Mathematics

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Question	Very Short Answer (VSA) 1 Mark	Short Answer– I (SA I) 2 Marks	Short Answer– II (SA II) 2 Marks	Long Answer (LA) 5 Marks	Total Marks	% Weightage
1.	**Remembering**	2	2	2	2	20	25%
2.	**Understanding**	2	1	1	4	23	29%
3.	**Application**	2	2	3	1	19	24%
4.	**High Order Thinking Skills**	-	1	4	-	14	17%
5.	**Inferential and Evaluative**	-	-	-	1	4	5%
	Total	6 × 1 = 6	6 × 2 = 12	10 × 3 = 30	8 × 4 = 32	80	100%

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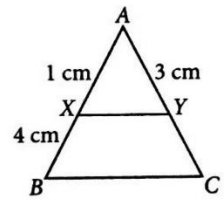
General Instructions:

(i) All question are compulsory
(ii) The question paper consists of 30 question divided into four section A, B, C and D.
(iii) Section A contains 6 questions of I mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

1. If one zero of $2x^2 - 3x + k$ is reciprocal to the other, then find the value of k.
2. If the perimeter of circle is equal to that of a square, then find the ratio of their areas.
3. In the given figure $XY || BC$. find the length of YC.

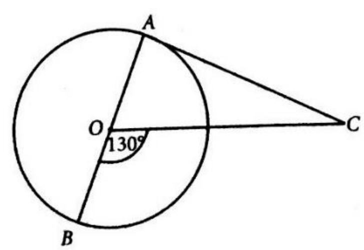


4. If the mean of the following distribution is 6, find the value of a.

x_i	2	4	6	10	$a + 5$
f_i	3	2	3	1	2

5. If $\cot A = \frac{12}{5}$, then find the value of $(\sin A + \cos A) \times \operatorname{cosec} A$.

6. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. $\angle BOC = 130°$, then find $\angle ACO$.



SECTION - B

7. Find the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively.
8. If the point $C(k, 4)$ divides the join of points $A(2, 6)$ and $B(5, 1)$ in the ratio 2 : 3, find the value of k.
9. Two dice are thrown at the same time. Find the probability of getting different numbers on the dice.
10. Find the roots of the quadratic equation $a^2b^2x^2 + b^2x - a^2x - 1 = 0$
11. Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cm/s in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour?
12. The 4^{th} term of an AP is zero. Prove that the 25th term of the AP is three times its 11^{th} term.

SECTION - C

13. On dividing the polynomial $3x^3 + 4x^2 + 5x - 13$ by a polynomial $g(x)$, the quotient and the remainder were $(3x + 10)$ and $(16x - 43)$ respectively. Find $g(x)$.

OR

Find the zeroes of $2s^2 - (1+2\sqrt{2})s + \sqrt{2}$ by factorisation method and verify the relation between the zeroes and the coefficients of the polynomial.

14. Solve the following equations for x and y.

$7^x + 5^y = 74,\ 7^{x+1} - 5^{y+1} = 218$

15. Draw a right angled ΔPQR, right angled at Q in which the sides PQ are of lengths 4 cm and 3 cm, respectively. Then, construct another triangle whose sides are 3/5 times of the corresponding sides of given triangle. Justify your construction.

16. Solve for $x : \frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, where $a, b, x \neq 0$ and $a + b + x \neq 0$

17. Calculate the mode for the following frequency distribution :

Class	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

OR

The mean of the following frequency distribution is 62. Find the missing frequency x.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8

18. If the p^{th} term of an A.P. is q and the q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.

19. Evaluate :

$$\frac{2}{3}\text{cosec}^2 58° - \frac{2}{3}\cot 58° \tan 32° - \frac{5}{3}\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$$

OR

Without using trigonometric tables, evaluate the following:

$(\sin^2 25° + \sin^2 65°) + \sqrt{3}\,(\tan 5° \tan 15° \tan 30° \tan 75° \tan 85°)$

20. A point P divides the line segment joining the points $A(3, -5)$ and $B(-4, 8)$ such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line $x + y = 0$, then find the value of k.

21. $ABCD$ is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If produced PQ meets BC at R, then prove that R is the mid-point of BC.

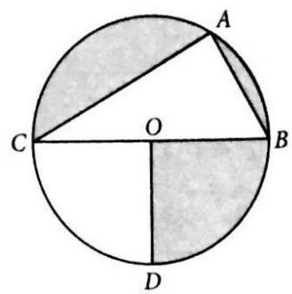
22. Using prime factorisation method, find HCF and LCM of 80, 124 and 144. Also, show that HCF × LCM ≠ Product of three numbers.

OR

A circular track around a sports ground has circumference of 1080 m. Two cyclists Paurush and Shreyan start together and cycled at constant speeds of 6m/s and 9 m/s respectively around the circular track. After how many minutes will they meet again at the starting point?

SECTION - D

23. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$. Find the area of the shaded region. [Take, $\pi = 3.14$]



24. The ratio of the sums of first m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of the m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

OR

The sum of the first p, q, r terms of an A.P. are a, b and c respectively. Show that

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0.$$

25. Two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices.

26. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is $r \sin \beta \operatorname{cosec} \alpha/2$.

27. The inner diameter of a cylindrical container is 7 cm and its top is of the shape of a hemisphere. If the height of the container is 16 cm, then find the actual capacity of the container. $\left[\text{Take, } \pi = \frac{22}{7}\right]$

OR

A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m^3 of air.

28. Mathematics teacher of a school decided to have maximum number of mixed sections for a team. Each section has to accommodate equal number of boys and equal number of girls. What is the maximum number of such sections if there are 372 boys and 444 girls?
Which type of quality predicted by teacher's act?

29. If $\tan A = a \tan B$ and $\sin A = b \sin B$, prove that $\cos^2 A = \dfrac{b^2 - 1}{a^2 - 1}$

OR

Prove that : $\left[\dfrac{1}{\sec^2\theta - \cos^2\theta} + \dfrac{1}{\csc^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta = \dfrac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$

30. A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 h less than the scheduled time and if the train was slower by 10 km/h, it would have taken 3h more than the scheduled time. Find the distance covered by the train.

Solution

1. Given, polynomial is $2x^2 - 3x + k$.

Let α and $\frac{1}{\alpha}$ be the two zeroes of the given polynomial.

Then, $\alpha \times \dfrac{1}{\alpha} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow 1 = \dfrac{k}{2} \Rightarrow k = 2$

2. Here, perimeter of circle = $2\pi r$

perimeter of square = $4a$

$\therefore\ 2\pi r = 4a \Rightarrow r = \dfrac{2a}{\pi}$

$\therefore$ Ratio of their areas = $\dfrac{\pi r^2}{a^2} = \dfrac{\pi}{a^2} \times \left(\dfrac{2a}{\pi}\right)^2$

$= \dfrac{\pi}{a^2} \times \dfrac{4a^2}{\pi^2} = \dfrac{4}{\pi} = \dfrac{4}{22} \times 7 = \dfrac{14}{11}$

3. $\because XY \| BC \quad \therefore\ \angle AXY = \angle ABC.$

Also, $\angle BAC = \angle XAY$

$\therefore\ \Delta XAY \sim \Delta BAC$

Now, $\dfrac{AX}{AB} = \dfrac{AY}{AC} = \dfrac{1}{5} = \dfrac{3}{AC}$

$\Rightarrow AC = 15 \quad \therefore\ YC = 12$ cm

4. Table for given distribution is

x_i	f_i	f_ix_i
2	3	6
4	2	8
6	3	18
10	1	10
$a + 5$	2	$2a + 10$

Here, $n = \Sigma f_i = 11, \Sigma f_ix_i = 2a + 52$

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \dfrac{2a+52}{11} = 6$ (∵ Mean = 6)

$\Rightarrow 2a + 52 = 66 \Rightarrow 2a = 66 - 52 = 14$

$\therefore a = \dfrac{14}{2} = 7$

5. We have

$\cot A = \dfrac{15}{5}$ then, $\sin A = \dfrac{5}{13}, \cos A = \dfrac{12}{13}$, cosec $A = \dfrac{13}{5}$

$\Rightarrow \left(\dfrac{5}{13} + \dfrac{12}{13}\right) \times \dfrac{13}{5} \Rightarrow \dfrac{17}{5}$

6. Given, $\angle BOC = 130°$

Since, AC is a tangent to the circle at A

$\therefore \angle OAC = 90°$ [∵ Radius is perpendicular to the tangent at point of contact]

Now, $\angle AOC + \angle BOC = 180°$[Linear pair]

$\Rightarrow \angle AOC = 180° - 130° = 50°$

In ΔAOC, $\angle AOC + \angle ACO + \angle OAC = 180°$ [Angle sum property]

$\Rightarrow \angle ACO = 180° - 50° - 90° = 40°$

7. Since the remainder are 4, 5 and 6 respectively

$\therefore$ we have to find the HCF of 445 – 4, 572 – 5 and 699 – 6 *i.e.*, 441, 567 and 693.

For the HCF of 441, 567 and 693, we have

$441 = 3 \times 3 \times 7 \times 7 = 3^2 \times 7^2$

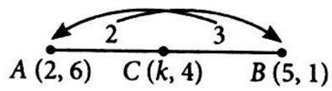
$567 = 3 \times 3 \times 3 \times 3 \times 7 = 3^4 \times 7$

$693 = 3 \times 3 \times 7 \times 11 = 3^2 \times 7 \times 11$

$\therefore$ HCF of 441, 567 and 693 $= 3^2 \times 7 = 63$

$\therefore$ The required number = 63

8.



Using section formula,

$$(k,4)=\left(\frac{3\times2+2\times5}{2+3},\frac{3\times6+2\times1}{2+3}\right)$$

$$=\left(\frac{6+10}{5},\frac{18+2}{5}\right)=\left(\frac{16}{5},\frac{20}{5}\right)$$

$$\Rightarrow\ (k,4)=\left(\frac{16}{5},4\right)$$

On comparing x-coordinate both sides, we get

$$k=\frac{16}{5}$$

9. Since the two dice are thrown simultaneously

$\therefore$ Total number of outcomes = 6 × 6 = 36

Number of outcomes for getting same numbers on both dice = 6

$\Rightarrow$ P(getting same number) $=\frac{6}{36}=\frac{1}{6}$

Now, P(getting same numbers) + P(getting different numbers) = 1

$\Rightarrow$ $\frac{1}{6}+P$ (getting different numbers) = 1

$\Rightarrow$ P(getting different number) $=1-\frac{1}{6}=\frac{5}{6}$

10. Given quadratic equation is

$a^2b^2x^2+b^2x-a^2x-1=0$

$\Rightarrow$ $b^2x\,(a^2x+1)-1\,(a^2x+1)=0$

$\Rightarrow$ $(a^2x+1)(b^2x-1)=0$

$\Rightarrow$ $a^2x+1=0$ or $b^2x-1=0$

$$\Rightarrow\ x=-\frac{1}{a^2}\ \text{ or }\ x=\frac{1}{b^2}$$

Hence, the roots of the given equations are $\frac{-1}{a^2}$ and $\frac{1}{b^2}$

11. Given, radius of tank, $r_1 = 40$ cm

Let height of water level in tank in half an hour = b_1

Also, internal radius of cylindrical pipe, $r_2 = 1$ cm

$\because$ Speed of water = 80 cm/s

$\therefore$ In 1 second water flow = 80 cm

$\therefore$ In 30 minutes water flow = $80 \times 60 \times 30 = 144000$ cm

Now, length of water column of pipe (b_2) = 144000 cm.

According to the question

Volume of water in cylindrical tank = Volume of water flow from the pipe in half an hour

$\Rightarrow \pi r_1^2 b_1 = \pi r_2^2 b_2$

$\Rightarrow 40 \times 40 \times b_1 = 1 \times 1 \times 144000$

$\therefore \quad b_1 = \frac{144000}{40 \times 40} = 90$ cm

Hence, the level of water in cylindrical tank rises 90 cm in half an hour.

12. Let a and d be the first term and common difference of the given AP, respectively.

Given, $a_4 = 0$

$\Rightarrow a + 3d = 0$ $\qquad [\because a_n = a + (n - 1)]$

$\Rightarrow a = -3d$ $\qquad$ ...(i)

To prove : $a_{25} = 3a_{11}$

Now, $a_{25} = a + (25 - 1)d = a + 24d$

$= - 3d + 24d$ [from (i)]

$\Rightarrow a_{25} = 21d$ $\qquad$ ...(ii)

Also, $a_{11} = a + (11 - 1)d$

$= a + 10d$

$= -3d + 10d$ $\qquad$ [From (i)]

$\Rightarrow a_{11} = 7d$

$\Rightarrow 3a_{11} = 21d$ $\qquad$ ...(iii)

[on multiplying both sides by 3]

From (ii) and (iii), we get $a_{25} = 3a_{11}$

13. Here, dividend $p(x) = 3x^3 + 4x^2 + 5x - 13$, quotient $q(x) = 3x + 10$ and remainder $r(x) = 16x - 43$

$\therefore$ By using division algorithm, we have

$g(x) \times (3x + 10) + 16x - 43 = 3x^3 + 4x^2 + 5x - 131$

$g(x) \times (3x + 10) = 3x^3 + 4x^2 + 5x - 13 - 16x + 43$

$$g(x) = \frac{3x^3 + 4x^2 - 11x + 30}{3x + 10}$$

$$\begin{array}{r|l} & \quad x^2 - 2x + 3 \\ \hline 3x + 10 & 3x^3 + 4x^2 - 11x + 30 \\ & 3x^3 + 10x^2 \\ & (-) \quad (-) \\ \hline & \quad -6x^2 - 11x + 30 \\ & \quad -6x^2 - 20x \\ & \quad (+) \quad (+) \\ \hline & \qquad 9x + 30 \\ & \qquad 9x + 30 \\ \hline & \qquad 0 \\ \hline \end{array}$$

Hence, $g(x) = x^2 - 2x + 3$.

OR

Consider, $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2} = 0$

$\Rightarrow 2s^2 - s - 2\sqrt{2}s + \sqrt{2} = 0$

$\Rightarrow s(2s - 1) - \sqrt{2}(2s - 1) = 0$

$\Rightarrow (s - \sqrt{2})(2s - 1) = 0$

$\Rightarrow s - \sqrt{2} = 0$ or $2s - 1 = 0$

$\Rightarrow s = \sqrt{2}$ or $s = \frac{1}{2}$

Thus, zeroes of the given polynomial are $\sqrt{2}$ and $\frac{1}{2}$.

Sum of zeroes $= \frac{1}{2} + \sqrt{2} = \frac{1 + 2\sqrt{2}}{2}$

$= \frac{-\{-(1 + 2\sqrt{2})\}}{2} = \frac{-\text{coefficient of } s}{\text{coefficient of } s^2}$

Product of zeroes

$= \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2} = \frac{\text{constant term}}{\text{coefficient of } s^2}$

Hence, the relationship between the zeroes and the coefficient of the polynomial is verified.

14. Let $7^x = p$ and $5^y = q$.

Since, we have, $7^x + 5^y = 74 \quad \Rightarrow \quad p + q = 74 \qquad$ ...(i)

and $7^{x+1} - 5^{y+1} = 218 \quad \Rightarrow \quad 7 \cdot 7^x - 5 \cdot 5^y = 218$

$\Rightarrow \quad 7p - 5q = 218 \qquad$ ...(ii)

On putting $p = 74 - q$ from eq. (i) in Eq. (ii), we get

$$7(74 - q) - 5q = 218$$

$\Rightarrow \quad -12q = 218 - 518 = -300 \quad \therefore \quad q = 25$

On substituting the value of q in Eq. (i), we get

$$p + 25 = 74 \quad \Rightarrow \quad p = 49$$

Thus, we get $p = 49$ and $q = 25$

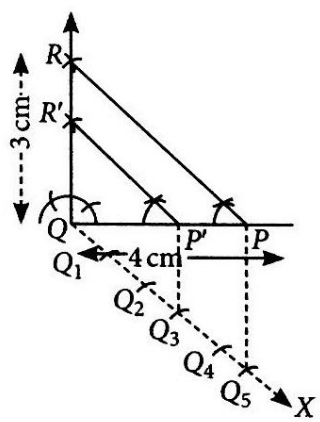
$\therefore \quad 7^x = 49$ and $5^y = 25$

$\Rightarrow \quad 7^x = 7^2$ and $5^y = 5^2$

$\therefore \quad x = 2$ and $y = 2 \qquad [\because m^a = m^b \Rightarrow a = b]$

Hence, the solution of the given system of equations is $x = 2$ and $y = 2$.

15.



Steps of Construction:

I. Construct the right triangle PQR such that $\angle Q = 90°$, $PQ = 4$ cm and $QR = 3$ cm.

II. Draw a ray QX such that an acute angle $\angle PQX$ is formed.

III. Mark 5 points Q_1, Q_2, Q_3, Q_4 and Q_5 on BX such that $BQ_1 = Q_1Q_2 = Q_2Q_3 = Q_4Q_5$.

IV. Join Q_5P.

V. Draw a line through Q_3 parallel to Q_5P, intersecting the extended line segment PQ at P'.

VI. Draw another line through P' parallel to PR intersecting the extended line segment QR at R'. Thus, $\Delta P'BR'$ is the required triangle.

16. We have, $\dfrac{1}{a+b+x}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}$

$\Rightarrow \dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{a+b+x}-\dfrac{1}{x} \Rightarrow \dfrac{a+b}{ab}=\dfrac{x-(a+b+x)}{(a+b+x)x}$

$\Rightarrow \dfrac{a+b}{ab}=\dfrac{x-a-b-x}{(a+b+x)x} \Rightarrow \dfrac{a+b}{ab}=\dfrac{-(a+b)}{(a+b+x)x}$

$\Rightarrow \dfrac{1}{ab}=\dfrac{-1}{(a+b+x)x}$

$\Rightarrow x^2+ax+bx+ab=0$

$\Rightarrow x(x+a)+b(x+a)=0$

$\Rightarrow (x+a)(x+b)=0$

$\Rightarrow x+a=0$ or $x+b=0$

$x=-a$ or $x=-b$

Hence, $x=-a$ or $-b$

17. The class 35 - 40 has maximum frequency. So, it is a modal class

$\therefore \quad l=35, h=5, f_1=50, f_0=34, f_2=42.$

Mode, $M=l+\left(\dfrac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$=35+\left(\dfrac{50-34}{2\times 50-34-42}\right)\times 5=35+\dfrac{16}{24}\times 5$

$=\left(35+\dfrac{10}{3}\right)=35+3.33=38.33$

Hence, mode = 38.33

OR

Class Interval	**Frequency (f_i)**	**Mid Value (x_i)**	f_ix_i
0 - 20	5	10	50
20 - 40	8	30	240
40 - 60	x	50	$50x$
60 - 80	12	70	840
80 - 100	7	90	630
100 - 120	8	110	880
	$n=\Sigma f_i = 40+x$		$\Sigma f_ix_i = 2640 + 50x$

Mean $\bar{x} = \frac{\sum f_i x_i}{n} \Rightarrow 62 = \frac{2640 + 50x}{40 + x}$

$\Rightarrow 62(40 + x) = 2640 + 50x$

$\Rightarrow 2480 + 62x = 2640 + 50x$

$\Rightarrow 2480 - 2640 = 50x - 62x \Rightarrow -160 = -12x$

$\Rightarrow x = \frac{160}{12} = 13.33 \simeq 13$ (approx.)

18. Let a be the first term and d be the common difference of the given A. P. Then,

p^{th} term $= q$

$\Rightarrow a + (p - 1)\,d = q$ ...(i)

q^{th} term $= p$

$\Rightarrow a + (q - 1)\,d = p$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$(q - p) = (p - q) \times d \Rightarrow d = -1$

Putting $d = -1$ in equation (i), we get

$a = p + q - 1$

n^{th} term $= a + (n - 1)\,d$

$= (p + q - 1) + (n - 1) \times (-1)$

$= (p + q - n)$

Hence proved.

19. We have, $\frac{2}{3}\text{cosec}^2 58° - \frac{2}{3}\cot 58° \tan 32°$

$- \frac{5}{3}\tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$

$= \frac{2}{3}\text{cosec}^2 58° - \frac{2}{3}\cot 58° \cot(90° - 32°)$

$- \frac{5}{3}\cot(90° - 13°)\cot(90° - 37°)$

$\times 1 \times \tan 53° \tan 77°$

$= \frac{2}{3}\text{cosec}^2 58° - \frac{2}{3}\cot 58° \cot 58°$

$-\frac{5}{3}\cot 77^\circ \cot 53^\circ \tan 53^\circ \tan 77^\circ$

$= \frac{2}{3}(\operatorname{cosec}^2 58^\circ - \cot^2 58^\circ) - \frac{5}{3}$

$(\cot 77^\circ \times \tan 77^\circ)(\cot 53^\circ \times \tan 53^\circ)$

$= \frac{2}{3} \times 1 - \frac{5}{3} \times 1 \times 1$

$[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1 \text{ and } \cot\theta \times \tan\theta = 1]$

$= \frac{2}{3} - \frac{5}{3} = \frac{-3}{3} = -1$

OR

We have, $(\sin^2 25^\circ + \sin^2 65^\circ)$

$+ \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ)$

$= \{\sin^2 25^\circ + \sin^2(90^\circ - 25^\circ)\} + \sqrt{3}\{\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan(90^\circ - 15^\circ) \tan(90^\circ - 5^\circ)\}$

$= (\sin^2 25^\circ + \cos^2 25^\circ) + \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \cot 15^\circ \cot 5^\circ)$

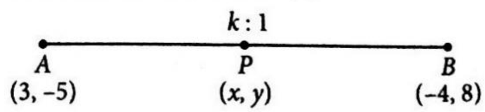
$= (1) + \sqrt{3}\left(\tan 5^\circ \tan 15^\circ \tan 30^\circ \times \frac{1}{\tan 15^\circ} \times \frac{1}{\tan 5^\circ}\right)$

$= 1 + \sqrt{3}\left(\frac{1}{\sqrt{3}}\right) = 1 + 1 = 2$

20. Let the coordinates of P be (x, y).

Also, $AP : PB = k : 1$

i.e., P divides AB in the ratio $k : 1$



$\therefore \quad x = \frac{k(-4) + 1(3)}{k+1} \quad \text{and} \quad y = \frac{k(8) + 1(-5)}{k+1}$

$\Rightarrow \quad x = \frac{-4k+3}{k+1} \quad \text{and} \quad y = \frac{8k-5}{k+1}$

$\therefore \quad (x, y) = \left(\frac{-4k+3}{k+1}, \frac{8k-5}{k+1}\right)$

P lies on the line $x + y = 0$

$\therefore \quad \frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0 \quad \Rightarrow \quad \frac{-4k+3+8k-5}{k+1} = 0$

$\Rightarrow \quad 4k - 2 = 0 \quad \Rightarrow \quad 4k = 2 \quad \Rightarrow \quad k = \frac{1}{2}$

21. Join BD,

We know that, diagonals of parallelogram bisect each other.

$\therefore$ O is the mid-point of AC and BD

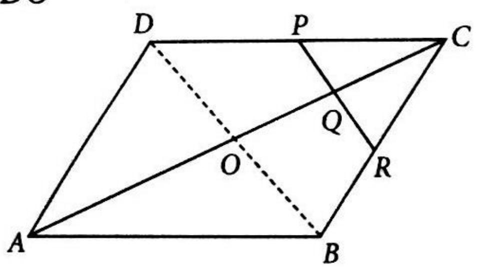
$\Rightarrow AO = OC = \frac{1}{2}AC$ also, $CQ = \frac{1}{4}AC$

$\Rightarrow \frac{OC}{CQ} = 2 \Rightarrow OC = 2\,CQ$

$\therefore$ Q is the mid-point of CO.

Now, in ΔDOC, P and Q are the mid-points of DC and OC

$\Rightarrow PQ \parallel DO$



Again, in ΔCOB, QR is a line segment through the mid-point of one side, parallel to another side. Thus bisects the third side.

22. Here, $80 = 2^4 \times 5$

$124 = 2^2 \times 3^{131}$

$144 = 2^4 \times 3^2$

Now, HCF of 80, 124 and 144 $= 2^2 = 4$

LCM of 80, 124 and 144 $= 2^4 \times 3^2 \times 5 \times 31 = 22320$

Product of HCF and LCM $= 4 \times 22320 = 89280$

Product of three numbers $= 80 \times 124 \times 144 = 1428480$

Clearly, HCF $\times$ LCF $\neq$ Product of three numbers.

OR

Circumference of circular track = 1080 m

Speed of two cyclists in 6 m/s and 9 m/s

$\therefore$ Time taken to go around the track once by

Paurush $= \frac{1080}{6} = 180$ sec.

Time taken to go around the track once by

Shreyan $= \frac{1080}{9} = 120$ sec.

The required number of minutes, when they meet again at the starting point is LCM of 180 and 120

Here, $180 = 2^2 \times 3^2 \times 5$

$120 = 2^3 \times 3 \times 5$

LCM of 180 and 120 $= 2^3 \times 3^2 \times 5 = 360$ seconds

$= 6$ minutes

Hence, they will meet again at the starting point after 6 minutes.

23. Given, $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$

$\because$ BOC is a diameter of the circle.

$\therefore$ $\angle BAC = 90°$

[$\because$ angle in a semi-circle is a right angle]

In right angled ΔBAC,

$BC^2 = AB^2 + AC^2$ [By Pythagoras theorem]

$\Rightarrow BC^2 = (7)^2 + (24)^2$

$\Rightarrow BC^2 = 49 + 576$

$\Rightarrow BC^2 = 625$

$\Rightarrow BC = 25$ cm [Taking positive square]

$\therefore$ Radius $(r) = \frac{25}{2} = 12.5$ cm

Now, area of shaded region

= Area of circle – Area of quadrant COD

– Area of ΔABC

$$= \pi r^2 - \frac{\pi r^2}{4} - \frac{1}{2} \times AB \times AC$$

$$= \frac{3}{4}\pi r^2 - \frac{1}{2} \times AB \times AC$$

$$= \frac{3}{4} \times 3.14 \times (12.5)^2 - \frac{1}{2} \times 7 \times 24$$

$= 367.97 - 84 = 283.97$ cm^2

24. Let a be the first term and d be the common difference of the given A.P. Then, the sums of first m and n terms are given by $S_m = \frac{m}{2}[2a+(m-1)d]$ and $S_n = \frac{n}{2}[2a+(n-1)d]$, respectively.

Since, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\Rightarrow [2a+(m-1)d]n = [2a+(n-1)d]m$$

$$\Rightarrow 2a(n-m) = d[(n-1)m-(m-1)n]$$

$$\Rightarrow 2a(n-m) = d(n-m) \Rightarrow d = 2a$$

Now, required ratio $= \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$$= \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$$

or $a_m : a_n = (2m-1):(2n-1)$ Hence proved.

OR

Let A be the first term and D be the common difference of the given A.P.

Given, a = Sum of first p terms

$$\Rightarrow a = \frac{p}{2}[2A+(p-1)D]$$

$$\left[\because S_m = \frac{n}{2}\{2a+(n-1)d\}\right]$$

$$\Rightarrow \quad \frac{a}{p} = \frac{1}{2}[2A + (p-1)D] \qquad \text{...(i)}$$

b = Sum of first q terms

$$\Rightarrow \quad b = \frac{q}{2}[2A + (q-1)D]$$

$$\Rightarrow \quad \frac{b}{q} = \frac{1}{2}[2A + (q-1)D] \qquad \text{...(ii)}$$

and c = Sum of first r terms

$$\Rightarrow \quad c = \frac{r}{2}[2A + (r-1)D]$$

$$\Rightarrow \quad \frac{c}{r} = \frac{1}{2}[2A + (r-1)D] \qquad \text{...(iii)}$$

On multiplying (i), (ii) and (iii) by $(q - r)$, $(r - p)$ and $(p - q)$ respectively and then adding, we get

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

$$= \frac{1}{2}[\{2A + (p-1)D\}(q-r) + \{2A + (q-1)D\}$$

$$(r - p) + \{2A + (r - 1)D\}(p - q)]$$

$$= \frac{1}{2}\ [2A(q - r + r - p + p - q) + D\{(p - 1)$$

$$(q - r) + (q - 1)(r - p) + (r - 1)(p - q)\}]$$

$$= \frac{1}{2}\ [2A \times 0 - D\{pq - pr - q + r + qr - qp - r + p$$

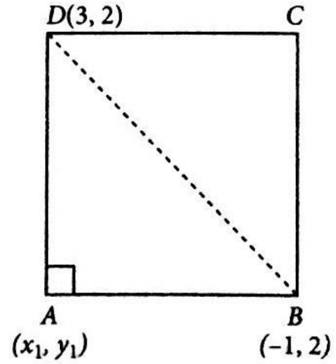
$$+ rp - rq - p + q\}]$$

$$= \frac{1}{2}[0 + D \times 0] = 0$$

Hence proved.

25. Let $ABCD$ be the given square with two opposite vertices as $B(-1, 2)$ and $D(3, 2)$.

Let coordinates of vertex A be $A(x_1, y_1)$.



Since adjacent sides of the square are equal.

$\therefore \quad |AB| = |AD|$

$\Rightarrow \quad AB^2 = AD^2$

$\Rightarrow \quad (-1 - x_1)^2 + (2 - y_1)^2 = (3 - x_1)^2 + (2 - y_1)^2$

$\Rightarrow \quad (1 + x_1)^2 = (3 - x_1)^2 \Rightarrow 1 + x_1^2 + 2x_1 = 9 + x_1^2 - 6x_1$

$\Rightarrow \quad 8x_1 = 8 \Rightarrow x_1 = 1.$

Also, ΔBAD is a right-angled triangle.

$\therefore \quad AB^2 + AD^2 = BD^2$

$(1 + x_1)^2 + (2 - y_1)^2 + (3 - x_1)^2 + (2 - y_1)^2$

$= (3 + 1)^2 + (2 - 2)^2$

$\Rightarrow \quad (1 + 1)^2 + 4 + y_1^2 - 4y_1 + (3 - 1)^2 + 4 + y_1^2 - 4y_1 = 16$

$\Rightarrow \quad 4 + 2y_1^2 - 8y_1 + 4 + 4 + 4 = 16$

$\Rightarrow \quad 2y_1(y_1 - 4) = 0 \Rightarrow y_1 = 0 \text{ or } y_1 = 4$

Thus, points are (1, 0) or (1, 4)

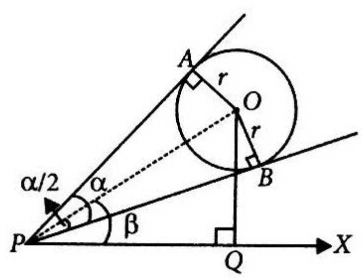
Hence, the coordinates of other two vertices are (1, 0) and (1, 4).

26. Let O be the centre of the balloon of radius r and P be the eye of the observer. Let PA, PB be tangents from P to the balloon. Then $\angle APB = \alpha$

$\therefore \quad \angle APO = \angle BPO = \alpha/2$

Let OQ be perpendicular from O on the horizontal PX. We are given that the angle of the elevation of the centre of the balloon is β. *i.e.*, $\angle OPQ = \beta$

In ΔOPB, right angled at B,



$$\sin\frac{\alpha}{2} = \frac{OB}{OP}$$

$$\Rightarrow \quad \sin\frac{\alpha}{2} = \frac{r}{OP}$$

$$\Rightarrow \quad OP = \frac{r}{\sin\frac{\alpha}{2}}$$

$$\Rightarrow \quad OP = r \operatorname{cosec}\frac{\alpha}{2} \qquad \text{...(i)}$$

Now, in ΔOPQ, right angled at Q,

$$\sin\beta = \frac{OQ}{OP} \Rightarrow OQ = OP\sin\beta \qquad \text{...(ii)}$$

Putting the value of OP from (i) in (ii), we get

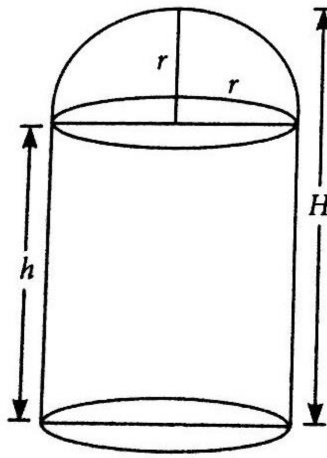
$OQ = r \operatorname{cosec} \alpha/2 \sin \beta$

Hence, the height of the centre of the balloon is $r \sin \beta \operatorname{cosec} \alpha/2$.

27. Given, inner diameter of cylinderical container = 7 cm

$\therefore$ Inner radius of cylindrical container

$= \frac{7}{2} = 3.5\,\text{cm}$ = Radius of hemisphere



Height of cylindrical container = 16 cm

Now, volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (3.5)^2 \times 16$

$= 22 \times 1.75 \times 16 = 616\ \text{cm}^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 = \frac{44}{3} \times 6.125$

$= \frac{269.5}{3} = 89.83\ \text{cm}^3$

$\therefore$ The actual capacity of the container

= Volume of cylindrical container – Volume of hemisphere

$= 616 - 89.83 = 526.17\ \text{cm}^3$

OR

Let r be the radius of the hemispherical dome and total height of building be H m.

$2r = \frac{2}{3}$ × Total height height of the building

$\therefore \quad r = \frac{\text{Diameter}}{2} = \frac{\frac{2}{3} \times H}{2}$

$\Rightarrow \quad r = \frac{1}{3} H\ \text{m}$

Let h be the height of the cylinder.

$\therefore \quad h = H - r$

$\Rightarrow \quad h = H - \frac{H}{3} = \frac{2}{3} H\ \text{m}$

Volume of the air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$= \frac{2}{3}\pi r^3 + \pi r^2 h$

$= \frac{2}{3}\pi\left(\frac{H}{3}\right)^3 + \pi\left(\frac{H}{3}\right)^2\left(\frac{2H}{3}\right) = \frac{8}{81}\pi H^3 \text{ m}^3$

Given, volume of the air inside the building

$= 67\frac{1}{21} \text{ m}^3$

$\therefore \quad \frac{8}{81}\pi H^3 = \frac{1408}{21}$

$\Rightarrow \quad H^3 = \frac{1408 \times 81 \times 7}{21 \times 8 \times 22} = 216$

$\Rightarrow H = 6$

Hence, height of the building is 6 m.

28. Since in each section equal number of boys and equal number of girls be accommodated, therefore, let us find out HCF of 372 and 444.

$372 = 2^2 \times 3 \times 31$

$444 = 2^2 \times 3 \times 37$

HCF of 372 and 444 is $2^2 \times 3 = 12$

Thus, number of boys in each section $= \frac{372}{12} = 31$

Number of girls in each section $= \frac{444}{12} = 37$

Hence, total number of sections are 12 and in each section number of boys are 31 and number of girls are 37. Equality is predicted by teacher's act.

29. Given, $\tan A = a \tan B \Rightarrow \tan B = \frac{1}{a}\tan A$

$\Rightarrow \quad \cot B = \frac{a}{\tan A}$ ...(i)

and $\sin A = b \sin B \Rightarrow \sin B = \frac{1}{b}\sin A$

$\Rightarrow \quad \operatorname{cosec} B = \frac{b}{\sin A}$ ...(ii)

We know that, $\operatorname{cosec}^2 B - \cot^2 B = 1$

$$\Rightarrow \frac{b^2}{\sin^2 A} - \frac{a^2}{\tan^2 A} = 1 \qquad \text{[From (i) and (ii)]}$$

$$\Rightarrow \frac{b^2}{\sin^2 A} - \frac{a^2 \cos^2 A}{\sin^2 A} = 1 \qquad \left[\because \tan A = \frac{\sin A}{\cos A}\right]$$

$$\Rightarrow \frac{b^2 - a^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow b^2 - a^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow b^2 - 1 = (a^2 - 1)\cos^2 A \qquad [\because \sin^2 A = 1 - \cos^2 A]$$

$$\Rightarrow \cos^2 A = \frac{b^2 - 1}{a^2 - 1}$$

OR

Given expresssion

$$= \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{1 - \cos^4\theta} + \frac{\sin^2\theta}{1 - \sin^4\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1 + \cos^2\theta(1 - \cos^2\theta)} + \frac{\sin^2\theta}{(1 - \sin^2\theta)(1 + \sin^2\theta)}\right]$$

$$\times \sin^2\theta \cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1 + \cos^2\theta)\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta(1 + \sin^2\theta)}\right]$$

$$\times \sin^2\theta \cos^2\theta$$

$$= \frac{\cos^4\theta}{1 + \cos^2\theta} + \frac{\sin^4\theta}{1 + \sin^2\theta}$$

$$= \frac{\cos^4\theta + \sin^2\theta\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^4\theta}{(1 + \cos^2\theta)(1 + \sin^2\theta)}$$

$$= \frac{\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta(\cos^2\theta + \sin^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$$

$$= \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$$

30. Let the actual speed of the train be x km/h and actual time taken be y h.

$\because$ Distance = Speed × Time

$\therefore$ Distance = xy km

According to the question,

$xy = (x + 10)(y - 2)$

$\Rightarrow xy = xy - 2x + 10y - 20$

$\Rightarrow 2x - 10y + 20 = 0$

$\Rightarrow x - 5y + 10 = 0$ ...(i)

and $xy = (x - 10)(y + 3)$

$\Rightarrow xy = xy + 3x - 10y - 30$

$\Rightarrow 3x - 10y - 30 = 0$ ...(ii)

On multiplying (i) by 3 and then subtracting (ii) from it, we get

$3 \times (x - 5y + 10) - (3x - 10y - 30) = 0$

$\Rightarrow -5y = -60$

$\therefore y = 12$

On putting $y = 12$ in (i), we get

$x - 5 \times 12 + 10 = 0$

$\Rightarrow x - 60 + 10 = 0 \Rightarrow x = 50$

Hence, the distance covered by the train

$= 50 \times 12 = 600$ km