- **Baye's Theorem.**

** EX-1

| A | B | C |
| --- | --- | --- |
| 2R | 1R | 3R |
| 3G | 4G | 2G |

Q—

a) what is the total probability of getting a Red ball "given that" A is chosen. $P(R/A) = \frac{2}{5}$

b) what is the prob of getting a red ball from bag 'A'.

$$P(A \cap R) = P(A)\,P(R/A).$$
$$= (1/3) \times (2/5)$$

c) What is the prob of getting a Red ball? $P(R)$ (Total probability)

$$P(R) = P(A \cap R) + P(B \cap R) + P(C \cap R).$$

d) what is the "conditional probability" that bag 'A' is chosen, given that (known that) red ball is drawn. $P(A/R)$

$$P(A/R) = \frac{P(A \cap R)}{P(A \cap R) + P(B \cap R) + P(C \cap R)} = \frac{P(A \cap R)}{P(R)}$$

$$\boxed{P(A/R) = \frac{P(A)\;P(R/A)}{P(A)\,P(R/A) + P(B)\,P(R/B) + P(C)\,P(R/C)}}$$

→ Baye's theorem (reverse theorem)

$$P(A/R) = \frac{P(A)\;P(R/A)}{P(R)}$$

$$\Rightarrow P(R)\,P(A/R) = P(A)\,P(R/A)$$

$$\boxed{P(R \cap A) = P(A \cap R)} \rightarrow \text{Reverse.}$$

Ex-2

| | |
|---|---|
| $2R_1$<br>$3G_1$ | $3R_2$<br>$2G_2$ |
| bag-A | bag-B |

what is the probability that drawn Ball from bag ~~B~~A is red given ~~$R_2$ is taken~~ we got a $R_2$ all ball.

SS

| | |
|---|---|
| $R_1R_2$ | $G_1R_2$ |
| $R_1G_2$ | $G_1G_2$ |

→ Probability of ~~getting~~ red ball from Bag B –

$$\boxed{P(R_2) = P(R_1 \cap R_2) + P(G_1 \cap R_2)} \rightarrow \text{Total probability}$$

$$= P(R_1)\, P(R_2/R_1) + P(G_1)\, P(R_2/G_1)$$

$$= \left(\frac{2}{5}\right)\left(\frac{4}{6}\right) + \left(\frac{3}{5}\right)\left(\frac{3}{6}\right) \Rightarrow \frac{4}{15} + \frac{3}{10} \Rightarrow \frac{17}{30}$$

prob of 1'st ball being ~~be~~ red given that second ball happend to be red,

$$\boxed{P(R_1/R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)}} \rightarrow \text{Bay's theorem.}$$

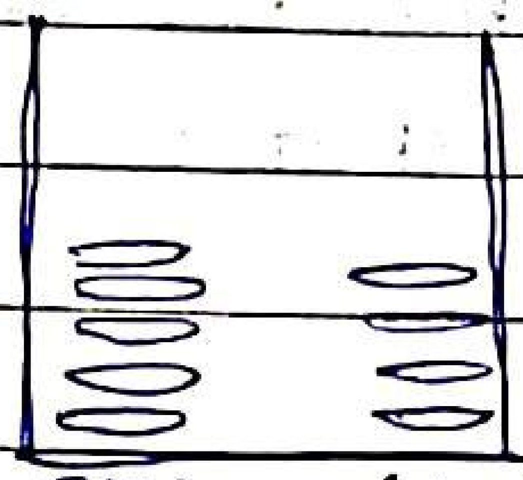
$$= \frac{P(R_1 \cap R_2)}{P(R_1 \cap R_2) + P(G_1 \cap R_2)}$$

$$= \frac{P(R_1)\, P(R_2/R_1)}{\left(4/15 + 3/10\right)}$$

$$= \frac{(2/5)(4/6)}{\frac{8+9}{30}} \Rightarrow \frac{4 \times 2}{17}$$

$$\Rightarrow \frac{8}{17}$$

Ex-3



5 Fair coin (F) — $P(H) = 1/2$, $P(T) = 1/2$

4 unfair coin (U) — $P(H) = 1/3$, $P(T) = 2/3$

- probability of getting head, $P(H) = P(F \cap H) + P(U \cap H)$

$$= P(F)\,P(H/F) + P(U)\,P(H/U)$$
$$= (5/9)(1/2) + (4/9)(1/3)$$

- probability of getting tail, $P(T) = P(F \cap T) + P(U \cap T)$

$$= P(F)\,P(T/F) + P(U)\,P(T/U)$$
$$= (5/9)(1/2) + (4/9)(2/3)$$

using Bayes theorem

① What is the Probability that the coin happed to be fair coin given that we got a Had.

$$P(F/H) = \frac{P(F \cap H)}{P(H)}$$

② What is the probability that the coin happed to be unfair coin given that we get a Had.

$$P(U/H) = \frac{P(U \cap H)}{P(H)}$$

③ What is the prob that the coin happed to be Fair coin given that we got a Tail.

$$P(F/T) = \frac{P(F \cap T)}{P(T)}$$

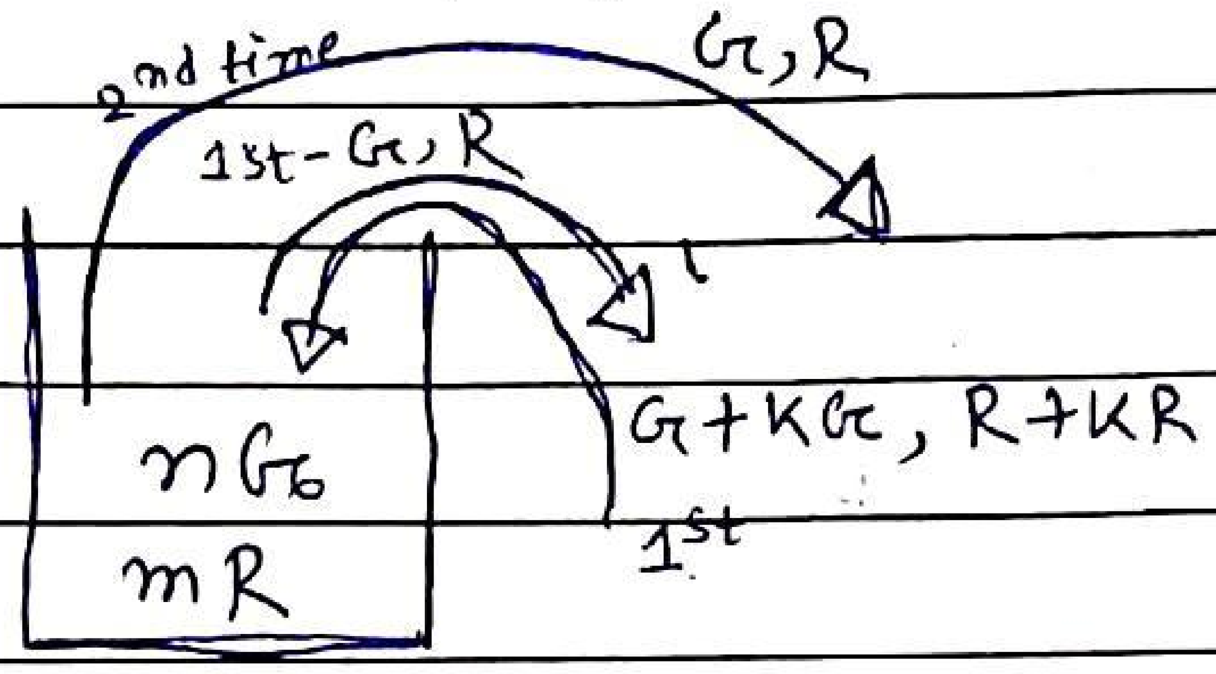
→ first find total probability after that goes for reverse probability.



(4) what is the probability that ~~we~~ ~~g~~ the coin happens to be unfair coin, given that we got a Tail -

$$P(U/T) = \frac{P(U \cap T)}{P(T)}$$

$\boxed{Ex-5}$




Bag -

SS =

| | |
|---|---|
| $G_1 R_2$ (1st 2nd) | $R_1 R_2$ |
| $G_1 G_2$ | $R_1 G_2$ (1st 2nd) |

$P(G_1) = \frac{n}{m+n}$

$P(R_1) = \frac{m}{m+n}$

$P(R_2/G_1) = \frac{m}{m+n+K}$

$P(R_2/R_1) = \frac{m+K}{m+n+K}$

(1) what is the probability that the second ball ~~w~~ happens to be red, $P(R_2)$

$$P(R_2) = P(G_1 \cap R_2) + P(R_1 \cap R_2)$$

$$= P(G_1)\,P(R_2/G_1) + P(R_1)\,P(R_2/R_1)$$

$$= \left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+K}\right) + \left(\frac{m}{m+n}\right)\left(\frac{m+K}{m+n+K}\right)$$

(2) ~~p~~ what is the probability that the second time drawnd ball happens to be green, $P(G_2)$

$$P(G_2) = P(G_1 \cap G_2) + P(R_1 \cap G_2)$$

$$= P(G_1)\,P(G_2/G_1) + P(R_1)\,P(G_2/R_1)$$

$$= \left(\frac{n}{m+n}\right)\left(\frac{n+K}{m+n+K}\right) + \frac{m}{(n+m)}\left(\frac{n}{m+n+K}\right)$$

Reverse –

③ What is the probability that the ball in first drawn to be Red, given that we got Red in second drawn –

$$P(R_1/R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)}$$

④ $$P(G_1/R_2) = \frac{P(G_1 \cap R_2)}{P(R_2)}$$

⑤ $$P(R_1/G_2) = \frac{P(R_1 \cap G_2)}{P(G_2)}$$

⑥ $$P(G_1/G_2) = \frac{P(G_1 \cap G_2)}{P(G_2)}$$

EX – 6 In answering a question on a multiple choice test, a student either knows or guesses. Let 'p' be a probability that she knows the answer and (1-P) be the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability 1/m, where 'm' is the number of multiple choice alternatives. What is the propability that she ~~knows~~ answer the question correct.

→ probability she answer it correctly, P(C)

KC | GC
 | GW

$$P(C) = P(K \cap C) + P(G \cap C)$$

$$= P(K)\,P(C/K) + P(G)\,P(C/G)$$

$$= (P)\cdot(1) + (1-P)\left(\frac{1}{m}\right)$$

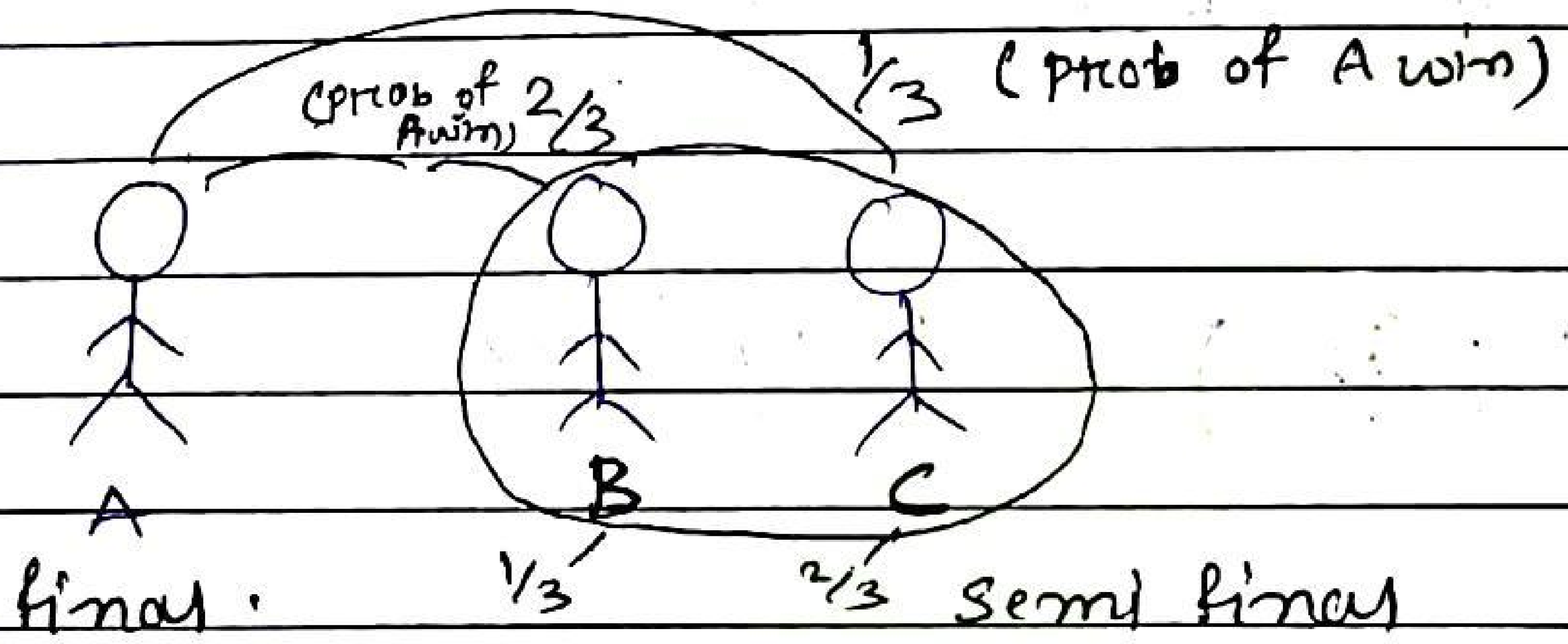
$$= P + (1-P)/m.$$

1. ~~prob~~ What is the probability that she ~~may~~ meight have gessed given that the answer is right.

$$P(G/C) = \frac{P(G\cap C)}{P(C)}$$ (Ex of reverse prob)

2. $$P(K/C) = \frac{P(K\cap C)}{P(C)}$$ (Ex of riverse prob).

$\boxed{EX-7}$




probability of B will win in semifinal = $\frac{1}{3}$

" " C " " " = $\frac{2}{3}$

Total ~~W~~ probability of A will win, P(A)

$$P(A) = P(B\cap A) + P(C\cap A)$$

$$= P(B)\, P(A/B) + P(C)\, P(A/C).$$

$$= (1/3)(2/3) + (2/3)(1/3)$$

$$= \frac{4}{9}$$

reverse theorem -

1. $$P(B/A) = \frac{P(B\cap A)}{P(A)} \Rightarrow \frac{2/9}{4/9} \Rightarrow 1/2$$

2. $$P(C/A) = \frac{P(C\cap A)}{P(A)} \Rightarrow \frac{2/9}{4/9} \Rightarrow 1/2.$$