Chapter 9. Factoring

Ex. 9.3

Answer 1CU.

Consider the polynomial x^2+6x+9.

Compare x^2+6x+9 with x^2+bx+c.

$b=6,$
$c=9$

Since b and c are positive,

$$x^2+6x+9=(x+m)(x+n)$$
$$=x^2+(m+n)x+mn$$

Therefore, $m+n, mn$ are positive, therefore, m,n must be positive.

That is the pair of factors of 9 must be positive.

Consider only positive factors of 9.

It is not necessary to check the sum of factor pairs -1 and -9 or -3 and -3.

Answer 2CU.

Consider the equation

$x^2+2x+1=0$

It is solved by using factoring x^2+2x+1 and used zero product property.

The zero product property is

$ab=0$, than

$a=0$, or

$b=0$ or both.

Compare x^2+2x+1 with x^2+bx+c

$b=2,$

$c=1$

$$x^2+2x+1=(x+m)(x+n)$$
$$=x^2+(m+n)x+m\cdot n$$

Since $m+n=2$,

$mn=1$ both positive, so find two numbers whose product is 1 and sum is 2.

Factors of 1	Sum of factors
1.1	2

The correct factors are $1,1$

$x^2+2x+1=(x+m)(x+n)$

$=(x+1)(x+1) \quad (m=1,n=1)$

$x^2+2x+1=0$

$\Rightarrow \quad (x+1)(x+1)=0$

$x+1=0$ or

$x+1=0$ (By zero product property)

Now solve equation separately.

$x+1=0$

$x+1-1=0-1$ (Subtract 1 on both sides)

$x=-1$

Also $x+1=0$

$x+1-1=0-1$ (Subtract 1 on each side)

$x=-1$

The solution set is $\boxed{\{-1,-1\}}$.

Answer 3CU.

Consider the equation $x^2 + 2x = 15$

Peter solved this as

$x^2 + 2x = 15$

$x = 15$

Or $x + 2 = 15$

$x = 13$

Aleta solved as

$x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$

$x - 3 = 0$

Or $x + 5 = 0$

$x = 3$

$x = -5$

The procedure of Aleta is true

Since Aleta used zero product property.

That is $ab = 0$ then

$a = 0$ or

$b = 0$ but not both.

But peter solved

$x(x+2) = 15$ then

$x = 15$ or

$x + 12 = 15$ is wrong.

Aleta is correct, the solution set is $\boxed{\{3, -5\}}$.

Answer 4CU.

Consider the trinomial $x^2+11x+24$

The objective is to factor the given trinomial,

Compare $x^2+11x+24$ with x^2+bx+c.

Here $b=11$,

$c=24$

Since

$$(x+m)(x+n)=x^2+(m+n)x+mn$$

Now find the two number whose product is 24 and whose sum is 11.

For this list all the factors of 24, and choose a pair of factors whose sum is 11.

Factors of 24	Sum of factors
$1,24$	25
$2,12$	14
$3,8$	11
$4,6$	10

The correct factors are $3,8$.

Hence

$$x^2+11x+24=(x+m)(x+n)$$

$$=(x+3)(x+8) \quad (m=3, n=8)$$

Check: Now check the result by multiplying the two factors using $FOIL$ method.

$$(x+3)(x+8)=\overset{F}{x\cdot x}+\overset{O}{x\cdot 8}+\overset{I}{3\cdot x}+\overset{L}{3\cdot 8}$$

($FOIL$ method)

$=x^2+11x+24$ True (Simplify)

Therefore factorization of $\boxed{x^2+11x+24}$ is $\boxed{(x+3)(x+8)}$.

Answer 5CU.

Consider the trinomial c^2-3c+2

The objective is to factor the given trinomial.

Compare c^2-3c+2 with x^2+bx+d.

$b=-3,$
$d=2$

$$c^2-3c+2=(c+m)(c+n)$$
$$=c^2+(m+n)c+mn$$

That is $m+n=-3$ is negative and

$mn=2$ is positive

So m and n must both be negative.

Now make a list of negative factors of 2, in those pair of factors, choose the factors whose sum is -3.

Factors of 2	Sum of factors
$-1,-2$	-3

The correct factors are $-1,-2$

Therefore, $c^2-3c+2=(c+m)(c+n)$

$$=(c+(-1))(c+(-2)) \quad (m=-1, n=-2)$$
$$=(c-1)(c-2)$$

Check: Check the results by multiplying two factors by $FOIL$ method.

$$(c-1)(c-2)=\overset{F}{c\cdot c}+\overset{O}{(-2)\cdot c}+\overset{I}{(-1)\cdot c}+\overset{L}{(-1)(-2)}$$

($FOIL$ method)

$=c^2-2c-c+2$ (Simplify)

$=c^2-3c+2$ True

Therefore, the factorization of $\boxed{c^2-3c+2}$ is $\boxed{(c-1)(c-2)}$.

Answer 6CU.

Consider the trinomial $n^2+13n-48$.

The objective is to factor given trinomial.

Compare $n^2+13n-48$ with x^2+bx+c.

Here $b=13$,

$c=-48$

$$\begin{aligned}n^2+13n-48&=(n+x)(n+y)\\&=n^2+(x+y)n+x\cdot y\end{aligned}$$

That is $x+y=13$ positive and

$xy=-48$ is negative.

So, either x or y negative, but not both.

Now make list of factors of -48, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 13.

Factors of -48	Sum of factors
-1.48	47
$1.-48$	-47
-2.24	22
$2.-24$	-22
-3.16	13
$3.-16$	-13
-4.12	8

$4, -12$	-8
$-6, 8$	2
$6, -8$	-2

The correct factors are $-3, 16$.

Therefore, $n^2 + 13n - 48 = (n + x)(n + y)$

$= (n + (-3))(n + 16) \quad (x = -3, y = 16)$

$= (n - 3)(n + 16)$

Check: Check the results by multiplying two factors by $FOIL$ method.

$(n-3)(n+16) = \overset{F}{n \cdot n} + \overset{O}{16 \cdot n} + \overset{I}{(-3) \cdot n} + \overset{L}{(-3) \cdot (16)}$

($FOIL$ method)

$= n^2 + 16n - 3n - 48$ (Simplify)

$= n^2 + 13n - 48$ True

Therefore, the factorization of $\boxed{n^2 + 13n - 48}$ is $\boxed{(n-3)(n+16)}$.

Answer 7CU.

Given trinomial is $p^2-2p-35$.

The objective is to factor given trinomial.

Compare $p^2-2p-35$ with ax^2+bx+c.

Here $b=-2$,

$c=-35$

$$p^2-2p-35=(p+m)(p+n)$$
$$=p^2+(m+n)p+mn$$

That is $m+n=-2$ is negative and

$mn=-35$ is also negative.

So, either m or n is negative but not both

Now make list of positive factors of -35, in those pair of factors, choose the factors whose sum is -2.

Factors of -35	Sum of factors
$1,-35$	-34
$7,-5$	2
$-7,5$	-2
$-1,35$	34

The correct factors are $-7,5$.

Therefore, $p^2-2p-35=(p+m)(p+n)$

$=(p+(-7))(p+5)$ $(m=-7,n=5)$

$=(p-7)(p+5)$

Check:- Check the result by multiplying the factors by $FOIL$ method.

$$(p-7)(p+5)=\overset{F}{p\cdot p}+\overset{O}{5\cdot p}+\overset{I}{(-7)p}+\overset{L}{(-7)5}$$

($FOIL$ method)

$=p^2+5p-7p-35$ (Simplify)

$=p^2-2p-35$ True

Therefore, the factorization of $\boxed{p^2-2p-35}$ is $\boxed{(p-7)(p+5)}$.

Answer 8CU.

Consider the trinomial $72+27a+a^2$

The given equation can be written as $a^2+27a+72$.

The objective is to factor the given trinomial

Compare $a^2+27a+72$ with x^2+bx+c.

Here $b=27$,

$c=72$

Since $(a+m)(a+n)=a^2+(m+n)a+mn$

Now find the number whose product is 72, and choose a pair of factors whose sum is 27.

Factors of 72	Sum of factors
1.72	73
2.36	38
3.24	27
18.4	22
6.12	18
8.9	17

The correct factors are $3, 24$.

Therefore, $a^2 + 27a + 72 = (a+m)(a+n)$

$= (a+3)(a+24) \quad (m=3, n=24)$

Check:- Check the results by multiplying two factors by $FOIL$ method.

$(a+3)(a+24) = \overset{F}{a \cdot a} + \overset{O}{24 \cdot a} + \overset{I}{3 \cdot a} + \overset{L}{3 \cdot 24}$

($FOIL$ method)

$= a^2 + 24a + 3a + 72$

(Simplify)

$= a^2 + 27a + 72$ True

Therefore, the factorized form of $\boxed{a^2 + 27a + 72}$ is $\boxed{(a+3)(a+24)}$.

Answer 9CU.

Consider the trinomial $x^2 - 4xy + 3y^2$

The objective is to factor the given trinomial.

Compare $x^2 - 4xy + 3y^2$ with $x^2 + bx + c$

$b = -4y,$

$c = 3y^2$

Since $(x+m)(x+n) = x^2 + (m+n)x + m \cdot n$

Now find two numbers whose product is $3y^2$ and sum is $-4y$

Since $b = -4y$ is negative $m+n$ is negative & mn is positive therefore both m, n are negative.

Factors of $3y^2$	Sum of factors
$-1.-3y^2$	$-1-3y^2$
$-3.-y^2$	$-3-y^2$
$-3y.-y$	$-4y$

Therefore, the correct factors are $-3y, -y$.

Hence $x^2 - 4xy + 3y^2 = (x+m)(x+n)$

$= (x + (-3y))(x + (-y))$

$(m = -3y, n = -y)$

$= (x - 3y)(x - y)$

Check: Check the result by multiplying two factors by $FOIL$ method.

$$(x-3y)(x-y) = \overset{F}{x \cdot x} + \overset{O}{(-y) \cdot x} + \overset{I}{(-3y)x} + \overset{L}{(-y) \cdot (-3y)}$$

($FOIL$ Method)

$= x^2 - xy - 3xy + 3y^2$

(Simplify)

$= x^2 - 4xy + 3y^2$ True

Therefore, the factorization of $\boxed{x^2 - 4xy + 3y^2}$ is $\boxed{(x-3y)(x-y)}$.

Answer 10CU.

Consider the equation

$n^2 + 7n + 6 = 0$

The objective is to solve the given equation.

For this first factor $n^2 + 7n + 6$ and then use zero product property.

Compare $n^2 + 7n + 6$ with $x^2 + bx + c$

Here $b = 7,$

$c = 6$.

Since $(n+k)(n+l) = n^2 + (k+l)n + kl$

Now find two numbers whose product is 6 and sum is 7.

For this list all the factors of 6 and choose a pair of factors whose sum is 7.

Factors of 6	Sum of factors
$1,6$	7
$2,3$	5

The correct factors are $1,6$

$n^2 + 7n + 6 = (n+k)(n+l)$

$= (n+1)(n+6) \quad (l = 1, k = 6)$

Therefore, $n^2 + 7n + 6 = 0$

$(n+1)(n+6) = 0$ (Factor)

$n + 1 = 0$

Or, $n+6=0$ (By zero product property)

Now solve each equation separately.

$n+1=0$

$n+1-1=0-1$ (Subtract 1 on each side)

$n=-1$

$n+6=0$

$n+6-6=0-6$ (Subtract 6 on each side)

$n=-6$

The solution set is $\{-1,-6\}$.

Check: To check the proposed solution set substitute n by $-6,-1$ in the given equation.

For $n=-1$,

$n^2+7n+6=0$

$(-1)^2+7(-1)+6=0$ (Put $n=-1$)

$1-7+6=0$ (Simplify)

$0=0$ True

For $n=-6$,

$n^2+7n+6=0$

$(-6)^2+7(-6)+6=0$ (Put $n=-6$)

$36-42+6=0$ (Simplify)

$42-42=0$

$0=0$ True

The solution set is $\boxed{\{-1,-6\}}$.

Answer 11CU.

Consider the equation

$$a^2+5a-36=0$$

The objective is to factor the given equation.

Compare $a^2+5a-36$ with x^2+bx+c

Here $b=5$,

$c=-36$

$$a^2+5a-36=(a+m)(a+n)$$
$$=a^2+(m+n)a+mn$$

This is $m+n=5$ Positive and

$mn=-36$ is negative.

So, either m or n negative but not both.

Now make list of factors of -36, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 5.

Factors of -36	Sum of factors
$1.-36$	-35
-1.36	35
-2.18	16
$2.-18$	-16
$6.-6$	0
-6.6	0

$3.-12$	-9
-3.12	9
$9.-4$	5
-9.5	-5

The correct factors are $9,-4$.

$$a^2+5a-36=(a+m)(a+n)$$

$$=(a+9)(a+(-4)) \quad (m=9,n=-4)$$

$$=(a+9)(a-4)$$

Therefore, $a^2+5a-36=0$

$(a+9)(a-4)=0$ (Factor)

$a+9=0$

(or) $a-4=0$ (By zero product property)

Now solve each equation separately.

$a+9=0$

$a+9-9=0-9$ (Subtract 9 on each side)

$a=-9$ (Simplify)

$a-4=0$

$a-4+4=0+4$ (Add 4 on each side)

$a=4$

The solution set is $\{-9, 4\}$.

Check:- To check the proposed solution set substitute a by $-9, 4$ in the given equation.

For $a = -9$,

$a^2 + 5a - 36 = 0$

$(-9)^2 + 5(-9) - 36 = 0$ (Put $a = -9$)

$81 - 45 - 36 = 0$ (Simplify)

$0 = 0$ True

For $a = 4$,

$a^2 + 5a - 36 = 0$

$(4)^2 + 5(4) - 36 = 0$ (Put $a = 4$)

$16 + 20 - 36 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{-9, 4\}}$.

Answer 12CU.

Consider the equation

$$p^2 - 19p - 42 = 0$$

The objective is to factor given equation.

Compare $p^2 - 19p - 42$ with $x^2 + bx + c$.

Here $b = -19,$

$c = -42$

$$p^2 - 19p - 42 = (p + x)(p + y)$$
$$= p^2 + (x + y)p + xy$$

That is $x + y = -19$ is negative,

$xy = -42$ is also negative.

So, either x (or) y is negative but not both.

Now make list of factors of -42 in those pair of factors, choose the factors whose sum is -19.

Factors of -42	Sum of factors
$1.-42$	-41
-1.42	41
$7.-6$	1
-7.6	-1
$2.-21$	-19
-2.21	19
-3.14	11

$3,-14$	-11

The correct factors are $2,-21$.

Therefore,

$p^2-19p-42=(p+x)(p+y)$

$=(p+2)(p+(-21))$

$(x=2, y=-21)$

$=(p+2)(p-21)$

So, $p^2-19p-42=0$

$(p+2)(p-21)=0$ (Factor)

$p+2=0$

(or) $p-21=0$ (By zero product property)

Now solve each equation separately.

$p+2=0$

$p+2-2=0-2$ (Subtract 2 on each side)

$p=-2$ (Simplify)

$p-21=0$

$p-21+21=0+21$ (Add 21 on each side)

$p=21$ (Simplify)

The solution set is $\{-2,21\}$.

Check:- Check the proposed solution set, substitute p by $-2,21$ in the given equation.

For $x=-2$,

$p^2-19p-42=0$

$(-2)^2-19(-2)-42=0$ (Put $x=-2$)

$4+38-42=0$ (Simplify)

$0=0$ True

For $x=21$,

$p^2-19p-42=0$

$(21)^2-19(21)-42=0$ (Put $x=21$)

$441-399-42=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{-2,21\}}$.

Answer 13CU.

Consider the equation

$y^2+9=-10y$

$y^2+9=-10y$

$y^2+9+10y=-10y+10y$ (Add $10y$ on each side)

$y^2+10y+9=0$ (Simplify)

The objective is to solve the given equation.

For this first factor $y^2+10y+9$ and then use zero product property.

Compare $y^2+10y+9$ with x^2+bx+c.

Here $b=10,$

$c=9$

Since $(y+m)(y+n)=y^2+(m+n)y+mn$

Now find two numbers whose product is 9 and sum is 10.

For this list all the factors of 9 and choose a pair of factors whose sum is 10.

Factors of 9	Sum of factors
1.9	10
3.3	6

The correct factors are $1,9$.

$y^2+10y+9=(y+m)(y+n)$

$=(y+1)(y+9)$ $(m=1, n=9)$

Therefore, $y^2+10y+9=0$

$(y+1)(y+9)=0$ (Factors)

$y+1=0$

(or) $y+9=0$ (By zero product property)

Now solve each equation separately.

$y+1=0$

$y+1-1=0-1$ (Subtract 1 on each side)

$y=-1$

$y+9=0$

$y+9-9=0-9$ (Subtract 9 on both sides)

$y=-9$

The solution set is $\{-1,-9\}$.

Check:- To check the proposed solution set substitute y by $-1,-9$ in the given equation.

For $y=-1$,

$y^2+10y+9=0$

$(-1)^2+10(-1)+9=0$ (Put $y=-1$)

$1-10+9=0$ (Simplify)

$0=0$ True

For $y=-9$,

$y^2+10y+9=0$

$(-9)^2+10(-9)+9=0$ (Put $y=-9$)

$81-90+9=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{-1,-9\}}$.

Answer 14CU.

Consider the equation

$9x + x^2 = 22$

The objective is to factor the given equation.

$9x + x^2 = 22$

$9x + x^2 - 22 = 22 - 22$ (Subtract 22 on each side)

$x^2 + 9x - 22 = 0$ (Simplify)

Compare $x^2 + 9x - 22$ with

$x^2 + bx + c = 0$

Here $b = 9$,

$c = -22$

$$x^2 + 9x - 22 = (x+m)(x+n)$$
$$= x^2 + (m+n)x + mn$$

That is $m + n = 9$ is positive and

$mn = -22$ is negative.

So, either m (or) n negative, but not both.

Now make list of factor of -22, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 9.

Factors of -22	Sum of factors
$1.-22$	-21
-1.22	21
$11.-2$	9
-11.2	-9

The correct factors are $11, -2$.

$x^2 + 9x - 22 = (x+m)(x+n)$

$= (x+11)(x+(-2))$

$(m = 11, n = -2)$

$= (x+11)(x-2)$

Therefore, $x^2 + 9x - 22 = 0$

$(x+11)(x-2) = 0$ (Factors)

$x + 11 = 0$

(or) $x - 2 = 0$ (By zero product property)

Now solve each equation separately.

$x + 11 = 0$

$x + 11 - 11 = 0 - 11$ (Subtract 11 on each side)

$x = -11$

$x - 2 = 0$

$x - 2 + 2 = 0 + 2$ (Add 2 on each side)

$x = 2$ (Simplify)

The solution set is $\{-11, 2\}$.

Check:- To check the proposed solution set substitute x by $-11, 2$ in the given equation.

For $x = -11$,

$x^2 + 9x - 22 = 0$

$(-11)^2 + 9(-11) - 22 = 0$ (Put $x = -11$)

$121 - 99 - 22 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{-11, 2\}}$.

Answer 15CU.

Consider the equation

$d^2 - 3d = 70$

The objective is to factor given equation.

$d^2 - 3d = 70$

$d^2 - 3d - 70 = 70 - 70$ (Subtract 70 on both sides)

$d^2 - 3d - 70 = 0$ (Simplify)

Compare $d^2 - 3d - 70$ with $x^2 + bx + c$.

Here $b = -3$,

$c = -70$

$$d^2 - 3d - 70 = (d+m)(d+n)$$
$$= d^2 + (m+n)d + mn$$

That is $m + n = -3$ is negative,

$mn = -70$ is also negative.

So either m (or) n is negative but not both.

Now make list of factors of -70 in those pair of factors, choose the factors whose sum is -3.

Factors of -70	Sum of factors
$1, -70$	-69
$-1, 70$	69
$10, -7$	3
$-10, 7$	-3
$2, -35$	-33

-2.35	33
$14.-5$	9
-14.5	-9

The correct factors are $-10, 7$.

$d^2 - 3d - 70 = (d+m)(d+n)$

$= (d + (-10))(d+7) \quad (m = -10, n = 7)$

$= (d-10)(d+7)$

Therefore, $d^2 - 3d - 70 = 0$

$\Rightarrow \quad (d-10)(d+7) = 0$ (Factors)

$d - 10 = 0$

(or) $d + 7 = 0$ (By zero product property)

Now solve equation separately.

$d - 10 = 0$

$d - 10 + 10 = 0 + 10$ (Add 10 on each side)

$d = 10$

$d + 7 = 0$

$d + 7 - 7 = 0 - 7$ (Subtract 7 on each side)

$d = -7$

The solution set is $\{10, -7\}$.

Check:- To check the proposed solution set, substitute d by $10, -7$ in the given equation.

For $d = 10$,

$d^2 - 3d - 70 = 0$

$(10)^2 - 3(10) - 70 = 0$ (Put $d = 10$)

$100 - 30 - 70 = 0$ (Simplify)

$0 = 0$ True

For $d = -7$,

$d^2 - 3d - 70 = 0$

$(-7)^2 - 3(-7) - 70 = 0$ (Put $d = -7$)

$49 + 21 - 70 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{10, -7\}}$.

Answer 16CU.

Consider that the product of two consecutive integers is 156.

The objective is to find the consecutive integers.

Let the consecutive integers be $x, x+1$.

Since the product of $x, x+1$ is 156.

$$x \cdot x+1 = 156$$

$$x(x+1) = 156$$

$x \cdot x + x \cdot 1 = 156$ (By distributive $a(b+c) = ab+ac$)

$$x^2 + x = 156$$

$x^2 + x - 156 = 156 - 156$ (Subtract 156 on both sides)

$$x^2 + x - 156 = 0$$

Now solve the above equation for x.

First factor the polynomial $x^2 + x - 156$.

Compare $x^2 + x - 156$ with $x^2 + bx + c$

Here $b = 1$,

$c = -156$.

$$\begin{aligned} x^2 + x - 156 &= (x+m)(x+n) \\ &= x^2 + (m+n)x + mn \end{aligned}$$

That is $m+n = 1$,

$$mn = -156$$

$m+n$ is positive and mn is negative. So either m or n negative but not both.

Now make list all the factors of -156, where one factor of each pair is negative in those pair of factors, choose the factors whose sum is 1.

Factors of -156	Sum of factors
-1.156	155
$1.-156$	-155
-2.78	76
$2.-78$	-76
-4.39	35
$4.-39$	-35
-3.52	49
$3.-52$	-49
-12.13	1
$12.-13$	-1

The correct factors are $-12,13$.

Therefore, $x^2+x-156=(x+m)(x+n)$

$=(x+(-12))(x+13) \quad (m=-12, n=13)$

$=(x-12)(x+13)$

Therefore, $x^2 + x - 156 = 0$

$\Rightarrow \quad (x-12)(x+13) = 0$

$\Rightarrow \quad x - 12 = 0$

Or $x + 13 = 0$ (By zero product property)

Now solve each equation separately

$x - 12 = 0$

$x - 12 + 12 = 0 + 12$ (Add 12 on each side)

$x = 12$

$x + 13 = 0$

$\Rightarrow \quad x + 13 - 13 = 0 - 13$ (Subtract 13 on each side)

$x = -13$

The solution set is $\{12, -13\}$

There are two consecutive integer sets.

For $x = 12$,

$x + 1 = 12 + 1$
$\quad = 13$

For $x = -13$,

$x + 1 = -13 + 1$
$\quad = -12$

Check: $12 \cdot 13 = 156$ True

$-12 \cdot -13 = 156$ True

Therefore the two consecutive integers are $\boxed{12, 13}$ or $\boxed{-13, -12}$.

Answer 17PA.

Consider the trinomial $a^2+8a+15$.

The objective is to factor the given trinomial.

Compare $a^2+8a+15$ with x^2+bx+c.

Here $b=8,$

$c=15$

Since

$(x+m)(x+n)=x^2+(m+n)x+mn$

Now find two numbers whose product is 15 and sum is 8.

For this list all the factors of 15 and choose a pair of factors whose sum is 8.

Factors of 15	Sum of factors
$1,15$	16
$3,5$	8

The correct factors are $3,5$.

Hence

$a^2+8a+15=(a+m)(a+n)$

$=(a+3)(a+5) \quad (m=3, n=5)$

Check: Check the result by multiply tow factors by $FOIL$ method.

$(a+3)(a+5)=\overset{F}{a\cdot a}+\overset{O}{5\cdot a}+\overset{I}{3\cdot a}+\overset{L}{3\cdot 5}$

($FOIL$ method)

$=a^2+8a+15$ True

Therefore, the factorization of $\boxed{a^2+8a+15}$ is $\boxed{(a+3)(a+5)}$.

Answer 18PA.

Consider the trinomial $x^2+12x+27$

The objective is to factor the given trinomial.

Compare $x^2+12x+27$ with x^2+bx+c.

Here $b=12$,

$c=27$

Since

$(x+m)(x+n)=x^2+(m+n)x+mn$

Now find two numbers whose product is 27 and sum is 12.

For this list all the factors of 27 and choose a pair of factors whose sum is 12.

Factors of 27	Sum of factors
$1,27$	28
$3,9$	12

The correct factors are $3,9$.

Hence

$x^2+12x+27=(x+m)(x+m)$

$=(x+3)(x+9)\quad (m=3,n=5)$

Check: Check the result by multiplying two factors by $FOIL$ method.

$(x+3)(x+9)=\overset{F}{x\cdot x}+\overset{O}{9\cdot x}+\overset{I}{3\cdot x}+\overset{L}{3\cdot 9}$ ($FOIL$ method)

$=x^2+12x+27$ True (Simplify)

Therefore, the factorization of $\boxed{x^2+12x+27}$ is $\boxed{(x+3)(x+9)}$.

Answer 1PA.

Consider the trinomial $c^2+12c+35$.

The objective is to factor the given trinomial.

Compare $c^2+12c+35$ with x^2+bx+d.

Where $b=12$,

$d=35$

Since $(x+m)(x+n)=x^2+(m+n)x+mn$

Now find two numbers whose product is 35 and sum is 12.

For this list all factors of 35 and choose a pair of factors whose sum is 12.

Factors of 35	Sum of factors
$1,35$	36
$5,7$	12

The correct factors are $5,7$.

Hence $c^2+12x+35=(c+m)(c+n)$

$=(c+5)(c+7) \quad (m=5, n=7)$

Check: Check the result by multiplying two factors using $FOIL$ method.

$(c+5)(c+7)=c\cdot c+7\cdot c+5\cdot c+5\cdot 7$

($FOIL$ method)

$=c^2+12c+5.7$ (Combine like terms)

$=c^2+12c+35$ True

Therefore, the factorization of $\boxed{c^2+12c+35}$ is $\boxed{(c+5)(c+7)}$.

Answer 20PA.

Consider the trinomial $y^2+13y+30$.

The objective is to factor the given trinomial.

Compare $y^2+13y+30$ with x^2+bx+c.

Where $b=13,$

$c=30$

Since $(x+m)(x+n)=x^2+(m+n)x+m\cdot n$

Now find two numbers whose product is 30, and sum is 13.

For this list all the factors of 30 and choose a pair of factors whose sum is 13.

Factors of 30	Sum of factors
$1,30$	31
$2,15$	17
$3,10$	13
$5,6$	11

The correct factors are $3,10$.

Hence, $y^2+13y+30=(y+m)(y+n)$

$=(y+3)(y+10)\quad(m=3,n=10)$

Check: Check the results by multiplying two factors by $FOIL$ method.

$$(y+3)(y+10)=\overset{F}{y\cdot y}+\overset{O}{10\cdot y}+\overset{I}{3\cdot y}+\overset{L}{3\cdot 10}$$

($FOIL$ method)

$=y^2+13y+3\cdot 10$ (Combine like terms)

$=y^2+13y+30$ True

<u>Therefore, the factorization of</u> $\boxed{y^2+13y+30}$ is $\boxed{(y+3)(y+10)}$.

Answer 21PA.

Consider the trinomial $m^2 - 22m + 21$.

The objective is to factor given trinomial.

Compare $m^2 - 22m + 21$ with $x^2 + bx + c$.

Here $b = -22,$

$c = 21$

$$m^2 - 22m + 21 = (m + r)(m + p)$$
$$= m^2 + (r + p)m + rp$$

That is $r + p = -22$ is negative and

$rp = 21$ is positive.

So r and p must both be negative.

Now make a list of negative factors of 21, in those pair of factors, chose the factors whose sum is -22.

Factors of 21	Sum of factors
$-1, -21$	-22
$-3, -7$	-10

The correct factors are $-1, -21$.

Therefore,

$$m^2 - 22m + 21 = (m + r)(m + p)$$
$$= (m + (-1))(m + (-21)) \quad (r = -1, p = -21)$$
$$= (m - 1)(m - 21)$$

Check:- Check the results by multiplying two factors by $FOIL$ method

$$(m-1)(m-21) = \overset{F}{m \cdot m} + \overset{O}{(-21) \cdot m} + \overset{I}{(-1) \cdot m} + \overset{L}{(-1) \cdot (-21)}$$

($FOIL$ method)

$= m^2 - 22m + 21$ True

Therefore, the factorization of $\boxed{m^2 - 22m + 21}$ is $\boxed{(m-1)(m-21)}$.

Answer 22PA.

Consider the trinomial $d^2 - 7d + 10$.

The objective is to factor the given trinomial.

Compare $d^2 - 7d + 10$ with $x^2 + bx + c$.

$$b = -7,$$
$$c = 10$$

$$d^2 - 7d + 10 = (d+m)(d+n)$$
$$= d^2 + (m+n)d + mn$$

That is $m + n = -7$.

$mn = 10, m + n$ is negative and mn is positive.

So m and n must both be negative.

Now make a list of negative factors of 10, in those pair of factors, choose the factors whose sum is -7.

Factors of 10	Sum of factors
$-10, -1$	-11
$-5, -2$	-7

The correct factors are $-5, -2$.

Hence $d^2 - 7d + 10 = (d+m)(d+n)$

$$= (d + (-5))(d + (-2)) \quad (m = -5, n = -2)$$
$$= (d-5)(d-2)$$

Check: Check the result by multiplying the factors by $FOIL$ method

$$(d-5)(d-2) = d \cdot \overset{F}{d} + (\overset{O}{-2}) \cdot d + (\overset{I}{-5}) d + (-5) \cdot (\overset{L}{-2})$$

($FOIL$ method)

$= d^2 - 2d - 5d + 10$ (Simplify)

$= d^2 - 7d + 10$ True

Therefore, the factorization of $\boxed{d^2 - 7d + 10}$ is $\boxed{(d-5)(d-2)}$.

Answer 23PA.

Consider the trinomial $p^2 - 17p + 72$

The objective is to factor given trinomial

Compare $p^2 - 17p + 72$ with $x^2 + bx + c$,

$b = -17,$

$c = 72$

$$p^2 - 17p + 72 = (p+m)(p+n)$$
$$= p^2 + (m+n)p + mn$$

That is $mn = -17$ is negative and

$mn = 72$ is positive

So m and n must both be negative.

Now make list of negative factors of 72, in those, pair of factors choose the factors whose sum is -17.

Factors of 72	Sum of factors
$-1, -72$	-73
$-2, -36$	-38
$-3, -24$	-27
$-4, -18$	-22
$-6, -12$	-18
$-8, -9$	-17

The correct factors are $-8,-9$.

Therefore,

$$p^2-17p+72=(p+m)(p+n)$$

$$=(p+(-8))(p+(-9)) \quad (m=-8, n=-9)$$

$$=(p-8)(p-9)$$

Check: Check the results by multiplying two factors by $FOIL$ method

$$(p-8)(p-9)=\overset{F}{p\cdot p}+\overset{O}{p\cdot(-9)}+\overset{I}{(-8)\cdot p}+\overset{L}{(-8)(-9)}$$

$$=p^2-9p-8p+72 \text{ (Simplify)}$$

$$=p^2-17p+72 \text{ True}$$

<u>Therefore the factorization of</u> $\boxed{p^2-17p+72}$ is $\boxed{(p-8)(p-9)}$.

Answer 24PA.

Consider the trinomial $g^2 - 19g + 60$

The objective is to factor gives trinomial.

Compare $g^2 - 19g + 60$ with $x^2 + bx + c$.

Here $b = -19,$

$c = 60$

$$g^2 - 19g + 60 = (g+m)(g+n)$$
$$= g^2 + (m+n)g + mn$$

That is $m + n = -19$ is negative and

$mn = 60$ is positive

So m and n must bots be negative.

Now make list of negative factors of 60, in those pair of factors, choose the factors whose sum is -19.

Factors of 60	Sum of factors
$-1, -60$	-61
$-2, -30$	-32
$-3, -20$	-23
$-4, -15$	-19
$-5, -12$	-17
$-6, -10$	-16

The correct factors are $-4, -15$.

Therefore,

$g^2 - 19g + 60 = (g+m)(g+n)$

$= ((g+(-14))g+(-15)) \quad (m=-4, n=-15)$

$= (g-4)(g-15)$

Check: Check the results by multiplying two factors by $FOIL$ Method

$(g-4)(g-15) = \overset{F}{g} \cdot g + (\overset{O}{-15}) \cdot g + (\overset{I}{-4}) \cdot g + (-4)(\overset{L}{-15})$

($FOIL$ method)

$= g^2 - 15g - 4g + 60$ (Simplify)

$= g^2 - 19g + 60$ True

Therefore, the factorization of $\boxed{g^2 - 19g + 60}$ is $\boxed{(g-4)(g-15)}$.

Answer 25PA.

Consider the trinomial $x^2 + 6x - 7$

The objective is to factor given trinomial.

Compare $x^2 + 6x - 7$ with $x^2 + bx + c$

Here $b = 6,$

$c = -7$

$x^2 + 6x - 7 = (x+m)(x+n)$

$= x^2 (m+n)x + mn$

That is $m + n = 6$ positive and

$mn = -7$ is negative.

So either m or n negative, but not both.

Now make list of factors of -7, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 6.

Factors of -7	Sum of factors
$-1, 7$	6
$1, -7$	-6

The correct factors are $-1, 7$.

Therefore, $x^2+6x-7=(x+m)(x+n)$

$=(x+(-1))(x+7) \quad (m=-1, n=7)$

$=(x-1)(x+7)$

Check:- Check the results by multiplying two factors by $FOIL$ method.

$(x-1)(x+7)=\overset{F}{x\cdot x}+\overset{O}{7\cdot x}+\overset{I}{(-1)\cdot x}+\overset{L}{(-1)\cdot 7}$

($FOIL$ method)

$=x^2+7x-x-7$ (Simplify)

$=x^2+6x-7$ True

Therefore, the factorization of $\boxed{x^2+6x-7}$ is $\boxed{(x-1)(x+7)}$.

Answer 26PA.

Consider the trinomial b^2+b-20.

The objective is to factor given trinomial.

Compare b^2+b-20 with x^2+cx+d.

Here $c=1$,

$d=-20$

$$b^2+b-20=(b+m)(b+n)$$
$$=b^2+(m+n)b+mn$$

That is $m+n=1$ positive and

$mn=-20$ is negative.

So, either m or n negative, but not both.

Now make list of factors of -20, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 1.

Factors of -20	Sum of factors
$-1, 20$	19
$1, -20$	-19
$4, -5$	-1
$-4, 5$	1
$10, -2$	8
$-10, 2$	-8

The correct factors are $-4, 5$.

Therefore,

$$b^2 + b - 20 = (b+m)(b+n)$$

$$= (b+(-4))(b+5) \quad (m=-4, n=5)$$

$$= (b-4)(b+5)$$

Check:- Check the results by multiplying two factors by $FOIL$ Method.

$$(b-4)(b+5) = b \cdot \overset{F}{b} + 5 \cdot \overset{O}{b} + (\overset{I}{-4}) \cdot b + (\overset{L}{-4}) \cdot 5$$

($FOIL$ Method)

$$= b^2 + 5b - 4b - 20 \text{ (Simplify)}$$

$$= b^2 + b - 20 \text{ True}$$

Therefore, the factorized form of $\boxed{b^2 + b - 20}$ is $\boxed{(b-4)(b+5)}$.

Answer 27PA.

Consider the trinomial $h^2+3h-40$

The objective is to factor given trinomial

Compare $h^2+3h-40$ with ax^2+bx+c.

Here $b=3$,

$c=-40$

$$h^2+3h-40=(h+x)(h+y)$$
$$=h^2+(x+y)h+xy$$

That is $x+y=3$, positive and

$xy=-40$ is negative.

So either x or y negative, but not both.

Now make list of factors of -40, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 3.

Factors of -40	Sum of factors
$1.-40$	-39
-1.40	39
$8.-5$	3
-8.5	-3
$2.-20$	-18
-2.20	18

$10,-4$	6
$-10,4$	-6

The correct factors are $8,-5$.

Therefore, $h^2+3h-40=(h+x)(h+y)$

$=(h+8)(h+(-5))$ $(m=8, n=-5)$

$=(h+8)(h-5)$

Check:- Check the results by multiplying two factors by $FOIL$ method.

$(h+8)(h-5)=\overset{F}{h}\cdot h+8\cdot\overset{O}{h}+(\overset{I}{-5})\cdot h+\overset{L}{8}(-5)$

($FOIL$ method)

$=h^2+8h-5h-40$ (Simplify)

$=h^2+3h-40$ True

Therefore, the factorized form of $\boxed{h^2+3h-40}$ is $\boxed{(h+8)(h-5)}$.

Answer 28PA.

Consider the trinomial $n^2+3n-54$

The objective is to factor given trinomial

Compare $n^2+3n-54$ with x^2+bx+c.

Here $b=3$,

$c=-54$

$$n^2+3n-54=(n+x)(n+y)$$
$$=n^2+(x+y)n+xy$$

That is $x+y=3$, positive and

$xy=-54$ is negative.

So either x or y negative, but not both.

Now make list of factors of -54, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 3.

Factors of -54	Sum of factors
$1.-54$	-53
-1.54	53
$9.-16$	3
$6.-9$	-3
$3.-18$	-15
-3.18	15
$2.-27$	-25

-2.27	25

The correct factors are $9, -6$.

Therefore, $n^2 + 3n - 54 = (n + x)(n + y)$

$= (n+9)(n+(-6)) \quad (x = 9, y = -6)$

$= (n+9)(n-6)$

Check:- Check the results by multiplying two factors by $FOIL$ method.

$(n+9)(n-6) = \overset{F}{n} \cdot n + (\overset{O}{-6}) \cdot n + \overset{I}{9} \cdot n + \overset{L}{9}(-6)$

($FOIL$ method)

$= n^2 - 6n + 9n - 54$ (Simplify)

$= n^2 + 3n - 54$ True

Therefore, the factorized form of $\boxed{n^2 + 3n - 54}$ is $\boxed{(n+9)(n-6)}$.

Answer 29PA.

Consider the trinomial $y^2 - y - 42$.

The objective is to factor given trinomial.

Compare $y^2 - y - 42$ with $x^2 + bx + c$.

Here $b = -1$,

$c = -42$

$$y^2 - y - 42 = (y+m)(y+n)$$
$$= y^2 + (m+n)y + mn$$

That is $m + n = -1$ is negative and

$mn = -42$ is also negative.

So, either m or n is negative but not both.

Now make list of positive factors of -42, in those pair of factors, choose the factors whose sum is -1.

Factors of -42	Sum of factors
$1, -42$	-41
$-1, 42$	41
$6, -7$	-1
$-6, 7$	1
$2, -21$	-19
$-2, 21$	19
$3, -14$	-11

$-3,14$	11

The correct factors are $6,-7$.

Therefore,

$$y^2-y-42=(y+m)(y+n)$$

$$=(y+6)(y+(-7)) \quad (m=6, n=-7)$$

$$=(y+6)(y-7)$$

Check:- Check the result, by multiplying the factors by $FOIL$ method.

$$(y+6)(y-7)=\overset{F}{y}\cdot y+\overset{O}{(-7)}\cdot y+\overset{I}{6}\cdot y+6\cdot\overset{L}{(-7)}$$

($FOIL$ method)

$$=y^2-7y+6y-42 \text{ (Simplify)}$$

$$=y^2-y-42 \text{ True}$$

Therefore, the factorized form of $\boxed{y^2-y-42}$ is $\boxed{(y+6)(y-7)}$.

Answer 30PA.

Consider the trinomial $z^2-18z-40$

The objective is to factor given trinomial.

Compare $z^2-18z-40$ with x^2+bx+c.

Here $b=-18,$

$c=-40$

$$z^2-18z-40=(z+m)(z+n)$$
$$=z^2+(m+n)z+mn$$

That is $m+n=-18$ is negative and

$mn=-40$ is also negative.

So, either m or n is negative, but not both.

Now make list of negative factors of -40, in those pair of factors, choose the factors whose sum is -18.

Factors of -40	Sum of factors
$1,-40$	-39
$-1,40$	39
$2,-20$	-18
$-2,20$	18
$10,-4$	6
$-10,4$	-6
$8,-5$	3

$-8,5$	-3

The correct factors are $2,-20$.

Therefore,

$$z^2-18z-40=(z+m)(z+n)$$

$$=(z+2)(z-20) \quad (m=2, n=-20)$$

Check:- Check the result, by multiplying the factors by $FOIL$ method.

$$(z+2)(z-20)=\overset{F}{z\cdot z}+\overset{O}{z\cdot(-20)}+\overset{I}{2(z)}+\overset{L}{2\cdot(-20)}$$

($FOIL$ Method)

$$=z^2-20z+2z-40$$

(Simplify)

$$=z^2-18z-40 \text{ True}$$

Therefore, the factorized form of $\boxed{z^2-18z-40}$ is $\boxed{(z+2)(z-20)}$.

Answer 31PA.

Consider the trinomial $-72+6w+w^2$

The given trinomial can be written as $w^2+6w-72$.

Compare $w^2+6w-72$ with x^2+bx+c

Here $b=6$,

$c=-72$

$$w^2+6w-72=(w+m)(w+n)$$
$$=w^2+(m+n)w+mn$$

That is $m+n=6$ is positive and

$mn=-72$ is negative.

So, either m (or) n negative, but not both.

Now make list of factors of -72, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 6.

Factors of -72	Sum of factors
$1.-72$	-71
-1.72	71
$8.-9$	-1
-8.9	1
-3.24	21
$3.-24$	-21
$6.-12$	-6

-6.12	6

The correct factors are $-6,12$.

Therefore, $w^2+6w-72=(w+m)(w+n)$

$=(w+(-6))(w+2)$

$(m=-6,n=12)$

$=(w-6)(w+12)$

Check:- Check the results, by multiplying two factors by $FOIL$ method.

$(w-6)(w+12)=\overset{F}{w}\cdot w+(\overset{O}{-6})\cdot w+\overset{I}{12}\cdot w+(\overset{L}{-6})\cdot 12$

($FOIL$ method)

$=w^2-6w+12w-72$

(Simplify)

$=w^2+6w-72$ True

Therefore, the factorized form of $\boxed{w^2+6w-72}$ is $\boxed{(w-6)(w+12)}$.

Answer 32PA.

Consider the trinomial $-30+13x+x^2$

The given trinomial can be written as $x^2+13x-30$.

Compare $x^2+13x-30$ with x^2+bx+c

Here $b=13,$

$c=-30$

$$x^2+13x-30=(x+m)(x+n)$$
$$=x^2+(m+n)x+mn$$

That is $m+n=13$ is positive and

$mn=-30$ is negative.

So, either m (or) n negative, but not both.

Now make list of factors of -30, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 13.

Factors of -30	Sum of factors
$1,-30$	-29
$-1,30$	29
$6,-5$	1
$-6,5$	-1
$2,-15$	-13
$-2,15$	13
$3,-10$	-7

-3.10	7

The correct factors are $-2,15$.

Therefore, $x^2+13x-30=(x+m)(x+n)$

$=(x+(-2))(x+15)$

$(m=-2,n=15)$

$=(x-2)(x+15)$

Check:- Check the results, by multiplying two factors by $FOIL$ method.

$(x-2)(x+15)=\overset{F}{x}\cdot x+\overset{0}{(-2)}\cdot x+\overset{I}{x}\cdot 15+\overset{L}{(-2)}\cdot 15$

($FOIL$ method)

$=x^2-2x+15x-30$

(Simplify)

$=x^2+13x-30$ True

Therefore, the factorized form of $\boxed{x^2+13x-30}$ is $\boxed{(x-2)(x+15)}$.

Answer 33PA.

Consider the trinomial $a^2+5ab+4b^2$.

The objective is to factor the given trinomial.

Compare $a^2+5ab+4b^2$ with x^2+cx+d.

Where $c=56,$

$d=46^2$

Since

$(a+m)(a+n)=a^2+(m+n)a+m\cdot n$

Now find two numbers whose product is 46^2 and sum is $5b$.

Factors of $4b^2$	Sum of factors
$2b.2b$	$4b$
$4b.b$	$5b$
$1.4b^2$	$4b^2+1$

The correct factors are $b,4b$.

Hence

$a^2+5ab+4b^2=(a+m)(a+n)$

$=(a+b)(a+4b)$ (Put $m=b,n=4b$)

Check: Check the result by multiply two factors by $FOIL$ method.

$(a+b)(a+4b)=\overset{F}{a\cdot a}+\overset{O}{a\cdot 4b}+\overset{I}{b\cdot a}+\overset{L}{b\cdot 4b}$ ($FOIL$ method)

$=a^2+4ab+ab+4b^2$ (Simplify)

$=a^2+5ab+4b^2$ True

Therefore, the factorization of $\boxed{a^2+5ab+4b^2}$ is $\boxed{(a+b)(a+4b)}$.

Answer 34PA.

Consider the trinomial $x^2 - 13xy + 36y^2$

The objective is to factor the given trinomial.

Compare $x^2 - 13y \cdot x + xy^2$ with $x^2 + bx + c$

$b = -13y,$

$c = 36y^2$

$$\begin{aligned} x^2 - 13xy + 36y^2 &= (x+m)(x+n) \\ &= x^2 + (m+n)x + mn \end{aligned}$$

That is $m + n = -13y$ is negative and

$mn = 36y^2$ is positive.

So, m, n must both be negative.

Now list of negative factors of $36y^2$, in those pair of factors, choose the factors whose sum is $-13y$.

$$\begin{aligned} 36y^2 &= 1 \cdot 36y^2 \\ &= 1 \cdot 3 \cdot 12 \cdot y^2 \\ &= 1 \cdot 3 \cdot 3 \cdot 4 \cdot y^2 \\ &= 1 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot y^2 \\ &= 1 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot y \cdot y \end{aligned}$$

Choose the factors,

$36y^2 = 9y \cdot 4y$

The negative factors are $-4y, -9y$.

The sum of the factors is

$-4y + -9y = -13y$

Therefore, the correct factors are $-4y, -9y$

Therefore,

$$x^2 - 13xy + 36y^2 = (x+m)(x+n)$$

$$= (x + -4y)(x + (-9y)) \quad (m = -4y, n = -9y)$$

$$= (x - 4y)(x - 9y)$$

Check: Check the result by multiplying the factors by $FOIL$ method.

$$(x-4y)(x-9y) = \overset{F}{x \cdot x} + \overset{O}{x \cdot (-9y)} + \overset{I}{x \cdot (-4y)} + \overset{L}{(-4y) \cdot (-9y)}$$

($FOIL$ method)

$= x^2 - 13xy + 36y^2$ True

Therefore, the factorization of $\boxed{x^2 - 13xy + 36y^2}$ is $\boxed{(x-4y)(x-9y)}$.

Answer 35PA.

Consider that the area of a rectangle

$= x^2 + 24x - 81$

The objective is to find an expression for the perimeter of a rectangle.

For this first factor $x^2 + 24x - 81$

Compare $x^2 + 24x - 81$ with $x^2 + bx + c$

Here $b = 24,$

$c = -81$

$$x^2 + 24x - 81 = (x+m)(x+n)$$
$$= x^2 + (m+n)x + m \cdot n$$

Here $m + n = 24$ positive and

$mn = -81$ is negative

So either m or n is negative but not both

Now make list all factors of -81 where one factor is negative, choose a pair of factors whose sum is 24.

Factors of -81	Sum of factors
-1.81	80
$1.-81$	-80
-3.27	24
$3.-27$	-27
$9.-9$	0

The correct factors are $-3, 27$

Therefore,

$x^2+24x-81=(x+m)(x+n)$

$=(x+(-3))(x+27) \quad (m=-3, n=27)$

$=(x-3)(x+27)$

Since area of rectangle

$=l\cdot b$.

Here $l=x+27,$

$b=x-3$

Also perimeter of rectangle

$p=2\cdot(l+b)$

$=2(x-27+x-3)$ (Substitute l,b)

$=2(x+x+24)$

$=2(2x+24)$ (Combine like terms)

$=2\cdot 2x+2\cdot 24$ (By distributive)

$=4x+48$

Therefore, perimeter of rectangle

$=\boxed{4x+48}$.

Answer 36PA.

Consider that area of a rectangle

$= x^2 + 13x - 90$

The objective is to find an expression for the perimeter of a rectangle.

For this first factor $x^2 + 13x - 90$.

Compare $x^2 + 13x - 90$ with $x^2 + bx + c$.

Here $b = 13,$

$c = -90$.

$$x^2 + 13x - 90 = (x+m)(x+n)$$
$$= x^2 + (m+n)x + m \cdot n$$

Here $m + n = 13$ is positive and

$mn = -90$ is negative.

So either m or n must be negative but not both

Now make list all factors of -90 where one factor is negative choose a pair of factors whose sum is 13.

Factors of -90	Sum of factors
$1.-90$	-89
-1.90	89
-2.45	43
$2.-45$	-43
-3.30	27
$3.-30$	-27
-5.18	13
$5.-18$	-13
-6.15	9
$6.-15$	-9
-9.10	1
$9.-10$	-1

The correct factors are $-5,18$.

Hence

$$x^2+13x-90=(x+m)(x+n)$$

$$=(x+(-5))(x+18) \quad (m=-5, n=18)$$

$$=(x-5)(x+18)$$

Since area of rectangle

$$=l \cdot b$$

Here $l = x+18,$

$$b = x-5$$

Also perimeter of rectangle

$$p=2(l+b)$$

$$=2(x+18+x-5) \quad (l=x+18, b=x-5)$$

$$=2(2x+13)$$ (Simplify)

$$=2\cdot 2x+2\cdot 13$$ (By distributive $a(b+c)=ab+ac$)

$$=4x+26$$

Therefore, perimeter of rectangle is $\boxed{4x+26}$.

Answer 37PA.

Consider the equation

$x^2 + 16x + 28 = 0$

The objective is to solve the given equation.

For this first factor $x^2 + 16x + 28$ and then use zero product property.

Compare $x^2 + 16x + 28$ with $x^2 + bx + c$.

Here $b = 16,$

$c = 28$

Since $(x+m)(x+n) = x^2 + (m+n)x + mn$

Now find two numbers whose product is 28 and sum is 16.

For this list all the factors of 28 and choose a pair of factors whose sum is 16.

Factors of 28	Sum of factors
1.28	29
7.4	11
14.2	16

The correct factors are $14, 2$.

$x^2 + 16x + 28 = (x+m)(x+n)$

$= (x+14)(x+2) \quad (m = 14, n = 2)$

Therefore, $x^2 + 16x + 28 = 0$

$(x+14)(x+2) = 0$ (Factor)

$x + 14 = 0$

(or) $x + 2 = 0$ (By zero product property)

Now solve each equation separately.

$x + 14 = 0$

$x+14-14=0-14$ (Subtract 14 on each side)

$x=-14$

$x+2=0$

$x+2-2=0-2$ (Subtract 2 on each side)

$x=-2$

The solution set is $\{-14,-2\}$.

Check:- To check the proposed solution set, substitute x by $-14,-2$ in the given equation.

For $x=-14$,

$x^2+16x+28=0$

$(-14)^2+16(-14)+28=0$ (Put $x=-14$)

$196-224+28=0$ (Simplify)

$0=0$ True

For $x=-2$,

$x^2+16x+28=0$

$(-2)^2+16(-2)+28=0$ (Put $x=-2$)

$4-32+28=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{-14,-2\}}$.

Answer 38PA.

Consider the equation

$b^2+20b+36=0$

The objective is to solve the given equation.

For this first factor $b^2+20b+36$ and then use zero product property.

Compare $b^2+20b+36$ with x^2+bx+c.

Here $b=20,$

$c=36$

Since $(b+m)(b+n)=b^2+(m+n)b+mn$

Now find two numbers whose product is 36 and sum is 20.

For this list all the factors of 36 and choose a pair of factors whose sum is 20.

Factors of 36	Sum of factors
1.36	37
2.18	20
3.12	15
9.4	13
6.6	12

The correct factors are $2,18$.

$b^2+20b+36=(b+m)(b+n)$

$=(b+2)(b+18) \quad (m=2,n=18)$

Therefore,

$b^2 + 20b + 36 = 0$

$(b+2)(b+18) = 0$ (Factor)

$b+2 = 0$ (or)

$b+18 = 0$ (By zero product property)

Now solve each equation separately.

$b+2 = 0$

$b+2-2 = 0-2$ (Subtract 2 on each side)

$b = -2$

$b+18 = 0$

$b+18-18 = 0-18$ (Subtract 18 on each side)

$b = -18$

The solution set is $\{-2, -18\}$.

Check: To check the proposed solution set substitute b by $-2, -18$ in the given equation.

For $b = -2$,

$b^2 + 20b + 36 = 0$

$(-2)^2 + 20(-2) + 36 = 0$ (Put $b = -2$)

$4 - 40 + 36 = 0$ (Simplify)

$0 = 0$ True

For $b = -18$,

$b^2 + 20b + 36 = 0$

$(-18)^2 + 20(-18) + 36 = 0$ (Put $b = -18$)

$324 - 360 + 36 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{-2, -18\}}$.

Answer 39PA.

Consider the equation

$$y^2+4y-12=0$$

The objective is to solve the given equation.

For this first factor $y^2+4y-12$ and then use zero product property.

Compare $y^2+4y-12$ with x^2+bx+c.

Here $b=4$,

$c=-12$

Since $(y+m)(y+n)=y^2+(m+n)y+mn$.

That is $m+n=4$ positive and

$mn=-12$ is negative. So, either m (or) n negative but not both.

Now make list of factors of -12, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 4.

Factors of -12	Sum of factors
$1,-12$	-11
$-1,12$	11
$3,-4$	-1
$-3,4$	1
$2,-6$	-4
$-2,6$	4

The correct factors are $-2,6$.

$$y^2+4y-12=(y+m)(y+n)$$

$=(y+(-2))(y+6)$ $(m=-2, n=6)$

$=(y-2)(y+6)$

Therefore, $y^2+4y-12=0$

$(y-2)(y+6)=0$ (Factors)

$y-2=0$

(or) $y+6=0$ (By zero product property)

Now solve each equation separately.

$y-2=0$

$y-2+2=0+2$ (Add 2 on both sides)

$y=2$

$y+6=0$

$y+6-6=0-6$ (Subtract 6 on both sides)

$y=-6$

The solution set is $\{2,-6\}$.

Check:- To check the proposed solution set, substitute y by $2,-6$ in given equation.

For $y=2$,

$y^2+4y-12=0$

$(2)^2+4(2)-12=0$ (Put $y=2$)

$4+8-12=0$

$0=0$ True

For $y=-6$,

$(-6)^2+4(-6)-12=0$ (Put $y=-6$)

$36-24-12=0$

$36-36=0$

$0=0$ True

Therefore, the solution set is $\boxed{\{2,-6\}}$.

Answer 40PA.

Consider the equation

$d^2+2d-8=0$

The objective is to solve the given equation.

For this first factor d^2+2d-8 and then use zero product property.

Compare d^2+2d-8 with x^2+bx+c.

Here $b=2$,

$c=-8$

Since $(d+m)(d+n)=d^2+(m+n)d+mn$

That is $m+n=2$ positive and

$mn=-8$ is negative.

So either m (or) n negative but not both.

Now make list of factors of -8, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 2.

Factors of -8	Sum of factors
$1,-8$	-7
$-1,8$	7
$2,-4$	-2
$-2,4$	2

The correct factors are $-2,4$.

$d^2+2d-8=(d+m)(d+n)$

$=(d-2)(d+4)\quad (m=-2,n=4)$

Therefore, $d^2+2d-8=0$

$(d-2)(d+4)=0$ (Factors)

$d-2=0$

(or) $d+4=0$ (By zero product property)

Now solve each equation separately,

$d-2=0$

$d-2+2=0+2$ (Add 2 on each side)

$d=2$

$d+4=0$

$d+4-4=0-4$ (Subtract 4 on each side)

$d=-4$

The solution set is $\{2,-4\}$.

Check: To check the proposed solution, substitute d by $2,-4$ in the given equation.

For $d=2$,

$d^2+2d-8=0$

$(2)^2+2(2)-8=0$ (Put $d=2$)

$4+4-8=0$ (Simplify)

$0=0$ True

For $d=-4$,

$d^2+2d-8=0$

$(-4)^2+2(-4)-8=0$ (Put $d=-4$)

$16-8-8=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{2,-4\}}$.

Answer 41PA.

Consider the equation

$a^2-3a-28=0$

The objective is to solve the given equation.

For this first factor $a^2-3a-28$ and then use zero product property.

Compare $a^2-3a-28$ with x^2+bx+c.

Here $b=-3$,

$c=-28$

Since $(a+m)(a+n)=a^2+(m+n)a+mn$

That is $m+n=-3$, negative and

$mn=-28$ is also negative.

So, either m (or) n is negative, but not both.

Now make list of factors of -28, in those pair of factors, choose the factors whose sum is -3.

Factors of -28	Sum of factors
$1,-28$	-27
$-1,28$	27
$4,-7$	-3
$-4,7$	3
$2,-14$	-12
$-2,14$	12

The correct factors are $4,-7$.

$a^2-3a-28=(a+m)(a+n)$

$=(a+4)(a+(-7))$ $(m=4, n=-7)$

$=(a+4)(a-7)$

Therefore, $a^2-3a-28=0$

$(a+4)(a-7)=0$ (Factor)

$a+4=0$

(or) $a-7=0$ (By zero product property)

Now solve each equation separately.

$a+4=0$

$a+4-4=0-4$ (Subtract 4 on each side)

$a=-4$

$a-7=0$

$a-7+7=0+7$ (Add 7 on each side)

$a=7$

The solution set is $\{-4,7\}$.

Check:- To check the proposed solution, substitute a by $-4,7$ in the given equation.

For $a=-4$,

$a^2-3a-28=0$

$(-4)^2-3(-4)-28=0$ (Put $a=-4$)

$16+12-28=0$

$0=0$ True

For $a=7$,

$a^2-3a-28=0$

$(7)^2-3(7)-28=0$ (Put $a=7$)

$49-21-28=0$

$0=0$ True

Therefore, the solution set is $\boxed{\{-4,7\}}$.

Answer 42PA.

Consider the equation

$g^2 - 4g - 45 = 0$

The objective is to solve the given equation.

For this first factor $a^2 - 3a - 28$ and then use zero product property

Compare $g^2 - 4g - 45$ with

$x^2 + bx + c = 0$

Here $b = -4$,

$c = -45$

Since $(g+x)(g+y) = g^2 + (x+y)g + xy$

That is $x + y = -4$ negative and

$xy = -45$ is also negative.

So, either m (or) n is negative, but not both.

Now make list of factors of -45, in those pair of factors, choose the factors whose sum is -4.

Factors of -45	Sum of factors
$1, -45$	-44
$-1, 45$	44
$9, -5$	4
$-9, 5$	-4
$3, -15$	-12
$-3, 15$	12

The correct factors are $-9, 5$.

$g^2 - 4g - 45 = (g + x)(g + y)$

$= (g + (-9))(g + 5)$ $(x = -9, y = 5)$

$= (g - 9)(g + 5)$

Therefore,

$g^2 - 4g - 45 = 0$

$(g - 9)(g + 5) = 0$ (Factors)

$g - 9 = 0$ (or)

$g + 5 = 0$ (By zero product property)

Now solve each equation separately.

$g - 9 = 0$

$g - 9 + 9 = 0 + 9$ (Add 9 on each side)

$g = 9$

$g + 5 = 0$

$g + 5 - 5 = 0 - 5$ (Subtract 5 on each side)

$g = -5$

The solution set is $\{9, -5\}$.

Check:- To check the proposed solution, substitute g by $9, -5$ in the given equation.

For $g=9$,

$g^2-4g-45=0$

$(9)^2-4(9)-45=0$ (Put $g=9$)

$81-36-45=0$ (Simplify)

$0=0$ True

For $g=-5$,

$g^2-4g-45=0$

$(-5)^2-4(-5)-45=0$ (Put $g=-5$)

$25+20-45=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{9,-5\}}$.

Answer 43PA.

Consider the equation

$m^2 - 19m + 48 = 0$

The objective is to solve the given equation.

For this first factor $m^2 - 19m + 48$ and then use zero product property.

Compare $m^2 - 19m + 48$ with $x^2 + bx + c$.

Here $b = 19,$

$c = 48$

Since $(m+x)(m+y) = m^2 + (x+y)m + xy$

That is $x + y = -19$ negative and

$xy = 48$ is positive.

So, $x \,\&\, y$ must both be negative.

Now make list of factors of 48, in those pair of factors, choose the factors whose sum is -19.

Factors of 48	Sum of factors
$-1.-48$	-49
$-2.-24$	-26
$-6.-8$	-14
$-16.-3$	-19
$-4.-12$	-16

The correct factors are $-16,-3$.

$m^3 - 19m + 48 = (m+x)(m+y)$

$= (m+(-16))(m+(-3)) \quad (x = -16, y = -3)$

$=(m-16)(m-3)$

Therefore,

$m^2-19m+48=0$

$(m-16)(m-3)=0$ (Factor)

$m-16=0$

(or) $m-3=0$ (By zero product property)

Now solve each equation separately.

$m-16=0$

$m-16+16=0+16$ (Add 16 on both sides)

$m=16$

$m-3=0$

$m-3+3=0+3$ (Add 3 on each side)

$m=3$

The solution set is $\{16,3\}$.

Check:- To check proposed solution, substitute m by $16,3$ in the given equation.

For $m=16$,

$m^2-19m+48=0$

$(16)^2-19(16)+48=0$ (Put $m=16$)

$256-304+48=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{16,3\}}$.

Answer 44PA.

Consider the equation

$n^2 - 22n + 72 = 0$

The objective is to solve the given equation.

For this first factor $n^2 - 22n + 72$ and then use zero product property.

Compare $n^2 - 22n + 72$ with $x^2 + bx + c$.

Here $b = -22$,

$c = 72$

Since $(n+x)(n+y) = n^2 + (x+y)n + xy$

That is $x + y = -22$ negative and

$xy = 72$ positive.

So, x and y must both be negative.

Now make list of factors of 72, in those pair of factors, choose the factors whose sum is -22.

Factors of 72	Sum of factors
$-1.-72$	-73
$-8.-9$	-17
$-36.-2$	-38
$-24.-3$	-27
$-18.-4$	-22
$-6.-12$	-18

The correct factors are $-18, -4$.

$n^2 - 22n + 72 = (n+x)(x+y)$

$= (n + (-18))(n + (-4))$ $(x = -18, y = -4)$

$= (n - 18)(n - 4)$

Therefore, $n^2 - 22n + 72 = 0$

$(n - 18)(n - 4) = 0$ (Factor)

$n - 18 = 0$

(or) $n - 4 = 0$ (By zero product property)

Now solve each equation separately.

$n - 18 = 0$

$n - 18 + 18 = 0 + 18$ (Add 18 on both sides)

$n = 18$

$n - 4 = 0$

$n - 4 + 4 = 0 + 4$ (Add 4 on both sides)

$n = 4$

The solution set is $\{18, 4\}$.

Check:- To check the proposed solution, substitute n by $18, 4$ in the given equation.

For $n = 18$,

$n^2 - 22n + 72 = 0$

$(18)^2 - 22(18) + 72 = 0$ (Put $n = 18$)

$324 - 396 + 72 = 0$ (Simplify)

$0 = 0$ True

For $n = 4$,

$n^2 - 22n + 72 = 0$

$(4)^2 - 22(4) + 72 = 0$ (Put $n = 4$)

$16 - 88 + 72 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set $\boxed{\{18, 4\}}$.

Answer 45PA.

Consider the equation

$z^2 = 18 - 7z$

The objective is to solve the given equation.

$z^2 = 18 - 7z$

$z^2 - 18 = 18 - 7z - 18$ (Subtract 18 on each side)

$z^2 - 18 = -7z$ (Simplify)

$z^2 - 18 + 7z = -7z + 7z$ (Add $7z$ on each side)

$z^2 + 7z - 18 = 0$ (Simplify)

The given equation can be written as

$z^2 + 7z - 18 = 0$

For this first factor $z^2 + 7z - 18$ and then use zero product property.

Compare $z^2 + 7z - 18$ with $x^2 + bx + c$.

Here $b = 7$,

$c = -18$

Since $(z+m)(z+n) = z^2 + (m+n)z + mn$

That is $m + n = 7$ positive,

$mn = -18$ negative.

So, either m (or) n negative, but not both.

Now make list of factors of -18, where one factor of each pair is negative, in those pair of factors, choose the factors whose sum is 7.

Factors of -18	Sum of factors
-1.18	17
$18.-1$	-17
-3.6	3
$3.-6$	-3
$9.-2$	7
-9.2	-7

The correct factors are $9,-2$.

$z^2+7z-18=(z+m)(z+n)$

$=(z+9)(z+(-2))$ $(m=9,n=-2)$

$=(z+9)(z-2)$

Therefore, $z^2+7z-18=0$

$(z+9)(z-2)=0$ (Factors)

$z+9=0$

(or) $z-2=0$ (By zero product property)

Now solve each equation separately.

$z+9=0$

$z+9-9=0-9$ (Subtract 9 on each side)

$z=-9$

$z-2=0$

$z-2+2=0+2$ (Add 2 on each side)

$z = 2$

The solution set is $\{-9, 2\}$.

Check:- To check the proposed solution, substitute z by $-9, 2$ in given equation.

For $z = -9$,

$z^2 + 7z - 18 = 0$

$(-9)^2 + 7(-9) - 18 = 0$ (Put $z = -9$)

$81 - 63 - 18 = 0$

$0 = 0$ True

For $z = 2$,

$z^2 + 7z - 18 = 0$

$(2)^2 + 7(2) - 18 = 0$ (Put $z = 2$)

$4 + 14 - 18 = 0$

$0 = 0$ True

Therefore, the solution set is $\boxed{\{-9, 2\}}$.

Answer 46PA.

Consider the equation

$h^2+15=-16h$

The equation can be written as

$h^2+15+16h=-16h+16h$ (Add $16h$ on each side)

$\Rightarrow \quad h^2+16h+15=0$ (Simplify)

The objective is to solve the given equation.

For this first factor $h^2+16h+15$ and then use zero product property.

Compare $h^2+16h+15$ with x^2+bx+c.

Here $b=16$,

$c=15$

Since $(h+m)(h+n)=h^2+(m+n)h+mn$

That is $m+n=16$ positive and

$mn=15$ is also positive.

Now find two numbers whose product is 15 and sum is 16.

For this list all the factors of 15 and choose a pair of factors whose sum is 16.

Factor of 15	Sum of factors
$1,15$	16
$3,5$	8

The correct factors are $1,15$.

$h^2+16h+15=(h+m)(h+n)$

$=(h+1)(h+15) \quad (m=1, n=15)$

Therefore,

$h^2+16h+15=0$

$(h+1)(h+15)=0$ (Factor)

$h+1=0$ (or)

$h+15=0$ (By zero product property)

Now solve each equation separately.

$h+1=0$

$h+1-1=0-1$ (Subtract 1 on each side)

$h=-1$

$h+15=0$

$h+15-15=0-15$ (Subtract 15 on each side)

$h=-15$

The solution set is $\{-1,-15\}$.

Check: To check the proposed solution, substitute h by $-1,-15$ in the given equation.

For $h=-1$,

$h^2+16h+15=0$

$(-1)^2+16(-1)+15=0$ (Put $h=-1$)

$1-16+15=0$ (Simplify)

$0=0$ True

For $h=-15$,

$h^2+16h+15=0$

$(-15)^2+16(-15)+15=0$ (Put $h=-15$)

$225-240+15=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\{-1,-5\}}$.

Answer 47PA.

Consider the equation

$24+k^2=10k$

$\Rightarrow \quad 24+k^2-10k=10k-10k$ (Subtract $10k$ on each side)

$\Rightarrow \quad 24+k^2-10k=0$ (Simplify)

The equation can be written as

$k^2 - 10k + 24 = 0$

The objective is to solve the given equation.

For this first factor $k^2 - 10k + 24$ and then use zero product property.

Compare $k^2 - 10k + 24$ with $x^2 + bx + c$.

Here $b = -10,$

$c = 24$

Since $(k+m)(k+m) = k^2 + (m+n)k + mn$

That is $m + n = -10$ negative and

$mn = 24$ is positive.

So, both m, n are negative.

For this list all the factors of 24 and choose a pair of factors whose sum is -10.

Factors of 24	Sum of factors
$-1, -24$	-25
$-6, -4$	-10
$-2, -12$	-14
$-8, -3$	-11

The correct factors are $-6, -4$.

$k^2 - 10k + 24 = (k+m)(k+n)$

$=(k+(-6))(k+(-4))$

$(m=-6, n=-4)$

$=(k-6)(k-4)$

Therefore,

$k^2-10k+24=0$

$(k-6)(k-4)=0$ (Factor)

$k-6=0$ (or)

$k-4=0$ (By zero product property)

Now solve each equation separately.

$k-6=0$

$k-6+6=0+6$ (Add 6 on each side)

$k=6$

$k-u=0$

$k-u+u=0+4$ (Add 4 on each side)

$k=4$

The solution set is $\{6,4\}$.

Check:- To check the proposed solution, substitute k by $6,4$ in the given equation.

For $k = 6$,

$k^2 - 10k + 24 = 0$

$(6)^2 - 10(6) + 24 = 0$ (Put $k = 6$)

$36 - 60 + 24 = 0$ (Simplify)

$0 = 0$ True

For $k = 4$,

$k^2 - 10k + 24 = 0$

$(4)^2 - 10(4) + 24 = 0$ (Put $k = 4$)

$16 - 40 + 24 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{6, 4\}}$.

Answer 48PA.

Consider the equation

$x^2 - 20 = x$

$x^2 - 20 - x = x - x$ (Subtract x on each side)

$\Rightarrow \quad x^2 - x - 20 = 0$ (Simplify)

The objective is to solve the given equation.

For this first factor $x^2 - x - 20$ and then use zero product property.

Compare $x^2 - x - 20$ with $x^2 + bx + c$.

Here $b = -1$,

$c = -20$

Since $(x+m)(x+n) = x^2 + (m+n)x + mn$

That is $m + n = -1$ negative and

$mn = -20$ is also negative.

So either m (or) n is negative but not both.

Now make list of factors of -20, in those pair of factors, choose the factors whose sum is -1.

Factors of -20	Sum of factors
-1.20	19
$1.-20$	-19
-4.5	1
$4.-5$	-1
$2.-10$	-8
-2.10	8

The correct factors are $4, -5$.

$x^2 - x - 20 = (x+m)(x+n)$

$= (x+4)(x+(-5)) \quad (m = 4, n = -5)$

$= (x+4)(x-5)$

Therefore, $x^2 - x - 20 = 0$

$(x+4)(x-5) = 0$ (Factors)

$x + 4 = 0$

(or) $x - 5 = 0$ (By zero product property)

Now solve each equation separately.

$x + 4 = 0$

$x + 4 - 4 = 0 - 4$ (Subtract 4 on each side)

$x = -4$

$x - 5 = 0$

$x - 5 + 5 = 0 + 5$ (Add 5 on each side)

$x = 5$

The solution set is $\{-4, 5\}$.

Check:- To check the proposed solution, substitute x by $-4, 5$ in the given equation.

For $x = -4$,

$x^2 - x - 20 = 0$

$(-4)^2 - (-4) - 20 = 0$ (Put $x = -4$)

$16 + 4 - 20 = 0$ (Simplify)

$0 = 0$ True

For $x = 5$,

$x^2 - x - 20 = 0$

$(5)^2 - 5 - 20 = 0$ (Put $x = 5$)

$25 - 5 - 20 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{-4, 5\}}$.

Answer 49PA.

Consider the equation

$c^2 - 50 = -23c$

$\Rightarrow \quad c^2 - 50 + 23c = -23c + 23c$ (Add $23c$ on each side)

$\Rightarrow \quad c^2 + 23c - 50 = 0$ (Simplify)

The objective is to solve the given equation.

For this first factor $c^2 + 23c - 50$ and then use zero product property.

Compare $c^2 + 23c - 50$ with $x^2 + bx + c$.

Here $b = 23,$

$c = -50$.

Since $(c+m)(c+n) = c^2 + (m+n)c + mn$

That is $m + n = 23$ positive and

$mn = -50$ is negative.

So, either m (or) n is negative but not both.

Now make list of factors of -50, in those pair of factors, choose the factors whose sum is 23.

Factors of -50	Sum of factors
$-1, 50$	49
$1, -50$	-49
$-2, 25$	23
$2, -25$	-23
$-10, 5$	-5
$10, -5$	5

The correct factors are $-2, 25$.

$c^2 + 23c - 50 = (c+m)(c+m)$

$= (c+(-2))(c+25) \quad (m = -2, n = 25)$

$= (c-2)(c+25)$

Therefore,

$c^2 + 23c - 50 = 0$

$(c-2)(c+25) = 0$ (Factor)

$c - 2 = 0$ (or)

$c + 25 = 0$ (By zero product property)

Now solve each equation separately.

$c - 2 = 0$

$c - 2 + 2 = 0 + 2$ (Add 2 on each side)

$c = 2$

$c + 25 = 0$

$c + 25 - 25 = 0 - 25$ (Subtract 25 on each side)

$c = -25$

The solution set is $\{2, -25\}$.

Check:- To check the proposed solution, substitute c by $2, -25$ in the given equation.

For $c = 2$,

$c^2 + 23c - 50 = 0$

$(2)^2 + 23(2) - 50 = 0$ (Put $c = 2$)

$4 + 46 - 50 = 0$ (Simplify)

$0 = 0$ True

For $c = -25$,

$c^2 + 23c - 50 = 0$

$(-25)^2 + 23(-25) - 50 = 0$ (Put $c = -25$)

$625 - 575 - 50 = 0$ (Simplify)

$0 = 0$ True

Therefore, the solution set is $\boxed{\{2, -25\}}$.

Answer 50PA.

Consider the equation

$y^2 - 29y = -54$

$\Rightarrow \quad y^2 - 29y + 54 = -54 + 54$ (Add 54 on each side)

$\Rightarrow \quad y^2 - 29y + 54 = 0$

The objective is to solve the given equation.

For this first find factor $y^2 - 29y + 54$ and then use zero product property

Compare $y^2 - 29y + 54$ with $x^2 + bx + c$

Here $b = -29$,

$c = 54$

Since $(y+m)(y+n) = y^2 + (m+n)y + mn$

That is $m + n = -29$ negative and

$mn = 54$ is positive.

So, $m \& n$ must be negative.

Now make list of factors of 54, in those pair of factors, choose factors whose sum is -29.

Factor of 54	Sum of factors
$-1. -54$	-55
$-9. -6$	-15
$-27. -2$	-29
$-18. -3$	-21

The correct factors are $-27, -2$.

$y^2 - 29y + 54 = (y+m)(y+n)$

$= (y + (-27))(y + (-9))$

$(m = -27, n = -2)$

$=(y-27)(y-2)$

Therefore,

$y^2-29y+54=0$

$(y-27)(y-2)=0$ (Factor)

$y-27=0$ (or)

$y-2=0$ (By zero product property)

Now solve each equation separately.

$y-27=0$

$y-27+27=0+27$ (Add 27 on each side)

$y=27$

$y-2=0$

$y-2+2=0+2$ (Add 2 on each side)

$y=2$

The solution set is $\{27,2\}$.

Check:- To check the proposed solution, substitute y by $27,2$ in the given equation.

For $y=27$,

$y^2-29y+54=0$

$(27)^2-29(27)+54=0$ (Put $y=27$)

$729-783+54=0$

$0=0$ True

For $y=2$,

$y^2-29y+54=0$

$(2)^2-29(2)+54=0$ (Put $y=2$)

$4-58+54=0$

$\Rightarrow \quad 0=0$ True

Therefore, the solution set is $\boxed{\{27,2\}}$.

Answer 51PA.

Consider the equation

$14p + p^2 = 51$

$\Rightarrow \quad 14p + p^2 - 51 = 51 - 51$ (Subtract 51 on each side)

$\Rightarrow \quad p^2 + 14p - 51 = 0$ (Simplify)

The objective is to solve the given equation.

For this first factor $p^2 + 14p - 51$ and then use zero product property.

Compare $p^2 + 14p - 51$ with $x^2 + bx + c$.

Here $b = 14$,

$c = -51$

Since $(p+x)(p+y) = p^2 + (x+y)p + xy$

That is $x + y = 14$ positive and

$xy = -51$ is negative.

So, either x (or) y is negative but not both.

Now make list of factors of -51, in those pair of factors, choose whose sum is 14.

Factors of -51	Sum of factors
$1.-51$	-50
-1.51	50
-17.3	-14
$17.-3$	14

The correct factors are $17, -3$.

$p^2 + 14p - 51 = (p+x)(p+y)$

$= (p+7)(p+(-3)) \quad (x = 17, y = -3)$

$=(p+17)(p-3)$

Therefore, $p^2+14p-51=0$

$(p+17)(p-3)=0$ (Factors)

$p+17=0$

(or) $p-3=0$ (By zero product property)

Now solve each equation separately.

$p+17=0$

$p+17-17=0-17$ (Subtract 17 on each side)

$p=-17$

$p-3=0$

$p-3+3=0+3$ (Add 3 on each side)

$p=3$

The solution set is $\{-17,3\}$.

Check:- To check the proposed solution, substitute p by $-17,3$ in each equation.

For $p=-17$,

$p^2+14p-51=0$

$(-17)^2+14(-17)-51=0$ (Put $p=-17$)

$289-238-51=0$

$0=0$ True

For $p=3$,

$p^2+148-51=0$

$(3)^2+14(3)-51=0$ (Put $p=3$)

$9+42-51=0$

$0=0$ True

Therefore, the solution set is $\boxed{\{-17,3\}}$.

Answer 52PA.

Consider the equation

$x^2 - 2x - 6 = 74$

$\Rightarrow \quad x^2 - 2x - 6 - 74 = 74 - 74$ (Subtract 74 on each side)

$\Rightarrow \quad x^2 - 2x - 80 = 0$

The objective is to solve the given equation for this first factor $x^2 - 2x - 80$ and then use zero product property.

Compare $x^2 - 2x - 80$ with $x^2 + bx + c$.

Here $b = -2,$

$c = -80$

Since $(x+m)(x+n) = x^2 + (m+n)x + mn$

That is $m + n = -2$ negative and

$mn = -80$ is also negative.

So, either m (or) n is negative but not both.

Now make list of factors of -80, in those pair of factors, choose whose sum is -2.

Factors of -80	Sum of factors
-1.80	79
$1.-80$	-79
$2.-40$	-38
-2.40	38
$4.-20$	-16
-4.20	16
$8.-10$	-2
-8.10	2
$16.-5$	11
-16.5	-11

The correct factors are $8,-10$.

$$x^2-2x-80=(x+m)(x+n)$$

$$=(x+8)(x+(-10)) \quad (m=8, n=-10)$$

$$=(x+8)(x-10)$$

Therefore, $x^2-2x-80=0$

$(x+8)(x-10)=0$ (Factor)

$x+8=0$

(or) $x-10=0$ (By zero product property)

Now solve each equation separately.

$x+8=0$

$x+8-8=0-8$ (Subtract 8 on each side)

$x=-8$

$x-10=0$

$x-10+10=0+10$ (Add 10 on each side)

$x=10$

The solution set is $\{-8,10\}$.

Check:- To check the proposed solution, substitute x by $-8,10$ in the given equation.

For $x=-8$,

$x^2-2x-80=0$

$(-8)^2-2(-8)-80=0$ (Put $x=-8$)

$64+16-80=0$

$0=0$ True

For $x=10$,

$x^2-2x-80=0$

$(10)^2-2(10)-80=0$ (True $x=10$)

$100-20-80=0$

$0=0$ True

Therefore, the solution set is $\boxed{\{-8,10\}}$.

Answer 53PA.

Consider the equation

$x^2-x+56=17x$

$\Rightarrow \quad x^2-x+56-17x=17x-17x$ (Subtract $17x$ on each side)

$\Rightarrow \quad x^2 - 18x + 56 = 0$ (Simplify)

The objective is to solve the given equation.

For this first factor $x^2 - 18x + 56$ and then use zero product property.

Compare $x^2 - 18x + 56$ with $x^2 + bx + c$

Here $b = -18$,

$c = 56$

Since $(x+m)(x+n) = x^2 + (m+n)x + mn$

That is $m + n = -18$ negative and

$mn = 56$ is positive.

So, m and n both are must be negative.

Now make list of factors of 56, in those pair of factors, choose whose factors of sum is -18.

Factors of 56	Sum of factors
$-1.-56$	-57
$-8.-7$	-15
$-2.-28$	-30
$-14.-4$	-18

The correct factors are $-14, -4$.

$x^2 - 18x + 56 = (x+m)(x+n)$

$= (x + (-14))(x + (-4))$

$(m = -14, n = -4)$

$= (x-14)(x-4)$

Therefore, $x^2 - 18x + 56 = 0$

$(x-14)(x-4) = 0$ (Factors)

$x - 14 = 0$ (or)

$x - 4 = 0$ (By zero product property)

Now solve each equation separately.

$x - 14 = 0$

$x - 14 + 14 = 0 + 14$ (Add 14 on each side)

$x = 14$

$x - 4 = 0$

$x - 4 + 4 = 0 + 4$ (Add 4 on each side)

$x = 4$

The solution set is $\{14, 4\}$.

Check:- To check the proposed solution, substitute x by $14, 4$ in the given equation.

For $x = 14$,

$x^2 - 18x + 56 = 0$

$(14)^2 - 18(14) + 56 = 0$ (Put $x = 14$)

$196 - 252 + 56 = 0$

$0 = 0$ True

For $x = 4$,

$x^2 - 18x + 56 = 0$

$(4)^2 - 18(4) + 56 = 0$ (Put $x = 4$)

$16 - 72 + 56 = 0$

$0 = 0$ True

Therefore, the solution set is $\boxed{14, 4}$.

Answer 54PA.

The Justices of the supreme court assemble to on the bench each day.

Each Justice shakes hands with each of other justices for a table of 36 hand shad shakes.

The total number of handshakes *h* possible for *n* people is given by

$$h=\frac{n^2-n}{2}$$

The objective is to find the number of Justices on the Supreme Court.

For $h=36$ handshakes,

$$h=\frac{n^2-n}{2}$$

$$\frac{n^2-n}{2}=h$$

$$\frac{n^2-n}{2}=36 \qquad [h=36]$$

$$\frac{n^2-n}{2}\cdot 2=36\cdot 2 \qquad [\text{Multiply with 2 on both sides}]$$

$$n^2-n=72$$

$$n^2-n-72=72-72 \qquad [\text{Subtract 72 on both sides}]$$

$$n^2-n-72=0$$

$$(n-9)(n+8)=0 \qquad [\text{Factor}]$$

By zero product property of $ab=0$ then $a=0$ or $b=0$ or both.

$$n-9=0 \text{ or } n+8=0$$

$$n-9=0$$

$$n-9+9=0+9 \qquad [\text{add 9 on both sides}]$$

$$n=9$$

$$n+8=0$$

$$n+8-8=0-8 [\text{Subtract 8 on both sides}]$$

$$n=-8$$

Since *n* always positive, $n=9$

Therefore, the number of Justices on the supreme Court is $\boxed{9}$

Answer 55PA.

Consider that the product of two consecutive even integers is 168.

The objective is to find the consecutive even integers

Let the consecutive even integers be $2x, 2(x+1)$.

Since the product of even integers is 168.

$$2x \cdot 2(x+1) = 168$$
$$4 \cdot x(x+1) = 168$$

$\frac{4 \cdot x(x+1)}{4} = \frac{168}{4}$ (Divide with 4 on each sides)

$$x(x+1) = 42$$

$x \cdot x + x \cdot 1 = 42$ (By distributive $a(b+c) = ab + ac$)

$$x^2 + x = 42$$

$x^2 + x - 42 = 42 - 42$ (Subtract 42 on both sides)

$$x^2 + x - 42 = 0$$

Now solve the above equation for x.

For this first factor $x^2 + x - 42$.

Compare $x^2 + x - 42$ with $x^2 + ax + b$.

Here $a = 1$,

$b = -42$

$$x^2 + x - 42 = (x+m)(x+n)$$
$$= x^2 + (m+n)x + m \cdot n$$

That is $m + n = 1$,

$mn = -42$

$m + n = 1$ is positive and mn is negative, so either m or n negative but not both

Now make list all the factors of -42, where one factor of factors whose sum is 1.

Factors of -42	Sum of factors
-1.42	41
$1.-42$	-41
$2.-21$	-19
-2.21	19
$3.-14$	-11
-3.14	11
-6.7	1
-7.6	-1

The correct factors are $-6,7$.

Therefore,

$$x^2+x-42=(x+m)(x+n)$$
$$=(x+(-6))(x+7) \quad (m=-6, n=7)$$
$$=(x-6)(x+7)$$

Therefore,

$$x^2+x-42=0$$
$$\Rightarrow \quad (x-6)(x+7)$$
$$=0$$

$x-6=0$

Or, $x+7=0$ (By zero product property)

Now solve each equation separately.

$x-6=0$

$x-6+6=0+6$ (Add 6 on each side)

$x=6$

$x+7=0$

$x+7-7=0-7$ (Subtract 7 on both sides)

$x=-7$

The solution set is $\{6,-7\}$.

For $x=6$, the consecutive integers are $2x,2(x+1)$

$2(6),2(6+1)\quad(x=6)$
$12,2\cdot 7$
$12,14$

For $x=-7$, the consecutive integers are $2(-7),2(-7+1)$

$-14,2(-6)$
$-14,-12$

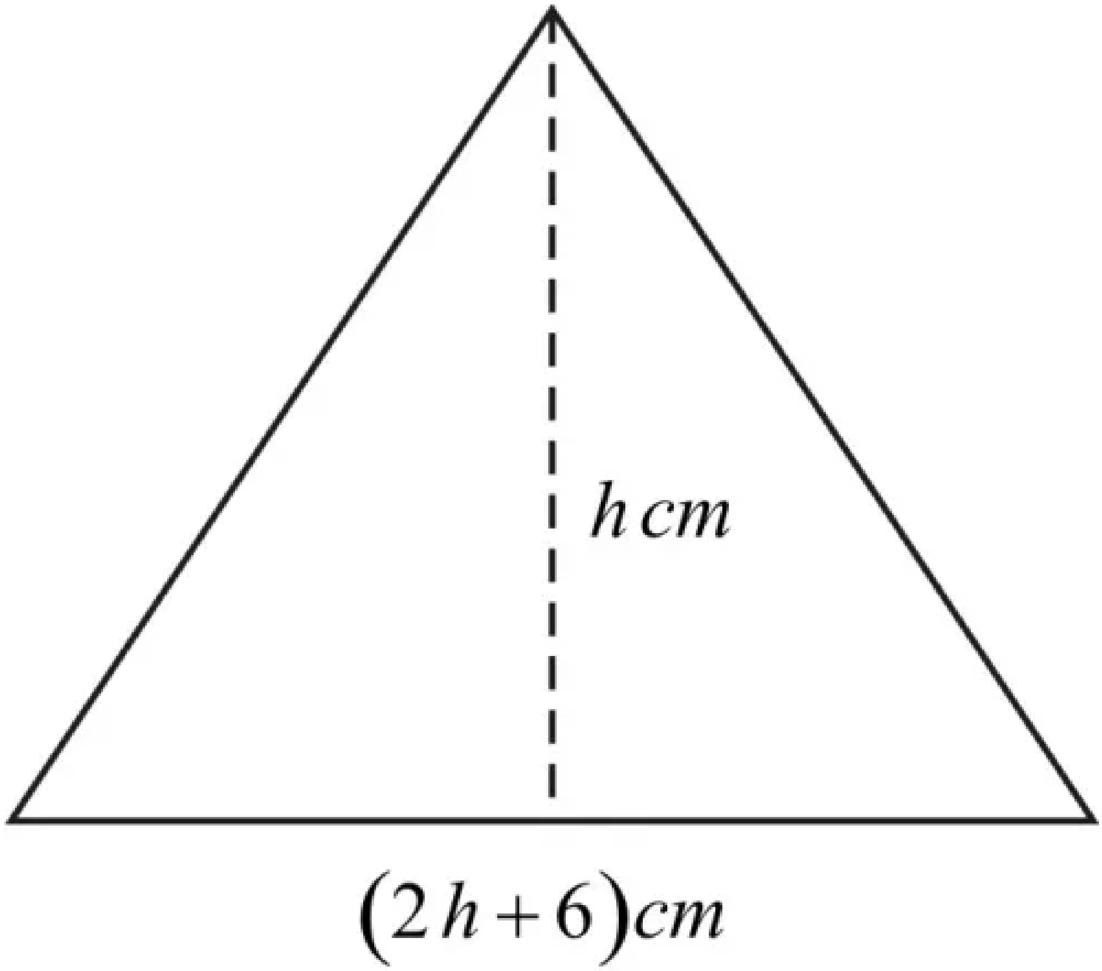
Check: $12\cdot 14=168$ True

$-12\cdot -14=168$ True

The two consecutive integers are $\boxed{12,14}$ or $\boxed{-14,-12}$.

Answer 56PA.

Consider the triangle



Given that area of triangle is 40 square centimeters.

And h is height of the triangle.

Since the area of a triangle with height h, base b is area

$$=\frac{1}{2}b\cdot h$$

Given that base $b=2h+6$

Height $h=h$

Area $=40$ square centimeters

$$\frac{1}{2}\cdot b\cdot h=40$$

$$\frac{1}{2}\cdot(2h+6)\cdot h=40$$

$$\frac{h}{2}\cdot(2h+6)=40$$ (Simplify)

2

$\frac{h}{2} \cdot 2h + \frac{h}{2} \cdot 6 = 40$ (By distributive $a(b+c) = ab + ac$)

$h^2 + h \cdot 3 = 40$ (Simplify)

$h^2 + 3h = 40$

$h^2 + 3h - 40 = 40 - 40$

$h^2 + 3h - 40 = 0$

Now we solve the above equation for h.

$h^2 - 5h + 8h - 40 = 0$

$h \cdot h - 5h + 8h - 8 \cdot 5 = 0$

$h(h-5) + 8(h-5) = 0$ (By distributive)

$(h+8)(h-5) = 0$ (By distributive)

By zero product property, of

$ab = 0$ then

$a = 0$ or

$b = 0$ or both

$h + 8 = 0$ or

$h-5=0$

$h+8=0$

$h+8-8=0-8$ (Subtract 8 on both sides)

$h=-8$

$h-5=0$

$h-5+5=0+5$ (Add 5 on each side)

$h=5$

Since height is a length it is positive

So consider $h=5$

Check:

Area $=\frac{1}{2}\cdot b\cdot h$

$=\frac{1}{2}(2hy+6)\cdot h$

$=\frac{1}{2}(2(5)+6)\cdot 5$

$=\frac{1}{2}(16)\cdot 5$

$=8\cdot 5$

$=40$ True

Therefore, height of the triangle is

$\boxed{h=5\,\text{cm}}$.

Answer 57PA.

Consider the trinomial $x^2+kx-19$

The objective is to find all value of *k* so that $x^2+kx-19$ can be factored.

For factor $x^2+kx-19$,

Compare it with x^2+bx+c

$$b=k, c=-19$$

$$\begin{aligned} x^2+kx-19 &= (x+m)(x+n) \\ &= x^2+(m+n)x+mn \end{aligned}$$

Now find two numbers *m*, *n* such that $m+n=k, mn=-19$

For find all the factors of $mn=-19$, and $m+n$ is nothing but the sum of factors.

Factors of -19	Sum of factors $m+n=k$
$-1\cdot 19$	18
$1\cdot -19$	-18

Thus the possible values of *k* are 18 or -18

Also in each case $x^2+bx-19$ is factored.

Therefore, the possible values of *k* are $\boxed{-18,18}$

Answer 58PA.

Consider the trinomial $x^2 + kx + 14$

The objective is to find all value of *k* so that $x^2 + kx + 14$ can be factored.

To factor $x^2 + kx + 14$, compare it with $x^2 + bx + c$

Where $k = b, c = 14$

$$\begin{aligned} x^2 + kx + 14 &= (x+m)(x+n) \\ &= x^2 + (m+n)x + mn \end{aligned}$$

$m + n = k, mn = 14$

Since $x^2 + kx + 14$ are factored using integers,

Find the two numbers *m*, *n* such that $m + n = k$ and $mn = 14$

For this list all the factors of 14, then *k* is nothing but sum of factors.

Factors of 14	Sum of factors
$1 \cdot 14$	15
$-1 \cdot -14$	-1
$2 \cdot 7$	9
$-2 \cdot -7$	-9

Thus the possible values of *k* are $-15, 15, -9, 9$

Therefore, the possible values of *k* are $\boxed{\pm 9, \pm 15}$

Answer 59PA.

Consider the trinomial $x^2 - 8x + k, \quad k > 0$

The objective is to find all value of *k* so that $x^2 - 8x + k,\ k > 0$ can be factored using integers.

Compare $x^2 - 8x + k$ with $x^2 + bx + c$

$b = -8, c = k$

$$x^2 - 8x + k = (x+m)(x+n)$$
$$= x^2 + (m+n)x + mn$$

$m + n = -8, mn = k$

Since $x^2 - 8x + k$ is a factored using integer,

For the two numbers *m*, *n* $m + n = -8$ and $mn = k > 0$

List all the pairs of number whose sum is -8 and whose product is $mn = k > 0$. It is possible only when both m, n are negative.

$m+n=-8$	$mn=k>0$
$-1 \cdot -7$	7
$-2 \cdot -6$	12
$-3 \cdot -5$	15
$-4 \cdot -4$	16

Thus the possible values of *k* are $7, 12, 15, 16$

Therefore, the values of *k* are $\boxed{7,12,15,16}$

Answer 60PA.

Consider the trinomial $x^2-5x+k,\ k>0$

The objective is to find all values of k so that $x^2-5x+k,\ k>0$ can be factored using integers.

Compare x^2-5x+k with x^2+bx+c.

Here $b=-8, c=k$

$$x^2-5x+k=(x+m)(x+n)$$
$$=x^2(m+n)x+mn$$

$m+n=-5$ and $mn=k>0$

Since $m+n=-5$ is negative

List all the pairs m,n so that $m+n=-5$ and $mn=k>0$

$m+n=-5$	$mn=k>0$
$-1\cdot-4$	4
$-2\cdot-3$	6

Thus the possible values of k are 4, 6.

Therefore, the values of k are $\boxed{4,6}$

Answer 61PA.

Consider that the length of a Rugby field is 52 meters longer than its width w.

The objective is to write an expression for the area of the rectangular field.

Width of the field $=w$

Let length of the field $=l$

Since length of field is 52 meters longer than width

So length =width plus 52

$=w+52$

Area of rectangle =length ·width

$=(w+52)\cdot w$

$=w(w+52)m^2$

Expression for area of Rugby field

$=\boxed{w(w+52)m^2}$.

Answer 62PA.

Consider that the length of a Rugby league field is 52 meters longer than its width *w*.

That is width of field = *w* meters

Length of field $=(w+52)$ meters

Also given Area of Rugby field $=8160$ square meters

The objective is to find the dimension of the filed

Since Area of rectangular filed = $\text{length}\cdot\text{width}$

$$8160=(w+52)\cdot w$$

$$w(w+52)=8160$$

$$w\cdot w+w\cdot 52=816 \qquad [\text{By distributive } a(b+c)=ab+ac]$$

$$w^2+52w=8160$$

$$w^2+52w-8160=8160-8160 \qquad [\text{Subtract 8160 on both sides}]$$

$$w^2+52w-8160=0$$

Now factor $w^2+52w-8160=0$

Compare $w^2+52w-8160$ with x^2+bx+c

$$b=52, c=-8160$$

$$w^2+52w-8160=(w+m)(w+n)$$

$$=w^2+(m+n)w+mn$$

Now find two numbers m,n such that $m+n=52$ positive and $mn=-8160$ negative.

Since $m+n$ is positive and mn is negative then one of *m* or *n* must be negative but not both.

For this list all the factors of -8160 in those choose a pair whose sum is 52

Factors of -8160	Sum of factors
$-1\cdot 8160$	8159
$1\cdot -8160$	-8159
$-2\cdot 4080$	4078
$2\cdot -4080$	-4078
$-3\cdot 2720$	2717
$3\cdot -2720$	-2717
$-4\cdot 2040$	✓ 2036
$120\cdot -68$	52

The connect factors are $120, -68$

$$\begin{aligned} w^2+52w-8160 &= (w+m)(w+n) \\ &= (w+120)(w-68) \quad [m=120, n=-68] \end{aligned}$$

$$w^2+52w-8160=0$$

$$(w+120)(w-68)=0$$

The zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$w+120=0 \text{ or } w-68=0$$

$$w+120=0$$

$$w+120-120=0-120 \quad [\text{Subtract 120 on both sides}]$$

$$w=-120$$

$$w-68=0$$

$$w-68+68=0+68 \quad [\text{Add 68 on both sides}]$$

$$w=68$$

Since width always positive take $w=68$ meters lengths

$$=w+52$$

$$=68+52$$

$$=120 \text{ meters}$$

Therefore, dimensions of Rugby league field are $\boxed{\begin{array}{l}\text{length}=120 \text{ meeters}\\ \text{width}=68 \text{ meters}\end{array}}$

Answer 64PA.

Consider the trinomial $x^2-17x+42$.

The objective is to factor the given trinomial

Compare $x^2-17x+42$ with x^2+bx+c

Here $b=-17,$

$c=42$

$$x^2-17x+42=(x+m)(x+n)$$

$$=x^2+(m+n)x+mn$$

Here $m+n=-17$ is negative and

$mn=42$ is positives

So both m and n must be negative.

For this list all negative factors of 42, in those choose a pair in which the sum is -17.

Factors of 42	Sum of factors
$-1,-42$	-43

$-2,-21$	-23
$-3,-14$	-17
$-7,-6$	-13

The correct factors are $-3,-14$.

Therefore,

$$x^2-17x+42=(x+m)(x+n)$$
$$=(x+(-3))(x+(-14)) \quad (m=-3, n=-14)$$
$$=(x-3)(x-14)$$

The factored form of $\boxed{x^2-17x+42}$ is $\boxed{(x-3)(x-14)}$.

Answer 65PA.

The equation is $p^2-13p-30=0$

The objective is to find the solution set of given equation

For this first factor $p^2-13p-30$

Compare $p^2-13p-30$ with ax^2+bx+c

Here $b=-13, c=-30$

$$2p^2-13p-30=(p+m)(p+n)$$
$$=2p^2+(m+n)p-mn$$

$$m+n=-13, mn=-30$$

Since $m+n, mn$ are negative then one of *m* or *n* must be negative but not both.

For this list all the factors of -30 in those choose a pair whose sum is -13

Factors of -30	Sum of factors
$-1 \cdot 30$	29
$1 \cdot -30$	-29
$-2 \cdot 15$	13
$2 \cdot -15$	✓ -13
$-3 \cdot 10$	7
$3 \cdot -10$	-7
$-5 \cdot 6$	1
$5 \cdot -6$	-1

The connect factors are $2, -15$

$$\begin{aligned} p^2 - 30p - 30 &= (p+m)(p+n) \\ &= (p+2)(p-15) \end{aligned}$$

$$\begin{aligned} p^2 - 13p - 30 &= 0 \\ (p+2)(p-15) &= 0 \end{aligned}$$

By zero product property is if $ab=0$ then $a=0$ or $b=0$ or both

$$p+2=0 \text{ or } p-15=0$$

$$p+2-2=0-2 \quad \text{[Subtract 2 on both sides]}$$

$$p=-2 \quad \text{[Negative solution]}$$

$$p-15=0$$

$$p-15+15=0+15 \quad \text{[Add 15 on both sides]}$$

$$p=15 \quad \text{[Positive solution]}$$

Therefore, the solution set of given equation is $\boxed{15}$

Answer 70MYS.

Consider the equation

$$(x+3)(2x-5)=0$$

The objective is to solve given equation and check the solution set.

The zero product property is, of

$ab=0$ then

$a=0$ or

$b=0$ or both.

$$(x+3)(2x-5)=0$$

$$\Rightarrow \quad x+3=0$$

Or, $2x-5=0$

Now solve each equality separately.

$$x+3=0$$

$x+3-3=0-3$ (Subtract 3 on bothsides)

$$x=-3$$

Also $2x-5=0$

$2x-5+5=0+5$ (Add 5 on each side)

$$2x=5$$

$\frac{2x}{2}=\frac{5}{2}$ (Divide with 2 on each side)

$$x=\frac{5}{2}$$

The solution set is $\left\{-3,\frac{5}{2}\right\}$.

Check: To check the proposed solution set, substitute each solution in the given equation.

For $x=-3$,

$(x+3)(2x-5)=0$

$(-3+3)(2(-3)-5)=0$ (Put $x=-3$)

$0(-6-5)=0$ (Simplify)

$0=0$ True.

For, $x=\frac{5}{2}$,

$(x+3)(2x-5)=0$

$\left(\frac{5}{2}+3\right)\left(2\cdot\frac{5}{2}-5\right)=0$ (Put $x=\frac{5}{2}$)

$\left(\frac{5}{2}+3\right)(5-5)=0$ (Simplify)

$\left(\frac{5}{2}+3\right)0=0$

$0=0$ True

Therefore, the solution set of given equation is $\boxed{\left\{-3,\frac{5}{2}\right\}}$.

Answer 71MYS.

Consider the equation $b(7b-4)=0$

The objective is to find the solution set of given equation.

The zero product property is, of

$ab=0$ then

$a=0$ or

$b=0$ or both

$b(7b-4)=0$

$b=0$

Or, $7b-4=0$ (By zero product property)

Now solve the above equations separately.

$b=0$,

$7b-4=0$

$7b-4+4=0+4$ (Add 4 on each side)

$7b=4$

$\frac{7b}{7}=\frac{4}{7}$ (Divide with 7 on both sides)

$b=\frac{4}{7}$

The solution set is $\left\{0,\frac{4}{7}\right\}$.

Check:- To check the proposed solution set, substitute each solution in the given equation.

For $b=0$,

$b(7b-4)=0$

$0(7(0)-4)=0$ (Put $b=0$)

$0(-4)=0$

$0=0$ True

For $b=\frac{4}{7}$,

$b(7b-4)=0$

$\frac{4}{7}\left(7\left(\frac{4}{7}\right)-4\right)=0$ (Put $b=\frac{4}{7}$)

$\frac{4}{7}(4-4)=0$ (Simplify)

$\frac{4}{7}(0)=0$

$0=0$ True

Therefore, the solution set of given equation is $\boxed{\left\{0,\frac{4}{7}\right\}}$.

Answer 72MYS.

Consider the equation

$5y^2 = -9y$

The objective is to find the solution set of given equation.

The zero product property is

If $ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$5y^2 = -9y$

$5y^2 + 9y = -9y + 9y$ (Add $9y$ on both sides)

$5y^2 + 9y = 0$ (Combine like terms)

$5 \cdot y \cdot y + 9y = 0$ (Since $y^2 = y \cdot y$)

$(5y+9)y = 0$ (By distributive property $(b+c)a = ba + ca$)

Hence $5y + 9 = 0$

Or $y = 0$ (By zero product property)

Now solve the above equations completely.

$5y + 9 = 0$

$\Rightarrow \quad 5y + 9 - 9 = 0 - 9$ (Subtract 9 on each side)

$5y = -9$ (Simplify)

$\frac{5y}{5} = \frac{-9}{5}$ (Divide with 5 on both sides)

$y = \frac{-9}{5}$

Also $y = 0$

The solution set is $\left\{0, \frac{-9}{5}\right\}$.

Check:- To check the proposed solution set, substitute each solution in the given equation and check whether it is true or false.

For $y=0$,

$5y^2=-9y$

$50^2=-9(0)$ (Put $y=0$)

$0=0$ True

For $y=\frac{-9}{5}$,

$5y^2=-9y$

For $y=\frac{-9}{5}$,

$5y^2=-9y$

$5\cdot\left(\frac{-9}{5}\right)^2=-9\left(\frac{-9}{5}\right)$ (Put $y=\frac{-9}{5}$)

$5\cdot\frac{-9}{5}\cdot\frac{-9}{5}=-9\cdot\frac{-9}{5}$ (Since $x^2=x\cdot x$)

$\frac{81}{5}=\frac{81}{5}$ True (Simplify)

Therefore, the solution set of given equation is $\boxed{\left\{0,\frac{-9}{5}\right\}}$.

Answer 73MYS.

Consider the set of monomials are $24, 36, 72$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first factor each monomial completely

$24 = 2 \cdot 12 \quad (24 = 2 \cdot 12)$

$= 2 \cdot 2 \cdot 6 \quad (12 = 2 \cdot 6)$

$= 2 \cdot 2 \cdot 2 \cdot 3 \quad (6 = 2 \cdot 3)$

$36 = 2 \cdot 18$ (Since $36 = 2 \cdot 18$)

$= 2 \cdot 2 \cdot 9 \quad (18 = 2 \cdot 9)$

$= 2 \cdot 2 \cdot 3 \cdot 3 \quad (9 = 3 \cdot 3)$

$72 = 2 \cdot 36$ (Since $72 = 2 \cdot 36$)

$= 2 \cdot 2 \cdot 18 \quad (36 = 2 \cdot 18)$

$= 2 \cdot 2 \cdot 2 \cdot 9 \quad (18 = 2 \cdot 9)$

$= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \quad (9 = 3 \cdot 3)$

$24 = (2) \cdot (2) \cdot 2 \cdot (3)$
$36 = (2) \cdot (2) \cdot 3 \cdot (3)$
$72 = (2) \cdot (2) \cdot 2 \cdot (3) \cdot 3$

Since the integers $24, 36, 72$ has two 2^x and one 3 as common prime factors.

The product of common prime factors

$= 2 \cdot 2 \cdot 3$
$= 12$

Therefore, the $\boxed{GCF}$ of $\boxed{24, 36, 72}$ is $\boxed{12}$.

Answer 74MYS.

Consider the set of monomials $9p^2q^5, 21p^3q^3$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first factor each monomial completely.

$9p^2q^5 = 3\cdot 3\cdot p^2q^5$ (Since $9 = 3\cdot 3$)

$= 3\cdot 3\cdot p\cdot p\cdot q^5 \quad (p^2 = p\cdot p)$

$= 3\cdot 3\cdot p\cdot p\cdot q\cdot q\cdot q\cdot q\cdot q$

$(q^5 = q\cdot q\cdot q\cdot q\cdot q)$

$21p^3q^3 = 3\cdot 7\cdot p^3q^3$ (Since $7\cdot 3 = 21$)

$= 3\cdot 7\cdot p\cdot p\cdot p\cdot q^3 \quad (p^3 = p\cdot p\cdot p)$

$= 3\cdot 7\cdot p\cdot p\cdot p\cdot q\cdot q\cdot q$

$(q^3 = q\cdot q\cdot q)$

Now circle the common factors.

$9p^2q^5 = (3)\cdot 3\cdot (p)\cdot (p)\cdot q\cdot (q)\cdot (q)\cdot (q)\cdot q$

$21p^3q^3 = (3)\cdot 7\cdot (p)\cdot (p)\cdot p\cdot (q)\cdot (q)\cdot (q)$

Since the monomials has one 3, two p^x and three q^x as common prime factors.

The product of prime factors

$= 3\cdot p\cdot p\cdot q\cdot q\cdot$

$= 3p^2q^3$

Therefore the $\boxed{GCF}$ of given set of monomials is $\boxed{3p^2q^3}$.

Answer 75MYS.

Consider the set of monomials are $30x^4y^5, 20x^2y^7, 75x^3y^4$.

The objective is to find the GCF of given set of monomials.

Since the GCF of two or more monomials is the product of their common factors when each monomial is in factored form.

For this first factor each monomial completely.

$30x^4y^5 = 2\cdot 15x^4y^5$ (Since $30 = 2\cdot 15$)

$= 2\cdot 3\cdot 5x^4y^5 \quad (15 = 3\cdot 5)$

$= 2\cdot 3\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot y^5 \quad \left(x^4 = x\cdot x\cdot x\cdot x\right)$

$= 2\cdot 3\cdot 5\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y$

$\left(y^5 = y\cdot y\cdot y\cdot y\cdot y\right)$

$20x^2y^7 = 2\cdot 10\cdot x^2y^7$ (Since $20 = 2\cdot 10$)

$= 2\cdot 2\cdot 5x^2y^7 \quad (10 = 2\cdot 5)$

$= 2\cdot 2\cdot 5\cdot x\cdot x\cdot y^7 \quad \left(x^2 = x\cdot x\right)$

$= 2\cdot 2\cdot 5\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y$

$\left(y^7 = y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\right)$

$75x^3y^4 = 3\cdot 25x^3y^4 \quad (75 = 3\cdot 25)$

$= 3\cdot 5\cdot 5x^3y^4 \quad (25 = 5\cdot 5)$

$= 3\cdot 5\cdot 5\cdot x\cdot x\cdot x\, y^4 \quad \left(x^3 = x\cdot x\cdot x\right)$

$= 3\cdot 5\cdot 5\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y$

$\left(y^4 = y\cdot y\cdot y\cdot y\right)$

Now circle the common factors

$$30x^4y^5 = 2\cdot3\cdot(5)\cdot(x)\cdot(x)\cdot x\cdot x\cdot(y)\cdot(y)\cdot(y)\cdot(y)\cdot y$$
$$20x^2y^7 = 2\cdot2\cdot(5)\cdot(x)\cdot(x)\cdot y\cdot y\cdot(y)\cdot(y)\cdot(y)\cdot(y)\cdot y$$
$$75x^3y^4 = 3\cdot5\cdot(5)\cdot(x)\cdot(x)\cdot x\cdot \quad (y)\cdot(y)\cdot(y)\cdot(y)$$

Since the monomials has one 5, two x^x and fair y^y as common prime factors.

The product of prime factors

$$= 5\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y$$
$$= 5x^2y^4$$

Therefore, the $\boxed{GCF}$ of given set of monomials is $\boxed{5x^2y^4}$.

Answer 78MYS.

Consider the polynomial $3y^2+2y+9y+6$.

The objective is to factor the given polynomial by graphing.

$$3y^2+2y+9y+6 = 3\cdot y\cdot y+2y+3\cdot3\cdot y+3\cdot2$$

(Since $y^2 = y\cdot y, 9 = 3\cdot3, 6 = 3\cdot2$)

$$= (3y+2)y+3(3y+2)$$

(By distributive $a(b+c) = ab+ac, (b+c)a = ba+ca$)

$$= y(3y+2)+3(3y+2)$$
$$= (y+3)(3y+2)$$

(By distributive $(b+c)a = ba+ca$)

Check: Now check the factors using $FOIL$ Method

$$(y+3)(3y+2) = \overset{F}{y\cdot3y}+\overset{O}{y\cdot2}+\overset{I}{3\cdot3y}+\overset{L}{3\cdot2}$$

(F = Firsts, O = Astsides, I = Inside, L = Last)

$$= 3y^2+2y+9y+6 \text{ True}$$

Therefore, the factorization of given polynomial is $\boxed{(y+3)(3y+2)}$.

Answer 79MYS.

Consider the polynomial $3a^2+2a+12a+8$

The objective is to factor the given polynomial by grouping.

$3a^2+2a+12a+8=3\cdot a\cdot a+2\cdot a+4\cdot 3\cdot a+4\cdot 2$

(Since $a^2=a\cdot a$)

$=(3a+2)a+4(3a+2)$

(By distributive $a(b+c)=ab+ac,(b+c)a=ba+ca$)

$=(3a+2)a+(3a+2)4$

(Simplify)

$=(3a+2)(a+4)$

(By distributive)

Check:- Now check the factors using $FOIL$ Method

$FOIL=$ Firsts outsides Insides Lasts

$(3a+2)(a+4)=\overset{F}{3}a\cdot a+\overset{O}{3}a\cdot 4+\overset{I}{2}\cdot a+2\cdot \overset{L}{a}$

$=3a^2+12a+2a+8$

(Simplify)

Therefore,

$\boxed{3a^2+2a+12a+8=(3a+2)(a+4)}$.

Answer 80MYS.

Consider the polynomial $4x^2+3x+8x+6$

The objective is to factor the given polynomial by grouping the distributive property is

$$a(b+c)=ab+ac$$
$$(b+c)a=ba+ca$$

$$4x^2+3x+8x+6=4\cdot x\cdot x+3\cdot x+4\cdot 2\cdot x+3\cdot 2$$

$$\left(x^2=x\cdot x\right)$$

$$=(4x+3)x+(4x+3)2$$

(By distributive)

$$=(4x+3)(x+2)$$

(By distributive)

Check:- Now check the factors using $FOIL$ Method

$FOIL=$ Firsts outsides Insides Lasts

$$(4x+3)(x+2)=\overset{F}{4}x\cdot x+\overset{O}{4}x\cdot 2+\overset{I}{3}\cdot x+\overset{L}{3}\cdot 2$$

$$=4x^2+8x+3x+6$$

(Simplify)

$=4x^2+3x+8x+6$ True

Therefore, the factorization of given polynomial is $\boxed{(4x+3)(x+2)}$.

Answer 81MYS.

Consider the polynomial $2p^2-6p+7p-21$

The objective is to factor given polynomial by grasping

The distributive property is

$$a(b+c)=ab+ac$$
$$(b+c)a=ba+ca$$

$$2p^2-6p+7p-21=2\cdot p\cdot p+2(-3)p+7\cdot p+7(-3)$$

(Because $p^2=p\cdot p$)

$$=2p(p+(-3))+7(p+(-3))$$

(By distributive)

$$=2p(p-3)+7(p-3)$$

(Simplify)

$$=(2p+7)(p-3)$$

(By distributive)

Check: Now check the factorization using $FOIL$ Method

$FOIL=$ First's outsides Insides Lasts

$$(2p+7)(p-3)=2p\cdot\overset{F}{p}+2p(\overset{O}{-3})+\overset{I}{7}\cdot p+7(\overset{L}{-3})$$

$=2p^2-6p+7p-21$ True

Therefore, the factorization of given polynomial is $\boxed{(2p+7)(p-3)}$.

Answer 82MYS.

Consider the polynomial $3b^2+7b-12b-28$

The objective is to factor the given polynomial.

The distributive properties are

$$a(b+c)=ab+ac$$
$$(b+c)a=ba+ca$$

$$3b^2+7b-12b-28=3\cdot b\cdot b+7\cdot b+(-12b)+(-28)$$

(Because $b\cdot b=b^2$)

$$=3\cdot b\cdot b+7\cdot b+(-4)3b+(-4)\cdot 7$$

$$=(3b+7)b+(-4)(3b+7)$$

(By distributive)

$$=(3b+7)b+(3b+7)(-4)$$

$$=(3b+7)(b+(-4))$$

(By distributive)

$$=(3b+7)(b-4)$$

Check: Now check the factorization using $FOIL$ Method

$$(3b+7)(b-4)=(3b+7)(b+(-4))$$

$$=\overset{F}{3b\cdot b}+\overset{O}{3b\cdot(-4)}+\overset{I}{7\cdot b}+\overset{L}{7(-4)}$$

$$=3b^2-12b+7b-28$$

(Simplify)

$$=3b^2+7b-12b-28$$

True

Therefore, the factorization of given polynomial is $\boxed{(3b+7)(b-4)}$.

Answer 83MYS.

Consider the polynomial $4g^2-2g-6g+3$.

The objective is to factor the given polynomial by grouping

The distributive property is

$$a(b+c)=ab+ac$$
$$(b+c)a=ba+ca$$

$$4g^2-2g-6g+3=2\cdot2\cdot g\cdot g+(-2g)+(-6g)+3$$

(Because $g\cdot g=g^2$)

$$=2\cdot g\cdot 2g+(-1)2g+3\cdot(-2)g+3$$

$$=2g\cdot 2g+2g(-1)+(-3)2g+(-3)(-1)$$

(Because $(-b)\cdot(-1)=1$)

$$=2g(2g+(-1))+(-3)(2g+(-1))$$

(By distributive)

$$=2g(2g-1)+(-3)(2g-1)$$

(Simplify)

$$=(2g+(-3))(2g-1)$$

(By distributive)

$$=(2g-3)(2g-1)$$

Check: The check the factorization use the $FOIL$ method

$FOIL=$ Firsts at sides Insides Lasts

$$(2g-3)(2g-1)=2g\cdot\overset{F}{2}g+2g\overset{O}{(-1)}+\overset{I}{(-3)}2g+\overset{L}{(-1)}(-3)$$

$$=4g^2-2g-6g+3$$

(Simplify) True

Therefore, the factorization of given polynomial is $\boxed{(2g-3)(2g-1)}$.