of the number in base x.

In other words, 342_8 will be a number of the form $8n + 2$ in base 10. You can use this principle for checking your conversion calculations.

Illustrations

1. The number of x digit numbers in nth base system will be

(a) n^x

(b) $n^x - 1$

(c) $n^x - n$

(d) $n^x - n^{(x-1)}$

Solution: Base → n, digit → x

So, required number of numbers = $n^x - n^{(x-1)}$

2. The number of 2 digit numbers in binary system is

(a) 2

(b) 90

(c) 10

(d) 4

Solution: By using the formula, we get the required number of numbers = $2^2 - 2^1$ = 2

⇒ Option (a)

3. The number of 5 digit numbers in binary system is

(a) 48

(b) 16

(c) 32

(d) 20

Solution: Required number of numbers = $2^5 - 2^4$ = 32 – 16 = 16

⇒ Option (b)

4. I celebrate my birthday on 12th January on earth. On which date would I have to celebrate my birthday if I were on a planet where binary system is being used for counting. (The number of days, months and years are same on both the planets.)

(a) 11th Jan

(b) 111th Jan

(c) 110th Jan

(d) 1100th Jan

Solution: On earth (decimal system is used).

12th Jan ⇒ 12th Jan

The 12th day on the planet where binary system is being used will be called

$(12)_{10} = (?)_2$

$$= \frac{1}{2^3}\frac{1}{2^2}\frac{0}{2^1}\frac{0}{2^0}$$

i.e., 1100th day on that planet

so, 12th January on earth = 1100th January on that planet

⇒ Option (d)

5. My year of birth is 1982. What would the year have been instead of 1982 if base 12 were used (for counting) instead of decimal system?

(a) 1182

(b) 1022

(c) 2082

(d) 1192

Solution: The required answer will be equal to $(1982)_{10} = (?)_{12}$.

$$= \frac{1}{12^3}\frac{1}{12^2}\frac{9}{12^1}\frac{2}{12^0} \rightarrow$$

$$= \frac{1}{12^3}\frac{1}{12^2}\frac{9}{12^1}\frac{2}{12^0} \rightarrow$$

$1 \times 12_3 + 1 \times 12_2 + 9 \times 12_1 + 2 \times 9_0 = 1728 + 144 + 108 + 2 = 1982$

Hence, the number $(1192)_{12}$ represents 1982 in our base system.

⇒ Option (d)

6. 203 in base 5 when converted to base 8, becomes

(a) 61

(b) 53

(c) 145

(d) 65

Solution: $(203)_5 = (?)_{10}$

$$= 2 \times 5_2 + 0 \times 5_1 + 3 \times 5_0$$

$$= 50 + 0 + 3 = 53$$

Now,

$$(53)_{10} = (?)8$$
$$= \frac{6}{8^1}\frac{5}{8^0}$$
$$= (203)_5 = (65)_8$$

⇒ Option (d)

7. $(52)_7 + 46_8 = (?)_{10}$

(a) $(75)_{10}$

(b) $(50)_{10}$

(c) $(39)_{39}$

(d) $(28)_{10}$

Solution: $(52)_7 = (5 \times 7_1 + 2 \times 7_0)_{10} = (37)_{10}$

also, $(46)_8 = (4 \times 8_1 + 6 \times 8_0)_{10} = (38)_{10}$

sum = $(75)_{10}$

⇒ Option (a)

8. $(23)_5 + (47)_9 = (?)_8$

(a) 70

(b) 35

(c) 64

(d) 18

Solution: $(23)_5 = (2 \times 5_1 + 3 \times 5_0)_{10} = (13)_{10}$

also, $(47)_9 = (4 \times 9_1 + 7 \times 9_0)_{10} = (43)_{10}$

sum = $(13)_{10} + (43)_{10} = (56)_{10} \rightarrow (70)_8$

⇒ Option (a)

9. $(11)_2 + (22)_3 + (33)_4 + (44)_5 + (55)_6 + (66)_7 + (77)_8 + (88)_9 = (?)_{10}$

(a) 396

(b) 276

(c) 250

(d) 342

Solution: $(11)_2 = (1 \times 2^1 + 1 \times 2^0)_{10} = (3)_{10}$

$(22)_3 = (2 \times 3^1 + 2 \times 3^0)_{10} = (8)_{10}$

$(33)_4 = (3 \times 4^1 + 3 \times 4^0)_{10} = (15)_{10}$

$(44)_5 = (4 \times 5^1 + 4 \times 5^0)_{10} = (24)_{10}$

$(55)_6 = (5 \times 6^1 + 5 \times 6^0)_{10} = (35)_{10}$

$(66)_7 = (6 \times 7^1 + 6 \times 7^0)_{10} = (48)_{10}$

$(77)_8 = (7 \times 8^1 + 7 \times 8^0)_{10} = (63)_{10}$

$(88)_9 = (8 \times 9^1 + 8 \times 9^0)_{10} = (80)_{10}$

sum = $(276)_{10}$

⇒ Option (b)

10. $(24)_5 \times (32)_5 = (?)_5$

(a) 1423

(b) 1422

(c) 1420

(d) 1323

Solution: $(24)_5 = 14_{10}$ and $32_5 = 17_{10}$. Hence, the required answer can be obtained by $14 \times 17 = 238_{10} = 1 \times 5^3 + 4 \times 5^2 + 2 \times 5^1 + 3 \times 5^0$ → 1423 as the correct answer.

Alternately, you could multiply directly in base 5 as follows:

$$\begin{array}{r} (2\ 4) \\ \times\quad (3\ 2) \\ \hline (1\ 4\ 2\ 3) \end{array}$$

Unit's digit of the answer would correspond to: $4 \times 2 = 8 \rightarrow 13_5$. Hence, we write 3 in the units place and carry over 1.

(**Note:** That in this process when we are doing 4×2, we are effectively multiplying individual digits of one number with individual digits of the other number. In such a case, we can write $4 \times 2 = 8$ by assuming that both the numbers are in decimal system as the value of a single digit in any base is equal.)

The tens digit will be obtained by: $2 \times 2 + 4 \times 3 = 16 + 1 = 17 \rightarrow 32_5$

Hence, we write 2 in the tens place and carry over 3 to the hundreds place.

Where we get $3 \times 2 + 3 = 9 \rightarrow 14$

Hence, the answer is 14.

⇒ Option (a)

11. In base 8, the greatest four digit perfect square is

(a) 9801

(b) 1024

(c) 8701

(d) 7601

Solution In base 10, the greatest 4 digit perfect square = 9801

In base 9, the greatest 4 digits perfect square = 8701

In base 8, the greatest 4 digits perfect square = 7601

Alternately, multiply $(77)_8 \times (77)_8$ to get 7601 as the answer.

I would encourage the student to try to multiply these two numbers in base 8 to get the value.

Unit's Digit

(A) The unit's digit of an expression can be calculated by getting the remainder while the expression is divided by 10.

Thus, for example if we have to find the units digit of the expression:

17 × 22 × 36 × 54 × 27 × 31 × 63

We try to find the remainder –

$$\frac{17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63}{10}$$

$$\xrightarrow{R} \frac{7 \times 2 \times 6 \times 4 \times 7 \times 3}{10}$$

$$= \frac{14 \times 24 \times 21}{10} \xrightarrow{R} \frac{4 \times 4 \times 1}{10} = \frac{16}{10} \xrightarrow{R} 6$$

Hence, the required answer is 6.

This could have been directly obtained by multiplying: 7 × 2 × 6 × 4 × 7 × 1 × 3 and only accounting for the unit's digit.

(B) Unit's digits in the contexts of powers —

Study the following table carefully.

Unit's digit when 'N' is raised to a power

Number Ending With	*Value of power* 1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2
3	3	9	7	1	3	9	7	1	3
4	4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	9	3	1	7
8	8	4	2	6	8	4	2	6	8
9	9	1	9	1	9	1	9	1	9
0	0	0	0	0	0	0	0	0	0

In the table above, if you look at the columns corresponding to the power 5 or 9 you will realise that the unit's digit for all numbers is repeated (i.e. it is 1 for 1, 2 for, 3 for 3....9 for 9).

This means that whenever we have any number whose unit's digit is 'x' and it is raised to a power of the form 4n + 1, the value of the unit's digit of the answer will be the same as the original units digit.

Illustrations: $(1273)^{101}$ will give a unit's digit of 3. $(1547)^{25}$ will give a unit's digit of 7 and so forth.

Thus, the above table can be modified into the form –

Value of power

Number Ending in	*If the value of the Power is* 4n + 1	4n + 2	4n + 3	4n
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

[Remember, at this point that we had said (in the Back to School section of Part 1) that all natural numbers can be expressed in the form $4n + x$. Hence, with the help of the logic that helps us build this table, we can easily derive the units digit of any number when it is raised to a power.]

A Special Case

Question: What will be the unit's digit of $(1273)^{122!}$?

Solution: 122! is a number of the form $4n$. Hence, the answer should be 1. [**Note:** 1 here is derived by thinking of it as 3 (for $4n + 1$), 9 (for $4n + 2$), 7 (for $4n + 3$), 1(for $4n$)]

EXERCISE FOR SELF-PRACTICE

Find the unit's digit in each of the following cases:

1. $2^2 \times 4^4 \times 6^6 \times 8^8$
2. $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \ldots \times 100^{100}$
3. $17 \times 23 \times 51 \times 32 + 15 \times 17 \times 16 \times 22$
4. $13 \times 17 \times 22 \times 34 + 12 \times 6 \times 4 \times 3 - 13 \times 33$
5. $37^{123} \times 43^{144} \times 57^{226} \times 32^{127} \times 52^{5!}$
6. $67 \times 37 \times 43 \times 91 \times 42 \times 33 \times 42$

   (a) 2

   (b) 6

   (c) 8

   (d) 4
7. $67 \times 35 \times 43 \times 91 \times 47 \times 33 \times 49$

   (a) 1

   (b) 9

   (c) 5

   (d) 6

8. $67 \times 35 \times 45 \times 91 \times 42 \times 33 \times 81$

(a) 2

(b) 4

(c) 0

(d) 8

9. $67 \times 35 \times 45 + 91 \times 42 \times 33 \times 82$

(a) 8

(b) 7

(c) 0

(d) 5

10. $(52)^{97} \times (43)^{72}$

(a) 2

(b) 6

(c) 8

(d) 4

11. $(55)^{75} \times (93)^{175} \times (107)^{275}$

(a) 7

(b) 3

(c) 5

(d) 0

12. $(173)^{45} \times (152)^{77} \times (777)^{999}$

(a) 2

(b) 4

(c) 8

(d) 6

13. $81 \times 82 \times 83 \times 84 \times 86 \times 87 \times 88 \times 89$

(a) 0

(b) 6

(c) 2

(d) 4

14. $82^{43} \times 83^{44} \times 84^{97} \times 86^{98} \times 87^{105} \times 88^{94}$

(a) 2

(b) 6

(c) 4

(d) 8

15. $432 \times 532 + 532 \times 974 + 537 \times 531 + 947 \times 997$

(a) 5

(b) 6

(c) 9

(d) 8

ANSWER KEY

6. (d)
7. (c)
8. (c)
9. (b)
10. (a)
11. (c)
12. (c)
13. (b)
14. (b)
15. (d)

Solutions and Shortcuts

1. The unit's digit would be given by the unit's digit of the multiplication of 4 × 6 × 6 × 6 = 4
2. 0
3. 7 × 3 × 1 × 2 + 0 → 2 + 0 = 2
4. 8 + 4 – 9 → 3
5. 3 × 1 × 9 × 8 × 6 = 6
6. 7 × 7 × 3 × 1 × 2 × 3 × 2 = 4
7. Since we have a 5 multiplied with odd numbers, the units digit would naturally be 5.
8. 5 × 2 → 0
9. 5 + 2 → 7

10. $2 \times 1 \rightarrow 2$

11. $5 \times 7 \times 3 \rightarrow 5$

12. $3 \times 2 \times 3 \rightarrow 8$

13. $2 \times 3 \times 4 \times 6 \times 7 \times 8 \times 9 \rightarrow 6$

14. $8 \times 1 \times 4 \times 6 \times 7 \times 4 \rightarrow 6$

15. $4 + 8 + 7 + 9 \rightarrow 8$

WORKED-OUT PROBLEMS

Problem 1.1 Find the number of zeroes in the factorial of the number 18.

Solution 18! contains 15 and 5, which combined with one even number give zeroes. Also, 10 is also contained in 18!, which will give an additional zero. Hence, 18! contains 3 zeroes and the last digit will always be zero.

Problem 1.2 Find the numbers of zeroes in 27!

Solution $27! = 27 \times 26 \times 25 \times \ldots \times 20 \times \ldots \times 15 \times \ldots \times 10 \times \ldots \times 5 \times \ldots \times 1$

A zero can be formed by combining any number containing 5 multiplied by any even number. Similarly, everytime a number ending in zero is found in the product, it will add an additional zero. For this problem, note that $25 = 5 \times 5$ will give 2 zeroes and zeroes will also be obtained by 20, 15, 10 and 5. Hence 27! will have 6 zeroes.

Short-cut method: Number of zeroes is $27! \rightarrow [27/5] + [27/25]$

where $[x]$ indicates the integer just lower than the fraction

Hence, $[27/5] = 5$ and $[27/5^2] = 1$, 6 zeroes

Problem 1.3 Find the number of zeroes in 137!

Solution $[137/5] + [137/5^2] + [137/5^3]$

$= 27 + 5 + 1 = 33$ zeroes

(Since the restriction on the number of zeroes is due to the number of fives.)

EXERCISE FOR SELF-PRACTICE

Problem

Find the number of zeroes in

(a) 81!

(b) 100!

(c) 51!

Answers

(a) 19

(b) 24

(c) 12

Problem 1.4 What exact power of 5 divides 87!?

Solution $[87/5] + [87/25] = 17 + 3 = 20$

Problem 1.5 What power of 8 exactly divides 25!?

Solution If 8 were a prime number, the answer should be $[25/8] = 3$. But since 8 is not prime, use the following process.

The prime factors of 8 is $2 \times 2 \times 2$. For divisibility by 8, we need three twos. So, everytime we can find 3 twos, we add one to the power of 8 that divides 25! To count how we get 3 twos, we do the following. All even numbers will give one 'two' at least $[25/2] = 12$

Also, all numbers in 25! divisible by 2^2 will give an additional two $[25/2^2] = 6$

Further, all numbers in 25! divisible by 2^3 will give a third two. Hence $[25!/2^3] = 3$

And all numbers in 25! divisible by 2^4 will give a fourth two. Hence $[25!/2^4] = 1$

Hence, total number of twos in 25! is 22. For a number to be divided by 8, we need three twos. Hence, since 25! has 22 twos, it will be divided by 8 seven times.

Problem 1.6 What power of 15 divides 87! exactly?

Solution $15 = 5 \times 3$. Hence, everytime we can form a pair of one 5 and one 3, we will count one.

87! contains – $[87/5] + [87/5^2] = 17 + 3 = 20$ fives

Also 87! contains – $[87/3] + [87/3^2] + [87/3^3] + [87/3^4] = 29 + \ldots$(more than 20 threes).

Hence, 15 will divide 87! twenty times since the restriction on the power is because of the number of 5s and not the number of 3s.

In fact, it is not very difficult to see that in the case of all factors being prime, we just have to look for the highest prime number to provide the restriction for the power of the denominator.

Hence, in this case we did not need to check for anything but the number of 5s.

EXERCISE FOR SELF-PRACTICE

(a) What is the highest power of 30 that will exactly divide 120!

Hint: $[128/5] + [128/5^2] + [128/5^3]$

(b) What is the highest power of 210 that will exactly divide 142!

Problem 1.7 Find the last digit in the expression $(36472)^{123!} \times (34767)^{76!}$.

Solution If we try to formulate a pattern for 2 and its powers and their units digit, we see that the units digit for the powers of 2 goes as: 2, 4, 8, 6, 2, 4, 8, 6, 2, 4, 8, 6 and so on. The number 2 when raised to a power of $4n + 1$ will always give a units digit of 2. This also means that the units digit for 2^{4n} will always end in 6. The power of 36472 is 123! . 123! can be written in the form 4n. Hence, $(36472)^{123!}$ will end in 6.

The second part of the expression is $(34767)^{76!}$. The units digit depends on the power of 7. If we try to formulate a pattern for 7 and its powers and their units digit, we see that the units digit for the powers of 7 go as: 7 9 3 1 7 9 3 1 and so on. This means that the units digit of the expression 7^{4n} will always be 1.

Since 76! can be written as a multiple of 4 as $4n$, we can conclude that the unit's digit in $(34767)^{76!}$ is 1.

Hence, the units digit of $(36472)^{123!} \times (34767)^{76!}$ will be 6.

Counting

Problem 1.8 Find the number of numbers between 100 to 200 if

(i) Both 100 and 200 are counted

(ii) Only one of 100 and 200 is counted

(iii) Neither 100 nor 200 is counted

Solution

(i) Both ends included-solution: 200 – 100 + 1 = 101

(ii) One end included-solution: 200 – 100 = 100

(iii) Both ends excluded-solution: 200 – 100 – 1 = 99

Problem 1.9 Find the number of even numbers between 122 and 242 if

(i) Both ends are included

(ii) Only one end is included

(iii) Neither end is included

Solution

(i) Both ends included—solution: (242 – 122)/2 = 60 + 1 = 61

(ii) One end included-solution: (242 – 122)/2 = 60

(iii) Both ends excluded-solution: (242 – 122)/2 – 1 = 59

EXERCISE FOR SELF-PRACTICE

(a) Find the number of numbers between 140 to 259, both included, which are divisible by 7.

(b) Find the number of numbers between 100 to 200, that are divisible by 3.

Problem 1.10 Find the number of numbers between 300 to 400 (both included), that are not divisible by 2, 3, 4, and 5.

Solution Total numbers: 101

Step 1: Not divisible by 2 = All even numbers rejected: 51

Numbers left: 50

Step 2: Of which: divisible by 3 = first number 300, last number 399. But even numbers have already been removed, hence count out only odd numbers between 300 and 400 divisible by 3. This gives us that:

First number 303, last number 399, common difference 6

So, remove: [(399 – 303)/6] + 1 = 17

∴ 50 – 17 = 33 numbers left.

We do not need to remove additional terms for divisibility by 4 since this would eliminate only even numbers (which have already been eliminated).

Step 3: Remove from 33 numbers left all odd numbers that are divisible by 5 and not divisible by 3.

Between 300 to 400, the first odd number divisible by 5 is 305 and the last is 395 (since both ends are counted, we have 10 such numbers as: [(395 – 305)/10 + 1 = 10].

However, some of these 10 numbers have already been removed to get to 33 numbers.

Operation left: of these 10 numbers, 305, 315…395, reduce all numbers that are also divisible by 3. Quick perusal shows that the numbers start with 315 and have common difference 30.

Hence, [(Last number – First number)/Difference + 1] = [(375 – 315)/30 + 1] = 3

These 3 numbers were already removed from the original 100. Hence, for numbers divisible by 5, we need to remove only those numbers that are odd, divisible by 5 but not by 3. There are 7 such numbers between 300 and 400.

So numbers left are: 33 – 7 = 26.

EXERCISE FOR SELF-PRACTICE

Find the number of numbers between 100 to 400 which are divisible by either 2, 3, 5 and 7.

Problem 1.11 Find the number of zeroes in the following multiplication: 5 × 10 × 15 × 20 × 25 × 30 × 35 × 40 × 45 × 50.

Solution The number of zeroes depends on the number of fives and the number of twos. Here, close scrutiny shows that the number of twos is the constraint. The expression can be written as

$5 \times (5 \times 2) \times (5 \times 3) \times (5 \times 2 \times 2) \times (5 \times 5) \times (5 \times 2 \times 3) \times (5 \times 7) \times (5 \times 2 \times 2 \times 2) \times (5 \times 3 \times 3) \times (5 \times 5 \times 2)$

Number of 5s – 12, Number of 2s – 8

Hence: 8 zeroes.

Problem 1.12 Find the remainder for $[(73 \times 79 \times 81)/11]$.

Solution The remainder for the expression: $[(73 \times 79 \times 81)/11]$ will be the same as the remainder for $[(7 \times 2 \times 4)/11]$

That is, $56/11 \Rightarrow$ remainder = 1

Problem 1.13 Find the remainder for $(3^{560}/8)$.

Solution $(3^{560}/8) = [(3^2)^{280}/8] = (9^{280}/8)$

$= [9.9.9\ldots(280 \text{ times})]/8$

remainder for above expression = remainder for $[1.1.1\ldots(280 \text{ times})]/8 \Rightarrow$ remainder = 1.

Problem 1.14 Find the remainder when $(2222^{5555} + 5555^{2222})/7$.

Solution This is of the form: $[(2222^{5555})/7 + (5555^{2222})/7]$

We now proceed to find the individual remainder of : $(2222^{5555})/7$. Let the remainder be R1.

When 2222 is divided by 7, it leaves a remainder of 3.

Hence, for remainder purpose $(2222^{5555})/7 \xrightarrow{R} (3^{5555}/7) = (3.3^{5554})/7$

$= [3(3^2)^{2777}]/7 = [3.(7 + 2)^{2777}]/7 \xrightarrow{R} (3.2^{2777})/7 = (3.2^2 \cdot 2^{2775})/7 = [3.2^2 \cdot (2^3)^{925}]/7$

$= [3.2^2 \cdot (8)^{925}]/7 \xrightarrow{R} (12/7)$ Remainder = 5

Similarly, $(5555^{2222})/7 \xrightarrow{R} (4^{2222})/7 = [(2^2)^{2222}]/7 = (2)^{4444}/7 = (2.2^{4443})/7 = [2.(2^3)^{1481}]/7 = [2.(8)^{1481}]/7 \xrightarrow{R} [2.(1)^{1481}]/7 \rightarrow 2$ (remainder)

Hence, $(2222^{5555})/7 + (5555^{2222})/7 \xrightarrow{R} (5 + 2)/7 \Rightarrow$ Remainder = 0

Problem 1.15 Find the GCD and the LCM of the numbers 126, 540 and 630.

Solution The standard forms of the numbers are:

126 → 3 × 3 × 7 × 2 → 32 × 7 × 2

540 → 3 × 3 × 3 × 2 × 2 × 5 → 22 × 33 × 5

630 → 3 × 3 × 5 × 2 × 7 → 2 × 32 × 5 × 7

For GCD we use intersection of prime factors and the lowest power of all factors that appear in all three numbers. 2 × 32 = 18.

For LCM → Union of prime factors and highest power of all factors that appear in any one of the three numbers ⇒ 22 × 33 × 5 × 7 = 3780.

EXERCISE FOR SELF-PRACTICE

Find the GCD, and the LCM of the following numbers:

(i) 360, 8400

(ii) 120, 144

(iii) 275, 180, 372, 156

(iv) 70, 112

(v) 75, 114

(vi) 544, 720

Problem 1.16 The ratio of the factorial of a number *x* to the square of the factorial of another number, which when increased by 50% gives the required number, is

1.25. Find the number *x*.

(a) 6

(b) 5

(c) 9

(d) None of these

Solution Solve through options: Check for the conditions mentioned. When we check for option (a) we get6 ! = 720 and $(4!)_2$ = 576 and we have 6 !/$(4!)_2$ = 1.25, which is the required ratio.

Hence, the answer is (a)

Problem 1.17 Three numbers A, B and C are such that the difference between the highest and the second highest two-digit numbers formed by using two of A, B and C is 5. Also, the smallest two two-digit numbers differ by 2. If B > A > C, then what is the value of B?

(a) 1

(b) 6

(c) 7

(d) 8

Solution Since *B* is the largest digit, option (a) is rejected. Check for option (b).

If *B* is 6, then the two largest two-digit numbers are 65 and 60 (Since, their difference is 5) and we have $B = 6$, $A = 5$ and $C = 0$.

But with this solution, we are unable to meet the second condition. Hence (b) is not the answer. We also realise here that C cannot be 0.

Check for option (c).

B is 7, then the numbers. are 76 and 71 or 75 and 70. In both these cases, the smallest two two-digit numbers do not differ by 2.

Hence, the answer is not (c).

Hence, Option (d) is the answer.

[To confirm, put $B = 8$, then the solution $A = 6$ and $C = 1$ satisfies the 2nd condition.]

Problem 1.18 Find the remainder when $2851 \times (2862)^2 \times (2873)^3$ is divided by 23.

Solution We use the remainder theorem to solve the problem. Using the theorem, we see that the following expressions have the same remainder.

$$\Rightarrow \frac{2851 \times (2862)^2 \times (2873)^3}{23}$$

$$\Rightarrow \frac{22 \times 10 \times 10 \times 21 \times 21 \times 21}{23}$$

$$\Rightarrow \frac{22 \times 8 \times 441 \times 21}{23} \Rightarrow \frac{22 \times 21 \times 8 \times 4}{23}$$

$$\Rightarrow \frac{462 \times 32}{23} \Rightarrow \frac{2 \times 9}{23} \Rightarrow \text{Remainder is } 18$$

Problem 1.19 For what maximum value of n will the expression $\frac{10200!}{504^n}$ be an integer?

Solution For $\frac{10200!}{504^n}$ to be a integer, we need to look at the prime factors of 504 →

$$504 = 3^2 \times 7 \times 8 = 2^3 \times 3^2 \times 7$$

We, thus, have to look for the number of 7s, the number of 2^3s and the number of 3^2s that are contained in 10200!. The lowest of these will be the constraint value for n.

To find the number of 2^3s, we need to find the number of 2s as

$$\left[\frac{10200}{2}\right]+\left[\frac{10200}{4}\right]+\left[\frac{10200}{8}\right]+\left[\frac{10200}{16}\right]+\left[\frac{10200}{32}\right]$$
$$+\left[\frac{10200}{64}\right]+\left[\frac{10200}{128}\right]+\left[\frac{10200}{256}\right]+\left[\frac{10200}{512}\right]+\left[\frac{10200}{1024}\right]$$
$$+\left[\frac{10200}{2048}\right]+\left[\frac{10200}{4096}\right]+\left[\frac{10200}{8192}\right]$$

where [?] is the greatest integer function.

= 5100 + 2550 + 1275 + 637 + 318 + 159 + 79 + 39 + 19 + 9 + 4 + 2 + 1

Number of twos = 10192

Hence, number of 2^3 = 3397

Similarly, we find the number of 3s as

Number of threes = $\left[\frac{10200}{3}\right]+\left[\frac{10200}{9}\right]+\left[\frac{10200}{27}\right]$

$$\left[\frac{10200}{81}\right]+\left[\frac{10200}{243}\right]+\left[\frac{10200}{729}\right]+\left[\frac{10200}{2187}\right]+\left[\frac{10200}{6561}\right]$$

= 3400 + 1133 + 377 + 125 + 41 + 13 + 4 + 1

Number of threes = 5094

∴ Number of 3^2 = 2547

Similarly we find the number of 7s as

$$\left[\frac{10200}{7}\right]+\left[\frac{10200}{49}\right]+\left[\frac{10200}{343}\right]+\left[\frac{10200}{2401}\right]$$

= 1457 + 208 + 29 + 4 = 1698.

Thus, we have, 1698 sevens, 2547 nines and 3397 eights contained in 10200!.

The required value of n will be given by the lowest of these three [The student is expected to explore why this happens]

Hence, answer = 1698

Short cut We will look only for the number of 7s in this case. *Reason:* $7 > 3 \times 2$. So, the number of 7s must always be less than the number of 2^3.

And $7 > 2 \times 3$, so the number of 7s must be less than the number of 3^2.

Recollect that earlier, we had talked about the finding of powers when the divisor only had prime factors. There we had seen that we needed to check only for the highest prime as the restriction had to lie there.

In cases of the divisors having composite factors, we have to be slightly careful in estimating the factor that will reflect the restriction. In the above example, we saw a case where even though 7 was the lowest factor (in relation to 8 and 9), the restriction was still placed by 7 rather than by 9 (as would be expected based on the previous process of taking the highest number).

Problem 1.20 Find the units digit of the expression: $78^{5562} \times 56^{256} \times 97^{1250}$.

Solution We can get the units digits in the expression by looking at the patterns followed by 78, 56 and 97 when they are raised to high powers.

In fact, for the last digit we just need to consider the units digit of each part of the product.

A number (like 78) having 8 as the units digit will yield units digit as

$78^1 \to 8$	$78^5 \to 8$	$8^{4n+1} \to 8$
$78^2 \to 4$	$78^6 \to 4$	$8^{4n+2} \to 4$
$78^3 \to 2$	$78^7 \to 2$	Hence, 78^{5562} will yield
$78^4 \to 6$	$78^8 \to 6$	four as the units digit

Similarly, $56^1 \to 6$ $\to 56^{256}$ will yield 6 as the units digit.

$56^2 \to 6$

$56^3 \to 6$

Similarly,

$97^1 \to 7$

$97^2 \to 9$

$97^3 \to 3$

$97^4 \to 1$

$7^{4n+1} \to 7$
$7^{4n+2} \to 9$
Hence, 97^{1250} will yield a units digit of 9.

Hence, the required units digit is given by $4 \times 6 \times 9 \to 6$ (answer).

Problem 1.21 Find the GCD and the LCM of the numbers *P* and *Q* where $P = 2^3 \times 5^3 \times 7^2$ and $Q = 3^3 \times 5^4$.

Solution GCD or HCF is given by the lowest powers of the common factors.

Thus, GCD = 5^3.

LCM is given by the highest powers of all factors available.

Thus, LCM = $2^3 \times 3^3 \times 5^4 \times 7^2$

Problem 1.22 A school has 378 girl students and 675 boy students. The school is divided into strictly boys or strictly girls sections. All sections in the school have the same number of students. Given this information, what are the minimum number of sections in the school?

Solution The answer will be given by the HCF of 378 and 675.

$$378 = 2 \times 3^3 \times 7$$

$$675 = 3^3 \times 5^2$$

Hence, HCF of the two is $3^3 = 27$

Hence, the number of sections is given by: $\frac{378}{27} + \frac{675}{27} = 14 + 25 = 39$ sections.

Problem 1.23 The difference between the number of numbers from 2 to 100 which are not divisible by any other number except 1 and itself and the numbers which are divisible by at least one more number along with 1 and itself.

(a) 25

(b) 50

(c) 49

(d) can not be determined

Solution From 2 to 100.

The number of numbers which are divisible by 1 and itself only = 25

Also, the number of numbers which are divisible by at least one more number except 1 and itself (i.e., composite numbers) 99 – 25 = 74

So, required difference = 74 – 25 = 49

⇒ Option (c)

Problem 1.24 If the sum of $(2n + 1)$ prime numbers where $n \in N$ is an even number, then one of the prime numbers must be

(a) 2

(b) 3

(c) 5

(d) 7

Solution For any $n \in N$, $2n + 1$ is odd.

Also, it is given in the problem that the sum of an odd number of prime numbers = even. Since all prime numbers except 2 are odd, the above condition will only be fulfilled if we have an (odd + odd + even) structure of addition. Since, the sum of the three prime numbers is said to be even, we have to include one even prime number. Hence, 2 being the only even prime number must be included.

If we add odd number of prime numbers, not including 2 (two), we will always get an odd number, because $\frac{\text{odd} + \text{odd} + \text{odd} + \text{----} + \text{odd}}{\text{(an odd number of times)}} = \text{odd number}$

⇒ Option (a)

Problem 1.25 What will be the difference between the largest and smallest four digit numbers made by using distinct single digit prime numbers?

(a) 1800

(b) 4499

(c) 4495

(d) 5175

Solution Required largest number → 7532

Required smallest number → 2357

Difference → 5175

⇒ Option (d)

Problem 1.26 The difference between the two three-digit numbers *XYZ* and *ZYX* will be equal to

(a) difference between *X* and *Z* i.e. $|x - z|$

(b) sum of *X* and z, i.e ($X + Z$)

(c) 9 × difference between X and Z

(d) 99 × difference between X and Z

Solution From the property of numbers, it is known that on reversing a three digit number, the difference (of both the numbers) will be divisible by 99. Also, it is known that this difference will be equal to 99 × difference between the units and hundreds digits of the three digit number.

⇒ Option (d)

Problem 1.27 When the difference between the number 842 and its reverse is divided by 99, the remainder will be

(a) 0

(b) 1

(c) 74

(d) 17

Solution From the property (used in the above question), we can say that the difference will be divisible by 99.

⇒ Remainder = 0 (zero)

⇒ Option (a)

Problem 1.28 When the difference between the number 783 and its reverse is divided by 99, the quotient will be

(a) 1

(b) 10

(c) 3

(d) 4

Solution The quotient will be the difference between extreme digits of 783, i.e. 7 – 3 = 4 (This again is a property which you should know.)

⇒ Option (d)

Problem 1.29 A long part of wood of same length when cut into equal pieces each of 242 cms, leaves a small piece of length 98 cms. If this part were cut into equal pieces each of 22 cms, the length of the leftover wood would be

(a) 76 cm

(b) 12 cm

(c) 11 cm

(d) 10 cm

Solution As 242 is divisible by 22, so the required length of left wood will be equal to the remainder when 98 is divided by 22:

Hence, 10 [98/22; remainder 10]

⇒ Option (d)

Problem 1.30 Find the number of numbers from 1 to 100 which are not divisible by 2.

(a) 51

(b) 50

(c) 49

(d) 48

Solution The 1st number from 1 to 100, not divisible by 2 is 1 and the last number from 1 to 100, not divisible by 2 is 99.

Every alternate number (i.e, at the gap of 2) will not be divisible by 2 from 1 to 99. (1, 2, 3, - - - - , 95, 97, 99)

$$\text{So, the required number of nos} = \frac{\text{last no.} - \text{first no.}}{\text{gap (or step)}} + 1$$

$$= \frac{99-1}{2} + 1 = 50$$

⇒ Option (b)

Alternate method

Total number of numbers from 1 to 100 = 100 (1)

Now, if we count number of numbers from 1 to 100 which are divisible by 2 and subtract that from the total number of numbers from 1 to 100, as a result we will find the number of numbers from 1 to 100 which are not divisible by 2.

To count the number of numbers from 1 to 100 which are divisible by 2:

The 1st number which is divisible by 2 = 2

The last number which is divisible by 2 = 100

(2, 4, 6, - - - - , 96, 98, 100)

Gap/step between two consecutive numbers = 2

So, the number of numbers which are divisible by 2 =

$$\frac{\text{last no.} - \text{first no.}}{\text{gap (or step)}} + 1 = \frac{100-2}{2} + 1 = 50 \quad (2)$$

So, from (1) & (2)

Required number of numbers = 100 – 50 = 50

⇒ Option (b)

Problem 1.31 Find the number of numbers from 1 to 100 which are not divisible by any one of 2 and 3.

(a) 16

(b) 17

(c) 18

(d) 33

Solution From 1 to 100

Number of numbers not divisible by 2 and 3 = Total number of numbers– number of numbers divisible by either 2 or 3. (1)

Now, total number of numbers = 100 (2)

For number of numbers divisible by either 2 or 3:

Number of numbers divisible by 2 = $\frac{\text{last no.} - \text{first no.}}{\text{gap (or step)}} + 1 = \frac{100-2}{2} + 1 = 50$ (3)

Now, the number of numbers divisible by 3 (but not by 2, as it has already been counted)

1st such number = 3 and the gap will be 6. Hence 2nd such number will be 9, 3rd number would be 15 and the last number would be 99. Hence this series is 3, 9, 15, ..., 93, 99.

So, the number of numbers divisible by 3 (but not by 2) =

$$\frac{\text{last such no.} - \text{first such no.}}{\text{gap/step}} + 1 = \frac{99-3}{6} + 1 = 17$$

Hence, the number of numbers divisible by either 2 or 3 = 50 + 17 = 67

So, from (1), (2) & (3) required number of numbers = 100 – 67 = 33

⇒ Option (d)

Problem 1.32 Find the number of numbers from 1 to 100 which are not divisible by any one of 2, 3, and 5.

(a) 26

(b) 27

(c) 29

(d) 32

Solution From the above question, we have found out that from 1 to 100, number of numbers divisible by 2 = 50 (1)

Number of numbers divisible by 3 (but not by 2) = 17 (2)

Now, we have to find out the number of numbers which are divisible by 5 (but not by 2 and 3). Numbers which are divisible by 5

(5) 10 15 20 (25) 30 (35) 40 45 50 (55) 60 (65) 70 75 80 (85) 90 (95) 100

That is, there are 7 such numbers. (iii)

Another way to find out the number of numbers that are divisible by 5 but not 2 and 3 is to first only consider odd multiples of 5.

You will get the series of 10 numbers: 5, 15, 25, 35, 45, 55, 65, 75, 85 and 95.

From amongst these, we need to exclude multiples of 3. In other words, we need to find the number of common elements between the above series and the series of odd multiples of 3, viz, 3, 9, 15, 21 …. 99.

This situation is the same as finding the number of common elements between the two series for which we need to first observe that the first such number is 15. Then the common terms between these two series will themselves form an arithmetic series and this series will have a common difference which is the LCM of the common differences of the two series. (In this case, the common difference of the two series are 10 and 6 respectively and their LCM being 30, the series of common terms between the two series will be 15, 45 and 75.) Thus, there will be 3 terms out of the 10 terms of the series 5, 15, 25…95 which will be divisible by 3 and hence, need to be excluded from the count of numbers which are divisible by 5 but not 2 or 3.

Hence, the required answer would be: 100 – 50 – 17 – 7 = 26

⇒ Option (a)

Problem 1.33 Find the number of numbers from 1 to 100 which are not divisible by any one of 2, 3, 5 and 7.

(a) 22

(b) 24

(c) 23

(d) 27

Solution From the above question, we have seen that from 1 to 100

Number of numbers divisible by 2 = 50 (1)

Number of numbers divisible by 3 but not by 2 = 17 (2)

Number of numbers divisible by 5 but not by 2 and 3 = 7 (3)

Number of numbers divisible by 7 but not by 2, 3 and 5;

such numbers are 7, 49, 77, 91 = 4 numbers (4)

Required number of numbers = Total number of numbers from 1 to 100 – {(1) + (2) + (3) + (4)}

= 100 – (50 + 17 + 7 + 4)

= 22

⇒ Option (a)

Problem 1.34 What will be the remainder when – 34 is divided by 5?

(a) 1

(b) 4

(c) 2

(d) – 4

Solution – 34 = 5 × (– 6) + (– 4)

Remainder = – 4, but it is wrong because remainder cannot be negative.

So, –34 = 5 × (– 7) + 1

⇒ Option (a)

Alternately, when you see a remainder of – 4 when the number is divided by 5, the required remainder will be equal to 5 – 4 = 1.

Problem 1.35 What will be the remainder when – 24.8 is divided by 6?

(a) 0.8

(b) 5.2

(c) –0.8

(d) – 5.2

Solution – 24.8 = 6 × (– 4) + (– 0.8)

Negative remainder, so it is not correct – 24.8 = 6 × (– 5) + 5.2

Positive value of remainder, so it is correct

⇒ Option (b)

Problem 1.36 If p is divided by q, then the maximum possible difference between the minimum possible and maximum possible remainder can be?

(a) $p - q$

(b) $p - 1$

(c) $q - 1$

(d) None of these

Solution $\frac{p}{q}$ minimum possible remainder = 0 (when q exactly divides P)

Maximum possible remainder = $q - 1$

So, required maximum possible difference = $(q - 1) - 0 = (q - 1)$

⇒ Option (c)

Problem 1.37 Find the remainder when 2^{256} is divided by 17.

(a) 0

(b) 1

(c) 3

(d) 5

Solution $\frac{2^{256}}{17} = \frac{(2^4)^{64}}{17} = \frac{16^{64}}{17} \Rightarrow R = 1$ ⇒ R = 1

$\therefore \frac{a^n}{a+1}; R = 1$

when $n \to$ even

⇒ Option (b)

Problem 1.38 Find the difference between the remainders when 7^{84} is divided by 342 and 344.

(a) 0

(b) 1

(c) 3

(d) 5

Solution $\frac{7^{84}}{342} = \frac{(7^3)^{28}}{342} = \frac{343^{28}}{342} \Rightarrow R = 1$

also, $\frac{7^{84}}{344} = \frac{(7^3)^{28}}{344} = \frac{343^{28}}{344} \Rightarrow R = 1$

The required difference between the remainders = 1 – 1 = 0

⇒ Option (a)

Problem 1.39 What will be the value of x for $\frac{(100^{17}-1)+(10^{34}+x)}{9}$; the remainder = 0

(a) 3

(b) 6

(c) 9

(d) 8

Solution $\frac{(100^{17}-1)+(10^{34}+x)}{9}$

$$100^{17}-1=\frac{1000\ldots00-1}{17\text{ zeroes}}=\frac{9999\ldots99}{16\text{ nines}}\Rightarrow\text{divisible by }9\Rightarrow R=0$$

Since the first part of the expression is giving a remainder of 0, the second part should also give 0 as a remainder if the entire remainder of the expression has to be 0. Hence, we now evaluate the second part of the numerator.

$$10^{34}+x=\frac{1000\ldots00+x}{34\text{ zeroes}}=\frac{1000\ldots00x}{33\text{ zeroes}}$$

Since the first part of the expression is giving a remainder of 0, the second part should also give 0 as a remainder if the entire remainder of the expression has to be 0. Hence, we now evaluate the second part of the numerator.

$$10^{34}+x=\frac{1000\ldots00+x}{34\text{ zeroes}}=\frac{1000\ldots00x}{33\text{ zeroes}}$$

with x at the right most place. In order for this number to be divisible by 9, the sum of digits should be divisible by 9.

⇒ $1 + 0 + 0 \ldots + 0 + x$ should be divisible by 9.

⇒ $1 + x$ should be divisible by 9 ⇒ $x = 8$

⇒ Option (d)

Space for Rough Work

LEVEL OF DIFFICULTY (I)

1. The last digit of the number obtained by multiplying the numbers $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$ will be

   (a) 0

   (b) 9

   (c) 7

   (d) 2

2. The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the changed number.

(a) 28

(b) 19

(c) 37

(d) 46

3. When we multiply a certain two-digit number by the sum of its digits, 405 is achieved. If you multiply the number written in reverse order of the same digits by the sum of the digits, we get 486. Find the number.

(a) 81

(b) 45

(c) 36

(d) 54

4. The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.

(a) 11, 4

(b) 12, 3

(c) 13, 2

(d) 10, 5

5. The difference between two numbers is 48 and the difference between the arithmetic mean and the geometric mean is two more than half of 1/3 of 96. Find the numbers.

   (a) 49, 1

   (b) 12, 60

   (c) 50, 2

   (d) 36, 84

6. If $A381$ is divisible by 11, find the value of the smallest natural number A.

   (a) 5

   (b) 6

   (c) 7

   (d) 9

7. If $381A$ is divisible by 9, find the value of smallest natural number A.

   (a) 5

   (b) 5

   (c) 7

   (d) 6

8. What will be the remainder obtained when $(9^6 + 1)$ will be divided by 8?

(a) 0

(b) 3

(c) 7

(d) 2

9. Find the ratio between the LCM and HCF of 5, 15 and 20.

(a) 8 : 1

(b) 14 : 3

(c) 12 : 2

(d) 12 : 1

10. Find the LCM of 5/2, 8/9, 11/14.

(a) 280

(b) 360

(c) 420

(d) None of these

11. If the number A is even, which of the following will be true?

(a) $3A$ will always be divisible by 6

(b) $3A + 5$ will always be divisible by 11

(c) $(A2 + 3)/4$ will be divisible by 7

(d) All of these

12. A five-digit number is taken. Sum of the first four digits (excluding the number at the units digit) equals sum of all the five digits. Which of the following will not divide this number necessarily?

(a) 10

(b) 2

(c) 4

(d) 5

13. A number 15*B* is divisible by 6. Which of these will be true about the positive integer B?

(a) *B* will be even

(b) *B* will be odd

(c) *B* will be divisble by 6

(d) Both (a) and (c)

14. Two numbers $P = 2^3.3^{10}.5$ and $Q = 2^5.3^1.7^1$ are given. Find the GCD of P and Q.

(a) 2.3.5.7

(b) $3.\ 2^2$

(c) $2^2.3^2$

(d) $2^3.3$

15. Find the units digit of the expression $25^{6251} + 36^{528} + 73^{54}$.

(a) 4

(b) 0

(c) 6

(d) 5

16. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$.

(a) 4

(b) 0

(c) 6

(d) 5

17. Find the units digit of the expression $11^{1} + 12^{2} + 13^{3} + 14^{4} + 15^{5} + 16^{6}$.

(a) 1

(b) 9

(c) 7

(d) 0

18. Find the units digit of the expression $11^{1}.12^{2}.13^{3}.\ 14^{4}.15^{5}.16^{6}$.

(a) 4

(b) 3

(c) 7

(d) 0

19. Find the number of zeroes at the end of 1090!

(a) 270

(b) 268

(c) 269

(d) 271

20. If 146! is divisible by 5^n, then find the maximum value of n.

(a) 34

(b) 35

(c) 36

(d) 37

21. Find the number of divisors of 1420.

(a) 14

(b) 15

(c) 13

(d) 12

22. Find the HCF and LCM of the polynomials $(x^2 - 5x + 6)$ and $(x^2 - 7x + 10)$.

(a) $(x - 2)$, $(x - 2)(x - 3)(x - 5)$

(b) $(x - 2)$, $(x - 2)(x - 3)$

(c) $(x - 3)$, $(x - 2)(x - 3)(x - 5)$

(d) $(x - 2)$, $(x - 2)(x - 3)(x - 5)^2$

Directions for Questions 23 to 25: Given two different prime numbers P and Q, find the number of divisors of the following:

23. $P.Q$

(a) 2

(b) 4

(c) 6

(d) 8

24. $P2Q$

(a) 2

(b) 4

(c) 6

(d) 8

25. $P3Q2$

(a) 2

(b) 4

(c) 6

(d) 12

26. The sides of a pentagonal field (not regular) are 1737 metres, 2160 metres, 2358 metres, 1422 metres and 2214 metres respectively. Find the greatest length of the tape by which the five sides may be measured completely.

(a) 7

(b) 13

(c) 11

(d) 9

27. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. Find the minimum total number of sections thus formed.

(a) 24

(b) 32

(c) 16

(d) 20

28. A milkman has three different qualities of milk. 403 gallons of 1st quality, 465 gallons of 2nd quality and 496 gallons of 3rd quality. Find the least possible number of bottles of equal size in which different milk of different qualities can be filled without mixing.

(a) 34

(b) 46

(c) 26

(d) 44

29. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?

(a) 9997

(b) 9793

(c) 9895

(d) 9487

30. Find the least number that when divided by 16, 18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7.

(a) 364

(b) 2254

(c) 2964

(d) 2884

31. Four bells ring at the intervals of 6, 8, 12 and 18 seconds. They start ringing together at 12'O' clock. After how many seconds will they ring together again?

(a) 72

(b) 84

(c) 60

(d) 48

32. For Question 31, find how many times will they ring together during the next 12 minutes. (including the 12 minute mark).

(a) 9

(b) 10

(c) 11

(d) 12

33. The units digit of the expression $125^{813} \times 553^{3703} \times 4532^{828}$ is

(a) 4

(b) 2

(c) 0

(d) 5

34. Which of the following is not a perfect square?

(a) 1,00,856

(b) 3,25,137

(c) 9,45,729

(d) All of these

35. Which of the following can never be in the ending of a perfect square?

(a) 6

(b) 00

(c) x 000 where x is a natural number

(d) 1

36. The LCM of 5, 8,12, 20 will not be a multiple of

(a) 3

(b) 9

(c) 8

(d) 5

37. Find the number of divisors of 720 (including 1 and 720).

(a) 25

(b) 28

(c) 29

(d) 30

38. The LCM of (16 – x2) and (x2 + x – 6) is

(a) (x – 3) (x + 3) (4 – x2)

(b) 4 (4 – x2) (x + 3)

(c) (4 – x2) (x – 3)

(d) None of these

39. GCD of x2 – 4 and x2 + x – 6 is

(a) x + 2

(b) x – 2

(c) x2 – 2

(d) x2 + 2

40. The number A is not divisible by 3. Which of the following will not be divisible by 3?

(a) 9 × A

(b) 2 × A

(c) 18 × A

(d) 24 × A

41. Find the remainder when the number 9^{100} is divided by 8.

(a) 1

(b) 2

(c) 0

(d) 4

42. Find the remainder of 2^{1000} when divided by 3.

(a) 1
(b) 2

(c) 4

(d) 6

43. Decompose the number 20 into two terms such that their product is the greatest.

(a) $x_1 = x_2 = 10$

(b) $x_1 = 5, x_2 = 15$

(c) $x_1 = 16, x_2 = 4$

(d) $x_1 = 8, x_2 = 12$

44. Find the number of zeroes at the end of 50!

(a) 13

(b) 11

(c) 5

(d) 12

45. Which of the following can be a number divisible by 24?

(a) 4,32,15,604

(b) 25,61,284

(c) 13,62,480

(d) All of these

46. For a number to be divisible by 88, it should be

(a) Divisible by 22 and 8

(b) Divisible by 11 and 8

(c) Divisible by 11 and thrice by 2

(d) All of these

47. Find the number of divisors of 10800.

(a) 57

(b) 60

(c) 72

(d) 64

48. Find the GCD of the polynomials $(x + 3)^2 (x - 2)(x + 1)^2$ and $(x + 1)^3 (x + 3)(x + 4)$.

(a) $(x + 3)^3 (x + 1)^2 (x - 2) (x + 4)$

(b) $(x + 3) (x - 2) (x + 1)(x + 4)$

(c) $(x + 3) (x + 1)^2$

(d) $(x + 1) (x + 3)^2$

49. Find the LCM of $(x + 3)(6x^2 + 5x + 4)$ and $(2x^2 + 7x + 3)(x + 3)$

(a) $(2x + 1)(x + 3)(3x + 4)$

(b) $(4x^2 - 1)(x + 3)^2(3x + 4)$

(c) $2(x + 3)^2(6x^2 + 5x + 4)(x + 1/2)$

(d) $(2x - 1)(x + 3)(3x + 4)$

50. The product of three consecutive natural numbers, the first of which is an even number, is always divisible by

(a) 12

(b) 24

(c) 6

(d) All of these

51. Some birds settled on the branches of a tree. First, they sat one to a branch and there was one bird too many. Next they sat two to a branch and there was one branch too many. How many branches were there?

(a) 3

(b) 4

(c) 5

(d) 6

52. The square of a number greater than 1000 that is not divisible by three, when divided by three, leaves a remainder of

(a) 1 always

(b) 2 always

(c) 0

(d) either 1 or 2

53. The value of the expression $(15^3 \cdot 21^2)/(35^2 \cdot 3^4)$ is

(a) 3

(b) 15

(c) 21

(d) 12

54. If $A = \left(\frac{-3}{4}\right)^3, B = \left(\frac{-2}{5}\right)^2, C = (0.3)^2, D = (-1.2)^2$ then

(a) $A > B > C > D$

(b) $D > A > B > C$

(c) $D > B > C > A$

(d) $D > C > A > B$

55. If $2 < x < 4$ and $1 < y < 3$, then find the ratio of the upper limit for $x + y$ and the lower limit of $x - y$.

(a) 6

(b) 7

(c) 8

(d) None of these

56. The sum of the squares of the digits constituting a positive two-digit number is 13. If we subtract 9 from that number, we shall get a number written by the same digits in the reverse order. Find the number.

(a) 12

(b) 32

(c) 42

(d) 52

57. The product of a natural number by the number written by the same digits in the reverse order is 2430. Find the numbers.

(a) 54 and 45

(b) 56 and 65

(c) 53 and 35

(d) 85 and 58

58. Find two natural numbers whose difference is 66 and the least common multiple is 360.

(a) 120 and 54

(b) 90 and 24

(c) 180 and 114

(d) 130 and 64

59. Find the pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13.

(a) 58 and 13 or 16 and 29

(b) 68 and 23 or 36 and 49

(c) 18 and 73 or 56 and 93

(d) 78 and 13 or 26 and 39

60. Find two natural numbers whose sum is 85 and the least common multiple is 102.

(a) 30 and 55

(b) 17 and 68

(c) 35 and 55

(d) 51 and 34

61. Find the pairs of natural numbers the difference of whose squares is 55.

(a) 28 and 27 or 8 and 3

(b) 18 and 17 or 18 and 13

(c) 8 and 27 or 8 and 33

(d) 9 and 18 or 8 and 27

62. Which of these is greater?

(a) 54_4 or 21_{12}

(b) $(0.4)_4$ or $(0.8)_3$

63. Is it possible for a common fraction whose numerator is less than the denominator to be equal to a fraction whose numerator is greater than the denominator?

(a) Yes

(b) No

64. What digits should be put in place of *c* in 38*c* to make it divisible by

(a) 2

(b) 3

(c) 4

(d) 5

(e) 6

(f) 9

(g) 10

65. Find the LCM and HCF of the following numbers: (54, 81, 135 and 189), (156, 195) and (1950, 5670 and 3900).

66. The last digit in the expansions of the three digit number (34x)43 and (34x)44 are 7 and 1, respectively. What can be said about the value of x?

(a) $x = 5$

(b) $x = 3$

(c) $x = 6$

(d) $x = 2$

Directions for Questions 67 and 68: Amitesh buys a pen, a pencil and an eraser for ₹41. If the least cost of any of the three items is ₹12 and it is known that a pen costs less than a pencil and an eraser costs more than a pencil, answer the following questions:

67. What is the cost of the pen?

(a) 12

(b) 13

(c) 14

(d) 15

68. If it is known that the eraser's cost is not divisible by 4, the cost of the pencil could be

(a) 12

(b) 13

(c) 14

(d) 15

69. A naughty boy Amrit watches an innings of Sachin Tendulkar and acts according to the number of runs he sees Sachin scoring. The details of these are given below.

1 run Place an orange in the basket

2 runs Place a mango in the basket

3 runs Place a pear in the basket

4 runs Remove a pear and a mango from the basket

(d) 7

70. In the famous Bel Air Apartments in Ranchi, there are three watchmen meant to protect the precious fruits in the campus. However, one day a thief got in without being noticed and stole some precious mangoes. On the way out however, he was confronted by the three watchmen, the first two of whom asked him to part with 1/3rd of the fruits and one more. The last asked him to part with 1/5th of the mangoes and 4 more. As a result he had no mangoes left. What was the number of mangoes he had stolen?

(a) 12

(b) 13

(c) 15

(d) None of these

71. A hundred and twenty digit number is formed by writing the first x natural numbers in front of each other as 12345678910111213… Find the remainder when this number is divided by 8.

(a) 6

(b) 7

(c) 2

(d) 0

72. A test has 80 questions. There is one mark for a correct answer, while there is a negative penalty of –1/2 for a wrong answer and –1/4 for an unattempted question. What is the number of questions answered correctly, if the student has scored a net total of 34.5 marks?

(a) 45

(b) 48

(c) 54

(d) Cannot be determined

73. For Question 72, if it is known that he has left 10 questions unanswered, the number of correct answers are

(a) 45

(b) 48

(c) 54

(d) Cannot be determined

74. Three mangoes, four guavas and five watermelons cost ₹ 750. Ten watermelons, six mangoes and nine guavas cost ₹1580. What is the cost of six mangoes, ten watermelons and four guavas?

(a) 1280

(b) 1180

(c) 1080

(d) Cannot be determined

75. From a number *M* subtract 1. Take the reciprocal of the result to get the value of '*N*'. Then which of the following is necessarily true?

(a) $0 \le M\ N \le 2$

(b) $M\ N > 3$

76. The cost of four mangoes, six guavas and sixteen watermelons is ₹500, while the cost of seven magoes, nine guavas and nineteen watermelons is ₹620. What is the cost of one mango, one guava and one watermelon?

(a) 120

(b) 40

(c) 150

(d) Cannot be determined

77. For the question above, what is the cost of a mango?

(a) 20

(b) 14

(c) 15

(d) Cannot be determined

78. The following is known about three real numbers, x, y and z.

$-4 \le x \le 4$, $-8 \le y \le 2$ and $-8 \le z \le 2$. Then the range of values that $M = xz/y$ can take is best represented by

(a) $-\infty < x < \infty$

(b) $-16 \le x \le 8$

(c) $-8 \le x \le 8$

(d) $-16 \le x \le 16$

79. A man sold 38 pieces of clothing (combined in the form of shirts, trousers and ties). If he sold at least 11 pieces of each item and he sold more shirts than trousers and more trousers than ties, then the number of ties that he must have sold is

(a) Exactly 11

(b) At least 11

(c) At least 12

(d) Cannot be determined

80. For Question 79, find the number of shirts he must have sold.

(a) At least 13

(b) At least 14

(c) At least 15

(d) At most 16

81. Find the least number which when divided by 12, 15, 18 or 20 leaves in each case a remainder 4.

(a) 124

(b) 364

(c) 184

(d) None of these

82. What is the least number by which 2800 should be multiplied so that the product may be a perfect square?

(a) 2

(b) 7

(c) 14

(d) None of these

83. The least number of 4 digits which is a perfect square is

(a) 1064

(b) 1040

(c) 1024

(d) 1012

84. The least multiple of 7 which leaves a remainder of 4 when divided by 6, 9, 15 and 18 is

(a) 94

(b) 184

(c) 364

(d) 74

85. What is the least 3 digit number that when divided by 2, 3, 4, 5 or 6 leaves a remainder of 1?

(a) 131

(b) 161

(c) 121

(d) None of these

86. The highest common factor of 70 and 245 is equal to

(a) 35

(b) 45

(c) 55

(d) 65

87. Find the least number, which must be subtracted from 7147 to make it a perfect square.

(a) 86

(b) 89

(c) 91

(d) 93

88. Find the least square number which is divisible by 6, 8 and 15.

(a) 2500

(b) 3600

(c) 4900

(d) 4500

89. Find the least number by which 30492 must be multiplied or divided so as to make it a perfect square.

(a) 11

(b) 7

(c) 3

(d) 2

90. The greatest 4-digit number exactly divisible by 88 is

(a) 8888

(b) 9768

(c) 9944

(d) 9988

91. By how much is three fourth of 116 greater than four fifth of 45?

(a) 31

(b) 41

(c) 46

(d) None of these

92. If 5625 plants are to be arranged in such a way that there are as many rows as there are plants in a row, the number of rows will be

(a) 95

(b) 85

(c) 65

(d) None of these

93. A boy took a seven digit number ending in 9 and raised it to an even power greater than 2000. He then took the number 17 and raised it to a power which leaves the remainder 1 when divided by 4. If he now multiplies both the numbers, what will be the unit's digit of the number he so obtains?

(a) 7

(b) 9

(c) 3

(d) Cannot be determined

94. Two friends were discussing their marks in an examination. While doing so they realised that both the numbers had the same prime factors, although Raveesh got a score which had two more factors than Harish. If their marks are represented by one of the options as given below, which of the following options would correctly represent the number of marks they got?

(a) 30,60

(b) 20,80

(c) 40,80

(d) 20,60

95. A number is such that when divided by 3, 5, 6, or 7 it leaves the remainder 1, 3, 4, or 5 respectively. Which is the largest number below 4000 that satisfies this property?

(a) 3358

(b) 3988

(c) 3778

(d) 2938

96. A number when divided by 2,3 and 4 leaves a remainder of 1. Find the least number (after 1) that satisfies this requirement.

(a) 25

(b) 13

(c) 37

(d) 17

97. A number when divided by 2, 3 and 4 leaves a remainder of 1. Find the second lowest number (not counting 1) that satisfies this requirement.

(a) 25

(b) 13

(c) 37

(d) 17

98. A number when divided by 2, 3 and 4 leaves a remainder of 1. Find the highest 2 digit number that satisfies this requirement.

(a) 91

(b) 93

(c) 97

(d) 95

99. A number when divided by 2,3 and 4 leaves a remainder of 1. Find the highest 3 digit number that satisfies this requirement.

(a) 991

(b) 993

(c) 997

(d) 995

100. A frog is sitting on vertex A of a square ABCD. It starts jumping to the immediately adjacent vertex on either side in random fashion and stops when it reaches point C. In how many ways, can it reach point C if it makes exactly 7 jumps?

(a) 1

(b) 3

(c) 5

(d) 0

101. Three bells ring at intervals of 5 seconds, 6 seconds and 7 seconds respectively. If they toll together for the first time at 9 a.m. in the morning, after what interval of time will they together ring again for the first time?

(a) After 30 seconds

(b) After 42 seconds

(c) After 35 seconds

(d) After 210 seconds

102. For the question above, how many times would they ring, together in the next 1 hour?

(a) 17

(b) 18

(c) 19

(d) None of these

103. A garrison has three kinds of soldiers. There are 66 soldiers of the first kind, 110 soldiers of the second kind and 242 soldiers of the third kind. It is desired to be arranging these soldiers in equal rows such that each row contains the same number of soldiers and there is only 1 kind of soldier in each row. What is the maximum number of soldiers who can be placed in each row?

(a) 11

(b) 1

(c) 22

(d) 33

104. For the question above, what are the minimum number of rows that would be required to be formed?

(a) 11

(b) 19

(c) 18

(d) None of these

105. A milkman produces three kinds of milk. On a particular day, he has 170 litres, 102 litres and 374 litres of the three kinds of milk. He wants to bottle them in bottles of equal sizes- so that each of the three varieties of milk would be completely bottled. How many bottle sizes are possible such that the bottle size in terms of litres is an integer?

(a) 1

(b) 2

(c) 4

(d) 34

106. For the above question, what is the size of the largest bottle which can be used?

(a) 1

(b) 2

(c) 17

(d) 34

107. For Question 105, what are the minimum number of bottles that would be required?

(a) 11

(b) 19

(c) 18

(d) None of these

108. Find the number of zeroes at the end of 100!

(a) 20

(b) 23

(c) 24

(d) 25

109. Find the number of zeroes at the end of 122!

(a) 20

(b) 23

(c) 24

(d) 28

110. Find the number of zeroes at the end of 1400!

(a) 347

(b) 336

(c) 349

(d) 348

111. Find the number of zeroes at the end of 380!

(a) 90

(b) 91

(c) 94

(d) 95

112. Find the number of zeroes at the end of 72!

(a) 14

(b) 15

(c) 16

(d) 17

113. The highest power of 3 that completely divides 40! is

(a) 18

(b) 15

(c) 16

(d) 17

114. $53!/3^n$ is an integer. Find the highest possible value of n for this to be true.

(a) 19

(b) 21

(c) 23

(d) 24

115. The highest power of 7 that completely divides 80! is

(a) 12

(b) 13

(c) 14

(d) 15

116. $115!/7^n$ is an integer. Find the highest possible value of n for this to be true.

(a) 15

(b) 17

(c) 16

(d) 18

117. The highest power of 12 that completely divides 122! is

(a) 54

(b) 56

(c) 57

(d) 58

118. $155!/20^n$ is an integer. Find the highest possible value of n for this to be true.

(a) 77

(b) 38

(c) 75

(d) 37

119. The minimum value of x so that $x^2/1024$ is an integer is

(a) 4

(b) 32

(c) 16

(d) 64

120. Find the sum of all 2 digit natural numbers which leave a remainder of 3 when divided by 7.

(a) 650

(b) 663

(c) 676

(d) 702

121. How many numbers between 1 and 200 are exactly divisible by exactly two of 3, 9 and 27?

(a) 14

(b) 15

(c) 16

(d) 17

122. A number N is squared to give a value of S. The minimum value of $N + S$ would happen when N is

(a) –0.3

(b) –0.5

(c) –0.7

(d) None of these

123. $L = x + y$ where x and y are prime numbers. Which of the following statement/s is/are true?

(i) The unit's digit of L cannot be 5

(ii) The unit's digit of L cannot be 0

(iii) L cannot be odd.

(a) All three

(b) Only (iii)

(c) only (ii)

(d) None of these

124. XYZ is a 3 digit number such that when we calculate the difference between the two three digit numbers $XYZ - YXZ$ the difference is exactly 90. How many possible values exist for the digits X and Y?

(a) 9

(b) 8

(c) 7

(d) 6

125. What is the sum of all even numbers between 1 and 100 (both included)?

(a) 2450

(b) 2500

(c) 2600

(d) 2550

126. The least number which can be added to 763 so that it is completely divisible by 57 is

(a) 35

(b) 22

(c) 15

(d) 25

127. The least number which can be subtracted from 763 so that it is completely divisible by 57 is

(a) 35

(b) 22

(c) 15

(d) 25

128. The least number which can be added to 8441 so that it is completely divisible by 57 is

(a) 42

(b) 15

(c) 5

(d) 52

129. The least number which can be subtracted from 8441 so that it is completely divisible by 57 is

(a) 3
(b) 4

(c) 5

(d) 6

130. Find the least number of 5 digits that is exactly divisible by 79.

(a) 10003

(b) 10033

(c) 10043

(d) None of these

131. Find the maximum number of 5 digits that is exactly divisible by 79.

(a) 99925

(b) 99935

(c) 99945

(d) 99955

132. The nearest integer to 773 which is exactly divisible by 12 is

(a) 768

(b) 772

(c) 776

(d) None of these

133. A number when divided by 84 leaves a remainder of 57. What is the remainder when the same number is divided by 12?

(a) 7

(b) 8

(c) 9

(d) Cannot be determined

134. A number when divided by 84 leaves a remainder of 57. What is the remainder when the same number is divided by 11?

(a) 2

(b) 7

(c) 8

(d) Cannot be determined

135. 511 and 667 when divided by the same number, leave the same remainder. How many numbers can be used as the divisor in order to make this occur?

(a) 14

(b) 12

(c) 10

(d) 8

136. How many numbers between 200 and 400 are divisible by 13?

(a) 14

(b) 15

(c) 16

(d) 17

137. A boy was trying to find 5/8th of a number. Unfortunately, he found out 8/5th of the number and realised that the difference between the answer he got and the correct answer is 39. What was the number?

(a) 38

(b) 39

(c) 40

(d) 52

138. The sum of two numbers is equal to thrice their difference. If the smaller of the numbers is 10, find the other number.

(a) 15

(b) 20

(c) 40

(d) None of these

139. $4_{11} + 4_{12} + 4_{13} + 4_{14} + 4_{15}$ is divisible by which of the following?

(a) 11

(b) 31

(c) 341

(d) All of these

140. The product of two numbers is 7168 and their HCF is 16. How many pairs of numbers are possible such that the above conditions are satisfied?

(a) 2

(b) 3

(c) 4

(d) 6

141. When 876 is added to another 3-digit number 2P3, we get a four digit number 10Q9 and 10Q9 is divisible by 11 then the value of P- *Q* is

142. There is a 22- digit number which consists of only one digit – from 1, 2, 3, 4, 5 or 6, e.g. 11111111.....11, 2222222....22, ...6666.....66. Such a number is always divisible by

143. The product of the factors of 72 is

144. The number of ways of expressing 72 as a product of 2 factors is

145. In how many ways can 144 be expressed as a product of two distinct factors?

146. How many numbers lie between 100 and 1000 which when divided by 7 leaves remainder 3 and when divided by 11 leaves remainder 4?

147. HCF of $3^{15}-1$ and $3^{25}-1$ is

148. The HCF of two natural numbers a, b is 10 and LCM of these numbers is 45. If a = 15 then b = ?

149. LCM and HCF of 10! and 15! are respectively

(a) 5! and 25!
(b) 5! and 30!

(c) 10! and 30!

(d) 15! and 10!

150. The remainder of $\frac{18^{116!}}{19}$ is

151. Given that $7x + y$ is a prime number for natural numbers x and y, then what is the minimum value of $(x + y)$?

152. If n is an odd digit then unit's digit of the product $171n \times 1414 \times 729 \times 2015$ will be

153. If unit's digit of the product $171n \times 1413 \times 729 \times 2015$ is 0 then the maximum number of values that 'n' may take will be

154. The first two, 2 digit numbers that divide $(21^{12346} - 1)$ are

155. What is the unit's digit of 1! + 2! + 3! + 4!+ 5! + 6! + 7! + ...+ 1000?

156. Find the unit's digit of $(35!)^{35!}$.

157. How many zeroes are there at the end of $(34!)^{6!}$?

158. The unit's digit of $7^{51^{31!}}$ is

159. If N^2 = 1234567654321, then N = ?

160. The LCM of two numbers is 421. What is the HCF of these two numbers?

161. Which of the following is greatest?

3^{50}, 4^{40}, 5^{30}, 6^{20}

162. What is the value of $M*N$ if $M39048458N$ is divisible by 8 and 11, where M and N are single digit integers?

163. How many times does the digit 4 appear when we count from 21 to 500?

164. Find the remainder when the sum of 15 consecutive natural numbers starting from 3671 is divided by 3670.

165. X is a number formed by writing 9 for 99 times. What will be the remainder of this number when divided by 7?

166. Find the remainder of $\frac{2^{41}}{41}$.

167. Find the remainder when 40! is divided by 41.

168. Find the remainder when $x^4 + 3x^3 + 4$ is divided by $x + 3$.

169. If X is a prime number then for how many values of X, $X^2 + 7$ is also a prime number?

170. If $X = 99^3 - 63^3 - 36^3$ then the number of factors of X is

171. If x is a natural number and $4 < x < 50$, then the largest n, such that $n!$ would always divide: $x(x^2 - 1)(x^2 - 4)(x^2 - 9)(x + 4)$ is

172. If X is a natural number and $X!$ ends with Y zeros then number of zeros at the end of $(5X)$ is

173. There are 90 questions in a test. Each correct answer fetches 1 mark, each wrong answer and unanswered question attract a penalty of ¼ marks and 1/8 marks respectively. Bilbo scored 23 marks in the test. What is the minimum possible number of the questions wrongly answered by him?

174. If $A = n^{2n^n}$, $B = n^{n^{2n}}$, $C = (n^{2n})^n$, $D = (n^n)^{n^2}$ when n is a natural number and $n \neq 1$.

Then arrange them in terms of their values.

175. How many numbers in the form of $2^n - 1$, which are less than 5000 are prime?

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. The arithmetic mean of two numbers is smaller by 24 than the larger of the two numbers and the GM of the same numbers exceeds by 12, the smaller of the numbers. Find the numbers.

   (a) 6 and 54

   (b) 8 and 56

   (c) 12 and 60

   (d) 7 and 55

2. Find the number of numbers between 200 and 300, both included, which are not divisible by 2, 3, 4 and 5.

   (a) 27

   (b) 26

   (c) 25

   (d) 28

3. Given x and n are integers, $(15n^3 + 6n^2 + 5n + x)/n$ is not an integer for what condition?

   (a) n is positive

   (b) x is divisible by n

   (c) x is not divisible by n

   (d) Both (a) and (c)

4. The unit digit in the expression $36^{234}*33^{512}*39^{180} - 54^{29}*25^{123}*31^{512}$ will be

(d) 5

5. The difference of $10^{25} - 7$ and $10^{24} + x$ is divisible by 3 for x = ?

(a) 3

(b) 2

(c) 4

(d) 6

6. Find the value of x in $\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{3x}}}} = x$.

(a) 1

(b) 3

(c) 6

(d) 12

(e) 9

7. If a number is multiplied by 22 and the same number is added to it, then we get a number that is half the square of that number. Find the number

(a) 45

(b) 46

(c) 47

(d) Data insufficient

8. $12^{55}/3^{11} + 8^{48}/16^{18}$ will give the digit at units place as

(a) 4

(b) 6

(c) 8

(d) 0

9. The mean of 1, 2, 2^2.... 2^{31} lies in between

(a) 2^{24} to 2^{25}

(b) 2^{25} to 2^{26}

(c) 2^{26} to 2^{27}

(d) 2^{29} to 2^{30}

10. *xy* is a number that is divided by *ab* where $xy < ab$ and gives a result 0.*xyxyxy*... then *ab* equals

(a) 11

(b) 33

(c) 99

(d) 66

11. A number *xy* is multiplied by another number *ab* and the result comes as *pqr*, where $r = 2y$, $q = 2(x + y)$ and $p = 2x$ where $x, y < 5$, q π 0. The value of *ab* may be

(a) 11

(b) 13

(c) 31

(d) 22

12. $[x]$ denotes the greatest integer value just below x and $\{x\}$ its fractional value. The sum of $[x]^3$ and $\{x\}^2$ is –7.91. Find x.

(a) –2.03

(b) –1.97

(c) –2.97

(d) –1.7

13. $16^5 + 2^{15}$ is divisible by

(a) 31
(b) 13

(c) 27

(d) 33

14. If $AB + XY = 1XP$, where $A > 0$ and all the letters signify different digits from 0 to 9, then the value of A is

(a) 6

(b) 7

(c) 9

(d) 8

Directions for Questions 15 and 16: Find the possible integral values of x.

15. $|x - 3| + 2|x + 1| = 4$

(a) 1

(b) –1

(c) 3

(d) 2

16. $x^2 + |x - 1| = 1$

(a) 1

(b) –1

(c) 0

(d) 1 or 0

17. If $4^{2n+1} + x$ and $4^{2n} - x$ are divisible by 5, n being an even integer, find the least value of x.

(a) 1

(b) 2

(c) 3

(d) 0

18. If the sum of the numbers $(a25)^2$ and a^3 is divisible by 9, then which of the following may be a value for a ?

(a) 1

(b) 7

(c) 9

(d) There is no value

19. If $| x - 4 | + | y - 4 | = 4$, then how many integer values can the set (x, y) have?

(a) Infinite

(b) 5

(c) 16

(d) 9

20. $[3^{32}/50]$ gives a remainder and {.} denotes the fractional part of that. The fractional part is of the form $(0 \cdot bx)$. The value of x could be

(a) 2

(b) 4

(c) 6

(d) 8

21. The sum of two numbers is 20 and their geometric mean is 20% lower than their arithmetic mean. Find the ratio of the numbers.

(a) 4 : 1

(b) 9 : 1

(c) 1 : 1

(d) 17 : 3

22. The highest power on 990 that will exactly divide 1090! is

(a) 101

(b) 100

(c) 108

(d) 109

23. If 146! is divisible by 6^n, then find the maximum value of n.

(a) 74

(b) 70

(c) 76

(d) 75

24. The last two digits in the multiplication of $35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$ is

(a) 00

(b) 40

(c) 30

(d) 10

25. The expression $333^{555} + 555^{333}$ is divisible by

(a) 2

(b) 3

(c) 37

(d) All of these

26. $[x]$ denotes the greatest integer value just below x and $\{x\}$ its fractional value. The sum of $[x]^2$ and $\{x\}^1$ is 25.16. Find x.

(a) 5.16

(b) –4.84

(c) Both (a) and (b)

(d) 4.84

27. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.

(a) 18

(b) 39

(c) 49

(d) 28

28. Find two numbers such that their sum, their product and the differences of their squares are equal.

(a) $\left(\frac{3+\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$ or $\left(\frac{3+\sqrt{2}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$

(b) $\left(\frac{3+\sqrt{7}}{2}\right)$ and $\left(\frac{1+\sqrt{7}}{2}\right)$ or $\left(\frac{3+\sqrt{6}}{2}\right)$ and $\left(\frac{1-\sqrt{6}}{2}\right)$

(c) $\left(\frac{3-\sqrt{5}}{2}\right)$ and $\left(\frac{1-\sqrt{5}}{2}\right)$ or $\left(\frac{3+\sqrt{5}}{2}\right)$ and $\left(\frac{1+\sqrt{5}}{2}\right)$

(d) None of these

29. The sum of the digits of a three-digit number is 17, and the sum of the squares of its digits is 109. If we subtract 495 from that number, we shall get a number consisting of the same digits written in the reverse order. Find the number.

(a) 773

(b) 863

(c) 683

(d) 944

30. Find the number of zeros in the product: $1_1 \times 2_2 \times 3_3 \times 4_4 \times$ ...... $98_{98} \times 99_{99}$ $\times\ 100_{100}$.

(a) 1200

(b) 1300

(c) 1050

(d) 1225

31. Find the pairs of natural numbers whose greatest common divisor is 5 and the least common multiple is 105.

(a) 5 and 105 or 15 and 35

(b) 6 and 105 or 16 and 35

(c) 5 and 15 or 15 and 135

(d) 5 and 20 or 15 and 35

32. The denominator of an irreducible fraction is greater than the numerator by 2. If we reduce the numerator of the reciprocal fraction by 3 and subtract the given fraction from the resulting one, we get 1/15. Find the given fraction.

(a) $\frac{2}{4}$

(b) $\frac{3}{5}$

(c) $\frac{5}{7}$

(d) $\frac{7}{9}$

33. A two-digit number exceeds by 19 the sum of the squares of its digits and by 44 the double product of its digits. Find the number.

(a) 72

(b) 62

(c) 22

(d) 12

34. The sum of the squares of the digits constituting a two-digit positive number is 2.5 times as large as the sum of its digits and is larger by unity than the trebled product of its digits. Find the number.

(a) 13 and 31

(b) 12 and 21

(c) 22 and 33

(d) 14 and 41

35. The units digit of a two-digit number is greater than its tens digit by 2, and the product of that number by the sum of its digits is 144. Find the number.

(a) 14

(b) 24

(c) 46

(d) 35

36. Find the number of zeroes in the product $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$.

(a) 8

(b) 9

(c) 12

(d) 13

37. The highest power of 45 that will exactly divide 123! is

(a) 28

(b) 30

(c) 31

(d) 59

38. Three numbers are such that the second is as much lesser than the third as the first is lesser than the second. If the product of the two smaller numbers is 85 and the product of two larger numbers is 115. Find the middle number.

(a) 9

(b) 8

(c) 12

(d) 10

39. Find the smallest natural number n such that $n!$ is divisible by 990.

(a) 3

(b) 5

(c) 11

(d) 12

40. $\sqrt{x}\,\sqrt{y} = \sqrt{xy}$ is true only when

(a) $x > 0, y > 0$

(b) $x > 0$ and $y < 0$

(c) $x < 0$ and $y > 0$

(d) All of these

Directions for Questions 41 to 60: Read the instructions below and solve the questions based on this.

In an examination situation, always solve the following type of questions by substituting the given options, to arrive at the solution.

However, as you can see, there are no options given in the questions here since these are meant to be an exercise in equation writing (which I believe is a primary skill required to do well in aptitude exams testing mathematical aptitude). Indeed, if these questions had options for them, they would be rated as LOD 1 questions. But since the option-based solution technique is removed here, these are placed in the LOD 2 category.

41. Find the two-digit number that meets the following criteria. If the number in the units place exceeds, the number in its tens by 2 and the product of the required number with the sum of its digits is equal to 144.

42. The product of the digits of a two-digit number is twice as large as the sum of its digits. If we subtract 27 from the required number, we get a number consisting of the same digits written in the reverse order. Find the number.

43. The product of the digits of a two-digit number is one-third that number. If we add 18 to the required number, we get a number consisting of the same digits written in the reverse order. Find the number.

44. The sum of the squares of the digits of a two-digit number is 13. If we subtract 9 from that number, we get a number consisting of the same digits written in the reverse order. Find the number.

45. A two-digit number is thrice as large as the sum of its digits, and the square of that sum is equal to the trebled required number. Find the number.

46. Find a two-digit number that exceeds by 12 the sum of the squares of its digits and by 16 the doubled product of its digits.

47. The sum of the squares of the digits constituting a two-digit number is 10, and the product of the required number by the number consisting of the same digits written in the reverse order is 403. Find the 2 numbers that satisfy these conditions.

48. If we divide a two-digit number by the sum of its digits, we get 4 as a quotient and 3 as a remainder. Now, if we divide that two-digit number by the product of its digits, we get 3 as a quotient and 5 as a remainder. Find the two-digit number.

49. There is a natural number that becomes equal to the square of a natural number when 100 is added to it, and to the square of another natural number when 169 is added to it. Find the number.

50. Find two natural numbers whose sum is 85 and whose least common multiple is 102.

51. Find 2 three -digit numbers whose sum is a multiple of 504 and the quotient is a multiple of 6.

52. The difference between the digits in a two-digit number is equal to 2, and the sum of the squares of the same digits is 52. Find all the possible numbers.

53. If we divide a given two-digit number by the product of its digits, we obtain 3 as a quotient and 9 as a remainder. If we subtract the product of the digits constituting the number, from the square of the sum of its digits, we obtain the given number. Find the number.

54. Find the three-digit number if it is known that the sum of its digits is 17 and the sum of the squares of its digits is 109. If we subtract 495 from this number, we obtain a number consisting of the same digits written in reverse order.

55. The sum of the cubes of the digits constituting a two-digit number is 243 and the product of the sum of its digits by the product of its digits is 162. Find the 2 two-digit numbers.

56. The difference between two numbers is 16. What can be said about the total numbers divisible by 7 that can lie in between these two numbers.

57. Arrange the following in descending order:

    1114, 110.109.108.107, 109.110.112.113

58. If $3 \le x \le 5$ and $4 \le y \le 7$. Find the greatest value of xy and the least value of x/y.

59. Which of these is greater?

(a) 200^{300} or 300^{200} or 400^{150}

(b) 5^{100} and 2^{200}

(c) 10^{20} and 40^{10}

60. The sum of the two numbers is equal to 15 and their arithmetic mean is 25 per cent greater than their geometric mean. Find the numbers.

61. Define a number K such that it is the sum of the squares of the first M natural numbers. (i.e. $K = 1^2 + 2^2 + ... + M^2$) where $M < 55$. How many values of M exist such that K is divisible by 4?

(a) 10

(b) 11

(c) 12

(d) None of these

62. M is a two digit number which has the property that:

the product of factorials of its digits > sum of factorials of its digits

How many values of M exist?

(a) 56

(b) 64

(c) 63

(d) None of these

63. A natural number when increased by 50% has its number of factors unchanged. However, when the value of the number is reduced by 75%, the number of factors is reduced by 66.66%. One such number could be

(a) 32

(b) 84

(c) 126

(d) None of these

64. Find the 28383rd term of the series: 123456789101112....

(a) 3

(b) 4

(c) 9

(d) 7

65. If you form a subset of integers chosen from between 1 to 3000, such that no two integers add up to a multiple of nine, what can be the maximum number of elements in the subset. (Include both 1 and 3000.)

(a) 1668

(b) 1332

(c) 1333

(d) 1336

66. The series of numbers (1,1/2,1/3,1/4 ....1/1972) is taken. Now two numbers are taken from this series (the first two) say x, y. Then the operation $x + y + x.y$ is performed to get a consolidated number. The process is repeated. What will be the value of the set after all the numbers are consolidated into one number?

(a) 1970

(b) 1971

(c) 1972

(d) None of these

67. K is a three digit number such that the ratio of the number to the sum of its digits is least. What is the difference between the hundreds and the tens digits of K?

(a) 9

(b) 8

(c) 7

(d) None of these

68. In Question 67, what can be said about the difference between the tens and the units digit?

(a) 0

(b) 1

(c) 2

(d) None of these

69. For the above question, for how many values of K will the ratio be the highest?

(a) 9

(b) 8

(c) 7

(d) None of these

70. A triangular number is defined as a number which has the property of being expressed as a sum of consecutive natural numbers starting with 1. How many triangular numbers less than 1000, have the property that they are the difference of squares of two consecutive natural numbers?

(a) 20

(b) 21

(c) 22

(d) 23

71. x and y are two positive integers. Then what will be the sum of the coefficients of the expansion of the expression $(x + y)^{44}$?

(a) 2^{43}

(b) $2^{43} + 1$

(c) 2^{44}

(d) $2^{44} - 1$

72. What is the remainder when $9 + 9^2 + 9^3 + \ldots 9^{2n+1}$ is divided by 6?

(a) 1

(b) 2

(c) 3

(d) 4

73. The remainder when the number 123456789101112 ......484950 is divided by 16 is

(a) 3

(b) 4

(c) 5

(d) 6

74. What is the highest power of 3 available in the expression 58!-38!?

(a) 17

(b) 18

(c) 19

(d) None of these

75. Find the remainder when the number represented by 22334 raised to the power $(1^2 + 2^2 + \ldots + 66^2)$ is divided by 5?

(a) 2

(b) 4

(c) 1

(d) None of these

76. What is the total number of divisors of the number $12^{33} \times 34^{23} \times 2^{70}$?

(a) 4658

(b) 9316

(c) 2744

(d) None of these

77. For Question 76, which of the following will represent the sum of factors of the number (such that only odd factors are counted)?

(a) $\frac{(3^{34}-1)}{2} \times \frac{(17^{24}-1)}{16}$

(b) $(3^{34} - 1) \times (17^{24} - 1)$

(c) $\frac{(3^{34}-1)}{33}$

(d) None of these

78. What is the remainder when $(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \ldots\ldots(1152!)^3$ is divided by 1152?

(a) 125

(b) 225

(c) 325

(d) 205

79. A set S is formed by including some of the first one thousand natural numbers. S contains the maximum number of numbers such that they satisfy the following conditions:

1. No number of the set S is prime.

2. When the numbers of the set S are selected two at a time, we always see co-prime numbers.

What is the number of elements in the set S?

(a) 11

(b) 12

(c) 13

(d) 7

Find the last two digits of the following numbers.

80. 101 × 102 × 103 × 197 × 198 × 199

(a) 54

(b) 74

(c) 64

(d) 84

81. 65 × 29 × 37 × 63 × 71 × 87

(a) 05

(b) 95

(c) 15

(d) 25

82. $65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 85$

(a) 25

(b) 35

(c) 75

(d) 85

83. $65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 62$

(a) 70

(b) 30

(c) 10

(d) 90

84. $75 \times 35 \times 47 \times 63 \times 71 \times 87 \times 82$

(a) 50

(b) 70

(c) 30

(d) 90

85. $(201 \times 202 \times 203 \times 204 \times 246 \times 247 \times 248 \times 249)^2$

(a) 36

(b) 56

(c) 76

(d) 16

86. Find the remainder when 7^{99} is divided by 2400.

(a) 1

(b) 343

(c) 49

(d) 7

87. Find the remainder when $(10^3 + 9^3)^{752}$ is divided by 12^3.

(a) 729

(b) 1000

(c) 752

(d) 1

88. Arun, Bikas and Chetakar have a total of 80 coins among them. Arun triples the number of coins with the others by giving them some coins from his own collection. Next, Bikas repeats the same process. After this, Bikas now has 20 coins. Find the number of coins he had at the beginning.

(a) 22

(b) 20

(c) 18

(d) 24

89. The super computer at Ram Mohan Roy Seminary takes an input of a number N and a X where X is a factor of the number n. In a particular case

N is equal to 83*p*796161*q* and *X* is equal to 11 where $0 < p < q$. Find the sum of remainders when *N* is divided by $(p + q)$ and *p* successively.

(a) 6

(b) 3

(c) 2

(d) 9

90. On 1 March 2016, Sherry saved ₹1. Everyday starting from 2 March 2016, he saved ₹1 more than the previous day. Find the first date after 1 March 2016 at the end of which his total savings will be a perfect square.

(a) 17 March 2016

(b) 18 April 2016

(c) 26 March 2016

(d) None of these

91. What is the rightmost digit preceding the zeroes in the value of 20^{53}?

(a) 2

(b) 8

(c) 1

(d) 4

92. What is the remainder when 2(8!) – 21(6!) divides 14(7!) + 14(13!)?

(a) 1

(b) 7!

(c) 8!

(d) 9!

93. How many integer values of x and y are there such that $4x + 7y = 3$, while $|x| < 500$ and $|y| < 500$?

(a) 144

(b) 141

(c) 143

(d) 142

94. If $n = 1 + m$, where m is the product of four consecutive positive integers, then which of the following is/are true?

(A) n is odd (B) n is not a multiple of 3

(C) n is a perfect square

(a) All three

(b) A and B only

(c) A and C only

(d) None of these

95. How many two-digit numbers less than or equal to 50, have the product of the factorials of their digits less than or equal to the sum of the factorials of their digits?

(a) 18

(b) 16

(c) 15

(d) None of these

96. A candidate takes a test and attempts all the 100 questions in it. While any correct answer fetches 1 mark, wrong answers are penalised as follows; one-tenth of the questions carry 1/10 negative marks each, one-fifth of the questions carry 1/5 negative marks each and the rest of the questions carry ½ negative mark each. Unattempted questions carry no marks. What is the difference between the maximum and the minimum marks that he can score?

(a) 100

(b) 120

(c) 140

(d) None of these

Directions for Questions 97 to 99: A mock test is taken at Mindworkzz. The test paper comprises of questions in three levels of difficulty—LOD1, LOD2 and LOD 3.

The following table gives the details of the positive and negative marks attached to each question type:

Difficulty level	Positive marks for answering the question correctly	Negative marks for answering the question wrongly
LOD 1	4	2
LOD 2	3	1.5
LOD 3	2	1

The test had 200 questions with 80 on LOD 1 and 60 each on LOD 2 and LOD 3.

97. If a student has solved 100 questions exactly and scored 120 marks, the maximum number of incorrect questions that he/she might have marked is

(a) 44

(b) 56

(c) 60

(d) None of these

98. If Amit attempted the least number of questions and got a total of 130 marks, and if it is known that he attempted at least one of every type, then the number of questions he must have attempted is

(a) 34

(b) 35

(c) 36

(d) None of these

99. In the above question, what is the least number of questions he might have got incorrect?

(a) 0

(b) 1

(c) 2

(d) None of these

100. Amitabh has a certain number of toffees, such that if he distributes them amongst ten children he has nine left, if he distributes amongst nine children he would have eight left, if he distributes amongst eight children he would have seven left … and so on until if he distributes amongst five children he should have four left. What is the second lowest number of toffees he could have with him?

(a) 2519

(b) 7559

(c) 8249

(d) 5039

101. If a positive integer 'n' is subtracted from the squares of three consecutive terms of an arithmetic progression, the numbers obtained are 108, 220 and 364 respectively. What is the sum of the digits of 'n'?

102. How many integers exist such that not only are they multiples of 904^{2008} but also are factors of 904^{2015}?

103. What is the remainder when $1^5 + 2^5 + 3^5 + \ldots\ldots + 96^5$ is divided by 194?

104. If $\left[\frac{s}{2}\right]+\left[\frac{s}{3}\right]+\left[\frac{s}{7}\right]=\frac{41}{42}s$ where[s] is the greatest integer less than or equal to 's' and $0 < s < 1000$, then find the number of possible values of 's'.

Directions for Questions 105 to 107:

In a zoo with 100 rabbits, there are three kinds of rabbits, weight-wise viz. 1 kg rabbits, 2 kg rabbits and 5 kg rabbits. There are a minimum of

10 and a maximum of 60 of each kind of rabbit. On a particular day, the zoo director transfers 40 rabbits from his stock and sends them off to the neighbouring zoo. On weighing these rabbits, it was found that the total weight of these 40 rabbits, was 148 kgs. It was also found that the weight of the remaining rabbits was 212 kgs.

105. What is the minimum number of 1 kg rabbits that were transferred?

106. What is the maximum possible number of 5 kg rabbits that remain?

107. If, a total of 26 5-kg rabbits were transferred, then what is the maximum possible numbers of 1 kg rabbits that remain in the zoo?

108. Let A be a two-digit number. The sum of the number A, the number formed by reversing the digits of A and the value of the product of the digits of A is found to equal 117. Then what is the sum of the digits of A?

109. Let M be the product of all natural numbers between 35 and 250 that have an odd number of factors. Find the highest power of 12 in M.

110. x and y are natural numbers such that they satisfy the equation $x + y + 21 = 3xy$. Find the maximum possible integral power of 6 in $(xy)!$.[n! is the product of the first 'n' natural numbers.]

111. Let X is the set of all the natural numbers each of which is equal to the number of its factors and Y is the set of all natural numbers from 1 to 100, each of which differ from the sum of its factors 1. Also, let x and y represent the number of elements in the sets respectively. Then find the value of $[y/x]$, where [] represents the greatest integer function.

112. A four digit number X has 15 factors. What is the number of factors of X^2?

113. How many four-digit odd numbers are possible such that the hundreds digit is two more than the tens digit?

114. Three natural numbers X, Y, Z are prime numbers less than 20 and are in arithmetic progression. If $X > Y > Z$, then how many possible values can we get for $X + Y + Z$?

115. A 100-digit number is multiplied by a 200-digit number and the product is multiplied with a 300-digit number and this product is again multiplied with a 400-digits number. What is the least number of digits in the product?

116. What are the last two digits of the number 3^{400}?

117. The unit digit in $1^7 \times 2^7 \times 3^7 \times \ldots\ldots \times 9^7 \times 11^7 \times 12^7 \ldots\ldots 19^7 \times 21^7 \ldots\ldots 99^7$ is

118. How many two digit numbers have their squares as 1 more than a multiple of 24?

119. There are 80 questions in a test. Each correct answer fetches 1 mark, each wrong answer and unanswered question attract a penalty of 1/4 mark and 1/8 mark respectively each. Frodo scored 23 marks in the test. What is the minimum possible number of the question wrongly answered by him?

120. 1777 has exactly 5 digits when converted to base 'x' from the decimal system. What is the minimum possible value of x?

(a) 3

(b) 4

(c) 6

(d) None of these

121. What will be the sum of all natural numbers between 101 and 1000 which on division by 2, 4, 6, 8, 10 leave remainders 1, 3, 5, 7, 9, respectively?

122. Units digit of which of the following is the same as the units digits of $a^{17} + b^{17}$ for any positive integer value of a, b?

(a) $a^{2} + b^{2}$

(b) $a^{12} + b^{12}$

(c) $a^{13} + b^{13}$

(d) $a^{10} + b^{10}$

123. How many 4-digit numbers are there in the decimal system, which have exactly 4 digits when expressed in base 6, base 7 and base 8?

(a) 158

(b) 248

(c) 296

(d) 368

124. What is the difference between the highest and the least 4-digit natural numbers that have exactly 4 digits when expressed in base 6 and base 7.

125. If n is a natural number, then what is the sum of all the possible distinct remainders when $9n + 6n + 4n + 11n$ is divided by 10?

126. If x, y, z and w are natural numbers, then what is the sum of all the possible remainders when $9x + 6y + 4z + 11z$ is divided by 10?

127. What is the maximum possible sum of the number of Mondays and Thursdays in two consecutive years?

128. In the previous question, find the minimum possible sum of the number of Mondays and Thursdays.

129. What will be the value of remainder when (111111111..........64terms)*(22222222..........55terms) is divided by 18?

(a) 0

(b) 1

(c) 2

(d) 17

130. Find the number of solutions of the equation: $x^2 - y^2 = 777314$:

Directions for Questions 131 and 132:

There was a table in a room and there were 100 coins (coin 1 to coin 100) in a row on the table. All the coins were heads up initially. You entered the room and turned all the coins, the second time you entered the room and turned every 2nd coin (coin 2, coin4, ...), the 3rd time you entered the room and turned every 3rd coin (coin 3, coin 6, coin 9, ...) and so on. If you visited the room 100 times and continued this sequence every time, then answer the following questions.

131. What were the states of 54th and 91st coin after the 100th visit? Type 1, if both are heads; Type 2, if the first one is head and the other is tail; Type 3, if the first one is tail and the second is tail; Type 4, if both are tails.

132. After your 100th visit, how many coins were in the heads-up position?

133. What is the digit at the hundredths place of the number $(225)^{40}$?

(a) 2

(b) 4

(c) 6

(d) 8

134. Consider the set of the first 14 natural numbers. Three numbers a, b, c are selected from this set such that $a > 3b > 4c$. How many such distinct triplets (a, b, c) are possible?

(a) 32

(b) 26

(c) 22

(d) 18

135. How many four-digit numbers having distinct digits using the first five natural numbers (1 to 5) can be formed such that the numbers formed are divisible by each of the digits used in the number?

(a) 0

(b) 1

(c) 2

(d) 3

Space for Rough Work

LEVEL OF DIFFICULTY (III)

1. The expression $7^{4n} – 5^{4n}$, where n is an even number, is always divisible by which of the following number?

   (i) 2

   (ii) 23

   (iii) 111

   (iv) 37

   (a) Only i

   (b) Only ii

   (c) i and iii

   (d) I, iii and iv

2. If the LCM of 3^{12}, 8^{8} and x is 12^{12}, then how many values are possible for x?

3. 'X' is a set of three numbers such that the sum and HCF of the three numbers in it are 105 and 5 respectively. If at least one number in X is less than or equal to 15, then how many such sets are possible?

4. Find the last two digits of: $15 \times 37 \times 63 \times 51 \times 97$ 17.

   (a) 35

   (b) 45

   (c) 55

   (d) 85

Directions for Questions 5 and 6: $15x\,3y\,7z$, a seven-digit number, is completely divisible by 44. 'x', 'y' and 'z' are single digit whole numbers.

5. Find the distinct possible values of $y + z - x$.

6. What is the maximum possible value of $(y + z - x)$.

Directions for Questions 7 and 8: If 'a' is an even natural number and 'b' is an odd natural number & $ab - 3a - 2b - 594 = 0$, then answer the following questions:

7. Find the total number of solutions of the given equation $ab - 3a - 2b - 594 = 0$.

8. Maximum possible value of $a + b$

Directions for Questions 9 and 10: 'x' is a 2-digit number more than 40 such that the tens digit of the number is a factor of x.

9. How many values are possible for x?

10. Find the sum of all values of x.

11. How many distinct five digit multiples of 11 can be formed using the digits 2, 3, 4, 5 and 6 ?

12. N is a 2-digit number, $N < 98$. When 910 is divided by N and $N + 2$ the remainder is 'x' in both cases. Then, the values of 'x' for maximum possible value of N =?

13. If P be the product of all the natural numbers between 50 and 290 that have an odd number of factors, what is the highest power of 6 in P?

14. Find the number of composite odd natural numbers less than 100 and relatively prime to 70.

15. If 51!/10! is completely divisible by 12^x, where x is a natural number, then what is the maximum possible value of x?

Directions for Questions 16 and 17: The sum of number of factors of N and N^2 is 60. If N is a three-digit number. Then answer the following questions:

16. How many values can N have?

17. What is the largest possible value of N?

18. If P and Q are two natural numbers such that

$$P^2 + 3Q^2 = 1376$$

How many solutions are possible for the above mentioned equation?

19. The last two-digits in the multiplication $122 \times 123 \times 125 \times 127 \times 129$ will be

(a) 20

(b) 50

(c) 30

(d) 40

20. A two digit number is divided by the sum of its digits. What is the maximum possible remainder?

21. X is the set of all the integers from –100 to 100. How many subsets of X having exactly 199 elements can be formed such that the sum of all the elements in each subset is either 1 or –1?

22. a, b, c, d, e, f, g and h are eight integers from the sets (–10, 10). Find the minimum possible value of $ab + bc + cd + de + ef + fg + gh + ha$.

23. The remainder obtained when $43^{101} + 23^{101}$ is divided by 66 is

(a) 2

(b) 10

(c) 5

(d) 0

24. Both the H.C.F and the difference of two numbers is 6. If the L.C.M of the two numbers is a 4-digit number, then what is the maximum possible value of the bigger number?

25. Both the H.C.F and the difference of two numbers is 6. If the L.C.M of the two numbers is a 4-digit number, then how many such pairs are possible?

Directions for Questions 26 and 27: N is a natural number which has 6 factors. If $1 \leq N \leq 50$, then answer the following questions:

Directions for Questions 26 and 27: N is a natural number which has 6 factors. If $1 \le N \le 50$, then answer the following questions:

26. How many values are possible for N?

27. Find the maximum possible value of N.

28. The numbers $(6 - p)$, $(6 - q)$, $(6 - r)$, $(6 - s)$ and $(6 - t)$ are all distinct integers whose product is equal to 6720. What is the value of $(p + q + r + s + t)$?

29. For how many positive integer values of x, will the expression $87y - 264x = 60$ be valid, if x and y are positive integers less than 100?

Directions for Questions 30 to 32: A two-digit number 'ab' having distinct digits when divided by the sum of the digits gives the same remainder as when a two-digit number that is formed by reversing the digits of original number is divided by the sum of the digits.

30. How many such two-digit numbers are possible?

31. Find the maximum value of the two digit number '*ab*'.

32. Find the minimum value of '*ab*'.

33. '*X*' is a 62-digit number. All the digits except the 26th digit from the right are the same. If '*X*' is divisible by 13, then which of the following can never be the unit's digit of '*X*'?

    (a) 4

    (b) 1

    (c) 5

    (d) 6

34. $P = \{2, 3, 5, 10, 20, 40, 80, 160, 320\}$. How many distinct values of '*S*' are possible, if '*S*' is defined as the sum of one or more elements of the set '*P*'?

35. Given that p, q, r and s are positive integers such that $q^{\frac{1}{2}} = p$ and $s^{\frac{1}{2}} = r$ If $s - q = 17$, then find the value of $p + r$.

36. $\frac{32^{32^{32}}}{9}$ will leave a remainder

(a) 4

(b) 7

(c) 1

(d) 2

Directions for Questions 37 to 39: *abc* is a three digit number and accb is a four digit number (*a*, *b*, *c* are single digit natural numbers). If $ab \times ac = abc$, then answer the following questions:

37. What is the value of *b*?

(a) 2

(b) 1

(c) 0

(d) Cannot be determined

38. What is the remainder when $bc \times accaaa$ is divided by *ac*?

39. If *abc* is divisible by 17, then what is the remainder when *abcbac* is divided by 14?

40. The remainder of $\frac{32^{32^{32}}}{7}$ is equal to:

(a) 4

(b) 2

(c) 1

(d) 3

41. n is a number, such that $2n$ has 28 factors and $3n$ has 30 factors. $6n$ has

(a) 35

(b) 32

(c) 28

(d) None of these

42. Suppose the sum of n consecutive integers is $x + (x + 1) + (x + 2) + (x + 3) + \dots + (x + (n - 1)) = 1000$, then which of the following cannot be true about the number of terms n

(a) The number of terms can be 16

(b) The number of terms can be 5

(c) The number of terms can be 25

(d) The number of terms can be 20

43. The remainder when $2_2 + 22_2 + 222_2 + 2222_2 + \dots\dots.(222\dots.49\text{ twos})_2$ is divided by 9 is

(a) 2

(b) 5

(c) 6

(d) 7

44. $N = 202 \times 20002 \times 200000002 \times 20000000000000002 \times 200000000\dots.2$ (31 zeroes) The sum of digits in this multiplication will be

(a) 112

(b) 160

(c) 144

(d) Cannot be determined

45. Twenty-five sets of problems on Data Interpretation– one each for the DI sections of 25 CATALYST tests were prepared by the AMS research team. The DI section of each CATALYST contained 50 questions of which exactly 35 questions were unique, i.e. they had not been used in the DI section of any of the other 24 CATALYSTs. What could be the maximum possible number of questions prepared for the DI sections of all the 25 CATALYSTs put together?

(a) 1100

(b) 975

(c) 1070

(d) 1055

46. In the above question, what could be the minimum possible number of questions prepared?

(a) 890

(b) 875

(c) 975

(d) None of these

Directions for Questions 47 to 49: At a particular time in the twenty-first century, there were seven bowlers in the Indian cricket team's list of 16 players short-listed to play the next world cup. Statisticians discovered that that if you looked at the number of wickets taken by any of the 7 bowlers of the current Indian cricket team, the number of wickets taken by them had a strange property.
The numbers were such that for any team selection of 11 players (having 1 to 7 bowlers) by using the number of wickets taken by each bowler and attaching coefficients of +1, 0, or –1 to each value available and adding the resultant values, any number from 1 to 1093, both included could be formed. If we denote W_1, W_2, W_3, W_4, W_5, W_6 and W_7 as the 7 values in the ascending order what could be the answer to the following questions?

47. Find the value of $W_1 + 2W_2 + 3W_3 + 4W_4 + 5W_5 + 6W_6$.

    (a) 2005

    (b) 1995

    (c) 1985

    (d) None of these

48. Find the index of the largest power of 3 contained in the product $W_1\ W_2\ W_3\ W_4\ W_5\ W_6\ W_7$.

    (a) 15

    (b) 10

    (c) 21

    (d) 6

49. If the sum of the seven coefficients is 0, find the smallest number that can be obtained.

(a) – 1067

(b) – 729

(c) – 1040

(d) – 1053

Directions for Questions 50 and 51: Answer these questions on the basis of the information given below.

In the ancient game of Honololo, the task involves solving a puzzle asked by the chief of the tribe. Anybody answering the puzzle correctly is given the hand of the most beautiful maiden of the tribe. Unfortunately, for the youth of the tribe, solving the puzzle is not a cakewalk since the chief is the greatest mathematician of the tribe.

In one such competition the chief called everyone to attention and announced openly:

"A three-digit number '*mnp*' is a perfect square and the number of factors it has, is also a perfect square. It is also known that the digits m, n and p are all distinct. Now answer my questions and win the maiden's hand."

50. If $(m + n + p)$ is also a perfect square, what is the number of factors of the six-digit number mnpmnp?

(a) 36

(b) 72

(c) 48

(d) Cannot be determined

51. If the fourth power of the product of the digits of the number mnp is not divisible by 5, what is the number of factors of the nine-digit number, *mnpmnpmnp*?

(a) 32

(b) 72

(c) 48

(d) Cannot be determined

52. In a cricket tournament organised by the ICC, a total of 15 teams participated. Australia, as usual, won the tournament by scoring the maximum number of points. The tournament is organised as a single round robin tournament—where each team plays with every other team exactly once. Three points are awarded for a win, 2 points are awarded for a tie/washed out match and 1 point is awarded for a loss. Zimbabwe had the lowest score (in terms of points) at the end of the tournament. Zimbabwe scored a total of 21 points. All the 15 national teams got a distinct score (in terms of points scored). It is also known that at least, one match played by the Australian team was tied/washed out. Which of the following is always true for the Australian team?

(a) It had at least two ties/washouts

(b) It had a maximum of 3 losses

(c) It had a maximum of 9 wins

(d) All of these

53. What is the remainder when 128^{1000} is divided by 153?

(a) 103

(b) 145

(c) 118

(d) 52

54. Find the remainder when $7^{51^{311}}$ is divided by 11.

(a) 6

(b) 4

(c) 7

(d) 3

55. Find the remainder when $50^{51^{52}}$ is divided by 11.

(a) 5

(b) 4

(c) 10

(d) 1

56. Find the remainder when $32^{33^{34}}$ is divided by 11.

(a) 5

(b) 9

(c) 6

(d) 3

57. Find the remainder when $30^{72^{87}}$ is divided by 11.

(a) 7

(b) 5

(c) 9

(d) 10

58. Find the remainder when $50^{56^{52}}$ is divided by 7.

(a) 5

(b) 4

(c) 6

(d) 2

59. Let S_m denote the sum of the squares of the first m natural numbers. For how many values of $m < 100$, is S_m a multiple of 4?

(a) 50

(b) 25

(c) 36

(d) 24

60. For the above question, for how many values will the sum of cubes of the first m natural numbers be a multiple of 5 (if $m < 50$)?

(a) 20

(b) 21

(c) 22

(d) None of these

61. How many integer values of x and y satisfy the expression $4x + 7y = 3$ where $|x| < 1000$ and $|y| < 1000$?

(a) 284

(b) 285

(c) 286

(d) None of these

62. N = 7777………………7777, where the digit 7 repeats itself 603 times. What is the remainder left when N is divided by 1144?

63. How many factors of 19! are there, whose unit digit is 5?

64. $N = 1! - 2! + 3! - 4! + \ldots + 47! - 48! + 49!, - 50! + 51!$ Then what is the unit digit of N^N?

65. How many numbers less than 100 have exactly four factors?

66. Two odd numbers have 36 factors each and the HCF of these two numbers is 225. What is the minimum possible LCM of these two numbers if the power of any prime factor in these two numbers is not more than 3?

67. Find the remainder when $[(7!)^{6!}]^{17777}$ is divided by 17?

68. A four digit number $wxyz$ is such that $x + y = 2w$ and $y + 6z = 2(w + x)$ & $w + 5z = 2y$. Find the sum of such four digit numbers which satisfy the given conditions.

69. Find the remainder when (17)(9!)+2 (18!) is divided by (9!)17408.

70. X is a number formed by writing the first 1002 natural numbers one after another from left to right then find the remainder when X is divided by 9.

71. X is a number formed by writing the first 1002 whole numbers one after another from left to right then a vertical line is drawn which divides the number such that the number of digits on either side of line is the same. Find the remainder when the number formed by the digits on the left of the vertical line, is divided by 625.

72. Amongst all the four digit natural numbers divisible by 24, how many have the number 24 in them?

73. If X is a natural number and $X < 100$ then the number of values of X for which $18X + 2$ & $12X + 1$ are relatively prime?

74. P is a natural number of at least 6 digits and its leftmost digit is 7. When this leftmost digit is removed from P, the number thus obtained is found to be 1/21 times of P. What is the product of the all the nonzero digits of P?

(a) 126

(b) 105

(c) 60

(d) 72

75. $X!$ is completely divisible by1151 but not by 1152. What is the sum of digits of largest such number X?

76. If 'a' is a natural number and HCF of a, $a + 5$ is 5. If the LCM of the two numbers is a three-digit number, then what is the difference between the maximum and minimum possible values of the smaller number?

(a) 25

(b) 35

(c) 40

(d) 45

77. How many times would 1 be used while writing all the natural numbers from 8 to 127 in the binary number system?

(a) 212

(b) 218

(c) 424

(d) 436

78. What is the maximum number of elements that one can pick from the set of natural numbers from 1 to 20 such that the product of no two of them results in a perfect square or perfect cube?

79. *abcdefghij* is a ten digit number with distinct digits such that $a > b > c$, d secutive odd digits. If $d + e + f = 9$, then what is the value of $\left[\frac{a \times b \times c \times d}{i}\right]$ (where [] denotes greatest integer function)?

(a) 42

(b) 0

(c) 54

(d) 66

80. $N = abc$ is a three digit number, the sum of whose digits is 1/7th of the product of its digits. Then how many possible sets of (a, b, c) are possible?

81. $(3132!)_{10} = (x)_{34}$, then what will be the number of consecutive zeroes at the end of 'x'?

(a) 124

(b) 167

(c) 194

(d) None of these

Space for Rough Work

ANSWER KEY

Level of Difficulty (I)

1. (a)
2. (a)
3. (b)
4. (b)
5. (a)

6. (c)
7. (d)
8. (d)
9. (d)
10. (d)
11. (a)
12. (c)
13. (d)
14. (d)
15. (b)
16. (c)
17. (b)
18. (d)
19. (a)
20. (b)
21. (d)
22. (a)
23. (b)
24. (c)
25. (d)
26. (d)
27. (c)
28. (d)
29. (b)
30. (d)

31. (a)
32. (b)
33. (c)
34. (d)
35. (c)
36. (b)
37. (d)
38. (d)
39. (b)
40. (b)
41. (a)
42. (a)
43. (a)
44. (d)
45. (c)
46. (d)
47. (b)
48. (c)
49. (c)
50. (d)
51. (a)
52. (a)
53. (b)
54. (c)

55. (d)

56. (b)

57. (a)

58. (b)

59. (d)

60. (d)

61. (a)

62. (a) $\rightarrow 21^{12}$ (b) $\rightarrow 0.8^{3}$

63. (b)

64. a. → 0, 2, 4, 6, 8

b. → 1, 4, 7

c. → 0, 4, 8

d. → 0, 5

e. → 4

f. → 7

g. → 0

65. LCM → 5670

HCF → 27

LCM → 780

HCF → 39

LCM → 737100

HCF → 30

66. (b)

67. (a)

68. (c)

69. (c)
70. (c)
71. (a)
72. (d)
73. (b)
74. (b)
75. (a)
76. (b)
77. (d)
78. (a)
79. (a)
80. (b)
81. (c)
82. (b)
83. (c)
84. (c)
85. (c)
86. (a)
87. (c)
88. (b)
89. (b)
90. (c)
91. (d)
92. (d)
93. (a)
94. (c)

95. (b)

96. (b)

97. (a)

98. (c)

99. (c)

100. (d)

101. (d)

102. (a)

103. (c)

104. (b)

105. (c)

106. (d)

107. (b)

108. (c)

109. (d)

110. (c)

111. (c)

112. (c)

113. (a)

114. (c)

115. (a)

116. (d)

117. (d)

118. (b)

119. (b)

120. (c)

121. (b)

122. (b)

123. (d)

124. (b)

125. (d)

126. (a)

127. (b)

128. (d)

129. (c)

130. (b)

131. (b)

132. (a)

133. (c)

134. (d)

135. (b)

136. (b)

137. (c)

138. (b)

139. (d)

140. (a)

141. 7

142. 11

143. 726

144. 6

145. 7

146. 12

147. 242

148. Data Inconsistent

149. d

150. 1

151. 5

152. 0

153. 5

154. 10 and 11

155. 3

156. 0

157. 5040

158. 1

159. 1111111

160. 1

161. 440

162. 24

163. 198

164. 120

165. 5

166. 2

167. 40

168. 4

169. 1

170. 96

171. 8

172. $X + Y$

173. 5

174. $B \geq A > D \geq C$

175. 4

Level of Difficulty (II)

1. (a)
2. (b)
3. (c)
4. (c)
5. (b)
6. (b)
7. (b)
8. (d)
9. (c)
10. (c)
11. (d)
12. (d)
13. (d)
14. (c)
15. (b)
16. (d)
17. (a)
18. (d)
19. (c)
20. (a)
21. (a)
22. (c)
23. (b)
24. (a)

25. (d)

26. (c)

27. (c)

28. (d)

29. (b)

30. (b)

31. (a)

32. (b)

33. (a)

34. (a)

35. (b)

36. (b)

37. (a)

38. (d)

39. (c)

40. (a)

41. (24)

42. (63)

43. (24)

44. (32)

45. (27)

46. (64)

47. (13, 31)

48. (23)

49. (1056)

50. (51, 34)

51. (144, 864)

52. (46, 64)

53. (63)

54. (863)

55. (36, 63)

56. may be 2 or 3 depending upon the numbers

57. 111^4 > 109.110.112.113 > 110.109.108.107

58. greatest → 35 least 3/7

59. (a) 200^{300}

(b) 5^{100}

(c) 10^{20}

60. (12, 3)

61. (c)

62. (c)

63. (b)

64. (a)

65. (d)

66. (c)

67. (b)

68. (a)

69. (a)

70. (b)

71. (c)

72. (c)

73. (d)

74. (a)

75. (b)

76. (d)

77. (a)

78. (b)

79. (b)

80. (c)

81. (b)

82. (c)

83. (d)

84. (a)

85. (c)

86. (b)

87. (d)

88. (b)

89. (d)

90. (d)

91. (a)

92. (b)

93. (c)

94. (a)

95. (a)

96. (c)

97. (b)

98. (a)

99. (a)

100. (d)

101. 9

102. 176

103. 0

104. 23

105. 1

106. 37

107. 10

108. 9

109. 8

110. 4

111. 12

112. 45

113. 360

114. 4

115. 997

116. 01

117. 0

118. 30

119. 6

120. (d)

121. 4312

122. (c)

123. 296

124. 295

125. 4

126. 8

127. 209

128. 208

129. (c)

130. 0

131. 1

132. 90

133. (c)

134. (c)

135. (a)

Level of Difficulty (III)

1. (d)
2. 325
3. 25
4. (a)
5. 12
6. 14
7. 12
8. 307
9. 12
10. 827
11. 12

12. 11

13. 8

14. 11

15. 19

16. 4 values

17. 784

18. 0

19. 50

20. 15

21. 200

22. –800

23. 0

24. 246

25. 28

26. 8

27. 50

28. 0

29. 1

30. 16

31. 84

32. 12

33. (a)

34. 384

35. 2195

36. 4

37. (d)

38. 0

39. 8

40. 4

41. (a)

42. (d)

43. (c)

44. (b)

45. (d)

46. (a)

47. (a)

48. (c)

49. (c)

50. (c)

51. (a)

52. (b)

53. (d)

54. (a)

55. (c)

56. (a)

57. (b)

58. (d)

59. (d)

60. (d)

61. (b)

62. 777

63. 1296

64. 1

65. 34

66. $3^3 \times 5^2 \times 7^2 \times 11 \times 13$

67. 1

68. 9723

69. 17.9!

70. 6

71. 601

72. 25

73. 99

74. (b)

75. 14

76. (d)

77. 436

78. 14

79. (c)

80. 2

81. (c)

Solutions and Shortcuts

Level of Difficulty (I)

1. The units digit in this case would obviously be '0' because the given expression has a pair of 2 and 5 in its prime factors.

2. When you read the sentence "when the digits are reversed, the number decreases by 54, you should automatically get two reactions going in your mind.

(i) The difference between the digits would be 54/9 = 6.

(ii) Since the number 'decreases'- the tens digit of the number would be larger than the units digit.

Also, since we know that the sum of the digits is 10, we get that the digits must be 8 and 2 and the number must be 82. Thus, the changed number is 28.

3. The two numbers should be factors of 405. A factor search will yield the factors. (look only for 2 digit factors of 405 with sum of digits between 1 to 19).

Also $405 = 5 \times 3^4$. Hence: 15×27

45×9 are the only two options.

From these factors pairs, only the second pair gives us the desired result.

i.e. Number × sum of digits = 405

Hence, the answer is 45.

4. You can solve this question by using options. It can be seen that Option (b) 12,3 fits the situation perfectly as their arithmetic mean = 7.5 and their geometric mean = 6 and the geometric mean is 20% less than the arithmetic mean.

6. For the number A381 to be divisible by 11, the sum of the even placed digits and the odds placed digits should be either 0 or a multiple of 11. This means that $(A + 8) – (3 + 1)$ should be a multiple of 11 – as it is not possible to make it zero. Thus, the smallest value that A can take (and in fact the only value it can take) is 7. Option (c) is correct.
7. For 381A to be divisible by 9, the sum of the digits $3 + 8 + 1 + A$ should be divisible by 9. For that to happen, A should be 6. Option (d) is correct.
8. 9^6 when divided by 8, would give a remainder of 1. Hence, the required answer would be 2.
9. LC*M* of 5, 15 and 20 = 60. HCF of 5, 15 and 20 = 5. The required ratio is 60:5 = 12:1
10. LCM of 5/2, 8/9 and 11/14 would be given by: (LCM of numerators)/(HCF of denominators)

    = 440/1 = 440
11. Only the first option can be verified to be true in this case. If A is even, 3A would always be divisible by 6 as it would be divisible by both 2 and 3. Options (b) and (c) can be seen to be incorrect by assuming the value of A as 4.
12. The essence of this question is in the fact that the last digit of the number is 0. Naturally, the number is necessarily divisible by 2, 5 and 10. Only 4 does not necessarily divide it.
13. B would necessarily be even, as the possible values of B for the three digit number 15B to be divisible by 6 are 0 and 6. Also, the condition stated in

Option (c) is also seen to be true in this case — as both 0 and 6 are divisible by 6. Thus, Option (d) is correct.

14. For the GCD, take the least powers of all common prime factors.

    Thus, the required answer would be $2^3 \times 3$.

15. The units digit would be given by 5 + 6 + 9 (numbers ending in 5 and 6 would always end in 5 and 6 irrespective of the power and 3^{54} will give a units digit equivalent to 3^{4n+2} which would give us a unit digit of 3^2 i.e. 9).

16. The respective units digits for the three parts of the expression would be:

    5 + 9 + 2 =16 → required answer is 6. Option (c) is correct.

17. The respective units digits for the six parts of the expression would be:

    1 + 4 + 7 + 6 + 5 + 6 =29 → required answer is 9. Option (b) is correct.

18. The respective units digits for the six parts of the expression would be:

    1 × 4 × 7 × 6 × 5 × 6 → required answer is 0. Option (d) is correct.

19. The number of zeroes would be given by adding the quotients when we successively divide 1090 by 5:

    1090/5 + 218/5 + 43/5 + 8/5 = 218 + 43 + 8 + 1 = 270. Option (a) is correct.

20. The number of 5's in 146! can be obtained by [146/5] + [29/5] + [5/5]= 29+5+1 = 35

21. $1420 = 142 \times 10 = 2^2 \times 71^1 \times 5^1$.

    Thus, the number of factors of the number would be (2 + 1) (1 + 1) (1 + 1) = 3 × 2 × 2 = 12.

Option (d) is correct.

22. $(x^2 - 5x + 6) = (x - 2)(x - 3)$

and $(x^2 - 7x + 10) = (x - 5)(x - 2)$

Required HCF = $(x - 2)$; required LCM = $(x - 2)(x - 3)(x - 5)$.

Option (a) is correct.

23. Since both P and Q are prime numbers, the number of factors would be $(1 + 1)(1 + 1) = 4$.

24. Since both P and Q are prime numbers, the number of factors would be $(2 + 1)(1 + 1) = 6$.

25. Since both P and Q are prime numbers, the number of factors would be $(3 + 1)(2 + 1) = 12$.

26. The sides of the pentagon being 1422, 1737, 2160, 2214 and 2358, the least difference between any two numbers is 54. Hence, the correct answer will be a factor of 54.

Further, since there are some odd numbers in the list, the answer should be an odd factor of 54.

Hence, check with 27, 9 and 3 in that order. You will get 9 as the HCF.

27. The HCF of 576 and 448 is 64. Hence, each section should have 64 children. The number of sections would be given by: $576/64 + 448/64 = 9 + 7 = 16$. Option (c) is correct.

28. The HCF of the given numbers is 31 and hence the number of bottles required would be

403/31 + 465/31 + 496/31 = 13 + 15 + 16 = 44. Option (d) is correct.

29. The LCM of the 4 numbers is 612. The highest 4 digit number which would be a common multiple of all these 4 numbers is 9792. Hence, the correct answer is 9793.

30. The LCM of 16, 18 and 20 is 720. The numbers which would give a remainder of 4, when divided by 16, 18 and 20 would be given by the series:

724, 1444, 2164, 2884 and so on. Checking each of these numbers for divisibility by 7, it can be seen that 2884 is the least number in the series that is divisible by 7 and hence is the correct answer. Option (d) is correct.

31. They will ring together again after a time which would be the LCM of 6, 8, 12 and 18. The required LCM = 72. Hence, they would ring together after 72 seconds. Option (a) is correct.

32. 720/72 = 10 times. Option (b) is correct.

33. $5 \times 7 \times 6 = 0$. Option (c) is correct.

34. All these numbers can be verified to not be perfect squares. Option (d) is correct.

35. A perfect square can never end in an odd number of zeroes. Option (c) is correct.

36. It is obvious that the LCM of 5,8,12 and 20 would never be a multiple of 9. At the same time it has to be a multiple of each of 3, 8 and 5. Option (b) is correct.

37. $720 = 2^4 \times 3^2 \times 5^1$. Number of factors = 5 × 3 × 2 = 30. Option (d) is correct.

38. $16 - x^2 = (4 - x)(4 + x)$ and $x^2 + x - 6 = (x + 3)(x - 2)$

The required LCM = $(4 - x)(4 + x)(x + 3)(x - 2)$.

Option (d) is correct.

39. $x^2 - 4 = (x - 2)(x + 2)$ and $x^2 + x - 6 = (x + 3)(x - 2)$

GCD or HCF of these expressions = $(x - 2)$.

Option (b) is correct.

40. If A is not divisible by 3, it is obvious that 2A would also not be divisible by 3, as 2A would have no '3' in it.

41. $9^{100}/8 = (8 + 1)^{100}/8$ → Since this is of the form $(a + 1)^n/a$, the Remainder = 1. Option (a) is correct.

42. $2^{1000}/3$ is of the form $(a)^{\text{EVEN POWER}}/(a + 1)$. The remainder = 1 in this case as the power is even. Option (a) is correct.

43. The condition for the product to be the greatest is if the two terms are equal. Thus, the break up in option (a) would give us the highest product of the two parts. Option (a) is correct.

44. 50/5 = 10, 10/5 = 2.

Thus, the required answer would be 10 + 2 = 12. Option (d) is correct.

45. For a number to be divisible by 24, it should be divisible by 3 and 8. Only Option (c) has this property.

46. Any number divisible by 88, has to be necessarily divisible by 11, 2, 4, 8, 44 and 22. Thus, each of the first three options is correct.

47. $10800 = 108 \times 100 = 3^3 \times 2^4 \times 5^2$.

The number of divisors would be: $(3 + 1)(4 + 1)(2 + 1) = 4 \times 5 \times 3 = 60$ divisors. Option (b) is correct.

48. The GCD (also known as HCF) would be obtained by multiplying the least powers of all common factors of the two polynomials. The common factors are$(x + 3)$ – least power 1, and $(x + 1)$ – least power 2. Thus, the answer would be $(x + 3)(x + 1)^2$. Option (c) is correct.

49. For the LCM of polynomials, write down the highest powers of all available factors of all the polynomials.

The correct answer would be $2(x + 3)^2(6x^2 + 5x + 4)(x + 1/2)$.

50. Three consecutive natural numbers, starting with an even number would always have at least three 2's as their prime factors and also would have at least one multiple of 3 in them. Thus, 6, 12 and 24 would each divide the product.

51. When the birds sat one on a branch, there was one extra bird. When they sat 2 to a branch, one branch was extra.

To find the number of branches, go through options. Checking option (a), if there were 3 branches, there would be 4 birds. (this would leave one bird without branch as per the question.)

When 4 birds would sit 2 to a branch there would be 1 branch free (as per the question). Hence, the answer (a) is correct.

52. The number would either be $(3n + 1)^2$ or $(3n + 2)^2$. In the expansion of each of these, the only term which would not be divisible by 3 would be the square of 1 and 2 respectively. When divided by 3, both of these give 1 as remainder.

53. The given expression can be written as:

$5^3 \times 3^3 \times 3^2 \times 7^2/5^2 \times 7^2 \times 3^4 = 5^3 \times 3^5 \times 7^2/5^2 \times 7^2 \times 3^4 = 15$. Option (b) is correct.

54. $D = 1.44, C = 0.09, B = 0.16$, while the value of A is negative.

Thus, $D > B > C > A$ is the required order. Option (c) is correct.

55. The upper limit for $x + y = 4 + 3 = 7$. The lower limit of $x - y = 2 - 3 = -1$. Required ratio = $7/-1 = -7$.

Option (d) is correct.

56. For the sum of squares of digits to be 13, it is obvious that the digits should be 2 and 3. So the number can only be 23 or 32. Further, the number being referred to has to be 32 since the reduction of 9, reverses the digits.

57. Trying the value in the options, you get that the product of $54 \times 45 = 2430$. Option (a) is correct.

58. Option (b) can be verified to be true as the LCM of 90 and 24 is indeed 360.

59. The pairs given in option (d), 78 and 13 and 26 and 39 meet both the conditions of LCM of 78 and HCF of 13. Option (d) is correct.

60. Solve using options. Option (d) 51 and 34 satisfies the required conditions.

61. $28^2 - 27^2 = 55$ and so also $8^2 - 3^2 = 55$. Option (a) is correct.

62. (a) $21^{12} = (21^3)^4$

Since $21^3 > 54$, $21^{12} > 54^4$.

(b) $(0.4)^4 = (4/10)^4 = 1024/10000 = 0.1024$

$(0.8)^3 = (8/10)^3 = 512/1000 = 0.512$

Hence, $(0.8)^3 > (0.4)^4$

63. This is never possible.

64. (a) c = 0, 2, 4, 6 or 8 would make 38c as even and hence divisible by 2.

(b) c = 1, 4 or 7 are possible values to make 38c divisible by 3.

(c) c = 0, 4 or 8 would make the number end in 80, 84 or 88 and would hence be divisible by 4.

(d) c = 0 or 5 would make the number 380 or 385 – in which case it would be divisible by 5.

(e) For the number to become divisible by 6, it should be even and divisible by 3. From the values 1, 4 and 7 which make the number divisible by 3, we only have c = 4 making it even. Thus, c = 4.

(f) For the number to be divisible by 9, $3 + 8 + c$ should be a multiple of 9. c = 7 is the only value of c which can make the number divisible by 9.

(g) Obviously c = 0 is the correct answer.

65. Use the standard process to solve for LCM and HCF.

66. For $34x^{43}$ to be ending in 7, x has to be 3 (as $43 = 4n + 3$). Option (b) is correct.

Solutions for Questions 67 and 68:

The given condition says that Pen<Pencil<Eraser.

Also, since the least cost of the three is ₹12, if we allocate a minimum of 12 to each we use up 36 out of the 41 available. The remaining 5 can be distributed as 0, 1, 4 or 0, 2 and 3 giving possible values of **Case 1** : 12, 13 and 16 or **Case 2:** 12, 14 and 15.

67. In both cases, the cost of the pen is 12.

68. If the cost of the eraser is not divisible by 4, it means that **Case 2** holds true. For this case, the cost of the pencil is 14.

69. Amrit would place eight oranges in the basket (as there are eight 1's).

For the mangoes, he would place six mangoes (number of 2's) and remove four mangoes (number of 4's) from the basket. Thus, there would be 2 mangoes and 8 oranges in the basket.

A total of 8 – 2 = 6 extra oranges in the basket. Option (c) is correct.

70. Solve using trial and error – Option (c) fits the situation as if we start with 15 mangoes, the following structure would take place:

Start with 15 mangoes → First watchman takes 1/3rd + 1 more = 5 + 1 = 6 mangoes → 9 mangoes left.

Second watchman takes → 1/3rd + 1 more = 3 + 1 = 4 mangoes → 5 mangoes left.

Third watchman takes → 1/5th + 4 more = 1 + 4 = 5 mangoes → 0 mangoes left.

71. The last 3 digits of the number would determine the remainder when it is divided by 8. The number up to the 120th digit would be 1234567891011… 646. 646 divided by 8 gives us a remainder of 6.

72. There would be multiple ways of scoring 34.5 marks. Think about this as follows:

If he solves 80 and gets all 80 correct, he would end up scoring 80 marks.

With every question that would go wrong, his score would fall down by 1.5 marks (he would lose the 1 mark he is gaining and further attract a penalty of 0.5 marks).

Also, for every question he does not attempt, his score would fall down by: 1.25 marks (he would lose the 1 mark he is gaining and further attract a penalty of 0.25 marks).

Thus, his score would drop @ 1.5 and @1.25 marks for every wrong and every unattempted question respectively.

Also, to get a total of 34.5 marks overall, he has to lose 45.5 marks.

There are many possible combinations of non-attempts and wrongs through which he can possibly lose 45.5 marks—for example:

17 wrongs (loses 25.5 marks) and 16 non-attempts (loses 20 marks)

12 wrongs (loses 18 marks) and 22 non-attempts (loses 27.5 marks)

Hence, we cannot answer this question uniquely and the answer is Option (d).

73. Continuing the thought process for the previous question our thinking would go as follows:

10 questions unanswered → loses 12.5 marks

To lose another 33 marks, he needs to get 22 incorrects.

Thus, the number of corrects would be 80 – 10 – 22 = 48. Option (b) is correct.

74. $3M + 4G + 5W = 750$ (1)

$6M + 9G + 10W = 1580$ (2)
Adding the two equations we get:

$9M + 13G + 15W = 2330$ (3)

Dividing this expression by 3, we get:

$3M + 4.33G + 5W = 776.666$ (4)

(4) – (1) → $0.33\,G = 26.666$ → $G = 80$

Now, if we look at the equation (1) and multiply it by 2, we get: $6M + 8G + 10W = 1500$. If we subtract the cost of 4 guavas from this we would get:

$6M + 4G + 10W = 1500 - 320 = 1180$

Option (b) is correct.

75. If you try a value of M as 5, N would become ¼. It can be seen that $5^{1/4}$ (which would be the value of M^N) would be around 1.4 and hence, less than 2.
If you try for possible values of M^N by increasing the value of M, you would get $6^{1/5}$, $7^{1/6}$, $8^{1/7}$, $9^{1/8}$ and so on. In each of these cases, you can clearly see that the value of M^N would always be getting consecutively smaller than the previous value.

If you tried to go for values of M such that they are lower than 5, you would get the following values for MN:

$4^{1/3}$, $3^{1/2}$ and the last value would be $2^{1/1}$. In this case, we can clearly see that the value of the expression M^N is increasing. However, it ends at the value of 2 (for $2^{1/1}$) and hence that is the maximum value that M^N can take. Option (a) is correct.

76. (7M + 9G + 19W) – (4M + 6G + 16W) = 120. Hence, 1M + 1G + 1W = 40

77. The cost of a mango cannot be uniquely determined here because we have only 2 equations between 3 variables, and there is no way to eliminate one variable.

78. Since the value of y varies between –8 to 2, it is evident that if we take a very small value for y, say 0.0000000000000000000000001, and we take normal integral values for x and z, the expression xz/y would become either positive or negative infinity (depending on how you manage the signs of the numbers x, y and z).

79. Ties < Trousers < Shirts. Since each of the three is minimum 11, the total would be a minimum of 33 (for all 3). The remaining 5 need to be distributed amongst ties, trousers and shirts so that they can maintain the inequality, Ties < Trousers < Shirts.

This can be achieved with 11 ties, and the remaining 27 pieces of clothing distributed between trousers and shirts such that the shirts are greater than the trousers.

This can be done in at least 2 ways: 12 trousers and 15 shirts; 13 trousers and 14 shirts.

If you try to go for 12 ties, the remaining 26 pieces of clothing need to be distributed amongst shirts and trousers such that the shirts are greater than the trousers and both are greater than 12.

With only 26 pieces of clothing to be distributed between shirts and trousers, this is not possible. Hence, the number of ties has to be exactly 11. Option (a) is correct.

80. The number of shirts would be at least 14 as the two distributions possible are: 11, 12, 15 and 11, 13, 14. Option (b) is correct.

81. The LCM of 12, 15, 18 and 20 is 180. Thus, the least number would be 184.

Option (c) is correct.

82. 2800 = 20 × 20 × 7. Thus, we need to multiply or divide with 7 in order to make it a perfect square.

83. The answer is given by $\sqrt{1024}$ = 32.

This can be experimentally verified as 30^2 = 900, 31^2 = 961 and 32^2 = 1024. Hence, 1024 is the required answer. Option (c) is correct.

84. First find the LCM of 6, 9, 15 and 18. Their LCM = 18 × 5 = 90.

The series of numbers which would leave a remainder of 4 when divided by 6, 9, 15 and 18 would be given by:

LCM + 4; 2 × LCM + 4; 3 × LCM + 4; 4 × LCM + 4; 5 × LCM + 4 and so on.

Thus, this series would be:

94, 184, 274, 364, 454....

The other constraint in the problem is to find a number which also has the property of being divisible by 7. Checking each of the numbers in the series above for their divisibility by 7, we see that 364 is the least value which is also divisible by 7. Option (c) is correct.

85. LCM of 2, 3, 4, 5 and 6 = 6 × 5 × 2 = 60 (Refer to the shortcut process for LCM given in the chapter notes).

Thus, the series 61, 121, 181, etc., would give us a remainder 1 when divided by 2, 3, 4, 5 and 6.

The least 3 digit number in this series is 121. Option (c) is correct.

86. 70 = 2 × 5 × 7; 245 = 5 × 7 × 7.

HCF = 5 × 7 = 35. Option (a) is correct.

87. 7056 is the closest perfect square below 7147. Hence, 7147 – 7056 = 91 is the required answer. Option (c) is correct.

88. The LCM of 6, 8 and 15 is 120 = 23 × 3 × 5. For a number to be a perfect square, all the prime factors should have even powers. Thus, if we multiply the above number by 2 × 3 × 5 = 30, we will get the required smallest perfect square. Thus, the correct answer is 120 × 30 = 3600.

89. 30492 = $2^2 \times 3^2 \times 7^1 \times 11^2$.

For a number to be a perfect square, each of the prime factors in the standard form of the number needs to be raised to an even power. Thus, we need to multiply or divide the number by 7 so that we either make it: $2^2 \times 3^2 \times 7^2 \times 11^2$ (if we multiply the number by 7) or

We make it: $2^2 \times 3^2 \times 11^2$ (if we divide the number by 7).

Option (b) is correct.

90. 88 × 113 = 9944 is the greatest 4 digit number exactly divisible by 88. Option (c) is correct.

91. 3/4th of 116 = ¾ × 116 = 87

4/5th of 45 = 4/5 × 45 = 36

Required difference = 51

Option (d) is correct.

92. The correct arrangement would be 75 plants in a row and 75 rows since 5625 is the square of 75.

93. $9^{\text{EVEN POWER}} \times 7^{4n+1} \rightarrow 1 \times 7 = 7$ as the units digit of the multiplication.

Option (a) is correct.

94. It can be seen that for 40 and 80 the number of factors are 8 and 10, respectively. Thus, Option (c) satisfies the condition.

95. In order to solve this question, you need to realise that remainders of 1, 3, 4 and 5 in the case of 3, 5, 6 and 7, respectively, means remainders of –2 in each case. In order to find the number which leaves remainder –2 when divided by these numbers, you need to first find the LCM of 3, 5, 6 and 7 and subtract 2 from them. Since the LCM is 210, the first such number which satisfies this condition is 208. However, the question has asked us to find the largest such number below 4000. So you need to look at multiples of the LCM and subtract 2. The required number is 3990 – 2 = 3988

96. The number would be given by the (LCM of 2, 3 and 4) +1 → which is 12 + 1 = 13. Option (b) is correct.

97. The number would be given by the 2 × (LCM of 2, 3 and 4) +1 → which is 24 + 1 = 25. Option (a) is correct.

98. In order to solve this, you need to find the last 2 digit number in the series got by the logic:

(LCM of 2, 3, 4) + 1; 2 × (LCM of 2, 3, 4) + 1; 3 × (LCM of 2, 3, 4) + 1 ...

i.e., you need to find the last 2 digit number in the series:

13, 25, 37, 49....

In order to do so, you can do one of the following:

(a) Complete the series by writing the next numbers as:

61, 73, 85, 97 to see that 97 is the required answer.

(b) Complete the series by adding a larger multiple of 12 so that you reach closer to 100 faster.

Thus, if you have seen 13, 25, 37..., you can straightaway add any multiple of 12 to get a number close to 100 in the series in one jump.

Thus, if you were to add 12 × 4 = 48 to 37, you would reach a value of 85 (and because you have added a multiple of 12 to 37, you can be sure that 85 would also be on the same series.)

Thus, the thinking in this case would go as follows:

13, 25, 37, ..., 85, 97. Hence, the number is 97.

If you look at the two processes above– it would seem that there is not much difference between the two, but the real difference would be seen and felt if you would try to solve a question which might have asked you to find the last 3 digit number in the series. (as you would see in the next question). In such a case, getting to the number would be much faster if you use a multiple of 12 to jump ahead on the series rather than writing each number one by one.

(c) For the third way of solving this, you can see that all the numbers in the series:

13, 25, 37… are of the form $12n + 1$. Thus, you are required to find a number which is of the form $12n + 1$ and is just below 100.

For this purpose, you can try to first see what is the remainder when 100 is divided by 12.

Since the remainder is 4, you can realise that the number 100 is a number of the form $12n + 4$.

Obviously then, if 100 is of the form $12n + 4$, the largest $12n + 1$ number just below 100 would occur at a value which would be 3 less than 100. (This occurs because the distance between $12n + 4$ and $12n + 1$ on the number line is 3.)

Thus, the answer is $100 - 3 = 97$.

Hence, Option (c) is correct.

99. In order to solve this, you need to find the last 3 digit number in the series obtained by the logic:

(LCM of 2, 3, 4) +1; 2 × (LCM of 2, 3, 4) +1; 3 × (LCM of 2, 3, 4) + 1 …

i.e., you need to find the last 3 digit number in the series:

13, 25, 37, 49….

In order to do so, you can do one of the following:

(a) Try to complete the series by writing the next numbers as:

61, 73, 85, 97, 109… However, you can easily see that this process would be unnecessarily too long and hence infeasible to solve this question.

(b) Complete the series by adding a larger multiple of 12 so that you reach closer to 1000 faster.

This is what we were hinting at in the previous question. If we use a multiple of 12 to write a number which will come later in the series, then we can reach close to 1000 in a few steps. Some of the ways of doing this are shown below:

(i) 13, 25, 37, …….. 997 (we add $12 \times 80 = 960$ to 37 to get to $37 + 960 = 997$ which can be seen as the last 3 digit number as the next number would cross 1000).

(ii) 13, 25, 37, …… (add 600)….637, …(add 120)… 757,,,,,, (add 120),,,,, 877, ….(add 120)….997. This is the required answer.

(iii) 13, 25, 37, ……(add 120)….157, …(add 120)…277……(add 120) …..397….(add 120)……517…. (add 120)…637……(add 120)…757,,,,,, (add 120),,,,, 877, ….(add 120)….997. This is the required answer.

What you need to notice is that all the processes shown above are correct. So while one of them might be more efficient than the other, as far as you ensure that you add a number which is a multiple of 12 (the common difference), you would always be correct.

(c) Of course, you can also do this by using remainders. For this, you can see that all the numbers in the series:

13, 25, 37….are of the form $12n + 1$. Thus, you are required to find a number which is of the form $12n + 1$ and is just below 1000.

For this purpose, you can try to first see what is the remainder when 1000 is divided by 12.

Since the remainder is 4, you can realise that the number 1000 is a number of the form $12n + 4$.

Obviously then, if 1000 is of the form $12n + 4$, the largest $12n + 1$ number just below 1000 would occur at a value which would be 3 less than 1000. (This occurs because the distance between $12n + 4$ and $12n + 1$ on the number line is 3.)

Thus, the answer is $1000 - 3 = 997$.

Hence, Option (c) is correct.

100. The logic of this question is that the frog can never reach point *C* if it makes an odd number of jumps. Since, the question has asked us to find out in how many ways can the frog reach point *C* in exactly 7 jumps, the answer would naturally be 0. Option (d) is correct.

101. They would ring together again after a time interval which would be the LCM of 5, 6 and 7. Since the LCM is 210, Option (d) is the correct answer.

102. Since they would ring together every 210 seconds, their ringing together would happen at time intervals denoted by the following series- 210, 420, 630, 840, 1050, 1260, 1470, 1680, 1890, 2100, 2310, 2520, 2730, 2940, 3150, 3360, 3570 – a total of 17 times. This answer can also be calculated by taking the quotient of $3600/210 = 17$. Option (a) is correct.

103. The maximum number of soldiers would be given by the HCF of 66, 110 and 242. The HCF of these numbers can be found to be 22 and hence, Option (c) is correct.

104. The minimum number of rows would happen when the number of soldiers in each row is the maximum. Since, the HCF is 22 the number of soldiers in each row is 22. Then the total number of rows would be given by:

66/22 + 110/22 + 242/22 = 3 + 5 + 11 = 19 rows Option (b) is correct.

105. The number of bottle sizes possible would be given by the number of factors of the HCF of 170, 102 and 374. Since, the HCF of these numbers is 34, the bottle sizes that are possible would be the divisors of 34 which are 1 litre, 2 litres, 17 litres and 34 litres, respectively. Thus, a total of 4 bottle sizes are possible. Option (c) is correct.

106. The size of the largest bottle that can be used is obviously 34 litres (HCF of 170, 102 and 374). Option (d) is correct.

107. The minimum number of bottles required would be: 170/34 + 102/34 + 374/34 = 5 + 3 + 11 = 19. Option (b) is correct.

(**Note:** The logic of how to think about Questions 108 to 118 has been given in the theory in the chapter. Please have a relook at that in case you have doubts about any of the solutions till Question 118.)

108. The answer would be given by the quotients of 100/5 + 100/25 = 20 + 4 = 24. Option (c) is correct.

109. 24 + 4 = 28. Option (d) is correct.

110. 280 + 56 + 11 + 2 = 349. Option (c) is correct.

111. 76 + 15 + 3 = 94. Option (c) is correct.

112. 14 + 2 = 16. Option (c) is correct.

113. 13 + 4 + 1 = 18. Option (a) is correct.

114. 17 + 5 + 1 = 23. Option (c) is correct.

115. 11 + 1 = 12. Option (a) is correct.

116. 16 + 2 = 18. Option (d) is correct.

117. The number of 3's in 122! = 40 +13 + 4 + 1 = 58. The number of 2's in 122! = 61 + 30 + 15 + 7 + 3 + 1 = 117. The number of 2^2s is hence equal to the quotient of 117/2 = 58. We have to choose the lower one between 58 and 58. Since both are equal, 58 would be the correct answer. Hence, Option (d) is correct.

118. The power of 20 which would divide 155! would be given by the power of 5's which would divide 155! since 20 = $2^2 \times 5$ and the number of 2^2s in any factorial would always be greater than the number of 5s in the factorial. 31 + 6 + 1 = 38. Option (b) is correct.

119. 1024 = 2^{10}. Hence, x has to be a number with power of 2 greater than or equal to 5. Since, we are asked for the minimum value, it must be 5. Thus, Option (b) is correct.

120. The two digit numbers that would leave a remainder of 3 when divided by 7 would be the numbers 10, 17, 24, 31, 38, 45, …94. The sum of these numbers would be given by the formula

(number of numbers × average of the numbers) = ? There are 13 numbers in the series and their average is 52. Thus, the required answer is 13 × 52 = 676. Option (c) is correct.

(**Note:** The logic used here is that of sum of an Arithmetic Progression and is explained in details in the next chapter).

121. All numbers divisible by 27 would also be divisible by 3 and 9. Numbers divisible by 9 but not by 27 would be divisible by 3 and 9 only and need to be counted to give us our answer.

The numbers which satisfy the given condition are: 9, 18, 36, 45, 63, 72, 90, 99, 117, 126, 144, 153, 171, 180 and 198. There are 15 such numbers.

Alternately, you could also think of this as:

Between 1 to 200, there are 22 multiples of 9. But not all these 22 have to be counted as multiples of 27 need to be excluded from the count. There are 7 multiples of 27 between 1 and 200. Thus, the answer would be given by 22 – 7 =15. Option (b) is correct.

122. The required minimum happens when we use (–0.5) as the value of N. $(-0.5)^2 + (-0.5) = 0.25 - 0.5 = -0.25$ is the least possible value for the sum of any number and its square. Option (b) is correct.

123. Each of the statements are false as we can have the sum of 2 prime numbers ending in 5, 0 and the sum can also be odd. Option (d) is correct.

124. This occurs for values such as: 213 – 123; 324 – 234, etc., where it can be seen that the value of X is 1 more than Y. The possible pairs of X and Y are: 2,1; 3,2… 9,8 – a total of eight pairs of values. Option (b) is correct.

125. The required sum would be given by the formula $n(n + 1)$ for the first n even numbers. In this case, it would be $50 \times 51 = 2550$. Option (d) is correct.

126. 763/57 leaves a remainder of 22 when it is divided by 57. Thus, if we were to add 35 to this number, the number we obtain would be completely divisible by 57. Option (a) is correct.

127. Since, 763/57 leaves a remainder of 22, we would need to subtract 22 from 763 in order to get a number divisible by 57. Option (b) is correct.

128. 8441/57 leaves a remainder of 5. Thus, if we were to add 52 to this number, the number we obtain would be completely divisible by 57. Option (d) is correct.

129. Since, 8441/57 leaves a remainder of 5. We would need to subtract 5 from 8441 in order to get a number divisible by 57. Option (c) is correct.

130. 10000 divided by 79 leaves a remainder of 46. Hence, if we were to add 33 to 10000 we would get a number divisible by 79. The correct answer is 10033. Option (b) is correct.

131. 100000 divided by 79 leaves a remainder of 65. Hence, if we were to subtract 65 from 100000, we would get a number divisible by 79. The correct answer is 99935. Option (b) is correct.

132. It can be seen that in the multiples of 12, the number closest to 773 is 768. Option (a) is correct.

133. Since 12 is a divisor of 84, the required remainder would be obtained by dividing 57 by 12. The required answer is 9. Option (c) is correct.

134. Since 11 does not divide 84, there are many possible answers for this question and hence we cannot determine one unique value for the answer. Option (d) is, thus, correct.

135. The numbers that can do so are going to be factors of the difference between 511 and 667, i.e. 156. The factors of 156 are 1, 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156. There are 12 such numbers. Option (b) is correct.

136. The multiples of 13 between 200 and 400 would be represented by the series:

208, 221, 234, 247, 260, 273, 286, 299, 312, 325, 338, 351, 364, 377 and 390

There are a total of 15 numbers in the above series. Option (b) is correct.

(**Note:** The above series is an Arithmetic Progression. The process of finding the number of terms in an Arithmetic Progression are defined in the chapter on Progressions.)

137. $8n/5 - 5n/8 = 39n/40 = 39$. Solve for n to get the value of $n = 40$. Option (c) is correct.

138. $x + y = 3(x - y) \rightarrow 2x = 4y$. If we take y as 10, we would get the value of x as 20. Option (b) is correct.

139. $4^{11} + 4^{12} + 4^{13} + 4^{14} + 4^{15} = 4^{11} (1 + 4^{1} + 4^{2} + 4^{3} + 4^{4}) = 4^{11} \times 341$. The factors of 341 are:

1, 11, 31 and 341. Thus, we can see that the values in each of the three options would divide the expression. $4^{11} + 4^{12} + 4^{13} + 4^{14} + 4^{15}$. Thus, Option (d) is correct.

140. Since the numbers have their HCF as 16, both the numbers have to be multiples of 16 (i.e. 24).

$$7168 = 2^{10} \times 7^{1}$$

In order to visualise the required possible pairs of numbers, we need to look at the prime factors of 7168 in the following fashion:

$7168 = 2^{10} \times 7^{1} = (2^{4} \times 2^{4}) \times 2^{2} \times 7^{1} = (16 \times 16) \times 2 \times 2 \times 7$. It is then a matter of distributing the 2 extra twos and the 1 extra seven in $2^{2} \times 7^{1}$ between the two numbers given by 16 and 16 inside the bracket. The possible pairs are:

32×224; 64×112; 16×448. Thus, there are 3 distinct pairs of numbers which are multiples of 16 and whose product is 7168. However, out of these, the pair 32×224 has its HCF as 32 and hence does not satisfy the given conditions. Thus, there are two pairs of numbers that would satisfy the condition that their HCF is 16 and their product is 7168. Option (a) is correct.

141. $876 + 2P3 = 10Q9$,

It is clear that there is no carryover obtained in the addition of the unit's digit, while the sum of hundredth digit of the numbers is same as the hundredths digit of the sum. It means $7 + P = Q$.

10Q9 is divisible by 11, so $(9 + 0) - (1 + Q) = 11n$, where $n = 0, 1, 2, 3, 4,$ .....

$$Q = 8 \;\&\; P = Q - 7 = 1.$$

Therefore, $P - Q = 7$

142. Since a number is divisible by 11 if the difference of the sum of digits in the odd places and sum of digits in the even places is 0 or divisible by 11. In the 22 digit number as described above, it is evident that there are 11 digits in the even place and 11 digits in the odd place. Since all the digits

are equal, it means that the difference between these digits would be 0. Hence, the given numbers would be always divisible by 11.

143. $72 = 2^3 \times 3^2$

Total number of factors = (3 + 1)(2 + 1) = 12. This would mean that there are 6 pairs of factors, each of which pair would have a product of 72.

So the product of all the factors = $(72)^{\frac{12}{2}} = (72)^6$

144. $72 = 2^3 \times 3^2$

Total number of factors of 72 = (3 + 1)(2 + 1) = 12

Total number of ways of expressing 72 as a product of two factors = $\frac{12}{2}$ = 6

145. $144 = 2^4 \times 3^2$

Total number of factors = (4 + 1)(2 + 1)=15

Number of ways of expressing 144 as a product of two factors = $\frac{15+1}{2}$ = 8

But the factors must be distinct, so we exclude the 12×12 case.

So the correct answer = 8 – 1 = 7

146. Let the number be N. According to the question:

$N = 7k + 3 = 11n + 4$ (where k, l = 0, 1, 2, 3, 4, 5,.....)

$$k = \frac{11n+1}{7}$$

Now put the minimum possible value of n for which k is an integer.

For $n = 5$, $k = 8$. So minimum possible value of $N = 7 \times 8 + 3 = 59$

Adding the LCM of 11 and 7 to 59, would get the next number. Thus, the next number would be 59 + 77= 136. Further numbers would be numbers belonging to the Arithmetic Progression, 136, 213, 290, ... 829, 906, 983. There are a total of 12 numbers which fulfill the given condition.

147. The HCF of $a^m - 1, a^n - 1 = a^{\text{HCF of } m,n} - 1$

The required HCF = $3^{\text{HCF of } 12,25} - 1 = 3^5 - 1 = 242$

148. The LCM must always be a multiple of the HCF. But here the LCM is not a multiple of the HCF, which is not possible. Hence, the correct answer would be that such a situation is not possible.

149. 10! is contained in 15!, so the LCM of 10! and 15! is 15!. By the same logic, their HCF is 10!.

150. $\frac{a^n}{a+1}$ leaves a remainder of 1, when n is even. Since 116! is an even number, so $\frac{18^{116!}}{19}$ leaves a remainder 1.

151. For $x = 1$, $y = 4$, $7x + y = 11$, which is a prime number. $x + y = 1 + 4 = 5$.

152. 1414 has 2 as a factor and 2015 has 5 as a factor so the unit's digit of the product must 0.

153. The unit's digit of the product $1413 \times 729 \times 2015$ is 5. So if the product has 0 as its unit's digit then n must be either 0 or even. So 'n' may take maximum 5 values.

151. For $x = 1, y = 4, 7x + y = 11$, which is a prime number. $x + y = 1 + 4 = 5$.

152. 1414 has 2 as a factor and 2015 has 5 as a factor so the unit's digit of the product must 0.

153. The unit's digit of the product 1413 × 729 × 2015 is 5. So if the product has 0 as its unit's digit then n must be either 0 or even. So 'n' may take maximum 5 values.

154. The given number is divisible by 10, 11.

Remainder when we divide the number by 10 = $1^{12346} - 1$ = 0

Remainder when we divide the number by 11 = $(-1)^{12346} - 1$ = 1 – 1 = 0

155. In the given expression, after 5! all the values would have a units' digit of 0. Thus, the units digit of the given expression just depends on the units' digit of 1! + 2! + 3! + 4!

1! + 2! + 3! + 4! = 1+ 2+ 6+ 24 = 33

So unit's digit of 1! + 2! + 3! + 4! + 5!+ …..+ 1000! = 3

156. 35! is perfectly divisible by 10. Then $(35!)^{35!}$ should also be divisible by 10. So unit's digit must be 0.

157. Number of zeroes at the end of 34! = [34/5] + [34/25] = 6 + 1 = 7

So number of zeroes at the end of $(34!)^{6!}$ = 7 × 6! = 5040.

158. $51!^{31!}$ is divisible by 4 & unit's digit of 7^{4k} is 1. So unit's digit of $7^{51!^{31!}}$ is 1.

159. $11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 = 123454321$

So $1234567654321 = (1111111)^2$

160. 421 is a prime number so 421 = 1× 421. So HCF of the numbers is 1.

161. $3^{50} = (243)^{10}$, $4^{40} = (256)^{10}$, $5^{30} = (125)^{10}$, $6^{20} = (36)^{10}$,
So 4^{40} is greatest of all.

162. A number is divisible by 8, if the number formed by the last three digits is divisible by 8, i.e. $58N$ is divisible by 8 $\Rightarrow N = 4$.

Again a number is divisible by 11, if the difference between the sum of digits at even places and sum of digits at the odd places is either 0 or divisible by 11, i.e. $(M + 9 + 4 + 4 + 8) - (3 + 0 + 8 + 5 + N) = M - N + 9 = M + 5$ (since $N = 4$) It cannot be zero hence, $M + 5 = 11 \Rightarrow M = 6$.

Hence, $M*N = 24$.

163. We can check for the appearance of 4 in the units place, the tens place and the hundreds place separately. So, 4 in the units place would occur once in each number of the arithmetic series: 24,34,44,54,…494 → A total of 48 times. Further, 4 would appear once in each number of the series': 40,41,42,…49 (Thus, 10 times); Similarly, it would appear 10 times in the tens place in the 140s, the 240s, the 340s and the 440s. Thus, a total of 50 appearances of 4, in the tens place. Also, 4 would appear in the hundreds' place exactly 100 times from 400 to 499. Thus, the correct answer would be: 48 + 50 + 100 = 198.

164. Here, we want the remainder of $\frac{3671+3672+3673+...+3685}{3670}$

$\Rightarrow 1 + 2 + 3 + \ldots + 15 = 120$

165. 999999 is divisible by 7. It means 999999......99 (6 × 16) times is divisible by 7.

So the required remainder = Remainder of $\frac{999}{7} \Rightarrow 5$

166. We know that $A^{p-1} \div p$, leaves a remainder of 1, when p is a prime number.

Here 41 is a prime number. Hence, $\frac{2^{40}}{41}$ leaves a remainder 1. Thus, the remainder of $2^{41} \div 41$ would be equal to the remainder of $2^{1} \div 41 \rightarrow 2$ (required remainder).

167. According to the Wilson theorem, if p is a prime number then $(p - 1)! + 1$ is a multiple of p.

Here 41 is a prime number so 40! + 1 is completely divisible by 41. This means that 40! leaves a remainder −1 when we divide it by 41 or it leaves a remainder 41 − 1 = 40.

168. $x + 3 = 0$ for $x = -3$

So the required remainder = $(-3)^4 + 3(-3)^3 + 4 = 4$

169. For any odd X, $X_2 + 7$ will be an even number. So for any odd X, $X_2 + 7$ cannot be a prime number. Only even prime number is 2 for which $2^2 + 7 = 11$ is also a prime number.

So required number of values = 1.

170. $X = 99_3 - 63_3 - 36_3$ or $99_3 + (-63)_3 + (-36)_3$

As we know that $a_3 + b_3 + c_3 = 3abc$ (if $a + b + c = 0$)

So $X = 3(99)(-63)(-36) = 2_2 \times 3_7 \times 7_1 \times 11_1$

Number of factors of $X = (2 + 1)(7 + 1)(1 + 1)(1 + 1) = 3 \times 8 \times 2 \times 2 = 96$

171. The given expression can be written as

$x(x^2 - 1)(x^2 - 4)(x^2 - 9)(x + 4) = (x - 3)(x - 2)(x - 1)x(x + 1)(x + 2)(x + 3)(x + 4)$

It is the product of eight consecutive natural numbers, so this product should be divisible by 8! Hence, the largest n, would be $n = 8$.

172. Number of zeroes at the end of

$$X! = \left[\frac{X}{5}\right] + \left[\frac{X}{25}\right] + \left[\frac{X}{125}\right] + \ldots = Y$$

Number of zeroes at the end of $5X! = \left[\frac{5X}{5}\right] + \left[\frac{5X}{25}\right] + \left[\frac{5X}{125}\right] + \ldots$ or

$$[X] + \left[\frac{X}{5}\right] + \left[\frac{X}{25}\right] + \ldots$$

So the number of zeros at the end of $5X!$ is $X + Y$.

Alternately, you could also solve this question through trial and error. Suppose, you take X as 10 and $5X$ as 50: You can see that in this case $Y = 2$ and the number of zeroes in $5X!$ is 12 which is also equal to $X + Y$. The relationship is maintained if you were to take $X = 20$ and $5X = 100$. Thus, 20! has 4 zeroes and 100! has 24 zeroes (again equal to $X + Y$).

173. He is losing a total of 67 marks in the test (from the all correct situation). Further, we know that if he answers a question wrongly, his score would drop by 1.25 marks from the maximum possible. The only other way for him to lose marks is if he leaves a question unanswered. In such a case, he is losing 1.125 marks (when he leaves a question unanswered, he gets a negative score of −0.125 marks instead of getting +1). Once we realise

this, we need to check whether 0 wrong answers are possible. In such a case, we can think of the following table to find the correct answer to the question.

Marks lost by wrong questions	Marks required to be lost by unanswered questions	Is that possible?
0 (0 wrong answers)	67	67/1.125 is not an integer. Hence, No.
1.25 (1 wrong answers)	65.75	65.75/1.125 is not an integer. Hence, No.
2.5 (2 wrong answers)	64.5	64.5/1.125 is not an integer. Hence, No.
3.75 (3 wrong answers)	63.25	63.25/1.125 is not an integer. Hence, No.
5 (4 wrong answers)	62	62/1.125 is not an integer. Hence, No.
6.25 (5 wrong answers)	60.75	60.75/1.125 is an integer. Hence, Yes.

Thus, the minimum number of wrong answers is 5.

174. For $n = 2$, $A = 2^{4^2} = 2^{16}$, $B = 2^{2^4} = 2^{16}$ So $A = B$ in this case. Also, for $n = 2$, $C = 28$ & D is 28.

However, if we go for values of n greater than 2, (3 for instance), we see that B = 3729, A = 3216, D = 327 and C = 318. We notice that this relationship continues for $n = 4$ too.

We can, thus, conclude that $B \geq A > D \geq C$

175. The numbers we need to check for are: 1,3,7,15,31,63,127,255, 511, 1023, 2047 and 4095. Out of these, the prime numbers are: 3,7,31 and 127. Hence, 4 numbers.

Level of Difficulty (II)

1. If a and b are two numbers, then their arithmetic mean is given by $(a + b)/2$ while their geometric mean is given by $(ab)0.5$. Using the options to meet the conditions we can see that for the numbers in the first option (6 and 54) the AM being 30, is 24 less than the larger number while the GM being 18, is 12 more than the smaller number. Option (a) is correct.

2. Use the principle of counting given in the theory of the chapter. Start with 101 numbers (i.e. all numbers between 200 and 300 both included) and subtract the number of numbers which are divisible by 2 (viz. [(300 – 200)/2] + 1 = 51 numbers), the number of numbers which are divisible by 3 but not by 2 (**Note:** This would be given by the number of terms in the series 201, 207, … 297. This series has 17 terms) and the number of numbers which are divisible by 5 but not by 2 and 3. (The numbers are 205, 215, 235, 245, 265, 275, 295. A total of 7 numbers).

   Thus, the required answer is given by 101 – 51 – 17 – 7 = 26. Option (b) is correct.

3. Since $15n3$, $6n2$ and $5n$ would all be divisible by n, the condition for the expression to not be divisible by n would be if x is not divisible by n. Option (c) is correct.

4. It can be seen that the first expression is larger than the second one. Hence, the required answer would be given by the (units digit of the first expression – units digit of the second expression) = 6 – 0 = 6. Option (c) is correct.

5. Suppose you were to solve the same question for $10^3 - 7$ and $10^2 + x$.

$10^3 - 7 = 993$ and $10^2 + x = 100 + x$

Difference = $893 - x$

For $10^4 - 7$ and $10^3 + x$

The difference would be $9993 - (1000 + x)$

$= (8993 - x)$

For $10^5 - 7$ and $10^4 + x$

Difference: $99993 - (10000 + x) = 89993 - x$

You should realise that the difference for the given question would be 8999 . . . . . 9 3 – x. For this difference to be divisible by 3, x must be 2 (since that is the only option which will give you a sum of digits divisible by 3).

6. The value of x should be such that the left hand side after completely removing the square root signs should be an integer. For this to happen, first of all the square root of 3x should be an integer. Only 3 and 12 from the options satisfy this requirement. If we try to put x as 12, we get the square root of 3x as 6. Then the next point at which we need to remove the square root sign would be 12+2(6) = 24 whose square root would be an irrational number. This leaves us with only 1 possible value ($x = 3$). Checking for this value of x we can see that the expression is satisfied as LHS = RHS.

7. If the number is n, we will get that $22n + n = 23n$ is half the square of the number n. Thus, we have

$$n^2 = 46n \rightarrow n = 46$$

8. $12^{55}/3^{11} = 3^{44}.4^{55} \rightarrow 4$ as units place.

Similarly, $8^{48}/16^{18} = 2^{72} \rightarrow 6$ as the units place.

Hence, 0 is the answer.

9. $1 + 2 + 2^2 + \ldots + 2^{31} = 2^{32} - 1$

Hence, the average will be: $\frac{2^{32}-1}{32} = 2^{27} - 1/2^5$ which lies between 2^{26} and 2^{27}.

Hence, the answer will be (c).

10. The denominator 99 has the property that the decimals it gives rise to are of the form 0.*xyxyxy*. This question is based on this property of 99. Option (c) is correct.

11. The value of b has to be 2 since, $r = 2y$. Hence, Option (d) is the only choice.

12. For $[x]^3 + \{x\}^2$ to give – 7.91,

$[x]^3$ should give – 8 (hence, $[x]$ should be –2]

Further, $\{x\}^2$ should be + 0.09.

Both these conditions are satisfied by –1.7.

Hence option (d) is correct.

13. $16^5 + 2^{15} = 2^{20} + 2^{15} = 2^{15}(2^5 + 1) \rightarrow$ Hence, is divisible by 33.

14. The interpretation of the situation $AB + XY = 1XP$ is that the tens digit in XY is repeated in the value of the solution (i.e. $1XP$). Thus, for instance if X was 2, it would mean we are adding a 2 digit number AB to a number in

the 20's to get a number in the 120's. This can only happen if AB is in the 90's which means that A is 9.

15. $|x - 3| + 2|x + 1| = 4$ can happen under three broad conditions.

(a) When $2|x + 1| = 0$, then $|x - 3|$ should be equal to 4.

Putting $x = -1$, both these conditions are satisfied.

(b) When $2|x + 1| = 2$, x should be 0, then $|x - 3|$ should also be 2. This does not happen.

(c) When $2|x + 1| = 4$, x should be + 1 or – 3, in either case $|x - 3|$ which should be zero does not give the desired value.

16. At a value of $x = 0$, we can see that the expression $x^2 + |x - 1| = 1 \rightarrow 0 + 1 = 1$. Hence, $x = 0$ satisfies the given expression. Also at $x = 1$, we get $1 + 0 = 1$. Option (d) is correct.

17. 4^{2n+1} represents an odd power of 4 (and hence would end in 4). Similarly, 4^{2n} represents an even power of 4 (and hence would end in 6). Thus, the least number 'x' that would make both $4^{2n+1} + x$ and $4^{2n} - x$ divisible by 5 would be for $x = 1$.

18. Check for each value of the options to see that the expression does not become divisible by 9 for any of the initial options. Thus, there is no value that satisfies the divisibility by 9 case.

19. The expression would have solutions based on a structure of:

4 + 0; 3 + 1; 2 + 2; 1 + 3 or 0 + 4.

There will be 2*1 = 2 solutions for 4 + 0 as in this case x can take the values of 8 and 0, while y can take a value of 4;

20. The numerator of $3^{32}/50$ would be a number that would end in 1. Consequently, the decimal of the form .bx would always give us a value of *x* as 2.

21. If we assume the numbers as 16 and 4 based on 4:1 (in option a), the AM would be 10 and the *GM* = 8 a difference of 20% as stipulated in the question. Option (a) is correct.

22. $990 = 11 \times 3^2 \times 2 \times 5$. The highest power of 990 which would divide 1090! would be the power of 11 available in 1090. This is given by [1090/11] + [1090/121] = 99 + 9 = 108.

23. For finding the highest power of 6 that divides 146!, we need to get the number of 3's that would divide 146!. The same can be obtained by: [146/3] + [48/3] + [16/3] + [5/3] = 70.

24. There would be two fives and more than 2 two's in the prime factors of the numbers in the multiplication. Thus, we would get a total of 2 zeroes.

25. Both 333^{555} and 555^{333} are divisible by 3,37 and 111. Further, the sum of the two would be an even number and hence divisible by 2. Thus, all the three options divide the given number.

26. Both the values of options (a) and (b) satisfy the given expression. As for 5.16, the value of $[x]^2 = 25$ and the value of $\{x\} = 0.16$. Thus, $[x]^2 + \{x\}^1 = 25.16$.

Similarly for a value of $x = -4.84$, the value of $[x] = -5$ and hence $[x]^2 = 25$ and the value of $\{x\} = 0.16$. Thus, $[x]^2 + \{x\}^1 = 25.16$

28. Solve this question through options. Also realise that $a \times b = a + b$ only occurs for the situation $2 \times 2 = 2 + 2$. Hence, clearly the answer has to be none of these.

29. 863 satisfies each of the conditions and can be spotted through checking of the options.

30. The number of zeroes would be given by counting the number of 5's. The relevant numbers for counting the number of 5's in the product would be given by:

5_{5}; 10_{10}, 15_{15}, 20_{20}, 25_{25}…and so on till 100_{100}.

The number of 5's in these values would be given by:

(5 + 10 + 15 + 20 + 50 + 30 + 35 + 40 + 45 + 100 + 55 + 60 + 65 + 70 + 150 + 80 + 85 + 90 + 95 + 200)

This can also be written as:

(5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + 55 + 60 + 65 + 70 + 75 + 80 + 85 + 90 + 95 + 100) + (25 + 50 + 75 + 100)

= 1050 + 250 = 1300

31. Option (a) is correct as the LCM of 5 and 105 is 105 and their HCF is 5. Also for the pair of values, 15 and 35 the HCF is 5 and the LCM is 105.

32. Solve using options. Using option $b = 3/5$ and performing the given operation we get:

2/3 – 3/5 = (10 – 9)/15 = 1/15. Option (b) is hence correct.

33. Both the conditions are satisfied for option (a) = 72 as the number 72 exceeds the sum of squares of the digits by 19 and also 72 exceeds the doubled product of its digits by 44.

34. Solve by checking the given options. 31 and 13 are possible values of the number as defined by the problem.

35. The given conditions are satisfied for the number 24.

36. The number of 2's in the given expression is lower than the number of 5's. The number of 2's in the product is 9 and hence that is the number of zeroes.

37. $45 = 3^2 \times 5$. Hence, we need to count the number of 3^2's and 5's that can be made out of 123!.

Number of 3's = 41 + 13 + 4 + 1 = 59 → Number of 3^2's = 29

Number of 5's = 24 + 4 = 28.

The required answer is the lower of the two (viz. 28 and 29). Hence, option (a) 28 is correct.

38. The first sentence means that the numbers are in an arithmetic progression. From the statements and a little bit of visualisation, you can see that 8.5, 10 and 11.5 can be the three values we are looking for – and hence the middle value is 10.

39. $990 = 11 \times 3^2 \times 5 \times 2$. For $n!$ to be divisible by 990, the value of n! should have an 11 in it. Since, 11 itself is a prime number, hence the value of n should be at least 11.

40. For the expression to hold true, x and y should both be positive.

41. Since, we are not given options here we should go ahead by looking within the factors of 144 (especially the two digit ones).

The relevant factors are 72,48, 36, 24, 18 and 12. Thinking this way creates an option for you where there is none available and from this list of numbers, you can easily identify 24 as the required answer.

Solutions for Questions 42 to 46:

Write simple equations for each of the questions and solve.

47. Since the sum of squares of the digits of the two digit number is 10, the only possibility of the numbers are 31 and 13.

48. If the number is 'ab', we have the following equations:

$(10a + b) = 4\,(a + b) + 3 \rightarrow 6a - 3b = 3$

$(10a + b) = 3\,(a \times b) + 5$

Obviously, we would need to solve these two equations in order to get the values of the digits (a) and (b) respectively. However, it might not be a very prudent decision to try to follow this process- as it might turn out to be too cumbersome.

A better approach to think here is:

From the first statement, we know that the number is of the form $4n + 3$. Thus, the number has to be a term in the series 11, 15, 19, 23, 27…

Also from the second statement, we know that the number must be a $3n + 5$ number.

Thus, the numbers could be 11, 14, 17, 20, 23….

Common terms of the above two series would be probable values of the number.

It can be seen that the common terms in the two series are: 11, 23, 35, 47, 59, 71, 83 and 95. One of these numbers has to be the number we are looking for.

If we try these values one by one, we can easily see that the value of the two digit number should be 23 since → 23/ (2 + 3) → Quotient as 4 and remainder = 3.

Similarly, if we look at the other condition given in the problem we would get the following-

23/6 → quotient as 3 and remainder = 5.

Thus, the value of the missing number would be 23.

49. We can see from the description that the number (say X) must be such that $X + 100$ and $X + 169$ both must be perfect squares. Thus, we are looking for two perfect squares which are 69 apart from each other. This would happen for 342 and 352 since their difference would be (35 – 34) (35 + 34) = 69.

50. Since their least common multiple is 102, we need to look for two factors of 102 such that they add up to 85. 51 and 34 can be easily spotted as the numbers.

51. If one number is x, the other should be $6x$ or $12x$ or $18x$ or $24x$ and so on. Also, their sum should be either 504 or 1008 or 1512. (**Note:** the next multiple of 504 = 2016 cannot be the sum of two three digit numbers.)

52. Obviously 46 and 64 are the possible numbers.

53. The key here is to look for numbers which are more than three times but less than four times the product of their digits. Also, the product of the digits should be greater than 9 so as to leave a remainder of 9 when the number is divided by the product of its digits.

In the 10s, 20s and 30s, the numbers 14,15,23 and 33 give us a quotient of 3, when divided by the product of their digits, but do not give us the required remainder. In the 40s, 43 is the only number which has a quotient of 3 when divided by 12 (product of its digits). But 43/12 does not give us a remainder of 9 as required.

In the 50s, the number 53 divided by 15 leaves a remainder of 8, while in the 60s, 63 divided by 18 gives us a remainder of 9 as required.

54. The first thing to use while solving this question is to look at the information that the sum of squares of the three digits is 109. A little bit of trial and error shows us that this can only occur if the digits are 8, 6 and 3. Using the other information, we get that the number must be 863 since, $863 - 495 = 368$.

55. It is obvious that the only condition where the cubes of 3 numbers add up to 243 is when we add the cubes of 3 and 6. Hence, the numbers possible are 36 and 63.

56. There would definitely be two numbers and in case we take the first number as $7n - 1$, there would be three numbers – (as can be seen when we take the first number as 27 and the other number is 43).

57. Between 111^4, $110 \times 109 \times 108 \times 107$, $109 \times 110 \times 112 \times 113$.

It can be easily seen that

$$111 \times 111 \times 111 \times 111 > 110 \times 109 \times 108 \times 107$$

also $109 \times 110 \times 112 \times 113 > 109 \times 110 \times 108 \times 107$

Further the product $111 \times 111 \times 111 \times 111 > 109 \times 110 \times 112 \times 113$ (since, the sum of the parts of the product are equal on the LHS and the RHS and the numbers on the LHS are closer to each other than the numbers on the RHS).

58. Both x and y should be highest for xy to be maximum. Similarly x should be minimum and y should be maximum for x/y to be minimum.

59. $200^{300} = (200^6)^{50}$

$300^{200} = (300^4)^{50}$
$400^{150} = (400^3)^{50}$

Hence 200^{300} is greater.

61. The sum of squares of the first n natural numbers is given by $n(n + 1)(2n + 1)/6$.

For this number to be divisible by 4, the product of $n(n + 1)(2n + 1)$ should be a multiple of 8. Out of n, $(n + 1)$ and $(2n + 1)$ only one of n or $(n + 1)$ can be even and $(2n + 1)$ would always be odd.

Thus, either n or $(n + 1)$ should be a multiple of 8.

This happens if we use n = 7, 8, 15, 16, 23, 24, 31, 32, 39, 40, 47, 48.
Hence, 12 such numbers.

62. In the 20s, the numbers are: 23 to 29

In the 30s, the numbers are: 32 to 39

Subsequently the numbers are 42 to 49, 52 to 59, 62 to 69, 72 to 79, 82 to 89 and 92 to 99.

A total of 63 numbers.

63. You need to solve this question using trial and error.

For 32, option (a):

$32 = 2^5$. Hence, 6 factors. On increasing by 50%, $48 = 2^4 \times 3^1$ has 10 factors. Thus, the number of factors is increasing when the number is increased by 50% which is not what the question is defining for the number. Hence, 32 is not the correct answer.

Checking for option (b) 84.

$84 = 2^2 \times 3^1 \times 7^1 \rightarrow (2 + 1)(1 + 1)(1 + 1) = 12$ factors

On increasing by 50% $\rightarrow 126 = 2^1 \times 3^2 \times 7^1 \rightarrow (1 + 1)(2 + 1)(1 + 1) = 12$ factors. (no change in number of factors).

Second Condition: When the value of the number is reduced by 75% → 84 would become 21 ($3^1 \times 7^1$) and the number of factors would be $2 \times 2 = 4$ – a reduction of 66.66% in the number of factors.

64. There will be 9 single digit numbers using 9 digits, 90 two digit numbers using 180 digits, 900 three digit numbers using 2700 digits. Thus, when the number 999 would be written, a total of 2889 digits would have been used up. Thus, we would need to look for the 25494th digit when we write all 4 digit numbers. Since, 25494/4 = 6373.5 we can conclude that the first 6373 four digit numbers would be used up for writing the first 25492 digits. The second digit of the 6374th four digit number would be

the required answer. Since, the 6374th four digit number is 7373, the required digit is 3.

65. In order to solve this question, think of the numbers grouped in groups of 9 as:

{1, 2, 3, 4, 5, 6, 7, 8, 9} {10, 11, 12.....18} and so on till {2989, 2990… 2997} – A total of 333 complete sets. From each set, we can take 4 numbers giving us a total of 333 × 4 = 1332 numbers.

Apart from this, we can also take exactly 1 multiple of 9 (any one) and also the last 3 numbers viz 2998, 2999 and 3000. Thus, there would be a total of 1332 + 4 = 1336 numbers.

66. It can be seen that for only 2 numbers (1 and 1/2), the consolidated number would be 1 + 1/2 + 1/2 = 2.

For 3 numbers (1, 1/2 , 1/3), the number would be 3. Thus, for the given series the consolidated number would be 1972.

67. The value of *K* would be 199 and hence, the required difference is 9 – 1 = 8.

68. 9 – 9 = 0 would be the difference between the units and the tens digits.

69. The highest ratio would be a ratio of 100 in the numbers, 100, 200, 300, 400, 500, 600, 700, 800 and 900. Thus a total of 9 numbers.

70. Basically every odd triangular number would have this property, that it is the difference of squares of two consecutive natural numbers. Thus, we need to find the number of triangular numbers that are odd.

3, 15, 21, 45, 55, 91, 105, 153, 171, 231, 253, 325, 351, 435, 465, 561, 595, 703, 741, 861, 903 – a total of 21 numbers.

71. The coefficients would be $^{44}C_0$, $^{44}C_1$, $^{44}C_2$ and so on till $^{44}C_{44}$. The sum of these coefficients would be 2^{44} (since the value of $^nC_0 + {}^nC_1 + {}^nC_2 + \ldots {}^nC_n = 2^n$).

72. The remainder of each power of 9 when divided by 6 would be 3. Thus, for (2n+1) powers of 9, there would be an odd number of 3's. Hence, the remainder would be 3.

73. The remainder when a number is divided by 16 is given by the remainder of the last 4 digits divided by 16 (because 10000 is a multiple of 16. This principle is very similar in logic to why we look at last 2 digits for divisibility by 4 and the last 3 digits for divisibility by 8). Thus, the required answer would be the remainder of 4950/16 which is 6.

74. $58! - 38! = 38!(58 \times 57 \times 56 \times 55 \times \ldots \times 39 - 1) \rightarrow 38!(3n - 1)$ since the expression inside the bracket would be a $3n - 1$ kind of number. Thus, the number of 3's would depend only on the number of 3's in 38! → 12 + 4 + 1 = 17.

75. The given expression can be seen as $(22334^{\text{ODD POWER}})/5$, since the sum of $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 66^2$ can be seen to be an odd number. The remainder would always be 4 in such a case.

76. $12^{33} \times 34^{23} \times 2^{70} = 2^{159} \times 3^{33} \times 17^{23}$. The number of factors would be $160 \times 34 \times 24 = 130560$. Thus, Option (d) is correct.

77. Option (a) is correct.

78. $1152 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^7 \times 3^2$. Essentially every number starting from 4!3 would be divisible perfectly by 1152 since each number after that would have at least 7 twos and 2 threes.

Thus, the required remainder is obtained by the first three terms:

(1 + 8 + 216)/1152 = 225/1152 gives us 225 as the required remainder.

79. We can take only perfect squares of prime numbers and the number 1. Thus, for instance we can take numbers like 1, 4, 9, 25, 49, 121, 169, 289, 361, 529, 841 and 961. A total of 12 such numbers can be taken.

80. (101 × 102 × 103 × 197 × 198 × 199)/100 → [1 × 2 × 3 × (–3) × (–2) × (–1)]/100 → –36 as remainder → remainder is 64.

81. [65 × 29 × 37 × 63 × 71 × 87]/100 → [–35 × 29 × 37 × –37 × –29 × –13]/100 → [35 × 29 × 37 × 37 × 29 × 13]/100 = [1015 × 1369 × 377]/100 → 15 × 69 × 77/100 → remainder as 95.

82. [65 × 29 × 37 × 63 × 71 × 87 × 85]/100 → [–35 × 29 × 37 × –37 × –29 × –13 × –15]/100 → [35 × 29 × 37 × 37 × 29 × 13 × –15]/100 = [1015 × 1369 × 377 × –15]/100 → [15 × 69 × 77 × – 15]/100 → remainder as 75.

83. [65 × 29 × 37 × 63 × 71 × 87 × 62]/100 → [–35 × 29 × 37 × –37 × –29 × –13 × 62]/100 → [35 × 29 × 37 × 37 × 29 × 13 × 62]/100 = [1015 × 1369 × 377 × 62]/100 → [15 × 69 × 77 × 62]/100 = [1035 × 4774]/100 → 35 × 74/100 → remainder as 90.

84. [75 × 35 × 47 × 63 × 71 × 87 × 82]/100 = [3 × 35 × 47 × 63 × 71 × 87 × 41]/2 → remainder = 1.

Hence, required remainder = 1 × 50 = 50.

85. For this question, we need to find the remainder of:

(201 × 202 × 203 × 204 × 246 × 247 × 248 × 249) × (201 × 202 × 203 × 204 × 246 × 247 × 248 × 249) divided by 100.

(201 × 202 × 203 × 204 × 246 × 247 × 248 × 249) × (201 × 202 × 203 × 204 × 246 × 247 × 248 × 249)/100 = (201 × 101 × 203 × 102 × 246 × 247 × 248 × 249) × (201 × 202 × 203 × 204 × 246 × 247 × 248 × 249)/25

→ (1 × 1 × 3 × 2 × –4 × –3 × –2 × –1) × (1 × 2 × 3 × 4 × –4 × –3 × –2 × –1)/25 = 144 × 576/25 → (19 × 1)/25 = remainder 19.

19 × 4 = 76 is the actual remainder (since we divided by 4 during the process of finding the remainder).

86. $7^4/2400$ gives us a remainder of 1. Thus, the remainder of $7^{99}/2400$ would depend on the remainder of $7^3/2400$ → remainder = 343.

87. The numerator can be written as $(1729)^{752}/1728$ → remainder as 1.

88. Bikas's movement in terms of the number of coins would be:
B → 3B (when Arun triples everyone's coins) → B.

Think of this as: When Bikas triples everyone's coins, and is left with 20 it means that the other 3 have 60 coins after their coins are tripled. This means that before the tripling by Bikas, the other three must have had 20 coins—meaning Bikas must have had 60 coins.

But 60 = 3B → B = 20.

89. For $83p796161q$ to be a multiple of 11 (here X is 11), we should have the difference between the sum of odd placed digits and even placed digits should be 0 or a multiple of 11.

$(8 + p + 9 + 1 + 1) - (3 + 7 + 6 + 6 + q) = (19 + p) - (22 + q)$. For this difference to be 0, p should be 3 more than q which cannot occur since $0 < p < q$.

The only way in which $(19 + p) - (22 + q)$ can be a multiple of 11 is if we target a value of –11 for the expression. One such possibility is if we take p as 1 and q as 9.

The number would be 8317961619. On successive division by $(p + q) = 10$ and 1 the sum of remainders would be 9.

90. $n(n + 1)/2$ should be a perfect square. The first value of n when this occurs would be for $n = 8$. Thus, on the 8 March, the required condition would come true.

91. We have to find the unit's digit of $2^{53} \rightarrow 2^{4n+1} \rightarrow 2$ as the units digit.

92. $[7!(14 + 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8)]/[7!(16 - 3)] = [(14 + 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8)]/[(13)] \rightarrow$ remainder 1.

Hence, the original remainder must be 7! (because for the sake of simplification of the numbers in the question, we have cut the 7! from the numerator and the denominator in the first step).

93. $x = 6$ and $y = -3$ is one pair of values where the given condition is met.

After that you should be able to spot that if you were to increase x by 7, y would decrease by 4. The number of such pairs would depend on how many terms are there in the series.

–498, –491, ... –1, 6, 13, 20, ... 489, 496. The series has $994/7 + 1 =$ 143 terms and hence there would be 143 pairs of values for (x, y) which would satisfy the equation.

94. All three conditions can be seen to be true.

95. Product of factorials < Sum of factorials would occur for any number that has either 0 or 1 in it.

The required numbers uptil and including 50 are: 10 to 19, 20, 21, 30, 31, 40, 41, 50. Besides for the number 22, the product of factorials of the digits would be equal to the sum of factorials of the digits. Thus, a total of 18 numbers.

96. The maximum marks he can score is: 100 (if he gets all correct).

The minimum marks he can score would be given by: $10 \times (-0.1) + 20 \times (-0.2) + 70 \times (-0.5) = -40$.

The difference between the two values would be $100 - (-40) = 140$ marks.

Solutions for Questions 97 to 99:

If a student solved 200 questions and got everything correct, he would score a total of 620 marks. By getting a LOD 1 question wrong, he would lose 4+2=6 marks, while by not solving an LOD 1 question, he would lose 4 marks.

Similarly for LOD 2 questions, loss of marks = 4.5 (for wrong answers) and loss of marks= 3 (for not solved).

Similarly for LOD 3 questions, loss of marks = 3 (for wrong answers) and loss of marks = 2 (for not solved)

Since, he has got 120 marks from 100 questions solved he has to lose 500 marks (from the maximum possible total of 620) by combining to lose marks through 100 questions not solved and some questions wrong.

97. It can be seen through a little bit of trial and error with the options, that if he got 44 questions of LOD 1 correct and 56 questions of LOD 3 wrong he would end up scoring $44 \times 4 - 56 \times 1 = 176 - 56 = 120$. In such a case, he would have got the maximum possible incorrects with the given score.

98. $32 \times 4 + 1 \times 3 - 1 \times 1 = 130$ (in this case, he has solved 32 corrects from LOD 1, 1 correct from LOD 2 and 1 incorrect from LOD 3). Thus, a total of 34 attempts.

99. In the above case, he gets 1 question incorrect. However, he can also get 130 marks by $30 \times 4 + 2 \times 3 + 2 \times 2$ where he gets 30 LOD 1 questions correct, and 2 questions correct each from LOD 2 and LOD 3).

The least number of incorrects would be 0.

100. The least number would be (LCM of 10, 9, 8, 7, 6 and 5 – 1) = 2519. The second least number = $2520 \times 2 - 1 = 5039$.

101. A quick scan of squares above the given numbers tells us that the required perfect squares are: 144, 256 and 400, which would be the squares of the numbers 12, 16 and 20 respectively. Hence, n would be $144 - 108 = 36$. The sum of digits of n is $3 + 6 = 9$.

102. The number $904 = 8 \times 113 = 2^3 \times 113$. This means that $904^{2008} = 2^{6024} \times 113^{2008}$. Also, $904^{2015} = 2^{6045} \times 113^{2015}$. Multiples of 904^{2008} would be numbers that would have $2^{6024 \text{ or more}}$ and $113^{2008 \text{ or more}}$. Also, factors of 904^{2015} would have $2^{6045 \text{ or less}}$ and $113^{2015 \text{ or less}}$. This means that the number of numbers that would be multiples of the first number and simultaneously be factors of the other number would be $22 \times 8 = 176$.

103. The expression $1^5 + 2^5 + 3^5 + \ldots\ldots + 96^5$ can be written as $(1 + 2 + \ldots + 96)(1^4 + \ldots\ldots)$. The first bracket in this expression would have a value of $96 \times 97 \div 2 = 48 \times 97$ (Using the formula for the sum of the first n natural numbers.). Since, 48×97 is a multiple of 194, the required remainder would be 0.

104. Since, the LHS of the above expression would always be an integer, the RHS too needs to be an integer. This condition is satisfied when *s* is a multiple of 42. Also, on checking for values of *s* as 42, 84, etc., we realise that for all values of s, the RHS gives a value that is equal to the LHS. So, in order to answer the question, we need to find out the number of multiples of 42 below 1000. We start from42 × 1 and end at 42 × 23. Hence, there are 23 such numbers.

Solutions for Questions 105 to 107:

The list of possible number of rabbits that were transferred for each weight category would be:

1 kg rabbits	2 kg rabbits	5 kg rabbits
1	16	23
4	12	24
7	8	25
10	4	26
13	0	27

Possible number of rabbits remaining in the zoo:

1 kg rabbits	2 kg rabbits	5 kg rabbits
1	28	31
4	24	32
7	20	33
10	16	34
13	12	35
16	8	36
19	4	37
22	0	38

The answers can be read off the tables:

105. The minimum number of 1 kg rabbits that were transferred was 1.

106. The maximum possible numbers of 5 kg rabbits that remain were 37. (**Note:** 38 is not possible, since the minimum number of 5 kg rabbits transferred out was 23 – and 23 + 38 would cross 60, which is not allowed in the question)

107. If 26, five kg rabbits were transferred, it means that there can be a maximum of 34, five kg rabbits that remain in the zoo. In such a case, we would get the highest numbers of 1 kg rabbits that remain in the zoo as 10. Hence, the correct answer for this question is '10'.

108. The best way to solve such questions is to do a selective trial and error. Scanning numbers, the first thing that becomes clear is that we cannot have both the digits of the number as even, in this context. This means that we need to scan only odd numbers. Scanning through the odd numbers, we find 36 and its reverse 63 giving us the required value as $36 + 63 + 3 \times 6 = 117$. Also, we can see that even $63 + 36 + 18$ gives us the same value. In both cases, the sum of the digits turns out to be 9.

109. The numbers that have an odd number of factors are the perfect squares. Thus, $M = 36 \times 49 \times 64 \times 81 \times 100 \times 121 \times 144 \times 169 \times 196 \times 225 = 2^{16} \times 3^{8} \times 5^{4} \times 7^{4} \times 11^{2} \times 13^{2}$. 12 is a number consisted only of 2's and 3's. Also, $12 = 2^{2} \times 3^{1} \rightarrow$ In the number M, we have eight 22s and eight 31s. Hence, the highest power of 12 in M is 8.

110. First through trial and error identify the two values of x and y that satisfy this. The first one that satisfies are the values 3 and 3, the next pair is 11 and 1. (This will also be the last pair!! Think why??)

Further, since we are looking for the maximum possible integral power of 6, we will use the pair 11 and 1. We need to find the highest power of 6 in 11! which is given by [11/3] + [11/9] = 3 + 1 = 4.

111. X has only 2 elements 1, 2 (so x = 2) and Y consists all the prime numbers. The prime numbers below 100 are 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89 and 97 → Y = 25. So the value of $[y/x] = [25/2] = 12$

112. If X has 15 factors, then it should either be of the form a^{14} or $b^2 \times c^4$ where a, b, c are prime numbers.

Case 1: When the number is of the form a^{14}, in this case the smallest possible number is $2^{14} = 16384$ which is a five digit number so this case is not possible.

Case 2: When the number is of the form $b^2 \times c^4$, the minimum possible number $3^2 \times 2^4 = 144$

$$X^2 = b^4 \times c^8$$

So the square of the number has (4 + 1).(8 + 1) = 5 × 9 = 45 factors.

113. If $wxyz$ is the four digit number then according to the question: $x = y + 2$, therefore, y can take any value from 0 to 7 for which x can take the respective values from 2 to 9. Since the number is odd, z can take five values (1, 3, 5, 7, 9). Now we have total 8 possibilities for x, y and five possibilities for z. For w, we have a total of 9 possibilities (from 1 to 9). So, the total possible numbers = 9 × 8 × 5 = 360.

114. X, Y, Z are in A.P. Let the common difference between them is d then following cases are possible.

Case 1: When $d = 1$, no triplet possible

Case 2: When $d = 2$, possible triplet is (3, 5, 7). Sum = 15

Case 3: When $d = 3$, no triplet possible

Case 4: When $d = 4$, possible triplet is (3, 7, 11). Sum = 21

Case 5: When $d = 5$, no triplet possible

Case 6: When $d = 6$, possible triplet is (5, 11, 17), (7, 13, 19). Sum = 33, 39

Case 7: When $d = 7$, no triplet possible

Case 8: When $d = 8$, possible triplet is (3, 11, 19). Sum = 33

For $d > 8$, no triplet possible. Hence, we see that the possible sums for X, Y and Z are 15, 21, 33 and 39 respectively. Hence, a total of 4 possibilities.

115. When a 'x' digit number is multiplied by a 'y' digit number, the product would have either '$x + y$' or '$x + y - 1$' digits.

So when we multiply a 100-digits number by a 200-digits number then the product will have 100 + 200 or 100 + 200 − 1 = 300 or 299 digits. Further, when this number is again multiplied by a 300 digit number, we get a 598 to 600 digit number. Finally, when this number is further multiplied by a 400 digit number, the answer would have a minimum of 997 digits to a maximum of 1000 digits. Hence, the correct answer is 997.

116. $3^{400} \Rightarrow (3^4)^{100} \Rightarrow (81)^{100}$

$\Rightarrow (1 + 80)^{100} \Rightarrow 1^{100} + 80^{100}$

Now we can easily see that in 80^{100} last two digits will always be 00.

Hence, 1 + 00 = 01.

117. This problem can be easily solved through Cyclicity principle. The units digits of all numbers starting from 1^7 up to 9^7 will be repeated ten times each for every range of ten numbers. Hence, we have to check out the units digit of $1^7 \times 2^7 \times \ldots 9^7$, i.e.

1^7 end in 1

2^7 end in 8

3^7 end in 7

4^7 end in 4

5^7 ends in 5.... Hence, $1^7 \times 2^7 \times \ldots 9^7$ would end in 0. Hence, the units digits would be 0 for the entire expression.

118. On division by 24, the square of a natural number, would leave a remainder of 1, only if the number is of the form $6n \pm 1$, where $n \in N$ (since n must not be divisible by 2 or 3).

In the two digit numbers, we have 15 numbers of the form $6n$ (6×2, 6×3 till 6×16). Hence, there are a total of 30 two digit numbers of the form $(6n \pm 1)$ [15 of the form $6n + 1$, 15 of the form of $6n - 1$].

119. Logical Solution: The maximum total marks, in the exam are 80 (if he gets all questions correct). From this number, he has two mechanisms for losing marks – For a wrong answer, he loses 1.25 marks for each wrong answer. For an unanswered question, he loses 1.125 marks per question. Since he has scored a total of only 23 marks, he has lost 57 marks (from the maximum possible total of 80). Since, we are trying to look for the minimum possible wrong answers, we can start trying to put the number of wrong answers as 0. In that case, we would need to lose the entire 57 marks due to not attempting questions. However, $57 \div 1.125$ is not an integer, hence 0 wrong answers is not possible. Going for 1 wrong answer, marks lost due to wrong answers = 1.25. Marks to be lost due to un-attempted questions = 57 – 1.25 = 55.75. Going further in this direction, we realise that if we put 6 incorrect questions, we would need 49.5 marks to be lost due to un-attempted questions. $49.5 \div 1.125$ = 44. Hence, we get 6 as the required answer.

120. In base-3, the decimal value of a five digit number must lie from 81 to 242.

$[(10000)_3 = 81 \ \& \ (22222)_3 = 242]$

Similarly in the base-4, the decimal value of a five-digit number must lie from 256 to 1023.

In base 5, the decimal value of the five-digit number must lie from 625 to 3124. Hence $x = 5$

Option (d) is correct.

121. Difference between the divisor and the remainder is 1 in each division. [2 – 1 = 4 – 3 = 6 – 5 = 8 – 7 = 10 – 9 = 1]

So the general form of the number will be LCM of [2, 4, 6, 8, 10] – 1 = 120k – 1 where 'k' is a natural number.

There are 8 such numbers between 101 to 1000: 119, 239, 359, 479, 599, 719, 839, 959.

So the required sum = 119 + 239 + 359 + 479 + 599 + 719 + 839 + 959 = 4312

122. Any digit (from 0 to 9) raised to the power of the type $4k + 1$ ($k \in N$) will always end in the same digit. It means for all possible values of x, x^{4k+1} has same unit digit as unit digit of x. So a has same unit digit as unit digit of a^5, a^9, a^{13}, a^{17}, a^{21},….. similarly b has same unit digit as b^5, b^9, b^{13}, b^{17}, b^{21},….. So $a^{13} + b^{13}$ has same unit digit as $a^{17} + b^{17}$.

123. The number of 4-digit numbers in decimal system will be from 10^3 = 1000 to $10^4 - 1$ = 9999, i.e., 9000 numbers.

The number of 4-digit numbers in base 8 will be from 8^3 = 512 to $8^4 - 1$ = 4095, i.e. 3584 numbers.

The number of 4-digit numbers in base 7 will be from 7^3 = 343 to $7^4 - 1$ = 2400, i.e. 2058 numbers.

The number of 4-digit numbers in base 6 will be from 6^3 = 216 to $6^4 - 1$ = 1295, i.e. 1080 numbers.

So the numbers from 1000 to 1295, would have 4 digits in each of base 6, 7 and 8. So, there are a total of 296 numbers possible.

124. The number of 4-digit numbers in decimal system will be from 10^3 = 1000 to $10^4 - 1$ = 9999.

The number of 4-digit numbers in base 7 will be from $7^3 = 343$ to $7^4 - 1 = 2400$.

The number of 4-digit numbers in base 6 will be from $6^3 = 216$ to $6^4 - 1 = 1295$.

So the common numbers are from 1000 to 1295. So, the required difference = 1295 – 1000 = 295.

125. $9^n + 11^n$ is divisible by 10 when n is odd but when n is even then it would leave a remainder of 2 (last digit of $9^2 + 11^2$).

$6^n + 4^n$ is divisible by 10 when n is odd but when n is even then it would leave a remainder of 2. (Last digit of $6^2 + 4^2$). Hence, whenever n is odd, there would be no remainder when 10 divides the expression.

When n is even, then the remainder would be 2 + 2 = 4.

So the sum of all possible remainders in this case will be 4.

126. There are four possible scenarios for the units' digit combination of the four numbers. These essentially arise from the even/odd combinations taking x and z:

Scenario 1: Both even: 1 + 6 +6 + 1 → Remainder = 4;

Scenario 2: Both odd: 9 + 6 + 4 + 1 → Remainder = 0

Scenario 3: x odd, z even: 9 + 6 + 6 + 1 → Remainder = 2

Scenario 4: x even, y odd: 1 + 6 + 4 + 1 → Remainder = 2

Thus, the sum of all remainders = 4 + 0 + 2 + 2 = 8.

127. In order to think of the maximum possible Mondays and Thursdays, we will need to consider one of the two years to be a leap year. In such a case, there are 104 weeks and 3 extra days in 2 years.

But these three extra days must be consecutive so if Monday is one of these three days then Thursday cannot be one of three days and vice-versa.

So maximum possible number of Mondays and Thursdays = 104 × 2 + 1 = 209.

128. In order to think of the minimum possible Mondays and Thursdays, we will need to consider none of the two years to be a leap year. In such a case, there are 104 weeks and 2 extra days in 2 years.

So minimum possible number of Mondays and Thursdays = 104 × 2 = 208. (In the event, that neither of these extra days is a Monday or a Thursday).

129. Remainder of

$$\frac{[(111111111.........64 terms)*(22222222.........55 terms)]}{18}$$

$$= 2 \times \text{Remainder of } \frac{[(111111111.........64 terms)*(111111.........55 terms)]}{9}$$

$$\rightarrow \text{Remainder} = 1$$

So required remainder = 2 × 1 =2

130. We can see from the expression that $(x - y)(x + y)$ is an even number. Both $x - y$ and $x + y$ are even numbers, so product of $(x - y)(x + y)$ must be divisible by 4. But 777314 is not divisible by 4 . So it has no solution. The correct answer would be 0.

Solutions for Questions 131 and 132:

Coin 1 was turned only once, coin 2 was turned twice, coin 3 was turned twice, coin 4 was turned thrice, coin 5 was turned twice, coin 6 was turned 4 times, coin 7 was turned twice, coin 8 was turned 4 times, while coin 9 was turned 3 times as so on.

We can easily see that each of the coins would be turned a number of times that would be equal to the number of factors the number on the coin has. Thus, it would be the coins having perfect square numbers that would be turned an odd number of times (hence show tails) and the rest of the coins were turned an even number of times (Hence, would be showing heads). There are a total 10 coins having perfect square numbers and these would be showing tails.

131. After the 100th visit, coin numbers 54 and 91 were in heads-up state.

132. After the 100th visit, there were 100 – 10 = 90 coins in heads-up state.

133. The last three digits of any number are the same as the remainder when we divide the number by 1000.

$$\frac{(225)^{40}}{1000} = \frac{5^{80} \cdot 9^{40}}{1000} = \frac{5^{77} \cdot 9^{40}}{8}$$

$9^{40} = (8 + 1)^{40}$ gives remainder 1 when we divide it by 8.

$\frac{5^{77}}{8} = \frac{5 \cdot (24+1)^{38}}{8}$ leaves a remainder 5.

So last three digits of $(225)^{40}$ = 125 × 5 = 625.

So the value of 100th digit is 6.

134. 'c' cannot be greater than 3 because if $c \geq 4$ then 'a' should be greater than or equal to 16 which is not possible. Similarly 'b' cannot be greater than 4.

c	b	a	Number of triplets
3	No possible values as $3b$ can be 13 or 14 only, but that gives us no value for b (as neither of these numbers is a multiple of 3)	No values possible	0
2	4	13, 14	2
2	3	10-14	5
1	2	7-14	8
1	3	10-14	5
1	4	13, 14	2

Total possible triplets = 0 + 2 + 5+ 8 + 5 + 2 = 22.

135. There are two cases in this situation:

If we include 5: If this number consists of the digit 5, then we would need to use at least one of the even digits between 2 and 4. In this case, the number would need to be even and also divisible by 5. For this to occur, the number should end in 0 – which is not a possibility in the given case. So 5 cannot be one of these four digits.

If we do not include 5: In this case the number, would be consisting of the digits 1,2,3 and 4. In such a case, we can easily realise that such a number cannot be a multiple of 3, since the sum of digits of the number is 10.

Level of Difficulty (III)

1. $a^n - b^n$ is always divisible by $(a - b)$. Also, if n is even then $a^n - b^n$ is divisible by $(a + b)$.

   $7^{4n} - 5^{4n}$ is divisible by $(7 - 5) = 2$, $7^4 - 5^4 = 1776$ and $7^4 + 5^4 = 3026$

   1776 is divisible by 37 and 111.

   Hence, the given expression is divisible by 2, 111 and 37. Hence, Option (d) is correct.

2. LCM of 3^{12}, 2^{24} and x is $2^{24} \times 3^{12}$.

   x can take 25 powers of 2 from 2^0 to 2^{24} and thirteen powers of 3, from 3^0 to 3^{12}.

   Hence, total possible values are $25 \times 13 = 325$.

3. Let the three numbers be $5a$, $5b$ and $5c$.

   $5a + 5b + 5c = 105$ or $a + b + c = 21$

   When either a, b or c is equal to 1. Then rest of the two numbers could be (19, 1), (18, 2), (17, 3), …, (10, 10). A total ten combinations are possible.

   When any a, b or c is equal to 2. Then rest of the two numbers could be (17, 2), (16, 3), …, (10, 9). Total eight combinations are possible.

   When either a, b or c is equal to 3. Then rest of the two numbers could be (15, 3), (14, 4), (13, 5), (12, 6), (11, 7), (10, 8), (9, 9). A total seven combinations are possible.

   Hence, total 25 combinations are possible.

4. $15 \times 37 \times 63 \times 51 \times 97 \times 17$ on division by 100 would give us a remainder that would be equal to its last 2 digits. First we can divide the numerator and the denominator by 5 to get the expression:

$\frac{3\times37\times63\times51\times97\times17}{20}$ using remainder theorem →

$\frac{3\times17\times3\times11\times17\times17}{20} = \frac{51\times33\times289}{20}$ using remainder theorem →

$\frac{11\times13\times9}{20} = \frac{143\times9}{20} \rightarrow \frac{3\times9}{20} = \frac{27}{20}$ → Remainder = 7. Hence, the required remainder = 7 × 5 = 35 which would also be the last two digits of the given number.

Hence, option (a) is correct.

Solutions for Questions 5 and 6:

5. Since the number is divisible by 44, then the number must be divisible by 4 and 11.

For the number to be divisible by 4, then z must be 2 or 6.

For the number to be divisible by 11, $(1 + x + y + z) - (5 + 3 + 7) = x + y + z - 14$ must be divisible by 11.

Case 1: $z = 2$

Then, $x + y + z - 14 = x + y - 12$ must be divisible by 11. Since, x and y are single digit whole numbers, this is only possible if $x + y - 12 = 0$ or if $x + y - 12 = -11$.

Thus, we get: $x + y = 12$ or $x + y = 1$

The following solutions would work for the above situations with x, y as

x	y	$y + z - x$
3	9	8
4	8	6
5	7	4
6	6	2
7	5	0
8	4	–2
9	3	–4
0	1	3
1	0	1

Case 2: When, $z = 6$

Then $x + y + z - 14$ or $x + y - 8$ must be a multiple of 11. For this to occur, $x + y - 8$ can again be 0.

(**Note:** $x + y - 8$ cannot reach 11, because the maximum value of $x + y$ can be 18. It cannot also reach –11, because $x + y$ would need to be –3 to get to –11, which is again not possible, since x and y are whole numbers.)

x	y	$y + z - x$
1	7	12
2	6	10
3	5	8
4	4	6
5	3	4
6	2	2
7	1	0
8	0	–2
0	8	14

Hence, $y + z - x$ can take total 12 values.

6. Maximum possible value of $y + z - x$ is 14.

Solutions for Questions 7 and 8:

7. $ab - 3a - 2b + 6 = 600$

   $(a - 2)(b - 3) = 600$

   As a is an even number and b is an odd number, $(a - 2)$ and $(b - 3)$ both must be even numbers. The following sets satisfy the given equation: (2, 300), (4, 150), (6, 100), (10, 60), (12, 50) and (20, 30).

   Hence, there are total $6 \times 2 = 12$ solutions of the given equation.

8. Maximum possible value of $a + b = 2 + 2 + 300 + 3 = 4 + 303 = 307$.

Solutions for Questions 9 and 10:

Let $x = ab = 10a + b$, as x is more than 40 so a 4.

According to the question 'a' is a factor of $10a + b$ then b must be divisible by 'a'.

In the 40s, there would be two such numbers 44 and 48. Likewise, 50 and 55; 60 and 66; 70 and 77; 80 and 88 and 90 and 99.

9. Twelve values are possible for x.

10. $99 + 88 + 48 + 77 + 66 + 55 + 44 + 90 + 80 + 70 + 60 + 50 = 827$

11. Let the five digit number be $abcde$.

    According to the question: $a + c + e - b - d = 0, 11$. However, a closer inspection shows us that the value of 11 is not achievable, no matter what you try to do with the numbers. It can be numerically visualised that $a + c + e - b - d = 0$ can be achieved only if we share 2, 3, 5 between a, c and e.

b and d need to be allocated 4 and 6 amongst them. This can be thought out algebraically also as follows:

For $a + c + e - b - d = 0$

$a + c + e + b + d = 2(b + d)$

$2 + 3 + 4 + 5 + 6 = 2(b + d)$

$2\,(b + d) = 20$ or $b + d = 10$

It is possible only when $b = 4$ or 6 and $d = 6$ or 4.

a, c and e can take total $3! = 6$ values by distributing 2, 3 and 5 amongst them.

Hence, total possible numbers= $2 \times 6 = 12$.

Hence, there are only twelve such numbers are possible.

12. $910 = Na + x$

$910 = (N + 2)\,b + x$, where 'a' and 'b' are natural numbers.

To maximise the value of 'N', the value of 'a' and 'b' must be equal to '$N + 2$' and 'N' respectively.

$910 = N\,(N + 2) + x$

$N\,(N + 2) = 910 - x$

$N^2 + 2N + 1 = 911 - x$

$(N + 1)^2 = 911 - x$

To maximise the value of N, x must be equal to 11.

$(N + 1)^2 = 911 - 11 = 900$

N = 29. Hence, the required answer would be the remainder of 910 divided by 29, which is 11.

13. Only perfect squares have odd number of factors. Hence, the required value of $P = 64 \times 81 \times 100 \times 121 \times 144 \times 169 \times 196 \times 225 \times 256 \times 289 = 2^{22} \times 3^{8} \times 5^{4} \times 11^{2} \times 13^{2} \times 17^{2}$

Hence, the maximum power of 6= minimum of (22, 8) = 8.

14. $70 = 2 \times 5 \times 7$

Hence, the number must be in the form of $3^{p} \times 11^{q} \times 13^{r} \times 17^{s} \times 19^{t} \times 23^{u} \times 29^{v} \times 31^{w} \ldots$

On substituting the values of p, q, r, s, ... we get the following composite numbers: 9, 27, 33, 39, 51, 57, 69, 81, 87, 93 and 99. Hence, there are a total of 11 numbers.

15. $51! = 2^{47} \times 3^{23} \times P$

$10! = 2^{8} \times 3^{4} \times Q$

$$\frac{51!}{10!} = \frac{2^{47} \times 3^{23} \times P}{2^{8} \times 3^{4} \times Q} = 2^{39} \times 3^{19} \times \left(\frac{P}{Q}\right)$$

As 51! is divisible by 10! completely so P/Q must be an integer and 51! must be divisible by $2^{2x} \times 3^{x}$.

Maximum possible value of $x = 19$.

Solutions for Questions 16 and 17:

16. As the number of factors of N^2 is odd. Hence, the number of factors of N must be odd. (Odd + Odd = even). So N must be a perfect square.

For $N = 2^{4} \times 3^{2}$, number of factors = (4 + 1) (2 + 1) = 15. $N^2 = 2^{8} \times 3^{4}$, number of factors= (8 + 1) (4 + 1) = 45

Hence, the total number of factors = 15 + 45 = 60

For $N = 3^4 \times 2^2$, number of factors = (4 + 1) (2 + 1) = 15, $N^2 = 3^8 \times 2^4$ number of factors = (8 + 1) (4 + 1) = 45

Hence, the total number of factors = 15 + 45 = 60

Similarly for $N = 5^2 \times 2^4$ and for $N = 2^4 \times 7^2$, number of factors of N and N^2 would be 15 + 45 = 60.

Thus, possible values of N = 144, 324, 400 and 784.

Hence, four values are possible for N.

17. 784.

18. $3Q^2 = 1376 - P^2$

$1376 - P^2$ must be a multiple of 3, which is not possible for any real value of P. Hence, the correct answer is 0.

19. Divide the given expression by 100 and find the remainder to get the answer. 50 would be the last two digits here.

20. The remainder will always be less than 18, since the sum of the digits (which would be the divisor) would at the maximum be 18. We would need to adopt a trial and error approach.

When the sum of digits is 18 then, only possible remainder is 9 (for 99).

When the sum of digits is 17 then, the largest possible remainder is 13 (for 98).

When the sum of digits is 16 then, the largest possible remainder is 15 (for 79).

When the sum of digits is 15 or below, we would not be able to get a remainder that is less greater than 15.

Hence, the maximum possible remainder is 15.

21. X has exactly 201 elements and the sum of all elements of X is equal to 0. As the subsets of X have exactly 199 elements. Hence, we have to leave rest of the two elements whose sum in either +1 or −1 out of that subset.

Sum '–1' for 199 elements can be obtained by removing two elements whose sum is 1.
This can be done by removing any of the following pairs of numbers: (0, 1), (–1, 2), (–2, 3),.....,(–99, 100). Similarly we can sum '+1' in 100 ways.

Hence, the required answer= 100 + 100 = 200.

22. The value of $ab + bc + cd + de + ef + gh + hi + ij + ja$ will be minimum when all the $a, b, c, d, e, f, g, h, i$ and j are 10, –10, 10, –10, ... –10 (As it is not given that all the integers are distinct or same).

Hence, minimum possible value of $ab + bc + cd + de + ef + fg + gh + ha$ = 10 × –10 + –10× 10 + ... +10 × –10 = –800

23. $43^{101} + 23^{101}$ is of the form $a^n + b^n$ with n odd. Such a number can be written to be a multiple of $(a + b)$. Thus, the given expression is a multiple of (43 + 23) = 66. Hence, the required remainder would be 0.

24. Let the numbers are $6k$ and $6(k + 1)$.

For the HCF to be 6, k and $k + 1$ are always co-prime.

LCM of 6k and $6(k + 1)$ is $6\,k\,(k + 1)$.

LCM of 6k and $6(k + 1)$ is $6k(k + 1)$.

$6k(k + 1)$ is a 5-digit number for $k \geq 41$.

Hence, the maximum possible value of k is 40 for which the LCM is a 4–digit number.

Hence, the required number = $6(k + 1) = 6 \times 41 = 246$.

25. Let the numbers be $6k$ and $6(k + 1)$.

For the HCF to be 6, k and $k + 1$ are always co-prime.

LCM of $6k$ and $6(k + 1)$ is $6k(k + 1)$.

$6k(k + 1)$ is a 4-digit number for $40 \geq k \geq 13$.

Hence, the required number of pairs = $40 - 13 + 1 = 28$.

26. N would have 6 factors if its standard form appears as any one of these:

Case 1: $a \times b2$ or **Case 2:** $c5$

Where a, b and c c are prime numbers.

Case 1: N = $a \times b2$

Possible values: when $a = 2$, $b = 3, 5$: and $N =$ **18, 50**

$a = 3, b = 2 - N = 12$

$a = 5, b = 2, 3 - N = 20, 45$

$a = 7, b = 2 - N = 28$

$a = 11, b = 2 - N = 44$

Case 2: $N = c5$

$c = 2 - N = 32$

Hence, total number of possible values of N are 8.

27. 50

28. $(6 - p) \times (6 - q) \times (6 - r) \times (6 - s) \times (6 - t) = 6720 = (6 - 2)(6 - 1)(6 + 0)(6 + 1)(6 + 2)$

$p = 2, q = 1, r = 0, s = -1, t = -2.$

$p + q + r + s + t = 2 + 1 + 0 - 1 - 2 = 0$

29. $87y - 264x = 60$

$87y - 60 = 264x \rightarrow 87y - 60 = (87 \times 3 + 3)x$

$87(y - 3x) = 60 + 3x$

As x and y are integers hence, $y - 3x$ must be integer.

For $y - 3x = 1$ $x = 9, y = 28$

For $y - 3x = 2$ $x = 38$ and $y = 116$ (not possible as x and y should both be below 100).

Hence, only one solution is possible.

Solutions for Questions 30 to 32:

30. Let the number be $10a + b$. Then, the number formed by reversing the digits would be $10b + a$. According to the question, when both these numbers are divided by their sum of digits (viz: $a + b$), the remainder is the same in both cases. This can be represented algebraically as:

$10a + b = p(a + b) + r, 10b + a = q(a + b) + r$

By subtracting the second equation from the first we get: $9(a - b) = (a + b)(p - q)$

Or: $p - q = 9(a - b)/(a + b)$

Since $p - q$, $(a - b)$ and $(a + b)$ are all integers, $(a + b)$ must be a multiple of 3.

By this logic, we get that the possible values of $(a + b) = 3, 6, 9, 12, 15, 18$. Of these

$a + b = 15$ is not possible because if $a + b = 15$ then in order for $(p - q)$ to be an integer, $(a - b)$ must be 5. We get from $a + b = 15$ and $1 - b = 5$, $a = 10$ and $b = 5$ which is not possible. Also for $a + b$ to be equal to 18, both a and b must be 9 each, which does not make sense in this question since we are talking about reversing of digits here.

Hence, using $a + b = 3, 6, 9$ or 12 we get the possible values of a and b as given in the following table.

Note: In order to make this table, we have to also check the same remainder condition being fulfilled when the digits are reversed:

Value of b	Value of a	Total possible values of the number
1	2, 5, 8	21, 51, 81
2	1, 4, 7	12, 42, 72
3	6	63
4	2, 5, 8	24, 54, 84
5	1, 4	15, 45
6	3	36
7	2	27
8	1, 4	18, 48
9	No values possible	
0	Reversal does not make sense	

30. There are 16 such numbers (8 such pairs).

31. Maximum possible value of '*ab*' = 84

32. Minimum possible value of '*ab*' = 12

33. Visualise the 62 digit number as X = pppppp.......(36 times) qp pppppp....24 times.

We know that if a digit is written 6 times side by side, then such a six digit number is always divisible by 7, 11 and 13.

∴ pppppp.......(36 times) and pppppp....24 times are divisible by 13.

For X to be divisible by 13, qp must be divisible by 13. Since, no two digit multiple of 13 has either 4 or 7 as its units digit. Hence, option (a) is correct.

34. Using the numbers (2, 3, 5), we can form distinct sum as 2, 3, 5, 2 + 5 = 7, 3 + 5 = 8 and 2 + 3 + 5 = 10 (6 – distinct sums)

If we include 10, we could not only obtain the above mentioned sum but also the following sums:

10 + 2 = 12, 10 + 3 = 13, 10 + 5 = 15, 10 + 7 = 17, 10 + 8 = 18, 10 + 10 = 20.

In other words, we can obtain 6 + 6 = 12 different sums if we use 2, 3, 5 and 10.

Similarly if we use 2, 3, 5, 10 and 20 we can obtain 12 + 12 = 24 different sums. Continuing with this logic, we can conclude that there will be a total of 384 distinct sums.

35. It is given that p and r are integers then q must be a perfect square and s must be a 4_{th} power of a natural number.

q = x^2 and $s = y^4$

$y^4 - x^2 = 17$

$(-x + y^2)(x + y^2) = 1 \times 17$

On solving the above equation, we get $x = 8$ and $y = 3$.

$q = 64$ and $s = 81$

$64^{\frac{1}{2}} = 8, r = 81^{\frac{7}{4}} = 2187$

$p + r = 8 + 2187 = 2195$

36. $32^{32^{32}} \div 9 \rightarrow 5^{32^{32}} \rightarrow \div 9 = 5^{6n+x} \div 9$. We write this in the form of 5^{6n+x} because 5^6 leaves a remainder of 1 when divided by 9. When we try to see 32^{32} as $6n + x$, we can find the value of x as the remainder of 2^{32} when divided by 6. The following thought process would help us find this value:

$2^{32} \div 6 = 2^{31} \div 3 \rightarrow$ Remainder = 2 (by the $a^n \div (a + 1)$ rule). Thus, $2^{32} \div 6$ would have a remainder of $2 \times 2 = 4$.

Hence, the required remainder would be $5^4 \div 9$, which is 4.

Solutions for Questions 37 to 39:

37. We are given that: $ab \times ac = abc$

This can be written as: $ab \times ac = ab \times 10 + c$

$ab\,(ac - 10) = c$

The right hand side of the above equation is a single digit number and the left hand side of the above equation can never be a single digit number. Hence, c must be equal to 0 and ac must be equal to 10. Hence, $a = 1$ and $c = 0$ we can not determine the exact value of b. Hence, option (d) is correct.

38. As $ac = 10$ and the last digit of bc must be 0 hence, $bc \times accaaa$ is perfectly divisible by ac. Hence, the required remainder is equal to 0.

39. $abc = 1b0$ and abc is divisible by 17 if and only if $b = 7$.

$abcbac = 170710$. If we divide 170710 by 14, we get 8 as the remainder.

40. $32^{32^{32}} \div 7 \rightarrow 4^{32^{32}} \div 7 = 4^{3n+x} \div 7$. We write this in the form of 4^{3n+x} because 4^3 leaves a remainder of 1 when divided by 7. When we try to see 32^{32} as $3n + x$, we can find the value of x as the remainder of 2^{32} when divided by 3. The following thought process would help us find this value:

$2^{32} \div 3 = 1$. (remainder)

Hence, the required remainder would be $4^1 \div$, 7 which is 4.

41. If we visualise the number as $2^p \times 3^q$, the number of factors would be $(p + 1)(q + 1)$; For $2n$, we realise that $2n = 2^{p+1} \times 3^q$ and its number of factors would be $(p + 2)(q + 1) = 28$. This has multiple possibilities based on the factors of 28. These are: 1×28; 2×14 and 4×7. Also, $3n = 2^p \times 3^{q+1}$ would have $(p + 1)(q + 2) = 30$ factors. Looking through the factor pairs of 30, we see 1×30, 2×15, 3×10 and 5×6. Considering both these lists, we can see that if we take p as 5 and q as 3, we get both the conditions fulfilled. Thus, $6n = 2^{p+1} \times 3^{q+1} = 2^6 \times 3^4$ would give us $7 \times 5 = 35$ factors.

42. Solve this question through the options. For *n* terms being 16 option (a), we would need an AP with 16 terms and common difference 1, that would add up to 1000. Since, the average value of a term of this AP turns out to be 1000 ÷ 16 = 62.5, we can create a 16 term AP as 55,56,57....62,63,64...70 that adds up to 1000. Hence, 16 terms is possible. Likewise, 5 terms gives the average as 200 and the 5 terms can be taken as 198,199,200,201,202. It is similarly possible for 25 terms with an average of 40, but is not possible for 20 terms with an average of 50. Hence, Option (d) is correct.

43. The first remainder would be 4, the second one would be given by 4 × 4 = 16/9 → 7, the third one 6 × 6 = 36/9 → 0. The fourth one, 8 × 8 = 64/9 → 1. Subsequent, remainders would be 1, 0, 7, 4, 0. This cycle would repeat for the next 9 numbers each time. Thus, the remainder for the first 45 numbers = (4 + 7 + 0 + 1 + 1 + 0 + 7 + 4 + 0) repeated 5 times → 120/9 → remainder = 3. The last 4 terms would then add 4 + 7 + 0 + 1 to the remainder. Thus, the final remainder = 15/9 → 6.

44. The product would be 32323232...repeated 32 times. Hence, the sum of digits would be 160.

45. For the maximum number of questions, we would need to keep the 15 questions that are not unique, to be shared between the least number of tests (i.e., 2 each). This would give us 12 sets of 15 questions each that are not unique. Also, the number of unique questions would be 35 × 25 = 875. Thus, the required maximum number of questions would be 875 + 180 =1055.

Solutions for Questions 47 to 49:

The given condition in the problem is a property of the numbers in the geometric series of the powers of 3.
The numbers from W_1 to W_7 would be 1,3,9,27,81,243 and 729. The answers can be got according to these values.

47. 1 + 2 × 3 + 3 × 9 + 4 × 27 +5 × 81 + 6 × 243 = 1 + 6 + 27 + 108 + 405 + 1458 = 2005.

48. $3_0 \times 3_1 \times 3_2 \times 3_3 \times 3_4 \times 3_5 \times 3_6 = 3_{21}$.

49. We would need to use the coefficients as 1,1,1,0, −1, −1 and −1 to W_1 to W_7 in that order to get:

1 + 3 + 9 +0 – 81 – 243 – 729 = −1040. (**Note:** In this question, we have to take the coefficients as defined in the problem as +1, 0 or −1 only).

Solutions for Questions 50 and 51

There are two 3 digit perfect square numbers that obey the factors are perfect squares rule - viz. 196 and 256. However, questions 50 and 51, both rule out the use of 256. Hence, for question 50, we are looking for the number of factors of 196196 = 2 × 2 × 7 × 7 × 7 × 13 × 11. This number would have 3 × 4 × 2 × 2 = 48 factors. Hence, Option (c) is correct. For question 51, we need the number of factors of 196196196 = 2 × 2 × 3 × 7 × 7 × 7 × 333667. This number gives us 3 × 2 × 3 × 2 = 36 factors.

(**Note:** The number 333667 is a prime number.)

52. Since each team scored a different number of points, it follows that the points scored by the 15 teams would be 21,22,23,24,...till 35. This is because, the total number of matches in the tournament is ${}^{15}C_2 = 105$ and there are 4 points for each match (either 3 + 1 or 2 + 2). Thus, the total number of points in the tournament is 105 × 4=420. The only way to fit in 420 points amongst 15 teams with each team getting different number of points and the least value for any team being 21 points would be to use the Arithmetic Progression 21,22,23,24...till 35. Once, we realise this, we know that Australia scored 35 points out of a maximum possible 14 × 3 = 42 (14 wins). This means that Australia is dropping 7 points. Each loss makes you drop 2 points as instead of 3 for a win, you receive only 1 point for a loss. Hence, it is not possible for Australia to lose 4 matches. The maximum losses Australia could have had is 3.

53. We can solve this by splitting the denominator into two co-prime numbers 9 and 17. First find the remainder of 1281000 on division by 9.

$128^{1000} \div 9 \xrightarrow{R} 2^{1000} \div 9 = (2^6)^{166} \times 2^4 \div 9 \rightarrow$ Remainder = 7. This means that 128^{1000} is a $9n + 7$ number.

Next find the remainder of 1281000 on division by 17.

$128^{1000} \div 17 \rightarrow 9^{1000} \div 17 = [(9^{16})^{62} \times 9^8] \div 17 \rightarrow$ Remainder = 1. This means that 1281000 is a $17n + 1$ number.

If we try to look for a number below 153, that is both $17n + 1$ as well as $9n + 7$, we would see that the number 52 fulfills this requirement. Hence, 52 is the required remainder of $128^{1000} \div 153$.

54. $\frac{50^{51^{52}}}{11} \rightarrow \frac{6^{51^{52}}}{11} = \frac{6^{10x} \times 6^1}{11} \rightarrow$ Remainder = 6

55. Use the −1 remainder rule for even powers. Thus,

$$\frac{32^{33^{34}}}{11} \rightarrow \frac{10^{\text{Odd Power}}}{11} \rightarrow \text{Remainder} = 10$$

56. $\frac{30^{72^{87}}}{11} \rightarrow \frac{8^{72^{87}}}{11} = \frac{8^{10x} \times 8^{2^{87}}}{11} \rightarrow \frac{1 \times 8^{2^{87}}}{11} \rightarrow \frac{8^{10x} \times 8^8}{11} \rightarrow$ Remainder =5

57. $\frac{50^{56^{62}}}{11} \rightarrow \frac{6^{56^{62}}}{11} = \frac{6^{10x} \times 6^6}{11} \rightarrow$ Remainder = 5

58. $\frac{33^{34^{35}}}{7} \rightarrow \frac{5^{34^{35}}}{7} = \frac{5^{6x} \times 5^4}{7} \rightarrow$ Remainder = 2

59. We need the expression $\frac{n(n+1)(2n+1)}{6}$ to be a multiple of 4. For this to occur, the numerator of the above expression should be a multiple of 8. In the expression $n(n + 1)(2n + 1)$, $2n + 1$ would always be an odd number. Also, amongst n and $(n + 1)$ one number would be odd and the other would be even. Since, we need $n(n + 1)(2n + 1)$ to be a multiple of 8, we would need either n or $(n + 1)$ to be a multiple of 8 (while at the same time it should be below 100). Thus, we get the number series n = 7,8,15,16,23,24,...95,96. Since there are 12 multiples of 8 below 100, the required answer is 12 × 2= 24.

60. We need the expression $\frac{[n(n+1)]^2}{2^2}$ to be a multiple of 5. For this to occur, the numerator of the above expression should be a multiple of 5. Either n

the numerator of the above expression should be a multiple of 5. Either n or $n + 1$ should be a multiple of 5. Below 50, there are 19 such instances. Hence, the correct answer is 19.

61. The first solution easily visible here would be at $x = -1$, and $y = 1$. In such equations, we should know that the value of x would change with the co-efficient of y, while the value of y would change with the coefficient of x (and the two values would move in the opposite directions since there is a 'plus' sign in the middle). Thus, the series of values of x from its highest positive value below 1000 to the lower limit of being just above −1000 would be 993, 986 ,...... 13, 6, −1,−8, −15,... −995. The number of terms in this series $= \frac{1988}{7} + 1 = 285$.

62. 1144 can be written as a product of three co-prime numbers – viz: 13 × 8 × 11. Further, the given number when divided by 11, leaves a remainder of 7, when divided by 13 leaves a remainder of 10 (because, if you divide 777777 by 13, there is no remainder. Hence, when you divide the given number by 13, the remainder would only depend on the remainder of the last three 7's. i.e. 777 ÷ 13). Also, the given number when divided by 8, leaves a remainder of 1. Hence, the given number is a number that is simultaneously $13n + 10$, $8n + 1$ and $11n + 7$. If we create a series of $13n + 10$, we can see that the series would be: 10, 23, 36, 49, 62… At 62, the number is $13n + 10$ as well as an $11n + 7$ number. The next such number would be 62 + 143 (because 143 is the LCM of 13 and 11). Thus, writing down the series of numbers that belong to $13n + 10$ and $11n + 7$ and checking when it also simultaneously becomes $8n + 1$, we can see that the series would be:

62, 205, 348, 491, 634, 777. The number 777 is also an $8n + 1$ number. Hence, the correct remainder is 777.

63. In order to solve this question, you would need to find the odd factors of 19! that are also multiples of 5.

$19! = 2^{16} \times 3^{8} \times 5^{3} \times 7^{2} \times 11^{1} \times 13^{1} \times 17^{1} \times 19^{1}$.

The required answer would be $1 \times 9 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 = 81 \times 16 = 1296$.

64. From 5! onwards, each of the numbers would have a units digit of 0. Hence, the units digit of the given number would depend on the units digit of 1! – 2! + 3! – 4!, which would be 1 (1 – 2 + 6 – 4). Since N is a number that has a unit's digit of 1, when it is raised to any power, the units digit would not change. Hence, the correct answer would be 1.

65. Exactly four factors would occur for numbers that can be represented by a single prime factor as p^3. Also, any number that can be represented by a product of two prime factors $p^1 \times q^1$ would also have four factors. There are four perfect cubes below 100. Besides, numbers like $2^1 \times 3^1$; $2^1 \times 5^1$; ....$2^1 \times 47^1$ (a total of 14 numbers). Next we consider numbers with the lower prime factor as 3. These would be $3^1 \times 5^1$; $3^1 \times 7^1$; ...$3^1 \times 31^1$ (a total of 9 numbers). Next, we consider numbers with their lower prime factors as 5. These would be $5^1 \times 7^1$; $5^1 \times 11^1$; ...$5^1 \times 19^1$ (a total of 5 numbers). For numbers starting with 7, we would get $7^1 \times 11^1$; $7^1 \times 13^1$; (a total of 2 numbers). But, out of the four perfect cubes, 1 and 64 do not have four factors. Hence, there are a total of 32 such numbers, below 100 that would have exactly 4 factors.

66. Since both the numbers are odd, there are no 2's in their prime factors. Since, their HCF is 225, both these numbers would necessarily have $3^2 \times 5^2$ inside them. From the information, that both these numbers have 36 factors, we can realise that 36 factors can only occur in cases where the prime factors of the numbers look as follows: $p^8 \times q^3$; $p^{11} \times q^2$; $p^2 \times q^3 \times r^2$ and $p^2 \times q^2 \times r \times s$. Amongst these, the best strategy to make smaller numbers satisfying the criteria would obviously be to use the structures: $p^2 \times q^3 \times r^2$ and $p^2 \times q^2 \times r \times s$. Since, the third prime factor of the two numbers cannot be the same (else it would change the HCF), we would need to introduce 7 and 11. Also, we would not try to increase the powers of 7 and 11 as they are comparatively larger as compared to 3 and 5. Thus, we can visualise the numbers: $3^2 \times 5^2 \times 11 \times 13$, $3^3 \times 5^2 \times 7^2$. The required smallest LCM would be $3^3 \times 5^2 \times 7^2 \times 11 \times 13$.

67. 7! = 5040. When we divide 7! by 17, it leaves a remainder 8. When $[(7!)^{6!}]^{17777}$ is divided by 17 it leaves a remainder that is the same as when we divide $[(8)^{6!}]^{17777}$ or $[(8)^{720}]^{17777}$ or $[(16)^{540}]^{17777}$ by 17. The remainder when $[(16)^{540}]^{17777}$ or $[(17 - 1)^{540}]^{17777}$ divided by 17 is 1 (since the power on 16 is even).

68. $x + y = 2w$ (1)

$y + 6z = 2(w + x)$ (2)

$w + 5z = 2y$ (3)

From equation (2) – equation (1), we get: $6z - x = 2x$ or $x = 2z$

Substituting $x = 2z$ in equation (1), we get : $2w - y = 2z$ (4)

Solving equation (3) and (4), we get $w = 3z$ and $y = 4z$

$z : y : x : w = 1 : 4 : 2 : 3$

So 3241 and 6482 are two possible values of $wxyz$.

So the required sum = 3241+ 6482 = 9723.

Alternately, once you have the relationship $x = 2z$, you can think of values and try to fit in the conditions of the other equations. z and x can take only 4 feasible values: viz 1, 2; 2, 4;3, 6 and 4, 8.

This gives us four possibilities for the numbers: _2_1; _4_2; _6_3; _8_4. The fourth of these, with $z = 4$ can be eliminated by looking at the third equation ($w + 5z = 2y$) as it would need y to be greater than 10.

For $z = 3$ and $x = 6$; we get $w = 1$ and $y = 8$ or $w = 3$ and $y = 9$ from the third equation. Both these values do not match the second equation.

For $z = 2$ and $x = 4$; we get $w = 2$ and $y = 6$ or $w = 4$ and $y = 7$ or $w = 6$ and $y = 8$ or $w = 8$ and $y = 9$. Checking for the second equation, only the values of $w = 6$ and $y = 8$ matches. Hence, we get the number 6482.

Likewise, when you check for $z = 1$ and $x = 2$, you would be able to find the number 3241.

69. $\frac{[17(9!)+2(18!)]}{9!.17408} = \frac{17.9!}{9!17.2^{10}} + \frac{2.18!}{9!.17.2^{10}}$

$$\frac{17.9!}{9!17.2^{10}} = \frac{1}{2^{10}} \text{ and remainder of } \frac{1}{2^{10}} = 1$$

Hence, the Remainder of $\left(\frac{17.9!}{9!17.2^{10}}\right) = 17.9!$

2.18! = 2.18.17.16.15.14.13.12.11.10.9!

In 18, 16, 14, 12, 10 the number of 2s are 1, 4, 1, 2 and 1, respectively.

So $2.18! = 2.18.17.16.15.14.13.12.11.10.9! = 2^{10}.17k.9!$ = Where k is an integer.

Remainder $\left(\frac{2^{10}.17k.9!}{9!(17)2^{10}}\right) = 0$

So the required remainder is $17.9! + 0 = 17.9!$.

70. Remainder when X is divided by 9, is same as remainder when sum of digits of X is divided by 9.

Sum of digit of first 999 natural numbers is 13500, which is divisible by 9.

Now sum of digits of 1000, 1001, 1002 are 1, 2 and 3, respectively.

Sum of digits of X is $9n + (1 + 2 + 3) = 9n + 6$. So the required remainder is 6.

71. X = 0123456789..........1001

X has a total of $10 + 2 \times 90 + 3 \times 900 + 2 \times 4 = 2898$ digits.

So there are total 1449 digits on the left of the vertical line, out of these 1449 numbers there are total $1449 - (10 + 180) = 1259$ digits are digits of 3-digit number.

$$1259 = 419 \times 3 + 2$$

On the left side of the vertical line, there are 419 3-digit numbers and 2 more digits.

419th 3-digit number = 518 and next two digits are 5, 1. Hence last four digits are 1851.

The remainder of any number divided by 625 is the remainder when last 4-digits of the number is divided by 625.

Required remainder = Remainder of $(1851/625) = 601$

72. If the 4- digit number is $abcd$, then three cases are possible for the number to have 24 in it.

Case 1: If $cd = 24$ then the numbers are of the form $ab24$

$$ab24 = 100ab + 24$$

24 is divisible by 24 and $100 \times ab$ must be divisible by 24. 100 is divisible so 4 then ab must be divisible by 6. Possible values of $ab = 12, 18, 24, \ldots,$ 96. So there are 15 such numbers possible.

Case 2: If $bc = 24$ then the number is of the form $a24b$.

$a24b$ must be divisible by 3 and 8. If $a24b$ is divisible by 8 then $24b$ is divisible by 8. Possible values of $b = 0, 8$.

Similarly $a + 2 + 4 + b = (a + b) + 6$ must be divisible by 3.

When $b = 0$ then a = 3, 6, 9 (3 possible cases)

When $b = 8$ then a = 1, 4, 7 (3 possible cases)

So there are total 6 possible cases.

Case 3: $ab = 24$ so the number should be of the form $24cd$.

$$24cd = 2400 + cd$$

2400 is divisible by 24, cd divisible by 24 when $cd = 00, 24, 48, 72, 96$ (5 possible cases).

However, the number 2424 occurred in cases 1 and 3 both. So the total possible numbers $= 5 + 6 + 15 - 1 = 25$.

73. If N divides both $18X + 2$, $12X + 1$ then their difference $6X + 1$ will also be divisible by N and difference of $12X + 1$ and $6X + 1$, i.e. $6X$ will also be divisible by n. If $6X$ is divisible by N then N can also divide $12X$. It means N divides both $12X$, $12X + 1$. Since $12X$ and $12X + 1$ both are consecutive numbers so $N = 1$. So the given numbers are relatively prime for all values of X, i.e. X would have 99 values.

74. Let $P = 7 \times 10^{5+n} + k$ where n, k are whole numbers.

After removal of the leftmost digit, the new number will be k.

According to the question:

$$7 \times 10^5 \times 10^n + k = 21k$$
$$7 \times 10^5 \times 10^n = 20k$$
$$\frac{7 \times 10^5 \times 10^n}{20} = k$$

$k = 35000 \times 10^n$

$P = 735000 \times 10^n$

The required product = $7 \times 3 \times 5 = 105$

75. $X!$ is completely divisible by 11^{51}. So the value of X should be less than $11 \times 51 = 561$

Highest power of 11 in 561! $\left[\frac{561}{11}\right] + \left[\frac{561}{11^2}\right] + \ldots = 51 + 4 = 55$

If we subtract $11 \times 3 = 33$ from 561, we get $561 - 33 = 528$. Highest power of 11 in 528! is $\left[\frac{528}{11}\right] + \left[\frac{528}{11^2}\right] = 52$

Highest power of 11 in $528 - 1 = 527!$ is $= \left[\frac{527}{11}\right] + \left[\frac{527}{11^2}\right] = 47 + 4 = 51$

So the required number is 527.

Sum of the digits = 5 + 2 + 7 = 14.

76. Let $a = 5k$ then $a + 5 = 5(k + 1)$. Both k, $k + 1$ are co-prime.

LCM of a, $a + 5 = 5.k.(k + 1)$

For $k = 4$, $5k(k + 1) = 100$.

So minimum possible value of smaller number is 20.
Maximum value of k for which the LCM is a three-digit number is 13.

Maximum possible value of the smaller number = 65

So the required difference = 65 − 20 = 45.

77. We need to look at writing the binary number system from $8 = (1000)_2$ to $127 = (1111111)_2$

There are 64 seven digit numbers in binary system are from 1000000 to 1111111. There are six digits after the leftmost 1. Each of these 6 digits can be filled by either 0 or 1 and both are equally probable in any position. So the number of 1 from 1000000 to 1111111 = $64 + 64 \times \frac{1}{2} \times 6 = 64 + 192 = 256$.

There are 32 six digit numbers in binary system are from 100000 to 111111. There are five digits after the leftmost 1. Each of these 5 digits can be filled by either 0 or 1 and both are equally probable in any position. So the number of 1 from 100000 to 111111 = $32 + 32 \times \frac{1}{2} \times 5 = 32 + 80 = 112$.

Similarly, from 10000 to 11111, there are $16 + \frac{16}{2} \times 4 = 16 + 32 = 48$.

Similarly, from 1000 to 1111, there are $8 + 8 \times \frac{1}{2} \times 3 = 20$

So total 1's = 20 + 48 + 112 + 256 = 436.

78. First of all pick all the prime numbers, i.e. {2, 3, 5, 7, 11, 13, 17, 19}. We cannot pick perfect square numbers, i.e. 4, 9, 16 and perfect cube numbers i.e. 8.

Now we are left with the numbers 6, 10, 12, 14, 15, 18, 20 out of which 12 and 18 will give perfect square numbers when we multiply them with 3 and 2, respectively. Also, we would need to take 1 into this list. So we have a total 14 such numbers {1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 20}. (**Note:** If you try to improve this solution, by taking the perfect squares and the perfect cubes in, you would first need to get rid of 1, in the list. Also, you can take only one perfect square number (as if you were to take two perfect squares, their product would be a perfect square too). With respect to the perfect cube 8, we can see that $8 \times 2 = 16$ is a perfect square. Hence, if we try to take in 8, we would need to remove 2 from our list. Thus, you can see that you can take this list to a maximum of 14 numbers.)

79. It is given that $d + e + f = 9$, now two cases are possible.

Case 1: When all three of them are odd. This case is not possible because if all three of them are odd then g, h, i, j cannot be odd digits.

Case 2: When only one number is odd. This case is possible and the digit which is odd among d, e, f can be 1 only. So the sum of rest of the two digits is 8 which is possible as 0 + 8 only.

$g = 9, h = 7, i = 5$ and $j = 3$ & $d = 8, f = 0$. So $e = 1$.

$a = 6, b = 4$ and $c = 2$

$$\left[\frac{a \times b \times c \times d}{i}\right] = \left[\frac{2 \times 4 \times 6 \times 8}{7}\right] = 54$$

80. If $N = abc$, then $a + b + c = abc/7$

As the product is divisible by 7 so one of the digits must be 7.

$a + b + 7 = ab.7/7$

$a + b + 7 = ab$

$ab - a - b = 7$

$ab - a - b + 1 = 8$

$a(b - 1) - (b - 1) = 8$

$(a - 1)(b - 1) = 8$. There are two possible ways to get 8 as a product of 2 digits. These are: 4×2 and 1×8

This gives us: $a = 5, b = 3$ and $a = 2, b = 9$

Possible sets of digits used in the numbers are (2, 9, 7), (3, 5, 7). Hence, the correct answer is 2 (i.e. 2 sets are possible)

81. In base 34, 10 means 34. In base 10, 10 is obtained by multiplying 2 and 5. In base 34, it is obtained by multiplying 2 and 17. Number of consecutive zeroes in base 34 at the end of the number is same as the number of 2's and 17's in 3132!. Since the number of 2's is much more than number of 17's, so we count number of 17's in 3132!

Maximum power of 17 in 3132! = $\left[\frac{3132}{17}\right] + \left[\frac{3132}{17^2}\right]$ = 184 + 10 = 194.