ALGORITHMS TEST 1

Number of Questions: 35 **Section Marks: 30**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Consider the given properties of Asymptotic Notations:
I. $f(n) = \theta(g(n))$ and $g(n) = \theta(h(n))$
$\Rightarrow$ $f(n) = \theta(h(n))$
II. $f(n) = \theta(g(n))$ if and only if $g(n) = \theta(f(n))$
III. $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$
IV. $f(n) = O(g(n))$ if and only if $g(n) = O(f(n))$
Which of the following are valid?
(A) I, II only (B) I, II, III only
(C) II, III only (D) I, II, III and IV

2. Consider the given Recurrence Relation
$T(n) = 2^n\ T(n/3) + n$
Which of the following is TRUE?
(A) Master theorem cannot be applied because 'a' is not constant.
(B) It comes under case 1 of Master theorem.
(C) It comes under case 2 of Master theorem.
(D) It comes under case 3 of Master theorem.

3. Let 'n' be the number of elements in the queue, then What is the Time complexity of following operations respectively, Enqueue( ), Dequeue ( ), IsEmptyQueue ( ), Delete Queue( )?
(A) $O(n)$, $O(1)$, $O(1)$, $O(n)$
(B) $O(1)$, $O(1)$, $O(n)$, $O(1)$
(C) $O(1)$, $O(1)$, O(1), $O(1)$
(D) $O(n)$, $O(n)$, $O(n)$, $O(1)$

4. Which of the following are Applications of Binary Trees?
I. Huffman coding trees are used in data compression Algorithms.
II. Priority Queues support search and deletion of minimum or maximum on 'n' number of items in (log n) time.
III. Expression trees are used in compilers.
IV. Binary search Tree supports search, insertion and deletion on 'n' number of items in (log n) time (average case)
(A) I, II only (B) I, III, IV only
(C) I, II, III only (D) I, II, III and IV

5. A Traversal is defined as follows:
1. Visit the root
2. While traversing Level 'i', keep all the elements at level '$i + 1$' in queue.
3. Go to the next Level and visit all the nodes at that level.
4. Repeat this until all the levels are completed.
The above defined traversal is
(A) Depth First Traversal
(B) Level order Traversal
(C) Binary Tree Traversal
(D) Binary search Tree Traversal

6. How many different binary trees are possible with '8' nodes?
(A) 256 (B) 128
(C) 248 (D) 64

7. A binary search tree is generated by inserting in order the following integers 66, 72, 46, 48, 9, 8, 40, 36, 18, 7, 5, 91, 88, 49, 6. The number of nodes in the Left sub tree and Right sub tree of the root respectively.
(A) (8, 6) (B) (9, 5)
(C) (10, 4) (D) (11, 3)

8. For a Full Binary tree of height 'h', the sum of the heights of all nodes is _____ ('n' is number of nodes)?
(A) $n - (h - 1)$ (B) $n - (h + 1)$
(C) $n + (h - 1)$ (D) $n + (h + 1)$

9. A graph 'G' has 29 edges and its complement $\overline{G}$ has 7 edges, what is the number of vertices present in graph G?
(A) 7 (B) 8
(C) 9 (D) 10

10. Consider the given statements:
I. Uses priority queue to store unvisited vertices by distance from source.
II. It uses greedy method, means pick the next closest vertex to the source.
III. Does not work with negative weights.
The above statements describe
(A) Bellman Ford Algorithm
(B) Dijkstra's Algorithm
(C) Breadth First search Algorithm
(D) Kruskals Algorithm

11. A complete bipartite graph $k_{m,n}$ is a bipartite graph that has each vertex from one set is adjacent to each vertex to another set, what is the minimum 'vertex cover' for $k_{m,n}$ graphs?
(A) MAX(m, n) (B) MIN(m, n)
(C) m (D) n

12. What is the number of Regions present in the bipartite graph $K_{5,2}$?
(A) 4 (B) 5
(C) 6 (D) 7

13. Consider the following:

	Best Case	Average Case	Worst Case
I.	$O(n)$	$O(n^2)$	$O(n^2)$
II.	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$

Which of the following is TRUE?
(A) Merge sort-II, Selection sort-II
(B) Merge sort-II, Heap sort-II
(C) Heap sort-I, Insertion sort-I
(D) Bubble sort-I, Insertion sort-II

14. Which of the following Algorithm uses Divide-and-Conquer strategy?
(A) Merge sort
(B) Quick sort
(C) Binary search and strassens multiplication
(D) All the above

15. For merging two sorted Lists of sizes 'K' and 'L' into a sorted list of size $K + L$, what is the number of comparisons required?
(A) $O(K)$ (B) $O(L)$
(C) $O(K*L)$ (D) $O(K + L)$

16. Consider the following:
$f(n) = n \log n$
$g(n) = \log (n!)$
$h(n) = 2^{\log_2^n}$
Which of the following is TRUE according to Rate of Growth?
(A) $g(n) \in \Omega\, h(n)$ and $g(n) \in O(f(n))$
(B) $g(n) \in O(f(n))$ and $f(n) \in \Omega(g(n))$
(C) $g(n) \in \Omega(f(n))$ and $f(n) \in \Omega(g(n))$
(D) $h(n) \in O(g(n))$ and $g(n) \in O(f(n))$

17. Consider the given code:

```
y = y + z;
for (i = 1; i < = n; i++)
k = k + 2;
for (i = 1; i < = n; i++)
{
for(j = 1; j < = n; j++)
x = x + 1;
}
```

What is the time complexity of the given code?
(A) $O(n^3)$ (B) $O(n^2)$
(C) $O(n \log n)$ (D) $O(n^2 \log n)$

18. Consider the given Recurrence Relation

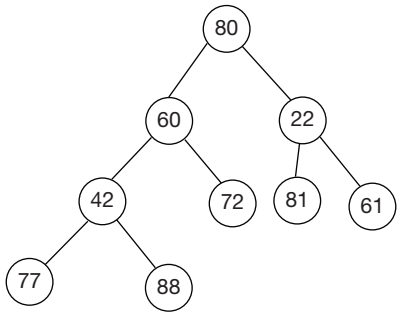
$$T(n) = 3T\left(\frac{n}{9}\right) + n^{0.52}.$$

What is the Time complexity?
(A) $O(n)$ (B) $O(n^2)$
(C) $O(n^{0.52})$ (D) $O(\sqrt{n})$

19. Consider an empty stack of integers. The numbers 6, 7, 1, 4, 3, 2, 8, 9 are pushed on to the empty stack in the above given order from Right to Left. Let Z denote a PUSH operation and 'W' denote a *POP* operation.
Which of the following is the sequence of integers Popped out after performing ZZZZWWZZWWZW?
(A) 3, 2, 4, 1, 7 (B) 3, 2, 1, 4, 9
(C) 3, 2, 1, 4, 8 (D) 3, 2, 1, 4, 7

20. The following sequence of operations is performed on a stack, PUSH(70), PUSH(50), POP, PUSH(30), PUSH(50), PUSH(70), POP, POP, PUSH(70), POP, POP, what is the sequence of values popped out?
(A) 50, 70, 50, 70, 70
(B) 50, 50, 70, 30, 70
(C) 50, 70, 50, 70, 30
(D) 50, 70, 50, 30, 70

21. Assume an algorithm for printing the level order data in Reverse order, for the Binary tree shown below:



(A) 80, 60, 22, 42, 72, 81, 61, 77, 88
(B) 80, 22, 60, 61, 81, 72, 42, 88, 77
(C) 77, 88, 42, 72, 81, 61, 60, 22, 80
(D) 88, 77, 61, 81, 72, 42, 22, 60, 80

22. Consider the given code:

```
int fun(struct BinaryTreeNode *root1,
struct Binary Tree Node *root2)
{
if(root1 = = NULL & & root2 = = NULL)
return 1;
if(root1 = = NULL || root2 = = NULL)
return 0;
return (root1 → data = = root2 →
   data &&
fun(root1 → left, root2 → left) &&
fun(root1 → right, root2 → right));
}
```

The above code describes, which of the following task?
(A) Finding the number of nodes with only one child
(B) Finding the number of nodes in a Tree.
(C) Finding, whether two binary trees are structurally identical or not
(D) Finding the number of Leaf nodes in a tree.

23. We are given a set of '6' distinct elements and an unlabelled binary tree with '6' nodes. In how many ways can we populate the tree with the given set, so that it becomes a binary search tree?
(A) 58 (B) 720
(C) 360 (D) 132

24. A binary tree is a tree data structure in which each node has atmost 2 children, that is, the degree of each node can be atmost '2'. If a binary tree has 'n' leaf nodes, then the number of nodes of degree 2 is?

(A) $n-1$ (B) $2n-1$

(C) $n+1$ (D) $\frac{n}{2}$

25. For a 5-ary tree (each node can contain maximum of 5 children), what is the maximum possible height with 50 nodes, Assume that height of a single node is '0'?

(A) 25 (B) 30

(C) 49 (D) 50

26. For a 7-ary tree (each node can contain maximum of 7 children), what is the maximum possible height with 60 nodes, If we have a restriction that atleast one node should have 7 children?

(A) 14 (B) 21

(C) 49 (D) 53

27. For a K-ary tree (each node can contain maximum of K children), what is the maximum possible nodes at height 'h'?

(A) Kh^{+1} (B) K^h

(C) Kh^{-1} (D) $2K^h$

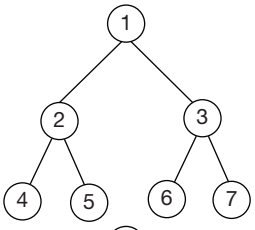
28. Consider any completer Binary tree, what are the minimum and maximum number of elements interms of height 'h'?

(A) 2^{h+1} and 2^{h+2} (B) 2^h and $2^{h+1}-1$

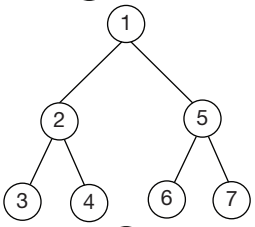
(C) $2^{h+1}-1$ and $2^{h+1}+1$ (D) 2^h and 2^{h+1}

29. Which of the following MIN-HEAP displays the elements in sequence (ascending order) if preorder traversal is applied on MIN-HEAP?

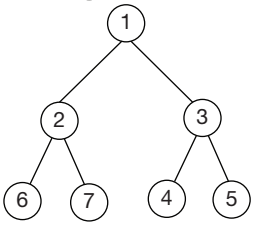
(A)



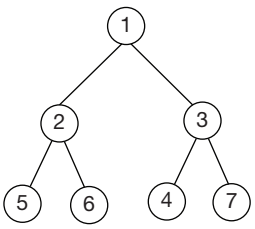
(B)



(C)

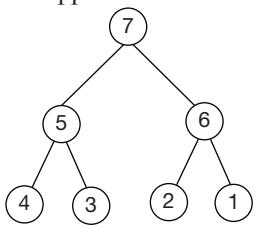


(D)

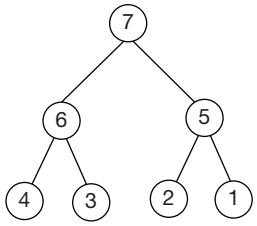


30. Which of the following MAX-HEAP displays the elements in sequence (descending order) if preorder traversal is applied on MAX-HEAP?

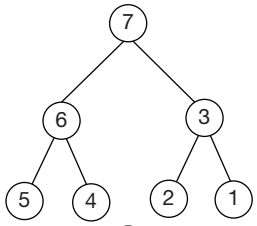
(A)



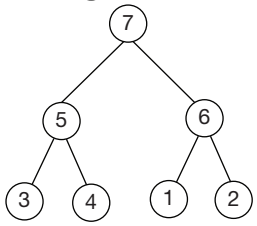
(B)



(C)



(D)



31. What is the minimum and maximum number of nodes that exist in MIN-HEAP/MAX-HEAP interms of height 'h'?

(A) 2^{h-1} and 2^{h+1} (B) 2^{h-1} and 2^h

(C) 2^h and $2^{h+1}-1$ (D) $2^{h+1}-1$ and 2^{h+1}

32. Which of the following cannot be the partition of array elements by using Quick sort Algorithm?

(A) $T(n)=T\left(\frac{6n}{10}\right)+T\left(\frac{4n}{10}\right)+\theta(n)$

(B) $T(n)=T\left(\frac{4n}{5}\right)+T\left(\frac{n}{5}\right)+\theta(n)$

(C) $T(n)=T(n-1)+T(1)+\theta(n)$

(D) $T(n)=T\left(\frac{4n}{5}\right)+T\left(\frac{4n}{5}\right)+\theta(n)$

33. A stable sorting is defined as, Assume that A is an array to be sorted, X and Y are having the same key and X appears earlier in the array than Y. That means X is at $A[i]$ and Y is at $A[j]$, where $i < j$, an algorithm is said to be stable if in the output X precedes Y. Which of the following is not stable sorting?

(A) Bubble sort (B) Insertion sort

(C) Merge sort (D) Quick sort

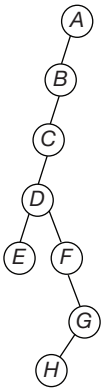
Common Data for Questions 34 and 35:
Consider the given Inorder, preorder, postorder traversals of a tree, but it is not known which is what order

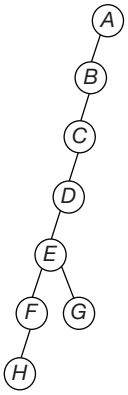
I. *ABCDEGFH*
II. *EDGHFCBA*
III. *EHFGDCBA*

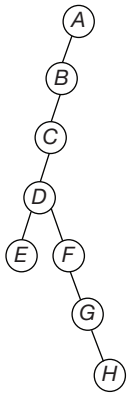
34. Which of the following is TRUE?

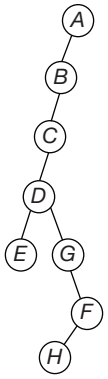
(A) I is pre order and II is post order
(B) I is pre order and III is In order
(C) I is pre order and II is In order
(D) II is post order and III is In order

35. Which of the following is the correct tree for the traversals given in the above question?

(A)


(B)


(C)


(D)


Answer Keys

1. B	**2.** A	**3.** C	**4.** D	**5.** B	**6.** C	**7.** D	**8.** B	**9.** C	**10.** B
11. B	**12.** B	**13.** B	**14.** D	**15.** D	**16.** D	**17.** B	**18.** C	**19.** D	**20.** C
21. C	**22.** C	**23.** D	**24.** A	**25.** C	**26.** D	**27.** B	**28.** B	**29.** B	**30.** C
31. C	**32.** D	**33.** D	**34.** C	**35.** D					

Hints and Explanations

1. Transitivity:
$f(n) = \theta(g(n))$ and $g(n) = \theta(h(n))$
$\Rightarrow$ $f(n) = \theta(h(n))$ valid for Ω and O notations also.
Symmetry:
$f(n) = \theta(g(n))$ if and only if $g(n) = \theta(f(n))$
Transpose symmetry:
$f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.
Choice (B)

2. $T(n) = aT(n/b) + f(n)$
'a' has to be constant. Choice (A)

3. Time complexity of EnQueue( ) = $O(1)$
Time complexity of DeQueue( ) = $O(1)$
Time complexity of IsEmptyQueue( ) = $O(1)$
Time complexity of DeleteQueue( ) = $O(1)$.
Choice (C)

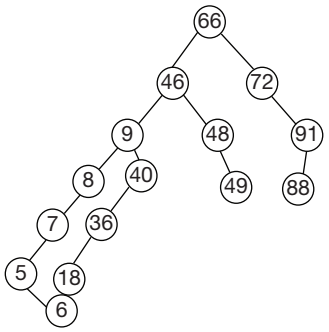
4. All the given statements I, II, III and IV are Applications of Binary Trees. Choice (D)

5. Depth First Traversal:
1. Pre order Traversal
2. In order Traversal
3. Post order Traversal

Breadth First Traversal:
1. Level order Traversal
The traversal defined in the given Questions is Level order traversal. Choice (B)

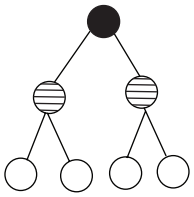
6. With 'n' nodes, it will have the maximum combinations of different trees.
$\Rightarrow$ $2^8 - 8 = 256 - 8 = 248$. Choice (C)

7. Root is '66', the numbers less than 66 will appear in Left sub tree and greater numbers appear in Right sub tree.
$\therefore$ (11, 3)

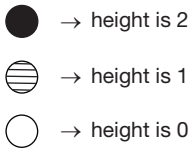


Choice (D)

8. Let us consider the following Full binary tree
$n = 7$ nodes



$n - (h + 1)$
$\Rightarrow \quad 7 - (2 + 1) = 7 - 3 = 4$
$h = 2$ (height)



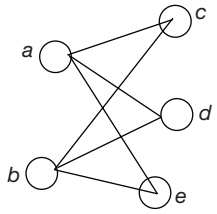
$2 + 1 + 1 + 0 + 0 + 0 + 0 = 4.$ Choice (B)

9. $\bar{G}(e) = \dfrac{V(V-1)}{2} - G(e)\, V^2 - V - 72 = 0$

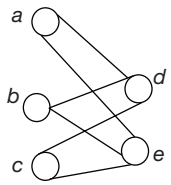
$\Rightarrow \quad V(V-9) + 8(V-9) = 0$
$\Rightarrow \quad V = 9.$ Choice (C)

10. The given statements describe Dijkstra's Algorithm.
Choice (B)

11. Let us take
$k_{2,3}$



Minimum vertex cover = $\{a, b\}$
$K_{3,2}$



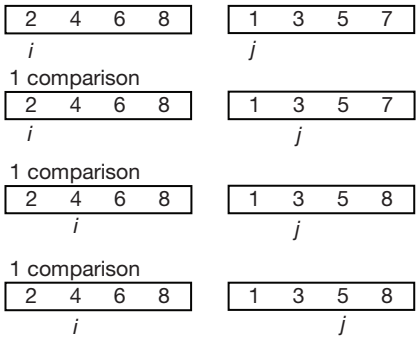
Minimum vertex cover = $\{d, e\}$
$\therefore \quad$ MIN(m, n). Choice (B)

12. Eulers formula: $|V| + |R| - |E| = 2$
$k_{5,2}$ is planar graph, As we know,
The number of vertices in $k_{m,n}$ graph is $m + n$
$\therefore \quad k_{5,2}$ has 7 vertices
The number of edges in $k_{m,n}$ graph is $m*n$
$\therefore \quad k_{5,2}$ has 10 edges
$|V| + |R| - |E| = 2$
$7 + R - 10 = 2$
$R - 3 = 2$
$R = 5$ Choice (B)

13. Merge sort - II
Heap sort - II
Insertion sort - I, Bubble sort - I, selection sort - I.
Choice (B)

14. All the algorithms follow Divide-and-conquer approach. Choice (D)

15. In the worst case, if there are 4 elements in each list, consider the following lists (if $(i < j)$
print 'i'
else
print 'j')



$\therefore \quad$ Almost $(K + L)$ comparisons are required.
Choice (D)

16. $n \log n \geq \log (n!) \geq 2^{\log_2^n}$
option (D) is true.
$2^{\log_2^n} \leq c * \log (n!)$
$\log (n!) \leq c * n \log n.$ Choice (D)

17. $y = y + 2$; // constant time
for $(i = 1; i <= n; i++)$ //executed '$n + 1$' times
$k = k + 2$; // executes n times
for $(i = 1; i \leq n; i++)$//outer loop executed '$n + 1$' times
for $(j = 1; j <= n; j++)$//inner loop executed '$n(n + 1)$' times
$x = x + 1$; // executes n times
Total time $= C_0 + C_1 n + C_2 n^2 = O(n^2)$. Choice (B)

18. $T(n) = 3T\left(\dfrac{n}{9}\right) + n^{0.52}$

$T(n) = aT\left(\dfrac{n}{b}\right) + f(n)$

$a = 3, b = 9, f(n) = n^{0.52}$
compare
$n^{\log_b^a}$ Vs $f(n)$
$n^{\log_9^3}$ Vs $n^{0.52}$
$n^{\log_9^{\sqrt{9}}}$ Vs $n^{0.52}$ $f(n)$ is greater than $n^{\log_b^a}$
case 3 of master theorem, so the time complexity is
$T(n) = \theta(f(n))$
$\Rightarrow \quad \theta(n^{0.52})$. Choice (C)

19. Given Integers
6, 7, 1, 4, 3, 2, 8, 9
From Right to Left:
9, 8, 2, 3, 4, 1, 7, 6
Z Z Z Z(4 push operations)

3
2
8
9

2 pop operations
⇒ 3, 2
Again 2 push operations

1
4
8
9

2 pop operations
⇒ 1, 4
One push operation

7
8
9

One pop operation
⇒ 7
∴ popped integers are 3, 2, 1, 4, 7. Choice (D)

20.

50
70

⇒ pop '50'

70
50
30
70

⇒ pop 70, 50

70
30
70

⇒ pop 70, 30
∴ popped sequence = 50, 70, 50, 70, 30.
Choice (C)

21. Level order data in reverse means,
Bottom level data elements from Left to Right should be taken first.
Level order Traversal is, 77, 88, 42, 72, 81, 61, 60, 22, 80. Choice (C)

22. Given two binary trees,
Returns true if they are structurally identical.
∴ If both trees are NULL then return true.
∴ If both trees are not NULL, then compare data and recursively check left and right subtree structures.
Choice (C)

23. With '6' distinct elements and 6 unlabelled binary tree nodes, we can have,

$$n = 6,\ \frac{1}{n+1}\times 2n_{c_n}$$

$$\frac{1}{6+1}\times\frac{(2\times 6)!}{(12-6)!\times 6!}\frac{1}{7}\times\frac{12\times 11\times 10\times 9\times 8\times 7}{6\times 5\times 4\times 3\times 2}$$

$= 11 \times 2 \times 3 \times 2 = 132.$ Choice (D)

24. A binary tree with 'n' leaves have $(n-1)$ internal nodes. So $(n-1)$ nodes will have degree '2'. Choice (A)

25. In 5-ary tree each node can contain 0 to 5 children and to get maximum height, we have to keep only one child for each parent, with 50 nodes the maximum possible height we can get is 49. Choice (C)

26. If we have a restriction that atleast one node should have 7 children, then we keep one node with 7 children and remaining all nodes with 1 child. In this case, with 'n' nodes the maximum possible height is $(n-7)$.
∴ $60 - 7 = 53$. Choice (D)

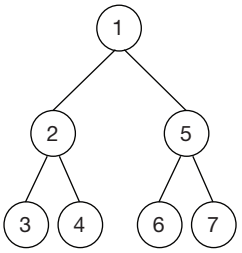
27. If we want to get minimum height, then we need to fill all nodes with maximum children.
Lets take a 4-ary tree.

Height (h)	Maximum Nodes at height, $h = 4^h$
0	1
1	4
2	4×4
3	$4 \times 4 \times 4$

Choice (B)

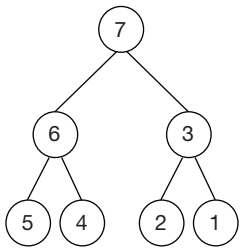
28. In a complete binary tree, all levels contain full nodes except possibly the lowest level.
Maximum = $2^{h+1} - 1$ elements
Minimum = 2^h. Choice (B)

29. Pre order traversal on the following Min-Heap produces elements in ascending order



∴ 1 2 3 4 5 6 7. Choice (B)

30. Pre order Traversal on the following MAX-HEAP produces elements in sequence (descending order)



$\therefore$ Pre order = 7 6 5 4 3 2 1. Choice (C)

31. A Heap is a complete binary tree. All the levels, except the lowest, are completely full. So the heap has atleast 2^h elements and atmost $2^{h+1} - 1$ elements. Choice (C)

32. The given array is partitioned into two non-empty sub arrays
In Option (D) 80 percent of elements are in First sub array that means in the second part of array there should be 20 percent elements that is $T\left(\frac{n}{5}\right)$ instead of $T\left(\frac{4n}{5}\right)$. Choice (D)

33. **Quick sort:**
The partitioning step can swap the location of records many times, and thus 2 elements with equal value could swap position in the final output.
Choice (D)

34. Pre order = *A B C D E G F H*
In order = *E D G H F C B A*
Post order = *E H F G D C B A*.
Choice (C)

35. The tree given in option (D) is correct.
Choice (D)