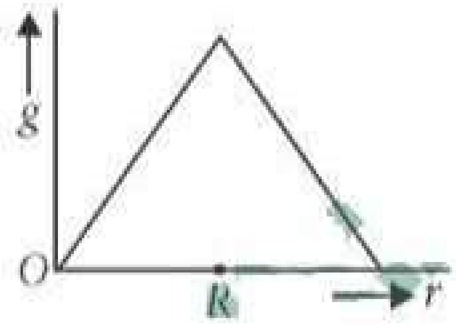
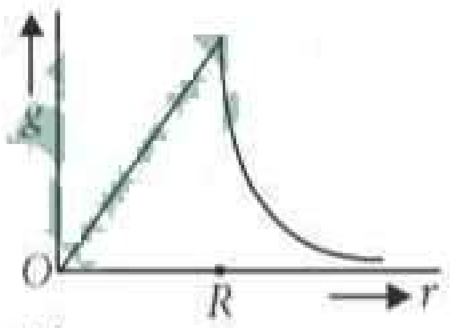
Chapter 7. Gravitation

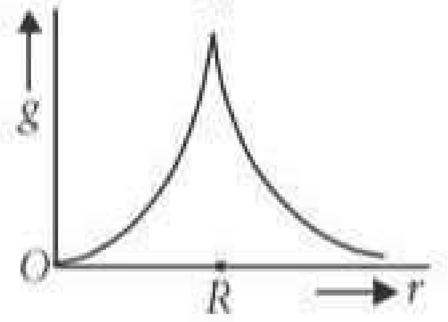
1. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then
   (a) $d = 1$ km (b) $d = \frac{3}{2}$ km
   (c) $d = 2$ km (d) $d = \frac{1}{2}$ km
   (NEET 2017)

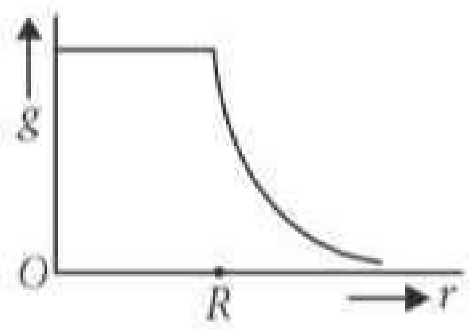
2. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will
   (a) move towards each other.
   (b) move away from each other.
   (c) will become stationary.
   (d) keep floating at the same distance between them.
   (NEET 2017)

3. Starting from the centre of the earth having radius R, the variation of g(acceleration due to gravity) is shown by

(a)


(b)


(c)


(d)


(NEET-II 2016)

4. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is
   (a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$
   (c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$
   (NEET-II 2016)

5. At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-1} and 6.0 m s^{-2} respectively? Take the radius of earth as 6400 km.
   (a) 1400 km (b) 2000 km
   (c) 2600 km (d) 1600 km
   (NEET-I 2016)

6. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is
   (a) 1 : 4 (b) $1:\sqrt{2}$
   (c) 1 : 2 (d) $1:2\sqrt{2}$
   (NEET-I 2016)

7. A remote-sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ ms^{-2}, then the orbital speed of the satellite is
   (a) 9.13 km s^{-1} (b) 6.67 km s^{-1}
   (c) 7.76 km s^{-1} (d) 8.56 km s^{-1}
   (2015)

8. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then,
   (a) the linear momentum of S remains constant in magnitude.
   (b) the acceleration of S is always directed towards the centre of the earth.

(c) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(d) the total mechanical energy of S varies periodically with time.

(2015)

9. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet *i.e.* $T^2 = Kr^3$ here K is constant. If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \dfrac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as

(a) $K = G$ (b) $K = \dfrac{1}{G}$
(c) $GK = 4\pi^2$ (d) $GMK = 4\pi^2$

(2015 Cancelled)

10. Two spherical bodies of mass M and $5M$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is

(a) $7.5R$ (b) $1.5R$
(c) $2.5R$ (d) $4.5R$

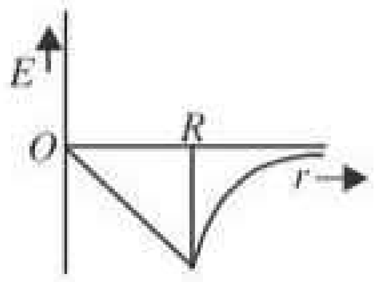
(2015 Cancelled)

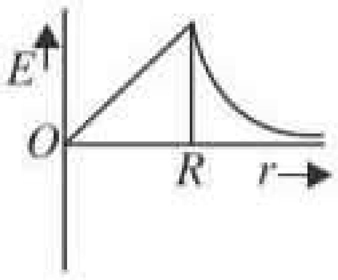
11. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole?

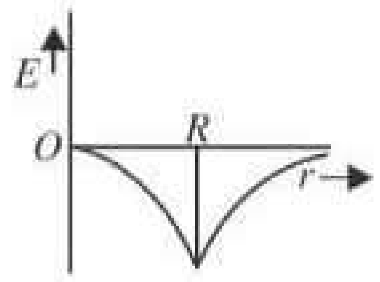
(a) 10^{-9} m (b) 10^{-6} m
(c) 10^{-2} m (d) 100 m

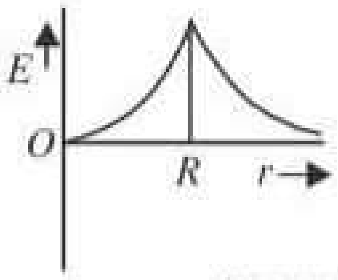
(2014)

12. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by

(a) 

(b) 

(c) 

(d) 

(2014)

13. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1 m, 2 m, 4 m, 8 m, ..., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be

(a) $-\dfrac{4}{3}G$ (b) $-4G$
(c) $-G$ (d) $-\dfrac{8}{3}G$

(NEET 2013)

14. A body of mass 'm' is taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be

(a) $3mgR$ (b) $\dfrac{1}{3}mgR$
(c) $mg2R$ (d) $\dfrac{2}{3}mgR$

(NEET 2013)

15. The radius of a planet is twice the radius of earth. Both have almost equal average mass-densities. V_P and V_E are escape velocities of the planet and the earth, respectively, then

(a) $V_P = 1.5\,V_E$ (b) $V_P = 2\,V_E$
(c) $V_E = 3\,V_P$ (d) $V_E = 1.5\,V_P$

(Karnataka NEET 2013)

16. A particle of mass 'm' is kept at rest at a height $3R$ from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The minimum speed with which it should be projected, so that it does not return back, is
(g is acceleration due to gravity on the surface of earth)

(a) $\left(\dfrac{GM}{2R}\right)^{1/2}$ (b) $\left(\dfrac{gR}{4}\right)^{1/2}$
(c) $\left(\dfrac{2g}{R}\right)^{1/2}$ (d) $\left(\dfrac{GM}{R}\right)^{1/2}$

(Karnataka NEET 2013)

17. The height at which the weight of a body becomes $\left(\dfrac{1}{16}\right)^{\text{th}}$, its weight on the surface of earth (radius R), is

(a) $5R$ (b) $15R$
(c) $3R$ (d) $4R$ *(2012)*

18. A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to

(a) $\frac{4GM_p}{D_p^2}$ (b) $\frac{GM_p m}{D_p^2}$

(c) $\frac{GM_p}{D_p^2}$ (d) $\frac{4GM_p m}{D_p^2}$ *(2012)*

19. A geostationary satellite is orbiting the earth at a height of $5R$ above the surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of $2R$ from the surface of the earth is

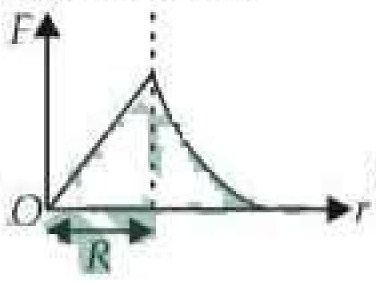
(a) 5 (b) 10
(c) $6\sqrt{2}$ (d) $\frac{6}{\sqrt{2}}$ *(2012)*

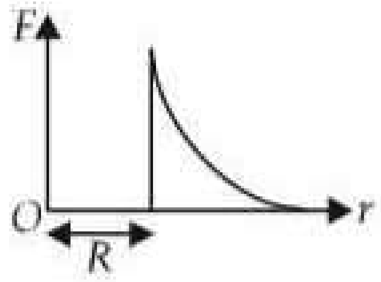
20. If v_e is escape velocity and v_o is orbital velocity of a satellite for orbit close to the earth's suface, then these are related by

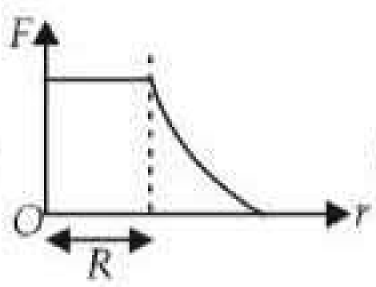
(a) $v_o = \sqrt{2}v_e$ (b) $v_o = v_e$
(c) $v_e = \sqrt{2v_o}$ (d) $v_e = \sqrt{2}v_o$

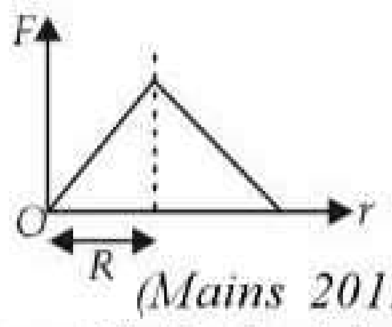
(Mains 2012)

21. Which one of the following plots represents the variation of gravitational field on a particle with distance r due to a thin spherical shell of radius R? (r is measured from the centre of the spherical shell)

(a) 

(b) 

(c) 

(d) 

(Mains 2012)

22. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

(a) $(r_1/r_2)^2$ (b) r_2/r_1
(c) $(r_2/r_1)^2$ (d) r_1/r_2 *(2011)*

23. A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum value of u so that the particle does not return back to earth, is

(a) $\sqrt{\frac{2GM}{R^2}}$ (b) $\sqrt{\frac{2GM}{R}}$

(c) $\sqrt{\frac{2gM}{R^2}}$ (d) $\sqrt{2gR^2}$

(Mains 2011)

24. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The magnitude of the gravitational potential at a point situated at $a/2$ distance from the centre, will be

(a) $\frac{GM}{a}$ (b) $\frac{2GM}{a}$

(c) $\frac{3GM}{a}$ (d) $\frac{4GM}{a}$

(Mains 2011, 2010)

25. The radii of circular orbits of two satellites A and B of the earth, are $4R$ and R, respectively. If the speed of satellite A is $3V$, then the speed of satellite B will be

(a) $\frac{3V}{4}$ (b) $6V$

(c) $12V$ (d) $\frac{3V}{2}$

(2010)

26. A man of 50 kg mass is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0.5 kg mass downwards with a speed 2 m/s. When the stone reaches the floor, the distance of the man above the floor will be

(a) 9.9 m (b) 10.1 m
(c) 10 m (d) 20 m *(2010)*

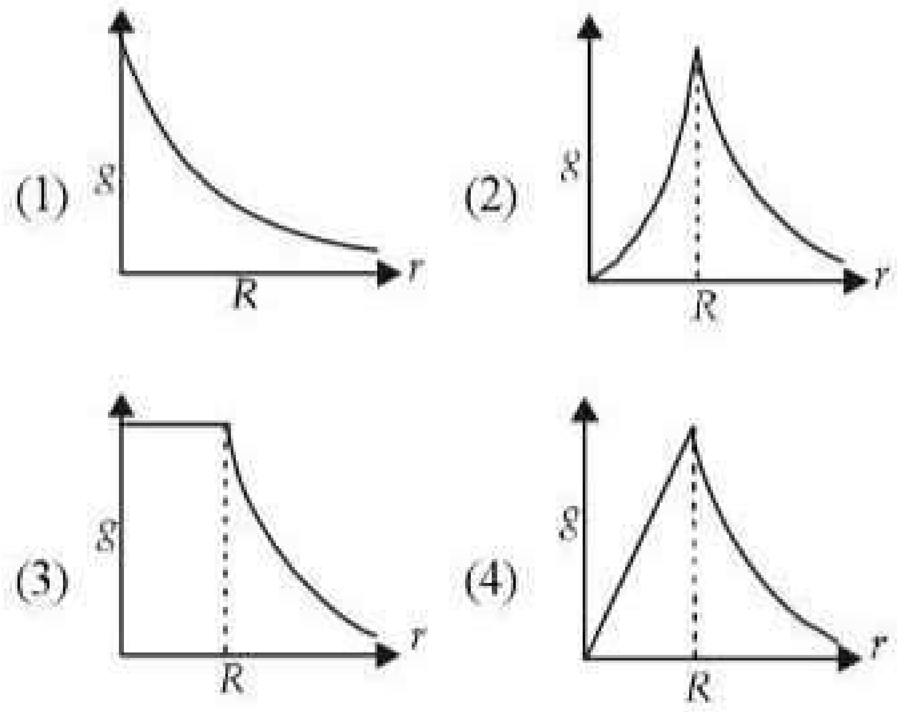
27. The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M, to transfer it from a circular orbit of radius R_1 to another of radius $R_2 (R_2 > R_1)$ is

(a) $GmM\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$ (b) $GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(c) $2GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (d) $\frac{1}{2}GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(Mains 2010)

28. The dependence of acceleration due to gravity g on the distance r from the centre of the earth, assumed to be a sphere of radius R of uniform density is as shown in figures below



The correct figure is

(a) (4) (b) (1)
(c) (2) (d) (3)

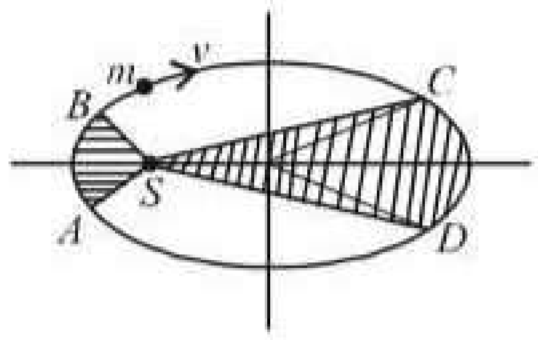
(Mains 2010)

29. (1) Centre of gravity (C.G.) of a body is the point at which the weight of the body acts
(2) Centre of mass coincides with the centre of gravity if the earth is assumed to have infinitely large radius.
(3) To evaluate the gravitational field intensity due to any body at an external point, the entire mass of the body can be considered to be concentrated at its C.G.
(4) The radius of gyration of any body rotating about an axis is the length of the perpendicular dropped from the C.G. of the body to the axis.
Which one of the following pairs of statements is correct?

(a) (4) and (1)
(b) (1) and (2)
(c) (2) and (3)
(d) (3) and (4)

(Mains 2010)

30. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then



(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$
(c) $t_1 = t_2$ (d) $t_1 > t_2$

(2009)

31. Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

(a) The potential energies of earth and satellite in the two cases are equal.
(b) S_1 and S_2 are moving with the same speed.
(c) The kinetic energies of the two satellites are equal.
(d) The time period of S_1 is four times that of S_2.

(2007)

32. The earth is assumed to be a sphere of radius R. A platform is arranged at a height R from the surface of the earth. The escape velocity of a body from this platform is fv, where v is its escape velocity from the surface of the Earth. The value of f is

(a) 1/2 (b) $\sqrt{2}$
(c) $1/\sqrt{2}$ (d) 1/3. *(2006)*

33. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then

(a) $g' = g/9$ (b) $g' = 27g$
(c) $g' = 9g$ (d) $g' = 3g$.

(2005)

34. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

(a) 1/2 (b) $1/\sqrt{2}$
(c) 2 (d) $\sqrt{2}$ *(2005)*

35. The density of a newly discovered planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R, the radius of the planet would be

(a) $2R$ (b) $4R$

(c) $\frac{1}{4}R$ (d) $\frac{1}{2}R$ *(2004)*

36. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) $3F$ (b) F

(c) $F/3$ (d) $F/9$

(2003)

37. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2 m on the surface of A. What is the height of jump by the same person on the planet B ?

(a) (2/9) m (b) 18 m

(c) 6 m (d) (2/3) m

(2003)

38. A body of mass m is placed on earth's surface which is taken from earth surface to a height of $h = 3R$, then change in gravitational potential energy is

(a) $\frac{mgR}{4}$ (b) $\frac{2}{3}mgR$

(c) $\frac{3}{4}mgR$ (d) $\frac{mgR}{2}$

(2003)

39. With what velocity should a particle be projected so that its height becomes equal to radius of earth?

(a) $\left(\frac{GM}{R}\right)^{1/2}$ (b) $\left(\frac{8GM}{R}\right)^{1/2}$

(c) $\left(\frac{2GM}{R}\right)^{1/2}$ (d) $\left(\frac{4GM}{R}\right)^{1/2}$

(2001)

40. For a planet having mass equal to mass of the earth but radius is one fourth of radius of the earth. Then escape velocity for this planet will be

(a) 11.2 km/sec (b) 22.4 km/sec

(c) 5.6 km/sec (d) 44.8 km/sec.

(2000)

41. Gravitational force is required for

(a) stirring of liquid

(b) convection

(c) conduction

(d) radiation. *(2000)*

42. A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be

(a) 36 N (b) 32 N

(c) 144 N (d) 50 N. *(2000)*

43. The escape velocity of a sphere of mass m is given by (G = Universal gravitational constant; M_e = Mass of the earth and R_e = Radius of the earth)

(a) $\sqrt{\frac{2GM_e m}{R_e}}$ (b)

(c) $\sqrt{\frac{GM_e}{R_e}}$ (d) $\sqrt{\frac{2GM_e + R_e}{R_e}}$

(1999)

44. The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and radius of the earth becomes half, the escape velocity becomes

(a) 22.4 km/s (b) 44.8 km/s

(c) 5.6 km/s (d) 11.2 km/s. *(1997)*

45. The period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun?

(a) 4 (b) 5

(c) 2 (d) 3 *(1997)*

46. What will be the formula of mass of the earth in terms of g, R and G ?

(a) $G\frac{R}{g}$ (b) $g\frac{R^2}{G}$

(c) $g^2\frac{R}{G}$ (d) $G\frac{g}{R}$. *(1996)*

47. A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball?

(a) it will fall down to the earth gradually

(b) it will go very far in the space

(c) it will continue to move with the same speed along the original orbit of spacecraft

(d) it will move with the same speed, tangentially to the spacecraft.

(1996)

48. The acceleration due to gravity g and mean density of the earth ρ are related by which of the following relations? (where G is the gravitational constant and R is the radius of the earth.)

(a) $\rho = \dfrac{3g}{4\pi GR}$ (b) $\rho = \dfrac{3g}{4\pi GR^3}$

(c) $\rho = \dfrac{4\pi gR^2}{3G}$ (d) $\rho = \dfrac{4\pi gR^3}{3G}$.

(1995)

49. Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is

(a) $\dfrac{1}{2}\sqrt{\dfrac{Gm}{R}}$ (b) $\sqrt{\dfrac{4Gm}{R}}$

(c) $\dfrac{1}{2R}\sqrt{\dfrac{1}{Gm}}$ (d) $\sqrt{\dfrac{Gm}{R}}$.

(1995)

50. The earth (mass $= 6 \times 10^{24}$ kg) revolves around the sun with an angular velocity of 2×10^{-7} rad/s in a circular orbit of radius 1.5×10^8 km. The force exerted by the sun on the earth, in newton, is

(a) 36×10^{21} (b) 27×10^{39}

(c) zero (d) 18×10^{25}.

(1995)

51. The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is about 10 times mass of mars. An object weighs 200 N on the surface of earth. Its weight on the surface of mars will be

(a) 20 N (b) 8 N

(c) 80 N (d) 40 N.

(1994)

52. The distance of two planets from the sun are 10^{13} m and 10^{12} m respectively. The ratio of time periods of the planets is

(a) $\sqrt{10}$ (b) $10\sqrt{10}$

(c) 10 (d) $1/\sqrt{10}$.

(1994, 1988)

53. If the gravitational force between two objects were proportional to $1/R$ (and not as $1/R^2$), where R is the distance between them, then a particle in a circular path (under such a force) would have its orbital speed v, proportional to

(a) R

(b) R^0 (independent of R)

(c) $1/R^2$ (d) $1/R$.

(1994, 1989)

54. A satellite in force free space sweeps stationary interplanetary dust at a rate of $dM/dt = \alpha v$, where M is mass and v is the speed of satellite and α is a constant. The acceleration of satellite is

(a) $\dfrac{-\alpha v^2}{2M}$ (b) $-\alpha v^2$

(c) $\dfrac{-2\alpha v^2}{M}$ (d) $\dfrac{-\alpha v^2}{M}$ *(1994)*

55. The escape velocity from earth is 11.2 km/s. If a body is to be projected in a direction making an angle 45° to the vertical, then the escape velocity is

(a) 11.2×2 km/s (b) 11.2 km/s

(c) $11.2/\sqrt{2}$ km/s (d) $11.2\sqrt{2}$ km/s

(1993)

56. A satellite A of mass m is at a distance of r from the surface of the earth. Another satellite B of mass $2m$ is at a distance of $2r$ from the earth's centre. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16

(c) 1 : 32 (d) $1:2\sqrt{2}$

(1993)

57. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite ?

(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$

(c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$

(1992)

58. The satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy ?

(a) $(3/4)mv^2$ (b) $(1/2)mv^2$

(c) mv^2 (d) $-(1/2)mv^2$

(1991)

59. A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct ?

(a) T is conserved (b) V is always positive

(c) E is always negative

(d) L is conserved but direction of vector L changes continously.

(1990)

60. For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be

(a) 11 km/s (b) $11\sqrt{3}$ km/s

(c) $\frac{11}{\sqrt{3}}$ km/s (d) 33 km/s

(1989)

61. The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major-axis of the orbit drawn from the sun is

(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 + r_2}{r_1 - r_2}$

(c) $\frac{2r_1r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$ *(1988)*

Answer Key

1. (c)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (a)
11. (c)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (d)
21. (b)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (b)
31. (b)	**32.** (c)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (a)	**40.** (b)
41. (b)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (a)	**49.** (d)	**50.** (a)
51. (c)	**52.** (b)	**53.** (b)	**54.** (d)	**55.** (b)	**56.** (d)	**57.** (a)	**58.** (d)	**59.** (c)	**60.** (a)
61. (c)									

EXPLANATIONS

1. **(c) :** The acceleration due to gravity at a height h is given as

$$g_h = g\left(1-\frac{2h}{R_e}\right)$$

where R_e is radius of earth.

The acceleration due to gravity at a depth d is given as

$$g_d = g\left(1-\frac{d}{R_e}\right)$$

Given, $g_h = g_d$

$$\therefore \quad g\left(1-\frac{2h}{R_e}\right) = g\left(1-\frac{d}{R_e}\right)$$

$\therefore \quad d = 2h = 2 \times 1 = 2\text{ km} (\because h = 1\text{ km})$

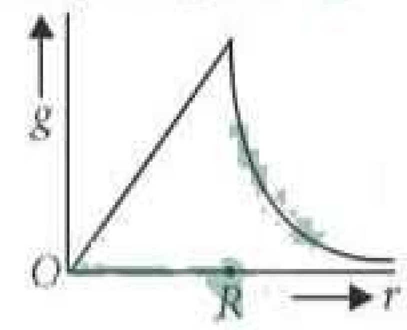
2. **(a) :** Since two astronauts are floating in gravitational free space. The only force acting on the two astronauts is the gravitational pull of their masses, $F = \dfrac{Gm_1m_2}{r^2}$,

which is attractive in nature.

Hence they move towards each other.

3. **(b) :** Acceleration due to gravity is given by

$$g = \begin{cases} \dfrac{4}{3}\pi\rho G r & ; r \le R \\[2ex] \dfrac{4}{3}\dfrac{\pi\rho R^3 G}{r^2} & ; r > R \end{cases}$$



4. **(b) :** Total energy of satellite at height h from the earth surface,

$$E = PE + KE$$

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 \quad \text{...(i)}$$

Also, $\dfrac{mv^2}{(R+h)} = \dfrac{GMm}{(R+h)^2}$

or, $v^2 = \dfrac{GM}{R+h}$ ...(ii)

From eqns. (i) and (ii),

$$E = -\frac{GMm}{(R+h)} + \frac{1}{2}\frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$$

$$= -\frac{1}{2}\frac{GM}{R^2} \times \frac{mR^2}{(R+h)}$$

$$= -\frac{mg_0R^2}{2(R+h)} \quad \left(\because g_0 = \frac{GM}{R^2}\right)$$

5. **(c) :** Gravitation potential at a height h from the surface of earth, $V_h = -5.4 \times 10^7\text{ J kg}^{-1}$

At the same point acceleration due to gravity,

$g_h = 6\text{ m s}^{-2}$

$R = 6400\text{ km} = 6.4 \times 10^6\text{ m}$

We know, $V_h = -\dfrac{GM}{(R+h)}$,

$$g_h = \frac{GM}{(R+h)^2} = -\frac{V_h}{R+h} \Rightarrow R+h = -\frac{V_h}{g_h}$$

$$\therefore \quad h = -\frac{V_h}{g_h} - R = -\frac{(-5.4\times10^7)}{6} - 6.4\times10^6$$

$$= 9 \times 10^6 - 6.4 \times 10^6 = 2600\text{ km}$$

6. **(d) :** As escape velocity, $v = \sqrt{\dfrac{2GM}{R}}$

$$= \sqrt{\frac{2G}{R}\cdot\frac{4\pi R^3}{3}\rho} = R\sqrt{\frac{8\pi G}{3}\rho}$$

$$\therefore \quad \frac{v_e}{v_p} = \frac{R_e}{R_p} \times \sqrt{\frac{\rho_e}{\rho_p}} = \frac{1}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}}$$

$(\because R_p = 2R_e \text{ and } \rho_p = 2\rho_e)$

7. **(c) :** The orbital speed of the satellite is

$$v_o = R\sqrt{\frac{g}{(R+h)}}$$

where R is the earth's radius, g is the acceleration due to gravity on earth's surface and h is the height above the surface of earth.

Here, $R = 6.38 \times 10^6\text{m}$, $g = 9.8\text{ m s}^{-2}$ and $h = 0.25 \times 10^6\text{ m}$

$$\therefore v_o = (6.38\times10^6\text{ m})\sqrt{\frac{(9.8\text{ m s}^{-2})}{(6.38\times10^6\text{ m} + 0.25\times10^6\text{ m})}}$$

$$= 7.76 \times 10^3\text{ m s}^{-1} = 7.76\text{ km s}^{-1}$$

8. **(b) :** The gravitational force on the satellite S acts towards the centre of the earth, so the acceleration of the satellite S is always directed towards the centre of the earth.

9. **(d) :** Gravitational force of attraction between sun and planet provides centripetal force for the orbit of planet.

$$\therefore \quad \frac{GMm}{r^2} = \frac{mv^2}{r}; \; v^2 = \frac{GM}{r} \qquad \ldots \text{(i)}$$

Time period of the planet is given by

$$T = \frac{2\pi r}{v}, \; T^2 = \frac{4\pi^2 r^2}{v^2} = \frac{4\pi^2 r^2}{\left(\frac{GM}{r}\right)} \qquad \text{(Using (i))}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} \qquad \ldots \text{(ii)}$$

According to question,

$$T^2 = Kr^3 \qquad \ldots \text{(iii)}$$

Comparing equations (ii) and (iii), we get

$$K = \frac{4\pi^2}{GM} \quad \therefore \; GMK = 4\pi^2$$

10. **(a)**

11. **(c) :** Light cannot escape from a black hole,

$v_{esc} = c$

$$\sqrt{\frac{2GM}{R}} = c \quad \text{or} \quad R = \frac{2GM}{c^2}$$

$$R = \frac{2 \times 6.67 \times 10^{-11} \text{ N m}^2\text{kg}^{-2} \times 5.98 \times 10^{24} \text{kg}}{(3 \times 10^8 \text{ m s}^{-1})^2}$$

$$= 8.86 \times 10^{-3} \text{ m} \approx 10^{-2} \text{ m}$$

12. **(a) :** For a point inside the earth *i.e.* $r < R$

$$E = -\frac{GM}{R^3} r$$

where M and R be mass and radius of the earth respectively.

At the centre, $r = 0$

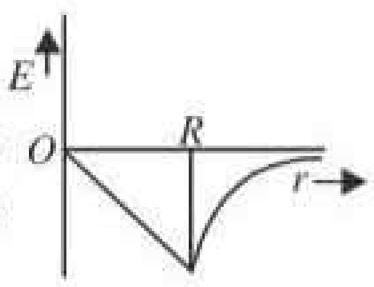
$\therefore \quad E = 0$

For a point outside the earth *i.e.* $r > R$,

$$E = -\frac{GM}{r^2}$$

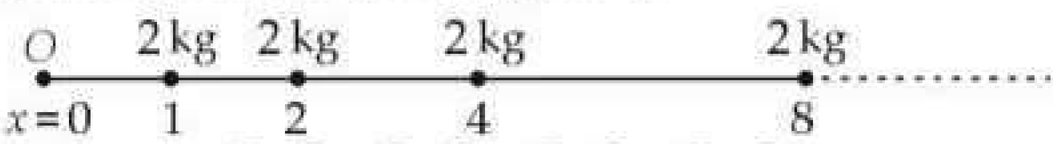
On the surface of the earth *i.e.* $r = R$,

$$E = -\frac{GM}{R^2}$$



The variation of E with distance r from the centre is as shown in the figure.

13. **(b) :** The resulting gravitational potential at the origin O due to each of mass 2 kg located at positions as shown in figure is



$$V = -\frac{G \times 2}{1} - \frac{G \times 2}{2} - \frac{G \times 2}{4} - \frac{G \times 2}{8} - \ldots$$

$$= -2G\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\right] = -2G\left[\frac{1}{1 - \frac{1}{2}}\right]$$

$$= -2G\left[\frac{2}{1}\right] = -4G$$

14. **(d) :** Gravitational potential energy at any point at a distance r from the centre of the earth is

$$U = -\frac{GMm}{r}$$

where M and m be masses of the earth and the body respectively.

At the surface of the earth, $r = R$

$$\therefore \quad U_i = -\frac{GMm}{R}$$

At a height h from the surface,

$$r = R + h = R + 2R \qquad (h = 2R \text{ (Given)})$$
$$= 3R$$

$$\therefore \quad U_f = -\frac{GMm}{3R}$$

Change in potential energy,

$$\Delta U = U_f - U_i$$

$$= -\frac{GMm}{3R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}\left(1 - \frac{1}{3}\right)$$

$$= \frac{2}{3}\frac{GMm}{R} = \frac{2}{3}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

15. **(b) :** Here, $R_P = 2R_E$, $\rho_E = \rho_P$

Escape velocity of the earth,

$$V_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2G}{R_E}\left(\frac{4}{3}\pi R_E^3 \rho_E\right)} = R_E\sqrt{\frac{8}{3}\pi G \rho_E} \qquad \ldots \text{(i)}$$

Escape velocity of the planet

$$V_P = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G}{R_P}\left(\frac{4}{3}\pi R_P^3 \rho_P\right)} = R_P\sqrt{\frac{8}{3}\pi G \rho_P} \qquad \ldots \text{(ii)}$$

Divide (i) by (ii), we get

$$\frac{V_E}{V_P} = \frac{R_E}{R_P}\sqrt{\frac{\rho_E}{\rho_P}} = \frac{R_E}{2R_E}\sqrt{\frac{\rho_E}{\rho_E}} = \frac{1}{2}$$

or $\quad V_P = 2V_E$

16. **(a) :** The minimum speed with which the particle should be projected from the surface of the earth so that it does not return back is known as escape speed and it is given by

$$v_e = \sqrt{\frac{2GM}{(R+h)}}$$

Here, $h = 3R$

$\therefore \quad v_e = \sqrt{\frac{2GM}{(R+3R)}} = \sqrt{\frac{2GM}{4R}} = \sqrt{\frac{GM}{2R}}$

$= \sqrt{\frac{gR}{2}} \qquad \left(\because g = \frac{GM}{R^2}\right)$

17. (c) : Acceleration due to gravity at a height h from the surface of earth is

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \quad \text{....(i)}$$

where g is the acceleration due to gravity at the surface of earth and R is the radius of earth.
Multiplying by m (mass of the body) on both sides in (i), we get

$$mg' = \frac{mg}{\left(1+\frac{h}{R}\right)^2}$$

$\therefore$ Weight of body at height h, $W' = mg'$
Weight of body at surface of earth, $W = mg$
According to question, $W' = \frac{1}{16}W$

$$\therefore \quad \frac{1}{16} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\left(1+\frac{h}{R}\right)^2 = 16 \quad \text{or} \quad 1+\frac{h}{R} = 4$$

$$\text{or} \quad \frac{h}{R} = 3 \quad \text{or} \quad h = 3R$$

18. (a) : Gravitational force acting on particle of mass m is

$$F = \frac{GM_p m}{(D_p/2)^2}$$

Acceleration due to gravity experience by the particle is

$$g = \frac{F}{m} = \frac{GM_p}{(D_p/2)^2} = \frac{4GM_p}{D_p^2}$$

19. (c) : According to Kepler's third law $T \propto r^{3/2}$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{R+2R}{R+5R}\right)^{3/2} = \frac{1}{2^{3/2}}$$

Since $T_1 = 24$ hours

$$\text{So,} \quad \frac{T_2}{24} = \frac{1}{2^{3/2}} \quad \text{or} \quad T_2 = \frac{24}{2^{3/2}} = \frac{24}{2\sqrt{2}} = 6\sqrt{2} \text{ hours}$$

20. (d) : Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$ ...(i)

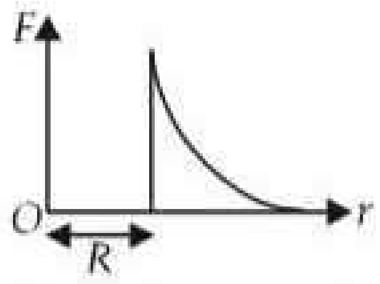
where M and R be the mass and radius of the earth respectively.
The orbital velocity of a satellite close to the earth's surface is

$$v_o = \sqrt{\frac{GM}{R}} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$v_e = \sqrt{2}v_o$$

21. (b) : Gravitational field due to the thin spherical shell
Inside the shell, *i.e.* (For $r < R$)
$F = 0$
On the surface of the shell, *i.e.* (For $r = R$)
$F = \frac{GM}{R^2}$
Outside the shell, *i.e.* (For $r > R$)
$F = \frac{GM}{r^2}$



The variation of F with distance r from the centre is as shown in the adjacent figure.

22. (b) : According to the law of conservation of angular momentum

$$L_1 = L_2$$

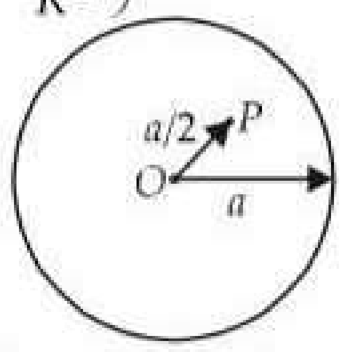
$$mv_1r_1 = mv_2r_2 \Rightarrow v_1r_1 = v_2r_2 \quad \text{or} \quad \frac{v_1}{v_2} = \frac{r_2}{r_1}$$

23. (b) : According to law of conservation of mechanical energy

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = 0 \quad \text{or} \quad u^2 = \frac{2GM}{R}$$

$$u = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

24. (c) : Here,
Mass of a particle = M
Mass of a spherical shell = M
Radius of a spherical shell = a
Let O be centre of a spherical shell.



Gravitational potential at point P due to particle at O is

$$V_1 = -\frac{GM}{a/2}$$

Gravitational potential at point P due to spherical shell is

$$V_2 = -\frac{GM}{a}$$

Hence, total gravitational potential at point P is

$$V = V_1 + V_2$$

$$= -\frac{GM}{a/2} + \left(\frac{-GM}{a}\right) = -\frac{2GM}{a} - \frac{GM}{a} = -\frac{3GM}{a}$$

$$|V| = \frac{3GM}{a}$$

25. (b) : Orbital speed of the satellite around the earth is

$$v = \sqrt{\frac{GM}{r}}$$

For satellite A

$$r_A = 4R,\ v_A = 3V$$

$$v_A = \sqrt{\frac{GM}{r_A}} \quad \text{...(i)}$$

For satellite B

$$r_B = R,\ v_B = ?$$

$$v_B = \sqrt{\frac{GM}{r_B}} \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we get

$$\therefore \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} \text{ or } v_B = v_A\sqrt{\frac{r_A}{r_B}}$$

Substituting the given values, we get

$$v_B = 3V\sqrt{\frac{4R}{R}} \text{ or } v_B = 6V$$

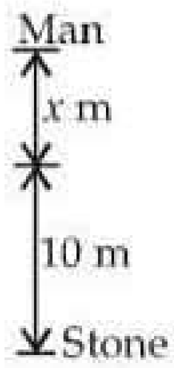
26. (b) : Since the man is in gravity free space, force on man + stone system is zero.

Therefore centre of mass of the system remains at rest. Let the man goes x m above when the stone reaches the floor, then

$$M_{man} \times x = M_{stone} \times 10$$

$$x = \frac{0.5}{50} \times 10$$

$$x = 0.1 \text{ m}$$



Therefore final height of man above floor $= 10 + x = 10 + 0.1 = 10.1$ m

27 (d)

28. (a) : The acceleration due to gravity at a depth d below surface of earth is

$$g' = \frac{GM}{R^2}\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

$g' = 0$ at $d = R$.

i.e., acceleration due to gravity is zero at the centre of earth.

Thus, the variation in value g with r is

For, $r > R$,

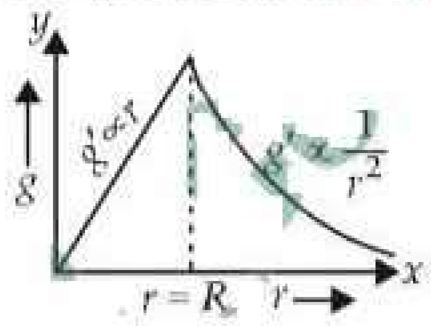
$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = \frac{gR^2}{r^2} \Rightarrow g' \propto \frac{1}{r^2}$$

Here, $R + h = r$

For $r < R$, $g' = g\left(1 - \frac{d}{R}\right) = \frac{gr}{R}$

Here, $R - d = r \Rightarrow g' \propto r$

Therefore, the variation of g with distance from centre of the earth will be as shown in the figure.



29. (a)

30. (b) : Equal areas are swept in equal time.

t_1, the time taken to go from C to $D = 2t_2$

where t_2 is the time taken to go from A to B.

As it is given that area $SCD = 2SAB$.

31. (b) : The satellite of mass m is moving in a circular orbit of radius r.

$\therefore$ Kinetic energy of the satellite, $K = \frac{GMm}{2r}$ ... (i)

Potential energy of the satellite, $U = \frac{-GMm}{r}$ ... (ii)

Orbital speed of satellite, $v = \sqrt{\frac{GM}{r}}$ ... (iii)

Time-period of satellite, $T = \left[\left(\frac{4\pi^2}{GM}\right)r^3\right]^{1/2}$ ... (iv)

Given $m_{S_1} = 4m_{S_2}$

Since M, r is same for both the satellites S_1 and S_2

$\therefore$ From equation (ii), we get $U \propto m$

$$\therefore \frac{U_{S_1}}{U_{S_2}} = \frac{m_{S_1}}{m_{S_2}} = 4 \text{ or, } U_{S_1} = 4U_{S_2}.$$

Option (a) is wrong.

From (iii), since v is independent of the mass of a satellite, the orbital speed is same for both satellites S_1 and S_2.

Hence option (b) is correct.

From (i), we get $K \propto m$

$$\therefore \frac{K_{S_1}}{K_{S_2}} = \frac{m_{S_1}}{m_{S_2}} = 4 \text{ or, } K_{S_1} = 4K_{S_2}.$$

Hence option (c) is wrong.
From (iv), since T is independent of the mass of a satellite, time period is same for both the satellites S_1 and S_2. Hence option (d) is wrong.

32. (c) : Escape velocity of the body from the surface of earth is $v = \sqrt{2gR}$

For escape velocity of the body from the platform potential energy + kinetic energy = 0

$$-\frac{GMm}{2R} + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow \; fv_{escape} = \sqrt{\frac{GM}{R^2} \cdot R} = \sqrt{gR} = fv$$

From the surface of the earth, $v_{\text{escape}} = \sqrt{2gR}$

$$\therefore \; fv_{\text{escape}} = \frac{v_{\text{escape}}}{\sqrt{2}}. \quad \therefore \quad f = \frac{1}{\sqrt{2}}.$$

33. (d) : $g = \dfrac{GM}{r^2} = \dfrac{G}{r^2}\left(\dfrac{4}{3} \times r^3\rho\right) = \dfrac{4}{3} \times \rho G r$

$$\frac{g'}{g} = \frac{3R}{R} \Rightarrow g' = 3g.$$

34. (a) : $-\dfrac{GMm}{R^2} + m\omega^2 R = 0$

$$\therefore \; \frac{GMm}{R^2} = m\omega^2 R$$

$$K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}mR^2\omega^2 = \frac{GMm}{2R}$$

$$P.E. = -\frac{GMm}{R}.$$

$$\therefore \; K.E. = \frac{|P.E.|}{2} \quad \text{or,} \quad \frac{K.E.}{|P.E.|} = \frac{1}{2}$$

35. (d) : From equation of acceleration due to gravity.

$$g_e = \frac{GM_e}{R_e^2} = \frac{G(4/3)\pi R_e^3}{R_e^2}\rho_e$$

$$g_e \propto R_e\rho_e$$

Acceleration due to gravity of planet $g_p \propto R_p\rho_p$

$R_e\rho_e = R_p\rho_p \Rightarrow R_e\rho_e = R_p 2\rho_e \Rightarrow R_p = \dfrac{1}{2}R$ $(\because R_e = R)$

36. (b) : The gravitational force does not depend upon the medium in which objects are placed.

37. (b) : The velocity of the mass while reaching the surface of both the planets will be same.

i.e., $\sqrt{2g'h'} = \sqrt{2gh}$

$$\sqrt{2 \times g \times h'} = \sqrt{2 \times 9g \times 2} \quad 2h' = 36 \Rightarrow h' = 18 \text{ m.}$$

38. (c) : Gravitational potential energy on earth's surface $= -\dfrac{GMm}{R}$, where M and R are the mass and radius of the earth respectively, m is the mass of the body and G is the universal gravitational constant.

Gravitational potential energy at a height $h = 3R$

$$= -\frac{GMm}{R+h} = -\frac{GMm}{R+3R} = -\frac{GMm}{4R}$$

$\therefore$ Change in potential energy

$$= -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4\;R} = \frac{3}{4}mgR$$

39. (a) : Use $v^2 = \dfrac{2gh}{1 + \dfrac{h}{R}}$ given $h = R$.

$$\therefore \; v = \sqrt{gR} = \sqrt{\frac{GM}{R}}.$$

40. (b) : $v_e = \sqrt{2gR} = \sqrt{\dfrac{2GM}{R}}$

If R is 1/4th then $v_e = 2\,v_{e\text{-earth}}$
$= 2 \times 11.2 = 22.4$ km/sec.

41. (b)

42. (b) : $F_{\text{surface}} = G\dfrac{Mm}{R_e^2}$

$$F_{R_e/2} = G\frac{Mm}{(R_e + R_e/2)^2} = \frac{4}{9} \times F_{\text{surface}} = \frac{4}{9} \times 72 = 32 \text{ N.}$$

43. (b)

44. (a) : Escape velocity of a body $(v_e) = 11.2$ km/s; New mass of the earth $M'_e = 2M_e$ and new radius of the earth $R'_e = 0.5R_e$.

Escape velocity $(v_e) = \sqrt{\dfrac{2GM_e}{R_e}} \propto \sqrt{\dfrac{M_e}{R_e}}$.

Therefore $\dfrac{v_e}{v'_e} = \sqrt{\dfrac{M_e}{R_e} \times \dfrac{0.5R_e}{2M_e}} = \sqrt{\dfrac{1}{4}} = \dfrac{1}{2}$

or, $v'_e = 2v_e = 22.4$ km/sec.

45. (a) : Period of revolution of planet A $(T_A) = 8T_B$.
According to Kepler's III law of planetary motion $T^2 \propto R^3$.

Therefore $\left(\dfrac{r_A}{r_B}\right)^3 = \left(\dfrac{T_A}{T_B}\right)^2 = \left(\dfrac{8T_B}{T_B}\right)^2 = 64$

or $\dfrac{r_A}{r_B} = 4$ or $r_A = 4r_B$.

46. (b)

47. (c) : Since no external torque is applied therefore, according to law of conservation of angular momentum, the ball will continue to move with the same angular velocity along the original orbit of the spacecraft.

48. (a) : Acceleration due to gravity $(g) = G \times \frac{M}{R^2}$

$= G\frac{(4/3)\pi R^3 \times \rho}{R^2} = G \times \frac{4}{3}\pi R \times \rho$ or $\rho = \frac{3g}{4\pi GR}$

49. (d) : The two masses, separated by a distance $2R$ are going round their common centre of mass, the centre of the circle.

Attractive force $= -G\frac{mm}{4R^2}$. But the two masses are going round the centre of mass or the reduced mass $\mu = \frac{mm}{m+m}$ is going round a circle of radius = distance of separation

$\therefore$ Centrifugal force $= \frac{m}{2}\omega^2.2R = \frac{m}{2}v^2.\frac{1}{2R}$

Now, $\frac{m}{2} \times \frac{v^2}{2R} = \frac{Gm^2}{4R^2} \Rightarrow v = \sqrt{\frac{Gm}{R}}$

50. (a) : Mass $(m) = 6 \times 10^{24}$ kg;
Angular velocity $(\omega) = 2 \times 10^{-7}$ rad/s and radius $(r) = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m.
Force exerted on the earth $= mR\omega^2$
$= (6 \times 10^{24}) \times (1.5 \times 10^{11}) \times (2 \times 10^{-7})^2$
$= 36 \times 10^{21}$ N.

51. (c) : Radius of earth $(R_e) = 6400$ km; Radius of mars $(R_m) = 3200$ km; Mass of earth $(M_e) = 10$ Mm and weight of the object on earth $(W_e) = 200$ N.

$$\frac{W_m}{W_e} = \frac{mg_m}{mg_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2 = \frac{1}{10} \times (2)^2 = \frac{2}{5}$$

or $W_m = W_e \times \frac{2}{5} = 200 \times 0.4 = 80$ N.

52. (b) : Distance of two planets from sun, $r_1 = 10^{13}$ m and $r_2 = 10^{12}$ m.
Relation between time period (T) and distance of the planet from the sun is $T^2 \propto r^3$ or $T \propto r^{3/2}$.

Therefore $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2} = 10\sqrt{10}$.

53. (b) : Centripetal force $(F) = \frac{mv^2}{R}$ and the gravitational force $(F) = \frac{GMm}{R^2} = \frac{GMm}{R}$ (where $R^2 \to R$). Since $\frac{mv^2}{R} = \frac{GMm}{R}$, therefore $v = \sqrt{GM}$.
Thus velocity v is independent of R.

54. (d) : Rate of change of mass $\frac{dM}{dt} = \alpha v$.
Retarding force = Rate of change of momentum
= Velocity × Rate of change in mass $= -v \times \frac{dM}{dt}$
$= -v \times \alpha v = -\alpha v^2$. (Minus sign of v due to deceleration)
Therefore, acceleration $= \frac{\alpha v^2}{M}$

55. (b) : Escape velocity does not depend on the angle of projection.

56. (d) : Time period does not depend on the mass. As $T^2 \propto r^3$,
$\frac{T_A}{T_B} = \frac{r^{3/2}}{2^{3/2} r^{3/2}} = 1 : 2\sqrt{2}$

57. (a) : $\frac{GMm}{r^2} = m\omega^2 r \Rightarrow r^3 = \frac{GM}{\omega^2} = \frac{gR^2}{\omega^2}$
$\therefore$ $r = (gR^2/\omega^2)^{1/3}$.

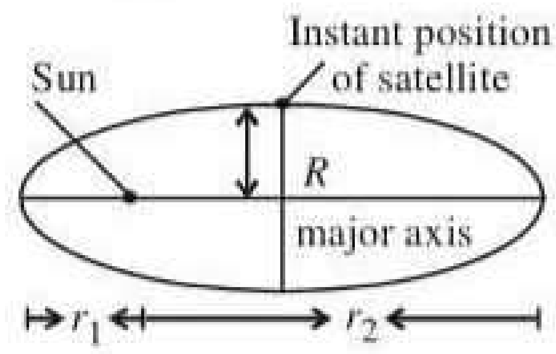
58. (d) : Total energy $= -$K.E. $= -\frac{1}{2}mv^2$

59. (c) : In a circular or elliptical orbital motion torque is always acting parallel to displacement or velocity. So, angular momentum is conserved. In attractive field, potential energy is negative. Kinetic energy changes as velocity increase when distance is less. But if the motion is in a plane, the direction of L does not change.

60. (a) : Since escape velocity $\left(v_e = \sqrt{2gR_e}\right)$ is independent of angle of projection, so it will not change.

61. (c) : Applying the properties of ellipse, we have

$$\frac{2}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2} \quad 1$$



$$R = \frac{2r_1 r_2}{r_1 + r_2}$$