9.3 Integrals of Irrational Functions

905. $$\int \frac{dx}{\sqrt{ax+b}} = \frac{2}{a}\sqrt{ax+b} + C$$

906. $$\int \sqrt{ax+b}\, dx = \frac{2}{3a}(ax+b)^{3/2} + C$$

907. $$\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b} + C$$

908. $$\int x\sqrt{ax+b}\, dx = \frac{2(3ax-2b)}{15a^2}(ax+b)^{3/2} + C$$

909. $$\int \frac{dx}{(x+c)\sqrt{ax+b}} = \frac{1}{\sqrt{b-ac}} \ln\left|\frac{\sqrt{ax+b}-\sqrt{b-ac}}{\sqrt{ax+b}+\sqrt{b-ac}}\right| + C,$$
$b-ac>0$.

910. $$\int \frac{dx}{(x+c)\sqrt{ax+b}} = \frac{1}{\sqrt{ac-b}} \arctan\sqrt{\frac{ax+b}{ac-b}} + C,$$
$b-ac<0$.

911. $$\int \sqrt{\frac{ax+b}{cx+d}}\, dx = \frac{1}{c}\sqrt{(ax+b)(cx+d)} -$$
$$-\frac{ad-bc}{c\sqrt{ac}} \ln\left|\sqrt{a(cx+d)}+\sqrt{c(ax+b)}\right| + C,\ a>0.$$

912. $$\int \sqrt{\frac{ax+b}{cx+d}}\, dx = \frac{1}{c}\sqrt{(ax+b)(cx+d)} -$$
$$-\frac{ad-bc}{c\sqrt{ac}} \arctan\sqrt{\frac{a(cx+d)}{c(ax+b)}} + C,\ (a<0,\ c>0).$$

913. $$\int x^2\sqrt{a+bx}\,dx=\frac{2\left(8a^2-12abx+15b^2x^2\right)}{105b^3}\sqrt{(a+bx)^3}+C$$

914. $$\int\frac{x^2dx}{\sqrt{a+bx}}=\frac{2\left(8a^2-4abx+3b^2x^2\right)}{15b^3}\sqrt{a+bx}+C$$

915. $$\int\frac{dx}{x\sqrt{a+bx}}=\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right|+C,\ a>0.$$

916. $$\int\frac{dx}{x\sqrt{a+bx}}=\frac{2}{\sqrt{-a}}\arctan\left|\frac{a+bx}{-a}\right|+C,\ a<0.$$

917. $$\int\sqrt{\frac{a-x}{b+x}}\,dx=\sqrt{(a-x)(b+x)}+(a+b)\arcsin\sqrt{\frac{x+b}{a+b}}+C$$

918. $$\int\sqrt{\frac{a+x}{b-x}}\,dx=-\sqrt{(a+x)(b-x)}-(a+b)\arcsin\sqrt{\frac{b-x}{a+b}}+C$$

919. $$\int\sqrt{\frac{1+x}{1-x}}\,dx=-\sqrt{1-x^2}+\arcsin x+C$$

920. $$\int\frac{dx}{\sqrt{(x-a)(b-a)}}=2\arcsin\sqrt{\frac{x-a}{b-a}}+C$$

921. $$\int\sqrt{a+bx-cx^2}\,dx=\frac{2cx-b}{4c}\sqrt{a+bx-cx^2}+$$
$$+\frac{b^2-4ac}{8\sqrt{c^3}}\arcsin\frac{2cx-b}{\sqrt{b^2+4ac}}+C$$

922. $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \ln\left|2ax+b+2\sqrt{a\left(ax^2+bx+c\right)}\right| + C,$
$a > 0.$

923. $\int \frac{dx}{\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{a}} \arcsin \frac{2ax+b}{4a} \sqrt{b^2-4ac} + C,\ a < 0.$

924. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \ln\left|x+\sqrt{x^2+a^2}\right| + C$

925. $\int x\sqrt{x^2+a^2}\,dx = \frac{1}{3}\left(x^2+a^2\right)^{3/2} + C$

926. $\int x^2\sqrt{x^2+a^2}\,dx = \frac{x}{8}\left(2x^2+a^2\right)\sqrt{x^2+a^2} -$
$-\frac{a^4}{8} \ln\left|x+\sqrt{x^2+a^2}\right| + C$

927. $\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left|x+\sqrt{x^2+a^2}\right| + C$

928. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + C$

929. $\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} + a \ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$

930. $\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2} + C$

931. $$\int \frac{x^2 dx}{\sqrt{x^2+a^2}} = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + C$$

932. $$\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$$

933. $$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

934. $$\int x\sqrt{x^2-a^2}\,dx = \frac{1}{3}\left(x^2-a^2\right)^{3/2} + C$$

935. $$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} + a\arcsin\frac{a}{x} + C$$

936. $$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

937. $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

938. $$\int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2} + C$$

939. $$\int \frac{x^2 dx}{\sqrt{x^2-a^2}} = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

940. $$\int \frac{dx}{x\sqrt{x^2-a^2}} = -\frac{1}{a}\arcsin\frac{a}{x} + C$$

941. $$\int \frac{dx}{(x+a)\sqrt{x^2-a^2}} = \frac{1}{a}\sqrt{\frac{x-a}{x+a}} + C$$

942. $$\int \frac{dx}{(x-a)\sqrt{x^2-a^2}} = -\frac{1}{a}\sqrt{\frac{x+a}{x-a}} + C$$

943. $$\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x} + C$$

944. $$\int \frac{dx}{(x^2-a^2)^{3/2}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$$

945. $$\int (x^2-a^2)^{3/2}\,dx = -\frac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2} + \\ + \frac{3a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$$

946. $$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$$

947. $$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}(a^2-x^2)^{3/2} + C$$

948. $$\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\arcsin\frac{x}{a} + C$$

949. $$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} + a\ln\left|\frac{x}{a+\sqrt{a^2-x^2}}\right| + C$$

950. $$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \arcsin\frac{x}{a} + C$$

951. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$

952. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin\frac{x}{a} + C$

953. $\int \frac{xdx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2} + C$

954. $\int \frac{x^2dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

955. $\int \frac{dx}{(x+a)\sqrt{a^2-x^2}} = -\frac{1}{2}\sqrt{\frac{a-x}{a+x}} + C$

956. $\int \frac{dx}{(x-a)\sqrt{a^2-x^2}} = -\frac{1}{2}\sqrt{\frac{a+x}{a-x}} + C$

957. $\int \frac{dx}{(x+b)\sqrt{a^2-x^2}} = \frac{1}{\sqrt{b^2-a^2}}\arcsin\frac{bx+a^2}{a(x+b)} + C,\ b > a.$

958. $\int \frac{dx}{(x+b)\sqrt{a^2-x^2}} = \frac{1}{\sqrt{a^2-b^2}}\ln\left|\frac{x+b}{\sqrt{a^2-b^2}\sqrt{a^2-x^2}+a^2+bx}\right| + C,$
$b < a.$

959. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

960. $\int (a^2-x^2)^{3/2}dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\arcsin\frac{x}{a} + C$

961. $$\int \frac{dx}{\left(a^2 - x^2\right)^{3/2}} = \frac{x}{a^2\sqrt{a^2 - x^2}} + C$$