ALGEBRAIC EXPRESSIONS & FACTORIZATION

◈ **Constant :** A symbol having a fixed numerical value is called a constant.

◈ **Variable :** A symbol which takes various numerical values is called a variabale.

Ex.1 We know that the perimeter P of a square of side s is given by $P = 4 \times s$. Here, 4 is a constant and P and s are variables.

Ex.2 The perimeter P of a rectangle of sides l and b is given by $P = 2(l + b)$. Here, 2 is a constant and l and b are variables.

◈ **Algebraic Expressions :** A combination of constants and variables connected by the signs of fundamental operation of addition, subtraction, multiplication and division is called an algebraic expression.

◈ **Terms :** Various parts of an algebraic expression which are separated by the signs of + or – are called the 'terms' of the expression.

Ex.3 $2x^2 - 3xy + 5y^2$ is an algebraic expression consisting of three terms, namely, $2x^2, -3xy$ and $5y^2$.

Ex.4 The expression $2x^3 - 3x^2 + 4x - 7$ is an algebraic expression consisting of four terms, namely, $2x^3$, $-3x^2$, $4x$ and -7.

◈ **Monomial :** An algebraic expression containing only one term is called a monomial.

Ex.5 $-5, 3y, 7xy, \frac{2}{3}x^2yz, \frac{5}{3}a^2bc^3$ etc. are all monomials.

◈ **Binomial :** An algebraic expression containing two terms is called a binomial.

Ex.6 The expression $2x - 3$, $3x + 2y$, $xyz - 5$ etc. are all binomials.

◈ **Trinomial :** An algebraic expression containing three terms is called a trinomial.

Ex.7 The expressions $a - b + 2x^2 + y^2 - xy$,
$x^3 - 2y^3 - 3x^2y^2z$ etc. are trinomial.

◈ **Factors :** Each terms in an algebraic expression is a product of one or more numbers (s) and /or literal (s). These number(s) and liteal(s) are known as the factors of that terms.
A constant factor is called a numerical factor, while a variable factor is known as a literal factor.

◈ **Coefficient :** In a term of an algebraic expression any of the factors with the sign of the term is called the coefficient of the other factors.

Ex.8 In $-5xy$, the coefficient of x is $-5y$; the coefficient of y is $-5x$ and the coefficient of xy is -5.

Ex.9 In $-x$, the coefficient of x is -1.

◈ **Constant Term :** A term of the expression having no literal factor is called a constant term.

Ex.11 In the algebraic expression $x^2 - xy + yz - 4$, the constant term is -4.

◈ **Like and Unlike Terms :** The terms having the same literal factors are called like or similar terms, otherwise they are called unlike terms.

Ex.12 In the algebraic exspression $2a^2b + 3ab^2 - 7ab - 4ba^2$, we have $2a^2b$ and $-4ba^2$ as like terms, whereas $3ab^2$ and $-7ab$ are unlike terms.

❖ EXAMPLES ❖

Ex.13 Add : $7x^2 - 4x + 5$, $-3x^2 + 2x - 1$ and $5x^2 - x + 9$.

Sol. We have,

Required sum

$= (7x^2 - 4x + 5) + (-3x^2 + 2x - 1) + (5x^2 - x + 9)$

$= 7x^2 - 3x^2 + 5x^2 - 4x + 2x - x + 5 - 1 + 9$ [Collecting like terms]

$= (7 - 3 + 5)x^2 + (-4 + 2 - 1)x + (5 - 1 + 9)$ [Adding like terms]

$= 9x^2 - 3x + 13$

Ex.14 Add : $5x^2 - \frac{1}{3}x + \frac{5}{2}$, $-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}$ and $-2x^2 + \frac{1}{5}x - \frac{1}{6}$.

Sol. Required sum

$= \left(5x^2 - \frac{1}{3}x + \frac{5}{2}\right) + \left(-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{3}\right) + \left(-2x^2 + \frac{1}{5}x - \frac{1}{6}\right)$ $= 5x^2 - \frac{1}{2}x^2 - 2x^2 - \frac{1}{3}x + \frac{1}{2}x + \frac{1}{5}x + \frac{5}{2} - \frac{1}{3} - \frac{1}{6}$ [Collecting like terms]

$= \left(5 - \frac{1}{2} - 2\right)x^2 + \left(-\frac{1}{3} + \frac{1}{2} + \frac{1}{5}\right)x + \left(\frac{5}{2} - \frac{1}{3} - \frac{1}{6}\right)$ [Adding like term]

$= \left(\frac{10 - 1 - 4}{2}\right)x^2 + \left(\frac{-10 + 15 + 6}{30}\right)x + \left(\frac{15 - 2 - 1}{6}\right)$

$= \frac{5}{2}x^2 + \frac{11}{30}x + 2$

(i) The product of two factors with like signs is positive and the product of two factors with unlike signs is negative

i.e., (a) $(+) \times (+) = +$

(b) $(+) \times (-) = -$

(c) $(-) \times (+) = -$

and, (d) $(-) \times (-) = +$

(ii) If a is any variable and m, n are positive integers, then

$a^m \times a^n = a^{m+n}$

For example , $a^3 \times a^5 = a^{3+5} = a^8$,

$y^4 \times y = y^{4+1} = y^5$ etc.

Ex.15 Find the product of the following pairs of polynomials :

(i) 4, 7x (ii) – 4a, 7a

(iii) – 4x, 7xy (iv) $4x^3$, – 3xy

(v) 4x, 0

Sol. We have,

(i) $4 \times 7x = (4 \times 7) \times x = 28 \times x = 28x$

(ii) $(-4a) \times (7a) = (-4 \times 7) \times (a \times a) = -28a^2$

(iii) $(-4x) \times (7xy) = (-4 \times 7) \times (x \times xy) = -28x^{1+1}y$

$= -28x^2y$

(iv) $(4x^3) \times (-3xy) = (4 \times -3) \times (x^3 \times xy)$

$= -12(x^{3+1}y) = -12x^4y$

(v) $4x \times 0 = (4 \times 0) \times x = 0 \times x = 0$

Ex.16 Find the areas of rectangles with the following pairs of monomials as their length and breadth respectively :

(i) (x, y) (ii) (10x, 5y)

(iii) $(2x^2, 5y^2)$ (iv) $(4a, 3a^2)$

(v) (3mn, 4np)

Sol. We know that the area of a rectangle is the product of its length and breadth.

Length	Breadth	Length × Breadth = Area
(i) x	y	$x \times y = xy$
(ii) 10 x	5y	$10x \times 5y = 50xy$
(iii) $2x^2$	$5y^2$	$2x^2 \times 5y^2 = (2 \times 5) \times (x^2 \times y^2) = 10x^2y^2$
(iv) 4a	$3a^2$	$4a \times 3a^2 = (4 \times 3) \times (a \times a^2) = 12a^3$
(v) 3mn	4np	$3mn \times 4np = (3 \times 4) \times (m \times n \times n \times p) = 12mn^2p$

Ex.17 Multiply :

(i) $3ab^2c^3$ by $5a^3b^2c$

(ii) $4x^2yz$ by $-\frac{3}{2}x^2yz^2$

(iii) $-\frac{8}{5}x^2yz^3$ by $-\frac{3}{4}xy^2z$

(iv) $\frac{3}{14}x^2y$ by $\frac{7}{2}x^4y$

(v) $2.1a^2bc$ by $4ab^2$

Sol. (i) We have,

$(3ab^2c^3) \times (5a^3b^2c)$

$= (3 \times 5) \times (a \times a^3 \times b^2 \times b^2 \times c^3 \times c)$

$= 15a^{1+3}b^{2+2}c^{3+1}$

$= 15a^4b^4c^4$

(ii) We have,

$(4x^2yz) \times \left(-\frac{3}{2}x^2yz^2\right)$

$= \left(4 \times -\frac{3}{2}\right) \times (x^2 \times x^2 \times y \times y \times z \times z^2)$

$= -6x^{2+2}y^{1+1}z^{1+2} = -6x^4y^2z^3$

(iii) We have,

$\left(-\frac{8}{5}x^2yz^3\right) \times \left(-\frac{3}{4}xy^2z\right)$

$= \left(-\frac{8}{5} \times -\frac{3}{4}\right) \times (x^2 \times x \times y \times y^2 \times z^3 \times z)$

$= \frac{6}{5}x^{2+1}y^{1+2}z^{3+1} = \frac{6}{5}x^3y^3z^4$

(iv) We have,

$\left(\frac{3}{14}x^2y\right) \times \left(\frac{7}{2}x^4y\right)$

$= \left(\frac{3}{14} \times \frac{7}{2}\right) \times (x^2 \times x^4 \times y \times y)$

$= \frac{3}{4}x^{2+4}y^{1+1} = \frac{3}{4}x^6y^2$

(v) We have,

$(2.1a^2bc) \times (4ab^2)$

$= (2.1 \times 4) \times (a^2 \times a \times b \times b^2 \times c)$

$= 8.4a^{2+1}b^{1+2}c = 8.4a^3b^3c$

Ex.18 Multiply :

(i) $-6a^2bc$, $2a^2b$ and $-\frac{1}{4}$

(ii) $\frac{4}{9}a^5b^2$, $10a^3b$ and 6

(iii) 3.15x and $-23x^2y$

(iv) $-x$, x^2yz and $-\frac{3}{7}xyz^2$

Sol. (i) We have,

$(-6a^2bc) \times (2a^2b) \times \left(-\frac{1}{4}\right)$

$= \left(-6 \times 2 - \frac{1}{4}\right) \times (a^2 \times a^2 \times b \times b \times c)$

$= 3a^{2+2}b^{1+1}c = 3a^4b^2c$

(ii) We have,

$\left(\frac{4}{9}a^5b^2\right) \times (10a^3b) \times (6)$

$= \left(\frac{4}{9} \times 10 \times 6\right) \times (a^5 \times a^3 \times b^2 \times b)$

$= \frac{80}{3}a^{5+3}b^{2+1} = \frac{80}{3}a^8b^3$

(iii) We have,

$(3) \times (15x) \times (-23x^2y)$

$= (3 \times 15 - 23) \times (x \times x^2 \times y)$

$= -1035x^{1+2}y = -1035x^3y.$

(iv) We have,

$(-x) \times (x^2yz) \times \left(\frac{-3}{7}xyz^2\right)$

$= \left(-1\times\frac{-3}{7}\right) \times (x \times x^2 \times x \times y \times y \times z \times z^2)$

$= \frac{3}{7}x^{1+2+1}y^{1+1}z^{1+2} = \frac{3}{7}x^4y^2z^3$

Ex.19 Multiply each of the following mnomials :

(i) 3xyz, 5x, 0 (ii) $\frac{6}{5}$ ab, $\frac{5}{6}$ bc, $\frac{12}{9}$ abc

(iii) $\frac{3}{4}x^2yz^2$, $0.5xy^2z^2$, $1.16x^2yz^3$, 2xyz

(vi) $20x^{10}y^{20}z^{30}$, $(10xyz)^2$

(v) $(-3x^2y)$, $(4xy^2z)$, $(-xy^2z^2)$ and $\left(\frac{4}{5}z\right)$

Sol. (i) We have,

$(3xyz) \times (5x) \times 0$

$= (3\times5\times0)\times (x\times x\times y\times z)$

$= 0 \times x^2yz = 0$

(ii) We have,

$\left(\frac{6}{5}ab\right) \times \left(\frac{5}{6}bc\right) \times \left(\frac{12}{9}abc\right)$

$\left(\frac{6}{5}\times\frac{5}{6}\times\frac{12}{9}\right) + (a\times a\times b\times b\times b\times c\times c)$

$= \frac{12}{9}a^{1+1}b^{1+1+1}c^{1+1} = \frac{4}{3}a^2b^3c^2$

(iii) We have,

$\left(\frac{3}{4}x^2yz^2\right) \times (0.5xy^2z^2) \times (1.16x^2yz^3) \times (2xyz)$

$= \left(\frac{3}{4}\times0.5\times1.16\times2\right)\times (x^2 \times x \times x^2 \times x \times y \times y^2 \times y \times y \times z^2 \times z^2 \times z^3 \times z)$

$= \left(\frac{3}{4}\times\frac{5}{10}\times\frac{116}{100}\times2\right)\times (x^{2+1+2+1} \times y^{1+2+1+1} \times z^{2+2+3+1})$

$= \frac{87}{100}x^6y^5z^8$

(iv) We have, $(20x^{10}y^{20}z^{30}) \times (10xyz)^2$

$= (20x^{10}y^{20}z^{30}) \times (10xyz) \times (10xyz)$

$= (20 \times 10 \times 10) \times (x^{10} \times x \times x \times y^{20} \times y \times y \times z^{30} \times z \times z)$

$= 2000x^{10+1+1}y^{20+1+1}z^{30+1+1}$

$= 2000x^{12}y^{22}z^{32}$

(v) We have, $(-3x^2y) \times (4xy^2z) \times (-xy^2z^2) \times \left(\frac{4}{5}z\right)$

$= \left(-3 \times 4 \times -1 \times \frac{4}{5}\right) \times (x^2 \times x \times x \times y \times y^2 \times y^2 \times z \times z^2 \times z)$

$= \frac{48}{5} x^{2+1+1} y^{1+2+2} z^{1+2+1} = \frac{48}{5} x^4 y^5 z^4$

Ex.20 Express the following product as a monomial:

$(x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$

Verify the product for x = 1

Sol. We have,

$(x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$

$= \left(1 \times 7 \times \frac{1}{5} \times -6\right) \times (x^3 \times x^5 \times x^2 \times x^4)$

$= -\frac{42}{5} x^{3+5+2+4} = -\frac{42}{5} x^{14}$

Verification : For x = 1, we have

L.H.S. $= (x^3) \times (7x^5) \times \left(\frac{1}{5}x^2\right) \times (-6x^4)$

$= (1)^3 \times \{7 \times (1^5)\} \times \left\{\frac{1}{5} \times (1)^2\right\} \times \{-6 \times (1)^4\}$

$= 1 \times 7 \times \frac{1}{5} \times -6 = -\frac{42}{5}$

and,

R.H.S. $= -\frac{42}{5} \times (1)^{14} = -\frac{42}{5}$

$\therefore$ L.H.S. = R.H.S.

Ex.21 Find the value of $(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$ for a = 1 and $b = \frac{1}{2}$.

Sol. We have,

$(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$

$= (5 \times -10 \times -2.1) \times (a^6 \times a \times a^2 \times b^2 \times b^3)$

$= \left(5 \times -10 \times -\frac{21}{10}\right) \times (a^6 \times a \times a^2 \times b^2 \times b^3)$

$= 105\, a^{6+1+2} b^{2+3} = 105a^9b^5$

Putting a = 1 and $b = \frac{1}{2}$, we have

$105a^9b^5 = 105 \times (1)^9 \times \left(\frac{1}{2}\right)^5$

$= 105 \times 1 \times \frac{1}{32} = \frac{105}{32}$

MULTIPLICATION OF A MONOMIAL & A BINOMIAL

Ex.22 Multiply : 2x by (3x + 5y)

Sol. We have,

$^{2} + 10xy$

Ex.23 Multiply : (7xy + 5y) by 3xy

Sol. We have,

$(7xy + 5y) \times 3xy$

$= 7xy \times 3xy + 5y \times 3xy$

$= 21x^{1+1}y^{1+1} + 15xy^{1+1} = 21x^2y^2 + 15xy^2$

Ex.24 Multiply : $-\frac{3ab^2}{5}$ by $\left(\frac{2a}{3} - b\right)$

Sol. We have,

$\left(-\frac{3ab^2}{5}\right) \times \left(\frac{2a}{3} - b\right)$

$= \left(-\frac{3ab^2}{5}\right) \times \frac{2a}{3} - \left(-\frac{3ab^2}{5}\right) \times b$

$= -\frac{3}{5} \times \frac{2}{3} a^{1+1}b^2 + \frac{3}{5}ab^{2+1} = -\frac{2}{5}a^2b^2 + \frac{3}{5}ab^3$

Ex.25 Multiply : $\left(3x - \frac{4}{5}y^2x\right)$ by $\frac{1}{2}xy$.

Sol.

Horizontal method

We have,

$\left(3x - \frac{4}{5}y^2x\right) \times \frac{1}{2}xy$

$= 3x \times \frac{1}{2}xy - \frac{4}{5}y^2x \times \frac{1}{2}xy$

$= \left(3 \times \frac{1}{2}\right) \times x \times x \times y -$

$\left(\frac{4}{5} \times \frac{1}{2}\right) \times y^2 \times y \times x \times x$

$= \frac{3}{2}x^2y - \frac{2}{5}y^3x^2 = \frac{3}{2}x^2y - \frac{2}{5}x^2y^3$

Column method

We have,

$3x - \frac{4}{5}y^2x$

$\times \frac{1}{2}xy$

$\frac{3}{2}x^2y - \frac{2}{5}x^2y^3$

Ex.26 Determine each of the following products and find the value of each for x = 2, y = 1.15, z = 0.01.

(i) $27x^2(1 - 3x)$ (ii) $xz(x^2 + y^2)$

(iii) $z^2(x - y)$ (iv) $(2z - 3x) \times (-4y)$

Sol. (i) We have,

$27x^2(1 - 3x)$

$= 27x^2 \times (1 - 3x)$

$= 27x^2 \times 1 - 27x^2 \times 3x$ [Expanding the bracket]

$= 27x^2 - 81x^3$

Putting x = 2, we have

$27x^2 (1 - 3x)$

$= 27\times (2)^2 \times (1 - 3\times2) = 27 \times 4 \times (1 - 6)$

$= 27 \times 4 \times - 5 = - 540$

(ii) We have,

$xz(x^2 + y^2)$

$= xz \times (x^2 + y^2)$

$= xz \times x^2 + xz \times y^2 = x^3z + xy^2z$

Putting x = 2, y = 1.15 and z = 0.01, we get

$xz (x^2 + y^2)$

$= 2 \times 0.01 \times \{(2)^2 + (1.15)^2\}$

$= 0.02 \times (4 + 1.3225) = 0.02 \times 5.3225 = 0.106450$

(iii) We have,

$z^2(x - y)$

$= z^2 \times (x - y)$

$= z^2 \times x - z^2 \times y = z^2x - z^2y$

Putting x = 2 y = 1.15 and z = 0.01, we get

$z^2(x - y)$

$= (0.01)^2 \times (2 - 1.15)$

$= (0.0001) \times (0.85) = 0.000085$

(vi) We have,

$(2z - 3x) \times (- 4y)$

$= (2z) \times (- 4y) - 3x \times (- 4y) = - 8zy + 12xy$

Putting x = 2, y= 1.15 and z = 0.01, we have

$(2z - 3x) \times - 4y$

$= [(2 \times 0.01) - (3 \times 2)] \times (-4 \times 1.15)$

$= (0.02 - 6) \times (-4.6) = - 5.98 \times -4.6 = 27.508$

Ex.27 Simplify the expression and evaluate them as directed :

(i) $x(x - 3) + 2$ for $x = 1$

(ii) $3y (2y - 7) - 3(y-4) - 63$ for $y = - 2$

Sol. (i) We have,

$x(x - 3) + 2 = x^2 - 3x + 2$

For x = 1, we have

$x^2 - 3x + 2 = (1)^2 - 3 \times 1 + 2 = 1 - 3 + 2$

$= 3 -3 = 0$

(ii) We have,

$3y(2y - 6) - 3 (y - 4) - 63$

$= (6y^2 - 21y) - (3y - 4) - 63$

$= 6y^2 - 21y - 3y + 12 - 63$

$= 6y^2 - 24y - 51$

For y = –2, we have

$6y^2 - 24y - 51 = 6 \times (-2)^2 - 24(-2) - 51$

$= 6 \times 4 + 24 \times 2 - 51 = 24 + 48 - 51 = 72 - 51= 21$

Ex.28 Subtract 3pq (p – q) from 2pq(p + q)

Sol. (i) We have,

$3pq\ (p - q) = 3p^2q - 3pq^2$

and, $2pq\ (p + q) = 2p^2q + 2pq^2$

Subtraction :

$2p^2q + 2pq^2$

$3p^2q - 3pq^2$

$\underline{- \quad +}$

$- p^2q + 5pq^2$

Ex.29 Add : (i) p(p – q), q (q – r) and r(r – p)

(ii) 2x (z – x – y) and 2y(z – y – x)

Sol. (i) We have,

$p\ (p - q) + q\ (q - r) + r\ (r - p)$

$= p^2 - pq + q^2 - qr + r^2 - rp$

$= p^2 + q^2 + r^2 - pq - qr - rp$

(ii) We have,

$2x(z - x - y) + 2y\ (z - y - x)$

$= 2xz - 2x^2 - 2xy + 2yz - 2y^2 - 2xy$

$= 2xz - 2x^2 - 4xy + 2yz - 2y^2$

Ex.30 Simplify each of the following expressions :

(i) $15a^2 - 6a\ (a - 2) + a(3 + 7a)$

(ii) $x^2(1 - 3y^2) + x(xy^2 - 2x) - 3y(y - 4x^2y)$

(iii) $4st(s - t) - 6s^2(t - t^2) - 3t^2(2s^2 - s) + 2st(s - t)$

Sol. (i) We have,

$15a^2 - 6a(a - 2) + a(3 + 7a)$

$= 15a^2 - 6a^2 + 12a + 3a + 7a^2$

$= 15a^2 - 6a^2 + 7a^2 + 12a + 3a = 16a^2 + 15a$

(ii) We have,

$x^2(1 - 3y^2) + x(xy^2 - 2x) - 3y(y - 4x^2y)$

$= x^2 \times 1 - 3y^2 \times x^2 + x \times xy^2 - x \times 2x - 3y$

$\times y + 3y \times 4\ x^2y$

$= x^2 - 3x^2y^2 + x^2y^2 - 2x^2 - 3y^2 + 12x^2y^2$

$= (x^2 - 2x^2) + (-3x^2y^2 + x^2y^2 + 12x^2y^2) - 3y^2$

$= -x^2 + 10x^2y^2 - 3y^2$

(iii) $4st(s - t) - 6s^2(t - t^2) - 3t^2(2s^2 - s) + 2st(s - t)$

$= 4st \times s - 4st \times t - 6s^2 \times t + 6s^2 \times t^2$

$- 3t^2 \times 2s^2 + 3t^2 \times s + 2st \times s - 2st \times t$

$= 4s^2t - 4st^2 - 6s^2t + 6s^2t^2 - 6s^2t^2$

$+ 3st^2 + 2s^2t - 2st^2$

$= (4s^2t - 6s^2t + 2s^2t) + (-4st^2 + 3st^2 - 2st^2)$

$+ (6s^2t^2 - 6s^2t^2)$

$= - 3st^2$

MULTIPLICATION OF TWO BINOMIALS

Ex.31 Multiply (3x + 2y) and (5x + 3y).

Sol. We have,

$(3x + 2y) \times (5x + 3y)$

$= 3x \times (5x + 3y) + 2y \times (5x + 3y)$

$= (3x \times 5x + 3x \times 3y) + (2y \times 5x + 2y + 3y)$

$= (15x^2 + 9xy) + (10xy + 6y^2)$

$= 15x^2 + 9xy + 10xy + 6y^2$

$= 15x^2 + 19xy + 6y^2$

Ex.32 Multiply (2x + 3y) and (4x – 5y)

Sol. We have,

$(2x + 3y) \times (4x - 5y)$

$= 2x \times (4x - 5y) + 3y \times (4x - 5y)$

$= (2x \times 4x - 2x \times 5y) + (3y \times 4x - 3y \times 5y)$

$= (8x^2 - 10xy) + (12xy - 15y^2)$

$= 8x^2 - 10xy + 12xy - 15y^2$

$= 8x^2 + 2xy - 15y^2$

Ex.33 Multiply (7a + 3b) and (2a + 3b) by column method.

Sol. We have,

$$\begin{array}{ll} \quad 7a + 3b & \\ \times \ 2a + 3b & \\ \hline 14a^2 + 6ab & \text{Multiplying } 7a + 3b \text{ by } 2a \\ \quad\quad + 21ab + 9b^2 & \text{Multiplying } 7a + 3b \text{ by } 3b \\ \hline 14a^2 + 27ab + 9b^2 & \text{Adding the like term} \end{array}$$

Ex.34 Multiply (7x–3y) by (4x–5y) by column method.

Sol. We have,

$$\begin{array}{ll} \quad 7x - 3y & \\ \times \ 4x - 5y & \\ \hline 28x^2 - 12xy & \text{Multiplying } 7x - 3y \text{ by } 4x \\ \quad\quad - 35xy + 15y^2 & \text{Multiplying } 7x - 3y \text{ by } -5y \\ \hline 28x^2 - 47xy + 15y^2 & \text{Adding the like terms} \end{array}$$

Ex.35 Multiply (0.5x – y) by (0.5x + y)

Sol. Horizontal Method:

We have,

$(0.5x - y) \times (0.5x + y)$

$= 0.5x\,(0.5x + y) - y\,(0.5x + y)$

$= 0.5x \times 0.5x + 0.5x \times y - y \times 0.5x - y \times y$

$= 0.25x^2 + 0.5xy - 0.5xy - y^2$

$= 0.25x^2 - y^2$

Column method:

We have,

$$\begin{array}{l} \quad 0.5x - y \\ \times \ 0.5x + y \\ \hline 0.25x^2 - 0.5xy \qquad \text{Multiplying } 0.5x - y \text{ by } 0.5x \\ \qquad\quad + 0.5xy - y^2 \qquad \text{Multiplying } 0.5x - y \text{ by } y \\ \hline 0.25x^2 - y^2 \qquad \text{Adding the like terms} \end{array}$$

Ex.36 Multiplying $\left(4x + \frac{3y}{5}\right)$ and $\left(3x - \frac{4y}{5}\right)$

Sol. Horizontal Method :

$\left(4x + \frac{3y}{5}\right) \times \left(3x - \frac{4y}{5}\right)$

$= 4x \times \left(3x - \frac{4y}{5}\right) + \frac{3y}{5} \times \left(3x - \frac{4y}{5}\right)$

$= 4x \times 3x - 4x \times \frac{4y}{5} + \frac{3y}{5} \times 3x - \frac{3y}{5} \times \frac{4y}{5}$

$= 12x^2 - \frac{16}{5}xy + \frac{9}{5}xy - \frac{12}{25}y^2$

$= 12x^2 - \frac{7}{5}xy - \frac{12}{25}y^2$

Column method:

We have,

$$\begin{array}{l} \quad 4x + \frac{3y}{5} \\ \times \ 3x - \frac{4y}{5} \\ \hline 12x^2 + \frac{9}{5}xy \qquad \text{Multiplying } 4x + \frac{3y}{5} \text{ by } 3x. \\ \qquad - \frac{16}{5}xy - \frac{12}{25}y^2 \qquad \text{Multiplying } 4x + \frac{3y}{5} \text{ by } -\frac{4y}{5}. \\ \hline 12x^2 - \frac{7}{5}xy - \frac{12}{25}y^2 \qquad \text{Adding the like terms} \end{array}$$

Ex.37 Find the value of the following products:

(i) $(x + 2y)(x - 2y)$ at $x = 1, y = 0$

(ii) $(3m - 2n)(2m - 3n)$ at $m = 1, n = -1$

(iii) $(4a^2 + 3b)(4a^2 + 3b)$ at $a = 1, b = 2$

Sol. (i) We have,

$(x + 2y)(x - 2y)$

$= x(x - 2y) + 2y(x - 2y)$

$= x \times x - x \times 2y + 2y \times x - 2y \times 2y$

$= x^2 - 2xy + 2yx - 4y^2$

$= x^2 - 4y^2$

When $x = 1, y = 0$, we get

$(x + 2y)(x - 2y)$

$= x^2 - 4y^2 = (1)^2 - 4 \times (0)^2 = 1 - 0 = 1.$

(ii) We have,

$(3m - 2n)(2m - 3n)$

$= 3m(2m - 3n) - 2n(2m - 3n)$

$= 3m \times 2m - 3m \times 3n - 2n \times 2m + 2n \times 3n$

$= 6m^2 - 9mn - 4mn + 6n^2$

$= 6m^2 - 13mn + 6n^2$

When $m = 1$, $n = -1$, we get

$(3m - 2n)(2m - 3n)$

$= 6m^2 - 13mn + 6n^2$

$= 6 \times (1)^2 - 13 \times 1 \times (-1) + 6 \times (-1)^2 = 6 + 13 + 6 = 25$

(iii) We have

$(4a^2 + 3b)(4a^2 + 3b)$

$= 4a^2 \times (4a^2 + 3b) + 3b \times (4a^2 + 3b)$

$= 4a^2 \times 4a^2 + 4a^2 \times 3b + 3b \times 4a^2 + 3b \times 3b$

$= 16a^4 + 12a^2b + 12a^2b + 9b^2$

$= 16a^4 + 24a^2b + 9b^2$

When, $a = 1$, $b = 2$, we get

$(4a^2 + 3b)(4a^2 + 3b)$

$= 16a^4 + 24a^2b + 9b^2$

$= 16 \times (1)^4 + 24 \times (1)^2 \times 2 + 9 \times (2)^2$

$= 16 + 48 + 36 = 100$

Ex.38 Simplify the following :

(i) $(2x + 5)(3x - 2) + (x + 2)(2x - 3)$

(ii) $(3x + 2)(2x + 3) - (4x - 3)(2x - 1)$

(iii) $(2x + 3y)(3x + 4y) - (7x + 3y)(x + 2y)$

Sol. (i) We have,

$(2x + 5)(3x - 2) + (x + 2)(2x - 3)$

$= 2x(3x - 2) + 5(3x - 2) + x(2x - 3) + 2(2x - 3)$

$= 6x^2 - 4x + 15x - 10 + 2x^2 - 3x + 4x - 6$

$= (6x^2 + 2x^2) + (-4x + 15x - 3x + 4x) + (-10 - 6)$

$= 8x^2 + 12x - 16$

(ii) We have,

$(3x + 2)(2x + 3) - (4x - 3)(2x - 1)$

$= \{3x(2x+3) + 2(2x + 3)\} - \{4x(2x - 1) - 3(2x - 1)\}$

$= (6x^2 + 9x + 4x + 6) - (8x^2 - 4x - 6x + 3)$

$= (6x^2 + 13x + 6) - (8x^2 - 10x + 3)$

$6x^2 + 13x + 6 - 8x^2 + 10x - 3$

$= -2x^2 + 23x + 3$

(iii) We have,

$(2x + 3y)(3x + 4y) - (7x + 3y)(x + 2y)$

$= \{2x(3x + 4y) + 3y(3x + 4y) - 7x(x + 2y) + 3y(x + 2y)\}$

$= (6x^2 + 8xy + 9xy + 12y^2) - (7x^2 + 14xy + 3xy + 6y^2)$

$= (6x^2 + 17xy + 12y^2) - (7x^2 + 17xy + 6y^2)$

(iii) $(x-1)(x+1)(x^2+1)(x^4+1)$

(iv) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

(v) $\left(x-\frac{y}{5}-1\right)\left(x+\frac{y}{5}+1\right)$

Sol. (i) We have,

$(x+2)(x-2)(x^2+4)$

$= \{(x+2)(x-2)\}(x^2+4)$

[By associativity of multiplication] $= (x^2-2^2)(x^2+4)$ [$\because (a+b)(a-b)=a^2-b^2$]

$= (x^2-4)(x^2+4)$

$= (x^2)^2-4^2$ [$\because (a+b)(a-b)=a^2-b^2$]

$= x^4-16$

(ii) We have,

$(2x+3y)(2x-3y)(4x^2+9y^2)$

$= \{(2x+3y)(2x-3y)\}(4x^2+9y^2)$

$= \{(2x+3y)(2x-3y)\}(4x^2+9y^2)$

$= \{(2x)^2-(3y)^2\}(4x^2+9y^2)$

[Using : $(a+b)(a-b)=a^2-b^2$]

$= (4x^2-9y^2)(4x^2+9y^2)$

$= (4x^2)^2-(9y^2)^2$

[Using : $(a+b)(a-b)=a^2-b^2$]

$= 16x^4-81y^4$.

(iii) We have,

$(x-1)(x+1)(x^2+1)(x^4+1)$

$= \{(x-1)(x+1)\}(x^2+1)(x^4+1)$

$= (x^2-1)(x^2+1)(x^4+1)$

$= \{(x^2-1)(x^2-1)\} + (x^4+1)$

$= \{(x^2)^2-1^2\}(x^4+1)$

$= (x^4-1)(x^4+1)$

$= \{(x^4)^2-1^2\}$

$= x^8-1$

(iv) We have

$\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$= \left\{\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\right\}\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$= \left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$= \left\{(x^2)^2-\left(\frac{1}{x^2}\right)^2\right\}\left(x^4+\frac{1}{x^4}\right)$

$= \left(x^4-\frac{1}{x^4}\right)\left(x^4+\frac{1}{x^4}\right)$

$= (x^4)^2 - \left(\frac{1}{x^4}\right)^2$

$= x^8 - \frac{1}{x^8}$

(v) We have,

$\left(x - \frac{y}{5} - 1\right)\left(x + \frac{y}{5} + 1\right)$

$= \left\{x - \left(\frac{y}{5} + 1\right)\right\}\ \left\{x + \left(\frac{y}{5} + 1\right)\right\}$

$= x^2 + \left(\frac{y}{5} + 1\right)^2$

$= x^2 - \left(\frac{y^2}{25} + \frac{2y}{5} + 1\right)$

$= x^2 - \frac{y^2}{25} - \frac{2y}{5} - 1$

Ex.50 Prove that:

$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$
$= (a - b)^2 + (b - c)^2 + (c - a)^2$

Sol. We have,

$LHS = 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$
$= (a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)$

[Re-arranging the terms]

$= (a - b)^2 + (b - c)^2 + (c - a)^2$
= R.H.S.

Hence, $2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$
$= (a - b)^2 + (b - c)^2 + (c - a)^2$

Ex.51 If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, prove that $a = b = c$.

Sol. We have,

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 2 \times 0$

[Multiplying both sides by 2]

$\Rightarrow (a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ac + a^2) = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\Rightarrow a - b = 0,\ b - c = 0,\ c - a = 0$

[∵ Sum of positive quantities is zero if and only if each quantity is zero]

$\Rightarrow a = b,\ b = c$ and $c = a$

$\Rightarrow a = b = c.$

Ex.52 Using the fomulae for squaring a binomial, evaluate the following :

(i) $(101)^2$ (ii) $(99)^2$ (iii) $(93)^2$

Sol. We have,

(i) $(101)^2 = (100 + 1)^2$

$= (100)^2 + 2 \times 100 \times 1 + (1)^2$

[Using : $(a + b^2) = a^2 + 2ab + b^2$]

$= 10000 + 200 + 1$

$= 10201$

(ii) $(99)^2 = (100 - 1)^2$

$= (100)^2 - 2 \times 100 \times 1 + (1)^2$

[Using : $(a - b)^2 = a^2 - 2ab + b^2$] $= 10000 - 200 + 1$

$= 9801$

(iii) $(93)^2 = (90 + 3)^2$

$= (90)^2 + 2 \times 90 \times 3t\ (3)^2$

$= 8100 + 540 + 9 = 8649$

Ex.53 Find the value of x, if

(i) $6x = 23^2 - 17^2$

(ii) $4x = 98^2 - 88^2$

(iii) $25x = 536^2 - 136^2$

Sol. (i) We have,

$6x = 23^2 - 17^2$

$\Rightarrow 6x = (23 + 17) \times (23 - 17)$

[Using : $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 6x = 40 \times 6$

$\Rightarrow \frac{6x}{6} = \frac{40 \times 6}{6}$ [Dividing both sides by 6]

$\Rightarrow x = 40$

(ii) We have,

$4x = 98^2 - 88^2$

$\Rightarrow 4x = (98 + 88) \times (98 - 88)$

[Using : $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow 4x = 186 \times 10$

$\Rightarrow \frac{4x}{4} = \frac{186 \times 10}{4}$ [Dividing both sides by 4]

$\Rightarrow x = \frac{1860}{4}$

$\Rightarrow x = 465$

(iii) We have,

$25x = 536^2 - 136^2$

$\Rightarrow 25x = (536 + 136) \times (536 - 136)$

[Using : $(a^2 - b^2) = (a + b)(a - b)$]

$\Rightarrow 25x = 672 \times 400$

$\Rightarrow \frac{25x}{25} = \frac{672 \times 400}{25}$ [Dividing both sides by 25]

$\Rightarrow x = 672 \times 16$

$\Rightarrow x = 10752$

(iv) $(x - 1)^2 - (x - 2)^2$

$= \{(x - 1) + (x - 2)\}\{(x - 1) - (x - 2)\}$

$= (2x - 3)(x - 1 - x + 2)$

$= (2x - 3) \times 1$

$= 2x - 3$

Ex.68 Factorize each of the following algebraic expression:

(i) $x^4 - 81y^4$ (ii) $2x^5 - 2x$

(iii) $3x^4 - 243$ (iv) $2 - 50x^2$

(v) $x^8 - y^8$ (vi) $a^{12}x^4 - a^4x^{12}$

Sol. (i) $x^4 - 81y^4 = (x^2)^2 - (9y^2)^2$

$= (x^2 - 9y^2)(x^2 + 9y^2)$

$= \{x^2 - (3y)^2\}(x^2 + 9y^2)$

$= (x - 3y)(x + 3y)(x^2 - 9y^2)$

(ii) $2x^5 - 2x = 2x(x^4 - 1)]$

$= 2x\{(x^2)^2 - 1^2\}$

$= 2x(x^2 - 1)(x^2 + 1)$

$= 2x(x - 1)(x + 1)(x^2 + 1)$

(iii) $3x^4 - 243 = 3(x^4 - 81)$

$= 3\{(x^2)^2 - 9^2\} = 3(x^2 - 9)(x^2 + 9)$

$= 3(x^2 - 3^2)(x^2 + 9)$

$= 3(x + 3)(x - 3)(x^2 + 9)$

(iv) $2 - 50x^2 = 2\{1 - 25x^2\}$

$= 2\{1^2 - (5x)^2\}$

$= 2(1 - 5x)(1 + 5x)$

(v) $x^8 - y^8 = \{(x^4)^2 - (y^4)^2\}$

$= (x^4 - y^4)(x^4 + y^4)$

$= \{(x^2)^2 - (y^2)^2\}(x^4 + y^4)$

$= (x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$

$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

$= (x - y)(x + y)(x^2 + y^2)$
$\{(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2\}$

$= (x - y)(x + y)(x^2 + y^2)\left\{(x^2 + y^2)^2 - \left(\sqrt{2}xy\right)^2\right\}$

$= (x - y)(x + y)(x^2 + y^2)\left(x^2 + y^2 - \sqrt{2}xy\right)$
$\left(x^2 + y^2 + \sqrt{2}xy\right)$

(vi) $a^{12}x^4 - a^4x^{12} = a^4x^4\left(a^8 - x^8\right)$ $= a^4x^4\{(a^4)^2 - (x^4)^2\}$

$= a^4x^4(a^4 + x^4)(a^4 - x^4)$

$= a^4x^4(a^4 + x^4)\{(a^2)^2 - (x^2)^2\}$

$= a^4x^4(a^4 + x^4)(a^2 + x^2)(a^2 - x^2)$

$= a^4x^4(a^4 + x^4)(a^2 + x^2)(a + x)(a - x)$

FACTORIZATION OF ALGEBRAIC EXPRESSIONS EXPRESSIBLE AS A PERFECT SQUARE

(i) $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$

(ii) $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

Ex.69 Factorize :

(i) $x^2 + 8x + 16$ (ii) $4a^2 - 4a + 1$

Sol. We have,

(i) $x^2 + 8x + 16 = x^2 + 2 \times x \times 4 + 4^2$

$= (x + 4)^2$ [Using : $a^2 + 2ab + b^2 = (a + b)^2$]

$= (x + 4)(x + 4)$

(ii) $4a^2 - 4a + 1 = (2a)^2 - 2 \times 2a \times 1 + (1)^2$

$= (2a - 1)^2$ [Using : $a^2 - 2ab + b^2 = (a - b)^2$]

$= (2a - 1)(2a - 1)$

Ex.70 Factorize :

(i) $4x^2 + 12xy + 9y^2$

(ii) $x^4 - 10x^2y^2 + 25y^4$

(iii) $a^4 - 2a^2b^2 + b^4$

Sol. We have,

(i) $4x^2 + 12xy + 9y^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2$

$= (2x + 3y)^2$

$= (2x + 3y)(2x + 3y)$

(ii) $x^4 - 10x^2y^2 + 25y^4$

$= (x^2)^2 - 2 \times x^2 \times 5y^2 + (5y^2)^2$

$= (x^2 - 5y^2)^2$

$= (x^2 - 5y^2)(x^2 - 5y^2)$

(iii) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$

$= (a^2 - b^2)^2$

$= \{(a - b)(a + b)\}^2 = (a - b)^2 (a + b)^2$

Ex.71 Factorize each of the following expressions:

(i) $x^2 - 2xy + y^2 - x + y$

(ii) $4a^2 + 12ab + 9b^2 - 8a - 12b$

(iii) $a^2 + b^2 - 2(ab - ac + bc)$

Sol. (i) $x^2 - 2xy + y^2 - x + y = (x^2 - 2xy + y^2)$

$= (x - y)^2 - (x - y)$

$= (x - y)\{(x - y) - 1\}$

$= (x - y)(x - y - 1)$

(ii) $4a^2 + 12ab + 9b^2 - 8a - 12b$

$= (2a)^2 + 2 \times 2a \times 3b + (3b)^2 - 4(2a + 3b)$

$= (2a + 3b)^2 - 4(2a + 3b)$

$= (2a + 3b)(2a + 3b - 4)$

(iii) $a^2 + b^2 - 2(ab - ac + bc)$

$= a^2 + b^2 - 2ab + 2ac - 2bc$

$= (a - b)^2 + 2c\ (a - b)$

$= (a - b)\ \{(a - b) + 2c\}$

$= (a - b)\ (a - b + 2c)$

Ex.73 Factorize each of the following expressions:

(i) $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$

(ii) $25x^2 - 10x + 1 - 36y^2$

(iii) $1 - 2ab - (a^2 + b^2)$

Sol. (i) $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$

$= (x^2 + 2xy + y^2) - (a^2 - 2ab + b^2)$

$= (x + y)^2 - (a - b)^2$

$= \{(x + y) + (a - b)\}\ \{(x + y) - (a - b)\}$

$= (x + y + a - b)\ (x + y - a + b)$

(ii) $25x^2 - 10x + 1 - 36y^2$

$= (5x)^2 - 2\times 5x \times 1 + 1^2 - (6y)^2$

$= (5x - 1)^2 - (6y)^2$

$= (5x - 1 + 6^2)\ (5x - 1 - 6y)$

(iii) $1 - 2ab - (a^2 + b^2) = 1 - (2ab + a^2 + b^2)$

$= 1 - (a + b)^2$

$= \{1 + (a + b)\}\ \{1 - (a + b)\}$

$= (1 + a + b)\ (1 - a - b)$

Ex.74 Factorize:

(i) $x^2 + 8x + 15$ (ii) $x^4 + x^2 + 1$ (iii) $x^4 + 4$

Sol. We have,

(i) $x^2 + 8x + 15 = (x^2 + 8x + 16) - 1$

[Replacing 15 by 16 – 1]

$= \{(x)^2 + 2 \times x \times 4 + 4^2\} - 1$

$= (x + 4)^2 - 1^2$

$= \{x + 4 + 1\}\ \{(x + 4) - 1\}$

$= (x + 5)\ (x + 3)$

(ii) $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$

[Adding and subtracting x^2]

$= (x^4 + 2x^2 + 1) - x^2$

$= \left((x^2)^2 + 2\times x^2\times 1 + 1^2\right) - x^2$

$= (x^2 + 1)^2 - x^2$

$= \{(x^2 + 1) + x\}\ \{(x^2 + 1) - x\}$

$= (x^2 + x + 1)\ (x^2 - x + 1)$

(iii) $x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2$

[Adding and subtracting $4x^2$]

$= \{(x^2)^2 + 2 \times x^2 \times 2 + 2^2\} - 4x^2$

$= (x^2 + 2)^2 - (2x)^2$

$= \{(x^2 + 2) + 2x\}\ \{(x^2 + 2) - 2x\}$

$= (x^2 + 2x + 2)\ (x^2 - 2x + 2)$

$$\left[\text{Adding and subtracting } \left(\frac{1}{2}\text{Coeff. of x}\right)^2 \text{ i.e., } \left(\frac{1}{a}\right)^2\right]$$

$$= -\ 2\left[\left\{x^2 + 2\times\frac{1}{4}\times x + \left(\frac{1}{4}\right)^2\right\} - \left\{\frac{1}{16} + 3\right\}\right]$$

$$= -2\left\{\left(x + \frac{1}{4}\right)^2 - \frac{49}{16}\right\}$$

$$= -\ 2\left\{\left(x + \frac{1}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right\}$$

$$= -\ 2\ \left\{\left(x + \frac{1}{4}\right) - \frac{7}{4}\right\}\left\{\left(x + \frac{1}{4}\right) + \frac{7}{4}\right\}$$

$$= -\ 2\left(x + \frac{1}{4} - \frac{7}{4}\right)\left(x + \frac{1}{4} + \frac{7}{4}\right)$$

$$= -\ 2\ \left(x - \frac{3}{2}\right)\ (x + 2)$$

$$= (-\ 2x + 3)\ (x + 2)$$

Polynomials : An algebraic expression in which the variables involved have only non-negative integral powers, is called a polynomial.

Degree of a polynomial in one variable: In a polynomial in one variable, the highest power of the variable is called degree.

Degree of a polynomial in two variable: In a polynomial in more than one variable the sum of the powers of the variables in each term is computed and the highest sum so obtained is called the degree of the polynomial.

Constant Polynomial : A polynomial consisting of a constant term only is called a constant polynomial. The degree of a constant polynomial is zero.

Linear Polynomial : A polynomial of degree 1 is called a linear polynomial.

Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial.

Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial.

Biquadratic Polynomials : A polynomial of degree 4 is called a biquadratic polynomial.

Ex.82 $\frac{2}{3}x^2 - \frac{3}{2}x^2 + x - 5$ is a polynomial in variables x whereas $\frac{1}{2}x^3 - 3x^2 + 5x^{1/2} + x - 1$ is not a polynomial, because it contains a term $5x^{1/2}$ which contains $\frac{1}{2}$ as the power of variable x, which is not a non-negative integer.

Ex.83 $3 - 2x^2 + 4x^2y + 8y - \frac{5}{3}xy^2$ is a polynomial in two variables x and y.

Ex.84 (i) $2x + 3$ is a polynomial in x of degree 1.

(ii) $2x^2 - 3x + \frac{7}{5}$ is polynomial in x of degree 2.

(iii) $\frac{2}{3}a^2 - \frac{7}{2}a^2 + 4$ is a polynomial in a dgree 3.

Ex.85 $3x^4 - 2x^3y^2 + 7xy^3 - 9x + 5y + 4$ is a polynomial in x and y of degree 5, whereas $\frac{1}{2} - 3x + 7x^2y - \frac{3}{4}x^2y^2$ is a polynomial of degree 4 in x and y.

Ex.86 $2 - \frac{3}{4}x, \frac{1}{2} + \frac{3}{5}y, 2 + 3a$ etc. are linear polynomials.

Ex.87 $2x^2 - 3x + 4,\ 2 - x + x^2,\ 2y^2 - \frac{3}{2}y + \frac{1}{4}$ are quadratic polynomials.

Ex.88 $x^3 - 7x + 2x - 3,\ 2 + \frac{1}{2}y - \frac{3}{2}y^2 + 4y^3$ are cubic polynomial.

Ex.89 $3x^4 - 7x^3 + x^2 - x + 9,\ 4 - \frac{2}{3}x^2 + \frac{3}{5}x^4$ are biquadratic polynomials.

DIVISION OF A MONOMIAL BY A MONOMIAL

While dividing a monomial by a monomial, we follow the following two rules:

Rule-1 Coefficient of the quotient of two monomial is equal to the quotient of their coefficients.

Rule-2 The variable part in the quotient of two monomials is equal to the quotient of the variables in the given monomials.

Ex.90 Divide :

(i) $12x^3y^3$ by $3x^2y$ (ii) $-15a^2bc^3$ by $3ab$

Sol. (i) We have,

$$\frac{12x^3y^2}{3x^2y} = \frac{12 \times x \times x \times x \times y \times y}{3 \times x \times x \times y} = 4 \times x \times y = 4xy$$

(ii) We have,

$$\frac{-15a^2bc^3}{3ab} = \frac{-15 \times a \times a \times b \times c \times c \times c}{3 \times a \times b} = -5ac^3$$

DIVISION OF A POLYNOMIAL BY A MONOMIAL

Step I Obtain the polynomial (dividend) and the monomial (divisor).

Step II Arrange the terms of the dividend in descending order of their degrees. For example, write $7x^2 + 4x - 3 + 5x^3$ as $5x^2 + 7x^2 + 4x - 3$.

Step III Divided each term of the plynomial by the given monomial by using the rules of division of a monomial by a monomial.

Ex.91 Divide :

(i) $9m^5 + 12m^4 - 6m^2$ by $3m^2$

(ii) $24x^3y + 20x^2y^2 - 4xy$ by $2xy$

Sol. (i) We have,

$$\frac{9m^5 + 12m^4 - 6m^2}{3m^2} = \frac{9m^5}{3m^2} + \frac{12m^4}{3m^2} - \frac{6m^2}{3m^2}$$

$$= 3m^3 + 4m^2 - 2$$

(ii) We have,

$$\frac{24x^3y + 20x^2y^2 - 4xy}{2xy}$$

$$= \frac{24x^3y}{2xy} + \frac{20x^2y^2}{2xy} - \frac{4xy}{2xy} = 12x^2 + 10xy - 2$$

DIVISION OF A POLYNOMIAL BY A BINOMIAL BY USING LONG DIVISION

Step I Arrange the terms of the dividend and divisor in descending order of their degrees.

Step II Divide the first term of the dividend by the first term of the divisor to otbain the first term of the quotient.

Step III Multiply the divisor by the first term of the quotient and subtract the result from the dividend to obtain the remainder.

Step IV Consider the remainder (if any) as dividend and repeat step II to obtain the second term of the quotient.

Step V Repeat the above process till we obtain a remainder which is either zero or a polynomial of degree less than that of the divisor.

Ex.92 Divide $6 + x - 4x^2 + x^3$ by $x - 3$.

Sol. We go through the following steps to perform the division:

Step I We write the terms of the dividend as well as of divisor in descending order of their degress. Thus, we write

$= 6 + x - 4x^2 + x^3$ as $x^3 - 4x^2 + x + 6$ and $x - 3$ as $x - 3$

Step II We divide the first term x^3 of the dividend by the first term x of the divisor and obtain $\frac{x^3}{x} = x^2$ as the first term of the quotient.

Step III We multiply the divisor $x - 3$ by the first term x of the quotient and subtract the result from the dividend $x^3 - 4x^2 + x + 6$. We obtain $-x^2 + x + 6$ as the remainder.

$$\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x - 3 & x^3 - 4x^2 + x + 6 \\
 & x^3 - 3x^2 \\
 & - \quad + \\
\hline
 & -x^2 - x + 6 \\
 & -x^2 + 3x \\
 & + \quad - \\
\hline
 & -2x + 6 \\
 & -2x + 6 \\
 & + \quad - \\
\hline
 & 0
\end{array}$$

Step IV We take $-x^2 + x + 6$ as the new dividend and repeat step II to obtain the second term $\left(\frac{x^2}{x} =\right)$ $-x$ of the quotient.

Step V We multiply the divisor $x - 3$ by the second term $-x$ of the quotient and subtract the result $-x^2 + 3x$ from the new dividend. We obtain $-2x + 6$ as the remainder.

Step VI Now we treat $-2x + 6$ as the new dividend and divide its first term $-2x$ by the first term x of the divisor to obtain $\frac{-2x}{x} = -2$ as the third term of the quotient.

Step VII We multiply the divisor $x - 3$ and the third term -2 of the quotient and subtract the result $-2x + 6$ from the the new dividend. We obtain 0 as the remainder.

Thus, we can say that

$(6 + x - 4x^2 + x^3) \div (x - 3) = x^2 - x - 2$

or, $\frac{6 + x - 4x^2 + x^3}{x - 3} = x^2 - x - 2$

The above procedure is displaced on the right side of the above step.

Note : In the above example, the remainder is zero. So, we can say that $(x - 3)$ is a factor of $6 + x - 4x^2 + x^3$.

Ex.93 Divide : $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

Sol. On dividing, we get

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
x^2 - 4x + 3 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 - 4x^2 + 3x \\
 & - \quad + \quad - \\
\hline
 & -2x^2 + 8x - 6 \\
 & -2x^2 + 8x - 6 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore$ $x^3 - 6x^2 + 11x - 6 = (x - 2)(x^2 - 4x + 3)$

Ex.94 Using division show that $3y^2 + 5$ is factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$.

Sol. On dividing $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$ by $3y^2 + 5$, we obtain

$$
\begin{array}{r|l}
 & 2y^3 + 5y^2 + 2y - 7 \\
\hline
3y^2 + 5 & 6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35 \\
 & 6y^5 \qquad + 10y^3 \\
 & - \qquad\quad - \\
\hline
 & 15y^4 + 6y^3 + 4y^2 + 10y - 35 \\
 & 15y^4 \qquad + 25y^2 \\
 & - \qquad\quad - \\
\hline
 & 6y^3 - 21y^2 + 10y - 35 \\
 & 6y^3 \qquad + 10y \\
 & - \qquad\quad - \\
\hline
 & -21y^2 - 35 \\
 & -21y^2 - 35 \\
 & + \qquad + \\
\hline
 & 0
\end{array}
$$

Since the remainder is zero. Therefore, $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$.

DIVISION AGLORITHM:

We know that if a number is divided by another number, then
Dividend = Divisor × Quotient + Remainder
Similarly, if a polynomial is divided by another polynomial, then
Dividend = Divisor × Quotient + Remainder
This is generally known as the division algorithm.

Ex.95 What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

Sol. We know that

Dividend = Quotient × Divisor + Remainder

$\Rightarrow$ Dividend – Remainder = Quotient × Divisor

Clearly, R.H.S of the above result is divisble by the divisor. Therefore, L.H.S. is also divisible by the divisor. Thus, if we subtract remainder from the dividend, then it will be exactly divisible by the divisior.

Dividing $8x^4 + 14x^3 - 2x^2 + 7x - 8$ by $4x^2 + 3x - 2$, we get

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
\hline
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 + 7x - 8 \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + 11x - 8 \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\
\hline
 & 14x - 10
\end{array}
$$

$\therefore$ Quotient = $2x^2 + 2x - 1$ and, Remainder = $14x - 10$

Thus, ifwe subtract the remainder $14x - 10$ from $8x^4 + 14x^3 - 2x^2 + 7x - 8$, it will be divisible by $4x^2 + 3x - 2$

Ex.96 Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

Sol. If $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$, then the remainder should be zero.

On dividing, we get

$$
\begin{array}{r|l}
 & x^2 + x + 7 \\
\hline
x^2 + 1 & x^4 + x^3 + 8x^2 + ax + b \\
 & x^4 \qquad + x^2 \\
 & - \qquad - \\
\hline
 & x^3 + 7x^2 + ax + b \\
 & x^3 \qquad + x \\
 & - \qquad - \\
\hline
 & 7x^2 + x(a-1) + b \\
 & 7x^2 \qquad + 7 \\
 & - \qquad - \\
\hline
 & x(a-1) + b - 7
\end{array}
$$

$\therefore$ Quotient = $x^2 + x + 7$ and,

Remainder = $x(a - 1) + b - 7$

Now, Remainder = 0

$\Rightarrow$ x (a − 1) + (b − 7) = 0

$\Rightarrow$ x (a − 1) + (b − 7) = 0x + 0

$\Rightarrow$ a − 1 = 0 and b − 7 =0

[Comparing coefficients of x and constant terms] $\Rightarrow$ a = 1 and b = 7

Ex.97 Divide $x^4 - x^3 + x^2 + 5$ by (x + 1) and write the quotient and remainder.

Sol. We have,

$x^4 - x^3 + x^2 + 5 = x^3 (x + 1) - 2x^2(x + 1)$

$+ 3x (x + 1) - 3 (x + 1) + 8$

$= (x + 1) (x^3 - 2x^3 + 3x - 3) + 8$

Hence, Quotient = $x^3 - 2x^2 + 3x - 3$ and, Remainder = 8.

Ex.98 Divide $12x^3 - 8x^2 - 6x + 10$ by (3x − 2). Also, write the quotient and the remainder.

Sol. We have,

$12x^3 - 8x^2 - 6x + 10$

$= 4x^2 (3x - 2) - 2 (3x - 2) + 6$

$= \{4x^2 (3x - 2) - 2 (3x - 2)\} + 6$

$= (3x - 2) (4x^2 - 2) + 6$

Hence, Quotient = $4x^2 - 2$ and, Remainder = 6.

Ex.99 Divide $6x^3 - x^2 - 10x - 3$ by (2x − 3).

Sol. We have,

$6x^3 - x^2 - 10x - 3$

$= 3x^2 (2x - 3) + 4x (2x - 3) - 1(2x - 3) - 6$

$= \{3x^2 (2x - 3)+ 4x (2x - 3) - 1 (2x - 3)\} - 6$

$= (2x - 3) (3x^2 + 4x - 1) - 6$

Hence, Quotient = $3x^2 + 4x - 1$ and, Remainder = − 6.

Division of Polynomials by using Factorization

Ex.100 Divide:

(i) $35a^2 + 32a - 99$ by 7a − 9

(ii) $ax^2 + (b + ac) x + bc$ by x + c

Sol. (i) We have,

$35a^2 + 32a - 99$

$= 35a^2 + 77a - 45a - 99$

$= 7a (5a + 11) - 9 (5a + 11) = (5a + 11) (7a - 9)$ ...(i)

$\therefore (35a^2 + 32a - 99) \div (7a - 9)$

$= \frac{35a^2 + 32a - 99}{7a - 9}$

$= \frac{(5a + 11)(7a - 9)}{(7a - 9)} = 5a + 11$ [Using (i)]

$\left[\begin{array}{l}\text{Just as numbers, we cancel common factor } (7a - 9) \\ \text{in numerator and denominator}\end{array}\right]$

(ii) We have,

$ax^2 + (b + ac)x + bc$

$= (ax^2 + bx) + (acx + bc)$

$= x\,(ax + b) + c\,(ax + b) = (ax + b)\,(x + c)$ ...(i)

$\therefore\ \left(ax^2 + (b + ac)x + bc\right) \div (x + c)$

$= \dfrac{ax^2 + (b + ac)x + bc}{(x + c)}$

$= \dfrac{(ax + b)(x + c)}{(x + c)}$ [Using (i)]

$= ax + b$ $\begin{bmatrix}\text{Cancelling common factor } (x + c)\\ \text{in numerator and denominator}\end{bmatrix}$

Ex.101 Divide: $a^{12} + a^6b^6 + b^{12}$ by $a^6 - a^3b^3 + b^6$

Sol. We have,

$a^{12} + a^6b^6 + b^{12}$

$= a^{12} + 2a^6b^6 + b^{12} - a^6b^6$

[Adding and subtracting a^6b^6]

$= (a^6 + b^6)^2 - (a^3b^3)^2$

$= (a^6 + b^6 - a^3b^3)\,(a^6 + b^6 + a^3b^3)$

$= (a^6 - a^3b^3 + b^6)\,(a^6 - a^3b^3 + b^6)$ ...(i)

$\therefore\ \dfrac{a^{12} + a^6b^6 + b^{12}}{a^6 - a^3b^3 + b^6}$

$= \dfrac{(a^6 - a^3b^3 + b^6)(a^6 + a^3b^3 + b^6)}{(a^6 - a^3b^3 + b^6)}$

$= a^6 + a^3b^3 + b^6$

[Cancelling $a^6 - a^3b^3 + b^6$ from N^r and D^r]

EXERCISE - 1

Q.1 Add the following algebraic expressions:

$2, \frac{2y}{3} - \frac{5y^2}{3} + \frac{5y^3}{2}, -\frac{4}{3} + \frac{2y^2}{3} - \frac{y}{2},$

$\frac{5y^3}{3} + 3y^2 + 3y + \frac{6}{5}$

Q.2 Subtract : $\left(-2y^2+\frac{1}{2}y-3\right)$ from $7y^2 - 2y + 10$.

Q.3 Subtract: $\frac{3}{2}x^2y + \frac{4}{5}y - \frac{1}{3}x^2yz$ from $\frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y$.

Q.4 Find the volume of the rectangular boxes with following length, breadth and height :

	Length	Breadth	Height
(i)	$2ax$	$3by$	$5cz$
(ii)	m^2n	n^2p	p^2m
(iii)	$2q$	$4q^2$	$8q^3$

Q.5 Find each of the following products:

(i) $(-2x^2) \times (7a^2x^7) \times (6a^5x^5)$

(ii) $(4s^2t) \times (3s^3t^3) \times (2st^4) \times (-2)$

Q.6 Multiply $-\frac{4}{3}xy^3$ by $\frac{6}{7}x^2y$ and verify your result for $x = 2$ and $y = 1$.

Q.7 Find the product of $-5x^2y$, $-\frac{2}{3}xy^2z$,

$\frac{8}{15}xyz^2$ and $-\frac{1}{4}$. Verify the result when $x = 1$, $y = 2$ and $y = q$.

Q.8 Find the product of $\frac{7}{2}s^2t$ and $s + t$. Verify the result for $s = \frac{1}{2}$ and $t = 5$.

Q.9 Find the following products:

(i) $100x \times (0.01x^4 - 0.01x^2)$

(ii) $121.5ab \times \left(ac+\frac{b}{10}\right)$

(iii) $0.1a \times (0.01a \times 0.001b)$

Q.10 Add:

(i) $5m(3 - m)$ and $6m^2 - 13m$

(ii) $4y(3y^2 + 5y - 7)$ and $2(y^3 - 4y^2 + 5)$

Q.11 (i) Subtract: $3l\,(l - 4m + 5n)$ from $4l\,(10n - 3m + 2l)$

(ii) Subtract : $3a\,(a + b + c) - 2b\,(a + b + c)$ from $4c(-a + b + c)$

Q.12 Multiply $\left(\frac{1}{5}x - \frac{1}{4}y\right)$ and $(5x^2 - 4y^2)$

Q.13 Multiply $(3x^2 + y^2)$ by $(x^2 + 2y^2)$.

Q.14 Multiply: $\{2m + (-n)\}$ by $\{-3m + (-5)\}$

Q.15 Find the product of $\left(y + \frac{2}{7}y^2\right)$ and $(7y - y^2)$ and verify the result for $y = 3$.

Q.16 Simplify the following:

(i) $\frac{1}{3}(6x^2 + 15y^2)\,(6x^2 - 15y^2)$

(ii) $9x^4(2x^3 - 5x^4) \times 5x^6\,(x^4 - 3x^2)$

Q.17 Multiply: $(2x^2 - 4x + 5)$ by $(x^2 + 3x - 7)$

Q.18 Find the product of the following binomials:

(i) $(6x^2 - 7y^2)\,(6x^2 - 7y^2)$

(ii) $\left(\frac{1}{2}x - \frac{1}{5}y\right)\left(\frac{1}{2}x - \frac{1}{5}y\right)$

Q.19 Find the product of the following binomials:

(i) $\left(\frac{3}{4}x + \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right)$

(ii) $\left(2a + \frac{3}{b}\right)\left(2a - \frac{3}{b}\right)$

(iii) $(a^2 + b^2)\,(-a^2 + b^2)$

(iv) $(-a + c)\,(-a - c)$

Q.20 If $x + \frac{1}{x} = 9$ and $x^2 + \frac{1}{x^2} = 53$, find the value of $x - \frac{1}{x}$.

Q.21 If $x + y = 12$ and $xy = 14$, find the value of $x^2 + y^2$.

Q.22 Simplify the following products:

(i) $(x^2 + x + 1)\,(x^2 - x + 1)$

(ii) $(x^2 + 2x + 2)\,(x^2 - 2x + 2)$

Q.23 Simplify the following by using: $(a + b)(a - b) = a^2 - b^2$.

(i) 68×72 (ii) 101×99

(iii) 67×73 (iv) $128^2 - 77^2$

Q.24 Find the greatest common factor of the monomials $6x^3a^2b^2c$, $8x^2ab^3c^3$ and $12a^3b^2c^2$.

Q.25 Factorize:

(i) $12x^3y^4 + 16x^2y^5 - 4x^5y^2$

(ii) $18a^3b^2 + 36ab^4 - 24a^2b^3$

Q.26 Factorize:

(i) $(x + y)(2x + 3y) - (2x + 3y) - (x + y)(x + 1)$

(ii) $(x + y)(2a + b) - (3x - 2y)(2a + b)$

Q.27 Factorize :

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 10y - 4$

(iii) $lm - lmn$

Q.28 Factorize:

(i) $a^2 + 2a + ab + 2b$

(ii) $x^2 - xz + xy - xz$

Q.29 Factorize each of the following expressions:

(i) $a^2 - b + ab - a$

(ii) $xy - ab + bx - ay$

(iii) $6ab - b^2 + 12ac - 2bc$

(iv) $a(a + b - c) - bc$

(v) $a^2x^2 + (ax^2 + 1)x + a$

(vi) $3ax - 6ay - 8by + 4bx$

Q.30 Factorize:

(i) $x^3 - 2x^2y + 3xy^2 - 6y^3$

(ii) $6ab - b^2 + 12ac - 2bc$

Q.31 Factorize :

(i) $x^4 - y^4$ (ii) $16x^4 - 81$

(iii) $x^4 - (y + z)^4$ (iv) $2x - 32x^5$

(v) $3a^4 - 48b^4$ (vi) $81x^4 - 121x^2$

Q.32 Factorize each of the following algebraic expressions:

(i) $16(2x - 1)^2 - 25z^2$

(ii) $4a^2 - 9b^2 - 2b - 3b$

(iii) $x^2 - 4x + 4y - y^2$

(iv) $3 - 12(a - b)^2$

(v) $x(x + z) - y(y + z)$

(vi) $a^2 - b^2 - a - b$

Q.33 Factorize :

(i) $4x^2 - 4xy + y^2 - 9z^2$

(ii) $16 - x^2 - 2xy - y^2$

(iii) $x^4 - (x - z)^4$

Q.34 Factorize :

(i) $4(x + y)^2 - 28y\,(x + y) + 49y^2$

(ii) $(2a + 3b)^2 + 2(2a + 3b)\,(2a - 3b) + (2a - 3b)^2$

Q.35 Factorize each of the following expressions:

(i) $9x^2 - 4y^2$

(ii) $36x^2 - 12x + 1 - 25y^2$

(iii) $a^2 - 1 + 2x - x^2$

Q.36 Factorize:

(i) $9 - a^6 + 2a^3b^3 - b^6$

(ii) $x^{16} - y^{16} + x^8 + y^8$

Q.37 Factorzie: $(2x + 3y)^2 - 5(2x + 3y) - 14$

Q.38 Factorize: $3m^2 + 24m + 36$

Q.39 Divide:

(i) $6x^4yz - 3xy^3z + 8x^2yz^4$ by $2xyz$

(ii) $\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^3 - \frac{1}{5}ab^3c^2$ by $\frac{1}{2}abc$

Q.40 Divide the polynomial $2x^4 + 8x^3 + 7x^2 + 4x + 3$ by $x + 3$.

Q.41 Divide $10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $2x^2 + 7x - 1$

Q.42 Divide $3y^5 + 6y^4 + 6y^3 + 7y^2 + 8y + 9$ by $3y^3 + 1$ and verify that Dividend = Divisor × Quotient + Remainder

Q.43 Divide $16x^4 + 12x^3 - 10x^2 + 8x + 20$ by $4x - 3$. Also, write the quotient and remainder.

Q.44 Divide $8y^3 - 6y^2 + 4y - 1$ by $4y + 2$. Also, write the quotient and the remainder.

Q.45 Divide: $a^4 - b^4$ by $a - b$

Q.46 Divide: $x^{4a} + x^{2a}y^{2b} + 4y^{4b}$ by $x^{2a} + x^ay^b + y^{2b}$

ANSWERS (EX. -1)

Sol.1 $\frac{28}{15} + \frac{19}{6}y + 2y^2 + \frac{25}{6}y^3$

Sol.2 $9y^2 - \frac{5}{2}y + 13$

Sol.3 $\frac{41}{15}x^2yz - \frac{5}{6}x^2y - \frac{3}{5}xyz - \frac{4}{5}y$

Sol.4 (i) 30 abcxyz (ii) $m^3n^3p^3$ (iii) $64q^6$

Sol.5 (i) $-84x^{14}a^7$ (ii) $-48s^6t^8$ (iii) $1000x^{14}y^{11}$

Sol.6 $-\frac{8}{7}x^3y^4$

Sol.7 $-\frac{4}{9}x^4y^4z^4$

Sol.8 $\frac{7}{2}s^3t + \frac{7}{2}s^2t^2$

Sol.9 (i) $x^5 - x^3$
(ii) $121.5a^2bc + 12.15ab^2$
(iii) $0.001a^2 + 0.0001ab$

Sol.10 (i) $2m + m^2$

Sol.11 (i) $25ln + 5l^2$
(ii) $-7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$

Sol.12 $x^3 - \frac{4}{5}xy^2 - \frac{5}{4}x^2y + y^3$

Sol.13 $3x^4 + 7x^2y^2 + 2y^4$

Sol.15 $7y^2 + y^3 - \frac{2}{7}y^4$

Sol.16 (i) $12x^4 - 75y^4$
(ii) $-225x^{18} + 90x^{17} + 675x^{16} - 270x^{15}$

Sol.17 $2x^4 + 2x^3 - 21x^2 + 43x - 35$

Sol.18 (i) $36x^4 - 84^2y^2 + 49y^4$ (ii) $\frac{1}{4}x^4 - \frac{xy}{5} + \frac{1}{25}y^2$

Sol.19 (i) $\frac{9}{16}x^2 - \frac{25}{36}y^2$ (ii) $4a^2 - \frac{9}{b^2}$
(iii) $b^4 - a^4$ (iv) $a^2 - c^2$

Sol.20 ± 5

Sol.21 116 **Sol.22** (i) $x^4 + x^2 + 1$, (ii) $x^4 - 2x^2 + 4$
Sol.23 (i) 4896 (ii) 9999 (iii) 4891 (iv) 10455

Sol.26 (i) $(x + y)(x + 3y - 1)$ (ii) $(-2x + 3y)(2a + b)$

Sol.28 (i) $(a + 2)(a + b)$ (ii) $(x + y)(x - z)$

Sol.29 (i) $(a + b)(a - 1)$ (ii) $(y + b)(x - a)$
(iii) $(b + 2c)(6a - b)$ (iv) $(a + b)(a - c)$
(v) $(x + a)(ax^2 + 1)$ (vi) $(3a + 4b)(x - 2y)$
Sol.30 (i) $(x - 2y)(x^2 + 3y^2)$ (ii) $(6a - b)(b + 2c)$
Sol.31 (i) $(x - y)(x + y)(x^2 + y^2)$
(ii) $(2x - 3)(2x + 3)(4x^2 + 9)$
(iii) $(x - y - z)(x + y + z)\{x^2 + (y + z)^2\}$
(iv) $2x(1 + 4x^2)(1 - 2x)(1 + 2x)$
(v) $3(a - 2b)(a + 2b)(a^2 + 4b^2)$
(vi) $x^2(9x - 11)(9x + 11)$
Sol.32 (i) $(8x - 5z - 4)(8x + 5z - 4)$
(ii) $(2a + 3b)(2a - 3b - 1)$
(iii) $(x - y)(x + y - 4)$
(iv) $3(1 + 2a - 2b)(1 - 2a + 2b)$
(v) $(x - y)(x + y + z)$
(vi) $(a + b)\{(a - b) - 1\}$
Sol.33 (i) $(2x - y + 3z)(2x - y - 3z)$
(ii) $(4 + x + y)(4 - x - y)$
(iii) $(2x^2 - 2xz + z^2)(2x - z)z$

Sol.34 (i) $(2x - 5y)^2$ (ii) $16a^2$

Sol.35 (i) $(3x + 2y)(3x - 2y)$
(ii) $(6x - 5y - 1)(6x + 5y - 1)$
(iii) $(a - 1 + x)(a + 1 - x)$

Sol.36 (i) $(a^3 - b^3 + 3)(-a^3 + b^3 + 3)$
(ii) $(x^8 + y^8)(x^8 - y^8 + 1)$
(iii) $(p + q - a + b)(p + q + a - b + 1)$

Sol.37 $(2x + 3y - 7)(2x + 3y + 2)$

Sol.38 $3(m + 2)(m + 6)$

Sol.39 (i) $3x^3 - \frac{3}{2}y^2 + 4xz^3$

(ii) $\frac{4}{3}abc + \frac{8}{3}bc^2 - \frac{2}{5}b^2c$

Sol.40 $(x + 3)(2x^3 + 2x^2 + x + 1)$

Sol.41 $(2x^2 + 7x - 1)(5x^2 - 9x + 3)$

Sol.44 $(4y + 2)\left(2y^2 - \frac{5}{2}y + \frac{9}{4}\right) - \frac{11}{2}$

Sol.45 $(a + b)(a^2 + b^2)$

Sol.46 $x^{2a} - x^a y^b + y^{2b}$

EXERCISE -2

Q.1 If $\left(x+\frac{1}{x}\right) = 3$, then $\left(x^2+\frac{1}{x^2}\right)$ is equal to -

(A) $\frac{10}{3}$ (B) $\frac{82}{9}$ (C) 7 (D) 11

Q.2 If $\left(x-\frac{1}{x}\right) = \frac{1}{2}$, then $\left(4x^2+\frac{4}{x^2}\right)$ is equal to -

(A) 7 (B) –7 (C) 9 (D) –9

Q.3 If $\left(x+\frac{1}{x}\right) = 4$, then $\left(x^4+\frac{1}{x^4}\right)$ is equal to -

(A) 196 (B) 194 (C) 192 (D) 190

Q.4 If $\left(x+\frac{1}{x}\right) = \sqrt{3}$, then the value of $\left(x^3+\frac{1}{x^3}\right)$ is -

(A) 0 (B) $3\sqrt{3}$

(C) $3(\sqrt{3}-1)$ (D) $3(\sqrt{3}+1)$

Q.5 If $\left(x+\frac{1}{x}\right) = 2$, then $\left(x^3+\frac{1}{x^3}\right)$ is equal to -

(A) 64 (B) 14 (C) 8 (D) 2

Q.6 If $\left(x^2+\frac{1}{x^2}\right) = 102$, the value of $\left(x-\frac{1}{x}\right)$ is -

(A) 8 (B) 10 (C) 12 (D) 13

Q.7 If $\left(x^4+\frac{1}{x^4}\right) = 322$, the value of $\left(x-\frac{1}{x}\right)$ is -

(A) 4 (B) 6 (C) 8 (D) $3\sqrt{2}$

Q.8 If $\left(x^3+\frac{1}{x^3}\right) = 52$, the value of $\left(x+\frac{1}{x}\right)$ is -

(A) 4 (B) 3 (C) 6 (D) 13

Q.9 If $\left(x^3-\frac{1}{x^3}\right) = 14$, the value of $\left(x-\frac{1}{x}\right)$ is -

(A) 5 (B) 4 (C) 3 (D) 2

Q.10 If x is an integer such that $\left(x+\frac{1}{x}\right) = \left(\frac{17}{4}\right)$, then the value of $\left(x-\frac{1}{x}\right)$ is -

(A) 4 (B) $\frac{13}{4}$ (C) $\frac{15}{4}$ (D) $\frac{1}{4}$

Q.11 If $\left(x^4+\frac{1}{x^4}\right) = 727$, the value of $\left(x^3-\frac{1}{x^3}\right)$ is

(A) 125 (B) 140 (C) 155 (D) 170

Q.12 If $\left(2x-\frac{3}{x}\right)=5$, the value of $\left(4x^2-\frac{9}{x^2}\right)$ is -

(A) 25 (B) 30 (C) 35 (D) 49

Q.13 If $\left(x+\frac{1}{x}\right)=3$, the value of $\left(x^6+\frac{1}{x^6}\right)$ is -

(A) 927 (B) 414 (C) 364 (D) 322

Q.14 If $t^2-4t+1=0$, then the value of $\left(t^3+\frac{1}{t^3}\right)$ is -

(A) 44 (B) 48 (C) 52 (D) 64

Q.15 If $x+y=7$ and $xy=12$, the value of (x^2+y^2) is -

(A) 25 (B) 29 (C) 37 (D) 49

Q.16 If $x+y=5$ and $xy=6$, the value of (x^3+y^3) is -

(A) 91 (B) 133 (C) 217 (D) 343

Q.17 If $x+y=5$ and $xy=6$, the value of (x^3-y^3) is -

(A) –19 (B) 19 (C) –63 (D) 63

Q.18 If $x^{1/3}+y^{1/3}+z^{1/3}=0$, then -

(A) $x+y+z=0$ (B) $(x+y+z)^3=27\,xyz$

(C) $x+y+z=3xyz$ (D) $x^3+y^3+z^3=0$

Q.19 If $a+b+c=0$, then $(a^3+b^3+c^3)$ is equal to -

(A) 0 (B) abc

(C) 3abc (D) $(ab+bc+ca)$

Q.20 If $a+b+c=0$, then the value of $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)$ is -

(A) 1 (B) 0 (C) –1 (D) 3

Q.21 If $x+y+z=9$ & $xy+yz+zx=23$, then the value of $(x^3+y^3+z^3-3xyz)$ is -

(A) 108 (B) 207 (C) 669 (D) 729

Q.22 If $\frac{5^x}{125}=1$, then x is equal to -

(A) 5 (B) 2 (C) 0 (D) 3

Q.23 If $3^x-3^{x-1}=18$, then the value of x^x is -

(A) 3 (B) 8 (C) 27 (D) 216

Q.24 If $2^x-2^{x-1}=16$, then the value of x^2 is -

(A) 4 (B) 9 (C) 16 (D) 25

Q.25 If x and y are non-zero rational unequal numbers, then $\frac{(x+y)^2-(x-y)^2}{x^2y-xy^2}$ is equal to -

(A) $\frac{1}{xy}$ (B) $\frac{1}{x-y}$ (C) $\frac{4}{x-y}$ (D) $\frac{2}{x-y}$

Q.26 If $\frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)}$

$= \frac{z}{(a-b)(a+b-2c)}$, the value of (x + y + z) is -

(A) a + b + c (B) $a^2 + b^2 + c^2$

(C) 0 (D) indeterminate

Q.27 Let $\frac{a}{b} - \frac{b}{a} = x : y$. If $(x - y) = \left(\frac{a}{b} + \frac{b}{a}\right)$, then x is equal to -

(A) $\frac{a+b}{a}$ (B) $\frac{a+b}{b}$

(C) $\frac{a-b}{a}$ (D) None of these

Q.28 If (x – 2) is a factor of $(x^2 + 3qx - 2q)$, then the value of q is -

(A) 2 (B) –2 (C) 1 (D) –1

Q.29 If $x^3 + 6x^2 + 4x + k$ is exactly divisible by (x + 2), then the value of k is -

(A) –6 (B) –7 (C) –8 (D) –10

Q.30 Let $f(x) = x^3 - 6x^2 + 11x - 6$. Then, which one of the following is not factor of f(x) ?

(A) x – 1 (B) x – 2 (C) x + 3 (D) x – 3

Q.31 The polynomial $(x^4 - 5x^3 + 5x^2 - 10x + 24)$ has a factor as -

(A) x + 4 (B) x – 2

(C) x + 2 (D) None of these

Q.32 Which of the following statements are correct ?

1. x + 3 is a factor of $x^3 + 2x^2 + 3x + 18$
2. x + 2 is a factor of $x^3 + 2x^2 - x - 2$
3. x + 1 is a factor of $x^3 + x^2 - 4x - 4$
4. x – 2 is a factor of $2x^3 - 3x + 4$

(A) 2, 3, 4 (B) 1, 3, 4

(C) 1, 2, 4 (D) 1, 2, 3

Q.33 $(x^{29} - x^{25} + x^{13} - 1)$ is divisible by -

(A) both (x – 1) & (x + 1)

(B) (x – 1) but not by (x + 1)

(C) (x + 1) but not by (x – 1)

(D) neither (x – 1) nor (x + 1)

Q.34 Value of k for which (x – 1) is a factor of $(x^3 - k)$ is -

(A) –1 (B) 1 (C) 8 (D) –8

Q.35 The factors of $(8x^3 - 27y^3)$ are -

(A) $(2x - 3y)(4x^2 + 9y^2 - 6xy)$

(B) $(2x - 3y)(4x^2 + 9y^2 + 6xy)$

(C) $(2x - 3y)(4x^2 - 9y^2 - 6xy)$

(D) $(2x - 3y)(4x^2 - 9y^2 + 6xy)$

Q.36 The factors of $(x^3 + y^3 + 2x^2 - 2y^2)$ are -

(A) $(x + y)(x^2 + y^2 + xy + 2x - 2y)$

(B) $(x + y)(x^2 + y^2 - xy + 2x - 2y)$

(C) $(x + y)(x^2 + y^2 + 2x - 2y)$

(D) None of these

Q.37 The factors of $(x^3 - 5x^2 + 8x - 4)$ are -

(A) $(x + 2)(x - 2)(x - 1)$

(B) $(x + 1)(x + 2)(x - 2)$

(C) $(x - 2)^2 (x - 1)$

(D) $(x - 2)^2 (x + 1)$

Q.38 The factors of $(x^4 + 4)$ are -

(A) $(x^2 + 2)^2$

(B) $(x^2 + 2)(x^2 - 2)$

(C) $(x^2 + 2x + 2)(x^2 - 2x + 2)$

(D) None of these

Q.39 $(x + y)^3 - (x - y)^3$ can be factorized as -

(A) $2y(3x^2 + y^2)$ (B) $2x(3x^2 + y^2)$

(C) $2y(3y^2 + x^2)$ (D) $2x(x^2 + 3y^2)$

Q.40 The factors of $(x^3 + 8y^3)$ are -

(A) $(x + 2y)(x^2 - 2xy + 4y^2)$

(B) $(x + 2y)(x^2 + 2xy + 4y^2)$

(C) $(x + 2y)(x - 2y)^2$

(D) $(x + 2y)^3$

Q.41 If $(x + 2)$ and $(x - 1)$ are the factors of $(x^3 + 10x^2 + mx + n)$, the values of m and n are -

(A) $m = 5, n = -3$ (B) $m = 17, n = -8$

(C) $m = 7, n = -18$ (D) $m = 23, n = -19$

Q.42 On dividing $(x^3 - 6x + 7)$ by $(x + 1)$, the remainder is -

(A) 2 (B) 12 (C) 0 (D) 7

Q.43 If $(x^5 - 9x^2 + 12x - 14)$ is divided by $(x - 3)$, the remainder is -

(A) 184 (B) 56 (C) 2 (D) 1

Q.44 When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x - 2)$, the remainder is -

(A) 3 (B) –3 (C) 2 (D) 0

Q.45 If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, the remainder is -

(A) 135 (B) –135 (C) 63 (D) –63

Q.46 If $(x^{11} + 1)$ is divided by $(x + 1)$, the remainder is -

(A) 0 (B) 2 (C) 11 (D) 12

Q.57 The factors of $(x^2 - 1 - 2a - a^2)$ are -
(A) $(x - a + 1)(x - a - 1)$
(B) $(x + a - 1)(x - a + 1)$
(C) $(x + a + 1)(x - a - 1)$
(D) None of these

Q.58 The factors of $(x^2 - 8x - 20)$ are -
(A) $(x + 10)(x - 2)$ (B) $(x - 10)(x + 2)$
(C) $(x - 5)(x + 4)$(D) $(x + 5)(x - 4)$

Q.59 The factors of $(x^2 - xy - 72y^2)$ are -
(A) $(x - 8y)(x + 9y)$ (B) $(x - 9y)(x + 8y)$
(C) $(x - y)(x + 72y)$ (D) $(x - 6y)(x + 12y)$

Q.60 The factors of $(x^2 - 11xy - 60y^2)$ are -
(A) $(x + 15y)(x - 4y)$
(B) $(x - 15y)(x + 4y)$
(C) $(15x + y)(4x - y)$
(D) None of these

Q.61 The factors of $(x^4 + x^2 + 25)$ are -
(A) $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$
(B) $(x^2 + 3x + 5)(x^2 + 3x - 5)$
(C) $(x^2 + x + 5)(x^2 - x + 5)$
(D) None of these

Q.62 The factors of $(x^4 - 7x^2y^2 + y^4)$ are -
(A) $(x^2 + y^2 - 3xy)(x^2 + y^2 + 3xy)$
(B) $(x^2 - y^2 - 3xy)(x^2 - y^2 + 3xy)$
(C) $(x^2 - 3xy + y^2)(x^2 - 3xy - y^2)$
(D) None of these

Q.63 The factors of $(x^2 + 4y^2 + 4y - 4xy - 2x - 8)$ are -
(A) $(x - 2y - 4)(x - 2y + 2)$
(B) $(x - y + 2)(x - 4y - 4)$
(C) $(x + 2y - 4)(x + 2y + 2)$
(D) None of these

Q.64 The factors of $(x^2 + xy - 2y^2)$ are -
(A) $(x - 2y)(x + y)$ (B) $(x + 2y)(x - y)$
(C) $(x - 2y)(x - y)$ (D) $(x + 2y)(x + y)$

Q.65 The factors of $(x^3 - x^2y - xy^2 + y^3)$ are -
(A) $(x + y)(x^2 + y^2 - xy)$
(B) $(x + y)(x^2 + y^2 + xy)$
(C) $(x + y)^2 (x - y)$
(D) $(x - y)^2 . (x + y)$

Q.66 The factors of $(216x^3 - 64y^3)$ are -

(A) $8(3x - 2y)(9x^2 + 4y^2 - 6xy)$

(B) $8(3x - 2y)(9x^2 - 4y^2 - 6xy)$

(C) $8(3x - 2y)(9x^2 + 4y^2)$

(D) $8(3x - 2y)(9x^2 + 4y^2 + 6xy)$

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	C	B	A	D	B	A	A	D	C	B	C	D	C	A	C	B	B	C	D
Q.No	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	A	D	C	D	C	C	D	D	C	C	B	D	B	B	B	B	C	C	A	A
Q.No	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	B	A	B	D	A	C	C	C	C	B	B	B	A	D	D	C	B	B	B
Q.No	61	62	63	64	65	66														
Ans.	A	A	A	B	D	D														

HINTS & SOLUTION - 1

Sol.1 Required sum

$= 2 + \frac{2y}{3} - \frac{5y^2}{3} + \frac{5y^3}{2} - \frac{4}{3} + \frac{2y^2}{3} - \frac{y}{2}$

$+ \frac{5y^3}{3} + 3y^2 + 3y + \frac{6}{5}$

$= 2 - \frac{4}{3} + \frac{6}{5} + \frac{2y}{3} - \frac{y}{2} + 3y - \frac{5y^2}{3} + \frac{2y^2}{3}$

$+ 3y^2 + \frac{5y^3}{2} + \frac{5y^3}{3}$

$= \left(2-\frac{4}{3}+\frac{6}{5}\right) + \left(\frac{2}{3}-\frac{1}{2}+3\right)y + \left(-\frac{5}{3}+\frac{2}{3}+3\right)y^2$

$+ \left(\frac{5}{2}+\frac{5}{2}\right)y^3$

$= \left(\frac{30-20+18}{15}\right) + \left(\frac{4-3+18}{6}\right)y + \left(\frac{-5+2+9}{3}\right)y^2$

$+ \left(\frac{15+10}{6}\right)y^3$

$= \frac{28}{15} + \frac{19}{6}y + 2y^2 + \frac{25}{6}y^3$

Sol.2 The required difference is given by

$(7y^2 - 2y + 10) - \left(-2y^2 + \frac{1}{2}y - 3\right)$

$= 7y^2 - 2y + 10 + 2y^2 - \frac{1}{2}y + 3$

$= 7y^2 + 2y^2 - 2y - \frac{1}{2}y + 10 + 3$

[Grouping like terms]

$= (7 + 2)y^2 + \left(-2-\frac{1}{2}\right)y + 13$

$= 9y^2 - \frac{5}{2}y + 13$

Sol.3 We have,

$\left(\frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y\right) - \left(\frac{3}{2}x^2y + \frac{4}{5}y - \frac{1}{3}x^2yz\right)$

$= \frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y - \frac{3}{2}x^2y - \frac{4}{5}y$

$+ \frac{1}{3}x^2yz$

$= \frac{12}{5}x^2yz + \frac{1}{3}x^2yz + \frac{2}{3}x^2y - \frac{3}{2}x^2y - \frac{3}{5}xyz - \frac{4}{5}y$

[Grouping like terms]

$$= \left(\frac{12}{5}+\frac{1}{3}\right)x^2yz + \left(\frac{2}{3}-\frac{3}{2}\right)x^2y - \frac{3}{5}xyz - \frac{4}{5}y$$

$$= \frac{41}{15}x^2yz - \frac{5}{6}x^2y - \frac{3}{5}xyz - \frac{4}{5}y$$

Sol.4 We know that the volume of a rectangular box is given by

Volume = Length × Breadth × Height

(i) Volume = $2ax \times 3by \times 5\,cz$

$= (2 \times 3 \times 5) \times (ax \times by \times cz)$

$= 30\,abcxyz$

(ii) Volume = $m^2n \times n^2p \times p^2m = m^{2+1}n^{1+2}p^{1+2}$

$= m^3n^3p^3$

(iii) Volume = $2q \times 4q^2 \times 8q^3 = (2 \times 4 \times 8)q^{1+2+3}$

$= 64q^6$

Sol.5 (i) We have,

$(-2x^2) \times (7a^2x^7) \times (6a^5x^5)$

$= (-2 \times 7 \times 6) \times (x^2 \times x^7 \times x^5 \times a^2 \times a^5)$

$= -84x^{2+7+5}a^{2+5} = -84x^{14}a^7$

(ii) We have,

$(4s^2t) \times (3s^3t^3) \times (2st^4) \times (-2)$

$= (4 \times 3 \times 2 \times -2) \times (s^2 \times s^3 \times s \times t \times t^3 \times t^4)$

$= -48s^{2+3+1}t^{1+3+4} = -48s^6t^8$

(iii) We have,

$(5x^6) \times (-10xy^4) \times (-2x^6y^6) \times (10xy)$

$= (5 \times -10 \times -2 \times 10) \times (x^6 \times x \times x^6 \times x \times y^4 \times y^6 \times y)$

$= 1000x^{6+1+6+1}y^{4+6+1} = 1000x^{14}y^{11}$

Sol.6 We have,

$$\left(-\frac{4}{3}xy^3\right) \times \left(\frac{6}{7}x^2y\right) = \left(-\frac{4}{3} \times \frac{6}{7}\right) \times (x \times x^2 \times y^3 \times y)$$

$$= -\frac{8}{7}x^{1+2}y^{3+1} = -\frac{8}{7}x^3y^4$$

Verification: For x = 2 and y = 1, we have

$$\text{L.H.S.} = \left(-\frac{4}{3}xy^3\right) \times \left(\frac{6}{7}x^2y\right)$$

$$= \left(-\frac{4}{3} \times 2 \times (1)^3\right) \times \left(\frac{6}{7} \times (2)^2 \times 1\right) = -\frac{8}{3} \times \frac{24}{7} = -\frac{64}{7}$$

and,

$$\text{R.H.S.} = -\frac{8}{7}x^3y^4 = -\frac{8}{7} \times 2^3 \times (1)^4 = -\frac{64}{7}$$

Hence, for x = 2 and y = 1, we have L.H.S. = R.H.S.

Sol.7 We have,

$$(-5x^2y)\ ^5 \times \left(-\frac{2}{3}xy^2z\right) \times \left(\frac{8}{15}xy^2z\right) \times \left(-\frac{1}{4}z\right)$$

$$= \left(-5\times-\frac{2}{3}\times\frac{8}{15}\times-\frac{1}{4}\right)\times (x^2 \times x \times x \times y \times y^2 \times z \times z^2)$$

$$= -\frac{4}{9}x^4y^4z^4$$

Verification : For x = 1, y = 2 and z = 3, we have

$$\text{L.H.S.} = (-5 \times (1)^2 \times 2) \times \left(-\frac{2}{3}\times 1\times (2)^2\times 3\right) \times\left(\frac{8}{15}\times 1\times 2\times (3)^2\right)\times\left(-\frac{1}{4}\times 3\right)$$

$$= (-5\times 1\times 2)\times\left(-\frac{2}{3}\times 1\times 4\times 3\right)\times\left(\frac{8}{15}\times 1\times 2\times 9\right)\times\left(-\frac{3}{4}\right)$$

$$= (-10) \times (-8) \times\left(\frac{48}{5}\right) \times \left(-\frac{3}{4}\right) = -576$$

and , $\text{R.H.S.} = \frac{4}{9}\times (1)^4 \times 2^4 \times 3^4 = \frac{4}{9}\times 1 \times 16 \times 81$

$= -576$

$\therefore$ L.H.S. = R.H.S.

Sol.8 We have,

$$\frac{7s^2t}{2}\times (s + t) = \frac{7}{2}s^2t\times s+\frac{7}{2}s^2t\times t = \frac{7}{2}s^3t+\frac{7}{2}s^2t^2$$

Verification: When $s = \frac{1}{2}$ and t = 5, we have

$$\text{L.H.S.} = \frac{7}{2}s^2t \times (s + t)$$

$$= \left\{\frac{7}{2}\times\left(\frac{1}{2}\right)^2\times 5\right\} \times \left(\frac{1}{2}+5\right)$$

$$= \left(\frac{7}{2}\times\frac{1}{4}\times 5\right)\times\frac{11}{2} = \frac{35}{8}\times\frac{11}{2} = \frac{385}{16}$$

$$\text{R.H.S.} = \frac{7}{2}s^3t + \frac{7}{2}s^2t^2$$

$$= \frac{7}{2}\times\left(\frac{1}{2}\right)^3\times 5 + \frac{7}{2}\times\left(\frac{1}{2}\right)^2\times (5)^2$$

$$= \frac{7}{2} \times \frac{1}{8} \times 5 + \frac{7}{2} \times \frac{1}{4} \times 25 = \frac{35}{16}+\frac{175}{8}$$

$$= \frac{35+350}{16} = \frac{385}{16}$$

$\therefore$ L.H.S. = R.H.S.

$= -4ac + 4bc + 4c^2 - \{3a^2 + ab + 3ac + 2b^2 - 2bc\}$
$= -4ac + 4bc + 4c^2 - 3a^2 - ab - 3ac - 2b^2 + 2bc$
$= -7ac + 6bc + 4c^2 - 3a^2 - ab - 2b^2$

Sol.12 We have,

$\left(\frac{1}{5}x - \frac{1}{4}y\right) \times (5x^2 - 4y^2)$

$= \frac{1}{5}x \times (5x^2 - 4y^2) - \frac{1}{4}y \times (5x^2 - 4y^2)$

$= \frac{1}{5}x \times 5x^2 - \frac{1}{5}x \times 4y^2 - \frac{1}{4}y \times 5x^2 + \frac{1}{4}y \times 4y^2$

$= x^3 - \frac{4}{5}xy^2 - \frac{5}{4}x^2y + y^3$

Sol.13 Column method:

We have,

$$\begin{array}{ll} \quad 3x^2 + y^2 & \\ \times \; x^2 + 2y^2 & \\ \hline 3x^4 + x^2y^2 & \text{Multiplying } 3x^2 + y^2 \text{ by } x^2 \\ \quad + 6x^2y^2 + 2y^4 & \text{Multiplying } 3x^2 + y^2 \text{ by } 2y^2 \\ \hline 3x^4 + 7x^2y^2 + 2y^4 & \text{Adding the like terms} \end{array}$$

Horizontal Method:

We have,

$(3x^2 + y^2)(x^2 + 2y^2)$
$= 3x^2 \times (x^2 + 2y^2) + y^2(x^2 + 2y^2)$
$= 3x^2 \times x^2 + 3x^2 \times 2y^2 + y^2 \times x^2 + y^2 \times 2y^2$
$= 3x^4 + 6x^2y^2 + x^2y^2 + 2y^4$
$= 3x^4 + 7x^2y^2 + 2y^4$

Sol.14 Horizontal Method:

We have,

$\{2m + (-n)\} \times \{-3m + (-5)\}$
$= (2m - n) \times (-3m - 5)$
$= 2m \times (-3m - 5) - n \times (-3m - 5)$
$= -6m^2 - 10m + 3mn + 5n$

Column method:

We have,

$2m + (-n) = 2m - n$ and $-3m + (-5) = -3m - 5$

$$\begin{array}{ll} \therefore \quad 2m - n & \\ \times \; -3m - 5 & \\ \hline -6m^2 + 3mn & \text{Multiplying } 2m - n \text{ by } -3m \\ \quad -10m + 5n & \text{Multiplying } 2m - n \text{ by } -5 \\ \hline -6m^2 + 3mn - 10m + 5n & \text{Adding the like terms} \end{array}$$

Sol.15 We have,

$$\left(y+\frac{2}{7}y^2\right) \times (7y - y^2)$$

$$= y \times (7y - y^2) + \frac{2}{7}y^2 \times (7y - y^2)$$

$$= y\times 7y - y \times y^2 + \frac{2}{7}y^2 \times 7y - \frac{2}{7}y^2 \times y^2$$

$$= 7y^2 - y^3 + 2y^3 - \frac{2}{7}y^4$$

$$= 7y^2 + y^3 - \frac{2}{7}y^4$$

Verification: When y = 3, we have

L.H.S. $= \left(y+\frac{2}{7}y^2\right) \times (7y - y^2)$

$$= \left(3+\frac{2}{7}\times(3)^2\right) \times (7 \times 3 - (3)^2)$$

$$= \left(3+\frac{2}{7}\times 9\right) \times (21- 9)$$

$$= \left(3+\frac{18}{7}\right) \times 12 = \left(\frac{21+18}{7}\right)\times 12$$

$$= \frac{39}{7}\times 12 = \frac{468}{7}$$

R.H.S. $= 7y^2 + y^3 - \frac{2}{7}y^4$

$$= 7 \times (3)^2 + (3)^2 - \frac{2}{7}\times(3)^4$$

$$= 7 \times 9 + 27 - \frac{2}{7}\times 81$$

$$= 63 + 27 - \frac{162}{7} = 90 - \frac{162}{7} = \frac{630-162}{7}$$

$$= \frac{468}{7}$$

∴ L.H.S. = R.H.S.

Sol.16 (i) We have,

$$\frac{1}{3}(6x^2 + 15y^2)(6x^2 - 15y^2)$$

$$= \left\{\frac{1}{3}\times(6x^2+15y^2)\right\} \times (6x^2 - 15y^2)$$

[By using associativity of multiplication]

$= \left(\frac{1}{3}\times 6x^2 + \frac{1}{3}\times 15y^2\right) \times (6x^2 - 15y^2)$

$\begin{bmatrix} \text{By using distributivity of} \\ \text{multiplication over addtion} \end{bmatrix}$

$= (2x^2 + 5y^2) \times (6x^2 - 15y^2)$
$= 2x^2 \times (6x^2 - 15y^2) + 5y^2 \times (6x^2 - 15y^2)$
$= 2x^2 \times 6x^2 - 2x^2 \times 5y^2 + 15y^2 \times 6x^2 - 5y^2 \times 15y^2$
$= 12x^4 - 30x^2y^2 + 30x^2y^2 - 75y^4$
$= 12x^4 - 75y^4$

(ii) We have,

$9x^4(2x^3 - 5x^4) \times 5x^6 (x^4 - 3x^2)$
$= 9x^4 \times (2x^3 - 5x^4) \times 5x^6 \times (x^4 - 3x^2)$
$= \{9x^4 \times (2x^3 - 5x^4)\} \times \{5x^6 \times (x^4 - 3x^2)\}$

$\begin{bmatrix} \text{By using associativity} \\ \text{of multiplication} \end{bmatrix}$

$= (9x^4 \times 2x^3 - 9x^4 \times 5x^4) \times (5x^6 \times x^4 - 5x^6 \times 3x^2)$ $= (18x^7 - 45x^8) \times (5x^{10} - 15x^8)$
$= 18x^7 \times (5x^{10} - 15x^8) - 45x^8(5x^{10} - 15x^8)$
$= 18x^7 \times 5x^{10} - 18x^7 \times 15x^8 - 45x^8 \times 5x^{10} + 45x^8 \times 15x^8$
$= 90x^{17} - 270x^{15} - 225x^{18} + 675x^{16}$
$= -225x^{18} + 90x^{17} + 675x^{16} - 270x^{15}$

Sol.17 Horizontal method:

We have,

$(2x^2 - 4x + 5) \times (x^2 + 3x - 7)$
$= 2x^2 (x^2 + 3x - 7) - 4x (x^2 + 3x - 7) + 5(x^2 + 3x - 7)$
$= (2x^4 + 6x^3 - 14x^2) + (-4x^3 - 12x^2 + 28x) + (5x^2 + 15x - 35)$
$= 2x^4 + 6x^3 - 4x^3 - 14x^2 - 12x^2 + 5x^2 + 28x + 15x - 35$
$= 2x^4 + 2x^3 - 21x^2 + 43x - 35$

Column Method:

We have,

$$\begin{array}{rl} 2x^2 - 4x + 5 & \\ \times \quad x^2 + 3x - 7 & \\ \hline 2x^4 - 4x^3 + 5x^2 \qquad\qquad & \text{Multiplying } 2x^2 - 4x + 5 \text{ by } x^2 \\ +6x^3 - 12x^2 + 15x \qquad & \text{Multiplying } 2x^2 - 4x + 5 \text{ by} -3x \\ -14x^2 + 28x - 35 & \text{Multiplying } 2x^2 - 4x + 5 \text{ by} -7 \\ \hline 2x^4 + 2x^3 - 21x^2 + 43x - 35 & \text{Adding the like terms} \end{array}$$

Sol.18 (i) We have,

$(6x^2 - 7y^2)(6y^2 - 7y^2)$

$= (6x^2 - 7y^2)^2 \qquad [\because a.a = a^2]$

$= (6x^2)^2 - 2 \times 6x^2 \times 7y^2 + (7y^2)^2$

$= 36x^4 - 84^2y^2 + 49y^4$

(ii) $\left(\frac{1}{2}x - \frac{1}{5}y\right)\left(\frac{1}{2}x - \frac{1}{5}y\right)$

$\left(\frac{1}{2}x - \frac{1}{5}y\right)^2 = \left(\frac{1}{2}x\right)^2 - 2\left(\frac{1}{2}x\right)\left(\frac{1}{5}y\right) + \left(\frac{1}{5}y\right)^2$

$= \frac{1}{4}x^4 - \frac{xy}{5} + \frac{1}{25}y^2$

Sol.19 (i) We have,

$\left(\frac{3}{4}x + \frac{5}{6}y\right)\left(\frac{3}{4}x - \frac{5}{6}y\right)$

$= \left(\frac{3}{4}x\right)^2 - \left(\frac{5}{6}y\right)^2$

[Using : $(a + b)(a - b) = a^2 - b^2$]

$= \frac{9}{16}x^2 - \frac{25}{36}y^2$

(ii) We have,

$\left(2a + \frac{3}{b}\right)\left(2a - \frac{3}{b}\right)$

$= (2a)^2 - \left(\frac{3}{b}\right)^2$ [Using: $(a + b)(a - b) = a^2 - b^2$]

$= 4a^2 - \frac{9}{b^2}$

(iii) We have,

$(a^2 + b^2)(-a^2 + b^2)$

$= (a^2 + b^2)\{-(a^2 - b^2)\}$

$= -(a^2 + b^2)(a^2 - b^2)$

$= -\{(a^2)^2 - (b^2)^2\} = -(a^4 - b^4) = -a^4 + b^4$

$= b^4 - a^4$

(iv) We have,

$(-a + c)(-a - c)$

$= \{-(a - c)\}\{-(a + c)\}$

$= (a - c)(a + c) = a^2 - c^2$

Sol.20 We have,

$\left(x + \frac{1}{x}\right)^2 + \left(x - \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$

$+ x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$

$\Rightarrow \left(x+\frac{1}{x}\right)^2+\left(x-\frac{1}{x}\right)^2 = x^2+2+\frac{1}{x^2}+x^2-2+\frac{1}{x^2}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2+\left(x-\frac{1}{x}\right)^2 = 2x^2+\frac{2}{x^2}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2+\left(x-\frac{1}{x}\right)^2 = 2\left(x^2+\frac{1}{x^2}\right)$

Putting the values of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$, we get

$9^2+\left(x-\frac{1}{x}\right)^2 = 2 \times 53$

$\Rightarrow 81+\left(x-\frac{1}{x}\right)^2 = 106$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 106-81$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 25$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 5^2$

$\Rightarrow x-\frac{1}{x} = \pm 5$

Sol.21 We have,

$(x+y)^2 = x^2+y^2+2xy$

Putting the values of x + y and xy, we otbain

$12^2 = x^2+y^2+2 \times 14$

$\Rightarrow 144 = x^2+y^2+28$

$\Rightarrow 144-28 = x^2+y^2$

$\Rightarrow x^2+y^2 = 116$

Sol.22 (i) We have,

$(x^2+x+1)(x^2-x+1)$

$\{(x^2+1)+x\}\{(x^2+1)-x\}$

$= (x^2+1)^2-x^2$

$= x^4+2x^2+1-x^2$

$= x^4+x^2+1$

(ii) We have,

$(x^2+2x+2)(x^2-2x+2)$

$= \{(x^2+2)+2x\}\{(x^2+2)-2x\}$

$= (x^2+2)^2-(2x)^2$

$= x^4+2x^2+4-4x^2$

$= x^4-2x^2+4$

Sol.23 (i) We have,

$$\frac{68+72}{2} = 70$$

So, we express 68 and 72 in terms of 70.

$\therefore\ 68 \times 72 = (70 - 2) \times (70 + 2)$

$= (70)^2 - 2^2 = 4900 - 4 = 4896$

(ii) We have,

$$\frac{101+99}{2} = 100$$

So, we express 101 and 99 in terms of 100.

$101 \times 99 = (100 + 1) \times (100 - 1) = (100)^2 - 1^2$

$= 10000 - 1 = 9999$

(iii) We have,

$$\frac{67+73}{2} = 70$$

So, we express 67 and 73 in terms of 70.

$67 \times 73 = (70 - 3) \times (70 + 3) = (70)^2 - (3)^2$

$= 4900 – 9 = 4891$

(iv) We have,

$128^2 - 77^2 = (128 + 77) \times (128 - 77)$

$= 205 \times 51 = 10455$

Sol.24 The numerical coefficients of the given monomials are 6, 8 and 12.

The greatest common factor of 6, 8 and 12 is 2.

The common literals appearing in the given monomials are a, b and c.

The smallest power of 'a' in the three monomials = 1

The smallest power of 'b' in the three monomials = 2

The smallest power of 'c' in the three monomials = 1

The monomial of common literals with smallest powers = $a^1b^2c^1 = ab^2c$

Hence, the greatest common factor = $2ab^2c$.

Sol.25 (i) The greatest common factor of the terms $12x^3y^4$, $16x^2y^5$ and $4x^5y^2$ of the expression $12x^3y^4 + 16x^2y^5 - 4x^5y^2$ is $4x^2y^2$.

Also, we can write

$12x^3y^4 = 4x^2y^2 \times 3xy^2$, $16x^2y^5 = 4x^2y^2 \times 4y^3$ and, $4x^5y^2 = 4x^2y^2 \times x^3$

$\therefore\ 12x^3y^4 + 16x^2y^5 - 4x^5y^2 = 4x^2y^2 \times 3xy^2 + 4x^2y^2 \times 4x^2y^2 \times 4y^3 - 4x^2y^2 \times x^3$

$4x^2y^2 (3xy^2 + 4y^3 - x^3)$

(ii) We have,

$18a^3b^2 + 36ab^4 - 24a^2b^3$

The greatest common factor of the terms $18a^3b^2$, $36ab^4$ and $24a^2b^2$ is $6ab^2$.

Also, we can write $18a^3b^2 = 6ab^2 \times 3a^2$,

$36ab^4 = 6ab^2 \times 6b^2$

and, $24a^2b^2 = 6ab^2 \times 4ab$

$\therefore\ 18a^3b^2 + 36ab^4 - 24a^2b^3$

$= 6ab^2 \times 3a^2 + 6ab^2 \times 6b^2 - 6ab^2 \times 4ab$

$= 6ab^2(3a^2 + 6b^2 - 4ab)$

Sol.26 We have,

(i) $(x + y)(2x + 3y) - (x + y)(x + 1)$

$= (x + y)(2x + 3y) - (x + 1)$

[Taking $(x + y)$ common]

$= (x + y)(2x + 3y - x - 1)$

$= (x + y)(x + 3y - 1)$

$= (x + y)(x + 3y - 1)$

(ii) $(x + y)(2a + b) - (3x - 2y)(2a + b)$

$= \{(x + y) - (3x - 2y)\}(2a + b)$

[Taking $(2a + b)$ common]

$= (x + y - 3x + 2y)(2a + b)$

$= (-2x + 3y)(2a + b)$

Sol.27 (i) We observe that there is no common factor among all terms. Also, there are four terms. So, let us think of grouping the terms in pairs in such a way that there are some factors common to them and after taking factors common from each pair same binomial is left inside thetwo brackets. We observe that first two terms have x as a common factor . Taking x common from them, we have,

$x^2 + xy = x(x + y)$

Also, 8 is a common factor from the last two terms. Taking 8 common from the last two terms, we have

$8x + 8y = 8(x + y)$

Clearly, $x + y$ is common from the two groups.

Thus, we group the terms as follows:

$x^2 + xy + 8x + 8y = (x^2 + xy) + (8x + 8y)$

(ii) We have, $15xy - 6x + 10y - 4$

Clearly, there is no common factor among all the terms. Also, there are four terms. So, let us think of grouping the terms in pairs in such a way that there are some factors common to the terms in each pair and after taking factors common from each pair same binomial is left inside the two brackets.

We observe that first two terms have 3x as a common factor. Taking 3x common from them, we have

$15xy - 6x = 3x(5y - 2)$

Last two terms have 2 as the common factor. Taking 2 common from these two, we have

$10y - 4 = 2(5y - 2)$

Clearly, $(5y - 2)$ is the binomial common from these two groups. Thus, we group the terms as follows:

$15xy - 6x + 10y - 4 = 3x(5y - 2) + 2(5y - 2)$

$= (3x + 2)(5y - 2)$

(iii) We have,

$lm - lmn = (n - 7) + (7lm - ln)$

$= (n - 7) + (7 - n) lm$

$= (n - 7) - (n - 7)lm$

$= (n - 7)(1 - lm)$

Sol.28 We have,

(i) $a^2 + 2a + ab + 2b = (a^2 + 2a) + (ab + 2b)$

[Grouping the terms]

$= a(a + 2) + (a + 2)b$

$= (a + 2)(a + b)$

$= (a + 2)(a + b)$ [Taking (a + 2) common]

(ii) $x^2 - xz + xy - yz = (x^2 - xz) + (xy - yz)$

[Grouping the terms]

$= x(x - z) + y(x - z)$

$= (x + y)(x - z)$ [Taking (x – z) common]

Sol.29 (i) We have,

$a^2 - b + ab - a = a^2 + ab - b - a$

$= (a^2 + ab) - (b + a)$

$= a(a + b) - (a + b)$

$= (a + b)(a - 1)$

(ii) $xy - ab + bx - ay = xy + bx - ab - ay$

$= x(y + b) - a(b + y)$

$= x(y + b) - a(y + b)$

$= (y + b)(x - a)$

(iii) $6ab - b^2 + 12ac - 2bc = 6ab + 12ac - b^2 - 2bc$ $= 6a(b + 2c) - b(b + 2c)$

$= (b + 2c)(6a - b)$

(iv) $a(a + b - c) - bc = a^2 + ab - ac - bc$

$= (a^2 + ab) - (ac + bc)$

$= a(a + b) - c(a + b)$

$= (a + b)(a - c)$

(v) $a^2x^2 + (ax^2 + 1)x + a = a^2x^2 + ax^3 + x + a$

$= ax^2(x + a) + (x + a)$

$= (x + a)(ax^2 + 1)$

(vi) $3ax - 6ay - 8by + 4bx = 3ax + 4bx - 6ay - 8by$

$= (3ax + 4bx) - (6ay + 8by)$

$= x(3a + 4b) - 2y(3a + 4b)$

$= (3a + 4b)(x - 2y)$

Sol.30 (i) We have,

$x^3 - 2x^2y + 3xy^2 - 6y^3 = (x^3 - 2x^2y) + (3xy^2 - 6y^3)$

$= x^2(x - 2y) + 3y^2(x - 2y)$

$= (x - 2y)(x^2 + 3y^2)$

(ii) We have,

$6ab - b^2 + 12ac - 2bc = b(6a - b) + 2c(6a - b)$

$= (6a - b)(b + 2c)$

Sol.31 We have,

(i) $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 - y^2)(x^2 + y^2)$

[Using : $a^2 - b^2 = (a - b)(a + b)$]

$= (x - y)(x + y)(x^2 + y^2)$

[Using : $a^2 - b^2 = (a - b)(a + b)$]

(ii) $16x^4 - 81 = (4x^2)^2 - (9)^2$

$= (4x^2 - 9)(4x^2 + 9)$

[Using : $a^2 - b^2 = (a - b)(a + b)$]

$= \{(2x)^2 - (3)^2\} + (4x^2 + 9)$

$= (2x - 3)(2x + 3)(4x^2 + 9)$

[Using : $a^2 - b^2 = (a - b)(a + b)$]

(iii) $x^4 - (y + z)^4 = (x^2)^2 - \{(y + z)^2\}^2$

$= \{x^2 - (y + z)^2\}\{x^2 + (y + z)^2\}$

$= \{x - (y + z)\}\{x + (y + z)\}\{x^2 + (y + z)^2\}$

$= (x - y - z)(x + y + z)\{x^2 + (y + z)^2\}$

(iv) $2x - 32x^5 = 2x(1 - 16x^4)$

$= 2x\{1^2 - (4x^2)^2\}$

$= 2x(1 + 4x^2)(1 - 4x^2)$

$= 2x(1 + 4x^2)\{1 - (2x)^2\}$

$= 2x(1 + 4x^2)(1 - 2x)(1 + 2x)$

(v) $3a^4 - 48b^4 = 3(a^4 - 16b^4)$

$= 3\{(a^2)^2 - (4b^2)^2\}$

$= 3(a^2 - 4b^2)(a^2 + 4b^2)$

$= 3\{a^2 - (2b)^2\}(a^2 + 4b^2)$

$= 3(a - 2b)(a + 2b)(a^2 + 4b^2)$

(vi) $81x^4 - 121x^2 = x^2(81x^2 - 121)$

$= x^2\{(9x)^2 - (11)^2\}$

$= x^2(9x - 11)(9x + 11)$

Sol.32 (i) $16(2x - 1)^2 - 25z^2 = \{4(2x - 1)\}^2 - (5z)^2$

$= \{4(2x - 1) - 5z\}\{4(2x - 1) + 5z\}$

$= (8x - 4 - 5z)(8x - 4 + 5z)$

$= (8x - 5z - 4)(8x + 5z - 4)$

(ii) $4a^2 - 9b^2 - 2a - 3b = \{(2a)^2 - (3b)^2\} - (2a + 3b)$

$= (2a - 3b)(2a + 3b) - (2a + 3b)$

$= (2a + 3b)\{(2a - 3b) - 1\}$

$= (2a + 3b)(2a - 3b - 1)$

(iii) $x^2 - 4x + 4y - y^2 = (x^2 - y^2) - (4x - 4y)$

$= (x - y)(x + y) - 4(x - y)$

$= (x - y)\{(x + y - 4)\}$

$= (x - y)(x + y - 4)$

(vi) $3 - 12(a - b)^2 = 3\{1 - 4(a - b)^2\}$

$= 3[1^2 - \{2(a - b)\}^2]$

$= 3[\{1 + 2(a - b)\}\{1 - 2(a - b)\}]$

$= 3(1 + 2a - 2b)(1 - 2a + 2b)$

(v) $x(x + z) - y(y + z) = x^2 + xz - y^2 - yz$

$= (x^2 - y^2) + (xz - yz)$

$= (x - y)(x + y) + z(x - y)$

$= (x - y)\ \{(x + y) + z\}$

$= (x - y)\ (x + y + z)$

(vi) $a^2 - b^2 - a - b = (a^2 - b^2) - (a + b)$

$= (a - b)\ (a + b) - (a + b)$

$= (a + b)\ \{(a - b) - 1\}$

Sol.33 We have,

(i) $4x^2 - 4xy + y^2 - 9z^2$

$= (4x^2 - 4xy + y^2) - 9z^2$

$= \{(2x^2) - 2 \times 2x \times y + y^2\} - (3z)^2$

$= (2x - y)^2 - (3z)^2$

$= (2x - y + 3z)\ (2x - y - 3z)$

(ii) $16 - x^2 - 2xy - y^2$

$= 16 - (x^2 + 2xy + y^2)$

$= 4^2 - (x + y)^2$

$= \{4 + (x + y)\}\{4 - (x + y)\}$

$= (4 + x + y)\ (4 - x - y)$

(iii) $x^4 - (x - z)^4 = (x^2)^2 - \{(x - z)^2\}^2$

$= \{x^2 + (x - z)^2\}\{x^2 - (x - z)^2\}$

$= (x^2 + x^2 - 2xz + z^2)\ [\{x + (x - z)\}\{x - (x - z)\}]$

$= (2x^2 - 2xz + z^2)\ (x + x - z)\ (x - x + z)$

$= (2x^2 - 2xz + z^2)\ (2x - z)z$

Sol.34 (i) $4(x + y)^2 - 28y\ (x + y) + 49y^2$

$= \{2(x + y)\}^2 - 2 \times 2\ (x + y) \times 7y + (7y)^2$

$= \{2(x + y) - 7y\}^2 = (2x + 2y - 7y)^2 = (2x - 5y)^2$

(ii) $(2a + 3b)^2 + 2\ (2a + 3b)\ (2a - 3b) + (2a - 3b)^2$

$= [(2a + 3b) + (2a - 3b)]^2 = (4a)^2 = 16a^2$

Sol.35 (i) $9x^2 - 4y^2 = (3x)^2 - (2y)^2 = (3x + 2y)\ (3x - 2y)$

(ii) $36x^2 - 12x + 1 - 25y^2$

$= (6x)^2 - 2 \times 6x \times 1 + 1^2 - (5y)^2$

$= (6x - 1)^2 - (5y)^2$

$= \{(6x - 1) - 5y\}\{(6x - 1) + 5y\}$

$= (6x - 1 - 5y)\ (6x - 1 + 5y)$

$= (6x - 5y - 1)\ (6x + 5y - 1)$

(iii) $a^2 - 1 + 2x - x^2 = a^2 - (1 - 2x + x^2)$

$= a^2 - (1^2 - 2 \times 1 \times x + (x)^2)$

$= a^2 - (1 - x)^2$

$= \{a - (1 - x)\}\ \{a + (1 - x)\}$

$= (a - 1 + x)\ (a + 1 - x)$

Sol.36 We have,

(i) $9 - a^6 + 2a^3b^3 - b^6 = 9 - (a^6 - 2a^3b^3 + b^6)$

$= 3^2 - \{(a^3)^2 - 2 \times a^3 \times b^3 + (b^3)^2\}^2$

$= \{3 + (a^3 - b^3)\}\ \{3 - (a^3 - b^3)\}$

$= (3 + a^3 - b^3)\ (3 - a^3 + b^3)$

$= (a^3 - b^3 + 3)\ (-a^3 + b^3 + 3)$

(ii) $x^{16} - y^{16} + x^8 + y^8 = \{(x^8)^2 - (y^8)^2\} + (x^8 + y^8)$

$= (x^8 - y^8)(x^8 + y^8) + (x^8 + y^8)$

$= (x^8 + y^8)(x^8 - y^8 + 1)$

(iii) $(p + q)^2 - (a - b)^2 + p + q - a + b$

$= \{(p + q)^2 - (a - b)^2\} + (p + q) - (a - b)$

$= \{(p + q) + (a - b)\}\{(p + q) - (a - b)\} + \{(p + q) - (a - b)\}$

$= (p + q + a - b)(p + q - a + b) + (p + q - a + b)$

$= (p + q - a + b)(p + q + a - b + 1)$

Sol.37 The given expression is $(2x + 3y)^2 - 5(2x + 3y) - 14$

Let $2x + 3y = a$. Then,

$(2x + 3y)^2 - 5(2x + 3y) - 14 = a^2 - 5a - 14$

$= a^2 - 7a + 2a - 14$

$= a(a - 7) + 2(a - 7)$

$= (a - 7)(a + 2)$

$= (2x + 3y - 7)(2x + 3y + 2)$

Sol.38 We have,

$3m^2 + 24m + 36$

$= 3(m^2 + 8m + 12)$

[Making coefficient of m^2 as 1]

$= 3\{m^2 + 8m + 4^2 - 4^2 + 12\}$

$\left[\text{Adding and subtracting} \left(\frac{8}{2}\right)^2 = 4^2\right]$

$= 3\{m^2 + 2 \times m \times 4 + 4^2 - 4\}$

$= 3\{(m + 4)^2 - 2^2\}$ [Completing the square]

$= 3\{(m + 4) - 2\}\{(m + 4) + 2\}$

$= 3(m + 4 - 2)(m + 4 + 2)$

$= 3(m + 2)(m + 6)$

Sol.39 (i) We have,

$$\frac{6x^4yz - 3xy^3z + 8x^2yz^4}{2xyz} = \frac{6x^4yz}{2xyz} - \frac{3xy^3z}{2xyz} + \frac{8x^2yz^4}{2xyz} = 3x^3 - \frac{3}{2}y^2 + 4xz^3$$

(ii) We have,

$$\frac{\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^3 - \frac{1}{5}ab^3c^2}{\frac{1}{2}abc}$$

$$= \frac{\frac{2}{3}a^2b^2c^2}{\frac{1}{2}abc} + \frac{\frac{4}{3}ab^2c^3}{\frac{1}{2}abc} - \frac{\frac{1}{5}ab^3c^2}{\frac{1}{2}abc}$$

$$= \frac{4}{3}abc + \frac{8}{3}bc^2 - \frac{2}{5}b^2c$$

Verification: We have,

Divisor × Quotient + Remainder

$= (3y^3 + 1)(y^2 + 2y + 2) + 6y^2 + 6y + 7$

$= 3y^3(y^2 + 2y + 2) + 1(y^2 + 2y + 2) + 6y^2 + 6y + 7$

$= 3y^5 + 6y^4 + 6y^3 + y^2 + 2y + 2 + 6y^2 + 6y + 7$

$= 3y^5 + 6y^4 + 6y^3 + 7y^2 + 8y + 9 =$ Dividend

Sol.43 We have,

$16x^4 + 12x^3 - 10x^2 + 8x + 20$

$= 4x^3(4x - 3) + 6x^2(4x - 3) + 2x(4x - 3) + \frac{7}{2}(4x - 3) + \frac{61}{2}$

$= (4x - 3)\left(4x^3 + 6x^2 + 2x + \frac{7}{2}\right) + \frac{61}{2}$

Hence, Quotient $= 4x^3 + 6x^2 + 2x + \frac{7}{2}$ and, Remainder $= \frac{61}{2}$

Sol.44 We have,

$8y^3 - 6y^2 + 4y - 1$

$= 2y^2(4y + 2) - \frac{5}{2}y(4y + 2) + \frac{9}{4}(4y + 2) - \frac{11}{2}$

$= \left\{2y^2(4y+2) - \frac{5}{2}y(4y+2) + \frac{9}{4}(4y+2)\right\} - \frac{11}{2}$

$= (4y + 2)\left(2y^2 - \frac{5}{2}y + \frac{9}{4}\right) - \frac{11}{2}$

Hence, Quotient $= 2y^2 - \frac{5}{2}y + \frac{9}{4}$ and, Remainder $= -\frac{11}{2}$.

Sol.45 We have,

$a^4 - b^4 = (a^2)^2 - (b^2)^2$

$\Rightarrow a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$

[Using: $x^2 - y^2 = (x + y)(x - y)$]

$\Rightarrow a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$...(i)

$\therefore (a^4 - b^4) \div (a - b)$

$= \frac{a^4 - b^4}{a - b}$

$= \frac{(a-b)(a+b)(a^2+b^2)}{(a-b)}$ [Using (i)]

$= (a + b)(a^2 + b^2)$

[Cancelling common factor $(a - b)$ in N^r and D^r]

Sol.46 We have,

$$x^{4a} + x^{2a}y^{2b} + y^{4b}$$
$$= (x^{2a})^2 + x^{2a}y^{2b} + (y^{2b})^2$$
$$=(x^{2a})^2 + 2x^{2a}y^{2b} + (y^{2b})^2 - x^{2a}y^{2b}$$
$$= (x^{2a} + y^{2b})^2 - (x^a y^b)^2$$
$$= (x^{2a} + y^{2b} + x^a y^b)\ (x^{2a} + y^{2b} - x^a y^b)$$
$$= (x^{2a} + x^a y^a + y^{2b})\ (x^{2a} - x^a y^b + y^{2b})$$

$$\therefore \quad \frac{x^{4a} + x^{2a}y^{2b} + y^{4b}}{x^{2a} + x^a y^b + y^{2b}}$$

$$= \frac{(x^{2a} + x^a y^b + y^{2b})(x^{2a} - x^a y^b + y^{2b})}{(x^{2a} + x^a y^b + y^{2b})}$$

$$= x^{2a} - x^a y^b + y^{2b}$$

HINTS & SOLUTION - 2

18.[B] Using $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$,

we get : $x^{1/3} + y^{1/3} + z^{1/3} = 0$

$\Rightarrow x + y + z = 3x^{1/3}\, y^{1/3}\, z^{1/3}$

$\Rightarrow (x + y + z)^3 = 27xyz$.

19.[C] $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.

20.[D] $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$;

$\therefore \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$

or $\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$.

21.[A] $(x^3 + y^3 + z^3 - 3xyz)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$

$= 9 \times (81 - 3 \times 23) = (2 \times 12) = 108$.

27.[D] $\frac{x}{y} = \frac{a^2 - b^2}{ab} \Rightarrow \frac{x + y}{x - y} = \frac{a^2 - b^2 + ab}{a^2 - b^2 - ab}$

Also, $(x - y) = \frac{a^2 + b^2}{ab}$ ....(i)

$\therefore x + y = \frac{(a^2 - b^2 + ab)}{(a^2 - b^2 - ab)} \cdot \frac{(a^2 + b^2)}{ab}$ ....(ii)

Adding (i) and (ii) we get :

$x = \frac{1}{2}\left(\frac{a^2 + b^2}{ab}\right) \cdot \left[1 + \frac{a^2 - b^2 + ab}{a^2 - b^2 - ab}\right]$

$= \frac{(a^4 - b^4)}{(a^2 - b^2 - ab)}$

29.[C] Since $(x + 2)$ is a factor, so $x = -2$ will make it zero.

$\therefore (-2)^3 + 6(-2)^2 + 4(-2) + k = 0$

or $k = -8$.

43.[A] Let $f(x) = x^5 - 9x^2 + 12x - 14$. This when divided by $(x - 3)$ gives remainder

$f(3) = 3^5 - 9 \times 3^2 + 12 \times 3 - 14 = 184$.

44.[B] Remainder is

$f(2) = 2^4 - 3 \times 2^3 + 2 \times 2^2 - 5 \times 2 + 7 = -3.$

49.[C] On actually dividing $x^3 + 5x^2 + 10k$ by $(x^2 + 2)$, the remainder obtained is $-2x + 10k - 10$.

$\therefore\ -2x + 10k - 10 = -2x$ or $k = 1$.

56.[D] $\frac{4}{x^4} + \frac{x^4}{81} = \left(\frac{2}{x^2}\right)^2 + \left(\frac{x^2}{9}\right)^2 + 2 \times \frac{2}{x^2} \times \frac{x^2}{9} - \frac{4}{9}$

$= \left(\frac{2}{x^2} + \frac{x^2}{9}\right)^2 - \left(\frac{2}{3}\right)^2$

$= \left(\frac{2}{x^2} + \frac{x^2}{9} + \frac{2}{3}\right)\left(\frac{2}{x^2} + \frac{x^2}{9} - \frac{2}{3}\right)$