Chapter 14

Theorems on area

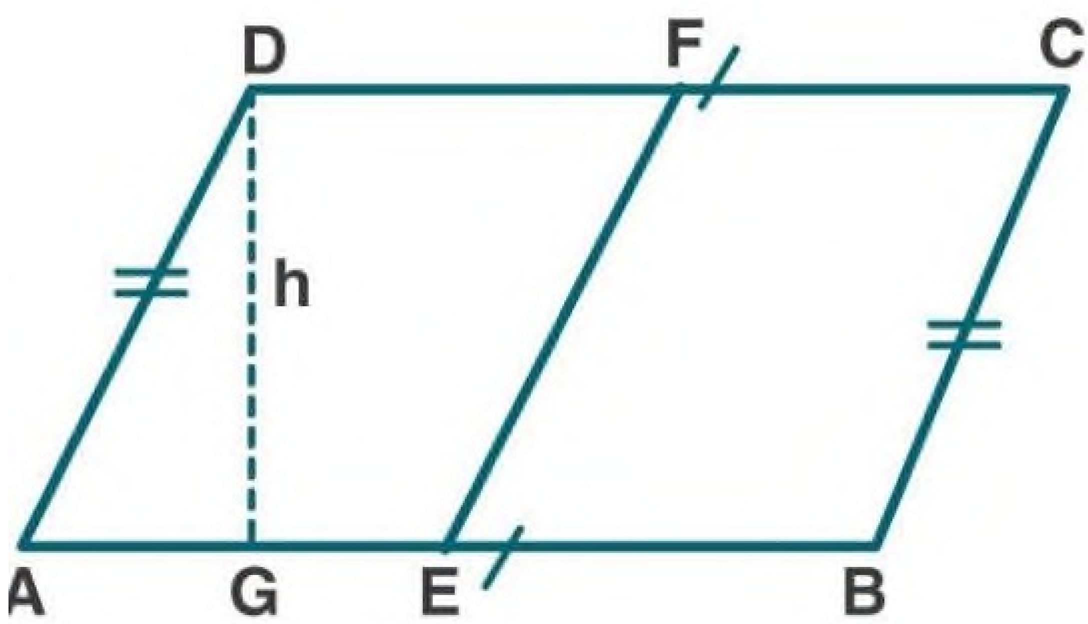
<u>Exercise 14</u>

1. prove that the line segments joining the mid – points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Solution

Let us consider ABCD be a parallelogram in which E and F are mid- points of AB and CD. join EF.

To prove : ar(∥ AEFD) = ar (∥ EBCF)



Let us construct DG ⊥ AG and let DG = h where, h is the altitude on side AB.

Proof :

ar(∥ ABCD) = AB × h

ar(∥ AEFD) = AE × h

$= \frac{1}{2} AB \times h$.....(1) [since, E is the mid- point of AB]

ar(∥ EBCF) = EF × h

$= \frac{1}{2} AB$ × h ....(2) [sine , E is the mid point of AB]

From(1) and (2)

ar(∥ ABFD) = ar (∥ EBCF)

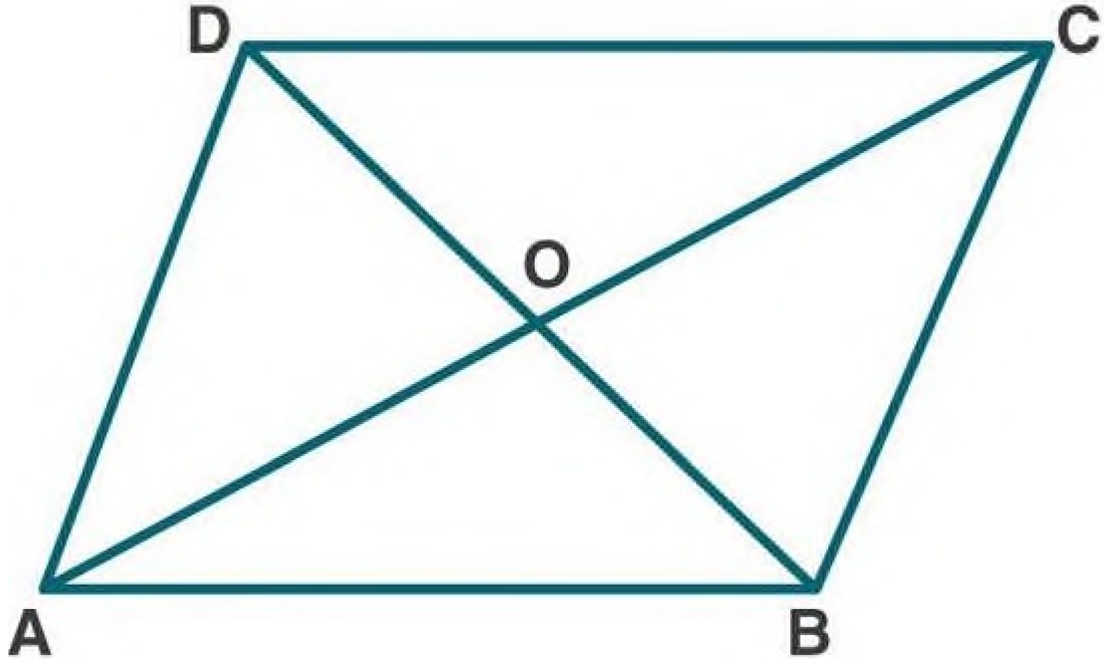
hence proved.

2. prove that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution

Let us consider in a parallelogram ABCD the diagonals AC and BD are cut at point O.

To prove: ar(Δ AOB) = ar(Δ BOC) = ar(Δ COD) = ar(Δ AOD)



Proof:

In parallelogram ABCD the diagonals bisect each other

AO = OC

In Δ ACD, O is the mid point of AC. DO is the median

ar(ΔAOD) = ar (COD) ......(1) [median of Δ divides it into two triangle of equal areas]

similarly , in Δ ABC

ar(Δ AOB) = ar(Δ COB).....(2)

in ΔADB

ar(ΔAOD) = ar(ΔAOB)...(3)

in ΔCDB

ar(ΔCOD) = ar(Δ COB)...(4)

from (1),(2) ,(3) and (4)

ar (ΔAOB) = ar(ΔBOC) = ar(ΔCOD) = ar(ΔAOD)

hence proved.

3. (a) in the figure (1) given below , AD is medium of ΔABC and P is any point on AD. Prove that

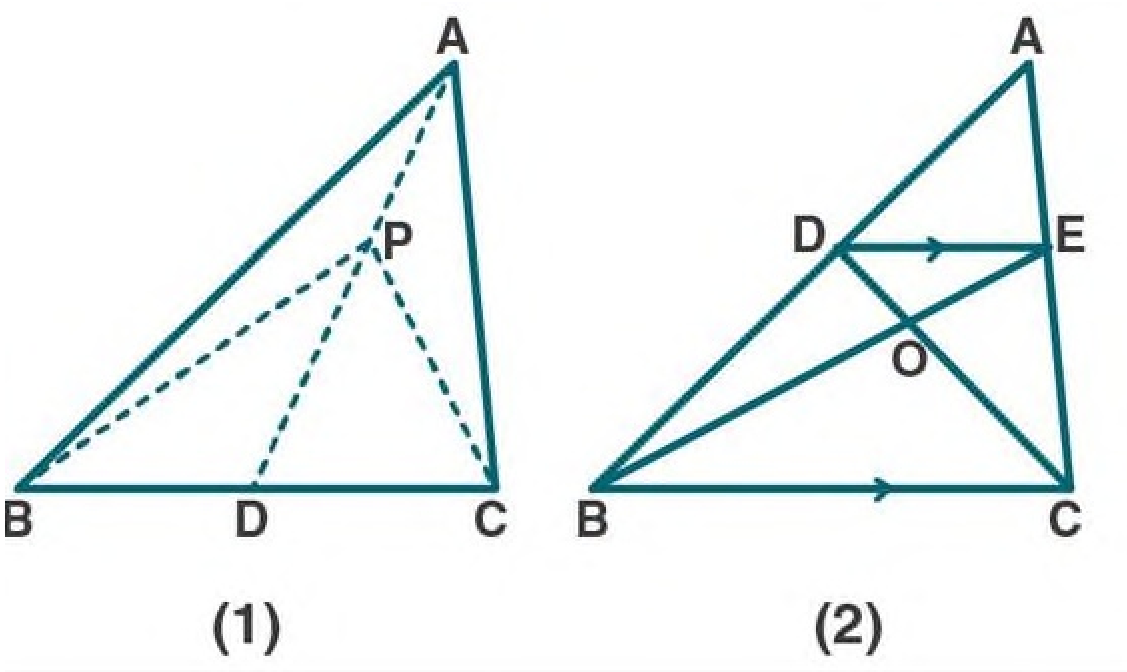
(i) area of ΔPBD = area of Δ PDC

(ii) area of Δ ABP = area of Δ ACP.

(b) in the figure(2) given below , DE∥ BC. Proved that

(i) area of ΔACD = area of Δ ABE

(ii) area of ΔOBD = area of Δ OCE.



Solution

(a) given :

Δ ABC in which AD is the median . P is any point on AD. Join PB and PC.

To prove :

(i) area of ΔPBD = area of Δ PDC

(ii) area of Δ ABP = area of Δ ACP

Proof :

From fig(1)

AD is a median of Δ ABC

So, ar (Δ ABD) = ar(Δ ADC) ...(1)

Also, PD is the median of Δ BPD

Similarly , ar(Δ PBD) = ar(Δ PDC) ....(2)

Now , let us subtract (2) from (1) we get

ar(ΔABD) – ar (Δ PBD) = ar (Δ ADC) – ar(Δ PDC)

or ar(ΔABP) = ar(ΔACP)

hence proved.

(b) given:

Δ ABC in which DE ‖ BC

To prove:

(i) area of Δ ACD = area of ΔABE

(ii) area of Δ OBD = area of Δ OCE.

Proof:

From fig(2)

Δ DEC and ΔBDE are on the same base DE and between the same ‖ line DE and BE.

ar(Δ DEC) = ar(Δ BDE)

now add ar(ADE) on both sides , we get

ar(ΔDEC) + ar(ΔADE) = ar(ΔBDE) + ar(ΔADE)

ar(ΔACD) = ar(ΔABE)

hence proved.

Similarly ,ar (Δ DEC) = ar(ΔBDE)

Subtract ar(ΔDOE) from both sides, we get

ar(ΔDEC) – ar(ΔDOE) = ar(ΔBDE) – ar(ΔDOE)

ar(Δ OBD) = ar(ΔOCE)

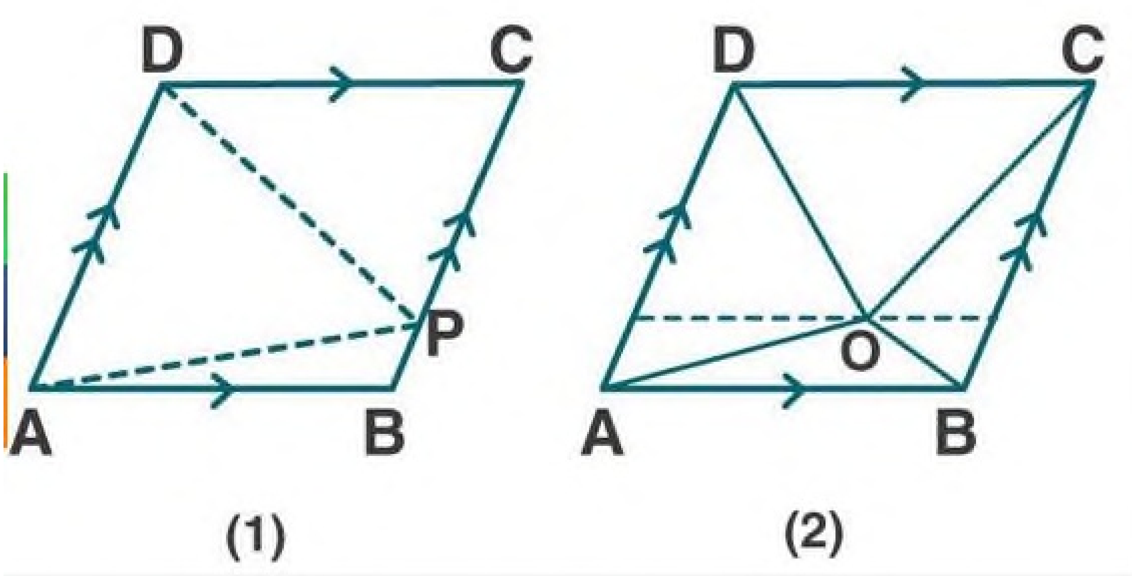
hence proved.

4. (a) in the figure(1) given below , ABCD is a parallelogram and P is any point in BC. Prove that : Area of ΔABP + area of ΔDPC = area of Δ APD.

(b) in the figure (2) given below, O is any point inside a parallelogram ABCD. Prove that

(i) area of ΔOAB + area of ΔOCD = $\frac{1}{2}$ area of ∥ gm ABCD

(ii) area of Δ OBC + area of ΔOAD = $\frac{1}{2}$ area of ∥ gm ABCD



Solution

(a) given :

From fig(1)

ABCD is a parallelogram and P is any point in BC

To prove :

Area of ΔABP + area of Δ DPC = area of Δ APD

Proof:

Δ APD and ∥ gm ABCD are on the same base AD and between the same ∥ lines AD and BC,

ar(ΔAPD) = $\frac{1}{2}$ar(∥ gmABCD)....(1)

in parallelogram ABCD

ar(∥ gm ABCD) = ar(ΔABP) + ar(ΔAPD) + ar(ΔDPC)

now , divide both sides by 2, we get

now , divide both sides by 2, we get

$\frac{1}{2}$ ar(∥ gm ABCD) = $\frac{1}{2}$ ar(Δ ABP) + $\frac{1}{2}$ar(ΔAPD) + $\frac{1}{2}$ar(ΔDPC)

...(2)

From (1) and (2)

Ar(Δ APD) = $\frac{1}{2}$ar(∥ gm ABCD)

Substituting (2) in (1)

ar(Δ APD) = $\frac{1}{2}$ar(ΔABP) + $\frac{1}{2}$ ar(ΔAPD) + $\frac{1}{2}$ar(ΔDPC)

ar (Δ APD) – $\frac{1}{2}$ar(ΔAPD) = $\frac{1}{2}$ar (Δ ABP) + $\frac{1}{2}$ar (ΔDPC)

$\frac{1}{2}$ar(ΔAPD) = $\frac{1}{2}$[ar(ΔABP) + ar(Δ DPC)]

Or ar(ΔABP) + ar(Δ DPC) = ar(Δ APD)

Hence proved.

(b) given:

From fig (2)

∥ gm ABCD in which O is any point inside it.

To prove:

(i) area of Δ OAB + area of ΔOCD = $\frac{1}{2}$ area of ∥ gmABCD

(ii) area of ΔOBC + area of ΔOAD = $\frac{1}{2}$ area of ∥ gm ABCD

Draw POQ ∥ AB through o. It meets AD at P and BC at Q.

Proof :

(i) AB ∥ PQ and AP ∥ BQ

ABQP is a ∥ gm

Similarly , PQCD is a ∥ gm

Now , ΔOAB and ∥ gm ABQP are on same base AB and between same ∥ lines AB and PQ

ar(ΔOAB) = $\frac{1}{2}$ar(∥ gm ABQP)....(1)

similarly , ar (Δ OCD) = $\frac{1}{2}$ar(∥ gm PQCD)...(2)

now adding (1) and (2)

ar(Δ OAB) + ar(ΔOCD) = $\frac{1}{2}$ar(∥ gm ABQP) + $\frac{1}{2}$ar(∥ gm PQCD)

= $\frac{1}{2}$[ar(∥ gm ABQP) + ar(∥ gm PQCD)]

= $\frac{1}{2}$ar(∥ gm ABCD)

ar(ΔOAB) + ar(ΔOCD) = $\frac{1}{2}$ ar(∥ gmABCD)

hence proved.

(ii) we know that,

ar(ΔOAB) + ar(ΔOBC) + ar(Δ OCD) + ar(Δ OAD) = ar(∥ gm ABCD)

[ar(ΔOAB) + ar(ΔOCD)] + [ar(ΔOBC) + ar(ΔOAD) = ar(∥ gm ABCD)

$\frac{1}{2}$ ar(∥ gm ABCD) + ar(Δ OBC) + ar(ΔOAD) = ar(∥gm ABCD)

ar(ΔOBC) + ar(ΔOAD) = ar(∥ gm ABCD) – $\frac{1}{2}$ar(∥ gm ABCD)

ar(ΔOBC) + ar(Δ OAD) = $\frac{1}{2}$ ar(∥ gm ABCD)

hence proved.

5. if E,F,G and H are mid- points of the sides AB, BC,CD and DA respectively of a parallelogram ABCD, prove that area of quad. EFGH = $\frac{1}{2}$ area of ∥ gm ABCD.

Solution

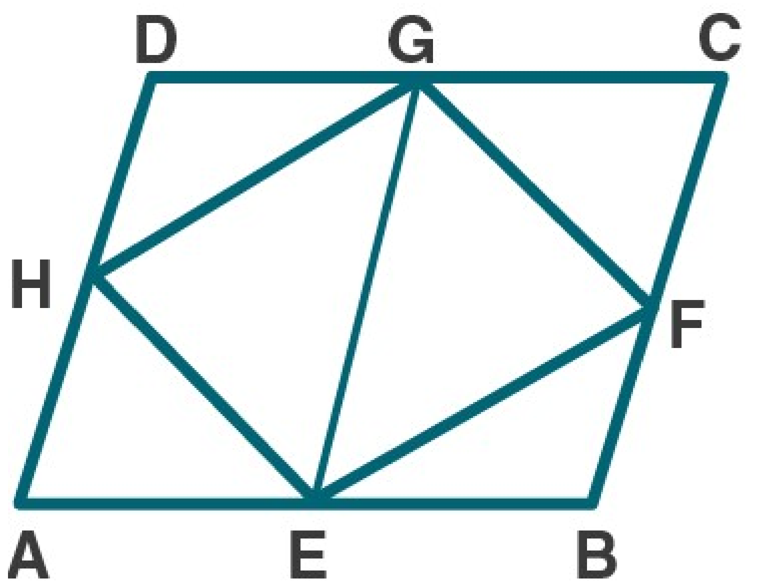
Given

In parallelogram ABCD, E,F,G,H are the mid – points of its sides AB,BC,CD and DA

Join EF,FG,GH and HE.

To prove :

Area of quad. EFGH = $\frac{1}{2}$ area of ∥ gm ABCD



Proof:

Let us join EG

We know that, E and G are mid- points of AB and CD

EG|| AD || BC

AEGD and EBCG are parallelogram

Now, || gm AEGD and Δ EHG are on the same base and between the parallel lines.

arΔEHG = $\frac{1}{2}$ ar ||gm AEGD.....(1)

similarly,

arΔEFG = $\frac{1}{2}$ar || gm EBCG....(2)

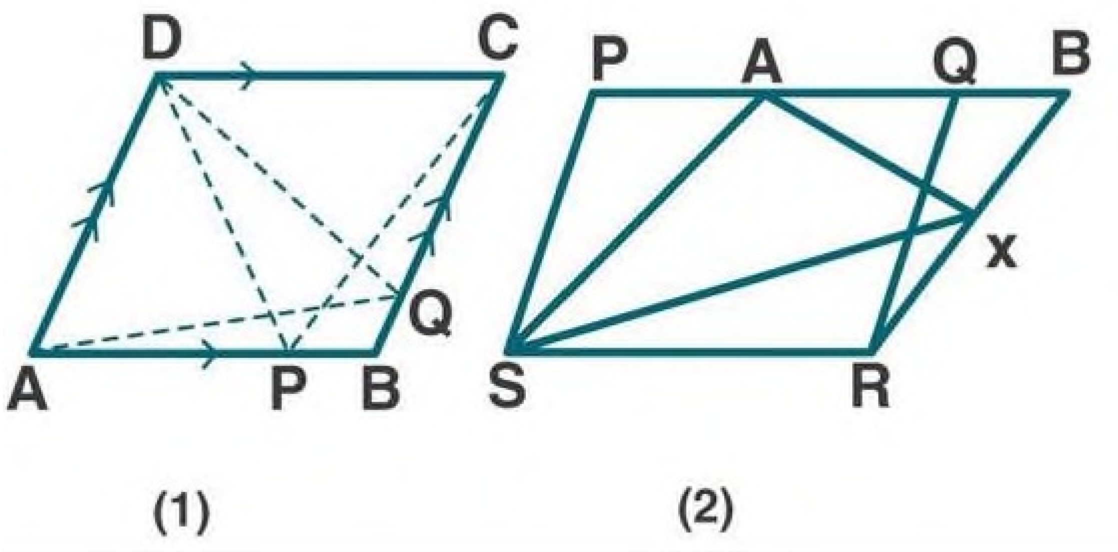
now by adding (1) and (2)

arΔ EHG + arΔ EFG = $\frac{1}{2}$ ar ||gm AEGD + $\frac{1}{2}$ ar || gm EBCG

area quad. EFGH = $\frac{1}{2}$ ar || gm ABCD

Hence proved.

6(a) In the figure(1) given below ,ABCD is a parallelogram. P,Q are any two points on the sides AB and BC respectively. Prove that, area of Δ CPD = area of Δ AQD.



(b) in the figure(2) given below, PQRS and ABRS are parallelograms and x is any point on the side BR. Show that area of ΔAXS = $\frac{1}{2}$ area of ||gm PQRS.

Solution

(a) given :

From fig (1)

∥ gm ABCD in which P is a point on AB and Q is a point on BC.

To prove:

Area of Δ CPD = area of Δ AQD.

Proof:

ΔCPD and ∥ gm ABCD are on the same base CD and between the same parallels AB and CD

ar(Δ CPD) = $\frac{1}{2}$ ar (∥ gm ABCD).....(1)

Δ AQD and ∥gm ABCD are on the same base AD and between the same parallels AD and BC.

ar(Δ AQD) = $\frac{1}{2}$ ar(∥ gm ABCD).....(2)

from (1) and (2)

ar (Δ CPD) = ar(Δ AQD)

hence proved.

(b) from fig (2)

Given:

PQRS and ABRS are parallelograms on the same base SR. X is any point on the side BR.

Join AX and SX.

To prove:

Area of Δ AXS = $\frac{1}{2}$ area of ∥gm PQRS

We know that , ||gm PQRS and ABRS are on the same SR and between the same parallels.

So , ar ||gm PQRS = ar ||gm ABRS....(1)

We know that, Δ AXS and ||gm ABRS are on the same base AS and between the same parallels.

So, arΔ AXS = $\frac{1}{2}$ ar ||gm ABRS

= $\frac{1}{2}$ ar ||gm PQRS [from (1)]

Hence proved.

7. D,E and F are mid – point of the sides BC, CA and AB respectively of a Δ ABC . prove that

(i) FDCE is a parallelogram

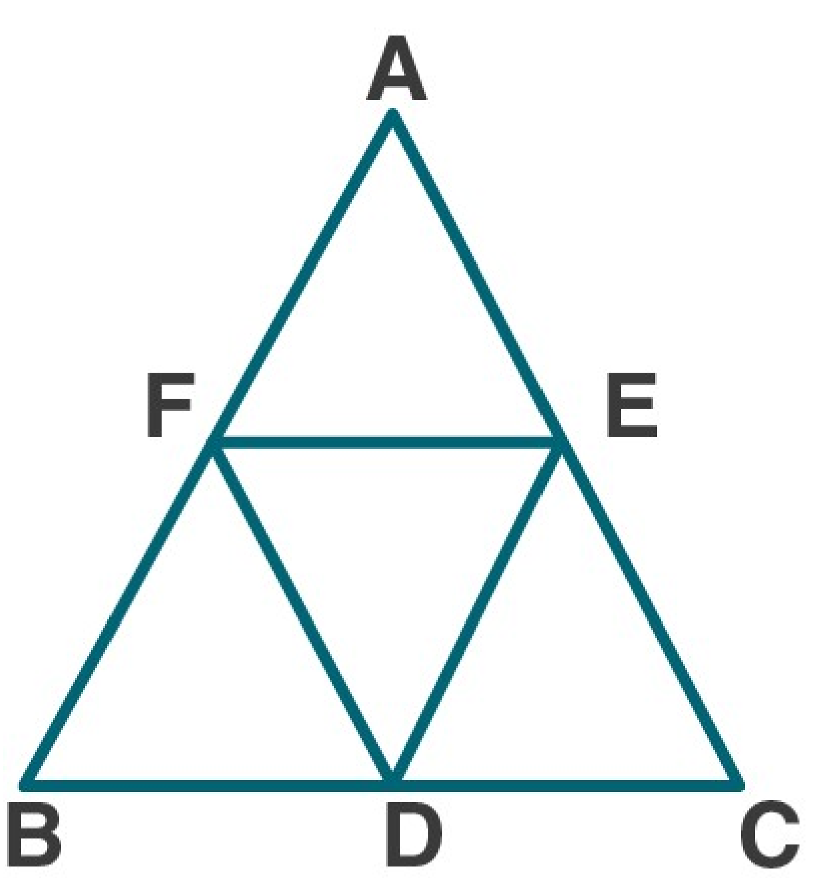
(ii) area of Δ DEF = $\frac{1}{4}$ area of Δ ABC

(iii) area of ||gm FDCE = $\frac{1}{2}$ area of Δ ABC

Solution :

Given :

D,E and F are mid- point of the sides BC, CA and AB respectively of a Δ ABC.



To prove:

(i) FDCE is a parallelogram

(ii) area of Δ DEF = $\frac{1}{4}$ area of Δ ABC

(iii) area of ∥ gm FDCE = $\frac{1}{2}$ area of Δ ABC

Proof :

(i) F and E are mid – points of AB and AC.

So, FE ∥ BC and FE = $\frac{1}{2}$ BC....(1)

Also, D is mid point of BC

CD = $\frac{1}{2}$ BC ....(2)

From (1) and (2)

FE ∥ BC and FE = CD

FE ∥ CD and FE = CD....(3)

Similarly,

D and F are mid- points of BC and AB.

So DF ∥ EC is a parallelogram.

Hence proved.

(ii) we know that, FDCE is a parallelogram

And DE is a diagonal of ∥gm FDCE

So, ar(Δ DEF) = ar(Δ DEC) ....(4)

Similarly, we know BDEF and DEAF are ∥ gm

So, ar(Δ DEF) = ar(Δ BDF) = ar(Δ AFE)....(5)

From(4) and (5)

ar(Δ DEF) = ar(Δ DEC) = ar(Δ BDF) = ar(Δ AFE)

now, ar(Δ ABC) = ar (Δ DEF) + ar(Δ DEF) + ar(Δ DEF) + ar(Δ DEF)

= 4 ar(Δ DEF)

ar(Δ DEF) = $\frac{1}{4}$ar(Δ ABC)....(6)

hence proved.

(iii) ar of ∥ gm FDCE = ar(Δ DEF) + ar(Δ DEC)

= ar(Δ DEF) + ar(ΔDEF)

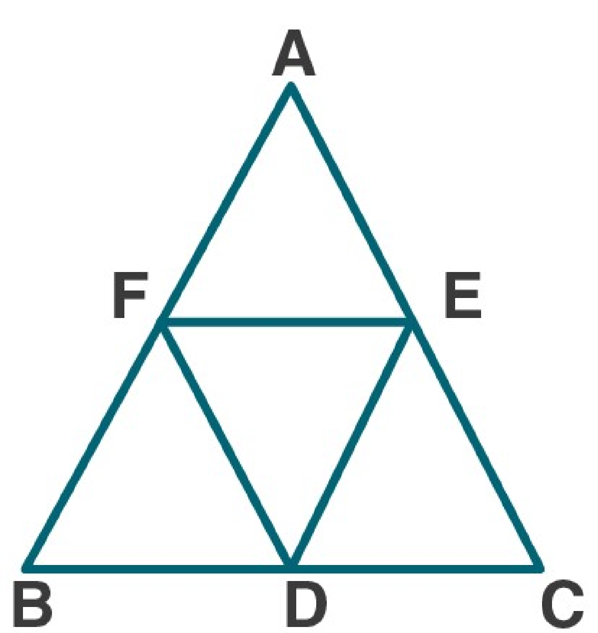
= 2 ar(Δ DEF) [from (4)]

= $2\left[\frac{1}{4} ar(\Delta\ ABC)\right]$ [from (6)]

ar of ∥ gm FDCE = $\frac{1}{2}$ ar of Δ ABC

hence proved.

8. in the given figure, D,E and F are mid- points of the sides BC, CA and AB respectively of Δ ABC. Prove that BCEF is a trapezium and area of trap. BCEF = $\frac{3}{4}$ area of Δ ABC.



Solution

Given:

In Δ ABC, D, E and F are mid – points of the sides BC, CA and AB.

To prove:

Area of trap. BCEF = $\frac{3}{4}$area of Δ ABC

Proof:

We know that D and E are the mid – points of BC and CA.

So , DE ∥ AB and $\frac{1}{2}$ AB

Similarly,

EF ∥ BC and $\frac{1}{2}$ BC

And FD ∥ AC and $\frac{1}{2}$AC

∴ BDEF, CDFE , AFDE are parallelograms which are equal in area.

ED, DF , EF are diagonals of these ∥gm which divides the corresponding parallelogram into two triangle equal in area.

Hence , BCEF is a trapezium.

Area of trap. BCEF = $\frac{3}{4}$ area of Δ ABC

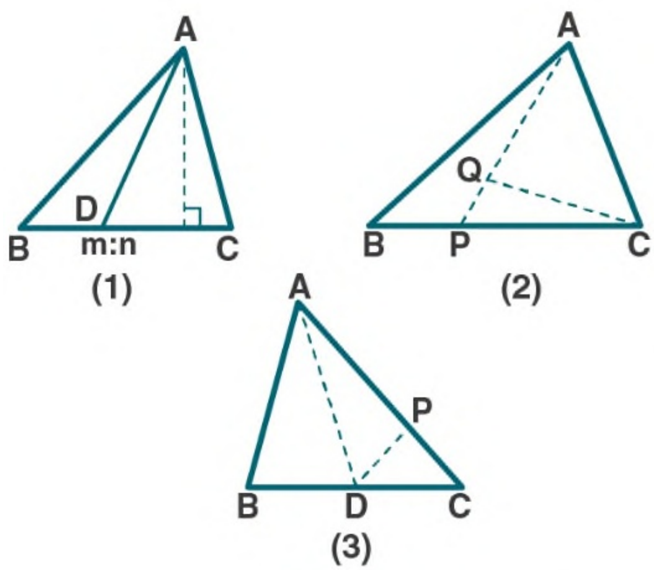
9. (a) in the figure (1) given below, the point D divides the side BC of ΔABC in the ratio m : n. Prove that area of Δ ABD : area of Δ ADC = m : n.

(b) in the figure(2) given below, P is a point on the side BC of ΔABC such that PC = 2BP, and Q is a point on AP such that QA = 5 PQ, find area of Δ AQC : area of Δ ABC.

(C) In the figure(3) given below, AD is a median of ΔABC and P is a point in AC such that area of Δ ADP : area of Δ ABD = 2:3 find

(i) AP: PC

(ii) area of Δ PDC : area of ΔABC.



Solution

(a) given:

From fig(1)

In ΔABC, the point D divides the side BC in the ratio m:n.

BD : DC = m:n

To prove:

Area of Δ ABD : area of Δ ADC = m :n

Proof:

Area of Δ ABD = $\frac{1}{2} \times base \times height$

ar(Δ ABD) = $\frac{1}{2} \times BD \times AE$.....(1)

ar(Δ ACD) = $\frac{1}{2} \times DC \times AE$.....(2)

let us divide (1) by (2)

$$\frac{\left[ar(\Delta\, ABD) = \frac{1}{2} \times BD \times AE\right]}{\left[ar(\Delta\, ACD) = \frac{1}{2} \times DC \times AE\right]} \quad \frac{[ar(\Delta\, ABD)]}{[ar(\Delta ACD)]} = \frac{BD}{DC}$$

= $\frac{m}{n}$ [it is given that, BD : DC = m : n]

Hence proved.

(b) given:

From fig(2)

In ΔABC, P is a point on the side BC such that PC = 2BP, and Q is a point on AP such that QA = 5 PQ.

To find:

Area of Δ AQC : area of ΔABC

Now,

It is given that: PC = 2BP

$\frac{PC}{2} = BP$

We know that , BC = BP + PC

Now substituting the values, we get

BC = BP + PC

$= \frac{PC}{2} + PC$

$= \frac{PC+2PC}{2}$

$= \frac{3PC}{2}$

$\frac{2BC}{3} = PC$

ar(Δ APC) = $\frac{2}{3}\ ar(\Delta ABC)$....(1)

it is given that QA = 5PQ

$\frac{QA}{5} = PQ$

We know that, QA = QA + PQ

So, QA = $\frac{5}{6}$ AP

ar(Δ AQC) = $\frac{5}{6}$ ar(ΔAPC)

= $\frac{5}{6}\left(\frac{2}{3}\ ar(\Delta ABC)\right)$ [from(1)]

ar(Δ AQC) = $\frac{5}{9}$ ar(Δ ABC)

$\frac{\text{ar}\Delta AQC}{ar(\Delta AQC)} = \frac{5}{9}$

Hence proved.

(c) Given:

From fig (3)

AD is a median of Δ ABC and P is point in AC such that area of ΔADP : area of ΔABD = 2 :3

To find:

(i) AP : PC

(ii) area of Δ PDC : area of ΔABC

Now,

(i) we know that AD is the median of ΔABC

ar(Δ ABD) = ar(Δ ADC) = $\frac{1}{2}$ ar(Δ ABC)......(1)

it is given that

ar(ΔADP) : ar(Δ ABD) = 2 : 3

AP : AC = 2 : 3

$\frac{AP}{AC} = \frac{2}{3}$

AP = $\frac{2}{3}$ AC

Now ,

PC = AC –AP

= AC – $\frac{2}{3}$AC

= $\frac{3AC - 2AC}{3}$

= $\frac{AC}{3}$.....(2)

So,

$\frac{AP}{PC} = \frac{\left(\frac{2}{3AC}\right)}{\frac{AC}{3}}$

= $\frac{2}{1}$

AP : PC = 2: 1

(ii) we know that form (2)

PC = $\frac{AC}{3}$

$\frac{PC}{AC} = \frac{1}{3}$

So,

$\frac{\text{ar}(\Delta\, PDC)}{ar(\Delta ADC)} = \frac{PC}{AC} = \frac{1}{3}$

$$\frac{\frac{ar(\Delta PDC)}{1}}{2ar(\Delta ABC)} = \frac{1}{3}$$

$$\frac{ar(\Delta PDC)}{ar(\Delta ABC)} = \frac{1}{3} \times \frac{1}{2}$$

$$= \frac{1}{6}$$

$\text{ar}(\Delta\, PDC): ar(\Delta\, ABC) = 1:6$

hence proved.

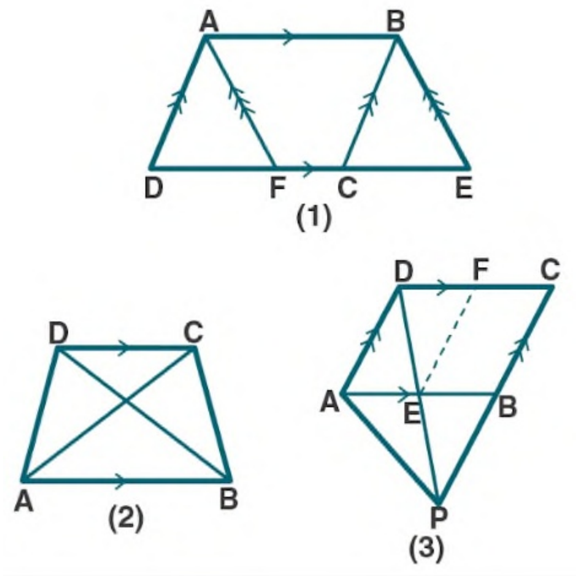
10.(a) in the figure (1) given below , area of parallelogram ABCD is 29 cm^2. Calculate the height of parallelogram ABEF if AB = 5.8 cm

(b) in the figure(2) given below , area of Δ ABD is 24sq. units. If AB = 8 units, find the height of ABC.

(c) in the figure (3) given below, E and F are mid-points of sides AB and CD respectively of parallelogram ABCD. If the area of parallelogram ABC is 36cm^2.

(i) state the area of Δ APD.

(ii) Name the parallelogram whose area is equal to the area of Δ APD.



Solution

(a) given :

From fig(1)

ar ∥ gm ABCD = 29 cm^2

to find :

height of parallelogram ABEF if AB = 5.8 cm

now let us find

we know that ∥ gm ABCD and ∥gm ABEF with equal bases and between the same parallels so that there area are same .

ar(∥ gm ABEF) = ar(∥ gm ABCD)

ar(∥ gm ABEF) = 29 cm^2 .....(1) [since , ar ∥gm ABCD = 29 cm^2]

also, ar (∥ gm ABEF = base × height)

29 = AB × height [from (1)]

29 = 5.8 $\times height$

Height = $\frac{29}{5.8}$

= 5

∴ height of parallelogram ABEF is 5 cm

(b) given:

From fig (2)

Area of Δ ABD is 24 sq. units . AB = 8 units

To find :

Height of ABC

Now, let us find

We know that ar Δ ABD = 24 sq. units ....(1)

So, ar Δ ABD = Δ ABC....(2)

From (1) and (2)

arΔ ABC = 24 sq. units

$\frac{1}{2} \times AB \times height$ = 24

$\frac{1}{2} \times 8 \times height$ = 24

$4 \times height$ = 24

$height = \frac{24}{4}$

= 6

∴ height of Δ ABC = 6 sq. units

(c) given :

From fig (3)

In ∥ gm ABCD, E and F are mid points of sides AB and CD respectively.

ar(∥ gm ABCD) = 36cm^2

to find:

(i) state the area of Δ APD.

(ii) Name the parallelogram whose area is equal to the area of Δ APD.

Now , let us find

(i) we know that Δ APD and ∥gm ABCD are on the same base AD and between the same parallel lines AD and BC.

ar(Δ APD) = $\frac{1}{2}$ $ar(\parallel gm\ ABCD)$....(1)

ar(∥ gm ABCD) = 36cm^2.....(2)

from(1) and (2)

ar(Δ APD) = $\frac{1}{2} \times 36$

= 18 cm^2

(ii) we know that E and F are mid – points of AB and CD

In Δ CPD, EF ∥ PC

Also, EF bisects the ∥ gm ABCD in two equal parts.

So EF ∥ AD and AE ∥ DF

AEFD is a parallelogram.

ar(∥ gm AEFD) = $\frac{1}{2}\, ar(\parallel gm\ ABCD)$.....(3)

from (1) and (3)

ar(Δ APD) = ar(∥ gm AEFD)

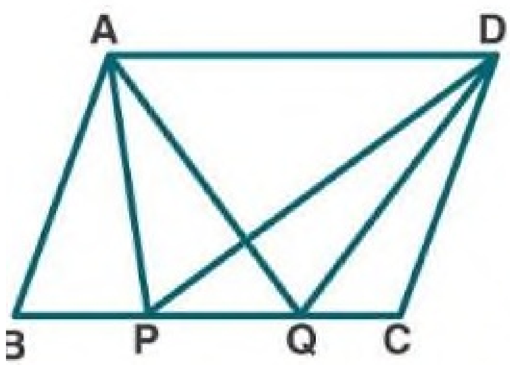
∴ AEFD is the required parallelogram which is equal to area of Δ APD.

11. (a) in the figure(1) given below, ABCD is a parallelogram. Points P and Q on BC trisect BC into three equal parts. Prove that:

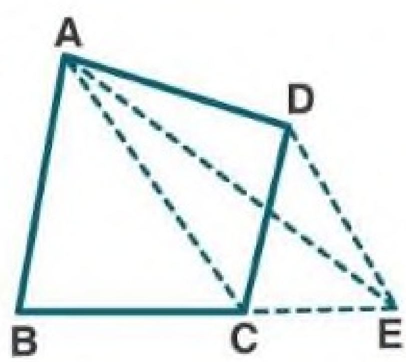
Area of Δ APQ = area of Δ DPQ = $\frac{1}{6}$ (area of ∥ gm ABCD)

(b) in the figure (2) given below, DE is drawn parallel to the diagonal AC of the quadrilateral ABCD to meet BC produced at the point E. Prove that area of quad. ABCD = area of Δ ABE

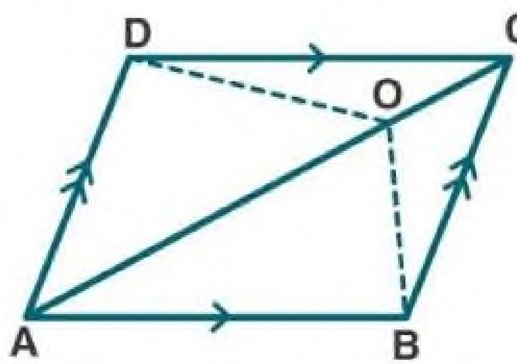
(c) in the figure(3) given below, ABCD is a parallelogram. O is any point on the diagonal AC of the parallelogram . show that the area of Δ AOB is equal to the area of Δ AOD.



(1)



(2)

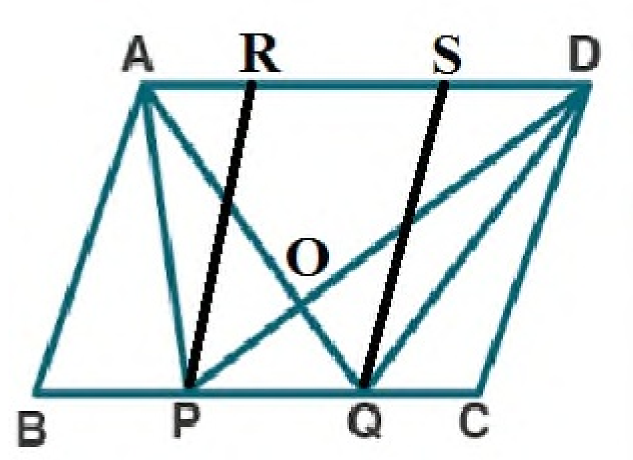


(3)

Solution

(a) Given:

From fig(1)



In ∥ gm ABCD, points P and Q trisect BC into three equal parts.

To prove :

Area of Δ APQ = area of Δ DPQ = $\frac{1}{6}$ (area of ∥ gm ABCD)

Firstly, let us construct: through p and Q, draw PR and QR parallel to AB and CD.

Proof:

ar(Δ APD) = ar(Δ AQD) [since, Δ APD and Δ AQD lie on the same base AD and between the same parallel lines AD and BC]

ar(Δ APD) – ar(Δ AOD) = ar(Δ AQD) – ar(Δ AOD) [on subtracting ar ΔAOD on both sides]

ar(Δ APO) = ar(Δ OQD)...(1)

ar(Δ APO) + ar(Δ OPQ) = ar(Δ OQD) + ar(Δ OPQ) [on adding ar ΔOPQ on both sides]

ar (ΔAPQ) = ar(Δ DPQ) ....(2)

we know that, Δ APQ and ∥gm PQRS are on the same base PQ and between same parallel lines PQ and AD.

ar(ΔAPQ) = $\frac{1}{2}$ ar(∥gm PQRS)....(3)

now,

$$\left[\frac{ar(\|gm\ ABCD)}{ar(\|gm\ PQRS)}\right] = \left[\frac{BC \times height}{PQ \times height}\right] = \left[\frac{3PQ \times height}{1PQ \times height}\right]$$

ar(∥ gm PQRS) = $\frac{1}{3}$ $ar(\|\ gmABCD)$....(4)

by using (2).(3),(4) we get

ar (ΔAPQ) = ar(Δ DPQ)

= $\frac{1}{2}$ ar (∥ gm PQRS)

= $\frac{1}{2} \times \frac{1}{3}$ $ar(\|\ gm\ ABCD)$

= $\frac{1}{6}$ ar(∥ gm ABCD)

Hence proved.

(b) given:

In the figure (2) given below, DE ∥ AC the diagonal of the quadrilateral ABCD to meet at point E on producing BC. Join AC, AE .

To prove:

Area of quad. ABCD = area of Δ ABE

Proof:

We know that , ΔACE and Δ ADE are on the same base AC and between the same parallelogram

ar(Δ ACE) = ar(Δ ADC)

now by adding ar(Δ ABC) on both sides, we get

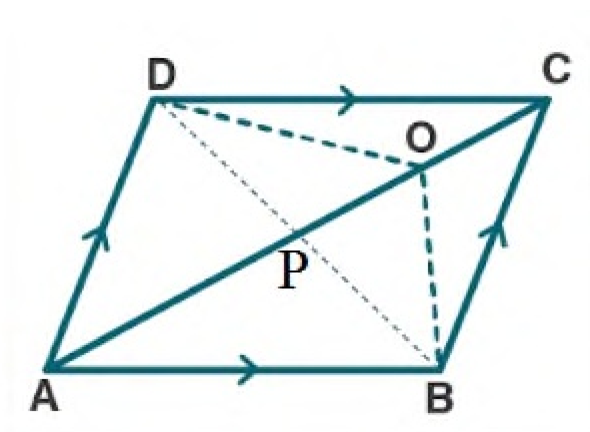
ar(Δ ACE) + ar(Δ ABC) = ar(Δ ADC) + ar(Δ ABC)

ar (Δ ABE) = ar quad. ABCD

hence proved.

(c) Given

From fig (3)



In ∥ gm ABCD, O is any point on diagonal AC.

To prove:

Area of Δ AOB is equal to the area of Δ AOD

Proof:

Let us join BD which meets AC at P.

In Δ ABD, AP is the median .

ar(Δ ABP) = ar(Δ ADP) .....(1)

similarly, ar(Δ PBO) = ar(Δ PDO)....(2)

now add (1) and (2) we get

ar(Δ ABO) = ar(Δ ADO).....(3)

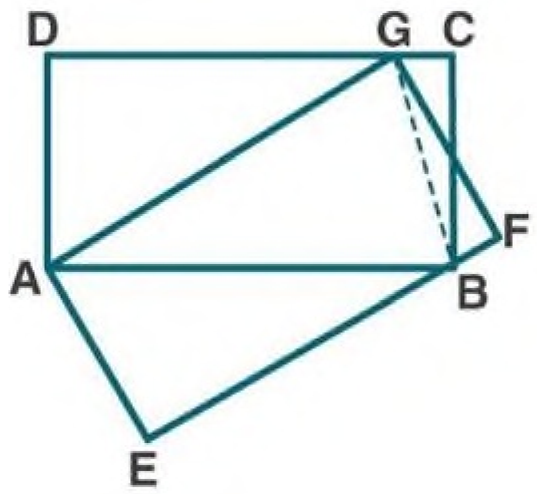
so,

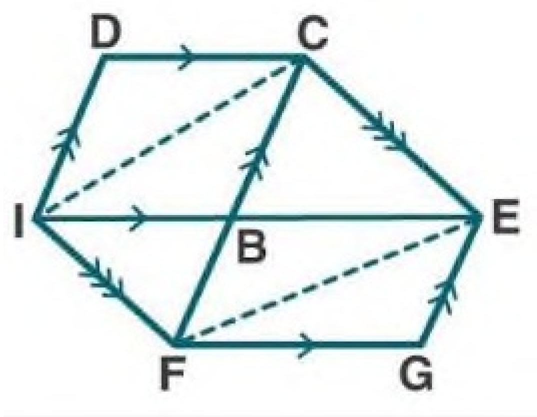
Δ AOB = arΔ AOD

$hence\ proved.$

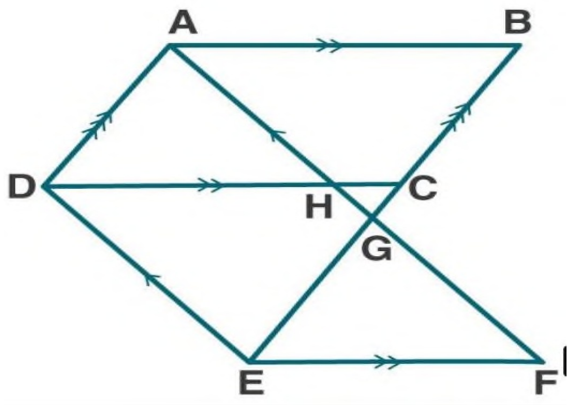
12. (a) In the figure given, ABCD and AEFG are two parallelograms. Prove that area of ‖ gm ABCD = area of ‖ gm AEFG.

(b) in the fig.(2) given below, the side AB of the parallelogram ABCD is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is completed prove that area of ‖ gm BFGE = area of ‖ gm ABCD.





(c) in the figure (3) given below AB ∥ $DC \parallel EF, AD \parallel BE$ and $DE \parallel AF$. Prove the area of DEFH is equal to the area of ABCD.



Solution

(a) Given

From fig(1)

ABCD and AEFG are two parallelograms as shown in the figure.

To prove:

Area of ∥ gm ABCD = area of ∥ gm AEFG

Proof:

Let us join BG.

We know that,

ar(Δ ABG) = $\frac{1}{2}$ $(ar \ \parallel gmABCD)$.....(1)

similarly,

ar(Δ ABG) = $\frac{1}{2}(ar \ \parallel gmAEFG)$....(2)

from (1) and (2)

$\frac{1}{2}(ar \parallel gmABCD) = \frac{1}{2}(ar \ \parallel gmAEFG)$

so,

ar(∥ gm ABCD = ar ∥ gm AEFG)

hence proved.

(b) given:

From fig (2)

A parallelogram ABCD in which AB is produced to E. A straight line through A is drawn parallel to CE to meet CB produced at F and parallelogram BFGE is completed.

To prove:

Area of ∥ gm BFGE = area of ∥ gm ABCD

Proof:

Let us join AC and EF.

We know that,

ar(ΔAFC) = ar(ΔAFE).....(1)

now subtract ar(Δ ABF) on both sides, we get

ar(Δ AFC) – ar(Δ ABF) = ar(Δ AFE) – ar(ΔABF)

or ar(Δ ABC) = ar(Δ BEF)

now multiply by 2 on both sides, we get

2. ar(Δ ABC) = 2. ar(Δ BEF)

Or ar(∥ gm ABCD) = ar(∥ gm BFGE)

Hence proved

(c) Given:

From fig(3)

AB $\parallel DC \parallel EF, AD \parallel BE \text{ and } DE \parallel AF$

To prove:

Area of DEFH = area of ABCD

Proof:

We know that,

$DE \parallel AF \text{ and } AD \parallel BE$

It is given that ADEG is a parallelogram .

So,

ar(∥ gm ABCD) = ar(∥ gm ADEG)....(1)

again , DEFG is a parallelogram

ar(∥ gm DEFH) = ar(∥ gm ADEG) .....(2)

From (1) and(2)

ar(∥ gm ABCD) = ar(∥gm DEFH)

or ar ABCD = ar DEFH

hence proved.

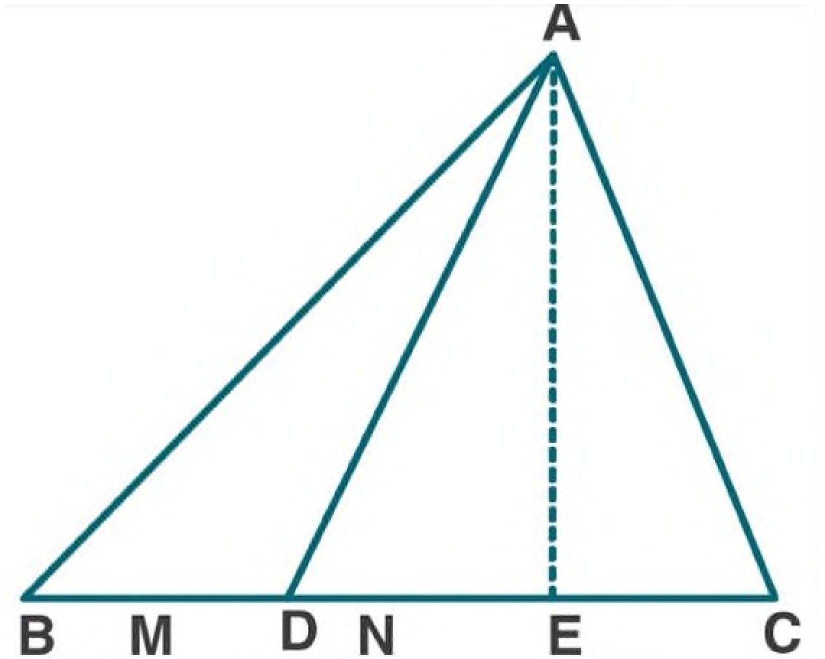
13. Any point D is taken on the side BC of , a Δ ABC and AD is produced to E such AD = DE, prove that area of Δ BCE = area of Δ ABC.

Solution

Given

In Δ ABC, D is taken on the side BC.

AD produced to E such that AD = DE



To prove:

Area of Δ BCE = area of Δ ABC

Proof:

In Δ ABE, it is given that AD = DE

So, BD is the median of Δ ABE

ar(Δ ABD) = ar(Δ BED) .....(1)

similarly,

in Δ ACE, CD is the median of Δ ACE

ar(Δ ACD) = ar(Δ CED).......(2)

by adding (1) and (2) , we get

ar(Δ ABD) + ar(Δ ACD) = ar(Δ BED) + ar(Δ CED)

or ar(Δ ABC) = ar(Δ BCE)

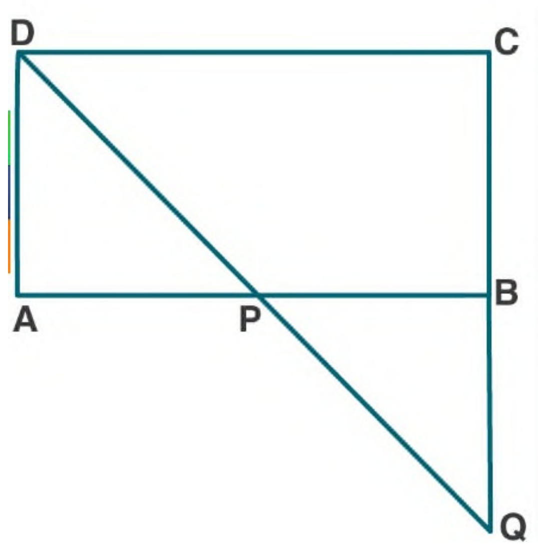
hence proved.

14. ABCD is a rectangle and P is mid- point of AB. DP is produced to meet CB at Q. Prove that area of rectangle Δ BCD = area of Δ DQC.

Solution

Given:

ABCD is a rectangle and P is mid-point of AB. DP is produced to meet CB at Q.



To prove :

Area of rectangle Δ BCD = area of Δ DQC

Proof:

In Δ APD and Δ BQP

AP = BP [since , D is the mid- point of AB]

∠DAP = ∠QBP [each angle is 90^o]

∠ APD = ∠BPQ [vertically opposite angles]

So, Δ APD ≅ Δ BQP [by using ASA postulate]

ar(Δ APD) = ar(Δ BQP)

now,

ar ABCD = ar(Δ APD) + ar PBCD

= ar (Δ BQP) + ar PBCD

= ar(Δ DQC)

Hence proved.

15.(a) In the figure (1) given below, the perimeter of parallelogram is 42 cm. Calculate the lengths of the sides of the parallelogram.

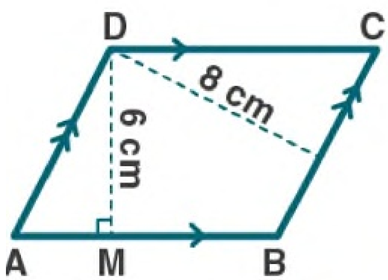
(b) In the figure(2) given below, the perimeter of Δ ABC is 37 cm . if the length of the altitudes AM, BN and CL are 5x, 6x

and 4x respectively, calculate the lengths of the sides of ΔABC.

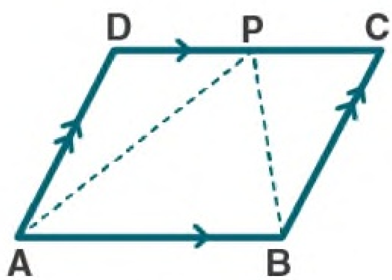
(c) In the fig.(3) Given below , ABCD is a parallelogram. P is a point on DC such that area of Δ DAP = 25cm^2 and area of ΔBCP = 15cm^2. Find

(i) area of ||gm ABCD

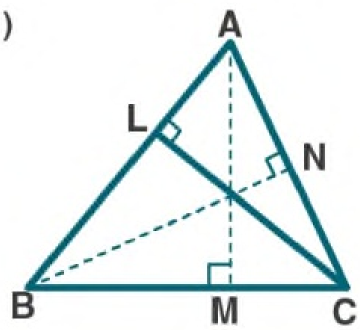
(ii) DP : PC



(1)



(3)



(2)

Solution

(a) given

The perimeter of parallelogram ABCD = 42cm

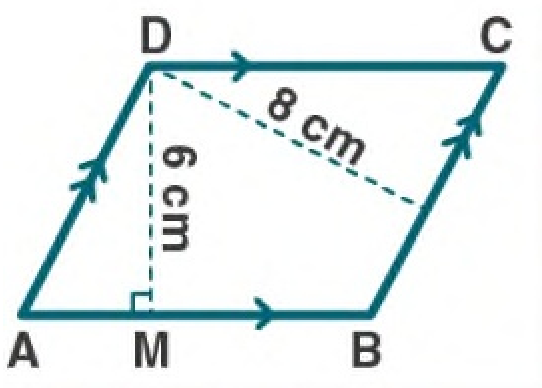
To find:

Lengths of the sides of the parallelogram ABCD.

From fig(1)

We know that,

AB= P



Then, perimeter of ∥ gm ABCD = 2(AB + BC)

42 = 2(p + BC)

$\frac{42}{2} = P + BC$

21 = P + BC

BC = 21- P

So, ar(∥ gm ABCD) = $AB \times DM$

= P × 6

= 6P.......(1)

Again , ar(∥ gm ABCD) = BC × DN

= (21 – P) × 8

= 8(21 – P) ....(2)

From (1) and (2) we get

6P = 8(21 –P)

6P = 168 – 8 P

6P + 8P = 168

14 P = 168

P = $\frac{168}{14}$

= 12

Hence, sides of ∥ gm are

AB = 12 cm and BC = (21 – 12)cm = 9cm

(b) given :

The perimeter of Δ ABC is 37 cm . the length of the altitudes AM, BN and CL are 5x, 6x and 4x respectively.

To find :

Length of the sides of Δ ABC .i.e, BC, CA and AB

Let us consider BC = P and CA = Q

From fig (2),

Then, perimeter of ΔABC = AB + BC + CA

= 37 = AB + P + Q

AB = 37 – P – Q

Area (Δ ABC) = $\frac{1}{2} \times base \ \times height$

$= \frac{1}{2} \times BC \ \times AM = \frac{1}{2} \times CA \times BN = \frac{1}{2} \times AB \times CL$

$= \frac{1}{2} \times P \ \times 5x = \frac{1}{2} \times Q \times 6x = \frac{1}{2}(37 - P - Q) \times 4x$

$= \frac{5p}{2} = 3Q = 2(37 - P - Q)$

Let us consider first two parts:

$\frac{5p}{2} = 3Q$

5P = 6Q

5P – 6Q = 0 ......(1)

25P – 30Q (multiplying by 5) .....(2)

Let us consider second and third parts:

3Q = 2(37 – P – Q)

3Q = 74 – 2P – 2Q

3Q + 2Q + 2P = 74

2P + 5Q = 74......(3)

12P + 30Q = 444(multiplying by 6)......(4)

By adding (2) and (4), we get

37P = 444

$P = \frac{444}{37}$

= 12

Now, substituting the value of P in equation (1) , we get

5P – 6Q = 0

5(12) – 6Q = 0

60 = 6Q

$Q = \frac{60}{6}$

$= 10$

Hence, BC = P = 12cm

CA = Q = 10cm

And AB = 37 –P – Q = 37 – 12 -10 = 15 cm

(c) given :

ABCD is a parallelogram . P is a point on DC such that area of Δ DAP = $25cm^2$ and area of Δ BCP = $15cm^2$

To find :

(i) area of ∥gm ABCD

(ii) DP : PC

Now let us find ,

From fig (3)

(i) we know that ,

ar(Δ APB) = $\frac{1}{2}$ ar(∥ gm ABCD)

then,

$\frac{1}{2}$ ar(∥ gm ABCD) = ar(Δ DAP) + ar(Δ BCP)

$= 25 + 15$

$= 40\ cm^2$

So, ar(|| gm ABCD) = $2 \times 40 = 80cm^2$

(ii) we know that

Δ ADP and ΔBCP are on the same base CD and between same parallel lines CD and AD.

$$\frac{ar(\Delta DAP)}{ar(\Delta BCP)} = \frac{DP}{PC}$$

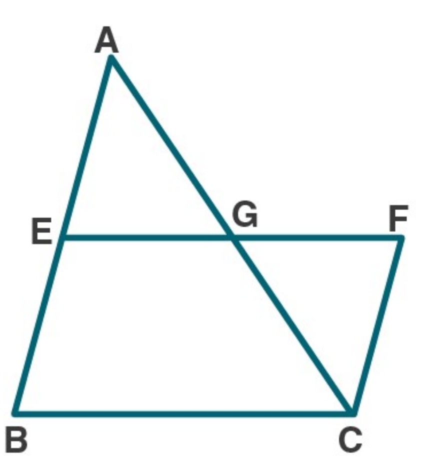
$$\frac{25}{15} = \frac{DP}{PC}$$

$$\frac{5}{3} = \frac{DP}{PC}$$

So, DP : PC = 5 : 3

16. In the adjoining figure, E is mid – point of the side AB of a triangle ABC and EBCF is a parallelogram. If the area of Δ ABC is 25sq. units, find the area of || gm EBCF.

Solution :



Let us consider EF, side of ||gm BCEF meets AC at G.

We know that, E is the mid- point and EF|| BC

G is the mid – point of AC.

So,

AG = GC

Now, in Δ AEG and ΔCFG,

The alternate angles are : ∠EAG, ∠GCF

Vertically opposite angles are : ∠EGA = ∠CGF

So, AG = GC

Proved.

∴ ΔAEG ≅ ΔCFG

ar(Δ AEG) = ar(Δ CFG)

now,

ar(|| gm EBCF) = ar BCGE + ar(Δ CFG)

= ar BCGE + ar(Δ AEG)

= ar(Δ ABC)

We know that, ar(Δ ABC) = 25 sq. units

Hence ,ar(∥ gm EBCF) = 25sq. units

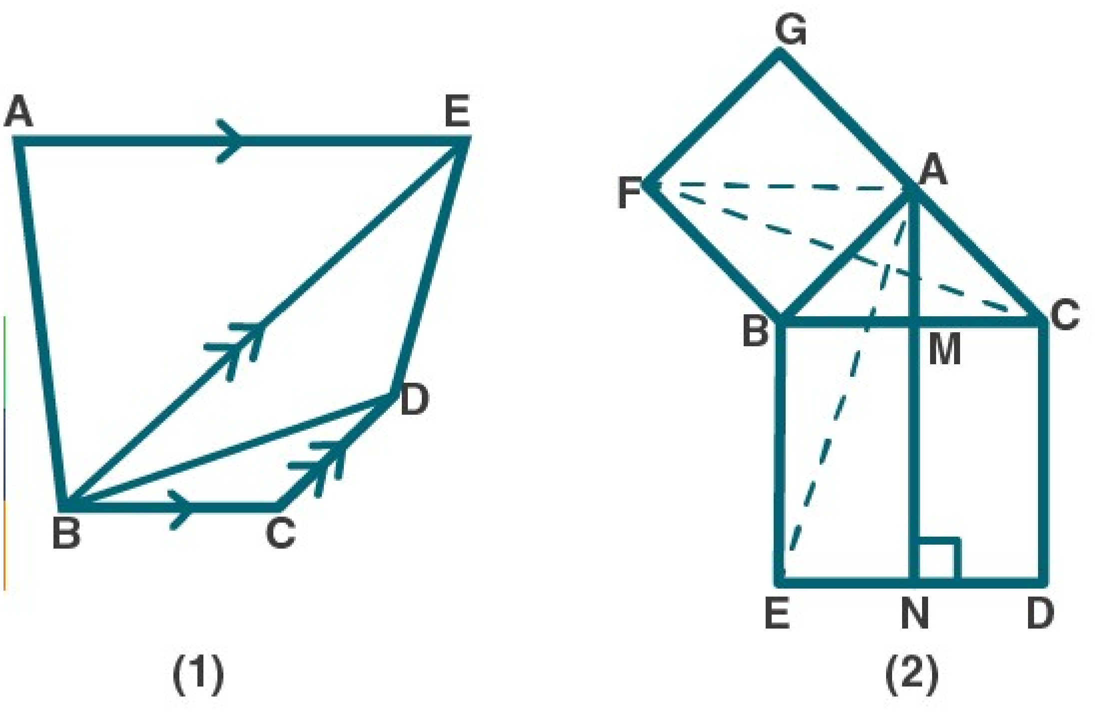
17. (a) In the figure (1) given below , BC ∥ AE and CD ∥ BE. Prove that:

Area of Δ ABC = area of Δ EBD.

(b) In the figure(2) given below, ABC is right angled triangle at A. AGFB is a square on the side AB and BCDE is a square on the hypotenuse BC. If AN ⊥ ED, prove that :

(i) Δ BCF ≅ ΔABE .

(ii) area of square ABFG = area of rectangle BENM.



Solution

(a) Given:

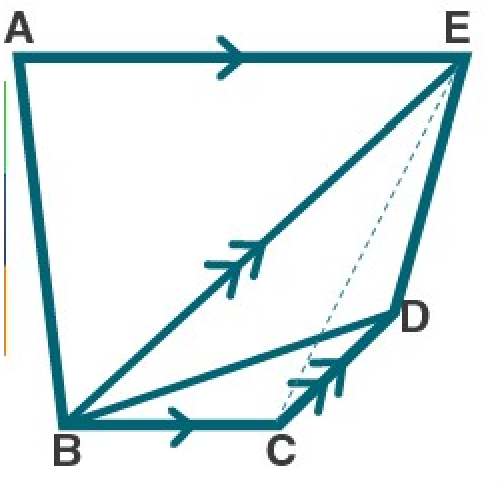
From fig (1)

BC ∥ AE and CD ∥ BE

To prove:

Area of Δ ABC = area of ΔEBD

Proof:



By joining CE.

We know that, from ΔABC and ΔEBC

ar(Δ ABC) = ar(Δ EBC).....(1)

from EBC and ΔEBD

ar(Δ EBC) = ar(Δ EBD).....(2)

from (1) and (2) , we get

ar(Δ ABC) = ar(Δ EBD)

hence proved.

(b) Given :

ABC is right angled triangle at A. Squares AGFB and BCDE are drawn on the side AB and hypotenuse BC of Δ ABC. AN ⊥ ED which meets BC at M.

To prove:

(i) ΔBCF ≅ Δ ABE.

(ii) area of square ABFG = area of rectangle BENM

From the figure(2)

(i) $\angle FBC = \angle FBA + \angle ABC$

So,

$\angle FBC = 90^o + \angle ABC$ .....(1)

$\angle ABE = \angle EAC + \angle ABC$

So,

$\angle ABE = 90^o + \angle ABC$ ....(2)

From (1) and (2) we get

$\angle FBC = \angle ABE$.......(3)

So BC = BE

Now in Δ BCF and ΔABE

BF = AB

By using SAS axiom rule of congruency.

$\therefore \Delta BCF \cong \Delta ABE$

Hence proved

(ii)we know that,

$\Delta BCF \cong \Delta ABE$

So, ar(Δ BCF) = ar(Δ ABE).....(4)

$\angle BAG + \angle BAC = 90^o + 90^o$

$= 180^o$

So, GAC is a straight line.

Now from Δ BCF and square AGFB

ar (Δ BCF) = $\frac{1}{2}$ ar(square AGFB)......(5)

From Δ ABE and rectangle BENM

ar(Δ ABE) = $\frac{1}{2}$ ar(rectangle BENM).....(6)

from(4),(5) and (6)

$\frac{1}{2}$ ar(square AGFB) = $\frac{1}{2}$ ar(rectangle BENM)

ar(square AGFB) = ar(rectangle BENM)

hence proved.