BCECE
ENGINEERING ENTRANCE EXAM
SOLVED PAPER 2006

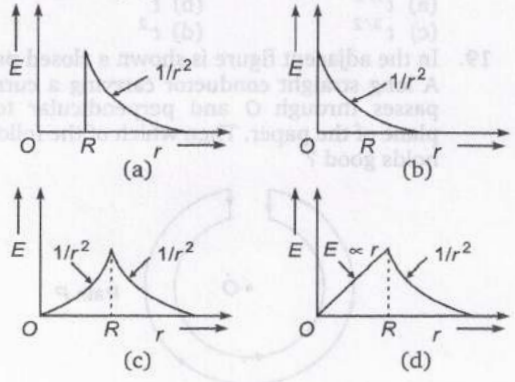
Physics

1. The length of a simple pendulum is about 100 cm known to an accuracy of 1mm. Its period of oscillation is 2s determined by measuring the time for 100 oscillations using a clock of 0.1 s resolution. What is the accuracy in the determined value of g ?
   (a) 0.2% (b) 0.5%
   (c) 0.1% (d) 2%

2. Young's modulus of the material of a wire is Y. On pulling the wire by a force F, the increase in its length is x. The potential energy of the stretched wire is :
   (a) $\frac{1}{2}Fx$ (b) $\frac{1}{2}Yx$
   (c) $\frac{1}{2}Fx^2$ (d) none of these

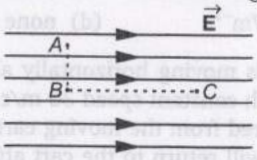
3. A charge situated at a certain distance along the axis of an electric dipole experiences a force F. If the distance of the charge from the dipole is doubled, the force acting on it will become :
   (a) $2F$ (b) $\frac{F}{2}$
   (c) $\frac{F}{4}$ (d) $\frac{F}{8}$

4. Which of the following plots represents the variation of the electric field with distance from the centre of a uniformly charged non-conducting sphere of radius R ?



5. A certain electrical conductor has a square cross-section, 2.0 mm on side, and is 12 m long. The resistance between its ends is 0.072 Ω. The resistivity of its material is equal to :
   (a) 2.4×10^{-6} Ωm (b) 1.2×10^{-6} Ωm
   (c) 1.2×10^{-8} Ωm (d) 2.4×10^{-8} Ωm

6. Figure shows three points A, B and C in a region of uniform electric field $\vec{E}$. The line AB is perpendicular and BC is parallel to the field lines. Then which of the following holds good ?

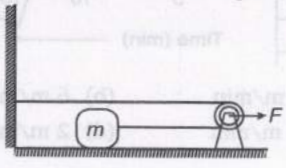


   (a) $V_A = V_B = V_C$ (b) $V_A = V_B > V_C$
   (c) $V_A = V_B < V_C$ (d) $V_A > V_B = V_C$
   where V_A, V_B and V_C represent the electric potential at the points A, B and C respectively.

7. The (x, y, z) coordinates of two points A and B are given respectively as $(0, 3, -1)$ and $(-2, 6, 4)$. The displacement vector form A to B may be given by :
   (a) $-2\hat{i}+6\hat{j}+4\hat{k}$ (b) $-2\hat{i}+3\hat{j}+3\hat{k}$
   (c) $-2\hat{i}+3\hat{j}+5\hat{k}$ (d) $2\hat{i}-3\hat{j}-5\hat{k}$

8. In the first second of its flight, rocket ejects 1/60 of its mass with a velocity of 2400 ms^{-1}. The acceleration of the rocket is :
   (a) 19.6 ms^{-2} (b) 30.2 ms^{-2}
   (c) 40 ms^{-2} (d) 49.8 ms^{-2}

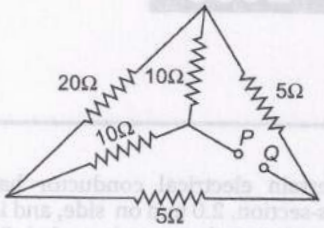
9. In the given figure the pulley is assumed massless and frictionless. If the friction force on the object of mass m is f, then its acceleration in terms of the force F will be equal to :



(a) $(F-f)/m$ (b) $\left(\frac{F}{2}-f\right)/m$

(c) F/m (d) none of these

10. The equivalent resistance between the points P and Q in the network shown in the figure is given by :



(a) 2.5 Ω (b) 7.5 Ω

(c) 10 Ω (d) 12.5 Ω

11. The magnetic field amplitude of an electromagnetic wave is 2×10^{-7} T. Its electric field amplitude, if the wave is travelling in free space is :

(a) 6 Vm^{-1} (b) 60 Vm^{-1}

(c) $\frac{1}{6}$ Vm^{-1} (d) none of these

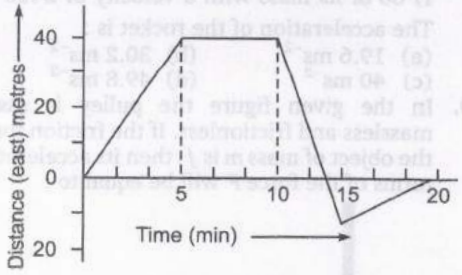
12. A cart is moving horizontally along a straight line with constant speed 30 m/s. A projectile is to be fired from the moving cart in such a way that it will return to the cart after the cart has moved 80 m. At what speed (relative to the cart) must the projectile be fired ?

(Take $g = 10$ m/s^2)

(a) 10 m/s (b) $10\sqrt{8}$ m/s

(c) $\frac{40}{3}$ m/s (d) None of these

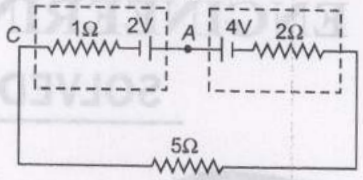
13. A boy begins to walk eastward along a street infront of his house, and the graph of his displacement from home is shown in the following figure. His average velocity for the whole time interval is equal to :



(a) 8 m/min (b) 6 m/min

(c) $\frac{8}{3}$ m/min (d) 2 m/min

14. What is the potential drop between points A and C in the following circuit ?



Resistances 1 Ω and 2 Ω represent the internal resistances of the respective cells.

(a) 1.75 V (b) 2.25 V

(c) $\frac{5}{4}$ V (d) $\frac{4}{5}$ V

15. The escape velocity of a projectile on the earth's surface is 11.2 kms^{-1}. A body is projected out with thrice this speed. The speed of the body far away from the earth will be :

(a) 22.4 kms^{-1} (b) 31.7 kms^{-1}

(c) 33.6 kms^{-1} (d) none of these

16. A body moves along a circular path of radius 10 m and the coefficient of friction is 0.5. What should be its angular speed in rad/s, if it is not to slip from the surface ? ($g = 9.8$ ms^{-2})

(a) 5 (b) 10 (c) 0.1 (d) 0.7

17. One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v, the net force on the particle (directed towards the centre) is :

(a) T (b) $T-\frac{mv^2}{l}$

(c) $T+\frac{mv^2}{l}$ (d) zero

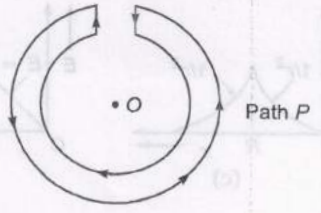
where T is the tension in the string.

18. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to :

(a) $t^{1/2}$ (b) t

(c) $t^{3/2}$ (d) t^2

19. In the adjacent figure is shown a closed path P. A long straight conductor carrying a current I passes through O and perpendicular to the plane of the paper. Then which of the following holds good ?



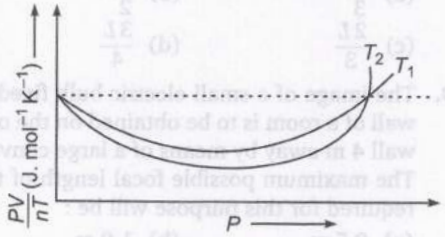
(a) $\int_p \vec{B}\cdot\vec{dl}=0$ (b) $\int_p \vec{B}\cdot\vec{dl}=\mu_0 I$

(c) $\int_p \vec{B}\cdot\vec{dl}>\mu_0 I$ (d) None of these

20. Two circular, similar, coaxial loops carry equal currents in the same direction. If the loops are brought nearer, what will happen ?

(a) Current will increase in each loop
(b) Current will decrease in each loop
(c) Current will remain same in each loop
(d) Current will increase in one and decrease in the other

21. The figure below shows the plot of $\frac{PV}{nT}$ *versus* P for oxygen gas at two different temperatures.



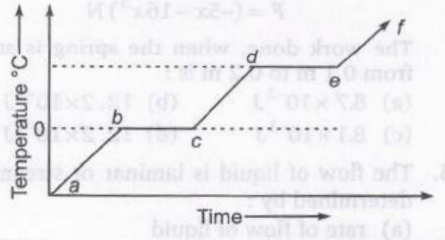
Read the following statements concerning the above curves :

(i) The dotted line corresponds to the ideal gas behaviour.
(ii) $T_1 > T_2$
(iii) The value of $\frac{PV}{nT}$ at the point where the curves meet on the y-axis is the same for all gases.

Which of the above statements is true ?

(a) (i) only (b) (i) and (ii) only
(c) All of these (d) None of these

22. The following figure represents the temperature *versus* time plot for a given amount of a substance when heat energy is supplied to it at a fixed rate and at a constant pressure.



Which parts of the above plot represent a phase change ?

(a) a to b and e to f (b) b to c and c to d
(c) d to e and e to f (d) b to c and d to e

23. A bar magnet has a coercivity $4\times10^3\,Am^{-1}$. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. The current carried by the solenoid should be :

(a) 8 A (b) 6 A (c) 4.5 A (d) 2 A

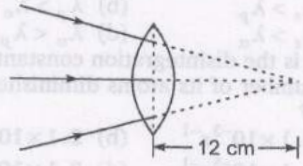
24. In a series LCR circuit the frequency of a 10 V AC voltage source is adjusted in such a fashion that the reactance of the inductor measures 15 Ω and that of capacitor 11 Ω. If $R = 3$ Ω, the potential difference across the series combination of L and C will be :

(a) 8 V (b) 10 V
(c) 22 V (d) 52 V

25. A circuit draws 330 W from a 110 V, 60 Hz AC line. The power factor is 0.6 and the current lags the voltage. The capacitance of a series capacitor that will result in a power factor of unity is equal to :

(a) 31 μF (b) 54 μF
(c) 151 μF (d) 201 μF

26. If the focal length of the lens is 20 cm, what is the distance of the image from the lens in the following figure ?



(a) 5.5 cm (b) 7.5 cm
(c) 12.0 cm (d) 20.0 cm

27. An open U-tube contains mercury. When 11.2 cm of water is poured into one of the arms of the tube, how high does the mercury rise in the other arm form its initial level ?

(a) 0.56 cm (b) 1.35 cm
(c) 0.41 cm (d) 2.32 cm

28. The change in the entropy of a 1 mole of an ideal gas which went through an isothermal process from an initial state (P_1, V_1, T) to the final state (P_2, V_2, T) is equal to :

(a) zero (b) $R \ln T$
(c) $R \ln \frac{V_1}{V_2}$ (d) $R \ln \frac{V_2}{V_1}$

29. An unpolarized beam of light is incident on a glass surface at an angle of incidence equal to the polarizing angle of the glass. Read the following statements :

(i) The reflected beam is completely polarized.
(ii) The refracted beam is partially polarized.
(iii) The angle between the reflected and the refracted beams is 90°.

Which of the above statements is/are true ?
(a) (i) only
(b) (ii) only
(c) (i) and (iii)
(d) All the statements are correct

30. The threshold frequency for certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, the cut-off voltage of the photoelectric current will be :
(a) 4.9 V (b) 3.0 V
(c) 2.0 V (d) 1.0 V

31. Frequencies higher than 10 MHz were found not being reflected by the ionosphere on a particular day at a place. The maximum electron density of the ionosphere on the day was near to :
(a) $1.5\times10^{10}\,m^{-3}$ (b) $1.24\times10^{12}\,m^{-3}$
(c) $3\times10^{12}\,m^{-3}$ (d) none of these

32. The de-Broglie wavelength of an electron, α-particle and a proton all having the same kinetic energy is respectively given as λ_e, λ_α and λ_p. Then which of the following is not true ?
(a) $\lambda_e > \lambda_p$ (b) $\lambda_p > \lambda_\alpha$
(c) $\lambda_e > \lambda_\alpha$ (d) $\lambda_\alpha < \lambda_p < \lambda_e$

33. What is the disintegration constant of radon, if the number of its atoms diminishes by 18% in 24 h ?
(a) $2.1\times10^{-3}\,s^{-1}$ (b) $2.1\times10^{-4}\,s^{-1}$
(c) $2.1\times10^{-5}\,s^{-1}$ (d) $2.1\times10^{-6}\,s^{-1}$

34. Which of the following statements is true for an *n*-type semiconductor ?
(a) The donor level lies closely below the bottom of the conduction band
(b) The donor level lies closely above the top of the valence band
(c) The donor level lies at the halfway mark of the forbidden energy gap
(d) None of the above

35. Carbon, silicon and germanium have four valence electrons each. These are characterized by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$. Which of the following statements is true ?
(a) $(E_g)_C = (E_g)_{Si} = (E_g)_{Ge}$
(b) $(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$
(c) $(E_g)_C < (E_g)_{Ge} > (E_g)_{Si}$
(d) $(E_g)_{Si} < (E_g)_{Ge} > (E_g)_C$

36. A particle executes SHM of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position ?
(a) 0.5 s (b) 1.0 s
(c) 1.5 s (d) 2.0 s

37. The speed of a wave on a string is 150 m/s when the tension is 120 N. The percentage increase in the tension in order to raise the wave speed by 20% is :
(a) 44% (b) 40%
(c) 20% (d) 10%

38. A straight rod of length L has one of its ends at the origin and the other at $x = L$. If the mass per unit length of the rod is given by $A\,x$, where A is constant, where is its mass centre ?
(a) $\frac{L}{3}$ (b) $\frac{L}{2}$
(c) $\frac{2L}{3}$ (d) $\frac{3L}{4}$

39. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 4 m away by means of a large convex lens. The maximum possible focal length of the lens required for this purpose will be :
(a) 0.5 m (b) 1.0 m
(c) 1.5 m (d) 2.0 m

40. The total energy of a satellite moving with an orbital velocity v around the earth is :
(a) $\frac{1}{2}mv^2$ (b) $-\frac{1}{2}mv^2$
(c) mv^2 (d) $\frac{3}{2}mv^2$

41. In hydrogen atom, the electron is moving round the nucleus with velocity 2.18×10^6 m/s in an orbit of radius 0.528 Å. The acceleration of the electron is :
(a) $9\times10^{18}\,m/s^2$ (b) $9\times10^{22}\,m/s^2$
(c) $9\times10^{-22}\,m/s^2$ (d) $9\times10^{12}\,m/s^2$

42. When a spring is stretched by a distance x, it exerts a force given by

$$F = (-5x - 16x^3)\,N$$

The work done, when the spring is stretched from 0.1 m to 0.2 m is :
(a) 8.7×10^{-2} J (b) 12.2×10^{-2} J
(c) 8.1×10^{-1} J (d) 12.2×10^{-1} J

43. The flow of liquid is laminar or stream line is determined by :
(a) rate of flow of liquid
(b) density of fluid
(c) radius of the tube
(d) coefficient of viscosity of liquid

44. If boiling point of water is 95°F, what will be reduction at celsius scale ?
(a) 7°C (b) 65°C (c) 63°C (d) 35°C

45. The motion of a particle varies with time according to the relation
$y = a(\sin \omega t + \cos \omega t)$.
(a) The motion is oscillatory but not SHM
(b) The motion is SHM with amplitude a
(c) The motion is SHM with amplitude $a\sqrt{2}$
(d) The motion is SHM with amplitude $2a$

46. Two closed organ pipes A and B have the same length. A is wider than B. They resonate in the fundamental mode at frequencies n_A and n_B respectively, then :
(a) $n_A = n_B$
(b) $n_A > n_B$
(c) $n_A < n_B$
(d) either (b) or (c) depending on the ratio of their diameters

47. Two waves having sinusoidal waveforms have different wavelengths and different amplitudes. They will be having :
(a) same pitch and different intensity
(b) same quality and different intensity
(c) different quality and different intensity
(d) same quality and different pitch

48. In double slit experiment, the distance between two slits is 0.6 mm and these are illuminated with light of wavelength 4800 Å. The angular width of first dark fringe on the screen distant 120 cm from slits will be :
(a) 8×10^{-4} rad (b) 6×10^{-4} rad
(c) 4×10^{-4} rad (d) 16×10^{-4} rad

49. If there were no atmosphere, the average temperature on the surface of the earth would be :
(a) lower (b) higher
(c) same as now (d) 0°

50. The ionisation energy of 10 times ionised sodium atom is :
(a) $\frac{13.6}{11}$ eV (b) $\frac{13.6}{112}$ eV
(c) $13.6\times(11)^2$ eV (d) 13.6 eV

Chemistry

1. When CH_3COOH reacts with CH_3—MgX :
(a) CH_3COX is formed
(b) hydrocarbon is formed
(c) acetone is formed
(d) alcohol is formed

2. A cyclic hydrocarbon molecule has all the carbon and hydrogen is a single plane. All the carbon-carbon bonds are of same length, less than 1.54 Å, but more than 1.34 Å. The C—C bond angle will be :
(a) 109°28′ (b) 100°
(c) 180° (d) 120°

3. Which will reduce zinc oxide to zinc ?
(a) Mg (b) Pb
(c) Cu (d) Fe

4. Some chemists at ISRO wished to prepare a saturated solution of a silver compound and they wanted it to have the highest concentration of silver ion possible. Which of the following compounds, would they use ?
$K_{sp}(AgCl) = 1.8\times10^{-10}$,
$K_{sp}(AgBr) = 5.0\times10^{-13}$,
$K_{sp}(Ag_2CrO_4) = 2.4\times10^{-12}$
(a) AgCl (b) AgBr
(c) Ag_2CrO_4 (d) None of these

5. By Wurtz reaction, a mixture of methyl iodide and ethyl iodide gives :
(a) butane
(b) ethane
(c) propane
(d) A mixture of the above three

6. Addition of $SnCl_2$ to $HgCl_2$ gives precipitate :
(a) white turning to red
(b) white turning to grey
(c) black turning to white
(d) none of the above

7. In fermentation by zymase, alcohol and CO_2 are obtained from :
(a) invert sugar (b) glucose
(c) fructose (d) all of these

8. The stability of ferric ion is due to :
(a) half filled f-orbitals
(b) half filled d-orbitals
(c) completely filled f-orbitals
(d) completely filled d-orbitals

9. Electron affinity is positive, when :
(a) O changes into O^-
(b) O^- changes into O^{2-}
(c) O changes into O^+
(d) electron affinity is always negative

10. Ionisation potential for a noble gas is :
(a) maximum in a period
(b) minimum in a period
(c) either minimum or maximum
(d) constant

11. Ethylamine on a acetylation gives :
(a) *N*-ethyl acetamide
(b) acetamide
(c) methyl acetamide
(d) none of the above

12. Strongest oxidising agent among halogen is :
(a) I_2 (b) Br_2
(c) Cl_2 (d) F_2

13. Which reagent can convert acetic acid into ethanol ?
(a) Na + alcohol (b) $LiAlH_4$ + ether
(c) H_2 + Pt (d) Sn + HCl

14. In presence of moisture, SO_2 can :
(a) act as oxidant (b) act as reductant
(c) gain electron (d) not act as reductant

15. Acetals are :
(a) ketones (b) diethers
(c) aldehyde (d) hydroxy aldehydes

16. The principle involved in the classification of basic radicals, is :
(a) common ion effect
(b) solubility product
(c) valency of radicals
(d) strength of salt

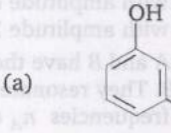
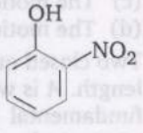
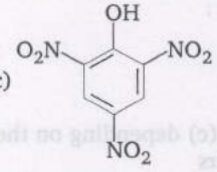
17. Formation of diethyl ether from ethanol is based on a :
(a) dehydration reaction
(b) dehydrogenation reaction
(c) hydrogenation reaction
(d) homolytic fission reaction

18. Hypo phosphorus acid, H_3PO_2 is :
(a) a monobasic acid
(b) a tribasic acid
(c) a dibasic acid
(d) not acidic at all

19. What is obtained when acetyl chloride is heated with benzene in presence of anhydrous $AlCl_3$?
(a) Acetyl benzoic acid
(b) Anisol
(c) Acetophenone
(d) Chlorobenzene

20. Which gas is used in airated water ?
(a) CO_2 (b) SO_2
(c) CO (d) Water vapours

21. The refluxing of $(CH_3)_2NCOCH_3$ with acid gives :
(a) $(CH_3)_2NH + CH_3COOH$
(b) $(CH_3)_2NCOOH + CH_4$
(c) $2CH_3OH + CH_3CONH_2$
(d) $2CH_3NH_2 + CH_3COOH$

22. Which is obtained on treating phenol, with dilute HNO_3 ?
(a) 
(b) 
(c) 
(d) None of these

23. Solder is an alloy of lead with :
(a) copper (b) zinc
(c) nickel (d) tin

24. Arrange NH_4^+, H_2O, H_3O^+, HF and OH^- in increasing order of acidic nature :
(a) $H_3\overset{+}{O} < NH_4^+ < HF < OH^- < H_2O$
(b) $NH_4^+ < HF < H_3O^+ < H_2O < OH^-$
(c) $HO^- < H_2O < NH_4^+ < HF < H_3O^+$
(d) $H_3O^+ > HF > H_2O > NH_4^+ > OH^-$

25. Which of the following radicals gives the apple green flame during flame test ?
(a) Ba^{2+} (b) Sr^{2+}
(c) Ca^{2+} (d) Cr^{3+}

26. When chlorine is passed through concentrated solution of KOH, the compound formed is :
(a) $KClO_4$ (b) $KClO_3$
(c) $KClO_2$ (d) KClO

27. The equilibrium constant K_p for the reaction, $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is :
(a) more than one (b) less than one
(c) equal to K_c (d) zero

28. What is the weight of oxygen that is required for the complete combustion of 2.8 kg of ethylene ?
(a) 9.6 kg (b) 96.0 kg
(c) 6.4 kg (d) 2.8 kg

29. The metal that does not displace hydrogen from an acid is :
(a) Ca (b) Al
(c) Zn (d) Hg

30. The decomposition of N_2O_5 occurs as, $2N_2O_5 \longrightarrow 4N_2 + O_2$, and follows Ist order kinetics, hence :
(a) the reaction is unimolecular
(b) the reaction is bimolecular
(c) $t_{1/2} \propto a^0$
(d) none of the above

31. Atomic radii of F and Ne, in Å, are given by :
(a) 0.72, 0.71
(b) 0.72, 1.6
(c) 1.6, 1.58
(d) 0.71, 0.72

32. Which pair has both members from the same period of periodic table ?
(a) Cl, Br (b) Ca, Cl
(c) Na, Ca (d) Na, Cl

33. When dilute aqueous solution of $AgNO_3$ (excess) is added to KI solution, positively charged sol of AgI is formed due to adsorption of :
(a) NO_3^- (b) O_2^-
(c) Ag^+ (d) K^+

34. Which of the following has largest ionic radius ?
(a) Cs^+ (b) Li^+
(c) Na^+ (d) K^+

35. For $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$ at 927°C, $\Delta H = 176$ kJ mol; then ΔE is :
(a) 180 kJ (b) 186.4 kJ
(c) 166.0 kJ (d) 160 kJ

36. The charge required to liberate one gram equivalent of an element is :
(a) 96500 F (b) 1 F
(c) 1 C (d) none of these

37. The shape of sulphate ion is :
(s) square planar (b) trigonal
(c) trigonal planar (d) tetrahedral

38. The H—H bond energy is 430 kJ mol^{-1} and Cl—Cl bond energy is 240 kJ mol^{-1}, ΔH for HCl is – 90 kJ. The H—Cl bond energy is about :
(a) 180 kJ mol^{-1}
(b) 360 kJ mol^{-1}
(c) 213 kJ mol^{-1}
(d) 425 kJ mol^{-1}

39. In the equation $H_2S + 2HNO_3 \longrightarrow 2H_2O + 2NO_2 + S$. The equivalent weight of hydrogen sulphide is :
(a) 18 (b) 68
(c) 34 (d) 17

40. The energy released in an atom bomb explosion is mainly due to :
(a) release of neutrons
(b) release of electrons
(c) greater mass of products than initial material
(d) lesser mass of products than initial material

41. Highest entropy is in :
(a) hydrogen (b) water
(c) graphite (d) mercury

42. Which one will liberate Br_2 from KBr ?
(a) I_2 (b) SO_2
(c) HI (d) Cl_2

43. Nuclides :
(a) have specific atomic numbers
(b) have same number of protons
(c) have specific atomic number and mass numbers
(d) are isotopes

44. Arrhenius equation is :
(a) $\Delta H = \Delta E + \Delta n_g RT$
(b) $\Delta G = \Delta H - T \cdot \Delta S$
(c) $K = Ae^{-E_a/RT}$
(d) none of the above

45. In which of the following compounds, the oxidation number of iodine is fractional ?
(a) IF_3 (b) IF_5 (c) I_3^- (d) IF_7

46. Non-directional orbital is :
(a) $4p$ (b) $4d$
(c) $4f$ (d) $3s$

47. A monoprotic acid in 1.00 M solution is 0.01% ionised. The dissociation constant of this acid is :
(a) 1×10^{-8} (b) 1×10^{-4}
(c) 1×10^{-6} (d) 1×10^{-5}

48. If both oxygen and helium gases are at the same temperature, the rate of diffusion of O_2 is very close to :
(a) 4 times that of He
(b) 2 times that of He
(c) 0.35 times that of He
(d) 8 times that of He

49. A white substance having alkaline nature in solution is :
(a) $NaNO_3$ (b) NH_4Cl
(c) Na_2CO_3 (d) Fe_2O_3

50. A solution of $FeCl_3$ in water acts as acidic solution due to :
(a) hydrolysis of Fe^{3+} (b) acidic impurities
(c) dissociation (d) ionisation

Mathematics

1. The unit's place digit in the number $13^{25} + 11^{25} - 3^{25}$ is :
(a) 0 (b) 1
(c) 2 (d) 3

2. The angle of intersection of the curves $y = x^2$, $6y = 7 - x^3$ at (1, 1) is :
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) none of these

3. The value of x for which the equation
$1 + r + r^2 + \ldots + r^x = (1 + r)(1 + r^2)(1 + r^4)(1 + r^8)$
holds is :
(a) 12 (b) 13
(c) 14 (d) 15

4. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x; then minimum value of $f(x)$:
(a) does not exist (b) is equal to 1
(c) is equal to 0 (d) is equal to -1

5. The value of a for which the sum of the squares of the roots of the equation
$x^2 - (a - 2)x - a - 1 = 0$
assumes the least value is :
(a) 0 (b) 1
(c) 2 (d) 3

6. A particle is dropped under gravity from rest from a height h ($g = 9.8$ m/s^2) and it travels a distance $\frac{9h}{25}$ in the last second the height h is :
(a) 100 m (b) 122.5 m
(c) 145 m (d) 167.5 m

7. The number of onto mappings from the set $A = \{1, 2, \ldots, 100\}$ to set $B = \{1, 2\}$ is :
(a) $2^{100} - 2$ (b) 2^{100}
(c) $2^{99} - 2$ (d) 2^{99}

8. Which of the following functions is inverse of itself ?
(a) $f(x) = \frac{1 - x}{1 + x}$ (b) $f(x) = 3^{\log x}$
(c) $f(x) = 3^{x(x+1)}$ (d) none of these

9. If $f(x) = \log(x + \sqrt{x^2 + 1})$, then $f(x)$ is :
(a) even function (b) odd function
(c) periodic function (d) none of these

10. The solution of $\log_{99}\{\log_2(\log_3 x)\} = 0$ is :
(a) 4 (b) 9
(c) 44 (d) 99

11. If $n = 1000!$, then the value of sum $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{1000} n}$ is :
(a) 0 (b) 1
(c) 10 (d) 10^3

12. If ω and ω^2 are the two imaginary cube roots of unity, then the equation whose roots are $a\omega^{317}$ and $a\omega^{382}$ is :
(a) $x^2 + ax - a^2 = 0$ (b) $x^2 + a^2 x + a = 0$
(c) $x^2 + ax + a^2 = 0$ (d) $x^2 - a^2 x + a = 0$

13. The value of $1 + \sum_{k=0}^{14}\left\{\cos\frac{(2k+1)}{15}\pi + i\sin\frac{(2k+1)}{15}\pi\right\}$ is :
(a) 0 (b) -1
(c) 1 (d) i

14. If $1, a_1, a_2, \ldots, a_{n-1}$ are roots of unity, then the value of $(1 - a_1)(1 - a_2)\ldots(1 - a_{n-1})$ is :
(a) 0 (b) 1
(c) n (d) n^2

15. If α, β are the roots of $ax^2 + bx + c = 0$, $\alpha + h, \beta + h$ are roots of $px^2 + qx + r = 0$; and D_1, D_2 are the respective discriminants of these equations, then $D_1 : D_2$ is equal to :
(a) $\frac{a^2}{p^2}$ (b) $\frac{b^2}{q^2}$
(c) $\frac{c^2}{r^2}$ (d) none of these

16. If a, b, c are three unequal numbers such that a, b, c are in AP and $b - a, c - b, a$ are in GP, then $a : b : c$ is :
(a) 1 : 2 : 3 (b) 1 : 3 : 4
(c) 2 : 3 : 4 (d) 1 : 2 : 4

17. The number of divisors of $3 \times 7^3, 7 \times 11^2$ and 2×61 are in :
(a) AP (b) GP
(c) HP (d) none of these

18. Suppose a, b, c are in AP and $|a|, |b|, |c| < 1$. If
$x = 1 + a + a^2 + \ldots$ to ∞,
$y = 1 + b + b^2 + \ldots$ to ∞,
$z = -1 + c + c^2 + \ldots$ to ∞
then x, y, z are in :
(a) AP (b) GP
(c) HP (d) none of these

19. $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots$ to ∞ is :
(a) $\frac{16}{35}$ (b) $\frac{11}{8}$
(c) $\frac{35}{16}$ (d) $\frac{7}{16}$

20. If the sum of first n natural numbers is $\frac{1}{78}$ times the sum of their cubes, then the value of n is :
(a) 11 (b) 12
(c) 13 (d) 14

21. If $p=\cos 55°$, $q=\cos 65°$ and $r=\cos 175°$, then the value of $\frac{1}{p}+\frac{1}{q}+\frac{r}{pq}$ is :
(a) 0 (b) -1
(c) 1 (d) none of these

22. The value of $\sin 20°\,(4+\sec 20°)$ is :
(a) 0 (b) 1
(c) $\sqrt{2}$ (d) $\sqrt{3}$

23. If $4\sin^{-1}x+\cos^{-1}x=\pi$, then x is equal to :
(a) 0 (b) 1/2
(c) $-1/2$ (d) 1

24. If the line $\frac{x}{a}+\frac{y}{b}=1$ moves such that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$, where c is a constant, then the locus of the foot of the perpendicular from the origin to the line is :
(a) straight line (b) circle
(c) parabola (d) ellipse

25. The straight line whose sum of the intercepts on the axes is equal to half of the product of the intercepts, passes through the point :
(a) (1, 1) (b) (2, 2)
(c) (3, 3) (d) (4, 4)

26. If the circle $x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle
$x^2+y^2-2x+8y-d=0$,
then $c+d$ is equal to :
(a) 60 (b) 50
(c) 40 (d) 30

27. The radius of the circle whose tangents at $x+3y-5=0$, $2x+6y+30=0$ is :
(a) $\sqrt{5}$ unit (b) $\sqrt{10}$ unit
(c) $\sqrt{15}$ unit (d) $\sqrt{20}$ unit

28. The latusrectum of the parabola $y^2=4ax$ whose focal chord is PSQ such that $SP=3$ and $SQ=2$ is given by :
(a) 24/5 (b) 12/5
(c) 6/5 (d) 1/5

29. If M_1 and M_2 are the feet of the perpendiculars from the foci S_1 and S_2 of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ on the tangent at a point P on the ellipse, then $(S_1M_1)(S_2M_2)$ is equal to :
(a) 16 (b) 9
(c) 4 (d) 3

30. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the hyperbola $4x^2-9y^2-36=0$ are at right angles, then $\frac{x_1x_2}{y_1y_2}$ is equal to :
(a) $\frac{9}{4}$ (b) $-\frac{9}{4}$
(c) $\frac{81}{16}$ (d) $-\frac{81}{16}$

31. The solution of $x\,dy-ydx+x^2e^x dx=0$ is :
(a) $\frac{y}{x}+e^x=c$ (b) $\frac{x}{y}+e^x=c$
(c) $x+e^y=c$ (d) $y+e^x=c$

32. The coefficient of x^2 in the binomial expansion of $\left(\frac{1}{3}x^{1/2}+x^{-1/4}\right)^{10}$ is :
(a) $\frac{70}{243}$ (b) $\frac{60}{423}$
(c) $\frac{50}{13}$ (d) none of these

33. The solution set of the equation
$\left[4\left(1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots\right)\right]^{\log_2 x}$
$=\left[54\left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\right)\right]^{\log_x 2}$ is :
(a) $\left\{4,\frac{1}{4}\right\}$ (b) $\left\{2,\frac{1}{2}\right\}$
(c) {1, 2} (d) $\left\{8,\frac{1}{8}\right\}$

34. If $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots$ and if $|x|<1$, then :
(a) $x=1-y+\frac{y^2}{2}-\frac{y^3}{3}+\ldots$
(b) $x=1+y+\frac{y^2}{2}+\frac{y^3}{3}+\ldots$
(c) $x=y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+\ldots$
(d) $x=y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+\ldots$

35. The length of perpendicular from (1, 6, 3) to the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is :

(a) 3 (b) $\sqrt{11}$

(c) $\sqrt{13}$ (d) 5

36. The plane $2x+3y+4z=1$ meets the coordinate axes in A, B, C. The centroid of the triangle ABC is :

(a) (2, 3, 4) (b) $\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$

(c) $\left(\frac{1}{6},\frac{1}{9},\frac{1}{12}\right)$ (d) $\left(\frac{1}{2},\frac{3}{3},\frac{3}{4}\right)$

37. The vector equation of the sphere whose centre is the point (1, 0, 1) and radius is 4, is :

(a) $|\vec{r}-(\hat{i}+\hat{k})|=4$ (b) $|\vec{r}+(\hat{i}+\hat{k})|=4^2$

(c) $\vec{r}\,(\hat{i}+\hat{k})=4$ (d) $\vec{r}\,(\hat{i}+\hat{k})=4^2$

38. The plane $2\lambda x-(1+\lambda)y+3z=0$ passes through the intersection of the planes :

(a) $2x-y=0$ and $y+3z=0$

(b) $2x-y=0$ and $y-3z=0$

(c) $2x+3z=0$ and $y=0$

(d) none of the above

39. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=\sqrt{37}, |\vec{b}|=3, |\vec{c}|=4$, then angle between $\vec{b}$ and $\vec{c}$ is :

(a) 30° (b) 45°

(c) 60° (d) 90°

40. If $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=2\hat{i}+\hat{j}$ the value of λ such that $\vec{a}+\lambda\vec{c}$ is perpendicular to $\vec{b}$ is :

(a) 1 (b) −1

(c) 0 (d) none of these

41. The total work done by two forces $\vec{F}_1=2\hat{i}-\hat{j}$ at $\vec{F}_2=3\hat{i}+2\hat{j}-\hat{k}$ acting on a particle when it is displace from the point $3\hat{i}+2\hat{j}+\hat{k}$ to $5\hat{i}+5\hat{j}+3\hat{k}$ is :

(a) 8 unit (b) 9 unit

(c) 10 unit (d) 11 unit

42. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors, and let $\vec{p}$ and $\vec{r}$ be vectors defined by the relations

$$\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]} \text{ and } \vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}.$$

Then, the value of the expression $(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}$ is equal to :

(a) 0 (b) 1

(c) 2 (d) 3

43. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$

$=(y-z)(z-x)(x-y)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$,

then n is equal to :

(a) 2 (b) −2

(c) −1 (d) 1

44. If $a_1, a_2, \ldots, a_n, \ldots$ are in GP and $a_1>0$ for each i, then determinant

$$\Delta=\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$$

is equal to :

(a) 0 (b) 1

(c) 2 (d) n

45. The values of a for which the system of equation $x+y+z=0$, $x+ay+az=0$, $x-ay+z=0$, possess non-zero solutions, are given by :

(a) 1, 2 (b) 1 , −1

(c) 1, 0 (d) none of these

46. If a square matrix A is such that $AA^T=I=A^T A$, then $|A|$ is equal to :

(a) 0 (b) ± 1

(c) ± 2 (d) none of these

47. $\int_a^b \frac{|x|}{x}dx$, $a<0<b$, is equal to :

(a) $|b|-|a|$ (b) $|b|+|a|$

(c) $|a-b|$ (d) none of these

48. A and B are two events. Odds against A are 2 to 1. Odds in favour of $A\cup B$ are 3 to 1. If $x\le P(B)\le y$, then ordered pair (x, y) is :

(a) $\left(\frac{5}{12},\frac{3}{4}\right)$ (b) $\left(\frac{2}{3},\frac{3}{4}\right)$

(c) $\left(\frac{1}{3},\frac{3}{4}\right)$ (d) none of these

49. In a series of three trials, the probability of exactly two successes in nine times is as large as the probability of three successes. Then, the probability of success in each trial is :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

50. An integer is chosen at random from first two hundred numbers. Then, the probability that the integer chosen is divisible by 6 or 8 is :

(a) $\frac{1}{4}$ (b) $\frac{2}{4}$

(c) $\frac{3}{4}$ (d) none of these

ANSWERS

PHYSICS

1. (a)	2. (a)	3. (d)	4. (d)	5. (d)	6. (b)	7. (c)	8. (c)	9. (d)	10. (b)
11. (b)	12. (c)	13. (d)	14. (b)	15. (b)	16. (d)	17. (a)	18. (b)	19. (b)	20. (b)
21. (c)	22. (d)	23. (a)	24. (a)	25. (b)	26. (b)	27. (c)	28. (d)	29. (c)	30. (c)
31. (b)	32. (a)	33. (d)	34. (a)	35. (b)	36. (a)	37. (a)	38. (b)	39. (b)	40. (b)
41. (b)	42. (a)	43. (a)	44. (d)	45. (c)	46. (c)	47. (a)	48. (a)	49. (a)	50. (c)

CHEMISTRY

1. (b)	2. (d)	3. (a)	4. (c)	5. (d)	6. (b)	7. (b)	8. (b)	9. (b)	10. (a)
11. (a)	12. (d)	13. (b)	14. (b)	15. (d)	16. (b)	17. (a)	18. (a)	19. (c)	20. (a)
21. (a)	22. (c)	23. (d)	24. (c)	25. (a)	26. (b)	27. (c)	28. (a)	29. (d)	30. (c)
31. (b)	32. (d)	33. (c)	34. (a)	35. (c)	36. (b)	37. (d)	38. (d)	39. (d)	40. (d)
41. (a)	42. (d)	43. (c)	44. (c)	45. (c)	46. (d)	47. (a)	48. (c)	49. (c)	50. (a)

MATHEMATICS

1. (b)	2. (c)	3. (d)	4. (d)	5. (b)	6. (b)	7. (a)	8. (a)	9. (b)	10. (b)
11. (b)	12. (c)	13. (c)	14. (c)	15. (a)	16. (a)	17. (a)	18. (c)	19. (c)	20. (b)
21. (a)	22. (d)	23. (b)	24. (b)	25. (b)	26. (b)	27. (b)	28. (a)	29. (b)	30. (d)
31. (a)	32. (a)	33. (a)	34. (d)	35. (c)	36. (c)	37. (a)	38. (b)	39. (c)	40. (b)
41. (d)	42. (d)	43. (c)	44. (a)	45. (b)	46. (b)	47. (d)	48. (a)	49. (c)	50. (a)

HINTS & SOLUTIONS

Physics

1. Time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

or $$g = \frac{4\pi^2 L}{T^2} \quad ...(i)$$

Differentiating Eq. (i), we have

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} \quad ...(ii)$$

Given, $L = 100\,\text{cm}, T = 2\,\text{s},$

$$\Delta T = \frac{0.1}{100} = 0.001\,\text{s},$$

$$\Delta L = 1\,\text{mm} = 0.1\,\text{cm}$$

Substituting the values in Eq. (ii), we have

$$\therefore \left|\frac{\Delta g}{g}\right|_{\max} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = \frac{0.1}{100} + 2\times\frac{0.001}{2}$$

Thus, maximum percentage error

$$\left|\frac{\Delta g}{g}\right|_{\max} \times 100 = \left(\frac{0.1}{100}\times 100\right) + \left(\frac{2\times 0.001}{2}\times 100\right)$$

$$= 0.1\% + 0.1\% = 0.2\%$$

2. The potential energy of the stretched wire is

$$U = \frac{1}{2}\times \text{stress}\times\text{strain}$$

$$= \frac{1}{2}\times F\times x = \frac{1}{2}Fx$$

3. The electric field at a point distance r from electric dipole is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \text{ or } E \propto \frac{1}{r^3}$$

where, p is dipole moment and r is distance of point from centre of dipole.

Force on that point is

$$F = EQ$$

or $$F \propto \frac{1}{r^3}$$

or $$\frac{F_2}{F_1} = \left(\frac{r_1}{r_2}\right)^3$$

or $$\frac{F_2}{F} = \left(\frac{r}{2r}\right)^3$$

or $$\frac{F_2}{F} = \frac{1}{8}$$

$\therefore$ $$F_2 = \frac{F}{8}$$

4. The electric field intensity at a point lying outside the sphere (non-conducting) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

where r is the distance of that point from centre of sphere.

$\therefore$ $$E \propto \frac{1}{r^2} \quad ...(i)$$

The electric field intensity at surface of sphere

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

or $$E \propto \frac{1}{R^2} \quad ...(ii)$$

R, being the radius of sphere.

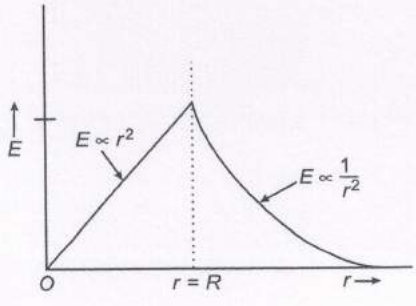
The electric field intensity at a point lying inside the sphere is

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}$$

or $$E \propto r \quad ... (iii)$$

Also at the centre of sphere $r = 0$. Hence, $E = 0$.

The graphical distribution is shown below :



5. The resistance R of a particular conductor is related to the resistivity ρ of its material by

$$R = \frac{\rho l}{A}$$

or $$\rho = \text{resistivity} = \frac{RA}{l}$$

Given, $R = 0.072\ \Omega$,

$A = 2\,\text{mm} \times 2\,\text{mm} = 4 \times 10^{-6}\,\text{m}^2$, $l = 12\,\text{mm}$

$$\therefore \quad \rho = \frac{0.072 \times 4 \times 10^{-6}}{12}$$

$$= 0.024 \times 10^{-6}\,\Omega\,\text{m}$$

$$= 2.4 \times 10^{-8}\,\Omega\,\text{m}$$

6. **Key Idea :** *Electric lines of force flow from higher potential region to lower potential region.*

The electric field intensity of a point in an electric field in a given direction is equal to the negative potential gradient in that direction, *i.e.*,

$$E = -\frac{dV}{dx}$$

The negative sign signifies that the potential decreases in the direction of electric field, *i.e.*, electric lines of force flow from higher potential region to lower potential region.

Since, AB is perpendicular to field lines, so A and B are at same potential.

Hence, $$V_A = V_B > V_C$$

7. The displacement vector from

$A \equiv (0, 3, -1)$ to $B \equiv (-2, 6, 4)$ is

$$= (-2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) - (3\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$$

8. Thrust force on the rocket

$$F_t = v_r\left(-\frac{dm}{dt}\right)$$

or $$ma = v_r\left(-\frac{dm}{dt}\right)$$

where v_r is the exhaust velocity of gases and $-\frac{dm}{dt}$ is the mass of the ejected per unit time.

Given, $v_r = 2400\ \text{ms}^{-1}$, $-\frac{dm}{dt} = \frac{1}{60}\ \text{kg/s}$,

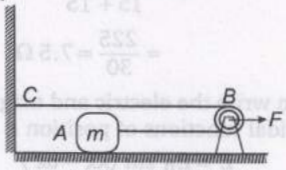
$m = 1\ \text{kg}$

Substituting the values in above relation

$$1 \times a = 2400 \times \frac{1}{60}$$

or $$a = 40\ \text{ms}^{-2}$$

9. To find the acceleration of the block we will need the tension in the string, that can be obtained by considering the pulley as the system.



The force acting on the pulley are

(i) F towards right

(ii) T towards left by the portion BC of the string and

(iii) T towards left by the portion BA of the string. Since there is no vertical motion. Then vertical force if any, add to zero.

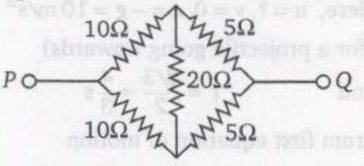
As the pulley is massless, the equation of motion is

$$F - 2T = 0 \text{ or } T = \frac{F}{2}$$

The acceleration of the block is $a = \frac{T}{m} = \frac{F}{2m}$

10. **Key Idea :** *The given circuit is a balanced Wheatstone bridge.*

The equivalent circuit is shown below :



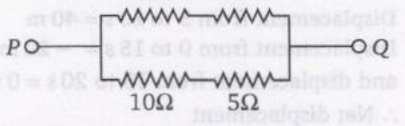
In the given circuit, the ratio of resistances in the opposite arms is same

$$\frac{P}{Q} = \frac{10}{10} = \frac{1}{1}$$

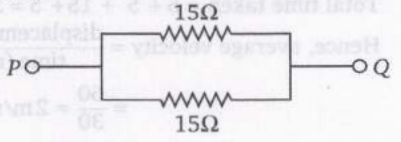
$$\frac{R}{S} = \frac{5}{5} = \frac{1}{1}$$

Hence, bridge is balanced.

The given circuit now reduces to



Here, $10\ \Omega$ and $5\ \Omega$ resistors are in series, therefore



Now, the two 15 Ω resistors are connected in parallel, hence equivalent resistance is

$$R = \frac{15 \times 15}{15 + 15}$$

$$= \frac{225}{30} = 7.5\ \Omega$$

11. We can write the electric and magnetic fields as sinusoidal functions of position x and time t.

$$E = E_0 \sin(kx - \omega t)$$

$$B = B_0 \sin(kx - \omega t)$$

In this E_0 and B_0 are the amplitudes of the fields. Further

$$c = \frac{E_0}{B_0}$$

or $$E_0 = B_0 c$$

Given, $B_0 = 2 \times 10^{-7}$ T, $c = 3 \times 10^8$ m/s

we have

$$E_0 = 2 \times 10^{-7} \times 3 \times 10^8 = 60\ \text{Vm}^{-1}$$

12. As seen from the cart the projectile moves vertically upwards and comes back.

The time taken by cart to cover 80 m

$$= \frac{s}{v} = \frac{80}{30} = \frac{8}{3}\ \text{s}$$

Here, $u = ?, v = 0, a = -g = 10\ \text{m/s}^2$

(for a projectile going upwards)

and $$t = \frac{8/3}{2} = \frac{4}{3}\ \text{s}$$

From first equation of motion

$$v = u + at$$

$$0 = u - 10 \times \frac{4}{3}$$

$$\therefore \quad u = \frac{40}{3}\ \text{m/s}$$

13. **Key Idea :** *The average velocity is defined as total displacement of the body in a particular time interval divided by the time interval.*

Displacement from 0 to 5 s = 40 m

Displacement from 5 to 10 s = 40 m

Displacement from 0 to 15 s = − 20 m

and displacement from 15 to 20 s = 0 m

∴ Net displacement

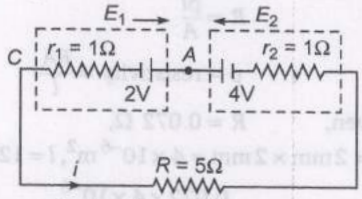
= 40 + 40 − 20 + 0 = 60 m

Total time taken = 5 + 5 + 15 + 5 = 30 min

Hence, average velocity $= \frac{\text{displacement (m)}}{\text{time (min)}}$

$$= \frac{60}{30} = 2\ \text{m/min}$$

14. Emf's E_1 and E_2 are opposing each other. Since, $E_2 > E_1$ current will move from right to left.



Current in circuit

$$i = \frac{E_2 - E_1}{R + r_1 + r_2} = \frac{4 - 2}{5 + 1 + 2} = \frac{2}{8} = 0.25\ \text{A}$$

The potential drop between points A and C is

$$V_A - V_C = E_1 + i r_1$$

$$= 2 + (0.25 \times 1) = 2.25\ \text{V}$$

15. **Key Idea :** *Applying conservation of energy.*

By law of conservation of energy

$$(U + K)_{\text{surface}} = (U + K)_{\infty}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2} m (3v_e)^2 = 0 + \frac{1}{2} m v^2$$

$$\Rightarrow \quad -\frac{GM}{R} + \frac{9 v_e^2}{2} = \frac{1}{2} v^2$$

Since, $$v_e^2 = \frac{2GM}{R}$$

$$\therefore \quad -\frac{v_e^2}{2} + \frac{9 v_e^2}{2} = \frac{1}{2} v^2$$

$$\Rightarrow \quad v^2 = 8 v_e^2$$

$$\therefore \quad v = 2\sqrt{2}\, v_e$$

$$= 2\sqrt{2} \times 11.2$$

$$= 31.7\ \text{kms}^{-1}$$

16. **Key Idea :** *The centripetal force to the body on circular path is supplied by the frictional force.*

For body to move on circular path, Frictional force provides the necessary centripetal force, *i.e.,* frictional force = centripetal force

or $$\mu mg = \frac{mv^2}{r} = mr\omega^2$$

or $$\mu g = r\omega^2$$

$$\therefore \quad 0.5 \times 9.8 = 10\, \omega^2$$

or $$\omega = 0.7\ \text{rad/s}$$

17. When particle moves a circle, then the resultant force must act towards the centre and its magnitude F must satisfy

$$F = \frac{mv^2}{l}$$

This resultant force is directed towards the centre and it is called centripetal force. This force originates from tension T.

$$\therefore \quad F = \frac{mv^2}{l} = T$$

18. Power delivered to the body

$$P = Fv$$
$$= mav$$
$$= ma(0 + gt) \qquad (\because u = 0)$$
$$= magt$$

or $P \propto t$

19. A long straight conductor carrying current I passes through O, then by symmetry, all points of the circular path are equivalent and hence the magnitude of magentic field should be same at these points.

The circulation of magnetic field along the circle is

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

(using Ampere's law)

20. When the loops are brought nearer, magnetic flux linked with each loop increases. Thus, the current will be induced in each loop in a direction opposite to its own current in order to oppose increase in magnetic flux. This is in accordance with Lenz's law. So, the current will decrease in each loop.

21. (i) The dotted line in the diagram shows that there is no deviation in the value of $\frac{PV}{nT}$ for different temperatures T_1 and T_2 for increasing pressure so, this gas behaves ideally. Hence, dotted line corresponds to 'ideal' gas behaviour.

(ii) At high temperature, the deviation of the gas is less and at low temperature the deviation of gas is more, In the graph, deviation for T_2 is greater than for T_1.

Thus, $T_1 > T_2$

(iii) Since, the two curves intersect at dotted line so, the value of $\frac{PV}{nT}$ at that point on the y-axis is same for all gases.

22. At 0°C from b to c, temperature of matter does not change but its state changes.

Similarly, from d to e state of matter changes without changing temperature. Hence, b to c and d to e show phase changes.

23. From relation $B = \mu_0 H$

or $\mu_0 ni = \mu_0 H$

or $\frac{Ni}{L} = H \qquad \left(\because n = \frac{N}{L}\right)$

or $i = \frac{HL}{N}$

Given, $H = 4 \times 10^3 \text{ Am}^{-1}$

$L = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$

$N = 60$ turns

$\therefore \quad i = \frac{4 \times 10^3 \times 12 \times 10^{-2}}{60} = 8 \text{ A}$

24. Given, $R = 3\ \Omega, X_L = 15\ \Omega, X_C = 11\ \Omega,$

$V_R = 10\text{V}$

$\therefore$ Current through the circuit

$$i = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$
$$= \frac{V_{rms}}{\sqrt{(3)^3 + (15-11)^2}}$$
$$= \frac{10}{\sqrt{9+16}}$$
$$= \frac{10}{5} = 2\text{A}$$

Since, L, C and R are connected in series combination the potential difference across R is

$$V_R = i \times R = 2 \times 3$$
$$= 6\text{V}$$

Across L,

$$V_L = iX_L = 2 \times 15 = 30\text{V}$$

Across C, $V_C = iX_C$

$$= 2 \times 11 = 22\text{V}$$

So, potential difference across series combination of L and C is

$$= V_L - V_C$$
$$= 30 - 22 = 8\text{ V}$$

25. **Ist Case :** From formula

$$R = \frac{V^2}{P}$$
$$= \frac{110 \times 110}{330}$$
$$= \frac{110}{3}\ \Omega$$

Since, current lags the voltage thus, the circuit contains resistance and inductance.

Power factor $\cos\phi = 0.6$

$$\frac{R}{\sqrt{R^2 + X_L^2}} = 0.6$$

$$\Rightarrow \quad R^2 + X_L^2 = \left(\frac{R}{0.6}\right)^2$$

$$\Rightarrow \quad X_L^2 = \frac{R^2}{(0.6)^2} - R^2$$

$$\Rightarrow \quad X_L^2 = \frac{R^2 \times 0.64}{0.36}$$

$$\therefore \quad X_L = \frac{0.8R}{0.6} = \frac{4R}{3} \quad \text{...(i)}$$

IInd Case : Now

$$\cos\phi = 1 \quad \text{(given)}$$

therefore, circuit is purely resistive, *i. e.*, it contains only resistance. This is the condition of resonance in which

$$X_L = X_C$$

$$\therefore \quad X_C = \frac{4R}{3} = \frac{4}{3} \times \frac{110}{3} = \frac{440}{9}\ \Omega$$

[from Eq. (i)]

or $$\frac{1}{2\pi f C} = \frac{440}{9}\ \Omega$$

$$\therefore \quad C = \frac{9}{2 \times 3.14 \times 60 \times 440}$$

$$= 0.000054\ \text{F} = 54\,\mu\text{F}$$

26. As shown in figure, the point on the right side of the lens at which rays converge will behave as virtual object of the lens.

$$\therefore \quad u = +12\ \text{cm},\ f = 20\ \text{cm}$$

From the relation

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\therefore \quad \frac{1}{20} = \frac{1}{v} - \frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20} + \frac{1}{12}$$

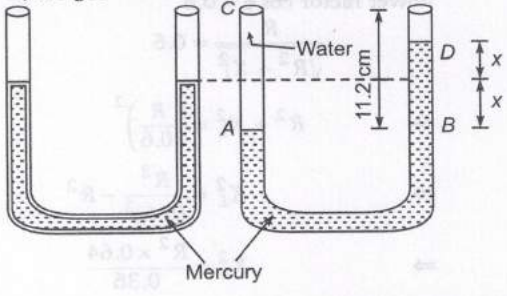
$$= \frac{3+5}{60} = \frac{8}{60}$$

$$\therefore \quad v = \frac{60}{8} = 7.5\ \text{cm}$$

So, image will be formed on same side of the virtual object at a distance of 7.5 cm from the lens.

27. **Key Idea :** *At the same level in the two limbs of a U-tube, pressure is same.*

On pouring water on left side, mercury rises x cm (say) from its previous level in the right limb of U-tube creating a difference of levels of mercury by $2x$ cm. Equating pressures at A and B, we get



$$P_A = P_B$$

$$\therefore 11.2 \times 10^{-2} \times \rho_{\text{water}} \times g = 2x \times \rho_{\text{mercury}} \times g$$

$$\Rightarrow 11.2 \times 10^{-2} \times 1000\ \text{kg/m}^3$$

$$= 2x \times 13600 \times \text{kg/m}^3$$

$$\Rightarrow \quad x = \frac{11.2 \times 10^{-2} \times 1000\ \text{m}}{2 \times 13600}$$

$$= 0.41\ \text{cm}$$

28. **Key Idea :** *In an isothermal process, there is no change in internal energy of gas, i.e., $\Delta U = 0$.*

The change in entropy of an ideal gas

$$\Delta S = \frac{\Delta Q}{T} \quad \text{...(i)}$$

In isothermal process, there is no change in internal energy of gas, *i.e.*, $\Delta U = 0$

$$\therefore \quad \Delta U = \Delta Q - W$$

$$\Rightarrow \quad 0 = \Delta Q - W$$

$$\Rightarrow \quad \Delta Q = W$$

i.e., ΔQ = work done by gas in isothermal process which went through from (P_1, V_1, T) to (P_2, V_2, T)

or $$\Delta Q = \mu RT \log_e \left(\frac{V_2}{V_1}\right) \quad \text{...(ii)}$$

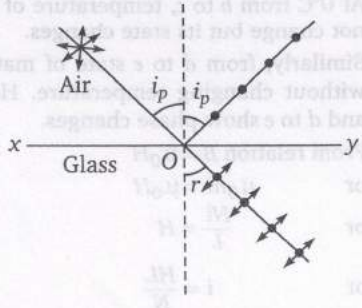
For 1 mole of an ideal gas, $\mu = 1$

So, from Eqs. (i) and (ii), we get

or $$\Delta S = R \log_e \left(\frac{V_2}{V_1}\right)$$

$$= R \ln \left(\frac{V_2}{V_1}\right)$$

29. According to the Brewster's theory the quantity of polarized light in reflected light depends upon angle of incidence. lf angle of incidence is equal to a particular angle called polarizing angle (i_p) then reflected beam is completely polarized. Also, when light is incident at polarizing angle then reflected and refracted rays are mutually perpendicular, *i.e.*,



$$i_p + r = 90°.$$

Here, i_p is angle of reflection, and r is angle of refraction.

30. Einstein's photoelectric equation is

$$KE = h\nu - \phi_0$$
$$eV_s = h(\nu - \nu_0)$$

where V_s is threshold or cut-off voltage.

$$V_s = \frac{h}{e}(\nu - \nu_0)$$

Here, $h = 6.6 \times 10^{-34}$ J-s, $e = 1.6 \times 10^{-19}$ C, $\nu = 8.2 \times 10^{14}$ Hz, $\nu_0 = 3.3 \times 10^{14}$ Hz

Substituting the values in the above relation, we have

$$V_s = \frac{6.6 \times 10^{-34}}{1.6 \times 10^{-19}}(8.2 - 3.3) \times 10^{14}$$
$$= 2\text{V}$$

31. The value of maximum frequency f which can be reflected from the layer at an angle of incidence i is given by

$$f^2 = \frac{80.6\,N}{\cos^2 i}$$

where, i is angle of incidence and N is electron density.

For the wave not reflected from ionosphere,

$$i = 0$$

$$\therefore \quad f^2 = \frac{80.6\,N}{\cos^2 0°} = \frac{80.6\,N}{1}$$

or
$$N = \frac{f^2}{80.6} = \frac{(10 \times 10^6)^2}{80.6}$$
$$= \frac{100}{80.6} \times 10^{12}$$
$$= 1.24 \times 10^{12}/\text{m}^3$$

32. de-Broglie wavelength

$$\lambda = \frac{h}{mv} \quad \text{or} \quad \lambda \propto \frac{1}{m}$$

$$\therefore \quad \lambda_e \propto \frac{1}{m_e}, \lambda_\alpha \propto \frac{1}{m_\alpha} \text{ and } \lambda_p \propto \frac{1}{m_p}$$

As we know that $m_e < m_p < m_a$

So, $\lambda_e > \lambda_p > \lambda_\alpha$

or $\lambda_e > \lambda_\alpha$ or $\lambda_p > \lambda_\alpha$ or $\lambda_e > \lambda_p$

33. Undisintegrated part

$$\frac{N}{N_0} = (100 - 18)\% = 82\%$$

Using relation

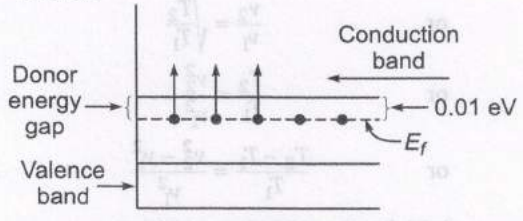
$$N = N_0(e^{-\lambda t})$$

$$\frac{82}{100} = e^{-(24 \times 60 \times 60\lambda)}$$

$$\therefore \quad 24 \times 60 \times 60 \times \lambda = \log\left(\frac{100}{82}\right)$$

or
$$\lambda = 2.1 \times 10^{-6}\ \text{s}^{-1}$$

34. The donor level is found only in n-type semiconductors. When the impurity of antimony is introduced, then each atom of germanium has an extra electron. The energy of these electrons is less than the least energy of conduction band and is greater than the highest value of energy of valence band. Thus, the electrons from a specific energy level below donor energy level. Donor level lies closely below the bottom of the conduction band as shown.



Note : Minimum donor energy level is 0.05 eV for Si and 0.01 eV for Ge. By giving this much amount of energy to the electron they become free and go to the conduction band.

35. Carbon (C), germanium (Ge) and silicon (Si) are semiconductors. They lie in IV A group of periodic table.

Now,
$$(E_g)_C = 5.2\,\text{eV}$$
$$(E_g)_{Ge} = 0.75\,\text{eV}$$
$$(E_g)_{Si} = 1.21\,\text{eV}$$

Thus, it is obvious that

$$(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$$

36. In order to find the time taken by the particle from -12.5 cm to $+12.5$ cm on either side of mean position, we will find the time taken by particle to go from $x = -12.5$ cm to $x = 0$ and to go from $x = 0$ to $x = +12.5$ cm.

Let the equation of motion be $x = A \sin \omega t$.

First, the particle moves from $x = -12.5$ cm to $x = 0$

$$\therefore \quad 12.5 = 25 \sin \omega t \qquad (\because A = 25\,\text{cm})$$

$$\Rightarrow \quad \frac{1}{2} = \sin \omega t$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\therefore \quad t = \frac{\pi}{6\omega}$$

Similarly to go from $x = 0$ to $x = 12.5$ cm

$$\omega t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{\pi}{6\omega}$$

$\therefore$ Total time taken from $x = -12.5$ cm to $x = 12.5$ cm

$$t = \frac{\pi}{6\omega} + \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$$

$$= \frac{\pi}{3\left(\frac{2\pi}{T}\right)} = \frac{T}{6} = \frac{3}{6} = 0.5\,s$$

37. Speed of wave on a string $v = \sqrt{\frac{T}{m}}$

or $$v \propto \sqrt{T}$$

or $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

or $$\frac{T_2}{T_1} = \frac{v_2^2}{v_1^2}$$

or $$\frac{T_2 - T_1}{T_1} = \frac{v_2^2 - v_1^2}{v_1^2} \quad \ldots(i)$$

Initially, $T_1 = 120$ N, $v_1 = 150$ m/s

$$v_2 = v_1 + \frac{20}{100}v_1 = v_1 + \frac{v_1}{5} = \frac{6v_1}{5}$$

$$= \frac{6}{5} \times 150 = 180 \text{ m/s}$$

So, from Eq. (i), we get

$$\frac{T_2 - T_1}{T_1} = \frac{(180)^2 - (150)^2}{(150)^2}$$

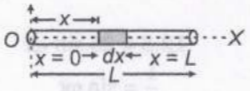
$$= \frac{30 \times 330}{150 \times 150} = 0.44$$

Hence, % increase in tension

$$= \left(\frac{T_2 - T_1}{T_1}\right) \times 100$$

$$= 0.44 \times 100 = 44\%$$

38. Let the mass of an element of length dx of rod located at distance x away from left end is $\frac{M}{L}dx$. The x-coordinate of the centre of mass is given by



$$x_{CM} = \frac{1}{M}\int x\, dm = 0$$

$$= \frac{1}{M}\int_0^L x\left(\frac{M}{L}dx\right)$$

$$= \left[\frac{1}{L}\frac{x^2}{2}\right]_0^L = \frac{L}{2}$$

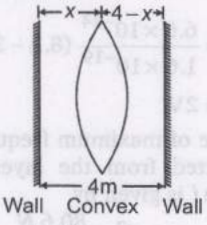
The y-coordinate is

$$Y_{CM} = \frac{1}{M}\int y\, dm = 0$$

and similarly, $z_{CM} = 0$

The centre of the mass is at $\left(\frac{L}{2}, 0, 0\right)$ or at the middle point of the rod, *i.e.*, at $\frac{L}{2}$.

39. Let a large convex lens is placed between two walls at a distance x from wall on which an electric bulb is fixed.



Using $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{4-x} - \frac{1}{-x} \quad [\because u = -x, v = 4 - x]$$

$$= \frac{4}{(4-x)(x)}$$

or $$f = \frac{(4-x)x}{4} \quad \ldots(i)$$

Now, magnification $m = \frac{v}{u} = \frac{4-x}{x}$

$$\Rightarrow \quad 1 = \frac{4-x}{x}$$

$$\Rightarrow \quad x = 4 - x$$

$$\Rightarrow \quad 2x = 4$$

$$\Rightarrow \quad x = 2\,\text{m}$$

Hence, from Eq. (i), we get

$$f = \frac{(4-2)(2)}{4}$$

$$= \frac{2 \times 2}{4} = 1\,\text{m}$$

40. **Key Idea :** *The necessary centripetal force to the satellite is provided by gravitational force of earth.*

Let a satellite is revolving around earth with orbital velocity v.

The gravitational potential energy of satellite is

$$U = -\frac{GM_e m}{R_e} \quad \ldots(i)$$

where, M_e is mass of earth,
m is mass of satellite
R_e is radius of earth
and G is gravitational constant.

The kinetic energy of satellite is

$$K = \frac{1}{2}\frac{GM_e m}{R_e} \quad \text{... (ii)}$$

$\therefore$ Total energy of satellite

$$E = U + K = -\frac{GM_e m}{R_e} + \frac{1}{2}\frac{GM_e m}{R_e}$$

$$= -\frac{1}{2}\frac{GM_e m}{R_e} \quad \text{... (iii)}$$

But we know that necessary centripetal force to the satellite is provided by the gravitational force, *i.e.*,

$$\frac{mv^2}{R_e} = \frac{GM_e m}{R_e^2}$$

or $$mv^2 = \frac{GM_e m}{R_e} \quad \text{... (iv)}$$

Hence, from Eqs. (iii) and (iv), we get

$$E = -\frac{1}{2}mv^2$$

41. Acceleration of electron moving round the nucleus is

$$a = \frac{v^2}{r} = \frac{(2.18 \times 10^6)^2}{0.528 \times 10^{-10}}$$

Given, $v = 2.18 \times 10^6$ m/s, $r = 0.528 \times 10^{-10}$ m

Substituting the values in the relation, we have

$$a = \frac{(2.18 \times 10^6)^2}{0.528 \times 10^{-10}}$$

$$\approx 9 \times 10^{22} \text{ m/s}^2$$

42. Force exerted on spring is given by

$$F = (-5x - 16x^3)\text{ N}$$

or $$F = -(5 + 16x^2)x \text{ N} \quad \text{...(i)}$$

Comparing Eq. (i) with

$$F = -kx$$

where k is a force constant.

We have $k = 5 + 16x^2$

Work done in stretching the spring from $x_1 = 0.1$ m to $x_2 = 0.2$ m is

$$W = \frac{1}{2}k_2 x_2^2 - \frac{1}{2}k_1 x_1^2$$

$$W = \frac{1}{2}[5 + 16x_2^2]\, x_2^2 - \frac{1}{2}[5 + 16x_1^2]\, x_1^2$$

Substituting the given values, we obtain

$$W = \frac{1}{2}[5 + 16(0.2)^2](0.2)^2$$

$$-\frac{1}{2}[5 + 16(0.1)^2](0.1)^2$$

$$= 2.82 \times 4 \times 10^{-2} - 2.58 \times 1 \times 10^{-2}$$

$$= 8.7 \times 10^{-2} \text{ J}$$

43. The streamline flow of a liquid is the flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceeding element passing through the same point. Hence, it is a regular flow. The path followed by each elements is called streamline. For laminar or streamline flow mass of the liquid entering the tube per second is equal to the mass of the liquid leaving the tube per second. Hence, the flow of liquid is laminar or streamline is determined by rate of flow of liquid.

44. The relation between Celsius scale and Fahrenheit scale is as derived below :

0°C ——— 100°C (100 equal parts)

32°F ——— 212°F (180 equal parts)

100 parts of Celsius scale = 180 parts of Fahrenheit scale

$\therefore$ 1 part of Celsius scale $= \frac{9}{5}$ parts of Fahrenheit scale

Hence, $$T_F = 32 + \frac{9}{5}T_C$$

or $$T_C = \frac{5}{9}(T_F - 32)$$

Given, $T_F = 95°\text{F}$

$\therefore$ $$T_C = \frac{5}{9}(95 - 32)$$

$$= 35°\text{C}$$

45. **Key Idea :** *Rearrange the given equation of particle motion.*

The given equation is written as

$$y = a(\sin \omega t + \cos \omega t)$$

or $$y = a\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin \omega t + \frac{1}{\sqrt{2}}\cos \omega t\right)$$

or $$y = a\sqrt{2}\left[\cos\frac{\pi}{4}\sin \omega t + \sin\frac{\pi}{4}\cos \omega t\right]$$

or $$y = a\sqrt{2}\sin(\omega t + \pi/4)$$

Thus, we have seen that the particle's motion is simple harmonic with amplitude $a\sqrt{2}$.

Note : We can represent the resultant equation in angular from as

$$\theta = \theta_0 \sin(\omega t + \pi/4)$$

where θ_0 is amplitude of angular SHM of particle.

46. In the fundamental mode of vibration

$$\frac{\lambda}{4} = (l + 0.3d)$$

where $0.3d$ is the necessary end correction.

Frequency of vibration, $n = \frac{v}{\lambda} = \frac{v}{4(l + 0.3d)}$

As l is same for both pipes, wider pipe (A) will resonate at a lower frequency, *i. e.*, $n_A < n_B$.

47. Pitch is that characteristic of sound which enables one to distinguish between a shrill sound and a grave sound. It depends upon the frequency of the source. While intensity of sound is proportional to the square of amplitude ($I = ka^2$), hence different amplitudes have different frequencies. Also quality depends upon the number, order and relative intensity of the overtone.

48. Destructive interference occurs when the path difference is an odd multiple of $\lambda/2$.

i.e., $\frac{xd}{D} = \frac{(2n-1)\lambda}{2}$

Angular width of first dark fringe is

$$\frac{2x}{D} = \frac{2(2n-1)\lambda}{2d}$$

Given, $n = 1$, $\lambda = 4800\ \text{Å} = 4800 \times 10^{-10}$ m,

$d = 0.6$ mm $= 0.6 \times 10^{-3}$ m

$$\therefore \quad \frac{2x}{D} = \frac{2(2\times1-1)\times 4800\times10^{-10}}{2\times0.6\times10^{-3}}$$

$$= 8 \times 10^{-4} \text{ rad}$$

49. Due to absence of atmosphere, there would have been no green house effect, *i. e.*,no back scattering of infrared radiations from atmosphere particles, which are emitted by earth. Due to which the temperature of earth would have decreased.

50. The energy of nth orbit of hydrogen like atom is,

$$E_n = -13.6\frac{Z^2}{n^2}$$

Here, $Z = 11$ for Na-atom. 10 electrons are removed already. For the last electron to be removed $n = 1$.

$$\therefore \quad E_n = \frac{-13.6\times(11)^2}{(1)^2} \text{ eV}$$

$$= -13.6 \times (11)^2 \text{ eV}$$

Chemistry

1. Carboxylic acids react with Grignard's reagent to give alkanes.

$$CH_3COOH + CH_3MgX \longrightarrow \underset{\text{methane}}{CH_4} + CH_3COOMgX$$

2. Cyclic hydrocarbon, with carbon-carbon bond length between 1.34 Å and 1.54 Å, is benzene in which due to resonance, C—C, bond length is 1.39 Å (*i. e.*, between 1.34 Å-1.54 Å). Benzene is a hexagonal molecule with bond-angle equal to 120°.

3. Magnesium is more electropositive than zinc, hence it can reduce Zn^{2+} (magnesium is placed above zinc in electrochemical series).

$$Mg + ZnO \longrightarrow MgO + Zn$$

4. For binary salts (like AgCl, AgBr),

$$s = \sqrt{K_{sp}}$$

$\therefore$ Solubility of AgCl $= \sqrt{1.8\times10^{-10}}$

$= 1.35 \times 10^{-7}$ mol/L

Solubility of AgBr $= \sqrt{5.0\times10^{-13}}$

$= 7.1 \times 10^{-7}$ mol/L

For, Ag_2CrO_4, $K_{sp} = 4s^3$

$\therefore$ Solubility of Ag_2CrO_4

$$= \sqrt[3]{\frac{K_{sp}}{4}} = \sqrt[3]{\frac{2.4\times10^{-12}}{4}}$$

$$= \sqrt[3]{600\times10^{-15}}$$

$= 8.44 \times 10^{-5}$ mol/L

As Ag_2CrO_4 has maximum solubility, it will give maximum Ag^+ ions in solution. Hence, it will be used.

5. Methyl iodide and ethyl iodide, on treatment with sodium in ethereal solution, give a mixture of propane, ethane and butane, as follows

$$CH_3\boxed{I + 2Na + I}CH_2CH_3 \xrightarrow{\text{ether}} \underset{\text{propane}}{CH_3CH_2CH_3} + 2NaI$$

$$CH_3I + 2Na + I{-}CH_3 \longrightarrow \underset{\text{ethane}}{CH_3{-}CH_3} + 2NaI$$

$$C_2H_5I + 2Na + I{-}C_2H_5 \longrightarrow \underset{\text{butane}}{C_2H_5C_2H_5} + 2NaI.$$

6. Stannous chloride ($SnCl_2$) is a good reducing agent. It reduces $HgCl_2$ into Hg (grey precipitate), in two steps.

$$SnCl_2 + 2HgCl_2 \longrightarrow SnCl_4 + \underset{\text{white}}{HgCl_2}\downarrow$$

$$SnCl_2 + Hg_2Cl_2 \longrightarrow SnCl_4 + 2\underset{\text{grey}}{Hg}\downarrow$$

7. Zymase enzyme act on glucose and give ethyl alcohol and carbondioxide.

$$C_6H_{12}O_6 \xrightarrow{\text{zymase}} \underset{\text{ethyl alcohol}}{2C_2H_5OH} + 2CO_2\uparrow$$

8. Fe^{3+} ion has the following configuration

$$Fe^{3+} = 1s^2, 2s^2\,2p^6, 3s^2\,3p^6\,3d^5$$

Hence, ferric ion is quite stable due to half-filled *d*-orbitals.

9. Electron affinity is the energy change, when an electron is added. When O^- changes into O^{2-}, the electron affinity is positive *i. e.*, change is endothermic. The reason is that O^- repels the incoming electron due to similar charge, hence, it needs energy to accept the electron. Hence, electron affinity is positive.

10. Ionisation potential is the energy required by an atom to lose electron and their ionisation potential is high.

11. Ethyl amine, on acetylation, given *N*-ethyl acetamide.

$$C_2H_5NH_2 \xrightarrow[\Delta]{CH_3COCl} \underset{N\text{-ethyl acetamide}}{C_2H_5NHCOCH_3}$$

12. Fluorine has maximum reduction electrode potential ($E°_{F/F^-} = 2.87$ V), hence, it is easily reduced inot F^- and consequently F_2 is the best oxidizing agent.

13. Lithium aluminium hydride is a powerful reducing agent. It reduces acetic acid into ethanol.

$$CH_3COOH \xrightarrow[\text{ether}]{LiAlH_4} CH_3CH_2OH$$

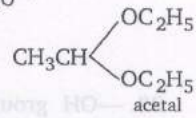
14. In presence of moisture, SO_2 acts as a reducing agent as gives nascent hydrogen. It reduces hydrogen peroxide into water.

$$SO_2 + 2H_2O \longrightarrow H_2SO_4 + 2H$$

$$H_2O_2 + 2H \longrightarrow 2H_2O$$

15. Aldehydes and ketones condense with alcohol to give aceta and ketals respectively. *e. g.*,

$$CH_3CHO + 2C_2H_5OH \xrightarrow{-H_2O}$$



acetal

16. Basic radicals have been classified into 6 groups on the basis of the solubility products of their salts with group reagent. The radicals have been grouped in the order of increasing solubility product. This is the reason, why $NiCl_2$ does not precipitate with 1st group chlorides or $Zn(OH)_2$ does not precipitate with $Al(OH)_3$ in IIIrd group.

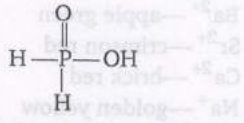
17. Ethanol, on dehydration in presence of conc. H_2SO_4 140° C, gives diethyl ether.

$$2C_2H_5OH \xrightarrow[140°\,C]{\text{conc. } H_2SO_4} \underset{\text{diethyl ether}}{C_2H_5{-}O{-}C_2H_5} + H_2O$$

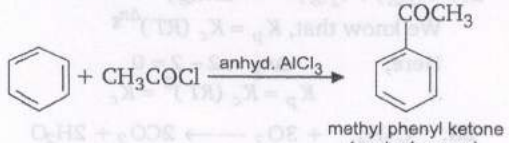
18. Hypophosphorus acid is a monobasic acid as it forms on one type of salts *e. g.*, sodium hydrogen phosphite (NaH_2PO_2)

$$NaOH + H_3PO_2 \longrightarrow NaH_2PO_2 + H_2O$$

Hypophosphorus acid has two hydrogen atoms attached phosphorus and one hydrogen atom attached to oxygen atom (which is ionisable), *i. e.*,



19. When, benzene is heated with acetyl chloride, in presence of anhydrous $AlCl_3$, electrophilic substitution takes place an acetophenone is obtained. The reaction is known as Friedel-Craft acylation.



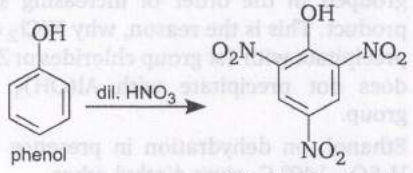
methyl phenyl ketone (acetophenone)

20. Carbondioxide gas remains present in airated water and soft-drinks.

21. Amides on acidic hydrolysis give acid and an amine. Hence, *N*-dimethylacetamide will give acetic acid and dimethyl amine on hydrolysis.

$$CH_3CON\begin{matrix}CH_3\\CH_3\end{matrix} \xrightarrow[H^+]{HOH} CH_3COOH + \overset{\oplus}{H_2N}\begin{matrix}CH_3\\CH_3\end{matrix}$$

22. —OH group is an activating group, hence increase electron density on *o*-and *p*-position in benzene ring. Thus phenol very easily undergoes nitration (electrophilic substitution and give trinitrophenol *i. e.*, picric acid.



phenol

2, 4, 6-trinitrophenol (picric acid)

23. Solder is an alloy of tin and lead. Its melting point is quite low, hence, it is very useful in stitching in ICs in various electrical instruments.

24. Among the given species, correct order of increasing acidic strength is

$$OH^- < H_2O < NH_4^+ < HF < H_3O^+$$

OH^- is basic (*i. e.*, it tends to gain a proton) and hence, is least acidic. H_2O is neutral species. H_3O^+ is most acidic as it readily lose proton.

25. When paste of salt in conc. HCl, is burnt on a spetchula, the following coloured flame appears for different radicals

Ba^{2+}—apple green
Sr^{2+}—crimson red
Ca^{2+}—brick red
Na^+—golden yellow

26. When chlorine is passed into hot concentrated solution of KOH, potassium chlorate is formed.

$$6KOH + 3Cl_2 \longrightarrow 5KCl + \underset{\text{potassium chlorate}}{KClO_3} + 3H_2O$$

27. $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

We know that, $K_p = K_c\,(RT)^{\Delta n_g}$

Here, $\Delta n_g = 2 - 2 = 0$

$\therefore$ $K_p = K_c\,(RT)^0 = K_c$

28. $\underset{12\times2+4=28}{C_2H_4} + \underset{96}{3O_2} \longrightarrow 2CO_2 + 2H_2O$

$\because$ The weight of oxygen required for complete combustion of 28 g ethylene = 96 g.

$\therefore$ Weight of oxygen required for combustion of 2.8 kg ethylene

$$= \frac{96 \times 2.8 \times 1000}{28 \times 1000}\ \text{kg} = 9.8\ \text{kg}$$

29. Mercury cannot displace hydrogen from acid. The reason can be explained on the basis of electrochemical series. The metal, which is placed above hydrogen in this series, can displace hydrogen from an acid. Mercury is placed below hydrogen in ECS, hence, it cannot displace hydrogen from an acid.

30. For an *n*th order reaction

$$t_{1/2} \propto \frac{1}{a^{n-1}}$$

$\therefore$ For 1st order reaction, $t_{1/2} \propto \frac{1}{a^{1-1}} \propto a^0$.

31. Atomic radii decrease in a period from left to right, hence, fluorine has a very less atomic radii (covalent atomic radii = 0.72 Å). But inert gases (like Ne) are monoatomic gases, hence, their covalent atomic radii cannot be found out. In fact, their calculated atomic radii is the van der Waals' radii, which is found almost double to covalent radii, hence, the van der Waals' radius of neon (Ne) is about 1.60 Å.

32. Sodium and chlorine are in same period.

$_{11}Na = 2, 8, 1$

$_{17}Cl = 2, 8, 7$

Both have 3-shells, hence, they both are [placed in 3rd period of periodic table.

33. When aqueous solution of $AgNO_3$ is added to KI solution, positively charged sol of AgI is obtained due to the adsorption of Ag^+ ions on AgI molecules.

$$\underset{\text{excess}}{AgNO_3} + KI \longrightarrow AgI + KNO_3$$

$$\downarrow Ag^+$$

$$[AgI]Ag^+$$

positively charged sol

34. Atomic and ionic radii increase from top to bottom in a group due to the inclusion of another shell at every step. Hence, Cs^+ ion will be the largest among given IA group ions (Na^+, Li^+ and K^+).

35. $CaCO_3(g) \rightleftharpoons CaO(s) + CO_2(g)$

$(\Delta n_g = 1 - 0 = 1)$

We know that,

$$\Delta H = \Delta E + \Delta n_g RT$$

$$176 = \Delta E + 1 \times 8.31 \times 10^{-3} \times (273 + 927)$$

$\therefore$ $\Delta E = 176 - 8.31 \times 10^{-3} \times 1200$

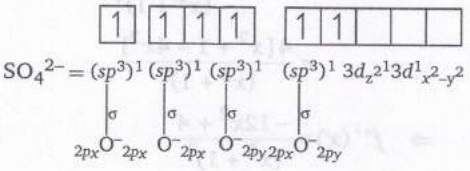
$= 176 - 9.97 = 166.03\ \text{kJ}$

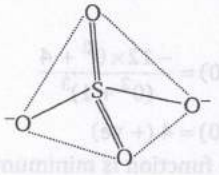
36. The charge required to liberate one gram equivalent of an element is always equal to 1 faraday (*i.e.*, = 96500 C).

37. Sulphate ion (SO_4^{2-}) has tetrahedral geometry, as in S-atom undergoes sp^3 hybridisation.

$$S = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^4$$

S in II excited state =





tetrahedral shape of SO_4^{2-}

38. The formation of one mole of HCl can be represented as,

$$\frac{1}{2}(H—H) + \frac{1}{2}(Cl—Cl) \longrightarrow H—Cl;$$

$$\Delta H = -90\ kJ$$

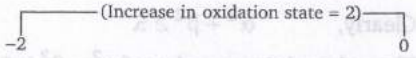
Hence, for the reaction,

Heat evolved = heat evolved in bond formation – heat required for bond breaking

$$\therefore \quad 90\ kJ = [H—Cl\ bond\ energy] - \left(\frac{1}{2}\times 430 + \frac{1}{2}\times 24\right)$$

$$\therefore (H—Cl\ bond\ energy) = 90 + (215 + 120)$$
$$= 425\ kJ\ mol^{-1}.$$

39.



$$H_2S + 2HNO_3 \longrightarrow 2H_2O + 2NO_2 + S$$

Hence, the equivalent weight of

$$H_2S = \frac{\text{molecular weight}}{\text{change in oxidation number}} = \frac{34}{2} = 17$$

40. The source of large energy, produced during atom bomb explosion, is the mass-defect occurring during the fission reaction, which is converted into energy equivalent to mass defect.

41. As the disorderness or randomness of particles in a system increases, its entropy increases. Hence, hydrogen has highest entropy as it is a gas and in gaseous state, randomness is maximum.

42. Chlorine can replace bromine from KBr solution, as it is placed above bromine in VIIA group in periodic table.

$$Cl_2 + 2KBr \longrightarrow 2KCl + Br_2$$

43. Nuclide have a definite number of protons and neutrons and consequently definite atomic number and mass-number. Such as oxygen nuclei contain 8 protons and 8 neutrons ($_8O^{16}$).

44. Arrhenius equation gives relation between rate constant and temperature of a reaction. It can be written in many forms, as

$$k = Ae^{-E_a/RT}$$

where, k = rate constant
E_a = activation energy
T = absolute temperature
A = frequency factor

45. Oxidation number of iodine in given species is as follows;

O.N. of iodine in $IF_3 = +3$

O.N. of iodine in $I_3^- = -\frac{1}{3}$

O.N. of iodine in $IF_5 = +5$

O.N. of iodine in $IF_7 = +7$

46. *s*-subshell has only one orbital and that is spherical, hence, *s*-orbitals are non-directional.

47. For weak electrolytes, according to Ostwald's dilution law

$$\alpha = \sqrt{KV}$$

Here, $\alpha = 0.01\% = 0.0001 = 1\times10^{-4}$

$$V = \frac{1}{C} = \frac{1}{1.0} = 1\ L$$

$$\therefore \quad K_a = \frac{\alpha^2}{V} = \frac{(1\times10^{-4})^2}{1} = 1\times10^{-8}$$

48. According to Graham's law of diffusion

$$\frac{r_{O_2}}{r_{He}} = \sqrt{\frac{M_{He}}{M_{O_2}}}$$

or $$= \sqrt{\frac{4}{32}} = \frac{1}{2.83}$$

$$\therefore \quad r_{O_2} = 0.35\, r_{He}$$

49. Na_2CO_3, on hydrolysis gives alkaline solution, as the solution contains strong base and weak acid.

$$\underset{\text{white}}{Na_2CO_3} \xrightarrow{2H_2O} \underset{\text{strong base}}{2NaOH} + \underset{\text{weak acid}}{H_2CO_3}$$

50. $FeCl_3$, on hydrolysis gives acidic solution due to the presence of strong acid (HCl) and weak base ($Fe(OH)_3$) in the solution.

$$FeCl_3 \xrightarrow{3H_2O} \underset{\text{weak base}}{Fe(OH)_3} + \underset{\text{strong acid}}{3HCl}$$

Hence, solution is acidic.

Mathematics

1. Unit place digit in the number $= 13^{25} = 3$
Unit place digit in the number $11^{25} = 1$
and unit place digit in the number $3^{25} = 3$
Thus, the unit place digit in the number
$13^{25} + 11^{25} - 3^{25} = 3 + 1 - 3 = 1.$

2. Given equation of curves are $y = x^2$ and $6y = 7 - x^3$.

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_{(1,1)} = 2(1) = 2$$

and $$m_2 = \left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{2}$$

Now, $m_1 m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$

$\therefore$ The angle of intersection of the curves is $\frac{\pi}{2}$.

3. $(1 + r)(1 + r^2)(1 + r^4)(1 + r^8)$
$= (1 + r + r^2 + r^3)(1 + r^4)(1 + r^8)$
$= (1 + r + r^2 + r^3 + r^4 + r^5 + r^6 + r^7)(1 + r^8)$
$= 1 + r + r^2 + \ldots + r^{11} + r^{12} + r^{13} + r^{14} + r^{15}$
Thus for $x = 15$, the equation
$1 + r + r^2 + \ldots + r^x$
$= (1 + r)(1 + r^2)(1 + r^4)(1 + r^8)$ holds.

4. $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ (for every real number x)
On differentiating w.r.t. x, we get

$$f'(x) = \frac{(x^2 + 1)(2x) - (x^2 - 1)2x}{(x^2 + 1)^2}$$

$$= \frac{2x[x^2 + 1 - x^2 + 1]}{(x^2 + 1)^2}$$

$$\Rightarrow \quad f'(x) = \frac{4x}{(x^2 + 1)^2} \quad \ldots(i)$$

Put $f'(x) = 0$ for minima or maxima

$$\therefore \quad \frac{4x}{(x^2 + 1)^2} = 0$$

$$\Rightarrow \quad x = 0$$

On differentiating Eq. (i) w.r.t. x, we get

$$f''(x) = \frac{(x^2 + 1)^2 4 - (4x)2(x^2 + 1)2x}{(x^2 + 1)^4}$$

$$= \frac{4(x^2 + 1)[x^2 + 1 - 4x^2]}{(x^2 + 1)^4}$$

$$= \frac{4[x^2 + 1 - 4x^2]}{(x^2 + 1)^3}$$

$$\Rightarrow \quad f''(x) = \frac{-12x^2 + 4}{(x^2 + 1)^3}$$

At $x = 0$

$$f''(0) = \frac{-12 \times 0^2 + 4}{(0^2 + 1)^3}$$

$$f''(0) = 4 \text{ (+ ve)}$$

Thus, the function is minimum at $x = 0$.
Then, minimum value of $f(x)$ at $x = 0$ is

$$f(0) = \frac{0^2 - 1}{0^2 + 1}$$

$$= -1$$

5. Given equation is
$x^2 - (a - 2)x - a - 1 = 0$
Let α, β be the roots of the given equation.
Then, $\alpha + \beta = a - 2,$
$\alpha\beta = -(a + 1)$

$$\therefore \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (a - 2)^2 + 2(a + 1)$$

$$= a^2 - 2a + 6$$

$$= (a - 1)^2 + 5$$

Clearly, $\alpha^2 + \beta^2 \geq 5$.
Thus, the minimum value of $\alpha^2 + \beta^2$ is 5, which it attains at $a = 1$.

6. Given, particle is dropped under gravity from rest of height h and $g = 9.8$ m/s^2 and travels a distance $\frac{9h}{25}$ in the last second.

$$\therefore \quad h_{nth} = u + \frac{1}{2}g(2n - 1)$$

$$= \frac{1}{2}g(2n - 1) \quad (\because u = 0)$$

$$\Rightarrow \quad \frac{9h}{25} = \frac{1}{2}g(2n - 1)$$

But $$h = \frac{1}{2}gn^2$$

$$\Rightarrow \quad \frac{9}{25} \times \frac{1}{2}gn^2 = \frac{1}{2}g(2n - 1)$$

$\Rightarrow \quad 9n^2 - 50n + 25 = 0$

$\Rightarrow \quad n = 5, \frac{5}{9}$

But $n = \frac{5}{9}$ is not possible.

$\therefore \quad n = 5\,s$

Then, $h = \frac{1}{2} \times 9.8 \times 5 \times 5$

$\Rightarrow \quad h = 122.5\ m/s$

7. If A and B are two sets having m and n elements respectively such that $1 \le n \le m$, then number of onto mappings from A to B

$$= \sum_{r=1}^{n} (-1)^{n-r}\, {}^{n}C_r\, r^m$$

Here, $m = 100$, $n = 2$

$\therefore$ The number of onto mappings from A to B

$$= \sum_{r=1}^{2} (-1)^{2-r}\, {}^{2}C_r\, r^{100}$$

$$= (-1)^{2-1} \cdot {}^{2}C_1 \cdot 1^{100} + (-1)^{2-2} \cdot {}^{2}C_2 \cdot 2^{100}$$

$$= -2 + 2^{100} = 2^{100} - 2.$$

Note : If set A has m elements and set B has n elements, then number of into functions from A to B is n^m.

8. Let us consider $f(x) = \frac{1-x}{1+x}$

$\therefore \quad fof = f[f(x)]$

$$= \frac{1 - f(x)}{1 + f(x)}$$

$$= \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}}$$

$$= \frac{1 + x - 1 + x}{1 + x + 1 - x}$$

$$= \frac{2x}{2}$$

$\Rightarrow \quad f[f(x)] = x$

$\Rightarrow \quad f(x) = f^{-1}(x)$

Hence, $f(x)$ is inverse of itself.

$\therefore$ Option (a) is correct.

9. Given that, $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

Now, $f(-x) = \log\left(-x + \sqrt{(-x)^2 + 1}\right)$

$$= \log\left(\sqrt{x^2 + 1} - x\right)$$

$$= -\log\left(\frac{1}{\sqrt{x^2 + 1} - x}\right)$$

$$= -\log\left[\frac{\sqrt{x^2 + 1} + x}{(\sqrt{x^2 + 1} - x)(\sqrt{x^2 + 1} + x)}\right]$$

$$= -\log\left[\frac{\sqrt{x^2 + 1} + x}{x^2 + 1 - x^2}\right]$$

$$= -\log\left(x + \sqrt{x^2 + 1}\right)$$

$\therefore \quad f(-x) = -f(x)\ \forall\ x$

Thus, the given function is an odd function.

10. Given that,

$$\log_{99}\{\log_2(\log_3 x)\} = 0$$

$\Rightarrow \quad \log_2(\log_3 x) = (99)^0$

$\Rightarrow \quad \log_2(\log_3 x) = 1$

$\Rightarrow \quad \log_3 x = 2$

$\Rightarrow \quad x = 3^2 = 9$

11. Given that, $n = 1000!$

Now, $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{1000} n}$

$$= \log_n 2 + \log_n 3 + \ldots + \log_n 1000$$

$$= \log_n 2 \cdot 3 \cdot 4 \cdot \ldots \cdot 1000$$

$$= \log_n (1000!) = \log_n n = 1$$

12. If ω and ω^2 are two imaginary cube roots of unity, then

$$1 + \omega + \omega^2 = 0$$

$\Rightarrow \quad \omega + \omega^2 = -1 \qquad \ldots(i)$

The sum of roots $= a\omega^{317} + a\omega^{382}$

$$= a(\omega^{317} + \omega^{382})$$

$$= a(\omega^2 + \omega) = -a \quad [\text{from (i)}]$$

The product of roots

$$= a\omega^{317} \times a\omega^{382} = a^2 \omega^{699} = a^2$$

Therefore, the required equation is

$x^2 - (\text{Sum of roots})\, x + (\text{Product of roots}) = 0$

$\Rightarrow \quad x^2 + ax + a^2 = 0.$

Note : Cube roots of -1 are $-1, -\omega, -\omega^2$.

13. $1 + \sum_{k=0}^{14} \left\{\cos\frac{(2k+1)}{15}\pi + i \sin\frac{(2k+1)}{15}\pi\right\}$

$$= 1 + \sum_{k=0}^{14} e^{i\frac{(2k+1)}{15}\pi}$$

$$= 1 + (\alpha + \alpha^3 + \alpha^5 + \ldots + \alpha^{29})$$

where, $\alpha = e^{i\pi/15}$

$$= 1 + \alpha\left[\frac{1 - (\alpha^2)^{15}}{1 - \alpha^2}\right]$$

$$= 1 + \alpha\left(\frac{1-\alpha^{30}}{1-\alpha^2}\right)$$

$$= 1 + \alpha\left(\frac{1-1}{1-\alpha^2}\right) = 1$$

$(\because \alpha^{30} = e^{i2\pi} = 1)$

14. Since $1, a_1, a_2, \ldots, a_{n-1}$ are the roots of $x^n - 1 = 0$

$\therefore x^n - 1 = (x-1)(x-a_1)(x-a_2)\ldots.$
$(x - a_{n-1})$

$$\Rightarrow \frac{x^n - 1}{x-1} = (x-a_1)(x-a_2)\ldots.(x-a_{n-1})$$

$$\Rightarrow (x-a_1)(x-a_2)\ldots.(x-a_{n-1})$$
$$= x^{n-1} + x^{n-2} + \ldots. + x^2 + x + 1$$

On putting $x = 1$ in the above equation, we get

$(1-a_1)(1-a_2)\ldots(1-a_{n-1})$
$= 1 + 1 + \ldots. + 1 + 1 + 1$

$\Rightarrow (1-a_1)(1-a_2)\ldots.(1-a_{n-1}) = n.$

Note : Sum of roots of unity is always zero.

15. Given α, β be the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$ and also D_1, D_2 discriminants of respectively equations.

Let $A = \alpha + h$ and $B = \beta + h$

Then, $A - B = (\alpha + h) - (\beta + h) = \alpha - \beta$

$$\Rightarrow (A-B)^2 = (\alpha - \beta)^2$$

$$\Rightarrow (A+B)^2 - 4AB = (\alpha+\beta)^2 - 4\alpha\beta$$

$$\Rightarrow \frac{q^2}{p^2} - \frac{4r}{p} = \frac{b^2}{a^2} - \frac{4c}{a}$$

$$\Rightarrow \frac{q^2 - 4rp}{p^2} = \frac{b^2 - 4ac}{a^2}$$

$$\Rightarrow \frac{D_2}{p^2} = \frac{D_1}{a^2}$$

$(\because D_1 = b^2 - 4ac,\ D_2 = q^2 - 4rp)$

$$\Rightarrow \frac{D_1}{D_2} = \frac{a^2}{p^2}$$

16. Given that, a, b, c are in AP

$$\Rightarrow 2b = a + c \quad \ldots(i)$$

and $b - a, c - b, a$ are in GP

$$\Rightarrow (c-b)^2 = (b-a)a$$

$$\Rightarrow c^2 + b^2 - 2bc = ab - a^2$$

$$\Rightarrow c^2 + \left(\frac{a+c}{2}\right)^2 - 2\left(\frac{a+c}{2}\right)c$$
$$= a\left(\frac{a+c}{2}\right) - a^2 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow c^2 + \frac{a^2 + c^2 + 2ac}{4} - ac - c^2 = \frac{a^2}{2} + \frac{ac}{2} - a^2$$

$$\Rightarrow \frac{a^2 + c^2 + 2ac - 4ac}{4} = \frac{ac - a^2}{2}$$

$$\Rightarrow (c-a)^2 = 2a(c-a)$$

$$\Rightarrow c - a = 2a$$

$$\Rightarrow c = 3a$$

From Eq. (i),

$$2b = a + 3a$$
$$b = 2a$$

$\therefore a : b : c = a : 2a : 3a$
$= 1 : 2 : 3$

17. Key Idea : *If a, b and c are the prime numbers, then the divisors of $a^{n_1}b^{n_2}c^{n_3}$ is*

$$(n_1+1)(n_2+1)(n_3+1)$$

The number of divisors of $3 \times 7^3 = 8$, the number of divisors of $7 \times 11^2 = 6$ and the number of divisors of $2 \times 61 = 4$

$\Rightarrow$ 8, 6, 4 are in AP with common difference -2.

18. Given a, b, c are in AP and $|a|, |b|, |c| < 1$

and $x = 1 + a + a^2 + \ldots$ to ∞

$y = 1 + b + b^2 + \ldots$ to ∞

$z = 1 + c + c^2 + \ldots$ to ∞

Then, $x = \frac{1}{1-a}, y = \frac{1}{1-b}, z = \frac{1}{1-c}$

$$\Rightarrow a = 1 - \frac{1}{x}, b = 1 - \frac{1}{y}, c = 1 - \frac{1}{z}$$

$$\Rightarrow \frac{1}{x} = 1 - a, \frac{1}{y} = 1 - b, \frac{1}{z} = 1 - c$$

But given a, b, c are in AP

$\therefore 1 - a, 1 - b, 1 - c$ are in AP

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are also in AP

$\Rightarrow x, y, z$ are in HP.

19. Given, $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$ to ∞

This sequence is arithmetico-geometric sequence.

Then, $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 1, r = \frac{1}{5}, d = 3$

$$\therefore \quad S_\infty = \frac{1}{1-\frac{1}{5}} + \frac{3\times\frac{1}{5}}{\left(1-\frac{1}{5}\right)^2}$$

$$= \frac{5}{4} + \frac{3}{5\times\frac{16}{25}} = \frac{5}{4} + \frac{15}{16} = \frac{35}{16}$$

Note : The sum of infinite arithmetico-geometric sequence is

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2},$$

where d is common difference, r is common ratio and a is first term.

20. Given, the sum of first n natural numbers $= \frac{1}{78}$ (the sum of their cubes)

$$\Rightarrow \quad \frac{n(n+1)}{2} = \frac{1}{78} \times \frac{n^2(n+1)^2}{4}$$

$$\Rightarrow \quad 156 = n(n+1)$$

$$\Rightarrow \quad n^2 + n - 156 = 0$$

$$\Rightarrow \quad (n+13)(n-12) = 0$$

$$\Rightarrow \quad n = 12 \quad (\because n \neq -13)$$

21. Given that

$p = \cos 55°$, $q = \cos 65°$ and $r = \cos 175°$

Then, $\frac{1}{p} + \frac{1}{q} + \frac{r}{pq} = \frac{p+q+r}{pq}$

$$= \frac{\cos 55° + \cos 65° + \cos 175°}{\cos 55° \cos 65°}$$

$$= \frac{\cos 55° + 2\cos\left(\frac{175°+65°}{2}\right)\cos\left(\frac{175°-65°}{2}\right)}{\cos 55° \cos 65°}$$

$$= \frac{\cos 55° + 2\cos 120° \cos 55°}{\cos 55° \cos 65°}$$

$$= \frac{\cos 55°\,(1 + 2\cos 120°)}{\cos 55° \cos 65°}$$

$$= \frac{\left(1 - 2\times\frac{1}{2}\right)}{\cos 65°} \qquad \left(\because \cos 120° = -\frac{1}{2}\right)$$

$$= \frac{0}{\cos 65°} = 0$$

22. $\sin 20°\,(4 + \sec 20°)$

$$= \sin 20°\left(4 + \frac{1}{\cos 20°}\right)$$

$$= \sin 20°\left(\frac{4\cos 20° + 1}{\cos 20°}\right)$$

$$= \frac{2\sin 40° + \sin 20°}{\cos 20°}$$

$$= \frac{\sin 40° + \sin 40° + \sin 20°}{\cos 20°}$$

$$= \frac{\sin 40° + 2\sin 30° \cos 10°}{\cos 20°}$$

$$= \frac{\sin 40° + \cos 10°}{\cos 20°}$$

$$= \frac{\cos 50° + \cos 10°}{\cos 20°}$$

$$= \frac{2\cos 30° \cos 20°}{\cos 20°}$$

$$= 2 \times \cos 30°$$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

23. Given, $4\sin^{-1} x + \cos^{-1} x = \pi$

$$\Rightarrow \quad 4\sin^{-1} x + \frac{\pi}{2} - \sin^{-1} x = \pi$$

$$\left(\because \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}\right)$$

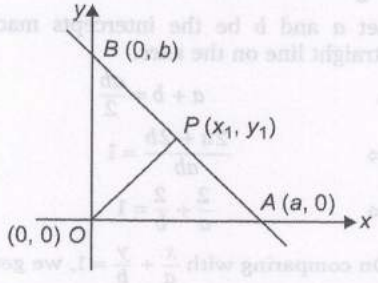
$$\Rightarrow \quad 3\sin^{-1} x = \pi - \frac{\pi}{2}$$

$$\Rightarrow \quad 3\sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \quad x = \sin\frac{\pi}{6}$$

$$\Rightarrow \quad x = \frac{1}{2}$$

24. Equation of line is $\frac{x}{a} + \frac{y}{b} = 1$



Let P be the foot of perpendicular from the origin to the line whose coordinate is (x_1, y_1).

Since $OP \perp AB$

$\therefore$ slope of $OP \times$ slope of $AB = -1$

$$\Rightarrow \quad \left(\frac{y_1}{x_1}\right)\left(\frac{b}{-a}\right) = -1$$

$$\Rightarrow \quad by_1 = ax_1 \qquad \ldots(i)$$

Since P lies on the line AB, then

$$\frac{x_1}{a}+\frac{y_1}{b}=1$$

$$\Rightarrow \quad bx_1+ay_1=ab \qquad ...(ii)$$

From Eqs. (i) and (ii),

$$x_1=\frac{ab^2}{a^2+b^2} \text{ and } y_1=\frac{a^2b}{a^2+b^2}$$

Now, $x_1^2+y_1^2=\left(\frac{ab^2}{a^2+b^2}\right)^2+\left(\frac{a^2b}{a^2+b^2}\right)^2$

$$\Rightarrow \quad x_1^2+y_1^2=\frac{a^2b^4}{(a^2+b^2)^2}+\frac{a^4b^2}{(a^2+b^2)^2}$$

$$\Rightarrow \quad x_1^2+y_1^2=\frac{a^2b^2(a^2+b^2)}{(a^2+b^2)^2}$$

$$\Rightarrow \quad x_1^2+y_1^2=\frac{a^2b^2}{(a^2+b^2)}$$

$$\Rightarrow \quad x_1^2+y_1^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$$

But $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ (given)

$$\therefore \quad x_1^2+y_1^2=c^2$$

Thus, the locus of $P(x_1, y_1)$ is

$$x^2+y^2=c^2$$

which is the equation of circle.

25. Given, the sum of intercepts of straight line $=\frac{1}{2}\times$ the product of intercepts.

Let a and b be the intercepts made by the straight line on the axes.

$$\therefore \quad a+b=\frac{ab}{2}$$

$$\Rightarrow \quad \frac{2a+2b}{ab}=1$$

$$\Rightarrow \quad \frac{2}{a}+\frac{2}{b}=1$$

On comparing with $\frac{x}{a}+\frac{y}{b}=1$, we get

$$x=2,\ y=2$$

The required point is (2, 2).

Hence, straight line passes through the point (2, 2).

26. Given that, circle $x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle

$$x^2+y^2-2x+8y-d=0$$

$\therefore$ The common chord of the given circle is

$$S_1-S_2=0$$

$$\Rightarrow \quad x^2+y^2+4x+22y+c-x^2-y^2+2x-8y+d=0$$

$$\Rightarrow \quad 6x+14y+c+d=0 \qquad ...(i)$$

So, Eq. (i) passes through the centre of the second circle *i.e.*, (1, – 4)

$$\therefore \quad 6-56+c+d=0$$

$$\Rightarrow \quad c+d=50$$

Note : If S_1 and S_2 are the equations of two circles, then equation of common chord is $S_1-S_2=0$.

27. The equation of tangents are given

$$x+3y-5=0 \qquad ...(i)$$

and $2x+6y+30=0$

or $x+3y+15=0 \qquad ...(ii)$

We know that the distance between two parallel lines is $\left|\frac{c_1-c_2}{\sqrt{a_1^2+b_1^2}}\right|$

$$=\left|\frac{-5-15}{\sqrt{1+9}}\right|$$

$$=\frac{20}{\sqrt{10}}$$

Hence, the radius of circle

$$=\frac{1}{2}\times\frac{20}{\sqrt{10}}$$

$$=\frac{10}{\sqrt{10}}$$

$$=\sqrt{10} \text{ unit}$$

Note : The distance between two parallel lines is $\frac{|c_1-c_2|}{\sqrt{a_1^2+b_1^2}}$.

28. **Key Idea :** *Since the semi-latus rectum of a parabola is the HM of segments of a focal chord.*

$$\therefore \text{ Semi-latus rectum} = \frac{2SP\cdot SQ}{SP+SQ}$$

$$=\frac{2\times3\times2}{3+2}=\frac{12}{5}$$

$\therefore$ Latus rectum of the parabola $=\frac{24}{5}$.

29. **Key Idea :** *The product of perpendiculars from the foci on any tangent to the ellipse* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ *is equal to* b^2.

We know that the product of perpendiculars from two foci S_1 and S_2 of an ellipse

$\frac{x^2}{9}+\frac{y^2}{16}=1$, on the tangent at any point P on the ellipse is equal to the square of the semi-minor axis.

$\therefore \quad (S_1 M_1)(S_2 M_2)=9.$

30. The equation of hyperbola is

$$4x^2-9y^2=36$$

$$\Rightarrow \quad \frac{x^2}{9}-\frac{y^2}{4}=1 \qquad \ldots(i)$$

The equation of the chord of contact of tangents from (x_1, y_1) and (x_2, y_2) to the given hyperbola are

$$\frac{xx_1}{9}-\frac{yy_1}{4}=1 \qquad \ldots(ii)$$

and $$\frac{xx_2}{9}-\frac{yy_2}{4}=1 \qquad \ldots(iii)$$

Lines (ii) and (iii) are at right angles

$$\therefore \quad \frac{4}{9}\cdot\frac{x_1}{y_1}\times\frac{4}{9}\cdot\frac{x_2}{y_2}=-1 \qquad (\because m_1 m_2=-1)$$

$$\Rightarrow \quad \frac{x_1 x_2}{y_1 y_2}=-\left(\frac{9}{4}\right)^2$$

$$=-\frac{81}{16}$$

31. Given equation is

$$x\,dy-y\,dx+x^2 e^x\,dx=0$$

$$\Rightarrow \quad x\,dy-y\,dx+x^2 e^x\,dx=0$$

$$\Rightarrow \quad \frac{x\,dy-y\,dx}{x^2}+e^x\,dx=0$$

$$\Rightarrow \quad d\left(\frac{y}{x}\right)+d(e^x)=0$$

On integrating both sides, we get

$$\frac{y}{x}+e^x=c$$

32. The general term in the expansion of $\left(\frac{1}{3}x^{1/2}+x^{-1/4}\right)^{10}$ is

$$T_{r+1}={}^{10}C_r\left(\frac{1}{3}x^{1/2}\right)^{10-r}(x^{-1/4})^r$$

$$={}^{10}C_r\,\frac{1}{3^{10-r}}\,x^{5-r/2}\,x^{-r/4}$$

$$={}^{10}C_r\,\frac{1}{3^{10-r}}\,x^{5-3r/4}$$

For coefficient of x^2, we take

$$5-\frac{3r}{4}=2$$

$$\Rightarrow \quad 3=\frac{3r}{4}$$

$$\Rightarrow \quad r=4$$

$$\therefore \text{ Coefficient of } x^2={}^{10}C_4\,\frac{1}{3^{10-4}}$$

$$=\frac{210}{729}$$

$$=\frac{70}{243}$$

33. $$\therefore \left[4\left(1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots\right)\right]^{\log_2 x}$$

$$=\left[54\left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\right)\right]^{\log_x 2}$$

$$\Rightarrow \left[4\left(\frac{1}{1+\frac{1}{3}}\right)\right]^{\log_2 x}=\left[54\left(\frac{1}{1-\frac{1}{3}}\right)\right]^{\log_x 2}$$

$$\Rightarrow \quad \left[4\left(\frac{3}{4}\right)\right]^{\log_2 x}=\left[54\times\frac{3}{2}\right]^{\log_x 2}$$

$$\Rightarrow \quad 3^{\log_2 x}=(81)^{\log_x 2}$$

$$\Rightarrow \quad 3^{\log_2 x}=3^{4\log_x 2}$$

$$\Rightarrow \quad \log_2 x=4\log_x 2$$

$$\Rightarrow \quad \log_2 x=\frac{4}{\log_2 x}$$

$$\Rightarrow \quad (\log_2 x)^2=4$$

$$\Rightarrow \quad \log_2 x=\pm 2$$

If $\log_2 x=+2$,

then $x=2^2=4$

and if $\log_2 x=-2$,

then $x=2^{-2}=\frac{1}{4}$

$\therefore$ Solution set of the equation $=\left\{4, \frac{1}{4}\right\}$.

34. Given, $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots.$

and $|x|<1$

Then, $y=\log_e(1+x)$

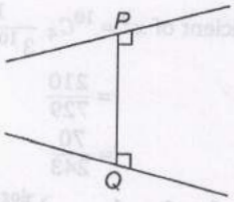
$$\Rightarrow \quad e^y=1+x \Rightarrow e^y-1=x$$

$$\Rightarrow \quad \left(1+\frac{y}{1!}+\frac{y^2}{2!}+\frac{y^3}{3!}+\ldots.\right)-1=x$$

$$\Rightarrow \quad \frac{y}{1!}+\frac{y^2}{2!}+\frac{y^3}{3!}+\ldots=x$$

35. Given, line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

$$\Rightarrow \quad \frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda \qquad \text{(say)}$$



$\Rightarrow \quad x = \lambda,\ y = 2\lambda + 1,\ z = 3\lambda + 2$

Therefore, direction ratios of PQ are

$\lambda - 1,\ 2\lambda + 1 - 6,\ 3\lambda + 2 - 3$

$\Rightarrow \quad \lambda - 1,\ 2\lambda - 5,\ 3\lambda - 1$

$\because PQ$ is perpendicular to the given line.

Therefore,

$1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \quad \lambda = 1$

$\therefore$ The coordinate of Q (1, 3, 5).

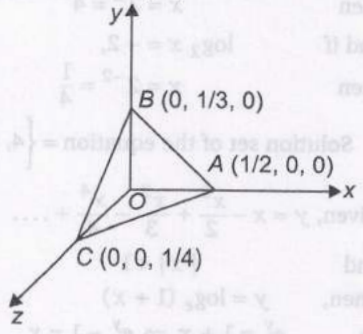
$\therefore$ Length of perpendicular

$= \sqrt{(1-1)^2 + (3-6)^2 + (5-3)^2}$

$= \sqrt{9+4} = \sqrt{13}$.

36. Key Idea : *If the plane* $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ *meets the coordinate axes, then the centroid of triangle is* $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$.

Given, plane $2x + 3y + 4z = 1$ meets the coordinate axes A, B, and C.



$\Rightarrow \quad \frac{2x}{1} + \frac{3y}{1} + \frac{4z}{1} = 1$

$\Rightarrow \quad \frac{x}{\frac{1}{2}} + \frac{y}{\frac{1}{3}} + \frac{z}{\frac{1}{4}} = 1$

Then, centroid of the triangle ABC is

$\left[\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right]$

$= \left(\frac{\frac{1}{2} + 0 + 0}{3}, \frac{0 + \frac{1}{3} + 0}{3}, \frac{0 + 0 + \frac{1}{4}}{3}\right)$

$= \left(\frac{1}{6}, \frac{1}{9}, \frac{1}{12}\right)$.

37. Given, centre of sphere $= (1, 0, 1)$ and radius $= 4$.

$\therefore$ Vector equation of sphere is $|\vec{\mathbf{r}} - \vec{\mathbf{a}}| = R$ and centre of sphere in vector form

$= \hat{\mathbf{i}} + 0\,\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$

Hence, the vector equation of sphere is

$|\vec{\mathbf{r}} - (\hat{\mathbf{i}} + \hat{\mathbf{k}})| = 4$.

38. We know that the equation of a plane through the intersection of the planes

$a_1x + b_1y + c_1z + d_1 = 0$

and $\quad a_2x + b_2y + c_2z + d_2 = 0$

is $(a_1x + b_1y + c_1z + d_1) + \lambda\,(a_2x + b_2y + c_2z + d_2) = 0$

where, λ is constant.

Thus, the equation of plane

$2x\lambda - (1 + \lambda)\,y + 3z = 0$

can be written as

$(2x - y)\,\lambda + (-y + 3z) = 0$

So, it is clear that the equation of plane passes through the intersection of planes $2x - y = 0$ and $y - 3z = 0$.

39. Given, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}| = \sqrt{37}, |\vec{\mathbf{b}}| = 3$ and $|\vec{\mathbf{c}}| = 4$

$\therefore \quad \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Rightarrow \quad \vec{\mathbf{a}} = -(\vec{\mathbf{b}} + \vec{\mathbf{c}})$

$\Rightarrow \quad |\vec{\mathbf{a}}|^2 = |-(\vec{\mathbf{b}} + \vec{\mathbf{c}})|^2$

$\Rightarrow \quad |\vec{\mathbf{a}}|^2 = |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\theta$

$= 9 + 16 + 24\cos\theta$

$\Rightarrow \quad 37 = 25 + 24\cos\theta$

$\Rightarrow \quad 24\cos\theta = 12$

$\Rightarrow \quad \theta = 60°$.

40. Given, $\vec{a} = \hat{i} + \hat{j} - \hat{k}, \vec{b} = -\hat{i} + \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j}$

$\because \quad (\vec{a} + \lambda \vec{c}) \perp \vec{b}$

$\therefore \quad (\vec{a} + \lambda \vec{c}) \cdot \vec{b} = 0$

$\Rightarrow [(\hat{i} + \hat{j} - \hat{k}) + \lambda (2\hat{i} + \hat{j})] \cdot (-\hat{i} + \hat{k}) = 0$

$\Rightarrow [(1 + 2\lambda)\hat{i} + (1 + \lambda)\hat{j} - \hat{k}] \cdot (-\hat{i} + \hat{k}) = 0$

$\Rightarrow \quad (1 + 2\lambda)(-1) + (-1) = 0$

$\Rightarrow \quad 2\lambda = -2$

$\Rightarrow \quad \lambda = -1.$

41. Given, forces $\vec{F}_1 = 2\hat{i} - \hat{j}, \vec{F}_2 = 3\hat{i} + 2\hat{j} - \hat{k}$ acting on a particle when it displaced from the point

$3\hat{i} + 2\hat{j} + \hat{k}$ to $5\hat{i} + 5\hat{j} + 3\hat{k}$.

i. e., its displacement $\vec{d} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\therefore \quad W = \vec{F} \cdot \vec{d}$

$= (\vec{F}_1 + \vec{F}_2) \cdot \vec{d}$

$= (5\hat{i} + \hat{j} - \hat{k}) \cdot (2\hat{i} + 3\hat{j} + 2\hat{k})$

$= 10 + 3 - 2$

$\Rightarrow \quad W = 11$ unit.

42. Given, $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ defined by the relations

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

and $$\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

$\therefore \quad \vec{a} \cdot \vec{p} = \dfrac{\vec{a} \cdot \vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$= \dfrac{\vec{a} \cdot (\vec{b} \times \vec{c})}{[\vec{a}\ \vec{b}\ \vec{c}]} = 1$

and $\vec{a} \cdot \vec{q} = \vec{a} \cdot \dfrac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

$= \dfrac{\vec{a} \cdot (\vec{c} \times \vec{a})}{[\vec{a}\ \vec{b}\ \vec{c}]} = 0$

Similarly, $\vec{b} \cdot \vec{q} = \vec{c} \cdot \vec{r} = 1$

and $\vec{a} \cdot \vec{r} = \vec{b} \cdot \vec{p} = \vec{b} \cdot \vec{r} = \vec{c} \cdot \vec{q} = \vec{c} \cdot \vec{p} = 0$

$\therefore (\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r}$

$= \vec{a} \cdot \vec{p} + \vec{b} \cdot \vec{p} + \vec{b} \cdot \vec{q} + \vec{c} \cdot \vec{q} + \vec{c} \cdot \vec{r} + \vec{a} \cdot \vec{r}$

$= 1 + 1 + 1 = 3.$

43. Given, $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$

$= (y - z)(z - x)(x - y)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

The degree of determinant

$= n + (n + 2) + (n + 3)$

$= 3n + 5$

and the degree of RHS = 2

$\therefore \quad 3n + 5 = 2$

$\Rightarrow \quad n = -1.$

44. $\because a_1, a_2, \ldots, a_n$ are in GP

$\Rightarrow a_n, a_{n+2}, a_{n+4}, \ldots$ are also in GP

Now, $(a_{n+2})^2, = a_n \cdot a_{n+4}$

$2 \log (a_{n+2}) = \log a_n + \log a_{n+4}$

Similarly $2 \log (a_{n+8}) = \log a_{n+6} + \log a_{n+10}$

Now, $\Delta = \begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$

Applying $C_2 \to 2C_2 - C_1 - C_3$

$= \begin{vmatrix} \log a_n & 2\log a_{n+2} - \log a_n - \log a_{n+4} & \log a_{n+4} \\ \log a_{n+6} & 2\log a_{n+8} - \log a_{n+6} - \log a_{n+10} & \log a_{n+10} \\ \log a_{n+12} & 2\log a_{n+14} - \log a_{n+12} - \log a_{n+16} & \log a_{n+16} \end{vmatrix}$

$= \begin{vmatrix} \log a_n & 0 & \log a_{n+4} \\ \log a_{n+6} & 0 & \log a_{n+10} \\ \log a_{n+12} & 0 & \log a_{n+16} \end{vmatrix} = 0.$

45. The given system of equations possess non-zero solutions.

$$\therefore \quad \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a \\ 1 & -a & 1 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \quad \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & a-1 \\ 0 & -a-1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1[0-(a^2-1)] = 0$$

$$\Rightarrow \quad a^2 = 1$$

$$\Rightarrow \quad a = \pm 1.$$

46. Given, A is a square matrix and $AA^T = I = A^T A$

$$\Rightarrow \quad |AA^T| = |I| = |A^T A|$$

$$\Rightarrow \quad |A||A^T| = 1 = |A^T||A|$$

$$\Rightarrow \quad |A|^2 = 1 \qquad (\because |A^T| = |A|)$$

$$\Rightarrow \quad |A| = \pm 1.$$

47. Given $\int_a^b \frac{|x|}{x}\,dx,\ a < 0 < b$

$$= \int_a^0 \frac{|x|}{x}\,dx + \int_0^b \frac{|x|}{x}\,dx$$

$$= \int_a^0 -1\,dx + \int_0^b 1\,dx$$

$$= a + b.$$

48. Given, A and B are two events.

Odds against A are 2 to 1, odds in favour of $A \cup B$ are 3 to 1 also $x \le P(B) \le y$

i.e., $$P(A) = \left(1 - \frac{2}{3}\right) = \frac{1}{3}$$

and $$P(A \cup B) = \frac{3}{4}$$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{3}{4} = \frac{1}{3} + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = P(B) - \frac{5}{12}$$

$$\Rightarrow \quad P(B) \ge \frac{5}{12}$$

Again, $$P(B) = \frac{5}{12} + P(A \cap B)$$

$$\Rightarrow \quad P(B) \le \frac{5}{12} + P(A)$$

$$\Rightarrow \quad P(B) \le \frac{5}{12} + \frac{1}{3}$$

$$\Rightarrow \quad P(B) \le \frac{3}{4}$$

Hence, $$x \le P(B) \le y$$

$$\therefore \quad \frac{5}{12} \le P(B) \le \frac{3}{4}.$$

49. Let x be the probability of success in each trial, then $(1 - x)$ will be the probability of failure in each trial.

Thus, probability of exactly two successes in a series of trials

$$= P(\bar{E}_1 E_2 E_3 + E_1 \bar{E}_2 E_3 + E_1 E_2 \bar{E}_3)$$

$$= (1-x)\,x \cdot x + x\,(1-x)\,x + x \cdot x\,(1-x)$$

$$= 3x^2(1-x)$$

and the probability of three successes

$$P(E_1 E_2 E_3) = x \cdot x \cdot x = x^3$$

According to question

$$9x^3 = 3x^2(1-x)$$

$$\Rightarrow \quad 3x = 1 - x$$

$$\Rightarrow \quad 4x = 1$$

$$\Rightarrow \quad x = \frac{1}{4}.$$

Hence, the probability of success in each trial is $\frac{1}{4}$.

50. Given, one integer is chosen at random from the first 200 positive integers and integer chosen is divisible by 6 or 8

$\therefore$ One integer can be chosen out of 200 integers in ${}^{200}C_1$ ways.

Let A be the event that an integer selected is divisible by 6 and B that it is divisible by 8.

Then, $$P(A) = \frac{33}{200},\ P(B) = \frac{25}{200}$$

and $$P(A \cap B) = \frac{8}{200}$$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{1}{4}.$$