Ray Optics and Optical Instruments



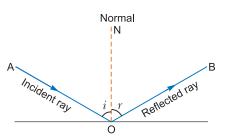
1. Optics The study of nature and propagation of light is called optics. Ray optics deals with particle nature of light whereas wave optics considers light as a wave.

2. Reflection of Light

When a light ray incident on a smooth surface bounces back to the same medium, it is called reflection of light.

Laws of regular Reflection

- (*i*) Angle of incidence is equal to the angle of reflection. i.e., i = r
- (*ii*) The incident ray, the reflected ray and the normal at the point of incidence, all lie in the same plane. These laws hold for any reflecting surface whether plane or curved. There is no change in wavelength and frequency



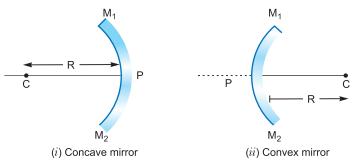
Spherical Mirror: A spherical mirror is simply a part cut off from the surface of a hollow sphere which has been made smooth and silver polished on one side.

Spherical mirrors are of two types:

during reflection.

(i) Concave mirror: If outer side or bulging side of the spherical surface is silver polished, it is called a concave mirror.

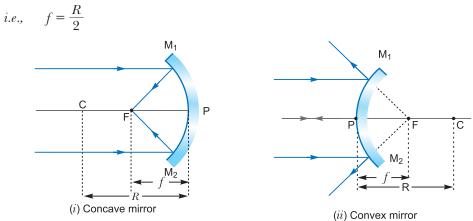
(ii) Convex mirror: If inner side of a spherical surface is silver polished, it is called a convex mirror. Relation between focal length and radius of curvature: The distance between centre (C) of spherical surface and its pole (P) is called the **radius of curvature**. It is denoted by R.



The rays parallel to the principal axis (CP) after striking the mirror meet at a point (F) (in concave mirror) or appear to be meeting at a point F (in convex mirror). This point is called the principal focus (F) of mirror. The distance of focus (F) from pole (P) of a mirror is called the focal length of



the mirror. It is denoted by f. The focal length f is half of the radius of curvature.



Mirror formula: The mirror formula is

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

where

u = distance of object from mirror; v = distance of image from mirror;

and

f =focal length of mirror.

Magnification produced by mirror: The ratio of the size of image to the size of object is called linear magnification produced by the mirror.

Magnification
$$M = \frac{h'}{h} = -\frac{v}{u} = -\frac{f}{u-f} = \frac{f-v}{f}$$

Where h' is the height of image and h is the height of object.

3. Refraction of Light

When a ray of light enters from one transparent medium into another, there is a change in speed and direction of the ray in the second medium. This phenomenon is called refraction of light. **Laws of refraction:**

- (*i*) The incident ray, the refracted ray and the normal to the surface separating the two media, all lie in the same plane.
- (*ii*) **Snell's Law:** For two media, the ratio of sine of angle of incidence to the sine of the angle of refraction is constant for a beam of particular wavelength, *i.e.*,

$$\frac{\sin i}{\sin r} = \text{constant} = \frac{n_2}{n_1} = {}_1n_2 \qquad \dots(i)$$

where n_1 and n_2 are absolute refractive indices of I and II media respectively and $_1n_2$ is a refractive index of second (II) medium with respect to first (I) medium.

Due to principle of reversibility of light,

$$\frac{\sin r}{\sin i} = {}_2n_1 \qquad \dots (ii)$$

Multiplying (i) by (ii), we get

$$1 = {}_{2}n_{1} \times {}_{1}n_{2}$$
 or ${}_{2}n_{1} = \frac{1}{{}_{1}n_{2}}$...(*iii*)

The frequency of light remains unchanged while passing from one medium to the other. **Refractive Index:**

The refractive index of a medium is defined as the ratio of speed of light in vacuum to the speed of light in a medium.



i.e.,

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$
$$= \frac{v\lambda_{air}}{v\lambda_{medium}} = \frac{\lambda_{air}}{\lambda_{medium}} \qquad \dots (iv)$$

 λ_{air} and λ_{medium} being wavelengths of light in air and medium respectively.

$$\therefore \qquad \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \left(= \frac{c/v_2}{c/v_1} \right) = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \qquad \dots (v)$$

Formation of image due to refraction: According to Snell's law, if $n_2 > n_1$, i > r. That is, if a ray of light enters from rarer medium to a denser medium, it is deviated towards the normal and if $n_2 < n_1$, i < r that is, if the ray of light enters from denser to a rarer medium it is deviated away from the normal.

Accordingly, if the ray of light starting from object O, in the given diagram in a denser medium travels along OP, it is deviated away from the normal along PQ. The ray PQ appears to come from I. Thus I is the virtual image of O. It can be shown that

$$n = \frac{\text{Real depth}(OM)}{\text{Apparent depth}(MI)} = \frac{t}{t-x} \qquad \dots (vi)$$

where x is the apparent shift.

:. The apparent shift, $x = (1 - \frac{1}{n})t$ (*vii*)

Refraction through a number of media: Let us consider the refraction of light ray through a series of media as shown in fig. The ray AB is incident on air-water interface at an angle *i*. The ray is deviated in water along *BC* towards the normal. Then it falls on water-glass interface and is again deviated towards normal along *CD*. If the last medium is again air, the ray emerges parallel to the incident ray. Let r_1 and r_2 be angles of refraction in water and glass respectively, then from Snell's law,

$$\frac{\sin i}{\sin r_1} = \frac{n_w}{n_a} = {}_a n_w \qquad \dots (viii)$$
$$\frac{\sin r_1}{\sin r} = \frac{n_g}{n} = {}_w n_g \qquad \dots (ix)$$

$$\frac{\sin r_2}{\sin i} = \frac{n_a}{n_g} = {}_g n_a \qquad \dots (x)$$

 $\begin{bmatrix} n_a = \text{refractive index of air} = 1 \\ n_w = \text{refractive index of water} \\ n_g = \text{refractive index of glass} \end{bmatrix}$

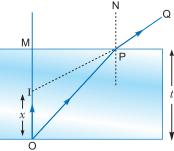
Multiplying (*viii*), (*ix*) and (*x*), we get $_{a}n_{w} \times _{w}n_{g} \times _{g}n_{a} = 1$

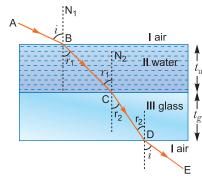
$${}_w n_g = \frac{1}{{}_a n_w \times {}_g n_a} = \frac{{}_a {}^n {}_g}{{}_a n_w} \qquad \dots (xi)$$

4. Critical Angle

When a ray of light is incident on the interface from denser medium to rarer medium, it is deviated away from the normal. When angle of incidence is increased, angle of refraction also increases and at a stage it becomes 90°.

The angle of incidence in denser medium for which the angle of refraction in rarer medium is 90° is called the **critical angle (C)** for the pair of media.





If n_r and n_d are refractive indices for rarer and denser media, then

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \text{ gives}$$
$$\frac{\sin C}{\sin 90^\circ} = \frac{n_r}{n_d} = {}_d n_r$$
$$\sin C = {}_d n_r = \frac{1}{{}_r n_d} = \frac{1}{n}$$

where $n_{d} = n$ and *n* is the refractive index of a denser medium with respect to a rarer medium.

5. Total Internal Reflection

When angle of incidence in the denser medium is greater than the critical angle, the incident ray does not refract into a rarer medium but is reflected back into the denser medium. This phenomenon is called total *internal reflection*. **The conditions for total internal reflection are**

- (i) The ray must travel from a denser into a rarer medium.
- (*ii*) The angle of incidence i > critical angle C.

The critical angle for water-air, glass-air and diamond-air interfaces are 49°, 42° and 24° respectively.

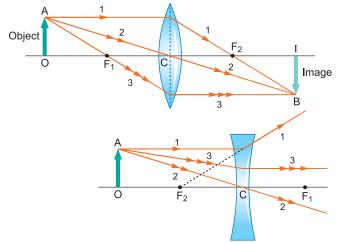
6. Spherical Lenses

There are two types of spherical lenses.

- (*i*) Convex lens (Converging lens)
- (*ii*) Concave lens (Diverging lens)

Rules of Image Formation in Lenses

- (*i*) The ray incident on lens parallel to the principal axis, after refraction through the lens, passes through the second focus (in convex lens) or appear to come from second focus (in concave lens).
- (ii) The ray incident on lens through optical centre C, after refraction, pass straight without any deviation.
- (*iii*) A ray directed towards the first focus incident on the lens, after refraction becomes parallel to the principal axis.



7. Thin Lens Formula

If u and v are object and image distances from a lens of focal length f, then thin lens formula is

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This equation holds for convex and concave lenses both, but proper signs of u, v and f are to be used according to sign convention of coordinate geometry. Focal length of a convex lens is taken as positive and of a concave lens is taken as negative.



Magnification produced by a lens

$$m = \frac{h'}{h} = \frac{v}{u} = \frac{f}{u+f}$$

where h' is the size of image and h is the size of object.

8. Lens Maker's Formula

If R_1 and R_2 are the radii of curvature of first and second refracting surfaces of a thin lens of focal length *f*, then lens makers formula is

$$\frac{1}{f} = ({}_1n_2 - 1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= (n - 1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where $_1n_2=n$ is refractive index of material of lens with respect to surrounding medium.

9. Power of a Lens

The power of a lens is its ability to deviate the rays towards its principal axis. It is defined as the reciprocal of focal length in metres.

Power of a lens,
$$P = \frac{1}{f(\text{in metre})}$$

Its unit is diopter and is represented as 'D'.

10. Lens Immersed in a Liquid

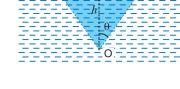
If a lens of refractive index n_g is immersed in a liquid of refractive index n_l then its focal length (f_l) in liquid, is given by

$$\frac{1}{f_l} = ({_l}n_g - 1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

 $_{l}n_{g} = \frac{n_{g}}{n_{l}}$

where

where $l^{\prime\prime}g = n_l$ If f_a is the focal length of lens in air, then $f_l = \frac{n_g - 1}{\frac{n_g}{n_l} - 1} \times f_a$



 n_1

n

0

 \cap

 n_1

 n_1

P₂

 n_2

Three cases arise:

- (*i*) If $n_g > n_l$, then f_l and f_a are of same sign but $f_l > f_a$. That is, the nature of lens remains unchanged, but its focal length increases and hence the power of lens decreases. In other words the convergent lens becomes less convergent and divergent lens becomes less divergent.
- (*ii*) If $n_g = n_l$, then $f_l = \infty$. That is, the lens behaves as a glass plate.
- (*iii*) If $n_g < n_l$, then f_l and f_a have opposite signs.

That is, the nature of lens changes. A convergent lens becomes divergent and vice versa.

11. Thin Lenses in Contact

If two or more lenses of focal lengths f_1, f_2 are placed in contact, then their equivalent focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \dots$$

The power of combination

$$P = P_1 + P_2 + \dots$$



12. Combination of a Lens and a Mirror

Consider a coaxial arrangement of a lens and a mirror. Let an object be placed in front of the lens. The incident rays, from the object, first undergo refraction at lens and are then incident on the mirror. To obtain the position of the image due to the combination, we can proceed as follows:

- (*i*) Using refraction formula, we can calculate where the image would have been formed, had there been only the lens. We then consider this image as an object for the mirror.
- (*ii*) Using the mirror formula, we can then locate the position of its final image formed by the mirror. This final position, would be the position of the image due to the combined effect of refraction at the lens and reflection at the mirror.

13. Refraction Through a Prism

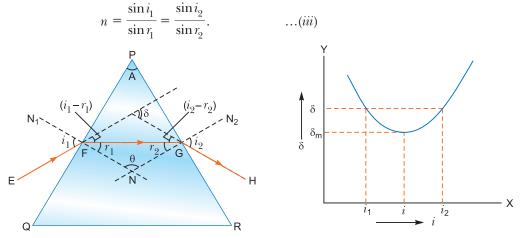
A prism is a transparent medium enclosed by two plane refracting surfaces. Let EF be the monochromatic ray incident on the face PQ of prism PQR of refracting angle A at angle of incidence i_1 . This ray is refracted along FG, r_1 being angle of refraction. The ray FG is incident on the face PR at angle of incidence r_2 and is refracted in air along GH. Thus GH is the emergent ray and i_2 is the angle of emergence. The angle between incident ray EF and emergent ray GH is called *angle of deviation* δ .

For a prism if A is the refracting angle of prism, then

and
$$r_1 + r_2 = A$$
 ...(*i*)
 $i_1 + i_2 = A + \delta$...(*ii*)

Clearly, deviation $\delta = i_1 + i_2 - A$, i_1 and i_2 may be inter-changed, therefore, there are two values of angles of incidence for same deviation δ .

If n is the refractive index of material of prism, then from Snell's law



If angle of incidence is changed, the angle of deviation δ changes as shown in fig. For a particular angle of incidence the deviation is minimum. This is called *angle of minimum deviation* δ_m .

Minimum deviation: At minimum deviation the refracted ray within a prism is parallel to the base. Therefore,

$$\begin{split} i_1 &= i_2 = i \text{ (say)} \\ r_1 &= r_2 = r \text{ (say)} \end{split}$$

Then from equations (i) and (ii),

$$r + r = A \text{ or } r = A/2 \qquad \dots (iv)$$

$$i + i = A + \delta_m \text{ or } i = \frac{A + \delta_m}{2} \qquad \dots (v)$$

 \therefore The refractive index of material of prism

$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)} \qquad \dots (vi)$$



For a thin prism, *viz*. $A \le 10^{\circ}$

$$\delta_m = (n-1) A.$$

14. Scattering of Light

The light is scattered by air molecules. According to Lord Rayleigh the intensity of scattered light

$$I \propto \frac{1}{(\text{wavelength})^4} \Rightarrow I \propto \frac{1}{\lambda^4}$$

As $\lambda_{blue} < \lambda_{red}$, accordingly blue colour is scattered the most and red the least, so sky appears **blue**. At the time of sunrise and sunset, blue colour is scattered the most and red colour enters our eyes, so sunrise and sunset appear red.

15. Optical Instruments (Microscopes and Telescopes)

A microscope is an optical instrument to see very small objects.

- (i) Simple Microscope: It consists of a convex lens of small focal length f.
 - If β = angle subtended by an image on eye

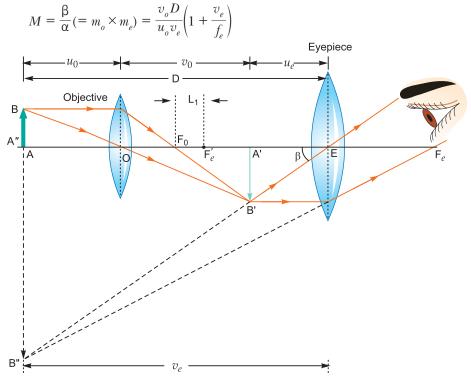
 α = angle subtended by an object on eye, when object is at a distance of distinct vision (D) Magnifying power,

$$M = \frac{\beta}{\alpha} = \frac{D}{v} \left(1 + \frac{v}{f} \right)$$

If the final image is at ∞ , $v = \infty$ then $M = \frac{D}{f}$.

If the final image is at a distance of distinct vision, $v = D, M = 1 + \frac{D}{f}$.

- (ii) Compound Microscope: A compound microscope essentially consists of two co-axial convex lenses of small focal lengths. The lens facing the object is called an objective lens while that towards eye is called the eye lens (eyepiece).
 - : Magnifying power of microscope,



Separation between lenses, $L = v_0 + u_e$

Special cases:

(a) When final image is formed at a distance of distinct vision, $v_e = D$

$$M = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e}\right) \text{ and } L = v_0 + u_o$$

The distance between second focal point of objective and first focal point of eye lens is called the tube length denoted by *L*,then

$$\frac{v_o}{u_o} = \frac{L}{f_0}$$

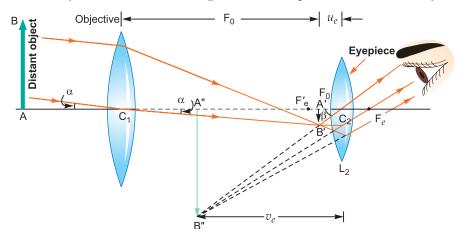
So,
$$M = -\frac{L}{f_0} \left(1 + \frac{D}{f_e} \right)$$

(b) When final image is formed at infinity, $v_e = \infty$, then

$$M = -\frac{v_o}{u_o} \times \frac{D}{f_e}$$
$$= -\frac{L}{f_o} \cdot \frac{D}{f_e} \text{ and } L = v_o + f_e$$

Telescope: It is an optical instrument to see distant objects.

(*iii*) Astronomical Telescope (Refracting Telescope): It is used to see magnified images of distant objects. An astronomical telescope essentially consists of two co-axial convex lenses. The lens facing the object has a large focal length and a large aperture and is called objective, while the lens towards eye has a small focal length and small aperture and is called eye lens.



The magnifying power of telescope is

$$M = \frac{\text{Angle subtended by final image at eye}}{\text{Angle subtended by object on eye}} = \frac{\beta}{\alpha}$$

$$= (m_0 \times m_e) = -\frac{f_0}{f_e} \left(1 + \frac{f_e}{v_e}\right)$$

and Length of telescope $L=f_0+u_e$

where u_{e} = distance of real image from eye lens

 $v_{\rm e}$ = distance of final image A' B' from eye lens

 f_0 = focal length of objective, f_e = focal length of eye lens



 α = angle subtended by an object at eye = $\frac{h}{f_0}$ β = angle subtended by an image at eye = $\frac{h}{f_e}$

Special cases:

(a) When final image is formed at a distance of distinct vision, then $v_e = D$

$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right) \text{and } L = f_o + u_e$$

(b) When final image is formed at infinity, then $v_e = \infty$

$$M = -\frac{f_o}{f_e}$$
 and $L = f_o + f_e$

Reflecting Telescope: In this telescope, a concave mirror is used as an objective in place of a convex lens. It is free from chromatic aberration and it has larger resolving power than refracting telescope.

16. Magnifying Power of Optical Instruments

The size of an object depends on the angle subtended by the object on eye. This angle is called visual angle. Greater the visual angle, greater the size of object. Stars are bigger than sun; but appear smaller because stars are much farther away than sun and they subtend smaller angles on eye.

The angle subtended on eye may be increased by using telescopes and microscopes. The telescopes and microscopes form the image of an object. The image subtends larger angle on eye; hence the object appears big. The magnification produced by optical instrument (telescope/microscope) is defined as the ratio of angle (β) subtended by image on eye and the angle (α) subtended by object on eye.

i.e., Angular magnification $M = \frac{\beta}{\alpha}$

Selected NCERT Textbook Questions

Reflection, Refraction and Total Internal Reflection

Q. 1. A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how should the screen be moved?

Ans. Given u = -27 cm, h = 2.5 cm

$$|R| = |2f| = 36 \text{ cm}$$

$$\Rightarrow f = -\frac{36}{2} = -18 \text{ cm (with sign convention)}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{18} + \frac{1}{27} = \frac{-3+2}{54} \Rightarrow v = -54 \text{ cm}$$

That is, image is formed in front of mirror at a distance 54 cm from the mirror. Therefore the screen must be placed at a distance 54 cm from the mirror.

Size of the image
$$h' = -\frac{v}{u} \times h = -\frac{(-54)}{-27} \times 2.5 \text{ cm.} = -5 \text{ cm}$$

The image is real, inverted and 5 cm long. If the candle is moved closer, the screen should have to be moved farther and farther. If the candle is brought less than 18 cm, the image will be virtual and cannot be collected on the screen.

Q. 2. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens if the needle is moved farther from the mirror.

Ans. Given u = -12 cm, f = +15 cm (convex mirror)

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \implies \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$
$$\frac{1}{v} = \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} \implies v = \frac{60}{9} = \frac{20}{3} = 6.67 \text{ cm}$$

That is image is formed at a distance 6.67 cm behind the mirror.

Magnification
$$m = -\frac{v}{u} = -\frac{20}{-3 \times 12} = \frac{5}{9}$$

Size of image $h' = mh = \frac{5}{9} \times 4.5 = 2.5 \text{ cm}$

The image is erect, virtual and has a size 2.5 cm.

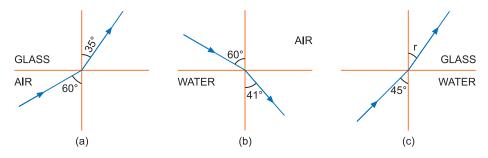
Its position is 6.67 cm behind the mirror when needle is moved farther, the image moves towards the focus and its size goes on decreasing.

Q. 3. A tank is filled with water to a height of 12.5 m. The apparent depth of the needle lying at the bottom of the tank as measured by a microscope is 9.4 cm. What is the refractive index of water ? If water is replaced by a liquid of refractive index 1.63 upto the same height, by what distance would the microscope be moved to focus on the needle again?

Ans. Refractive index,
$$n = \frac{\text{Real depth}(H)}{\text{Apparent depth}(h)}$$

Given H = 12.5 cm, h = 9.4 cm

- ∴ Refractive index of water, $n_w = \frac{12.5}{9.4} = 1.33$ Refractive index of liquid, $n_l = 1.63$
- \therefore Apparent height with liquid in tank, $h = \frac{H}{n_l} = \frac{12.5}{1.63} = 7.7 \text{ cm}$
- :. Displacement of microscope, x = 9.4 7.7 = 1.7 cm
- Q. 4. Fig. (a) and (b) show refraction of an incident ray in air at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle (r) of refraction of an incident ray in water at 45° with the normal to a water-glass interface [fig. (c)].



Ans. Snell's law of refraction is $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = {}_1n_2$

Fig. (a)
$$\frac{\sin 60^{\circ}}{\sin 35^{\circ}} = \frac{n_g}{n_a} = {}_a n_g$$

 \Rightarrow Refractive index of glass with respect to air, ${}_a n_g = \frac{\sin 60^{\circ}}{\sin 35^{\circ}} = \frac{0.8660}{0.5736} = 1.51$

Fig. (b)
$$\frac{\sin 60^{\circ}}{\sin 41^{\circ}} = \frac{n_w}{n_a} = {}_a n_u$$



Refractive index of water with respect to air, $_a n_w = \frac{\sin 60^\circ}{\sin 41^\circ} = \frac{0.8660}{0.6561} = 1.32$

Fig.(c)
$$\frac{\sin 45^{\circ}}{\sin r} = \frac{{}^{a}n_{g}}{{}^{a}n_{w}}$$

$$\Rightarrow \quad \sin r = \frac{{}^{a}n_{w}}{{}^{a}n_{g}} \times \sin 45^{\circ} = \frac{1.32}{1.51} \times 0.7071 = 0.6181$$

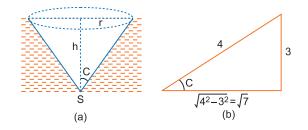
$$\Rightarrow \quad r = \sin^{-1} (0.6181) = \mathbf{38^{\circ}}$$

- Q. 5. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is $\frac{4}{3}$.
- **Ans.** The light rays starting from bulb can pass through the surface if angle of incidence at surface is less than or equal to critical angle (*C*) for water-air interface. If *h* is depth of bulb from the surface, the light will emerge only through a circle of radius *r* given by $r = h \tan C$, where h = 80 cm = 0.80 m

But

...

$$r = 0.80 \times \left(\frac{3}{\sqrt{7}}\right)$$



 \therefore Area of circular surface of water,

 $\sin C = \frac{1}{a n_w} = \frac{3}{4}$

 $\tan C = \frac{3}{\sqrt{7}}$

$$A = \pi r^2 = 3.14 \times \left(0.8 \times \frac{3}{\sqrt{7}}\right)^2 = 3.14 \times 0.64 \times \frac{9}{7} = 2.6 \text{ m}^2$$

- **Q. 6.** Use the mirror equation to show that
 - (a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f. [CBSE Delhi 2015, (F) 2017, 2019 (55/3/3)]
 - (b) a convex mirror always produces a virtual image independent of the location of the object.
 - (c) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. [CBSE (AI) 2011]
- **Ans.** (a) Mirror equation is $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ or $\frac{1}{v} = \frac{1}{f} \frac{1}{u}$

For a concave mirror, f is negative, *i.e.*, f < 0.

For a real object (on the left of mirror), u < 0

$$\therefore \qquad 2f < u < f \text{ or } \frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

or
$$-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f} \text{ or } \frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f}$$

or
$$\frac{1}{2f} < -\frac{1}{v} < 0 \quad i.e, \frac{1}{v} \text{ is negative.}$$

This implies that v is negative.

Also from above inequality 2f > v

or
$$|2f| < |v|$$
 (:: 2f and v are negative)

- Hence, the real image is formed beyond 2f.
- (b) For a convex mirror, f is positive, i.e., f > 0.For a real object on the left, u is negative

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

As u is negative and f is positive; $\frac{1}{v}$ must be positive, so v must be positive i. e., image lies behind the mirror. Hence, image is virtual whatever the value of u may be.

 $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ (c) For a mirror, ...(i) For a concave mirror, f is negative *i.e.*, f < 0As *u* is also negative, so f < u < 0

This implies, $\frac{1}{f} - \frac{1}{u} > 0$ Then from (i) $\frac{1}{v} > 0$ or v is positive. *i.e.*, image is on the right and hence **virtual**. Magnification, $m = -\frac{v}{u} = -\frac{f}{u-f}$

As *u* is negative and *f* is positive, magnification $m = \frac{|f|}{|f| - |u|} > 1$ *i.e.*, image is enlarged.

Q.7. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Ans. Apparent thickness of slab =
$$\frac{\text{Real thickness}}{\text{Refractive index}} = \frac{H}{n}$$

Displacement of pin, $x = \left(H - \frac{H}{n}\right) = H\left(1 - \frac{1}{n}\right)$ Here H = 15 cm, n = 1.5, $x = H\left(1 - \frac{1}{n}\right) = 15\left(\frac{1.5 - 1}{1.5}\right)$ cm = 5 cm

Thus the pin appears to be raised by 5 cm. The answer does not depend upon the location of slab.

Refraction at Spherical Surface and by Lenses

Q. 8. A double convex lens is made of a glass of refractive index 1.55, with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm.

[CBSE (AI) 2017]

Ans. Given, f = 20 cm and n = 1.55

Let the radius of the curvature of each of two surfaces of the lens be *R*.

If R_1 and R, then $R_2 = -R$

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\Rightarrow \qquad \frac{1}{20} = (1.55 - 1) \left[\frac{1}{R} + \frac{1}{R} \right] \qquad \Rightarrow \qquad \frac{1}{20} = \frac{0.55 \times 2}{R}$$

$$\Rightarrow \qquad \frac{1}{20} = \frac{1.10}{R} \qquad \Rightarrow \qquad R = 20 \times 1.10$$

$$\therefore \qquad R = 22 \text{ cm}$$

Q.9. A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, (b) a concave lens of focal length 16 cm?

(a) Point P acts as a virtual object for convex lens. Ans. Given u = +12 cm, f = +20 cm

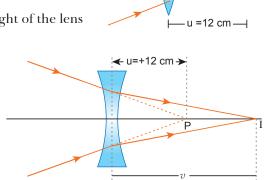


$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ gives } \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} + \frac{1}{12}$$
$$= \frac{3+5}{60}$$
$$\Rightarrow \quad v = \frac{60}{8} = 7.5 \text{ cm}$$

This implies that the image is formed to the right of the lens and is real.

(b) In this case,
$$u = +12 \text{ cm}, f = -16 \text{ cm}$$

 $\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ gives } \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$
 $= -\frac{1}{16} + \frac{1}{12} = \frac{-3+4}{48}$
 $v = 48 \text{ cm}$



С

This shows that the image is formed at a distance of 48 cm to the right of concave lens and is real.

Q. 10. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved farther from the lens?

Ans. Size of object
$$h = 3.0$$
 cm,
 $u = -14$ cm,
 $f = -21$ cm (concave lens)
 \therefore Formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$

or
$$\frac{1}{v} = \frac{1}{-21} + \frac{1}{-14} = -\frac{2+3}{42}$$
 or $v = -\frac{42}{5} = -8.4$ cm

Size of image $h' = \frac{v}{u}h = \frac{-8.4}{-14} \times 3.0 \text{ cm} = 1.8 \text{ cm}$

That is, image is formed at a distance of 8.4 cm in front of lens. The image is virtual, erect and of size 1.8 cm. As the object is moved farther from the lens, the image goes on shifting towards focus and its size goes on decreasing. The image is never formed beyond the focus of the concave lens.

- Q. 11. What is the focal length of a combination of a convex lens of focal length 30 cm and a concave lens of focal length 20 cm in contact? Is the system a converging or a diverging lens? Ignore thickness of lenses.
- **Ans.** Given $f_1 = +30$ cm, $f_2 = -20$ cm

The focal length (F) of combination of given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$
$$F = \frac{f_1 f_2}{f_1 + f_2} = \frac{30 \times (-20)}{30 - 20} = -60 \text{ cm}$$

 \Rightarrow

That is, the focal length of combination is 60 cm and it acts like a diverging lens.

- Q. 12. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?
- Ans. For a fixed distance *D* between object and image for its real image

$$D = |u| + |v| ...(i) x = v - u ...(ii)$$

...(11)

From equation (i) and (ii),

 \Rightarrow

$$v = \frac{D+x}{2} \qquad \qquad u = \frac{D-x}{2}$$

Sign convention: u is negative and v is positive.

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{2}{D+x} + \frac{2}{D-x} = \frac{4D}{D^2 - x^2}$$
$$f = \frac{D^2 - x^2}{4D}$$

where x is the separation between two positions of lens.

For maximum
$$f, x = 0$$

 \therefore $f_{max} = \frac{D}{4}$
Given $D = 3$ m
 $f = \frac{3}{4}$ m = **0.75 m**

- Q. 13. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.
- **Ans.** Given separation between object and screen, D = 90 cm Separation between two positions of lens, x = 20 cm

$$\therefore Focal length of lens, f = \frac{D^2 - x^2}{4D} = \frac{(90)^2 - (20)^2}{4 \times 90} = \frac{8100 - 400}{4 \times 90}$$
$$= \frac{7700}{4 \times 90} = 21.4 \text{ cm}$$

Refraction of light through prism

- Q. 14. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. By rotating the prism, the minimum angle of deviation is measured to be 40°. What is the refractive index of the prism ? If the prism is placed in water (refractive index 1.33), predict the new minimum angle of deviation of a parallel beam of light. The refracting angle of prism is 60° (use: sin 50° = 0.7660 and sin 35°=0.576). [HOTS]
- **Ans.** Key idea: Refractive index of prism material and ${}_w n_g = \frac{n_g}{n_w}$ Given angle of prism $A = 60^\circ$, Minimum angle of deviation $\delta_m = 40^\circ$

Refractive index
$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left(\frac{60+40}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 50^{\circ}}{\sin 30^{\circ}} = \frac{0.7660}{0.5} = 1.532.$$

When prism is placed in water, its refractive index becomes

$$_{w}n_{g} = \frac{n_{g}}{n_{w}} = \frac{1.532}{1.33} = 1.152$$

If δ'_m is the new angle of deviation, then

$$_{w}n_{g} = \frac{\sin\left(\frac{A+\delta_{m}'}{2}\right)}{\sin A/2} = \frac{\sin\left(\frac{60^{\circ}+\delta_{m}'}{2}\right)}{\sin 30^{o}}$$



$$1.152 = \frac{\sin\left(\frac{60^{\circ} + \delta'_m}{2}\right)}{0.5}$$
$$= \sin\frac{60^{\circ} + \delta'_m}{2} = 1.152 \times 0.5 = 0.576$$
$$\frac{60^{\circ} + \delta'_m}{2} = 35^{\circ} \text{ or } \delta'_m = 10^{\circ}$$

- Q. 15. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of prism is 1.524.
- Ans. **Key idea :** For just total internal reflection from prism, the ray must be incident at critical angle on the second face.

Given angle of prism, $A = 60^{\circ}$, n = 1.524If *C* is the critical angle for total internal reflection, then $\sin C = \frac{1}{n} = \frac{1}{1.524} = 0.6561$ $C = \sin^{-1}(0.6561) = 41^{\circ}$ Let *i* be the angle of incidence at first face of prism *AB*. The ray follows the path PQRS For just total internal reflection at the other face AC $r_2 = C = 41^{\circ}$ $\vec{r_1} + r_2 = A$ $r_1 = A - r_2 = 60^\circ - 41^\circ = 19^\circ$

From Snell's law, $n = \frac{\sin i}{\sin r}$

$$\Rightarrow$$

 $= 1.524 \sin 19^{\circ} = 1.524 \times 0.3256 = 0.4962$

Angle of incidence $i = \sin^{-1} (0.4962) = 29^{\circ} 45'$.

 $\sin i = n \sin r$

Microscopes and Telescopes

Q. 16. A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (i) the least distance of distinct vision (D = 25 cm) and (ii) infinity?

What is the magnifying power of the microscope in each case ?

(i) When final image is formed at least distance of distinct vision (
$$D = 25$$
 cm) :

For eye lens : Here $v_e = -25$ cm

$$\therefore \qquad \frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\Rightarrow \qquad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{6.25} = \frac{-1-4}{25}$$
or
$$u_e = -5 \text{ cm}$$
As
$$L = |v_0| + |u_e| \Rightarrow |v_0| = L - |u_e| = 15 - 5 = 10 \text{ cm}$$
For objective lens:
$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$$\Rightarrow \qquad \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{10} - \frac{1}{2} = -\frac{2}{5} \Rightarrow u_0 = -\frac{5}{2} = -2.5 \text{ cm}$$

60

Ray Optics and Optical Instruments 351 That is distance of object from objective is 2.5 cm.

Magnification, $M = \frac{v_0}{u_0} \left(1 + \frac{D}{f_e} \right)$ = $\frac{10}{2.5} \left(1 + \frac{25}{6.25} \right) = 4 \times 5 = 20$

(ii) When final image is formed at infinity:

In this case $L = v_0 + f_e \Rightarrow v_0 = L - f_e = 15 - 6.25 = 8.75$ cm For objective lens :

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$$\Rightarrow \qquad \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{8.75} - \frac{1}{2} = \frac{2 - 8.75}{2 \times 8.75}$$

$$u_0 = -\frac{2 \times 8.75}{6.75}$$

$$\therefore$$
 $u_0 = -2.59 \text{ cm}, |u_0| = 2.59 \text{ cm}$

$$M = \frac{v_0}{u_0} \cdot \frac{D}{f_e} = \frac{8.75}{2.59} \cdot \left(\frac{25}{6.25}\right) = 13.5$$

- Q. 17. A person with a normal near point (25 cm) using a compound microscope with an objective of focal length 8.0 mm and an eye-piece of focal length 2.5 cm can bring an object placed 9.0 mm from the objective in sharp focus. What is the separation between the two lenses ? What is the magnifying power of the microscope ?
- **Ans.** Given focal length of objective, $f_0 = 8 \text{ mm}$ Focal length of eye-piece, $f_e = 2.5 \text{ cm} = 25 \text{ mm}$ **For objective lens :**

Magnification,

Distance of object from objective, $u_0 = -9 \text{ mm}$

From lens formula
$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$
, we get
 $\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{8} - \frac{1}{9} = +\frac{1}{72} \implies v_0 = 72 \,\mathrm{mm}$

For eye-lens if final image is formed at least distance of distinct vision, then

$$v_e = -D = -25 \text{ cm} = -250 \text{ mm}$$

$$\therefore \qquad \frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{250} - \frac{1}{25} = -\frac{11}{250}$$

$$\therefore \qquad u_e = -\frac{250}{11} \text{ mm} = -22.7 \text{ mm}$$

Separation between lenses, $L = |v_0| + |u_e| = 72 \text{ mm} + 22.7 \text{ mm}$ = 94.7 mm = **9.47 cm**

Magnifying power,
$$M = \frac{v_0}{u_0} \left(1 + \frac{D}{f_e}\right)$$

= $\frac{72}{9} \left(1 + \frac{25 \text{ cm}}{2.5 \text{ cm}}\right) = 8(1+10) = 88$



- Q. 18. A small telescope has an objective lens of focal length 144 cm and an eye piece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eye-piece?
- **Ans.** Given $f_0 = 144 \text{ cm}, f_e = 6.0 \text{ cm}$

Magnifying power of telescope, $M = -\frac{f_0}{f_e} = -\frac{144}{6.0} = -24$

Negative sign shows that the final image is real and inverted.

Separation between objective and eye-piece :

 $L = f_0 + f_e = 144 + 6.0 = 150 \text{ cm}$

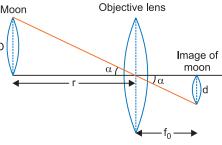
- Q. 19. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eye-piece of focal length 1.0 cm is used, what is the angular magnification of the telescope?
 - (b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10⁶ m and radius of lunar orbit is 3.8×10⁸ m.
 [CBSE (AI) 2011, Delhi 2014, 2015, 2019 (55/1/1)]
- **Ans.** (a) Given $f_0 = 15 \text{ m}, f_e = 1.0 \text{ cm} = 1.0 \times 10^{-2} \text{ m}$
 - Angular magnification of telescope,

$$m = -\frac{f_0}{f_e} = -\frac{15}{1.0 \times 10^{-2}} = -1500$$

Negative sign shows that the final image is real and Moon inverted.

(b) Let D be diameter of moon, d diameter of image D of moon formed by objective and r the distance of moon from objective lens, then from Fig.

$$\frac{D}{r} = \frac{d}{f_0}$$
$$d = \frac{D}{r}. f_0 = \frac{3.48 \times 10^6}{3.8 \times 10^8} \times 15 \text{ m} = 0.137 \text{ m} = 13.7 \text{ cm}$$



- Q. 20. A small telescope has an objective lens of focal length 140 cm and an eye-piece of focal length 5.0 cm. What is the
 - (a) magnifying power of telescope for viewing distant objects when the telescope is in normal adjustment (*i.e.*, when the final image is at infinity)?
 - (b) the final image is formed at the least distance of distinct vision (D = 25 cm)?
 - (c) What is the separation between the objective and eye lens when final image is formed at infinity?
 - (d) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?
 - (e) What is the height of the final image of the tower if it is formed at the least distance of distinct vision D = 25 cm?

Ans. Given $f_0 = 140 \text{ cm}, f_e = 5 \text{ cm}.$

 \Rightarrow

(a) When final image is at infinity,

magnifying power, $M = -\frac{f_0}{f_e} = -\frac{140}{5.0} = -28$

Negative sign shows that the image is real and inverted.

(b) When final image is at the least distance of distinct vision,

magnifying power,
$$M = \frac{f_0}{f_e} \left(1 + \frac{f_e}{D} \right) = \frac{140}{5.0} \left(1 + \frac{5.0}{25} \right) = 33.6$$

(c) Separation between objective and eye lens when final image is formed at infinity $L = f_0 + f_e = 140 \text{ cm} + 5.0 \text{ cm} = 145 \text{ cm}$

(d) Angle subtended by 100 m tall tower at 3 km away is

$$\alpha = \tan \alpha = \frac{100}{3 \times 10^3} = \frac{1}{30} \text{ rad}$$

Let h be the height of image of tower formed by objective. The angle subtended by image produced by objective will also be equal to α .

$$\alpha = \frac{h}{f_o} = \frac{h}{140} \implies \frac{h}{140} = \frac{1}{30}$$

 $h = \frac{140}{30} = \frac{14}{3} = 4.67 \text{ cm}$

(e) Magnification produced by eyepiece $m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$

For eyepiece, $m_e = \frac{\text{height of final image}(h_2)}{\text{height of image formed by objective}(h_1)}$

Height of final image $= h_2 = m_e h_1 = 6 \times 4.67$ cm = 28.02 cm

Q. 21. An angular magnification of 30X is desired using an objective of focal length 1.25 cm and an eye-piece of focal length 5 cm. How would you set up the compound microscope?

[CBSE Sample Paper 2018]

Ans. The final image is formed at the least distance of distinct vision,

 $\therefore D = 25 \text{ cm}, f_e = 5 \text{ cm}$

Angular magnification of the eye lens is

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Total magnification

$$m = m_{o} \times m_{o}$$
$$30 = m_{o} \times 6$$

A

$$\therefore \qquad \text{Angular magnification of the objective lens is} \\ m_o = \frac{30}{6} = 5 \\ \text{Also,} \qquad m_o = \frac{v_o}{-u_o} \implies v_o = -5u_o \implies f_o = 1.25 \text{ cm} \\ \text{Using,} \quad \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \implies \frac{1}{-5u_o} - \frac{1}{u_o} = \frac{1}{1.25} \implies \frac{-6}{5u_o} = \frac{1}{1.25} \\ u_o = \frac{-6 \times 1.25}{5} = -1.5 \text{ cm} \end{cases}$$

The object should be placed 1.5 cm from the objective to obtain the desired magnification. Now, $v_0 = -5u_0 = -5 \times (-1.5) = 7.5$ cm

Using
$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

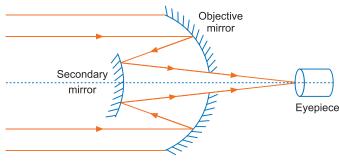
 $\therefore \quad \frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5} \implies \frac{1}{u_e} = \frac{-1-5}{25} = \frac{-6}{25}$
 $u_e = \frac{-25}{6} = -4.17 \text{ cm}$

Separation between the lenses $d = |v_0| + |u_e| = 7.5 + 4.17 = 11.67$ cm

Thus to obtain, the desired magnification the separation between the lenses must be 11.67 cm and the objective must be placed at a distance 1.5 cm in front of the objective lens.



Q. 22. A Cassegrain telescope uses two mirrors as shown in fig. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and of the small mirror is 140 mm, where will the final image of an object at infinity be?



Ans. Given $r_1 = 220 \text{ mm}$,

$$f_1 = \frac{r_1}{2} = 110 \text{ mm} = 11 \text{ cm}$$

 $r_2 = 140 \text{ mm}, f_2 = \frac{r_2}{2} = 70 \text{ mm} = 7.0 \text{ cm}$

Distance between mirrors, $d_{1} = 20 \text{ mm} = 2.0 \text{ cm}$

The parallel incident rays coming from distant object fall on the concave mirror and try to be focused at the principal focus of concave mirror *i.e.*,

$$v_1 = -f_1 = -11 \text{ cm}$$

But in the path of rays reflected from concave mirror, a convex mirror is placed. Therefore the image formed by the concave mirror, acts as a virtual object for convex mirror.

For convex mirror $f_2 = -7.0$ cm, $u_2 = -(11 - 2) = -9$ cm

$$\therefore \qquad \frac{1}{f_2} = \frac{1}{v_2} + \frac{1}{u_2}$$

$$\Rightarrow \qquad \frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2} = -\frac{1}{7} + \frac{1}{9}$$

$$v_2 = -\frac{63}{9} \text{ cm} = -31.5 \text{ cm}$$

This is the distance of final image formed by the convex mirror.

Thus, the final image is formed at a distance 31.5 cm from the smaller (convex) mirror behind the bigger mirror.

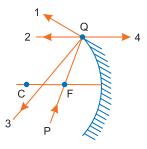
Multiple Choice Questions

Choose and write the correct option(s) in the following questions.

- 1. An object is placed at a distance of 40 cm from a concave mirror of focal length 15 cm. If the object is displaced through a distance of 20 cm towards the mirror, the displacement of the image will be
 - (a) 30 cm away from the mirror
 - (b) 36 cm away from the mirror
 - (c) 30 cm towards the mirror
 - (d) 36 cm towards the mirror
- 2. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (Fig. given alongside). Which of the four rays correctly shows the direction of reflected ray?

[NCERT Exemplar]

- (a) 1 (b) 2
- (c) 3 (d) 4



355

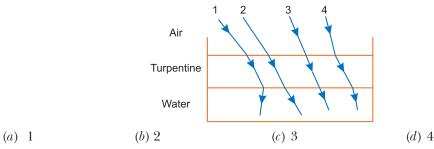
[1 mark]

3. A concave mirror of focal length 15 cm forms are image having twice the linear dimensions of the object. The position of the object, when the image is virtual, will be

(b) 7.5 cm

- (a) 22.5 cm
- (c) 30 cm (d) 45 cm
- A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is

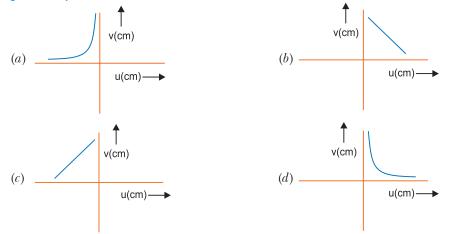
 (a) blue
 (b) green
 (c) violet
 (d) red
- 5. The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in the figure, the path shown is correct? [NCERT Exemplar]



- 6. Why is refractive index in a transparent medium greater than one?
 - (a) Because the speed of light in vacuum is always less than speed in a transparent medium
 - (b) Because the speed of light in vacuum is always greater than the speed in a transparent medium
 - (c) Frequency of wave changes when it crosses medium
 - (*d*) None of the above
- 7. Transmission of light in optical fibre is due to
 - (a) scattering
 - (b) diffraction
 - (c) refraction
 - (d) multiple total internal reflection
- 8. You are given four sources of light each one providing a light of a single colour red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statements is correct if the source of yellow light is replaced with that of other lights without changing the angle of incidence? [NCERT Exemplar]
 - (a) The beam of red light would undergo total internal reflection.
 - (b) The beam of red light would bend towards normal while it gets refracted through the second medium.
 - (c) The beam of blue light would undergo total internal reflection.
 - (d) The beam of green light would bend away from the normal as it gets refracted through the second medium.
- 9. Which of the following is not due to total internal reflection ?
 - (a) Working of optical fibre
 - (b) Difference between apparent and real depth of a pond
 - (c) Mirage on hot summer days
 - (*d*) Brilliance of diamond
- 10. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image [NCERT Exemplar]
 - (a) moves away from the lens with an uniform speed 5 m/s.
 - (b) moves away from the lens with an uniform accleration.



- (c) moves away from the lens with a non-uniform acceleration.
- (d) moves towards the lens with a non-uniform acceleration.
- 11. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will [NCERT Exemplar]
 - (a) act as a convex lens only for the objects that lie on its curved side.
 - (b) act as a concave lens for the objects that lie on its curved side.
 - (c) act as a convex lens irrespective of the side on which the object lies.
 - (d) act as a concave lens irrespective of side on which the object lies.
- 12. A student measures the focal length of a convex lens by putting an object pin at a distance u'from the lens and measuring the distance v' of the image pin. The graph between u' and v'plotted by the student should look like



- 13. Focal length of a convex lens of refractive index 1.5 is 2 cm. Focal length of lens, when immersed in a liquid of refractive index of 1.25 will be (c) 5 cm (d) 2.5 cm
 - (a) 10 cm (b) 7.5 cm

(*d*) f' = f and f'' = 2 f

14. An equiconvex lens is cut into two halves along (i) XOX' and (ii) YOY' as shown in the figure. Let f, f' and f'' be the of the focal lengths of complete lens of each half in case (i) and of each half in case (ii) respectively. Choose the correct statement from the following :

(a)
$$f' = 2f$$
 and $f'' = f$ (b) $f' = f$ and $f'' = f$

(c)
$$f' = 2f$$
 and $f'' = 2f$

15. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is [NCERT Exemplar] $(d) 2.5^{\circ}$ (a) 7.5° (b) 5° (c) 15°

16. The refractive index of the material of a prism is $\sqrt{2}$ and the angle of the prism is 30°. One of the two refracting surfaces of the prism is made a mirror inwards, by silver coating. A beam of monochromatic light entering the prism from the other face will retrace it path (after reflection from the silvered surface) if its angle of incidence of the prism is

(a)
$$60^{\circ}$$
 (b) 45° (c) 30° (d) zero

- 17. A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue Blue wavelengths are 1.39, 1.44 and 1.47 respectively. Green Red The prism will
 - (*a*) not separate the three colours at all



R

X'

X

45°

C

- (b) separate the red colour part from the green and blue colours
- (c) separate the blue colour part from the red and green colours
- (d) separate all the three colours from one another
- 18. A thin prism having refracting angle 10° is made of glass of refractive index 1.42. This prism is combined with another thin prism of glass of refractive index 1.7. This combination produces dispersion without deviation. This refracting angle of second prism should be (a) 6° (b) 8° (c) 10° $(d) 4^{\circ}$
- 19. The sky would appear red instead of blue if
 - (a) atmospheric particles scatter blue light more than red light
 - (b) atmospheric particles scatter all colours equally
 - (c) atmospheric particles scatter red light more than blue light
 - (*d*) the sun was much hotter
- 20. The reddish appearance of rising and setting sun is due to
 - (a) reflection of light (b) diffraction of light
 - (c) scattering of light (d) interference of light
- 21. A setting sun appears to be at an altitude higher than it really is. This is because of
 - (a) absorption of light (b) reflection of light
 - (c) refraction of light (d) dispersion of light
- 22. For relaxed eye, the magnifying power of a microscope is

(a)
$$\frac{v_0}{u_0} \times \frac{D}{f_e}$$
 (b) $\frac{v_0}{u_0} \times \frac{f_e}{D}$ (c) $\frac{u_0}{v_0} \times \frac{D}{f_e}$ (d) $\frac{u_0}{v_0} \times \left(-\frac{D}{f_e}\right)$

23. If the focal length of objective lens is increased then magnifying power of

- (a) microscope will increase but that of telescope decrease
- (b) microscope and telescope both will increase
- (c) microscope and telescope both will decrease
- (d) microscope will decrease but that of telescope will increase

24. Four lenses of focal length ± 15 cm and ± 150 cm are available for making a telescope. To produce the largest magnification, the focal length of the eyepiece should be

- (a) +15 cm(b) + 150 cm(c) -150 cm(d) -15 cm
- 25. The magnifying power of a telescope is 9. When it is adjusted for parallel rays the distance between the objective and evepiece is 20 cm. The focal length of lenses are

<i>(a)</i>	11 cm, 9 cm	(<i>b</i>) 10 cm, 10 cm	(c) 15	cm, 5 cm	(<i>d</i>) 18 cm, 2 cm	
Answers						
1. (<i>b</i>)	2. (<i>b</i>)	3. (<i>b</i>)	4. (<i>d</i>)	5. (<i>b</i>)	6. (<i>b</i>)	
7. (d)	8. (<i>c</i>)	9. (<i>b</i>)	10. (<i>c</i>)	11. (<i>c</i>)	12. (<i>a</i>)	
13. (<i>c</i>)	14. (<i>d</i>)	15. (<i>a</i>)	16. (<i>b</i>)	17. (b)	18. (<i>a</i>)	
19. (<i>c</i>)	20. (<i>c</i>)	21. (<i>c</i>)	22. (<i>a</i>)	23. (<i>d</i>)	24. (<i>a</i>)	

25. (*d*)

Fill in the Blanks

- 1. When the refractive index of the material of the lens is greater than that of the surroundings, then biconvex lens acts as a
- **2.** The power of a lens is defined as the of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.



[1 mark]

- 3. A lens of power of –4.0 D means a concave lens of focal length ______ cm.
- 4. When we apply the sign convention, we see that, for erect and virtual image formed by a convex or concave lens, *m* is
- 5. The angle between the emergent ray and the direction of the incident ray is called the
- At the minimum deviation, the refraction ray inside the prism becomes parallel to the **6**.
- 7. In the visible spectrum, red light is at the long wavelength end (~ 700 nm) while the is at the short wavelength end (~ 400 nm).
- 8. The largest telescope in India is in Kavalur, Tamil Nadu. It is a diameter reflecting telescope (cassegrain).
- The amount of scattering is inversely proportional to the 9. power of the wavelength.
- 10. For the same angle of incidence, the angles of refraction in three different medium A, B and C are 15°, 25° and 35° respectively. In medium velocity of light will be minimum.

Answers

- **3.** −25 cm **1.** converging lens 2. tangent 4. positive 7. violet light 8. 2.34 m **5.** angle of deviation 6. base 9. fourth
- **10.** A

Very Short Answer Questions

Q. 1. When light travels from an optically denser medium to a rarer medium, why does the critical angle of incidence depend on the colour of light? [CBSE Ajmer 2015]

The refractive index is different for different colour wavelength as $n = a + \frac{b}{\lambda^2}$. Hence, critical Ans.

angle sin $i_c = \frac{1}{n}$ would also be different for different colour of light.

- Q. 2. How does the angle of minimum deviation of a glass prism vary if the incident violet light is replaced by red light? [CBSE 2019 (55/3/1)]
- **Ans.** The angle of minimum deviation decreases, if violet light is replaced by red light *i.e.* $\delta r < \delta v$.
- Why does bluish colour predominate in a clear sky? [CBSE Delhi 2010; Allahabad 2015] **Q.** 3.
- Ans. The colour of the sky, as seen from the earth, is due to the scattering of sunlight by molecules of earth's atmosphere. The amount of scattering is inversely proportional to the fourth power of the wavelength, *i.e.*,

$$I \propto \frac{1}{\lambda^4}$$

Thus, shorter wavelengths are scattered much more than longer wavelengths. Since $\lambda_{\rm B} << \lambda_{\rm R}$. Hence, the bluish colour predominates in the clear sky.

Q. 4. A ray of light falls on a transparent sphere with centre C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the angle of refraction at A if refractive index of the material of the sphere is $\sqrt{3}$. [CBSE (F) 2014]



[1 mark]

$$\sin r = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

 $\sin r = \sin 30^\circ \implies r = 30^\circ$

Angle of refraction = **30°**.

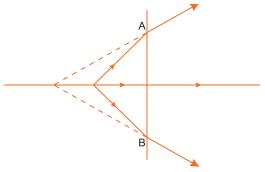
Q. 5. For the same angle of incidence, the angle of refraction in two media A and B are 25° and 35° respectively. In which one of the two media is the speed of light lesser?

[CBSE Bhubaneshwar 2015]

Ans.
$$n = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$
$$\frac{n_A}{n_B} = \frac{\sin i / \sin r_A}{\sin i / \sin r_B} = \frac{\sin r_B}{\sin r_A} = \frac{v_1 / v_A}{v_1 / v_B}$$
$$\frac{\sin r_B}{\sin r_A} = \frac{v_B}{v_A}$$
$$r_A < r_B \qquad \sin r_A < \sin r_B \implies v_A < v_B$$

Speed of light in *A* is lesser.

Q. 6. The line *AB* in the ray diagram represents a lens. State whether the lens is convex or concave. [*CBSE Chennai 2015*]



Ans. It is a concave or diverging lens.

Reason: The refracted ray is bending away from the principal axis.

Q. 7. The focal length of an equiconvex lens is equal to the radius of curvature of either face. What is the value of refractive index of the material of the lens? [CBSE Panchkula 2015]

Ans.
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

 $\frac{1}{f} = (n-1)\left(\frac{2}{f}\right)$ (: $f = R$)
 $\frac{1}{2} = (n-1)$
 $n = 1.5$

Q. 8. How does focal length of a lens change when red light incident on it is replaced by violet light? Give reason for your answer. [CBSE (F) 2012]

Ans. We know
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

 $f \propto \frac{1}{(n-1)}$ and $n_v > n_r$

The increase in refractive index would result in decrease of focal length of lens. Hence, we can say by replacing red light with violet light, decreases the focal length of the lens used.



Q. 9. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? [CBSE Delhi 2015]

Ans. Concave lens, in medium of high refractive index, behaves as a convex lens (or a converging lens).

Reason:

Since

$$n_m > n_g$$

 $\frac{1}{f_m} = +$ ve value

 $\frac{1}{f} = \left(\frac{n_g}{n_m} - 1\right) \left(-\frac{1}{R} - \frac{1}{R}\right)$

So, $f_m > 0$. Hence acts a convex lens.

- Q. 10. Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid? [CBSE Delhi 2012]
- **Ans.** When $n_L = n_g$

where $n_{\rm L}$ = Refractive index of liquid and $n_{\rm g}$ = Refractive index of glass

- Q. 11. A converging lens of refractive index 1.5 is kept in a liquid medium having same refractive index. What is the focal length of the lens in this medium?
- Ans. The focal length of lens in a liquid-medium is given by

$$\begin{split} \frac{1}{f_l} &= (_l n_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \left(\frac{n_g}{n_l} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ \text{Given } n_l &= n_g = 1.5 \\ \therefore \qquad \frac{1}{f_l} &= 0 \quad \text{or} \quad f_l = \infty \end{split}$$

i.e., focal length of converging lens is **infinity** *i.e.*, glass lens behaves as a glass plate.

- Q. 12. Out of blue and red light which is deviated more by a prism? Give reason. [CBSE Delhi 2010]
- Ans. Blue is deviated more than red. **Reason:** Deviation caused by a prism $\delta = (n - 1)$ A and Refractive index (*n*) is more for blue than red.
- Q. 13. A ray of light passes through an equilateral glass prism such that the angle of incidence is equal to angle of emergence and each of these angles is equal to $\frac{3}{4}$ of angle of prism. What is the value of angle of deviation? [CBSE Patna 2015]

Ans. In prism
$$i + e = A + D =$$
 and $i = e = \frac{3}{4}A$ (given)
So, $A + D = \frac{3}{4}A + \frac{3}{4}A$

$$\Rightarrow$$

 $D = \frac{3A}{2} - A = \frac{A}{2}$

Since $A = 60^{\circ}$ (being an equilateral glass prism)

So,
$$D = \frac{60^{\circ}}{2} = 30^{\circ}$$

Q. 14. Why does the sun look reddish at sunset or sunrise?

[CBSE (F) 2015, (Central) 2016, 2019 (55/2/1)]

Ans. During sunset or sunrise, the sun is just above the horizon, the blue colour gets scattered most by the atmospheric molecules while red light gets scattered least, hence sun appears red.

Reason: Scattering intensity $I \propto \frac{1}{\lambda^4}$ and $\lambda_B << \lambda_R$. Thus, the sun appears red due to least scattering of red light as it has longest wavelength.

Q. 15. Why can't we see clearly through fog? Name the phenomenon responsible for it.

[CBSE (North) 2016]

Ans. Scattering of light: When light falls on fog then scattering takes place so the particles of fog becomes visible. Visible light cannot pass through fog.

Q. 16. You are given following three lenses. Which two lenses will you use as an eyepiece and as an objective to construct an astronomical telescope? Give reason.

Lenses	Power (D)	Aperture (cm)	
L_1	3	8	
L_2	6	1	
L_3	10	1 [CBSE Delhi 2009, CBSE (AI) 2017]]
Objective :	Lens L_1		

Eyepiece : Lens L_3

Ans.

Reason: The objective lens should have large aperture (here, 8 cm) and large focal length $\left(f = \frac{1}{\text{Power}}\right)$ while the eyepiece should have small aperture and small focal length.

- Q. 17. Does the magnifying power of a microscope depend on the colour of the light used? Justify your answer. [CBSE (F) 2017]
- **Ans.** Yes, since magnification depends upon focal length and focal length depends on the colour and different colours have different wavelengths (*i.e.*, different refractive indices).

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$
(By Lens Mal

Also, magnification of compound microscope

$$M = \frac{-L}{f_0} \left(1 + \frac{D}{f_e} \right)$$

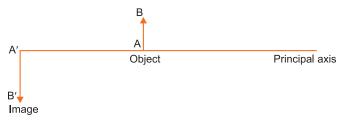
By Lens Makers Formula)

- Q. 18. (a) Explain briefly how the focal length of a convex lens changes with increase in wavelength of incident light.
 - (b) What happens to the focal length of convex lens when it is immersed in water? Refractive index of the material of lens is greater than that of water. [HOTS] [CBSE (South) 2016]
- **Ans.** (*a*) Focal length increases with increase of wavelength.

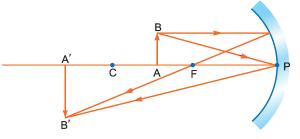
$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right) \frac{2}{R}$$
 as wavelength increases, $\frac{n_2}{n_1}$ decreases hence focal length increases.

(b) As
$$n_2 > n_1$$
, $\left(\frac{n_2}{n_1} - 1\right)$ decreases so, focal length increases.
$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\frac{2}{R}$$

Q. 19. Redraw the diagram given below and mark the position of the centre of curvature of the spherical mirror used in the given set up. [CBSE Sample Paper]



Ans. If the object is in between focus 'F' and centre of curvature 'C', image would be beyond the centre of curvature, inverted real and magnified.





Q. 20. An equi-convex lens has refractive index 1.5. Write its focal length in terms of radius of curvature R. [HOTS]

Ans.
$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right) \implies \frac{1}{f} = \frac{1}{R} \implies f = R.$$

- Q. 21. A concave mirror and a converging lens have the same focal length in air. Which one of the two will have greater focal length when both are immersed in water? [HOTS]
- Ans. Converging lens; the focal length of a spherical mirror remains unaffected.

For converging lens, $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right) \left(\frac{1}{R_2} + \frac{1}{R_2}\right)$ When it is interpolation

 n_2 (in water) $< n_2$ (air) $\left(\frac{n_2}{n_1} - 1\right)$ decreases hence focal length of converging lens increases in water.

Q. 22. A concave lens is placed in water. Will there be any change in focal length? Give reason. [HOTS]

Ans. Focal length of lens in water
$$f_w = \frac{n_g - 1}{\frac{n_g}{n_w} - 1} f_a$$

$$As n_g > n_w, \frac{n_g}{n_w} > 1, \text{ so } f_w > f_a$$

That is, focal length of lens in water will increase, but the nature of lens will remain unchanged.

- Q. 23. For which colour the magnifying power of a simple microscope is highest? For which colour it is lowest?
- **Ans.** It is highest for violet and lowest for red colour since $M = 1 + \frac{D}{f}$ and $f_V < f_R$
- Q. 24. A telescope has been adjusted for relaxed eye. You are asked to adjust it for least distance of distinct vision, then how will you change the distance between two lenses? [HOTS]
- Ans. For relaxed eye, $L = f_0 + f_e$ For least distance of distinct vision

$$L' = f_0 + u_e , u_e < f_e$$

Therefore, L' < L, that is, the distance will be decreased.

- Q. 25. Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefronts emerging from the final image? [HOTS] [NCERT Exemplar]
- The focal point of a convergent lens is the position of real image formed by this lens, when Ans. object is at infinity. When another convergent lens of short focal length is placed on the other side, the combination will form a real point image at the combined focus of the two lenses. The wavefronts emerging from the final image will be spherical.
- Q. 26. Will the focal length of a lens for red light be more, same or less than that for blue light?
- [HOTS] [NCERT Exemplar] **Ans.** As the refractive index for red is less than that for blue $\frac{1}{f} \propto n-1$, parallel beams of light incident on a lens will be bent more towards the axis for blue light compared to red. Thus the focal length

for red light will be more than that for blue. Q. 27. An unsymmetrical double convex thin lens forms the image of a point object on its axis. Will the position of the image change if the lens is reversed? [HOTS] [NCERT Exemplar]

Ans. No, the reversibility of the lens makes equation.

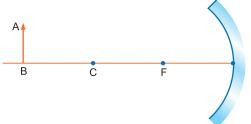
For convex lens,
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = -(n-1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

On reversing the lens, values of R_1 and R_2 are reversed and so their signs. Hence, for a given position of object (u), position of image (v) remains unaffected.

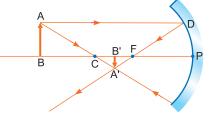
- Q. 28. State the condition under which a large magnification can be achieved in an astronomical telescope. [CBSE 2019 (55/3/1)]
 - **Ans.** The condition under which a large magnification can be achieved in an astronomical telescope is $f_0 >> f_e$, focal length of objective must be greater than focal length of eyepiece.

Short Answer Questions–I

Q. 1. An object *AB* is kept in front of a concave mirror as shown in the figure. [CBSE (AI) 2012]



- (i) Complete the ray diagram showing the image formation of the object.
- (*ii*) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black?
- **Ans.** (*i*) Image formed will be inverted diminished between *C* and *F*.



(ii) There will be no change in the position of the image but its intensity will be reduced.

Q. 2. For paraxial rays, show that the focal length of a spherical mirror is one-half of its radius of curvature. [CBSE 2019 (55/3/1)]

According to the law of reflection, Ans. Angle of incidence (i) = Angle of reflection (r).... $\angle ABC = \angle FBC$ $\angle ABC = \angle BCF$ (alternate angles) But $\angle FBC = \angle BCF$ ·.. ...(i) Triangle *BCF* is isosceles. Hence, CF = FBIf aperture of mirror is small, then point *B* is very near to *P*, so FB = FP...(*ii*) From equations (i) and (ii), CF = FP $FP = \frac{FP + CF}{2} = \frac{PC}{2}$ $f = \frac{R}{2}$ ÷. or

Principal axis C F F R

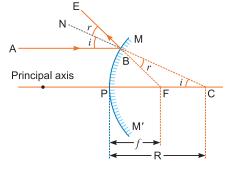
[2 marks]

Thus, the focal length of a spherical mirror (concave mirror) is half of its radius of curvature.

Q. 3. For paraxial rays, show that the focal length of a convex mirror is one half of its radius of curvature. Ans. According to the law of reflection, Angle of incidence = Angle of reflection $\therefore \quad \angle ABN = \angle EBN$ Also $\angle FBC = \angle EBN$ (vertically opposite angles)

and
$$\angle ABN = \angle FCB$$
 (corresponding angles)

 $\therefore \qquad \angle FBC = \angle FCB$





∴ Triangle *FCB* is isosceles

 $\therefore \qquad FC = BF \qquad \dots (i)$

If aperture of mirror is small, then point B is very near to the point P

$$\therefore PF = BF$$
$$\therefore PF = \frac{PF + BF}{2}$$
$$PF + FC$$

$$= \frac{PF + FC}{2} = \frac{PC}{2}$$
$$f = \frac{R}{2}$$

That is, the focal length of a convex mirror is half of its radius of curvature.

Q. 4. The following data was recorded for values of object distance and the corresponding values of image distance in the experiment on study of real image formation by a convex lens of power + 5 D. One of these observations is incorrect. Identify this observation and give reason for your choice:

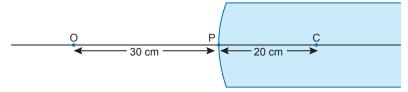
S. No.	1	2	3	4	5	6
Object distance (cm)	25	30	35	45	50	55
Image distance (cm)	97	61	37	35	32	30

Ans. Power of lens = +5 D

Focal length of lens,
$$f = \frac{1}{P} = \frac{1}{5} = 0.20 \text{ m} = 20 \text{ cm}$$

The observations at serial number (3) *i.e.*, (object distance 35 cm and image distance 37 cm is incorrect), because if the object is placed at a distance between f and 2f its image will be formed beyond 2f, while in this observation the object and image distances, both are between f and 2f.

Q. 5. A spherical convex surface of radius of curvature 20 cm, made of glass (n = 1.5) is placed in air. Find the position of the image formed, if a point object is placed at 30 cm in front of the convex surface on the principal axis. [CBSE Sample Paper 2018]



Ans. Here, $R = +20 \text{ cm}, n_1 = 1.0, n_2 = 1.5, u = -30 \text{ cm}$ Using, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ $\frac{1.5}{v} - \frac{1.0}{-30} = \frac{1.5 - 1.0}{20}$ $\Rightarrow \qquad \frac{1.5}{v} + \frac{1}{30} = \frac{0.5}{20} = \frac{1}{40}$ $\Rightarrow \qquad \frac{1.5}{v} = \frac{1}{40} - \frac{1}{30} \Rightarrow \qquad \frac{1.5}{v} = \frac{3 - 4}{120}$ $\Rightarrow \qquad \frac{1.5}{v} = \frac{-1}{120}$ $\Rightarrow \qquad v = -180.0 \text{ cm}$

Q. 6. A converging and a diverging lens of equal focal lengths are placed co-axially in contact. Find the power and the focal length of the combination. [CBSE (AI) 2010]

Ans. Let focal length of converging and diverging lenses be + f and -f respectively.

Power of converging lens $P_1 = \frac{1}{f}$ Power of diverging lens $P_2 = -\frac{1}{f}$

- \therefore Power of combination $P = P_1 + P_2 = \frac{1}{f} \frac{1}{f} = \mathbf{0}$
- :. Focal length of combination $F = \frac{1}{P} = \frac{1}{0} = \infty$ (infinite)

Q. 7. An object is kept in front of a concave mirror of focal length 15 cm. The image formed is real and three times the size of the object. Calculate the distance of the object from the mirror.

[CBSE 2019 (55/4/1)]

- Ans. Here, m = -3 and f = -15 cm $m = -\frac{v}{u} = -3$ \therefore v = 3u $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $\frac{1}{-15} = \frac{1}{3u} + \frac{1}{u}$ \Rightarrow u = -20 cm
- Q. 8. Calculate the radius of curvature of an equi-concave lens of refractive index 1.5, when it is kept in a medium of refractive index 1.4, to have a power of -5D? [CBSE 2019 (55/1/1)]
- Ans. We know that

$$P = \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

According to question P = -5 D,

$$n_2 = 1.5, n_1 = 1.4$$

Also, lens is equiconcave $R_1 = -R$, $R_2 = R$

$$-5 = \left(\frac{1.5 - 1.4}{1.4}\right) \left(-\frac{1}{R} - \frac{1}{R}\right)$$
$$-5 = -\frac{0.1}{1.4} \times \frac{2}{R} \implies 5 = \frac{1}{14} \times \frac{2}{R}$$
$$\Rightarrow \frac{1}{R} = 35 \implies R = \frac{1}{35} \text{ m} = \frac{100}{35} \text{ cm} = \frac{20}{7} \text{ cm} = 2.86 \text{ cm}$$

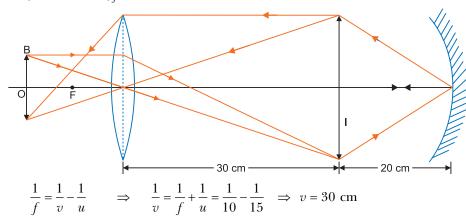
Q. 9. Calculate the distance d, so that a real image of an object at O, 15 cm in front of a convex lens of focal length 10 cm be formed at the same point O. The radius of curvature of the mirror is 20 cm. Will the image be inverted or erect?

OR

An object is placed 15 cm in front of a convex lens of focal length 10 cm. Find the nature and position of the image formed. Where should a concave mirror of radius of curvature 20 cm be placed so that the final image is formed at the position of the object itself?

[CBSE Panchkula 2015]







For image to be formed at O, the rays incident on mirror should form the image at centre of curvature. It will be so if the image I formed by the lens lies at the centre of curvature of the mirror, then the final image of mirror will be at centre of curvature and inverted, this image will be object for the lens.

 $\therefore \quad d = |v| + |R| = 30 + 20 = 50 \text{ cm}$ The image is inverted.

Q. 10. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and an eye piece is 36 cm and the final image is formed at infinity. Calculate the focal length of the objective and the focal length of the eye piece?

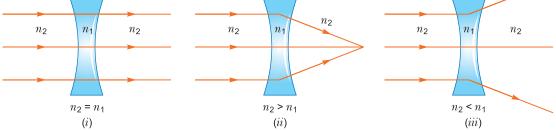
[CBSE Sample Paper 2018]

Ans. Magnification
$$m = f_0/f_e = 5$$

 $f_0 = 5f_e$
Now, length of the tube, $L = f_0 + f_e$
 $36 = 5f_e + f_e$
 $6f_e = 36 \text{ cm}$
 $f_e = 6 \text{ cm}$
 $\therefore \qquad f_0 = 5 \times 6 = 30 \text{ cm}$

Q. 11. The refractive index of a material of a concave lens is n_1 . It is immersed in a medium of refractive index n_2 . A parallel beam of light is incident on the lens. Trace the path of emergent rays when (i) $n_2 = n_1$ (ii) $n_2 > n_1$ (iii) $n_2 < n_1$.

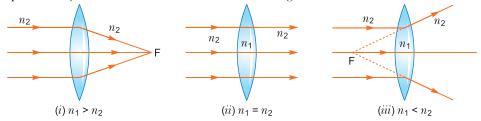
Ans. $\frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right) \left(-\frac{1}{R_2} - \frac{1}{R_2}\right)$ (*i*) for $n_1 = n_2$ $f = \infty$ (*ii*) for $n_1 < n_2$ f > 0 (*iii*) for $n_1 > n_2$ f < 0The path of rays in three cases is shown in fig.



Q. 12. A convex lens made of a material of refractive index n_1 is kept in a medium of refractive index n_2 . Parallel rays of light are incident on the lens. Complete the path of rays of light emerging from the convex lens if: (i) $n_1 > n_2$ (ii) $n_1 = n_2$ (iii) $n_1 < n_2$.

Ans.
$$\frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right) \left(\frac{1}{R_2} + \frac{1}{R_2}\right)$$

In case (i) $n_1 > n_2$, the lens behaves as convergent lens. In case (ii) $n_1 = n_2$, the lens behaves as a plane plate. In case (iii) $n_1 < n_2$, the lens behaves as a divergent lens. The path of rays in all the three cases is shown in fig.



Ray Optics and Optical Instruments 367

Q. 13. The radii of curvature of both the surfaces of a lens are equal. If one of the surfaces is made plane by grinding, then will the focal length of lens change? Will the power change? [CBSE Guwahati 2015]

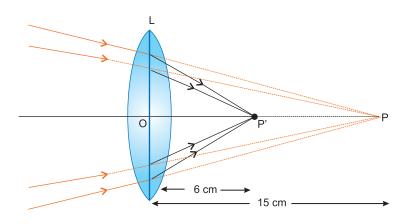
Ans. Focal length of lens $\frac{1}{f} = (n-1)\left(\frac{1}{R} + \frac{1}{R}\right) \implies f = \frac{R}{2(n-1)}$

When one surface is made plane, $\frac{1}{f} = (n-1)\left(\frac{1}{R} + \frac{1}{\infty}\right)$

$$\therefore f' = \frac{R}{(n-1)} = 2f.$$
 That is, the focal length will be doubled.

As $P = \frac{1}{f}$, so power will be halved.

Q. 14. A beam of light converges at a point P. Now a convex lens is placed in the path of the convergent beam at 15 cm from P. At what point does a beam converge if the convex lens has a focal length 10 cm? [CBSE 2019 (55/4/1)]



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$
 (lens formula)

Here u = +15 cm; f = +10 cm

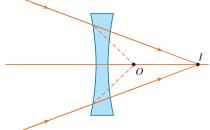
$$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{15}$$
$$\Rightarrow \quad v = 6 \text{ cm}$$

- Q. 15. A lens is placed in the path of a beam of light which converges to the point *O* in the absence of the lens. The distance between the lens and the point is 15 cm, what distance from the point *O* will the beam converge if the lens is a concave lens of focal length 25 cm.
- Ans. In the case of concave lens,

$$f = 25 \text{ cm}, u = +15 \text{ cm}, v = ?$$

$$v = \frac{uf}{u+f} = \frac{15 \times (-25)}{15 - 25} = +37.5 \text{ cm}$$

The distance *OI* = 37.5 – 15 = **22.5 cm**

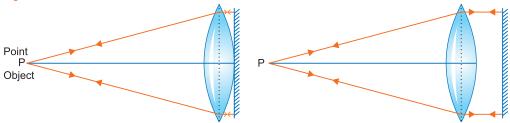


Q. 16. A convex lens is placed in contact with a plane mirror. A point object at a distance of 20 cm on the axis of this combination has its image coinciding with itself. What is the focal length of the lens? [CBSE Delhi 2014]



Ans.

Ans. The focal length of the lens = 20 cm Explanation:



As the image of this combination coincides with the object itself, the rays from the object, after refraction from the lens should fall normally on the plane mirror, so that they retrace their path. So the rays from the point object after refraction from the lens must form parallel beam. Hence the rays must be originating from the focus.

- Q. 17. (*i*) State the condition under which a large magnification can be achieved in an astronomical telescope. [CBSE 2019 (55/3/1)]
 - (*ii*) Give two reasons to explain why a reflecting telescope is preferred over a refracting telescope. [CBSE (F) 2017]
 - Ans. (i) (a) When final image is formed at least distance of distinct vision, magnification

$$m = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

(b) Magnification in normal adjustment,

$$m = \frac{f_o}{f_e}$$

Clearly, for large magnification

$$f_o >> f_e$$

- (ii) Reflecting telescope is preferred over refracting telescope because
 - (a) No chromatic aberration, because mirror is used.
 - (b) Spherical aberration can be removed by using a parabolic mirror.
 - (c) Image is bright because no loss of energy due to reflection.
 - (d) Large mirror can provide easier mechanical support.
- Q. 18. Calculate the speed of light in a medium whose critical angle is 45°.[CBSE Patna 2015]Does critical angle for a given pair of media depend on wave length of incident light? Give reason.
- **Ans.** Critical angle in the medium, $i_C = 45^{\circ}$

So, refractive index,
$$n = \frac{1}{\sin i_C} = \frac{1}{\sin 45^\circ}$$

$$\Rightarrow$$

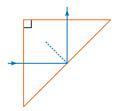
$$n = \sqrt{2}$$

Refractive index, $n = \frac{c_0}{c_m}$

$$\sqrt{2} = \frac{3 \times 10^8}{c_m}$$
$$c_m = \frac{3 \times 10^8}{\sqrt{2}} = 2.1 \times 10^8 \text{ m/s}$$

Yes, critical angle for a pair of media depends on wavelength, because $n = a + \frac{b}{\lambda^2}$, where *a* and *b* are constants of the media.

Q. 19. A ray of light incident normally on one face of a right isosceles prism is totally reflected as shown in figure. What must be minimum value of refractive index glass? Give relevant calculations. [CBSE Delhi 2016]



Ans. The critical angle depends on refractive index *n* as

$$\sin i_{c} = \frac{1}{n}$$
For total internal reflection,

$$\angle i \ge \angle i_{c} \text{ (critical angle)}$$

$$\Rightarrow \quad 45^{\circ} \ge \angle i_{c} \quad \Rightarrow \quad \angle i_{c} \le 45^{\circ}$$

$$\Rightarrow \quad \sin i_{c} \le \sin 45^{\circ} \Rightarrow \quad \sin i_{c} \le \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{1}{\sin i_{c}} \ge \sqrt{2} \quad \Rightarrow \quad n \ge \sqrt{2}$$

Hence, the minimum value of refractive index must be $\sqrt{2}$.

Q. 20. An equilateral glass prism has a refractive index 1.6 in air. Calculate the angle of minimum deviation of the prism, when kept in a medium of refractive index $4\sqrt{2}/5$.

[CBSE 2019 (55/1/1/)]

$$n = \frac{n_2}{n_1} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \frac{1.6}{\frac{4\sqrt{2}}{5}} = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin\frac{60^\circ}{2}} \Rightarrow \frac{5 \times 1.6}{4\sqrt{2}} = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin 30^\circ}$$

$$\Rightarrow \frac{8 \times 0.5}{4\sqrt{2}} = \sin\left(\frac{60^\circ + \delta_m}{2}\right) \Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

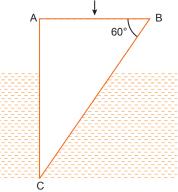
$$\Rightarrow \sin(45^\circ) = \sin\left(\frac{60^\circ + \delta_m}{2}\right) \Rightarrow \frac{60^\circ + \delta_m}{2} = 45^\circ \Rightarrow \delta_m = 90^\circ - 60^\circ = 30^\circ$$

$$\therefore \delta_m = 30^\circ$$

- Q. 21. (a) A ray of light is incident normally on the face AB of a right-angled glass prism of refractive index $_an_g = 1.5$. The prism is partly immersed in a liquid of unknown refractive index. Find the value of refractive index of the liquid so that the ray grazes along the face BC after refraction through the prism.
 - (b) Trace the path of the rays if it were incident normally on the face AC. [HOTS] [CBSE Ajmer 2015]
 - **Ans.** (*a*) From Snell's law

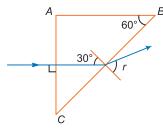
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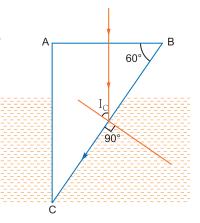
$$an_g \sin i_c = an_l \sin 90^\circ$$
$$1.5 \times \sin 60^\circ = an_l$$
$$an_l = 1.5 \times \frac{\sqrt{3}}{2} = 1.3$$





(b) The ray strikes at an angle of $30^{\circ} < i_c$. So, the ray of light deviates apart from the normal, as it moves from denser to rarer medium.





Q. 22. A ray of light incident on an equilateral glass prism propagates parallel to the base line of the prism inside it. Find the angle of incidence of this ray. Given refractive index of material of glass prism is $\sqrt{3}$. [CBSE Bhubaneshwar 2015]

Ans. From the figure, we see $r = 30^{\circ}$

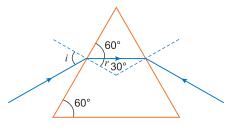
We know

$$\Rightarrow \qquad n_{21} = \frac{\sin i}{\sin r}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{\sin i}{\sin 30^{\circ}}$$

$$\Rightarrow \qquad \sin i = \sqrt{3} \sin 30^{\circ} = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \qquad i = 60^{\circ}$$



- Q. 23. A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when the angle of incidence is $\frac{3}{4}$ th of the angle of prism. Calculate the speed of light in the prism. [CBSE (AI) 2017]
 - **Ans.** Angle of prism, $A = 60^{\circ}$ (Since prism is an equilateral glass prism) We are given that

$$i = \frac{3}{4}A = \frac{3}{4} \times 60^{\circ}$$
$$i = 45^{\circ}$$

At minimum deviation,

...

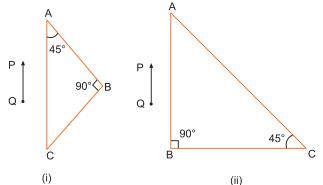
$$r = \frac{A}{2} = 30^{\circ}$$

$$n = \frac{\sin i}{\sin r} = \frac{\sin 45^{\circ}}{\sin 30^{\circ}} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

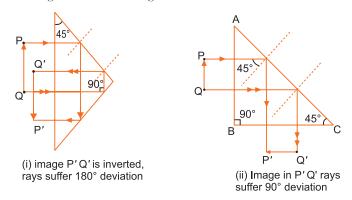
 \therefore Speed of light in the prism is given by

$$v = \frac{c}{n} = \frac{3 \times 10^8}{\sqrt{2}} = 2.1 \times 10^8 \text{ m/s}$$

Q. 24. A right-angled crown glass prism with critical angle 41° is placed before an object, PQ in two positions as shown in the figures (i) and (ii). Trace the paths of the rays from P and Q passing through the prisms in the two cases.



Ans. The formation of images is shown in figures (*i*) and (*ii*).



Short Answer Questions–II

[3 marks]

- Q. 1. (i) What is total internal reflection? Under what conditions does it occur?
 - (ii) Find a relation between critical angle and refractive index.
 - (iii) Name one phenomenon which is based on total internal reflection.

[CBSE (East) 2016, 2019 (55/1/1)]

Ans. (i) When a ray of light travels from an optically denser medium into a rarer medium at an angle greater than the critical angle, it reflects back into the denser medium. This phenomenon is called total internal reflection.

Conditions for total internal reflection:

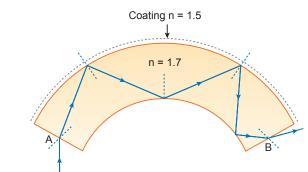
- (a) Light must travel from denser medium to rarer medium.
- (b) Angle of incidence in denser medium must be greater than critical angle.
- (*ii*) $\frac{1}{n} = \frac{\sin i}{\sin r}$, for total internal reflection to occur $i \ge i_c$; at critical angle, angle of refraction,

$$r=90^{\circ}$$
 hence $\frac{1}{n} = \frac{\sin i_c}{\sin 90^{\circ}} \Rightarrow n = \frac{1}{\sin i_c}$

- (*iii*) (a) Mirage (b) optical fibre (c) sparkling of diamond (d) shinning of air bubbles in water(*e*) totally reflecting prism.
- Q. 2. (i) Name the phenomenon on which the working of an optical fibre is based.
 - (ii) What are the necessary conditions for this phenomenon to occur?
 - (*iii*) Draw a labelled diagram of an optical fibre and show how light propagates through the optical fibre using this phenomenon. [*CBSE* (*South*) 2016, 2019 (55/2/3)]

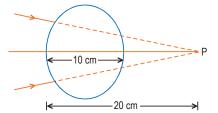


- **Ans.** (*i*) Working of an optical fibre is based on total internal reflection.
 - (*ii*) (*a*) Rays of light have to travel from optically denser medium to optically rarer medium and(*b*) Angle of incidence in the denser medium should be greater than critical angle.



Q. 3. A converging beam of light travelling in air converges at a point P as shown in the figure. When a glass sphere of refractive index 1.5 is introduced in between the path of the beam, calculate the new position of the image. Also draw the ray diagram for the image formed.

[CBSE 2019 (55/3/1)]



Ans. Given, u = 20 cm

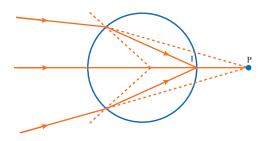
(iii)

$$n_1 = 1$$

 $R = \frac{10}{2} = 5 \text{ cm}$

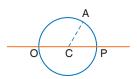
As the light passes from rare to denser medium, so

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$
$$\frac{1.5}{v} - \frac{1}{20} = \frac{1.5 - 1}{5}$$
$$\frac{1.5}{v} = \frac{1}{10} + \frac{1}{20}$$
$$\frac{1.5}{v} = \frac{2 + 1}{20}$$
$$v = +10 \text{ cm}$$



Thus, the image is formed at the other end (I) of the diameter.

Q. 4. A point 'O' marked on the surface of a glass sphere of diameter 20 cm is viewed through glass from the position directly opposite to the point O. If the refractive index of the glass is 1.5, find the position of the image formed. Also, draw the ray diagram for the image formed. Also, draw the ray diagram for the image.



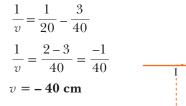
[CBSE 2019 (55/3/1)]

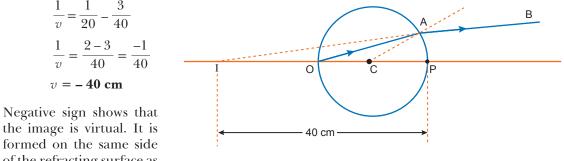
Ans. The mark O on the surface of glass sphere acts as object. The incident ray OA is in glass and refracted ray AB is in air. I is the image of O.

Thus,
$$n_1 = 1, n_2 = 1.5$$

u = -20 cm (Minus sign is taken for refraction at concave surface) As light passes from denser to rarer medium, so

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}$$
$$\frac{1}{v} + \frac{1.5}{20} = \frac{1 - 1.5}{-10}$$



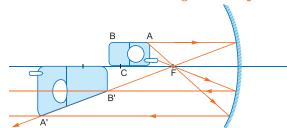


of the refracting surface as the object at a distance of 40 cm from the pole P.

Q. 5. How is the working of a telescope different from that of a microscope?

[CBSE Delhi 2012, 2019 (55/2/3)]

- **Ans.** Difference in working of telescope and microscope:
 - (i) Objective of telescope forms the image of a very far off object at or within the focus of its eyepiece. The microscope does the same for a small object kept just beyond the focus of its objective.
 - (*ii*) The final image formed by a telescope is magnified relative to its size as seen by the unaided eye while the final image formed by a microscope is magnified relative to its absolute size.
 - (iii) The objective of a telescope has large focal length and large aperture while the corresponding parameters for a microscope have very small values.
- Q. 6. (a) A mobile phone lies along the principal axis of a concave mirror. Show, with the help of a suitable diagram, the formation of its image. Explain why magnification is not uniform.
 - (b) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect this will have on the image of the object? Explain. [CBSE Delhi 2014]



The position of the image of different parts of the mobile phone depends on their position with respect to the mirror. The image of the part which is on the plane perpendicular to principal axis will be on the same plane. It will be of the same size, *i.e.*, B'C = BC. The images of the other parts of the phone are getting magnified as when the object is placed between C and F it gets magnified.

- (b) Taking the laws of reflection to be true for all points of the remaining (uncovered) part of the mirror, the image will be that of the whole object. As the area of the reflecting surface has been reduced, the intensity of the image will be low (in this case half).
- **Q.** 7. (a) Calculate the distance of an object of height h from a concave mirror of radius of curvature 20 cm, so as to obtain a real image of magnification 2. Find the location of image also.
 - (b) Using mirror formula, explain why does a convex mirror always produce a virtual [CBSE Delhi 2016] image.

Ans. (a)
$$R = -20$$
 cm and $m = -2$

Focal length $f = \frac{R}{2} = -10$ cm Magnification $m = -\frac{v}{u} = -2$ (given) $\therefore v = 2u$ Using mirror formula



Ans.

(a)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \implies \frac{1}{2u} + \frac{1}{u} = -\frac{1}{10}$$

$$\implies \frac{3}{2u} = -\frac{1}{10} \implies u = -15 \text{ cm}$$

$$\therefore \qquad v = 2 \ (-15) = -30 \text{ cm}$$
(b) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
Using sign convention for convex mirror we get

f > 0, u < 0

 $\therefore \text{ From the formula: } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ As *f* is positive and *u* is negative, *v* is always positive, hence image is always virtual.

Q. 8. What are optical fibres? Mention their one practical application.

[CBSE Delhi 2011, Guwahati 2015]

Ans. Optical Fibre: An optical fibre is a device based on total internal reflection by which a light signal may be transmitted from one place to another with a negligible loss of energy. It is a very long and thin pipe of quartz (n = 1.7) of thickness nearly $\approx 10^{-4}$ m coated all around with a material of refractive index 1.5. A large number of such fibres held together form a *light pipe* and are used for communication of light signals. When a light ray is incident on one end at a small angle of incidence, it suffers refraction from air to quartz and strikes the quartz-coating interface at an angle more than the critical angle and so suffers total internal reflection and strikes the opposite face again at an angle greater than critical angle and so again suffers total internal reflection. Thus the ray within the fibre suffers multiple total internal reflections and finally strikes the other end at an angle less than critical angle for quartz-air interface and emerges in air.

As there is no loss of energy in total internal reflection, the light signal is transmitted by this device without any appreciable loss of energy.

Application : Optical fibre is used to transmit light signal to distant places.

For diagram, Refer to Question 2 (iii) on Page 372.

- **O.9.** A convex lens made up of glass of refractive index 1.5 is dipped, in turn, in (i) a medium of refractive index 1.65, (ii) a medium of refractive index 1.33.
 - (a) Will it behave as a converging or a diverging lens in the two cases?

[CBSE (AI) 2011] (b) How will its focal length change in the two media?

Ans. Focal length of lens in liquid (l)

$$f_l = \frac{n_g - 1}{\frac{n_g}{n_l} - 1} f_d$$

(a) (i) $n_g = 1.5$, $n_l = 1.65$ $\frac{n_g}{n_l} = \frac{1.5}{1.65} < 1$, so f_l and f_a are of opposite sign, so convex lens in liquid $n_l = 1.65$

behaves as a diverging lens

(*ii*)
$$n_g = 1.5, n_l = 1.33$$

 $\therefore \quad \frac{n_g}{n_l} = \frac{1.5}{1.33} > 1$

so f_l and f_a are of same sign, so convex lens in liquid ($n_l = 1.33$) behaves as a convergent lens.

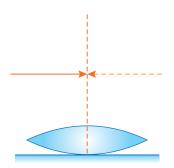
(b) (i) Focal length,
$$f_1 = \frac{1.5 - 1}{\frac{1.5}{1.65} - 1} f_a = -5.5 f_a$$

(Focal length becomes negative and its magnitude increases)

(*ii*) Focal length,
$$f_2 = \frac{1.5 - 1}{\frac{1.5}{1.33} - 1} f_a = 4f_a$$
 (Focal length increases)

Ray Optics and Optical Instruments 375

Q. 10. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x. On removing the liquid layer and repeating the experiment, the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of *x* and *y*. [CBSE 2018]



Ans. Let n_l denote the refractive index of the liquid. When the image

of the needle coincides with the lens itself; its distance from the lens, equals the relevant focal length.

With liquid layer present, the given set up, is equivalent to a combination of the given (convex) lens and a concave plane/plano concave 'liquid lens'.

We have
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and
$$\frac{1}{f} = \left(\frac{1}{f_1} + \frac{1}{f_2}\right)$$

As per the given data, we then have

ar

$$\frac{1}{f_2} = \frac{1}{y} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{(-R)}\right) = \frac{1}{R}$$

$$\therefore \qquad \frac{1}{x} = (n_l - 1)\left(-\frac{1}{R}\right) + \frac{1}{y} = \frac{-n_l}{y} + \frac{2}{y}$$

$$\therefore \qquad \frac{n_l}{y} = \frac{2}{y} - \frac{1}{x} = \left(\frac{2x - y}{xy}\right)$$
or
$$n_l = \left(\frac{2x - y}{x}\right)$$

Q. 11. A biconvex lens of glass of refractive index 1.5 having focal length 20 cm is placed in a medium of refractive index 1.65. Find its focal length. What should be the value of the refractive index of the medium in which the lens should be placed so that it acts as a plane sheet of glass?

[CBSE Bhubaneshwar 2015]

From lens formula, when lens in a medium Ans.

 $\frac{1}{f_{m}} = \left(\frac{n_g}{n_m} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$...(i) When lens in air $\frac{1}{f_a} = (n_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \qquad \dots (ii)$ From equation (i) and (ii), we get $\frac{f_a}{f_a} = \frac{\left(\frac{n_g}{n_m} - 1\right)}{\left(\frac{n_g}{n_m} - 1\right)}$

$$\int_{m}^{f_{m}} (n_{g} - 1)$$

$$\frac{20 \text{ cm}}{f_{m}} = \frac{\left(\frac{1.5}{1.65} - 1\right)}{(1.5 - 1)}$$

$$\Rightarrow \qquad f_{m} = \frac{20 \times (1.5 - 1)}{\left(\frac{1.5}{1.65} - 1\right)} = \frac{20 \times 0.5 \times 1.65}{-0.15} = -110 \text{ cm}$$

 $n_m = 1.65$

If lens in the medium behave as a plane sheet of glass. Then $f_m = \infty$



$$\frac{1}{\infty} = \left(\frac{n_g}{n_m} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\Rightarrow \left(\frac{n_g}{n_m} - 1\right) = 0 \quad \Rightarrow \quad n_g = n_m = 1.5$$

The refractive index of the medium must be 1.5.

- Q. 12. A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index 1.6. If it is immersed in a liquid of refractive index 1.3, find its new focal length. [CBSE (F) 2017]
 Ans For spherical lens (thin) having same medium in both sides
- Ans. For spherical lens (thin) having same medium in both sides

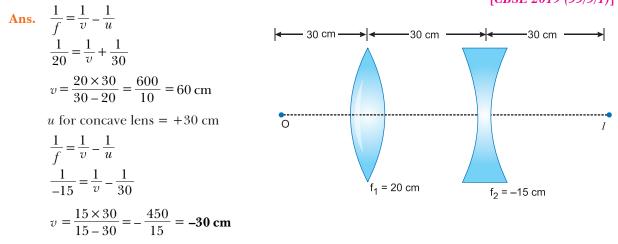
$$\frac{1}{f_{eq}} = (n_{net} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \qquad \text{where } n_{net} = \frac{n_{lens}}{n_{med.}}$$
$$\frac{1}{f_{eq}} = \left(\frac{1.6}{1.3} - 1 \right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \qquad \dots (i)$$

Also,
$$\frac{1}{f_a} = \frac{1}{20} = (1.6 - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$
...(*ii*)
 $\Rightarrow \qquad \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{20 \times 0.6} = \frac{1}{12}$

Substituting in (i)

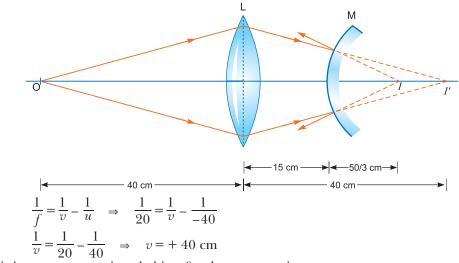
$$\Rightarrow \qquad \frac{1}{f_{eq}} = \frac{0.3}{1.3} \times \frac{1}{12} \qquad \Rightarrow \qquad f_{eq} = \frac{12 \times 1.3}{0.3} = 52 \text{ cm}$$

Q. 13. A convex lens of focal length 20 cm and a concave lens of focal length 15 cm are kept 30 cm apart with their principal axes coincident. When an object is placed 30 cm in front of the convex lens, calculate the position of the final image formed by the combination. Would this result change if the object were placed 30 cm in front of the concave lens? Give reason. [CBSE 2019 (55/5/1)]



No, the result will not change from principle of reversibility.

- Q. 14. A convex lens of focal length 20 cm is placed coaxially with a convex mirror of radius of curvature 20 cm. The two are kept 15 cm apart. A point object is placed 40 cm in front of the convex lens. Find the position of the image formed by this combination. Draw the ray diagram showing the image formation. [CBSE (AI) 2014]
- **Ans.** For convex lens, u = -40 cm, f = 20 cm



This image acts as a virtual object for the convex mirror.

$$\therefore \quad u = 40 - 15 = 25 \,\mathrm{cm} \quad \Rightarrow \quad f = \frac{20}{2} = +10 \,\mathrm{cm}$$

Using mirror formula,

 \Rightarrow

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \implies \frac{1}{10} = \frac{1}{v} + \frac{1}{25}$$
$$\frac{1}{v} = \frac{1}{10} - \frac{1}{25} \implies v = \frac{50}{3} \text{ cm} \simeq 16.67 \text{ cm}$$

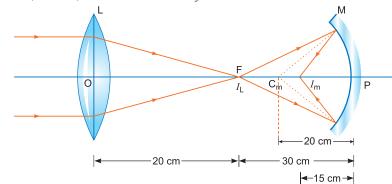
Hence, the final image is a virtual image formed at a distance of 16.67 cm.

- Q. 15. A convex lens of focal length 20 cm is placed coaxially with a concave mirror of focal length 10 cm at a distance of 50 cm apart from each other. A beam of light coming parallel to the principal axis is incident on the convex lens. Find the position of the final image formed by this combination. Draw the ray diagram showing the formation of the image. [CBSE (AI) 2014]
- Ans. For the convex lens,

...

$$u = \infty, f = 20 \text{ cm}$$
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$
$$v = 20 \text{ cm}$$

For the concave mirror, the image formed by the lens acts as the object. Hence, u = -(50 - 20) cm = -30 cm and f = -10 cm



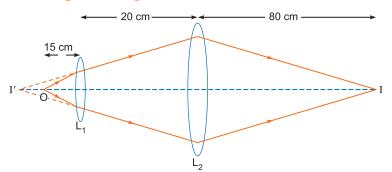
Using mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \Rightarrow \qquad \frac{1}{v} + \frac{1}{-30} = \frac{1}{-10}$$
$$\Rightarrow \qquad \frac{1}{v} - \frac{1}{30} = -\frac{1}{10} \qquad \Rightarrow \qquad v = -15 \text{ cm}$$



The lens-mirror combination, therefore, forms a real image I_m at a distance of 15 cm to the left of the concave mirror or at a distance of 35 cm to the right of the convex lens.

Q. 16. In the following diagram, an object 'O' is placed 15 cm in front of a convex lens L_1 of focal length 20 cm and the final image is formed at 'T' at a distance of 80 cm from the second lens L_2 . Find the focal length of the L_2 . [CBSE (F) 2016]



Ans. Let focal length of lens L_2 be *x* cm Now, for lens, L_1

u = -15 cm; f = +20 cm; v = ?

Using lens formula

$$\begin{aligned} \frac{1}{v} - \frac{1}{u} &= \frac{1}{f} \implies \frac{1}{v} = \frac{1}{f} + \frac{1}{u} \\ &= \frac{1}{20} + \frac{1}{-15} = \frac{15 - 20}{300} = \frac{-5}{300} = \frac{-1}{60} \end{aligned}$$

 \Rightarrow v = -60 cm

i.e., 60 cm from lens in the direction of object.

Now, for lens, L_2

The image formed by lens L_1 , will act as object for lens L_2

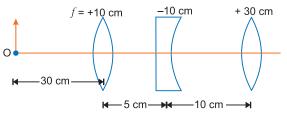
$$u = -60 + (-20) = -80 \text{ cm}$$

 $v = +80 \text{ cm} \text{ (given)}$ and $f = x \text{ cm}$
Applying lens formula for lens L_2

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \implies \frac{1}{x} = \frac{1}{80} - \frac{1}{(-80)} = \frac{1}{80} + \frac{1}{80}$$
$$\implies \frac{1}{x} = \frac{2}{80} \implies x = 40 \text{ cm}$$

Hence, focal length of lens L_2 is 40 cm.

Q. 17. Find the position of the image formed of an object 'O' by the lens combination given in the [CBSE (F) 2011, 2019 (55/4/1)] figure.



Ans. For first lens, $u_1 = -30 \text{ cm}, f_1 = +10 \text{ cm}$

$$\therefore \qquad \text{From lens formula, } \frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1} \\ \Rightarrow \qquad \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30}$$



 \Rightarrow $v_1 = 15 \text{ cm}$

The image formed by the first lens serves as the object for the second. This is at a distance of (15-5) cm = 10 cm to the right of the second lens. Though the image is real, it serves as a virtual object for the second lens, which means that the rays appear to come from it for the second lens.

For second lens, $f_2 = -10$ cm, $u_2 = 15 - 5 = +10$ cm

:.
$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{10} \Rightarrow v_2 = \infty$$

The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens.

For third lens, $f_3 = +30$ cm, $u_3 = \infty$

From lens formula, $\frac{1}{v_2} = \frac{1}{f_3} + \frac{1}{u_3} = \frac{1}{30} + \frac{1}{\infty}$ $v_3 = 30 \text{ cm}$

The final image is formed at a distance 30 cm to the right of third lens.

- Q. 18. (i) A screen is placed at a distance of 100 cm from an object. The image of the object is formed on the screen by a convex lens for two different locations of the lens separated by 20 cm. Calculate the focal length of the lens used.
 - (*ii*) A converging lens is kept coaxially in contact with a diverging lens both the lenses being of equal focal length. What is the focal length of the combination? [CBSE (North) 2016]
 - **Ans.** (*i*) For first position of the lens, we have

$$\frac{1}{f} = \frac{1}{y} - \frac{1}{(-x)} \qquad \Rightarrow \quad \frac{1}{f} = \frac{1}{y} + \frac{1}{x} \dots (i)$$

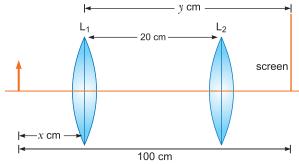
For second position of lens, we have

$$\frac{1}{f} = \frac{1}{y - 20} - \frac{1}{[-(x + 20)]}$$
$$\frac{1}{f} = \frac{1}{y - 20} + \frac{1}{x + 20} . \qquad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{(y-20)} + \frac{1}{(x+20)}$$
$$\frac{x+y}{xy} = \frac{(x+20) + (y-20)}{(y-20)(x+20)}$$
$$\frac{x+y}{xy} = \frac{x+y}{(y-20)(x+20)}$$
$$\therefore \qquad xy = (y-20) (x+20)$$
$$\Rightarrow \qquad xy = xy - 20x + 20y - 400$$
$$\Rightarrow \qquad 20x - 20y = -400$$
$$\therefore \qquad x-y = -20$$
Also,
$$x + y = 100$$
On solving, we have
$$x = 40 \text{ cm and } y = 60 \text{ cm}$$

 $\frac{1}{f} = \frac{1}{60} - \frac{1}{-40} = \frac{5}{120} \implies f = 24$ cm





...

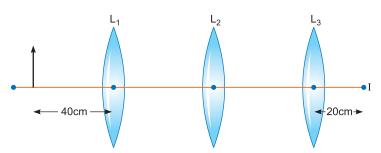
(ii) Let focal length of the combination be f.

$$\therefore \qquad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \qquad \frac{1}{f} = \frac{1}{f} + \left(-\frac{1}{f}\right)$$

$$\Rightarrow \qquad \frac{1}{f} = 0 \Rightarrow f = \text{infinite.}$$

Q. 19. You are given three lenses L_1 , L_2 and L_3 each of focal length 20 cm. An object is kept at 40 cm in front of L_1 , as shown. The final real image is formed at the focus '*I*' of L_3 . Find the separations between L_1 , L_2 and L_3 . [CBSE (AI) 2012]



Ans. Given $f_1 = f_2 = f_3 = 20$ cm For lens L_1 , $u_1 = -40$ cm

By lens formula $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{20} + \frac{1}{-40} \Rightarrow v_1 = 40 \text{ cm}$

For lens L_3 , $f_3 = 20$ cm, $v_3 = 20$ cm, $u_3 = ?$

By lens formula,
$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3} \implies \frac{1}{20} - \frac{1}{u_3} = \frac{1}{20}$$

 $\frac{1}{u_3} = 0 \implies u_3 = \infty$

Thus lens L_2 should produce image at infinity.

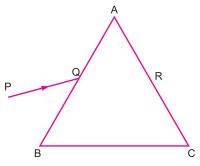
Hence, for L_2 , its objective should be at focus. The image formed by lens L_1 is at 40 cm on the right side of lens L_1 which lies at 20 cm left of lens L_2 *i.e.*, focus of lens L_2 .

Hence, the distance between L_1 and $L_2 = 40 + 20 = 60$ cm.

As the image formed by lens L_2 lies at infinity, then the distance between lens L_2 and L_3 does not matter.

Hence, the distance between L_2 and L_3 can have any value.

Q. 20. A ray PQ incident on the face AB of a prism ABC, as shown in the figure, emerges from the face AC such that AQ = AR. Draw the ray diagram showing the passage of the ray through the prism. If the angle of the prism is 60° and refractive index of the material of the prism is $\sqrt{3}$, determine P the values of angle of incidence and angle of deviation. [CBSE Panchkula 2015]

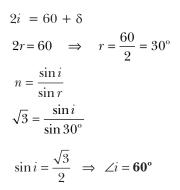


Ans.

$$\angle A = 60^{\circ}$$
 and $n = \sqrt{3}$
 $i + e = A + \delta$

Since *QR* is parallel to *BC* hence this is the case of minimum deviation.

$$i = e$$



Substitute in (*i*), we have

 $120 = 60 + \delta \Longrightarrow \ \delta = 60^{\circ}$

Q. 21. A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS such that AQ = AR. If the angle of prism $A = 60^{\circ}$ and refractive index of material of prism is $\sqrt{3}$, calculate angle θ . [CBSE North 2016]

 θ is the angle of minimum deviation.

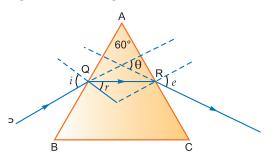
Ans. Given, AQ = AR, we have

...

$$QR \mid BC$$

At the minimum deviation, the refracted ray inside the prism becomes parallel to its base.

 $n = \frac{\sin\left(\frac{A+\theta}{2}\right)}{\sin\left(\frac{A}{2}\right)} \implies \sqrt{3} = \frac{\sin\left(\frac{60^{\circ}+\theta}{2}\right)}{\sin 30^{\circ}}$ $\sin\left(\frac{60^{\circ}+\theta}{2}\right) = \frac{\sqrt{3}}{2} \implies \sin\left(\frac{60^{\circ}+\theta}{2}\right) = \sin 60^{\circ}$ $\frac{60^{\circ}+\theta}{2} = 60^{\circ} \implies \theta = 60^{\circ}$



60° ~~∫δ

Q

в

R

R

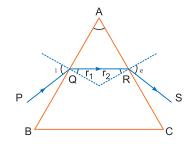
C

R

Q. 22. Figure shows a ray of light passing through a prism. If the refracted ray *QR* is parallel to the base *BC*, show that

(*i*)
$$r_1 = r_2 = A/2$$
,

(*ii*) angle of minimum deviation, $D_m = 2i - A$.



Ans. (*i*) We know that $r_1 + r_2 = A$

[CBSE (F) 2014]

...(i)

S

C



Since QR is parallel to BC

So, $r_1 = r_2$ and i = e

 \Rightarrow $r_1 = r_2 = A / 2$ Therefore, $2r_1$ or $2r_2 = A$

(*ii*) D_m = Deviation at the first face + Deviation of the second face

$$= (i - r_1) + (e - r_2) = (i + e) - (r_1 + r_2)$$

= 2i - A (: i = e)

Q. 23. A compound microscope uses an objective lens of focal length 4 cm and eyepiece lens of focal length 10 cm. An object is placed at 6 cm from the objective lens. Calculate the magnifying power of the compound microscope. Also calculate the length of the microscope. [CBSE (AI) 2011]

Ans. Given $f_o = 4 \text{ cm}, f_e = 10 \text{ cm}$ $u_0 = -6 \text{ cm}$

=

Magnifying power of microscope

$$M = -\frac{|v_o|}{|u_o|} \left(1 + \frac{D}{f_e}\right)$$

From lens formula $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$$\Rightarrow \frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12}$$

$$\Rightarrow \qquad v_o = 12 \text{ cm}$$

$$\therefore \qquad m = -\frac{12}{6} \left(1 + \frac{25}{10} \right) = -2 \times 3.5 = -7$$

Negative sign shows that the image is inverted. Length of microscope $L = |v_o| + |u_e|$

For eye lens
$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

 $\Rightarrow \qquad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{10} \quad (v_e = D = -25 \text{ cm}, u_e = ?)$

$$\Rightarrow$$
 $u_e = -\frac{50}{7}$ cm = -7.14 cm

:.
$$L = |v_o| + |u_e| = 12 + 7.14 = 19.14 \text{ cm}$$

Q. 24. The total magnification produced by a compound microscope is 20. The magnification produced by the eye piece is 5. The microscope is focussed on a certain object. The distance between the objective and eyepiece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal length of the objective and the eye piece. [CBSE Delhi 2014]

Ans. Here,
$$M = -20$$
, $m_e = 5$, $v_e = -20$ cm

For eyepiece,
$$m_e = \frac{v_e}{u_e}$$

 $\Rightarrow 5 = \frac{-20}{u_e} \Rightarrow u_e = \frac{-20}{5} = -4 \text{ cm}$

Using lens formula,

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \implies -\frac{1}{20} + \frac{1}{4} = \frac{1}{f_e}$$
$$\implies \frac{-1+5}{20} = \frac{1}{f_e} \implies f_e = 5 \text{ cm}$$

Now, total magnification

$$M = m_e \times m_o$$

$$-20 = 5 \times m_o \implies m_o = -4$$
Also
$$|v_o| + |u_e| = 14$$

$$|v_o| + |-4| = 14$$

$$v_o = 14 - 4 = 10 \text{ cm}$$

$$m_o = 1 - \frac{v_o}{f_o} \implies -4 = 1 - \frac{10}{f_o}$$

$$-5 = -\frac{10}{f_o} \implies f_o = 2 \text{ cm}.$$

Q. 25. A small telescope has an objective lens of focal length 150 cm and eyepiece of focal length 5 cm. What is the magnifying power of the telescope for viewing distant objects in normal adjustment?

If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens? [CBSE Allahabad 2015]

Ans. If the telescope is in normal adjustment, *i.e.*, the final image is at infinity.

$$M = \frac{f_o}{f_e}$$

Since $f_o = 150 \text{ cm}, f_e = 5 \text{ cm}$

$$\therefore \qquad M = \frac{150}{5} = 30$$

If tall tower is at distance 3 km from the objective lens of focal length 150 cm. It will form its image at distance v_o So,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{150 \text{ cm}} = \frac{1}{v_o} - \frac{1}{(-3 \text{ km})}$$

$$\frac{1}{v_o} = \frac{1}{1.5 \text{ m}} - \frac{1}{3000 \text{ m}}$$

$$v_o = \frac{3000 \times 1.5}{3000 - 1.5} = \frac{4500}{2998.5} = 1.5 \text{ m}$$
Magnification,
$$m_o = \frac{I}{O} = \frac{h_i}{h_o} = \frac{v_o}{u_o}$$

$$\frac{h_i}{100 \text{ m}} = \frac{1.5 \text{ m}}{3 \text{ km}} = \frac{1.5}{3000}$$

$$h_i = \frac{1.5 \times 100}{3000} = \frac{1}{20} \text{ m}$$

$$h_i = 0.05 \text{ m}$$

Q. 26. An amateur astronomer wishes to estimate roughly the size of the sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eyepiece of focal length 10 cm. By adjusting the distance of the eyepiece from the objective, he obtains an image of the sun on a screen 40 cm behind the eyepiece. The diameter of the sun's image is measured to be 6.0 cm. Estimate the sun's size, given that the average earth-sun distance is 1.5×10^{11} m.

[CBSE 2019 (55/5/1)]

Ans. For eyepiece.

Given,
$$v_e = 40 \text{ cm}, f_e = 10 \text{ cm}$$



$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

or $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{40} - \frac{1}{10}$
 $\Rightarrow u_e = \frac{-40}{3}$ cm

Magnification produced by eye piece is

$$m_e = \frac{v_e}{|u_e|} = \frac{40}{40/3} = 3$$

Diameter of the image formed by the objective is

$$d = 6/3 = 2 \text{ cm}$$

If D be the diameter of the sun then the angle subtended by it on the objective will be

$$a = \frac{D}{1.5 \times 10^{11}} \text{rad}$$

...

Angle subtended by the image at the objective

= angle subtended by the sun

$$\therefore \alpha = \frac{\text{Size of image}}{f_0} = \frac{2}{200} = \frac{1}{100} \text{ rad}$$
$$\therefore \frac{D}{1.5 \times 10^{11}} = \frac{1}{100}$$
$$\Rightarrow D = 1.5 \times 10^9 \text{ m}$$

Q. 27. An object is placed 40 cm from a convex lens of focal length 30 cm. If a concave lens of focal length 50 cm is introduced between the convex lens and the image formed such that it is 20 cm from the convex lens, find the change in the position of the image.

[CBSE Chennai 2015] [HOTS]

Ans. For the convex lens, $f_1 = +30$ cm and object distance $u_1 = -40$ cm, therefore,

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$
$$\frac{1}{+30} = \frac{1}{v_1} - \frac{1}{-40}$$
$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{40} = \frac{1}{120}$$

 $\Rightarrow v_1 = + 120 \text{ cm}, \text{ a real image is formed.}$ On introducing a concave lens, $f_2 = -50 \text{ cm}$ and $u_2 = 120 - 20 = + 100 \text{ cm}$ from the concave lens

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2} \qquad \frac{1}{-50} = \frac{1}{v_2} - \frac{1}{+100}$$
$$\frac{1}{v_2} = -\frac{1}{50} + \frac{1}{100} = -\frac{1}{100}$$
$$v_2 = -100 \text{ cm}$$

A virtual image is formed at the distance of 100 cm from the concave lens.

The change in position between the real image and the virtual image is 100 cm+100 cm=+200 cm to the left of its original position.

Ray Optics and Optical Instruments 385

 n_1

- Q. 28. A biconvex lens with its two faces of equal radius of curvature R is made of a transformation refractive index n_1 . It is kept in contact with a medium of refractive refractive
 - (a) Find the equivalent focal length of the combination.
 - (b) Obtain the condition when this combination acts as a diverging lens.
 - (c) Draw the ray diagram for the case $n_1 > (n_2 + 1)/2$, when the object is kept far away from the lens. Point out the nature of the image formed by the system. [CBSE Patna 2015] [HOTS]
 - **Ans.** (*a*) If refraction occurs at first surface

$$\frac{n_1}{v_1} - \frac{1}{u} = \frac{(n_1 - 1)}{R} \qquad \dots (i)$$

If refraction occurs at second surface, and the image of the first surface acts as an object

$$\frac{n_2}{v} - \frac{n_1}{v_1} = \frac{n_2 - n_1}{-R} \qquad \dots (ii)$$

On adding equation (i) and (ii), we get

$$\frac{n_2}{v} - \frac{1}{u} = \frac{2n_1 - n_2 - 1}{R}$$

If rays are coming from infinity, *i.e.*, $u = -\infty$ then v = f

$$\frac{n_2}{f} + \frac{1}{\infty} = \frac{2n_1 - n_2 - 1}{R} \implies f = \frac{n_2 R}{2n_1 - n_2 - 1}$$

(b) If the combination behave as a diverging system then f < 0. This is possible only when

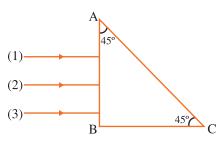
$$\Rightarrow \qquad \begin{array}{l} 2n_1 - n_2 - 1 < 0 \\ 2n_1 < n_2 + 1 \\ \Rightarrow \qquad n_1 < \frac{\left(n_2 + 1\right)}{2} \end{array}$$

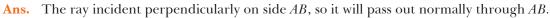
(c) If the combination behaves as a converging lens then f > 0. It is possible only when

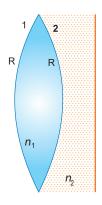
$$\begin{array}{ccc} & 2n_1-n_2-1>0\\ \Rightarrow & 2n_1->n_2+1\\ \Rightarrow & n_1>\frac{(n_2+1)}{2} \end{array}$$

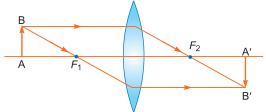
Nature of the image formed is real.

Q. 29. Three rays (1, 2, 3) of different colours fall normally on one of the sides of an isosceles right angled prism as shown. The refractive index of prism for these rays is 1.39, 1.47 and 1.52 respectively. Find which of these rays get internally reflected and which get only refracted from AC. Trace the paths of rays. Justify your answer with the help of necessary calculations. [CBSE (F) 2016] [HOTS]











On face AC, $i = 45^{\circ}$ For total internal reflection to take place at face AC, Angle of incidence > critical angle

$$45^{\circ} > i_{c}$$

$$\sin 45^{\circ} > \sin i_{c}$$

$$\frac{1}{\sqrt{2}} > \frac{1}{n} \qquad \left[\therefore i_{c} = \sin^{-1} \left(\frac{1}{n} \right) \right]$$

$$\Rightarrow \qquad \sqrt{2} < n \qquad \Rightarrow \qquad 1.414 < n$$

Hence, rays 2, 3 will undergo TIR and path of ray will be as shown.

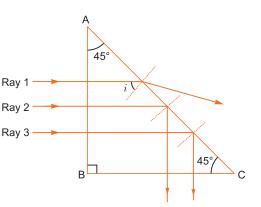
Ray 1 is refracted from AC.

- Q. 30. A ray of light incident on one of the faces of a glass prism of angle 'A' has angle of incidence 2A. The refracted ray in the prism strikes the opposite face which is silvered, the reflected ray from it retraces its path. Trace the ray diagram and find the relation between the refractive index of the material of the prism and the angle of the prism. [CBSE Chennai 2015] [HOTS]
- Ans. From Snell's law

$$n = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin r} \qquad \dots(i)$$

In ΔXQR , $(90^{\circ} - r) + A + 90^{\circ} = 180^{\circ}$
or $r = A \qquad \dots(ii)$
From Eq. (i) and (ii), we get
 $n = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A} = \frac{2\sin A\cos A}{\sin A} = 2\cos A$
 $\therefore \qquad A = \cos^{-1}(n/2)$

. . .



- Q. 31. A ray PQ incident normally on the refracting face BA is refracted in the prism BAC made of material of refractive index 1.5. Complete the path of ray through the prism. From which face will the ray emerge? Justify your answer. [CBSE Central 2016] [HOTS]
- **Ans.** For face $AB, \angle i = 0^\circ$, $\therefore \angle r = 0^\circ$, the ray will pass through AB undeflected Now, at face AC

Here,
$$i_c = \sin^{-1}\left(\frac{1}{n}\right)$$

 $= \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}(0.66)$

 $\angle i$ on face AC is 30° which is less than $\angle i_c$. Hence, the ray get refracted.

And, applying Snell's law at face AC

$$\sin 30^{\circ} \times \frac{3}{2} = \sin r \times 1$$

$$\Rightarrow \quad \sin r = \frac{1}{2} \times \frac{3}{2} \quad \Rightarrow \quad r = \sin^{-1} \left(\frac{3}{4}\right) = \sin^{-1} (0.75)$$

And, clearly r > i, as ray passes from denser to rarer medium.

Q. 32. Trace the path of a ray of light passing through a glass prism (ABC) as shown in the figure. If the refractive index of glass is $\sqrt{3}$, find out of the value of the angle of emergence from the prism. [CBSE (F) 2012] [HOTS]

В



60°

90°

60°

С

C

_ Normal

B

റ്

≺60°

30

Ans. Given $n_g = \sqrt{3}$

$$i = 0$$

At the interface *AC*,

By Snell's Law

$$\frac{\sin i}{\sin r} = \frac{n_g}{n_a}$$

But $\sin i = \sin 0^\circ = 0$, hence r = 0At the interface AB, $i = 30^\circ$

Applying Snell's Law

$$\frac{\sin 30^{\circ}}{\sin e} = \frac{n_a}{n_g} = \frac{1}{\sqrt{3}} \implies \sin e = \sqrt{3} \sin 30^{\circ} \implies e = 60^{\circ}$$

- Q. 33. A ray of light incident on the face *AB* of an isosceles triangular prism makes an angle of incidence (*i*) and deviates by angle β as shown in the figure. Show that in the position of minimum deviation $\angle \beta = \angle \alpha$. Also find out the condition when the refracted ray QR suffers total internal reflection. [*CBSE 2019 (55/2/2)*]
- Ans. For minimum deviation

$$r_{1} + r_{2} = A; \qquad r_{1} = r_{2}$$
Also,
$$(90 - \beta) + (90 - \beta) = A$$

$$\Rightarrow \qquad 180 - 2\beta = A$$

$$\Rightarrow \qquad 2\beta = 180 - A$$

$$\Rightarrow \qquad 2\beta = 2\alpha$$

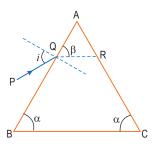
$$\Rightarrow \qquad \beta = \alpha$$
We have,
$$r_{1} + r_{2} = A$$

$$r_{1} + i_{c} = A \qquad (Take r_{2} = i_{c})$$

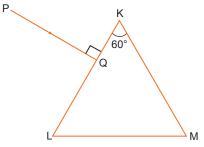
$$i_{c} = A - r_{1}$$

$$i_{c} = A - (90 - \beta)$$

1



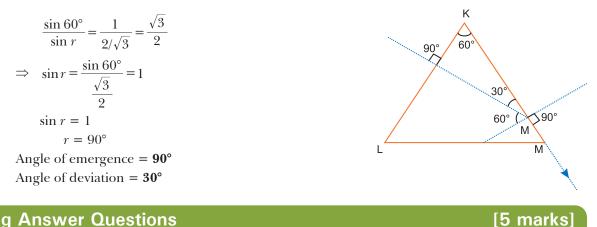
Q. 34. A triangular prism of refracting angle 60° is made of a transparent material of refractive index $2/\sqrt{3}$. A ray of light is incident normally on the face KL as shown in the figure. Trace the path of the ray as it passes through the prism and calculate the angle of emergence and angle of deviation. [CBSE 2019 (55/2/1)]



Ans. When light ray incident on face *KL*, it is pass undeviated, because it is normal to the surface and incident on face *KM*. The angle of incidence for face *KM* is equal to 60°.

$$\frac{\sin 60^{\circ}}{\sin r} = \frac{n_2}{n_1} \qquad \qquad \begin{bmatrix} n_2 = \text{Second medium} = \text{air} \\ n_1 = \text{Glass medium} = 2/\sqrt{3} \end{bmatrix}$$



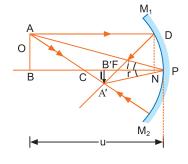


Long Answer Questions

- (i) Derive the mirror formula. What is the corresponding formula for a thin lens? Q. 1.
 - (ii) Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed. [CBSE Delhi 2011]
- (i) Mirror Formula: M_1M_2 is a concave mirror having pole P, focus F and centre of Ans. curvature C.

An object AB is placed in front of mirror with point B on the principal axis. The image formed by mirror is A'B'. The perpendicular dropped from point of incidence D on principal axis is DN

In $\triangle ABC$ and $\triangle A' B' C$	
$\angle ABC = \angle A' B' C$	(each equal to 90°)
$\angle ACB = \angle A'CB'$	(opposite angles)
Both triangles are similar.	
$\therefore \frac{AB}{A'B'} = \frac{BC}{B'C}$	(<i>i</i>)
Now in ΔDNF and $\Delta A' B' F$	
$\angle DNF = \angle A'B'F$	(each equal to 90°)
$\angle DFN = \angle A'FB'$	(opposite angles)
$\therefore \text{Both triangles are simil} \\ DN FN AB$	



... $\frac{DN}{A'B'} = \frac{FN}{B'F} \text{ or } \frac{AB}{A'B'} = \frac{FN}{B'F} (\because AB = DN) \dots (ii)$ Comparing (i) and (ii), we get

$$\frac{BC}{B'C} = \frac{FN}{B'F} \qquad \dots (iii)$$

If aperture of mirror is very small, the point N will be very near to P, so FN = FP

$$\therefore \frac{BC}{B'C} = \frac{FP}{B'F} \text{ or } \frac{PB-PC}{PC-PB'} = \frac{FP}{PB'-PF} \quad \dots (iv)$$

By sign convention

Distance of object from mirror PB = -uDistance of image from mirror PB' = -vFocal length of mirror PF = -fRadius of curvature of mirror PC = -R = -2f Substituting these values in (iv), we get

$$\frac{-u - (-2f)}{-2f - (-v)} = \frac{-f}{-v - (-f)}$$
$$\frac{-u + 2f}{-2f + v} = \frac{-f}{-v + f}$$

 $\Rightarrow 2f^2 - vf = -uf + uv + 2f^2 - 2vf \quad \text{or} \quad vf + uf = uv$

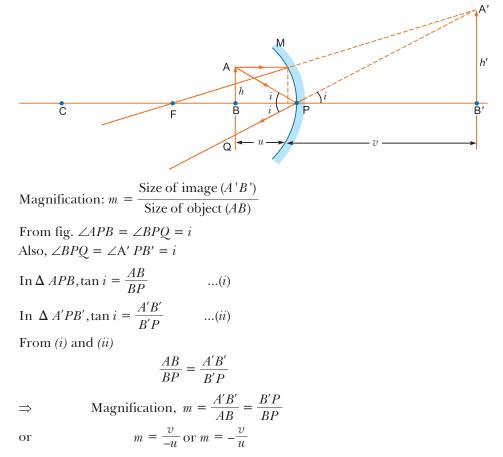
Dividing both sides by *uvf* we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

The corresponding formula for thin lens is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

(ii) **Ray Diagram:** The ray diagram of image formation for an object between focus *(F)* and pole *(P)* of a concave mirror is shown in fig.



Q. 2. With the help of a ray diagram, show the formation of image of a point object due to refraction of light at a spherical surface separating two media of refractive indices n_1 and n_2 ($n_2 > n_1$) respectively. Using this diagram, derive the relation

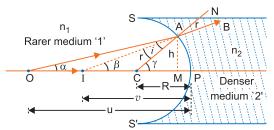
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_1 - n_2}{R}$$

Write the sign conventions used. What happens to the focal length of convex lens when it is immersed in water?



Ans. Formula for Refraction at Spherical Surface

Concave Spherical Surface: Let SPS' be a spherical refracting surface, which separates media '1' and '2'. Medium '1' is rarer and medium '2' is denser. The refractive indices of media '1' and '2' are n_1 and n_2 respectively ($n_1 < n_2$). Let P be the pole and C the centre of curvature and PC the principal axis of spherical refracting surface.



O is a point-object on the principal axis. An incident ray OA, after refraction at A on the spherical surface bends towards the normal CAN and moves along AB. Another incident ray OP falls on the surface normally and hence passes undeviated after refraction. These two rays, when produced backward meet at point I on principal axis. Thus I is the virtual image of O.

Let angle of incidence of ray OA be *i* and angle of refraction be *r i.e.*,

$$\angle OAC = i \quad \text{and} \quad \angle NAB = r$$
Let
$$\angle AOP = \alpha, \ \angle AIP = \beta \text{ and} \ \angle ACP = \gamma$$
In triangle *OAC*
$$\gamma = \alpha + i \quad \text{or} \quad i = \gamma - \alpha$$
In triangle *AIC*,
$$\gamma = \beta + r \quad \text{or} \quad r = \gamma - \beta$$
From Snell's law
$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$
...(ii)

If point A is very near to P, then angles i, r, α , β , γ will be very small, therefore sin i=i and sin r=r

Substituting values of i and r from (i) and (ii) we get

$$\frac{\gamma - \alpha}{\gamma - \beta} = \frac{n_2}{n_1} \quad \text{or} \quad n_1 (\gamma - \alpha) = n_2 (\gamma - \beta) \qquad \dots (iv)$$

The length of perpendicular AM dropped from A on the principal axis is h i.e., AM = h. As angles α , β and γ are very small, therefore

$$\tan \alpha = \alpha$$
, $\tan \beta = \beta$, $\tan \gamma = \gamma$

Substituting these values in equation *(iv)*

$$n_1 (\tan \gamma - \tan \alpha) = n_2 (\tan \gamma - \tan \beta)$$
 ...(v)

As point A is very close to P, point M is coincident with P

$$\tan \alpha = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AM}{MO} = \frac{h}{PO}$$
$$\tan \beta = \frac{AM}{MI} = \frac{h}{PI}, \quad \tan \gamma = \frac{AM}{MC} = \frac{h}{PC}$$

Substituting this value in (v), we get

$$n_1 \left(\frac{h}{PC} - \frac{h}{PO}\right) = n_2 \left(\frac{h}{PC} - \frac{h}{PI}\right)$$
$$\frac{n_1}{PC} - \frac{n_1}{PO} = \frac{n_2}{PC} - \frac{n_2}{PI} \qquad \dots (vi)$$

or

In In

Let u, v and R be the distances of object O, image I and centre of curvature C from pole P. By sign convention PO, PI and PC are negative, *i.e.*, u = -PO, v = -PI and R = -PC

> **Ray Optics and Optical Instruments** 391

Substituting these values in (vi), we get

$$\frac{n_1}{(-R)} - \frac{n_1}{(-u)} = \frac{n_2}{(-R)} - \frac{n_2}{(-v)} \text{ or } \frac{n_1}{R} - \frac{n_1}{u} = \frac{n_2}{R} - \frac{n_2}{v}$$
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

or

Sign Conventions:

(*i*) All the distances are measured from optical centre (P) of the lens.

(ii) Distances measured in the direction of incident ray of light are taken positive and vice-versa. As we know

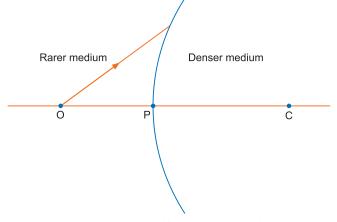
$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

When convex lens is immersed in water, refractive index *n* decreases and hence focal length will increase *i.e.*, the focal length of a convex lens increases when it is immersed in water.

Q. 3. A spherical surface of radius of curvature *R*, separates a rarer and a denser medium as shown in the figure.

Complete the path of the incident ray of light, showing the formation of a real image. Hence derive the relation connecting object distance 'u', image distance 'v', radius of curvature R and the refractive indices n_1 and n_2 of two media.

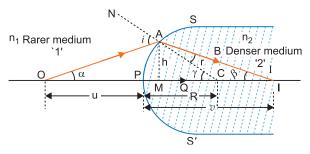
Briefly explain, how the focal length of a convex lens changes, with increase in wavelength of incident light.



[CBSE Delhi 2014; Central 2016; (F) 2017; Sample Paper 2016]

Ans. Relation of object and image distances of a convex spherical surface: Let *SPS'* be the convex spherical refracting surface, separating the two media of refractive indices n_1 and n_2 respectively

 $(n_1 < n_2)$ *i.e.*, medium '1' is rarer and medium '2' is denser. Let *P* be the pole, *C* the centre of curvature and *PC* the principal axis of convex refracting surface. *O* is a distant point object on the principal axis. The ray *OA* starting from *O* is incident on point *A* of the spherical surface, *CAN* is normal at point *A* of the surface. Due to going from rarer to denser medium the ray *OA* deviates along the normal *CAN* and is refracted along the



direction *AB*. The another ray *OP* starting from *O* is incident normally on the spherical surface and passes undeviated after refraction along *PQ*. Both the rays *AB* and *PQ* meet at point *I* on the principal axis, *i.e.*, *I* is the real image of point object *O*.

Let *i* be the angle of incidence of ray *OA* and *r* the angle of refraction in the denser medium *i.e.*, $\angle OAN = i$ and $\angle CAI = r$. Let $\angle AOP = \alpha$, $\angle AIP = \beta$ and $\angle ACP = \gamma$



In triangle *OAC*,
$$i = \gamma + \alpha$$
 ...(*i*)

 $\gamma = \beta + r$ or $r = \gamma - \beta$ In triangle AIC, ...(*ii*)

 $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$

From Snell's law

If point *A* is very close to *P*, then angles *i*, *r*, α , β and γ will be very small, therefore

 $\sin i = i$ and $\sin r = r$

From equation (iii),

$$\frac{i}{r} = \frac{n_2}{n_1}$$

~

Substituting values of i and r from (i) and (ii), we get

$$\frac{\gamma + \alpha}{\gamma - \beta} = \frac{n_2}{n_1} \text{or } n_1(\gamma + \alpha) = n_2(\gamma - \beta) \qquad \dots (iv)$$

Let *h* be the height of perpendicular drawn from *A* on principal axis *i.e.*, AM = h. As α , β and γ are very small angles.

$$\tan \alpha = \alpha$$
, $\tan \beta = \beta$ and $\tan \gamma = \gamma$

h

Substituting these values in (iv)

$$n_1(\tan\gamma + \tan\alpha) = n_2(\tan\gamma - \tan\beta)$$
 ...(v)

...(*iii*)

As point A is very close to point P, point M is coincident with P.

From figure

$$\tan \alpha = \frac{AM}{OM} = \frac{h}{OP}$$
$$\tan \beta = \frac{AM}{MI} = \frac{h}{PI}$$
$$\tan \gamma = \frac{AM}{MC} = \frac{h}{PC}$$
in (v) we get

Substituting these values in (v), we get

or
$$n_1 \left(\frac{h}{PC} + \frac{h}{OP}\right) = n_2 \left(\frac{h}{PC} - \frac{h}{PI}\right)$$
$$\dots(vi)$$

If the distances of object O, image I, centre of curvature C from the pole be u, v and R respectively, then by sign convention PO is negative while PC and PI are positive. Thus,

$$u = -PO, \quad v = +PI, \quad R = +PO$$

Substituting these values in (vi), we get

...

$$\frac{\overline{R}}{R} - \frac{\overline{u}}{u} = \frac{\overline{R}}{R} - \frac{\overline{v}}{v}$$
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The focal length of a convex lens is given by

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

 $n_1\left(\frac{1}{R} - \frac{1}{u}\right) = n_2\left(\frac{1}{R} - \frac{1}{v}\right)$

 $n_1 \quad n_1 \quad n_2 \quad n_2$

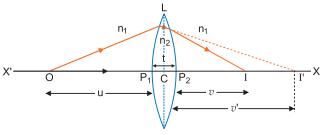
According to Cauchy's formula

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \dots$$

Then *n* varies inversely as λ .

When wavelength increases, the refractive index n decreases; so focal length of lens increases with increase of wavelength.

- Q. 4. Draw a ray diagram for formation of image of a point object by a thin double convex lens having radii of curvature R_1 and R_2 . Hence, derive lens maker's formula for a double convex lens. State the assumptions made and sign convention used. [CBSE (F) 2013, (Central) 2016, 2020 (55/2/1)]
- Lens Maker's Formula: Suppose L is Ans. a thin lens. The refractive index of the material of lens is n_2 and it is placed in a medium of refractive index n_1 . The optical centre of lens is C and X ' X is the principal axis. The radii of curvature of the surfaces of the lens are R_1 and R_2 and their poles are P_1 and P_2 The thickness of



lens is t, which is very small. O is a point object on the principal axis of the lens. The distance of O from pole P_1 is u. The first refracting surface forms the image of O at I' at a distance v' from P_1 . From the refraction formula at spherical surface

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \qquad \dots (i)$$

The image *I*' acts as a virtual object for second surface and after refraction at second surface, the final image is formed at I. The distance of I from pole P_2 of second surface is v. The distance of virtual object (I') from pole P_9 is (v' - t).

For refraction at second surface, the ray is going from second medium (refractive index n_2) to first medium (refractive index n_1), therefore from refraction formula at spherical surface

$$\frac{n_1}{v} - \frac{n_2}{(v'-t)} = \frac{n_1 - n_2}{R_2} \tag{ii}$$

For a thin lens *t* is negligible as compared to v' therefore from (*ii*)

$$\frac{n_1}{v} - \frac{n_2}{v'} = -\frac{n_2 - n_1}{R_2} \qquad \dots (iii)$$

Adding equations (i) and (iii), we get

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$
$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

or

i.e. $\frac{1}{v} - \frac{1}{u} = ({}_1n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \dots (iv)$ where ${}_1n_2 = \frac{n_2}{n_1}$ is refractive index of second medium (*i.e.*, medium of lens) with respect to first medium. If the object O is at infinity, the image will be formed at second focus *i.e.*,

$$\text{if } u = \infty, v = f_2 = f$$

Therefore from equation (iv)

$$\begin{split} & \frac{1}{f} - \frac{1}{\infty} = (_1 n_2 - 1) \bigg(\frac{1}{R_1} - \frac{1}{R_2} \bigg) \\ & i.e., \qquad \qquad \frac{1}{f} = (_1 n_2 - 1) \bigg(\frac{1}{R_1} - \frac{1}{R_2} \bigg) \\ & \dots(v) \end{split}$$

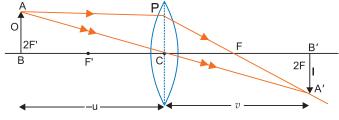
This formula is called *Lens-Maker's formula*.



If first medium is air and refractive index of material of lens be *n*, then $_1n_2 = n$, therefore the modified equation (*v*) may be written as

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \qquad \dots (vi)$$

- Q. 5. Draw a ray diagram to show the formation of real image of the same size as that of the object placed in front of a converging lens. Using this ray diagram establish the relation between u, v and f for this lens.
- **Ans.** Thin Lens Formula: Suppose an object *AB* of finite size is placed normally on the principal axis of a thin convex lens (fig.). A ray *AP* starting from *A* parallel to the principal axis, after refraction through the lens, passes through the second focus *F*. Another ray *AC* directed towards the



optical centre C of the lens, goes straight undeviated. Both the rays meet at A' Thus A' is the real image of A. The perpendicular A' B' dropped from A' on the principal axis is the whole image of AB.

Let distance of object *AB* from lens = u

Distance of image A'B' from lens = v

But PC = AB

Focal length of lens = f. We can see that triangles *ABC* and *A'B'C'* are similar

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \qquad \dots (i)$$

Similarly triangles *PCF* and *A'B'F* are similar

From (i) and (ii), we get $\frac{CB}{CB'} = \frac{CF}{FB'}$ From sign convention, CB = -u, CB' = v, CF = fand FB' = CB' - CF = v - f

Substituting this value in *(iii)*, we get, $-\frac{u}{v} = \frac{f}{v-f}$ or -u(v-f) = vf or -uv + uf = vf

Dividing throughout by uvf, we get $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Q. 6. Derive the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a thin concave lens, using the necessary ray diagram.

Ans. The formation of image by a concave lens '*L* is shown in fig. *AB* is object and *A*' *B*' is the image. Triangles *ABO* and *A*' *B*' *O* are similar

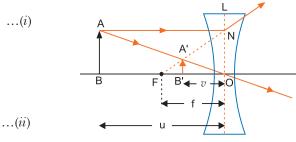
$$\frac{AB}{A'B'} = \frac{OB}{OB'}$$

Also triangles *NOF* and *A' B' F* are similar

$$\frac{NO}{A'B'} = \frac{OF}{FB'}$$

But
$$NO = AB$$

$$\frac{AB}{A'B'} = \frac{OF}{FB'}$$



Ray Optics and Optical Instruments



...(iii)

...(iv)

Comparing equation (i) and (ii)

$$\frac{OB}{OB'} = \frac{OF}{FB'} \Rightarrow \frac{OB}{OB'} = \frac{OF}{OF - OB'}$$

Using sign conventions of coordinate geometry

$$OB = -u, OB' = -v, OF = -$$
$$\frac{-u}{-v} = \frac{-f}{-f + v} \Rightarrow uf - uv = vf$$
$$uv = uf - vf$$

 \Rightarrow

Dividing throughout by *uvf*, we get

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

This is the required lens formula.

Q. 7. Define power of a lens. Write its units. Deduce the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ for two thin lenses kept in contact coaxially. [*CBSE* (*F*) 2012, 2019(55/4/3)]

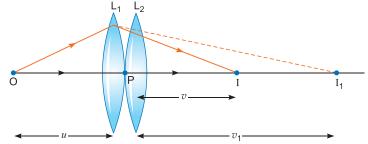
Ans. Power of lens: It is the reciprocal of focal length of a lens.

$$P = \frac{1}{f} (f \text{ is in metre})$$

Unit of power of a lens is Diopter.

An object is placed at point O. The lens L_1 produces an image at I_1 which serves as a virtual object for lens L_2 which produces final image at I.

Given, the lenses are thin. The optical centres (P) of the lenses L_1 and L_2 is co-incident.



For lens L_1 , we have

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$
...(*i*)

For lens
$$L_2$$
, we have $\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$

...(*ii*)

Adding equations (i) and (ii), we have

$$\frac{1}{v_1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$
...(iii)

If two lenses are considered as equivalent to a single lens of focal length f, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \dots (iv)$$

From equation (iii) and equation (iv), we can write

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

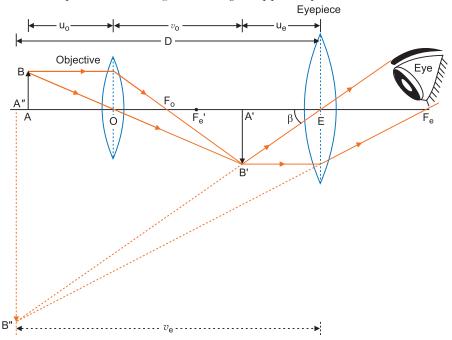


- Q. 8. (a) Draw the labelled ray diagram for the formation of image by a compound microscope. Derive an expression for its total magnification (or magnifying power), when the final image is formed at the near point. [CBSE Delhi 2009, 2010, 2013, 2019 (55/5/1)]
 Why both objective and eyepiece of a compound microscope must have short focal lengths?
 - (b) Draw a ray diagram showing the image formation by a compound microscope. Hence obtain expression for total magnification when the image is formed at infinity.

[CBSE Delhi 2013]

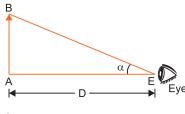
Ans. (a) Compound Microscope: It consists of a long cylindrical tube, containing at one end a convex lens of small aperture and small focal length. This is called the objective lens (O). At the other end of the tube another co-axial smaller and wide tube is fitted, which carries a convex lens (E) at its outer end. This lens is towards the eye and is called the eye-piece. The focal length and aperture of eyepiece are somewhat larger than those of objective lens. Cross-wires are mounted at a definite distance before the eyepiece. The entire tube can be moved forward and backward by the rack and pinion arrangement.

Adjustment: First of all the eyepiece is displaced backward and forward to focus it on crosswires. Now the object is placed just in front of the objective lens and the entire tube is moved by rack and pinion arrangement until there is no parallax between image of object and cross wire. In this position the image of the object appears quite distinct.



Working : Suppose a small object *AB* is placed slightly away from the first focus F_0' of the objective lens. The objective lens forms the real, inverted and magnified image *A' B'* which acts as an object for eyepiece. The eyepiece is so adjusted that the image *A' B'* lies between the first focus F_e' and the eyepiece *E*. The eyepiece forms its image *A'' B''* which is virtual, erect and magnified. Thus the final image *A'' B''* formed by the microscope is inverted and magnified and its position is outside the objective and eyepiece towards objective lens.

Magnifying power of a microscope is defined as the ratio of angle (β) subtended by final image on the eye to the angle (α) subtended by the object on eye, when the object is placed at the least distance of distinct vision, *i.e.*,



Magnifying power $M = \frac{\beta}{\alpha}$(*i*)

As object is very small, angles α and β are very small and so tan $\alpha = \alpha$ and tan $\beta = \beta$. By definition the object *AB* is placed at the least distance of distinct vision.

$$\alpha = \tan \alpha = \frac{AB}{EA}$$

By sign convention EA = -D, $\therefore \quad \alpha = \frac{AB}{-D}$ and from figure $\beta = \tan \beta = \frac{A'B'}{EA'}$

If u_e is distance of image A' B' from eye-piece E, then by sign convention, $EA' = -u_e$ and so, $\beta = \frac{A'B'}{(-u_e)}$

Hence magnifying power $M = \frac{\beta}{\alpha} = \frac{A'B'/(-u_e)}{AB(-D)} = \frac{A'B'}{AB} \cdot \frac{D}{u_e}$

By sign conventions, magnification of objective lens

$$\frac{A'B'}{AB} = \frac{v_0}{(-u_0)}$$
$$M = -\frac{v_0}{u_0} \cdot \frac{D}{u_e} \qquad \dots (ii)$$

Using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for eye-lens, (*i.e.*, using $f = f_e$, v_e , $u = -u_e$), we get

$$\frac{1}{f_e} = \frac{1}{-v_e} - \frac{1}{(-u_e)} \quad \text{or} \qquad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{v_e}$$

Magnifying power $M = -\frac{v_0}{u_0} D\left(\frac{1}{f_e} + \frac{1}{v_e}\right)$

or

$$M = -\frac{v_0}{u_0} \left(\frac{D}{f_e} + \frac{D}{v_e} \right)$$

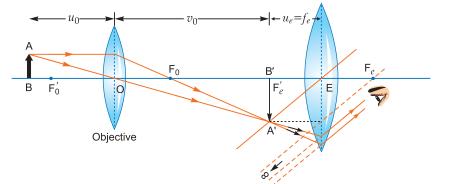
When final image is formed at the distance of distinct vision, $v_e = D$

Magnification,
$$M = -\frac{v_0}{u_0} \left(1 + \frac{D}{f_e}\right)$$

For greater magnification of a compound microscope, f_e should be small. As $f_0 < f_e$, so f_0 is small. Hence, for greater magnification both f_0 and f_e should be small with f_0 to be smaller of the two.



(b) If image A'B' is exactly at the focus of the eyepiece, then image A"B" is formed at infinity.



If the object *AB* is very close to the focus of the objective lens of focal length f_o , then magnification M_o by the objective lens

$$M_o = \frac{L}{f_0}$$

where L is tube length (or distance between lenses L_o and L_e) Magnification M_e by the eyepiece

$$M_e = \frac{D}{f_e}$$

where D = Least distance of distinct vision

Total magnification, $m = M_o M_e = \left(\frac{L}{f_o}\right) \left(\frac{D}{f_e}\right)$

Q. 9. Explain with the help of a labelled ray diagram, how is image formed in an astronomical telescope. Derive an expression for its magnifying power. [CBSE (F) 2014, 2019 (55/1/1)]

OR

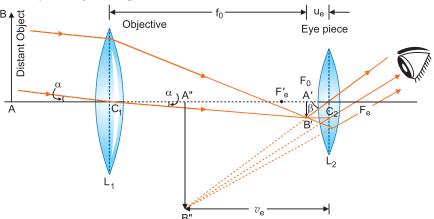
Draw a ray diagram showing the image formation of a distant object by a refracting telescope. Define its magnifying power and write the two important factors considered to increase the magnifying power.

Describe briefly the two main limitations and explain how far these can be minimised in a reflecting telescope. [CBSE (F) 2015]

Ans. Astronomical (Refracting) Telescope:

Construction: It consists of two co-axial cylindrical tubes, out of which one tube is long and wide, while the other tube is small and narrow. The narrow tube may be moved in and out of the wide tube by rack and pinion arrangement. At one end of wide tube an achromatic convex lens L_1 is placed, which faces the object and is so called *objective (lens)*. The focal length and aperture of this lens are kept large. The large aperture of objective is taken that it may collect sufficient light to form a bright image of a distant object. The narrow tube is towards eye and carries an achromatic convex lens L_2 of small focal length and small aperture on its outer end. This is called **eye-lens or eyepiece.** The small aperture of eye-lens is taken so that the whole light refracted by it may reach the eye. Cross-wires are fitted at a definite distance from the eye-lens.

Due to large focal length of objective lens and small focal length of eye lens, the final image subtends a large angle at the eye and hence the object appears large. The distance between the two lenses may be arranged by displacing narrow tube in or out of wide tube by means of rack and pinion arrangement. **Adjustment:** First of all the eyepiece is moved backward and forward in the narrow tube and focused on the cross-wires. Then the objective lens is directed towards the object and narrow tube is displaced in or out of wide tube until the image of object is formed on cross-wires and there is no parallax between the image and cross-wires. In this position a clear image of the object is seen. As the image is formed by refraction of light through both the lenses, this telescope is called the *refracting telescope*.



Working: Suppose AB is an object whose end A is on the axis of telescope. The objective lens (L_1) forms the image A' B' of the object AB at its second principal focus F_0 This image is real, inverted and diminished. This image A' B' acts as an object for the eye-piece L_2 and lies between first focus F_e and optical centre C_2 of lens L_2 . Therefore eye-piece forms its image A' B' which is virtual, erect and magnified.

Thus the final image $A^{"}B^{"}$ of object AB formed by the telescope is magnified, inverted and lies between objective and eyepiece.

Magnifying Power: The magnifying power of a telescope is measured by the ratio of angle (β) subtended by final image on the eye to the angle (α) subtended by object on the eye, *i.e.*,

Magnifying power $M = \frac{\beta}{\alpha}$

As α and β are very small angles, therefore, from figure.

The angle subtended by final image A'' B'' on eye

 β = angle subtended by image *A*' *B*' on eye

$$=\tan\beta=\frac{A'B'}{C_{0}A'}$$

As the object is very far (at infinity) from the telescope, the angle subtended by object at eye is same as the angle subtended by object on objective lens.

$$\alpha = \tan \alpha = \frac{A'B'}{C_1 A'}$$
$$M = \frac{\beta}{\alpha} = \frac{A'B'/C_2 A'}{A'B'/C_1 A'} = \frac{C_1 A'}{C_2 A'}$$

If the focal lengths of objective and eye-piece be f_o , and f_e , distance of image A' B' from eye-piece be u_e , then by sign convention

$$C_1 A' = + f_0, C_2 A' = -u_e$$

 $M = -\frac{f_0}{u_e}$...(i)

If v_e is the distance of $A^{"}B^{"}$ from eye-piece, then by sign convention, f_e is positive, u_e and v_e both are negative. Hence by lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we have



$$\frac{1}{f_e} = \frac{1}{-v_e} - \frac{1}{(-u_e)} \text{ or } \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{v_e}$$

Substituting this value in (i), we get

$$M = -f_0 \left(\frac{1}{f_e} + \frac{1}{v_e}\right) \qquad \dots (ii)$$

This is the general formula for magnifying power. In this formula only numerical values of f_0 , f_e and v_e are to be used because signs have already been used.

Length of Telescope: The distance between objective and eye-piece is called the length (*L*) of the telescope. Obviously

$$L = L_1 L_2 = C_1 C_2 = f_o + u_e \qquad ...(iii)$$

Now there arise two cases:

(i) When the final image is formed at minimum distance (D) of distinct vision : then $v_e = D$

$$M = -f_0 \left(\frac{1}{f_e} + \frac{1}{D}\right) = -\frac{f_0}{f_e} \left(1 + \frac{f_e}{D}\right) \qquad \dots (iv)$$

Length of telescope $L = f_o + u_e$

(*ii*) In normal adjustment position, the final image is formed at infinity : For relaxed eye, the final image is formed at infinity. In this state, the image *A' B'* formed by objective lens should be at first the principal focus of eyepiece, *i.e.*,

$$u_e = f_o \text{ and } v_e = \infty$$

$$\therefore \qquad \text{Magnifying power, } M = -f_o \left(\frac{1}{f_e} + \frac{1}{\infty}\right) = -\frac{f_o}{f_e}$$

Length of telescope = $f_0 + f_e$.

For large magnifying power, f_o should be large and f_e should be small.

For high resolution of the telescope, diameter of the objective should be large.

Factors for increasing the magnifying power

- 1. Increasing focal length of objective
- 2. Decreasing focal length of eye piece

Limitations

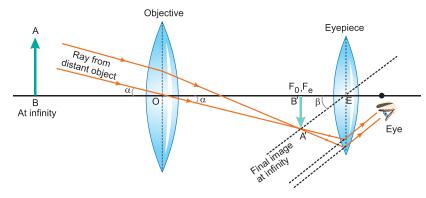
- 1. Suffers from chromatic aberration
- 2. Suffers from spherical aberration
- 3. Small magnifying power
- 4. Small resolving power

Advantages:

- (a) No chromatic aberration, because mirror is used.
- (b) Easy mechanical support (less mechanical support is required, because mirror weighs much less than a lens of equivalent optical quality.)
- (c) Large gathering power.
- (d) Large magnifying power.
- (e) Large resolving power.
- (f) Spherical aberration can be removed by using parabolic mirror.
- Q. 10. (i) Draw a labelled ray diagram to obtain the real image formed by an astronomical telescope in normal adjustment position. Define its magnifying power.
 [CBSE 2019 (55/1/2)]
 - $(ii)\,$ You are given three lenses of power 0.5 D, 4 D and 10 D to design a telescope.
 - (a) Which lenses should be used as objective and eyepiece? Justify your answer.
 - (b) Why is the aperture of the objective preferred to be large? [CBSE (Central) 2016]



Ans. (i)



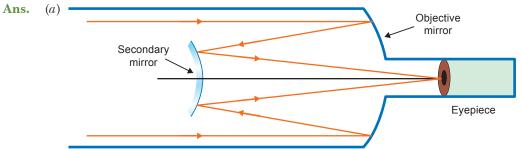
Definition: It is the ratio of the angle (β) subtended at the eye by the final image, to the angle (α)subtended by the object on the eye, *i.e.*, $M = \frac{\beta}{\alpha}$

- (*ii*) (*a*) Objective = 0.5 D
 - Eye lens = 10 D

This choice would give higher magnification as

$$M = \frac{f_0}{f_e} = \frac{P_e}{P_0}$$

- (*b*) The aperture of the objective lens is preferred to be large that it may collect sufficient light to form a brighter image of a distant object.
- Q. 11. (a) With the help of a labelled ray diagram, explain the construction and working of a Cassegrain reflecting telescope.
 - (b) An amateur astronomer wishes to estimate roughly the size of the sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eyepiece of focal length 10 cm. By adjusting the distance of the eyepiece from the objective, he obtains an image of the sun on a screen 40 cm behind the eyepiece. The diameter of the sun's image is measured to be 6.0 cm. Estimate the Sun's size, given that the average earth-sun distance is 1.5×10^{11} m. [CBSE 2019 (55/5/1)]



It consists for large concave (primary) paraboidal mirror having in its central part a hole. There is a small convex (secondary) mirror near the focus of concave mirror. Eye pieces if placed near the hole of the concave mirror.

The parallel rays from distant object are reflected by the large concave mirror. These rays fall on the convex mirror which reflects these rays outside the hole. The final magnified image in formed.

(b) For eyepiece.

Given, $v_{e} = 40 \text{ cm}, f_{e} = 10 \text{ cm}$



$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$
or
$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{40} - \frac{1}{10}$$

$$\Rightarrow \qquad u_e = \frac{-40}{3} \text{ cm}$$

Magnification produced by eye pieces is

$$m_e = \frac{v_e}{|u_e|} = \frac{40}{40/3} = 3$$

Diameter of the image formed by the objective is

$$l = 6/3 = 2 \text{ cm}$$

Ξ

If D be the diameter of the sun then the angle subtended by it on the objective will be

$$a = \frac{D}{1.5 \times 10^{11}} \text{rad}$$

Angle subtended by the image at the objective

= angle subtended by the sun

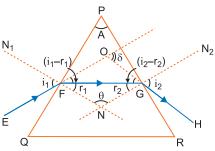
$$\therefore \alpha = \frac{\text{Size of image}}{f_0} = \frac{2}{200} = \frac{1}{100} \text{ rad}$$
$$\therefore \frac{D}{1.5 \times 10^{11}} = \frac{1}{100}$$
$$\Rightarrow D = 1.5 \times 10^9 \text{ m}$$

Q. 12. Draw a graph to show the angle of deviation δ with the variation of angle of incidence *i* for a monochromatic ray of light passing through a prism of refracting angle A. Deduce the relation

$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

- [CBSE Delhi 2011, 2016; (F) 2011, 2017; Sample Paper 2016]
- **Ans.** Graph of deviation in δ with variation in angle of incidence i: The homogeneous transparent medium (such as glass) enclosed by two plane refracting surfaces is called a prism. N_1 The angle between the refracting surfaces is called the refracting angle or angle of prism. The section cut by a plane perpendicular to the refracting surfaces is called the principal section of the prism.

Let PQR be the principal section of the prism. The refracting angle of the prism is A.



A ray of monochromatic light EF is incident on face PQ at angle of incidence i_1 The refractive index of material of prism for this ray is n. This ray enters from rarer to denser medium and so is deviated towards the normal FN_1 and gets refracted along the direction FG. The angle of refraction for this face is r_1 The refracted ray FG becomes incident on face PR and is refracted away from the normal GN_2 and emerges in the direction GH. The angle of incidence on this face is r_2 (into prism) and angle of refraction (into air) is i_2 . The incident ray EF and emergent ray GH when produced meet at O. The angle between these two rays is called angle of deviation ' δ '.

$\angle OFG = i_1 - r_1$ and $\angle OGF = i_2 - r_2$	
In Δ <i>FOG</i> , δ is exterior angle	
$\delta = \angle OFG + \angle OGF = (i_1 - r_1) + (i_2 - r_2)$	
$= (i_1 + i_2) - (r_1 + r_2)$	(<i>i</i>)
The normals FN_1 and GN_2 on faces PO and PR respectively when pro-	duced meet at N Let

The normals FN_1 and GN_2 on faces PQ and PR respectively, when produced meet at N. Let $\angle FNG = \theta \ln \Delta FGN$, $r_1 + r_2 + \theta = 180^\circ$...(*ii*) In quadrilateral PFNG, $\angle PFN = 90^\circ$, $\angle PGN = 90^\circ$

$$A + 90^{\circ} + \theta + 90^{\circ} = 360^{\circ} \text{ or } A + \theta = 180^{\circ} \dots (iii)$$

Comparing (ii) and (iii),
$$r_1 + r_2 = A$$
 ...(iv)

Substituting this value in (i), we get

$$\begin{split} \delta &= i_1 + i_2 - A & \dots(v) \\ i_1 + i_2 &= A + \delta & \dots(vi) \end{split}$$

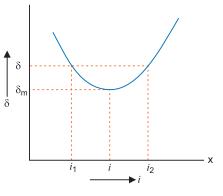
or

From Snell's law
$$n = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2}$$
 ...(vii)

Minimum Deviation: From equation (v), it is clear that the angle of deviation depends upon the angle of incidence i_1 As the path of light is reversible, therefore if angle of incidence be i_2 then angle of emergence will be i_1 . Thus for two angles of incidence

 i_1 and i_2 there will be one angle of deviation.

If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot *i* (*on X*-axis) and δ (on *Y*-axis), we get a curve as shown in figure. Clearly if angle of incidence is gradually increased, δ from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase. Obviously for one angle of deviation (δ) there are two angles of incidences i_1 and i_2 , but *for one*



and only one particular value of angle of incidence (i), the angle of deviation is the minimum. This minimum angle of deviation is represented by δ_m . For minimum deviation i_1 and i_2 become coincident, *i.e.*, $i_1 = i_2 = i$ (say)

So from (vii) $r_1 = r_2 = r$ (say)

Hence from (*iv*) and (*vi*), we get r + r = A or r = A / 2 or

and

$$i + i = A + \delta_m \text{ or } i = \frac{A + \delta_m}{2}$$

Hence from Snell's law,
$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + o_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$



Self-Assessment Test

Time allowed: 1 hour

- 1. Choose and write the correct option in the following questions.
 - (i) Match the corresponding entries of column 1 with column 2. [Where m is the magnification produced by the mirror]

Column 1	Column 2	
(A) $m = -2$	(<i>p</i>) Convex mirror	
(B) $m = -\frac{1}{2}$	(q) Concave mirror	
(C) $m = +2$	(r) Real image	
(D) $m = +\frac{1}{2}$	(s) Virtual image	
(a) $A \rightarrow p$ and s; $B \rightarrow q$ and r; $C \rightarrow q$ and s; $D \rightarrow q$ and r		
(b) $A \rightarrow r$ and s; $B \rightarrow q$ and s; $C \rightarrow q$ and r; $D \rightarrow p$ and s		
(c) $A \rightarrow q$ and r; B –	$\rightarrow q$ and r ; C $\rightarrow q$ and s ; D $\rightarrow p$ and s	
(d) $A \rightarrow p$ and r; $B \rightarrow p$ and s; $C \rightarrow p$ and q; $D \rightarrow r$ and s		

- (ii) An astronomical telescope has objective and evepiece of focal length 40 cm and 4 cm respectively. To view an object 200 cm away from the objective, the lenses must be separated by a distance (a) 50.0 cm (b) 54.0 cm (c) 37.3 cm (d) 46.0
- (*iii*) The angle of incidence for a ray of light at a refracting surface of a prism is 45°. The angle of prism is 60°. If they ray suffers minimum deviation through the prism, the angle of minimum deviation and refractive index of the material of the prism respectively, are.
 - (b) $30^{\circ}; \frac{1}{\sqrt{2}}$ (c) $45^{\circ}; \frac{1}{\sqrt{2}}$ (d) $30^{\circ};\sqrt{2}$ (a) $45^{\circ};\sqrt{2}$

2. Fill in the blanks.

- (*i*) When the refractive index of the material of the lens is greater than that of the surroundings, then a biconcave lens acts as a
- *(ii)* In a reflecting type telescope, a concave mirror of large aperture is used as in place of a convex lens.
- 3. A biconvex lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging lens? Give reason. 1
- 4. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason.
- 5. For the same angle of incidence the angles of refraction in three different media A, B and C are 15°, 25° and 35° respectively. In which medium the velocity of light is minimum? 1
- 6. Two monochromatic rays of light are incident normally on the face AB of an isosceles rightangled prism ABC. The refractive indices of the glass prism for the two rays '1' and '2' are respectively 1.3 and 1.5. Trace the path of these rays after entering through the prism. Explain briefly. 2

 $(3 \times 1 = 3)$



 $(2 \times 1 = 2)$

- 7. A biconvex lens has a focal length $\frac{2}{3}$ times the radius of curvature of either surface. Calculate the refractive index of lens material.
- 8. Light from a point source in air falls on a convex spherical glass surface of refractive index 1.5 and radius of curvature 20 cm. The distance of light source from the glass surface is 100 cm. At what position is the image formed?
- 9. Find the radius of curvature of the convex surface of a plano-convex lens, whose focal length is 0.3 m and the refractive index of the material of the lens is 1.5.
- 10. Draw a ray diagram to show the image formation of a distant object by a refracting telescope. Write the expression for its angular magnification in terms of the focal lengths of the lenses used. State the important considerations required to achieve large resolution and their consequent limitations.

OR

- (a) Plot a graph for angle of deviation as a function of angle of incidence for a triangular prism.
- (b) Derive the relation for the refractive index of the prism in terms of the angle of minimum deviation and angle of prism.
- 11. A screen is placed 90 cm from as object. The image of the object on the screen is formed by a convex lens at two different positions separated by 20 cm. Calculate the focal length of the lens.
 [CBSE 2019 (55/5/1)] 3
- 12. A convex lens of focal length 20 cm is placed coaxially with a convex mirror of radius of curvature 20 cm. The two are kept at 15 cm from each other. A point object lies 60 cm in front of the convex lens. Draw a ray diagram to show the formation of the image by the combination. Determine the nature and position of the image formed.
- 13. Draw a labelled ray diagram to show the image formation by an astronomical telescope.

Derive the expression for its magnifying power in normal adjustment. Write two basic features which can distinguish between a telescope and a compound microscope. 5

Answers

- **1.** (*i*) (*c*) (*ii*) (*b*) (*iii*) (*d*)
- **2.** (*i*) diverging lens (*ii*) objective
- **7.** $n = \frac{7}{4}$

9. *R* = 15 cm

