CHAPTER 02

PERT and CPM

2.1 Programme Evaluation and Review Technique (PERT)

- PERT stands for "Project/Programme Evaluation and Review Technique".
- PERT involves uncertainty into the project completion time.
- It is a numerical technique used in the projects in which time can not be estimated accurately such as research and development projects. It is an event **oriented network**. Cost is assumed to be directly proportional to time.

Three time estimates are made in PERT:

1. **Optimistic time (t_o)**: This is the minimum possible time in which an activity can be completed under the most ideal conditions
2. **Pessimistic time (t_p)**: This is the maximum time required to complete an activity under the worst possible conditions.
3. **Most likely time (t_m)** : This is the time required to complete an activity under normal working conditions. Its value lies between t_o and t_p. It is near to the expected time.

Do You Know?

The most likely time (t_m) is based on experience and judgement being based on the time required if the activity is repeated a number of times under essentially the same conditions. This time signifies the most frequently occurring time. It reflects a situation "things are as usual, nothing exciting".

2.2 Mean Time, Standard Deviation and Variance of an Activity

2.2.1 Mean Time or Expected Time or Average Time

- In PERT each activity is assumed to follow β-distribution curve of time.
- This is calculated from β-distribution curve of time at which probability of activity is just 50%. Time taken by various activities follow β-distribution.

Hence value of expected time is calculated by weighted average as,

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

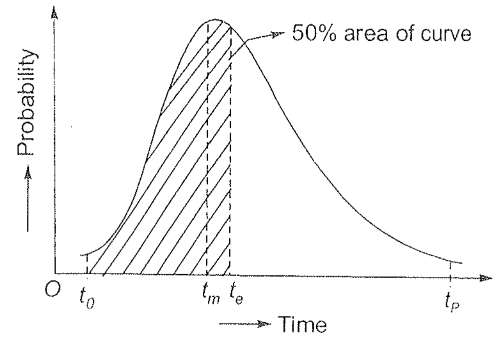


Fig. 2.1 β-curve with skewness to the left

2.2.2 Standard Deviation of An activity (σ)

- This is the measurement of uncertainty, which is approximately one sixth of time range i.e.,

$$\sigma = \frac{t_p - t_o}{6}$$

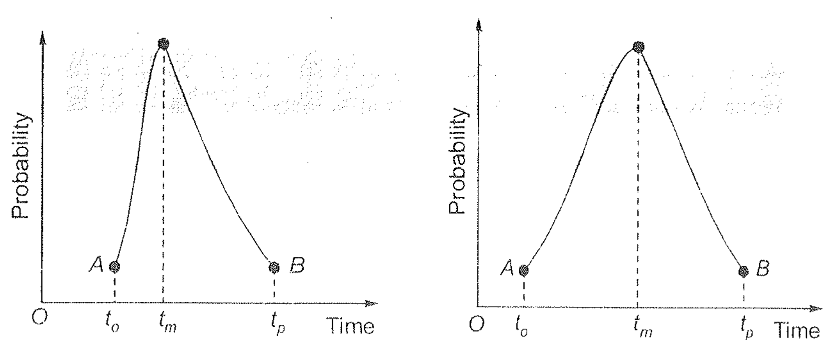
- It can be seen above that 'σ' is affected by relative distance from the most optimistic estimate to the most pessimistic estimate.
- Therefore, wide range in time estimates represents greater uncertainly.

Do You Know? In a limiting case, certainty of an activity duration occurs only when the three time estimates coincide, so that the standard deviation and the variance both vanish. Consequently the activity duration becomes certain which is the case of CPM. Hence, a PERT is a general case whereas CPM is the particular case of PERT.

2.2.3 Variance of an Activity (σ^2)

- Square of standard deviation is variance of an activity. It is to be noted that higher the uncertainty about a process, greater is the standard deviation and hence greater is the variance of a project.

Do You Know? A beta distribution is the one which is not symmetrical about its apex. Figure below shows two beta distribution curves, one having skew to the left (Beta distribution for optimistic estimator) and other having skew to the right (Beta distribution curve for the pessimistic estimator).



(a) Optimistic Estimation (b) Pessimistic Estimation

Fig. 2.2 Two different forms of β-distribution curves

2.3 Central Limit Theorem

The theorem states that a project consists of a large number of activities, where each activity has its own mean time (t_e), standard deviation (σ), variance (σ^2) and also its own β-distribution curve. The distribution of time for the project as a whole will approximately be a normal distribution, i.e. mean time or expected time of a project is

$t_e = t_{e_1} + t_{e2} + t_{e3} + \ldots\ldots\ldots$ along critical path and the variance is,

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots\ldots\ldots$$

along critical path. Hence standard deviation of the project as a whole

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots\ldots\ldots}$$

along critical path.

Critical path: The time wise longest path is the critical path. In this path, any type of delay in any event will cause delay in the project. These are shown by double lines or dark lines in a network.

1-2-3-4 is the critical path of following net work.

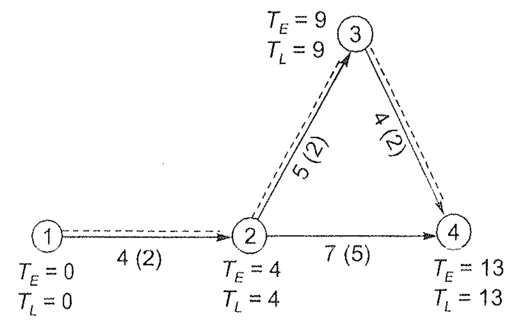


Fig. 2.3 Critical path

Time computation of Events:

1. **Earliest Expected Occurrence Time (EOT):** The time at which an event is expected to occur earliest.
   - The time at which an event can be expected to occur earliest.
   - An event occurs when all the activities leading to it are completed.
   - It is generally denoted by T_E. It is calculated by forward path.

$$T_E^j = T_E^i + t_{ij} \quad \text{...(when there is only one path)}$$

$$T_E^j = (T_E^i + t_{ij})_{max} \quad \text{...(when there are more than one path)}$$

Here,

T_E^j = EOT of event j

T_E^i = EOT of event i

t_i^j = Expected time of activity i-j

2. **Latest Allowable Occurrence Time (LOT):** The latest allowable time at which an event must occur to keep the project on schedule.

   It is generally denoted by T_L. This is calculated through backward path.

$$T_L^i = T_L^j - t_{ij} \quad \text{...(when there is only one path)}$$

$$T_L^i = (T_L^j - t_{ij})_{min} \quad \text{...when there are more than one path)}$$

Where

T_L^j = LOT of event j

T_L^i = LOT of event i

t_{ij} = expected time of activity i-j

NOTE: The latest allowable occurrence time of the finish event is equal to the schedule completion time of the project.

- **Slack** : Slack is defined as the difference between latest allowable time (T_L) and earliest expected time (T_E) of an event.

  Slack for any event $j = T_L^j - T_E^j$

  Slack for any event $i = T_L^i - T_E^i$

- Slack may be positive, zero or negative.
- When "Slack is greater than zero". It indicates project is ahead of schedule and availability of excess resources. Such events are sub critical.
- If slack is zero, it indicates work is on schedule and events are critical. Resources are just adequate.
- If slack is negative, it indicates work is behind schedule and may cause delay in project completion. Events are super critical. Extra resources are required.
- The path having minimum or zero slack value is the "critical path" which is also time wise longest path.

Do You Know?

If a project has more than one critical path then severity is calculated by calculating standard deviation of the project from each path. (Having lesser value of σ have more deterministic approach hence critical path will be having more value of σ).

Probability of a project to be completed in schedule time :

- Probability of a project to get complete in expected time (T_e) is just 50% and probability of completion increases or decreases with an increase or decrease of scheduled time respectively.
- As the probability distribution curve for the entire project is normal distribution curve.
- Probability can be calculated by using probability factor (Z) which is defined as,

$$Z = \frac{T_s - T_e}{\sigma}$$

Value of Z	Probability (%)
0	50.00
+1	84.13
+2	97.72
+3	99.87
-1	15.87
-2	2.28
-3	0.13

It should be noted that:

1. At $Z = 0$, $T_s = T_e$ and probability = 50%

   $Z < 0$, $T_s < T_e$ probability < 50% and vice versa.
2. For intermediate value of Z, interpolation is required.
3. The time and probability distribution for project as a whole is a normal distribution.
4. Area under the curve is equal to unity.
5. 68% of the area lies within '$t_e - \sigma$' and '$t_e + \sigma$' range. This is called probable range.
6. Area bounded by $t_e + 2\sigma$ and $t_e - 2\sigma$ is about 95%
7. Area bounded by $t_e - 3\sigma$ and $t_e + 3\sigma$ is about 99.7% which is approximately equal to area bounded by t_o and t_p, which is maximum probable range.

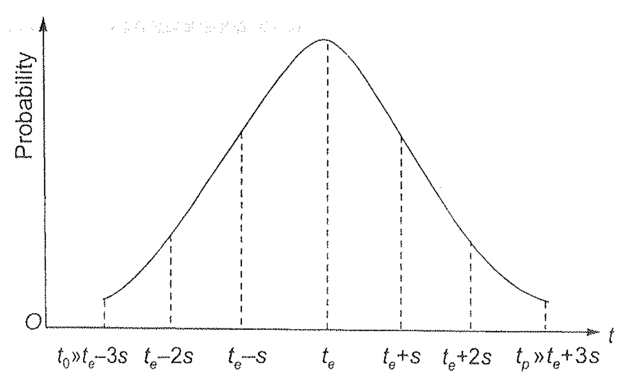


Fig. 2.4 Probability Distribution Curve

$$\sigma = \frac{t_p - t_o}{6}$$

2.4 Critical Path Method (CPM)

This is based on deterministic approach in which only one time estimate is made for activity completion. Network diagram in CPM is activity oriented.

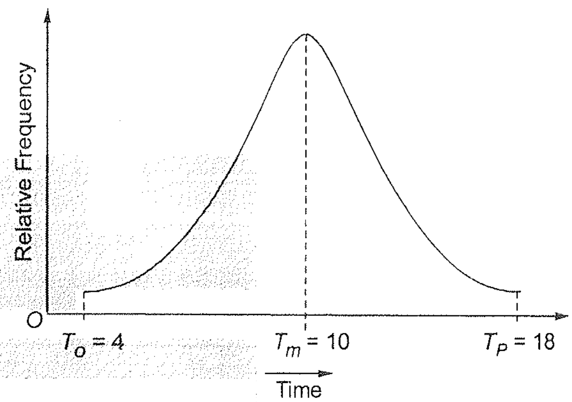
- It is activity oriented network.
- It is used for repetitive type of work and has deterministic approach.

2.5 Comparison between PERT and CPM

PERT

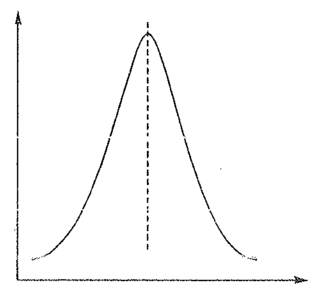
1. It is event oriented.
2. It has probabilistic approach. The probability distribution is of the type of β distribution
3. Three types of times are estimated on the basis of which an expected time t_e is derived.
4. Cost is directly proportional to time. Hence efforts are made to minimize the time so as to result in the minimum cost.
5. It is suitable for newer type of projects which have not been performed in the past and no exact assessment of time and cost are available.

   Examples: Research work, launching of space aircraft, development of missile programme etc.



CPM

1. It is an activity oriented network.
2. It has deterministic approach. Probability value approaches to one here.
3. Only one time is calculated i.e. activity duration '*t*'.
4. Time and cost are related by the following curve given. From this curve optimum time is derived which results in the minimum cost.
5. It is suitable for repetitive type of work where time and cost can be evaluated with fair degree of accuracy.



Examples:

- Construction work,
- Maintenance work,
- Civil engineering projects.

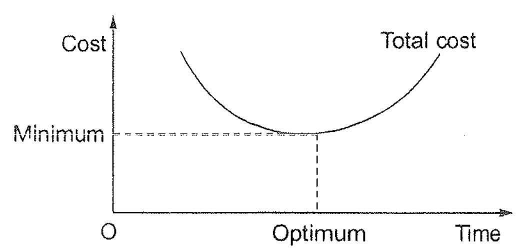


Fig. 2.5 Time-cost Relationship

Event Times in CPM

1. **Earliest occurrence time (T_E):** Time at which an event may occur as early as possible.
2. **Latest allowable time (T_L):** Time at which event may occur as late as possible without delaying the overall project completion time.

   These are similar to PERT and are calculated in the same fashion.

Activity Times in CPM

1. **Earliest start time (EST) :** It is the earliest possible time at which an activity can be started.

   For an activity $i-j$, earliest event time of event i, i.e. T_E^i is EST of activity $i-j$.

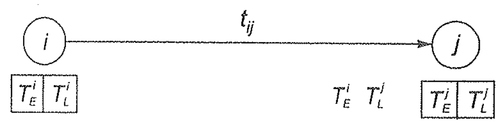


Fig.2.6 EST and EFT

2. **Earliest finish time (EFT) :** It is the earliest possible time by which an activity can be completed.

   For an activity $i-j$

$$\text{EFT} = \text{EST} + t_{ij} = T_E^i + t_{ij}$$

$$t_{ij} = \text{Activity duration}$$

3. **Latest start time (LST) :** This is the latest possible time at which an activity can be started without delaying the overall project

$$\therefore \quad \text{LST} = \text{LFT} - \text{Activity duration}$$

$$\therefore \quad \text{LST} = T_L^j - t_{ij}$$

$$\text{LFT} = \text{Latest finish time of activity } i-j = T_L^j$$

4. **Latest finish time (LFT) :** This is the latest time by which an operation or activity must be completed without delaying the project.

   For an activity $i-j$, latest allowable time of head event j, i.e. T_{Lj} is LFT of activity $i-j$.

NOTE: LST of an activity is to be calculated on the basis of latest occurrence time of its head event and not on the brass of latest occurrence time of its tail event.

Float

- It is associated with activity times
- It is analogous to slack of events in PERT.
- It is the range within which start or finish time of an activity may fluctuate without affecting the project completion time.
- Floats are of following types:

1. **Total float:** The time span by which starting or finishing of an activity can be delayed without delaying the completion of the project.

   It is the maximum available time in excess of the activity completion time.

   Total float is given by F_T

$$F_T = (T_L^j - T_E^i) - t_{ij}$$

   Total float is given by F_T

$$F_T = (T_L^j) - (T_E^i + t_{ij})$$

$$F_T = (T_L^j) - (T_E^i + t_{ij})$$

$$F_T = \text{LFT} - \text{EFT}$$

or

$$F_T = (T_L^j - t_{ij}) - T_E^i$$

$$F_T = \text{LST} - \text{EST}$$

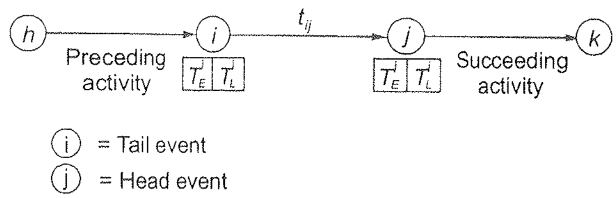


Fig. 2.7 Total float

- Total float of an activity affects total float of succeeding as well as preceding activities.

Do you know? Total float of an activity constrains the finishing of preceding activity and starting of succeeding activity.

2. **Free float (F_F)** : The delay which can be made without delaying succeeding activities. It affects only preceding activities. It is denoted by F_F. It is assumed that all activities start as early as possible.

   Free float is given by

$$F_F = (T_E^j - T_E^i) - t_{ij}$$

$$\Rightarrow \quad F_F = F_T - S_j$$

   where S_j is head event slack.

Do You Know? In free float, preceding activity is not allowed to occur at its latest time and hence total float of preceding activity is affected. However the succeeding activity can start at its earliest start time and hence its total float is not affected.

3. **Independent Float (F_{ID})**: It is the minimum excess available time which exists without affecting any of succeeding or preceding activities. It is denoted by F_{ID}.
   - It is the excess of minimum available time over the activity duration.

$$F_{ID} = (T_E^j - T_L^i) - t_{ij}$$

$$F_{ID} = F_F - S_i$$

   where S_i is tail event slack.

4. **Interfering float (F_{INT})** : It is similar to head event slack.

$$F_{INT} = S_j = F_T - F_F$$

Critical Path

In CPM analysis, the path along which total floats are zero or minimum is called as critical path. All activities on this path are critical. There can be more than one critical paths.

Subcritical Path: It is the path joining all subcritical activities. For a subcritical activity total float is greater than zero i.e.,

$$F_T > 0$$

Supercritical Path: It is the path joining all super critical activities. For a supercritical activities total float is less than zero i.e.,

$$F_T < 0.$$

2.6 CPM Systems

Mainly two systems are used in CPM analysis:

1. **A-O-A system** (Activity on arrow system)

   An activity is graphically represented by an arrow.

   The tail end and head end of arrow represent start and finish of an activity respectively.

Do you know? Dummies have been used to represent the constraint and consequent interdependence. But number of dummies must be minimum for an efficient network.

2. A-O-N System (Activity on node system or precedence diagram). Activity is represented by a circle or a node. Events have no places. Arrows are used only to show the dependency relationship between activity nodes.
   - When two or more activities start parallely then an activity called DEBUT (D_O) is provided at the beginning. Like wise a finish activity (F_O) is provided at the end when more than one activities finish parallely. Activities D & F has zero duration.

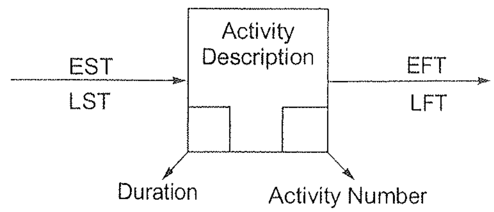
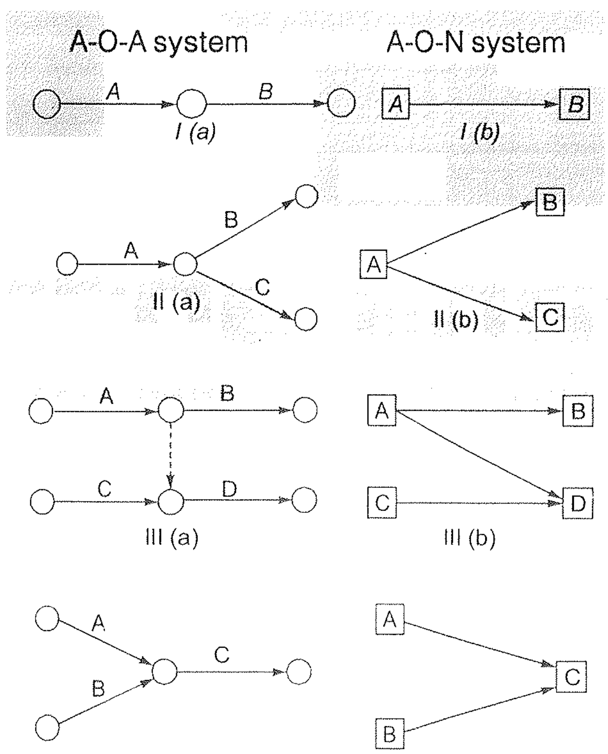


Fig. 2.8 Debut Activity

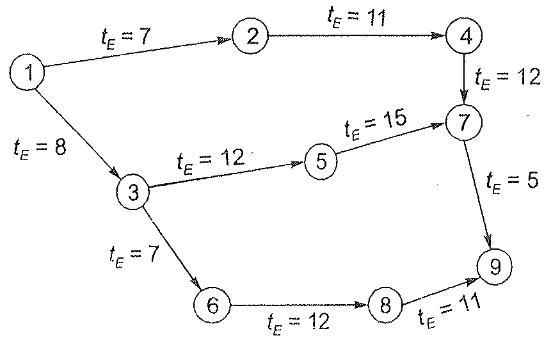
2.6.1 Advantages of A-O-N System over A-O-A System

1. A-O-N system eliminates the use of dummy activities.
2. It is more helpful for projects having more overlapping activities.
3. It is a self sufficient and self -explanatory. All activity times (EST, EFT, LST, LFT) are represented on the diagram.
4. Revision and modifications are easier.
5. Pre-operations and post-operations of activities under consideration are distinctly visible.

Examples:



Example 2.1 Consider the network as shown below with expected time of completion of each activity (t_E) (in days) is shown on arrows. Calculate the earliest expected time of the network for the completion of work.



Solution:

Here event 7 is approached by two paths viz. path along $1 \to 2 \to 4 \to 7$ and $1 \to 3 \to 5 \to 7$.

For path $1 \to 2 \to 4 \to 7$,

$$t_E \text{ for event } 7 = 7 + 11 + 12 = 30 \text{ days}$$

For path $1 \to 3 \to 5 \to 7$,

$$t_E \text{ for event } 7 = 8 + 12 + 15 = 35 \text{ days}$$

Thus event 7 cannot be considered to occur until all activities of path $1 \to 3 \to 5 \to 7$ have completed i.e., event 7 cannot occur prior to 35 days. Thus earliest expected time for event 7 is 35 days.

In this network, event 9 is the terminal event which is preceded by path $7 \to 9$ and $3 \to 6 \to 8 \to 9$.

For path $7 \to 9$,

$$T_E \text{ for event } 9 = 35 + 5 = 40 \text{ days}$$

For path $3 \to 6 \to 8 \to 9$

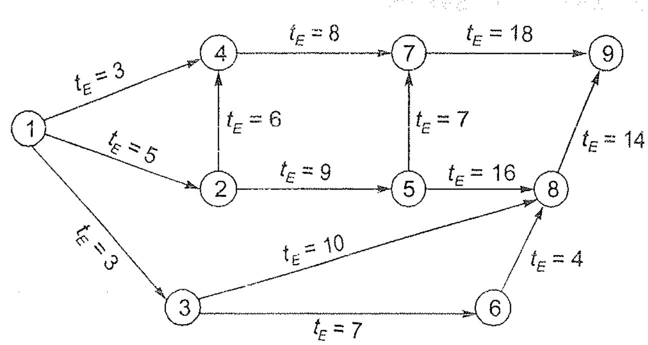
$$T_E \text{ for event } 9 = 8 + 7 + 12 + 11 = 38 \text{ days}$$

Thus earliest expected time of event 9

= Earliest expected time for completion of work

= 40 days (larger of two values viz. 40 days and 38 days)

Example 2.2 Compute the earliest expected time of completion of a project whose network is shown below. The earliest expected time of each activity is shown on the arrows (in weeks).



Solution:

The network commences from event 1 and culminates at event 9.

Thus earliest expected time of completion of event 1 $(T_E^1) = 1$

Earliest expected time for event

$$2(T_E^2) = T_E^1 + t_E^{12} = 0 + 5 = 5 \text{ weeks}$$

Similarly earliest expected time for event

$$3(T_E^3) = T_E^1 + t_E^{13} = 0 + 3 = 3 \text{ weeks}$$

Event 4 is approached by paths $1 \to 4$ and $1 \to 2 \to 4$.

For these two paths, $T_E^4 = T_E^1 + t_E^{14} = 0 + 3 = 3$ weeks (for path $1 \to 4$)

$T_E^4 = T_E^2 + t_E^{24} = 5 + 6 = 11$ weeks (for path $1 \to 2 \to 4$)

Thus, $T_E^4 = 11$ weeks (Larger of 3 weeks and 11 weeks)

Thus,

$$T_E^1 = 0$$
$$T_E^2 = 5 \text{ weeks}$$
$$T_E^3 = 3 \text{ weeks}$$

$$T_E^4 = \begin{bmatrix} T_E^2 + t_E^{24} = 5 + 6 = 11 \text{ weeks} \\ T_E^1 + t_E^{14} = 0 + 3 = 3 \textit{ weeks} \end{bmatrix} = 11 \text{ weeks}$$

$$T_E^5 = T_E^2 + t_E^{25} = 5 + 9 = 14 \text{ weeks}$$
$$T_E^6 = T_E^3 + t_E^{36} = 3 + 7 = 10 \text{ weeks}$$

$$T_E^7 = \begin{bmatrix} T_E^5 + t_E^{57} = 14 + 7 = 21 \text{ weeks} \\ T_E^4 + t_E^{47} = 11 + 8 = 19 \textit{ weeks} \end{bmatrix} = 21 \text{ weeks}$$

$$T_E^7 = \begin{bmatrix} T_E^5 + t_E^{5-7} = 14 + 7 = 21 \text{ weeks} \\ T_E^4 + t_E^{4-7} = 11 + 18 = 19 \textit{ weeks} \end{bmatrix} = 21 \text{ weeks}$$

Now,

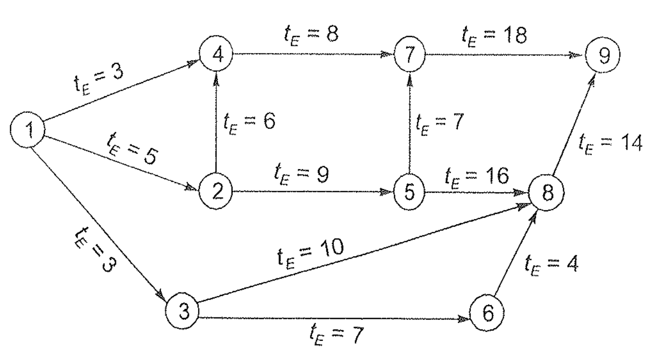
$$T_E^8 = \begin{bmatrix} T_E^5 + t_E^{5-8} = 14 + 16 = 30 \text{ weeks} \\ T_E^6 + t_E^{6-8} = 10 + 4 = 14 \text{ weeks} \\ T_E^3 + t_E^{3-8} = 3 + 10 = 13 \text{ weeks} \end{bmatrix} = 30 \text{ weeks}$$

Now,

$$T_E^9 = \begin{bmatrix} T_E^7 + t_E^{7-9} = 21 + 18 = 39 \text{ weeks} \\ T_E^8 + t_E^{8-9} = 30 + 14 = 44 \textit{ weeks} \end{bmatrix} = 44 \text{ weeks}$$

So, earliest expected time of completion of project = 44 weeks

Example 2.3 In Question No. 2.2 above, compute the latest allowable occurrence time for the network.



Solution:

Here since the schedule time of completion of project (T_S) is not given and thus it is usual to assume that $T_S = T_L$.

Commencing from the last event i.e., event 9 and moving backwards, the latest allowable occurrence times (T_L) are as computed below:

For event 9, $\quad T_L^9 = T_E^9 = 44$ weeks

For event 8, $\quad T_L^8 = T_L^9 - t_E^{89} = 44 - 14 = 30$ weeks

For event 6, $\quad T_L^6 = T_L^8 - t_E^{68} = 30 - 4 = 26$ weeks

For event 3,
$$T_L^3 = \begin{bmatrix} T_L^6 - t_E^{36} = 26 + 7 = 19 \text{ weeks} \\ T_L^8 - t_E^{38} = 30 + 10 = 20 \text{ weeks} \end{bmatrix} = 19 \text{ weeks}$$

For event 7, $\quad T_L^7 = T_L^9 - t_E^{79} = 44 - 18 = 26$ weeks

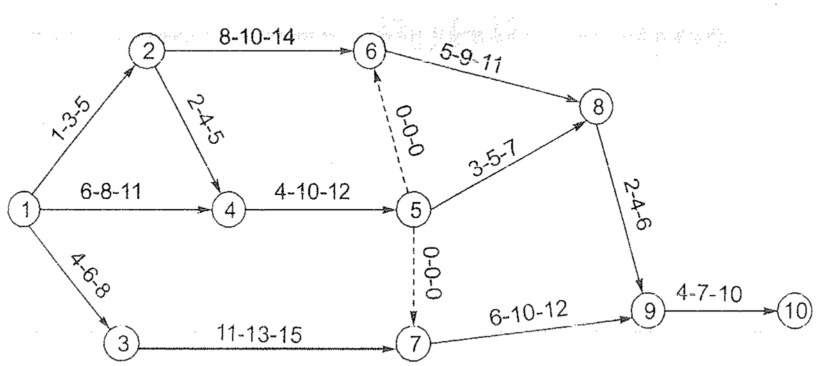
For event 5,
$$T_L^5 = \begin{bmatrix} T_L^8 - t_E^{58} = 30 + 16 = 14 \text{ weeks} \\ T_L^7 - t_E^{57} = 26 + 7 = 19 \text{ weeks} \end{bmatrix} = 14 \text{ weeks}$$

For event 4, $\quad T_L^4 = T_L^7 - t_E^{47} = 26 - 8 = 18$ weeks

For event 2,
$$T_L^2 = \begin{bmatrix} T_L^5 - t_E^{25} = 14 + 9 = 5 \text{ weeks} \\ T_L^4 - t_E^{24} = 18 + 6 = 12 \text{ weeks} \end{bmatrix} = 5 \text{ weeks}$$

For event 1,
$$T_L^1 = \begin{bmatrix} T_L^3 - t_E^{13} = 19 + 3 = 16 \text{ weeks} \\ T_L^2 - t_E^{12} = 5 + 5 = 0 \\ T_L^4 - t_E^{14} = 18 - 3 = 15 \text{ weeks} \end{bmatrix} = 0$$

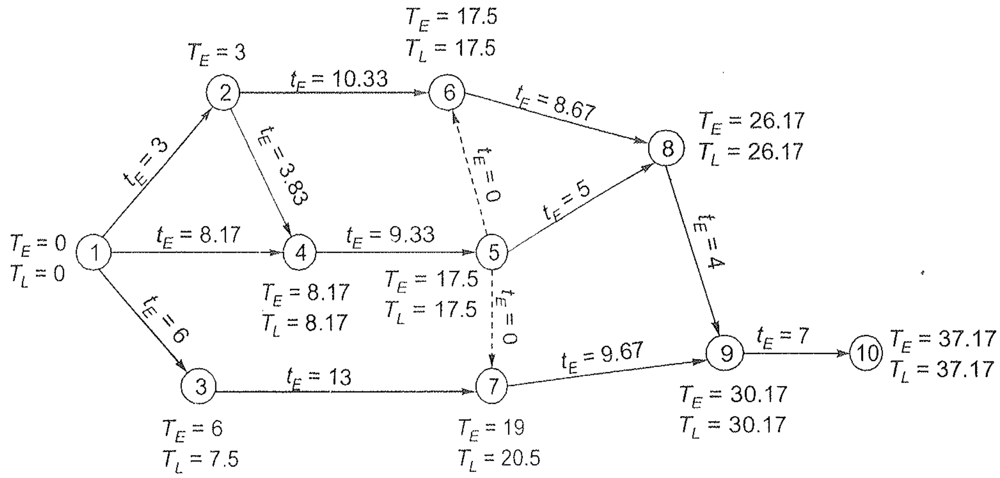
Example 2.4 In the network as shown, the three times viz. optimistic time, most likely time and the pessimistic time (in days) are shown on the arrows. Compute the earliest expected times and the latest allowable occurrence times of various events.



Solution:

The expected time of an activity is given by,

$$t_E^{ij} = \frac{t_0^{ij} + 4t_L^{ij} + t_P^{ij}}{6}$$



Activity *i-j*							
Successor event *j*	**Predecessor event *i***	t_o^{ij}	t_L^{ij}	t_P^{ij}	t_E^{ij}	T_E^j	T_L^j
10	9	4	7	10	7	37.17	37.17
9	8	2	4	6	4	30.17	30.17
9	7	6	10	12	9.67	30.17	30.17
8	6	5	9	11	8.67	26.17	26.17
8	5	3	5	7	5	26.17	26.17
7	5	0	0	0	0	19	20.5
7	3	11	13	15	13	19	20.5
6	5	0	0	0	0	17.5	17.5
6	2	8	10	14	10.33	17.5	17.5
5	4	4	10	12	9.33	17.5	17.5
4	2	2	4	5	3.83	8.17	8.17
4	1	6	8	11	8.17	8.17	8.17
3	1	4	6	8	6	6	7.5
2	1	1	3	5	3	3	4.34

Example 2.5 For the network given in Question No. 2.4, compute the slack of each event and the critical path of the network.

Solution:

Slack of an event is the difference of latest allowable occurrence time (T_L) and the earliest expected time (T_E) i.e.,

$$\text{Slack} = T_L - T_E$$

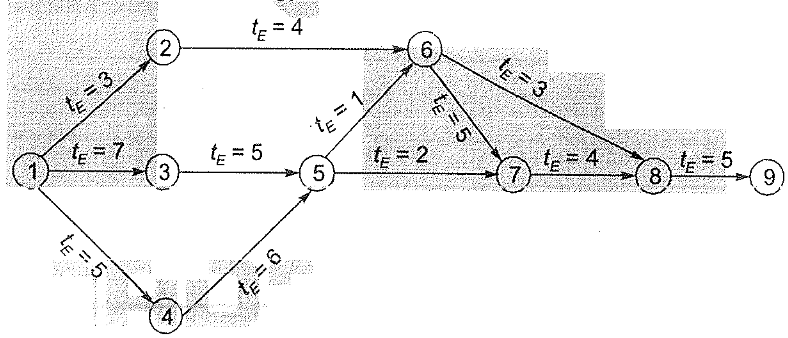
The computation of slack of various events is tabulated below:

Activity *i-j*								
Successor event *j*	**Predecessor event *i***	t_0^{ij}	t_L^{ij}	t_P^{ij}	t_E^{ij}	T_E^j	T_L^j	**Slack =** $T_L^j - T_E^j$
10	9	4	7	10	7	37.17	37.17	0
9	8	2	4	6	4	30.17	30.17	0
9	7	6	10	12	9.67	30.17	30.17	0
8	6	5	9	11	8.67	26.17	26.17	0
8	5	3	5	7	5	26.17	26.17	0
7	5	0	0	0	0	19	20.5	1.5
7	3	11	13	15	13	19	20.5	1.5
6	5	0	0	0	0	17.5	17.5	0
6	2	8	10	14	10.33	17.5	17.5	0
5	4	4	10	12	9.33	17.5	17.5	0
4	2	2	4	5	3.83	8.17	8.17	0
4	1	6	8	11	8.17	8.17	8.17	0
3	1	4	6	8	6	6	7.5	1.5
2	1	1	3	5	3	3	4.34	1.34

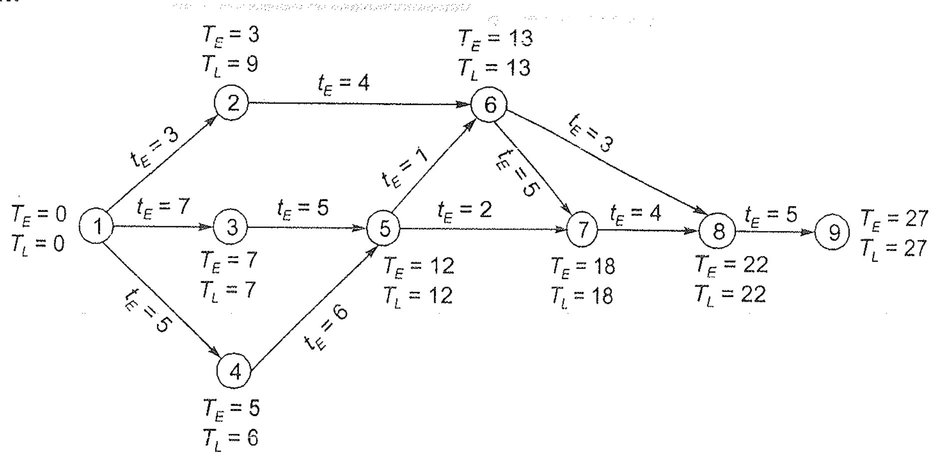
Critical path is the one having zero or the medium slack. Thus in the network given, critical path is 1–4–5–6–8–9–10.

1–4–5–6–8–9–10 is also the longest path time wise.

Example 2.6 For the network as shown below, determine the critical path. The duration of each activity (in days) is shown on the arrows.



Solution:



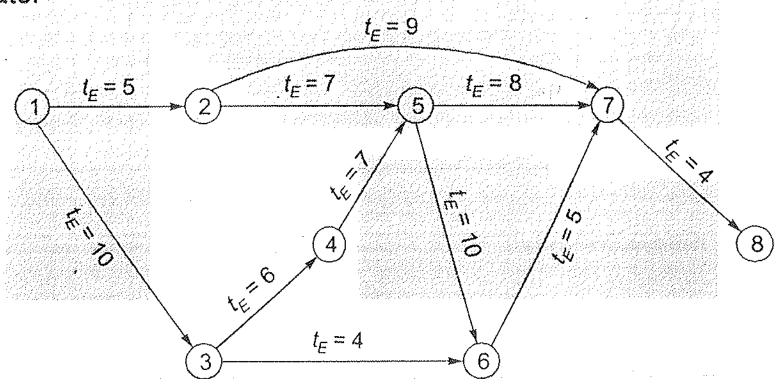
Along the critical path, slack = 0 i.e., $T_L - T_E = 0$

Thus T_L and T_E for each event needs to be computed and the same is shown in the network.

Event No.	Earliest expected time			Latest occurrence time				Slack (S) $S = T_L - T_E$
	Predecessor event (i)	Activity time (t_E^{ij})	T_L^{i}	Successor event (j)	(t_E^{ij})	T_L^{i}	T_L	
1.	—	—	0	2, 3, 4	3, 7, 5	6, 0, 1	0	0
2.	1	3	3	6	4	9	9	6
3.	1	7	7	5	5	7	7	0
4.	1	5	5	5	6	6	6	1
5.	3 4	5 6	12 11	6 7	1 2	12 16	12	0
6.	2 5	4 1	7 13	7 8	5 3	13 19	13	0
7.	5 6	2 5	14 18	8	4	18	18	0
8.	6 7	3 4	16 22	9	5	22	22	0
9.	8	5	27	—	—	27	27	0

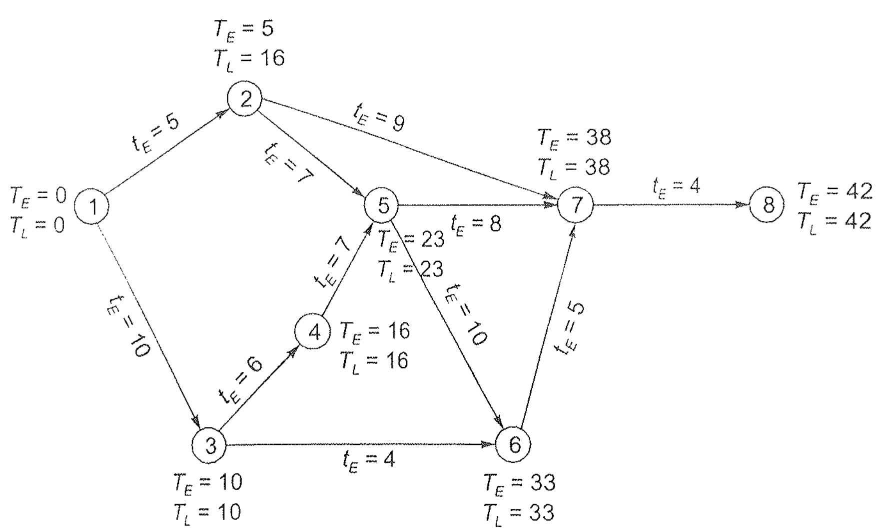
∴ Critical path is 1–3–5–6–7–8–9, which is also the longest path time wise.

Example 2.7 A network is shown below with activity times (in days) marked on the arrows. Compute the various floats.



Solution:

The earliest start time (EST) and latest finish time (LFT) of each event is computed and is shown in the network.



Activitiy $(i–j)$	Duration (days) t^{ij}	Earliest Time		Latest Time		Total Float (F_T)	Free Float (F_F)	Independent Float (F_{ID})
		Start (EST)	Finish (LFT)	Start (LST)	Finish (LFT)			
1-2	5	0	5	11	16	11	0	0
1-3	10	0	10	0	10	0	0	0
2-5	7	5	12	16	23	11	11	0
2-7	9	5	14	29	38	24	24	13
3-4	6	10	16	10	16	0	0	0
3-6	4	10	14	29	33	19	19	19
4-5	7	16	23	16	23	0	0	0
5-6	10	23	33	23	33	0	0	0
5-7	8	23	31	30	38	7	7	7
6-7	5	33	38	33	38	0	0	0
7-8	4	38	42	38	42	0	0	0

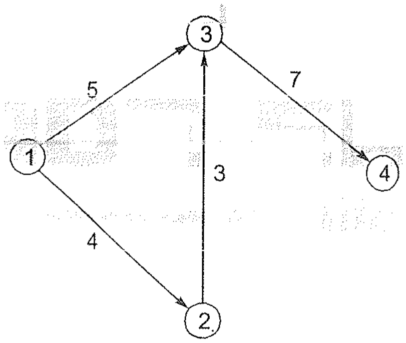
Critical path will have total float as zero for each activity.

∴ Critical path is $1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8$.

NOTE

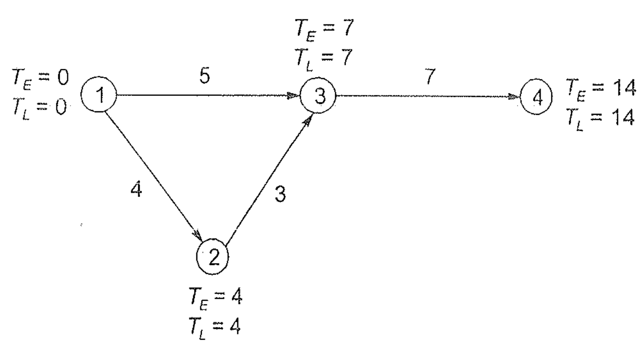
- Slack condition gives only necessary condition while float concept provide both necessary and sufficient condition.
- As in above example, event (5) and (6) having zero slack but activity 5–7 is not critical.

Example 2.8 Find out the project completion time and critical path for the network shown below. The activity durations are in days.



Solution:

The network can be redrawn as shown below:



Activity (i-j)	Duration (t^{ij}, days)	Earliest Time		Latest Time		Total float (F_T)
		Start (EST)	Finish (EFT)	Start (LST)	Finish (LFT)	
1-2	4	0	4	0	4	0
1-3	5	0	5	2	7	2
2-3	3	4	7	4	7	0
3-4	7	7	14	7	14	0

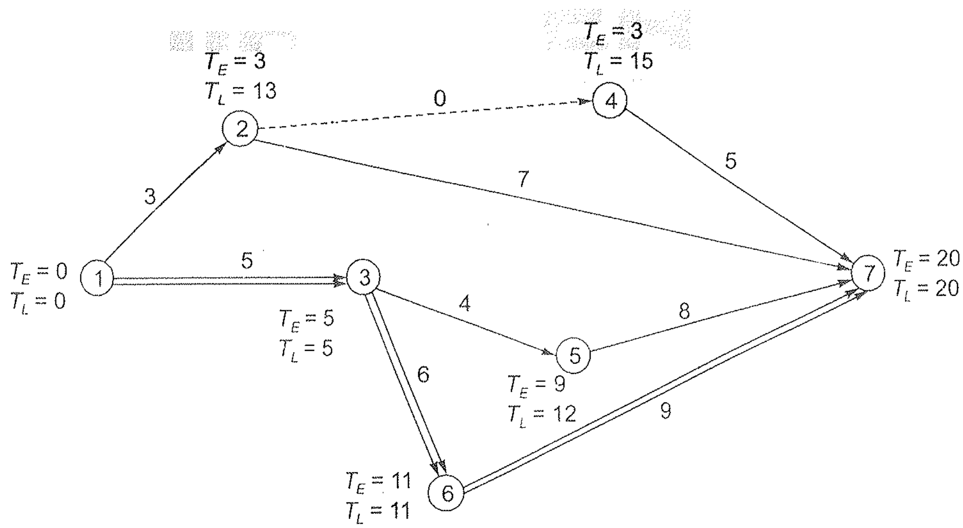
∴ Project completion time = 14 days.

Critical is the one which is having zero total floats i.e., path 1 → 2 → 3 → 4 is critical.

Example 2.9 Determine the project completion time and identify the critical path for a project with the following activities.

Activity	Duration (days)	Remarks
1-2	3	1-2 and 1-3 are starting activities and can be taken up simultaneously.
1-3	5	
2-4	0	Dummy activity
2-7	7	2-7 succeeds 1-2
3-5	4	1-3 preceeds both 3-5 and 3-6.
3-6	6	3-5 and 3-6 can be taken up simultanecusly
4-7	5	4-7 succeeds 1-2
5-7	8	5-7 follows 3-5
6-7	9	3-6 preceeds 6-7

Solution:



(Activity durations are in days)

With the network data given the question, the network can be drawn as shown below:

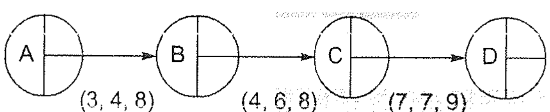
Activitiy (i–j)	Duration (days)	Earliest Time		Latest Time		Total Float (F_T)
		Start (EST)	Finish (EFT)	Start (LST)	Finish (LFT)	
1-2	3	0	3	10	13	10
1-3	5	0	5	0	5	0
2-4	0	3	3	15	15	12
2-7	7	3	10	13	20	10
3-5	4	5	9	8	12	3
3-6	6	5	11	5	11	0
4-7	5	3	8	15	20	12
5-7	8	9	17	12	20	3
6-7	9	11	20	11	20	0

∴ Project completion time = 20 days.

Critical path is $1 \rightarrow 3 \rightarrow 6 \rightarrow 7$.

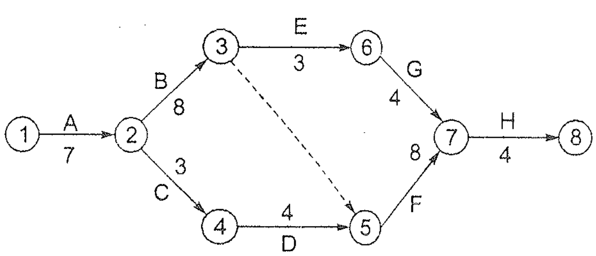
Objective Brain Teasers

Q.1 The optimistic, likely and pessimistic time estimates for the PERT network of a project are shown in the given figure. The expected duration of the project is



(a) 14 days (b) 17 days
(c) 17.83 days (d) 18.67 days

Q.2 The flow net of activities of a project is shown in the below figure. The duration of the activities are written along the arrows:



The critical path of the activities is along
(a) 1-2-4-5-7-8 (b) 1-2-3-5-7-8
(c) 1-2-3-6-7-8 (d) 1-2-4-5-3-6-7-8

Q.3 The area under the β-distribution curve is divided into two equal halves by vertical ordinate through
(a) expected time
(b) optimistic time
(c) most likely time
(d) pessimistic time

Q.4 Consider the following statements of network:
1. Only one time estimate is required for each activity.
2. Three time estimates for each activity.
3. Time and cost are both controlling factors.
4. It is built-up of event-oriented diagram.

Which of the above statements are correctly applicable to CPM network?
(a) 1 and 3 (b) 1 and 2
(c) 2 and 4 (d) 3 and 4

Q.5 In PERT, slack is computed as the difference between which one of the following?
(a) Latest allowable time and the pessimistic time
(b) Earliest expected time and latest allowable time

(c) Earliest expected time and the pessimistic time
(d) Latest allowable time and the earliest feasible time

Q.6 In the PERT analysis, which one of the following is followed by the time estimates of activities and probability of their occurrence?
(a) Normal distribution
(b) Poisson's distribution
(c) β-distribution
(d) Binomial distribution

Q.7 What does higher standard deviation imply in cost analysis?
(a) Higher uncertainty
(b) Lower uncertainty
(c) Nothing to do with uncertainty
(d) Extra costs are likely

Q.8 What does the critical path in PERT represent?
1. The shortest path for the earliest completion of the project.
2. The longest path of the network from the initial to final event.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q.9 In a PERT network, the activity durations are given as t_0 (optimistic time), t_p (pessimistic time) and t_m (most likely time). What is the variance of the activity?
(a) $\dfrac{t_0 + 4t_m + t_p}{6}$ (b) $\left(\dfrac{t_p - t_o}{6}\right)$
(c) $\left(\dfrac{t_p - t_o}{6}\right)^2$ (d) None of these

Answers

1. (c)	2. (b)	3. (a)	4. (a)	5. (b)
6. (c)	7. (a)	8. (b)	9. (c)	

Hints & Solution

4. (a)
Three time estimates for each activity are required in PERT network. PERT is built up of event oriented diagram.