DAY FIFTEEN

Heat and Thermodynamics

Learning & Revision for the Day

- Heat
- Thermometry
- Thermal Expansion
- Specific Heat Capacity
- Calorimetry
- Change of State
- Heating Curve
- Zeroth Law of Thermodynamics
- First Law of Thermodynamics
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Heat

Heat is a form of energy which characterises the thermal state of matter. It is transferred from one body to the other due to temperature difference between them.

Heat is a scalar quantity with dimensions $[ML^2T^{-2}]$ and its SI unit is joule (J) while practical unit is calorie (cal); $1 \text{ cal} = 4.18 \text{ J}$

If mechanical energy (work) is converted into heat then, the ratio of work done (W) to heat produced (Q) always remains the same and constant.

i.e. $$\frac{W}{Q} = \text{constant} = J \quad \text{or} \quad W = JQ$$

The constant J is called **mechanical equivalent of heat**.

Temperature

The factor that determines the flow of heat from one body to another when they are in contact with each other, is called temperature. Its SI unit is kelvin.

Thermometer

An instrument used to measure the temperature of a body is called a thermometer. For construction of thermometer, two fixed reference point **ice point** and **steam point** are taken. Some common types of thermometers are as follows:

1. **Liquid (mercury) thermometer** Range of temperature: –50°C to 350°C
2. **Gas thermometer (Nitrogen gas)** Range of temperature: –200°C to 1600°C
3. **Pyrometers** Range of temperature: –800°C to 6000°C

Scales of Temperature

Three most common scales are Celsius scale or Centigrade scale, Fahrenheit scale and Kelvin scale (Absolute scale).

Scale	Ice point / lower reference point	Steam point / Upper reference point	Unit
Celsius	0	100	°C
Fahrenheit	32	212	°F
Kelvin	273.15	373.15	K

Relation between C, F and K scales is

$$\frac{C}{5}=\frac{F-32}{9}=\frac{K-273.15}{5}$$

In general, $$\frac{\text{Temperature of } X-\text{Ice point of } X}{\text{Steam point of } X-\text{Ice point of } X}$$

$$=\frac{\text{Temperature of } Y-\text{Ice point of } Y}{\text{Steam point of } Y-\text{Ice point of } Y}$$

Thermometry

The branch dealing with measurement of temperature is called thermometry.

Let thermometric properties at temperatures 0°C (ice point), 100°C (steam point) and t°C (unknown temperature) are X_0, X_{100}, and X_t, respectively. Then,

$$\frac{X_t-X_0}{t}=\frac{X_{100}-X_0}{100} \quad \text{or} \quad \frac{X_t-X_0}{X_{100}-X_0}=\frac{t}{100}$$

Thus, $$t=\left(\frac{X_t-X_0}{X_{100}-X_0}\right)\times 100^\circ\,\text{C}$$

Thermal Expansion

Almost all substances (solid, liquid and gas) expand on heating and contract on cooling. The expansion of a substance on heating is called thermal expansion of substance.

Thermal Expansion of Solids

Thermal expansion in solids is of three types:

1. **Linear Expansion** Thermal expansion along a single dimension of a solid body is defined as the linear expansion.

   If a rod is having length l_0 at temperature T, then elongation in length of rod due to rise in temperature by ΔT is,

   $$\Delta l = l_0\,\alpha\,\Delta T \quad \text{or} \quad \alpha=\frac{\Delta l}{l_0\times\Delta T}$$

   where, α is the coefficient of linear expansion whose value depends on the nature of the material.

   Final length, $l_f = l_0 + l_0\alpha\,\Delta T$

   $= l_0(1+\alpha\,\Delta T)$

2. **Superficial Expansion or Areal Expansion** Expansion of solids along two dimensions of solid objects is defined as superficial expansion.

   Coefficient of superficial expansion,

   $$\beta=\frac{\Delta A}{A_0\times\Delta T}$$

   Final area, $A_f = A_0(1+\beta\Delta T)$

   where, A_0 is the area of the body at temperature T.

3. **Volume or Cubical Expansion** Expansion of solids along three dimensions of solids objects is defined as cubical expansion.

   Coefficient of volume or cubical expansion,

   $$\gamma=\frac{\Delta V}{V_0\times\Delta T}$$

   Final volume, $V = V_0(1+\gamma\Delta T)$.

   where, V_0 is the volume of the body at temperature T.

NOTE

- The coefficients α, β and γ for a solid are related to each other.

  $$\alpha=\frac{\beta}{2}=\frac{\gamma}{3}$$

- As temperature increases, density decreases according to relation,

  $$\rho=\frac{\rho_0}{1+\gamma\Delta T}$$

  or $$\rho=\rho_0(1-\gamma\Delta T) \qquad \text{[valid for small } \Delta T]$$

Thermal Expansion of Liquid

Liquids do not have linear and superficial expansion but these only have volume expansion.

Liquids have two coefficients of volume expansion

1. Coefficient of apparent expansion,

   $$\gamma_a=\frac{\text{Apparent expansion in volume}}{\text{Initial volume}\times\Delta T}=\frac{(\Delta V)_a}{V\times\Delta T}$$

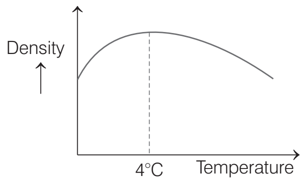
2. Coefficient of real expansion,

   $$\gamma_r=\frac{\text{Real expansion in volume}}{\text{Initial volume}\times\Delta T}=\frac{(\Delta V)_r}{V\times\Delta T}$$

Anomalous/Exceptional Behaviour of Water

Generally, density of liquids decreases with increase in temperature but for water as the temperature increases from 0 to 4°C, its density increases and as temperature increases beyond 4°C, the density decreases.

The variation in the density of water with temperature is shown in the figure given below.



Anomalous behaviour of water

Thermal Expansion of Gases

Gases have no definite shape, therefore, gases have only volume expansion.

1. The coefficient of volume expansion at constant pressure, $\alpha = \frac{\Delta V}{V_0} \times \frac{1}{\Delta T}$

   Final volume, $V' = V(1 + \alpha \Delta T)$
2. The coefficient of pressure expansion at constant volume, $\beta = \frac{\Delta p}{p} \times \frac{1}{\Delta T}$

   Final pressure, $p' = p(1 + \beta \Delta T)$.

Specific Heat Capacity

The quantity of heat required to raise the temperature of unit mass of a substance by 1°C is called specific heat.

Specific heat, $s = \frac{Q}{m \times \Delta T}$

The SI unit of specific heat is J kg^{-1} k^{-1}.

- Specific heat capacity can have any value from 0 to ∞. For some substances under particular situations, it can have negative values also.
- The product of mass of the body and specific heat is termed as **heat capacity**, $C = m \times s$.

Molar Heat Capacity

The amount of heat required to change the temperature of a unit mole of substance by 1°C is termed as its molar heat capacity,

$$C_m = \frac{Q}{\mu \Delta T}$$

Generally, for gases, two molar heat capacities are very common—molar heat capacity at constant pressure (C_p) and molar heat capacity at constant volume (C_V).

Water Equivalent of a Substance

Water equivalent of certain amount of substance is defined as the amount of water, which when replaced by the substance requires the same amount of heat for the same rise in temperature.

$$m_w = \frac{mS}{S_w},$$

where, m_w = water equivalent of substance whose mass is m,

S = specific heat capacity of substance

and S_w = specific heat capacity of water

Calorimetry

Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body and provided no heat is allowed to escape to the surrounding.

A device in which heat measurement can be made is called a calorimeter.

If temperature changes, heat exchanged is given by

$$Q = ms\Delta T$$

As temperature of the body increases, it means heat is taken by the body, otherwise given out by the body.

Change of State

When we supply heat (energy) to a body and its temperature does not change, then the energy consumed by the body is used up in changing its phase and the process is called change of state.

Latent Heat

In case of phase change, heat is consumed during melting and boiling while released during freezing and condensation.

The heat required to change the phase of a system is proportional to the mass of the system i.e.

$$Q \propto m$$

$$Q = mL$$

where, L is the latent heat, which is defined as the amount of heat required to change the phase of the unit mass of a substance at given temperature.

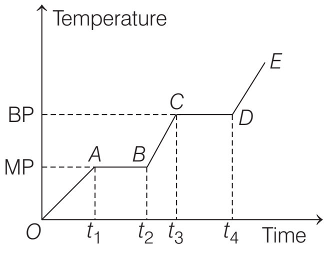
- In case of ice, the latent heat of fusion of ice is 80 cal/gm.
- In case of water, the latent heat of vapourisation is 536 cal/gm.

Sublimation

- A substance can sometimes change directly from solid to gaseous phase, this process is termed as **sublimation**. Corresponding latent heat is termed as **latent heat of sublimation L_s**. The reverse process can also occur.
- Very pure water can be cooled several degrees below the freezing temperature without freezing, the resulting unstable state is described as **supercooled**. When this supercooled water is disturbed (either by dropping dust particles etc.), it crystallises within a second or less.
- A liquid can sometimes be **superheated** above its normal boiling temperature. Any small disturbance such as agitation causes local boiling with bubble formation.

Heating Curve

If we supply energy to a body in solid state (temperature<melting point) at a constant rate, then the curve drawn between temperature and time is termed as the heating curve.



Heating curve

OA represents heating of the solid,

$$S_{\text{solid}} \propto \frac{1}{\text{Slope of } OA}$$

AB represents melting of the solid,

$$\text{length of } AB \propto L_f$$

BC represents heating of the liquid,

$$S_{\text{liquid}} \propto \frac{1}{\text{Slope of } BC}$$

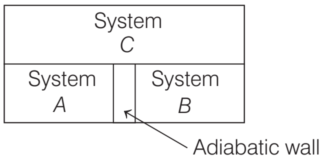
CD represents boiling (vaporisation) of the liquid, length of $CD \propto L_V$, *DE* represents heating of the gaseous phase,

$$S_{\text{gas}} \propto \frac{1}{\text{Slope of } DE}$$

Zeroth Law of Thermodynamics

When there is no exchange of heat between two objects placed in contact, then both are called in **thermal equilibrium**.

According to this law, if two systems A and B , separated by an adiabatic wall, are separately and independently in thermal equilibrium with a third system C, then the systems A and B are also in a state of thermal equilibrium with each other.



Three system of thermal equilibrium

Basic Terms Used Thermodynamics

Internal Energy

Internal energy of a system is defined as the sum of the total kinetic energy of all its constituent particles and sum of all the potential energies of interaction among these particles.

The internal energy of an ideal gas is totally kinetic and it is given by

$$U = \frac{3}{2}\mu RT$$

and change in internal energy

$$\Delta U = \frac{3}{2}\mu R\Delta T.$$

For non-ideal gases, internal energy depends not only on the temperature but also on the pressure.

Work

Consider a system in a cylinder with movable piston, whose volume can be changed (a gas, liquid or solid). Suppose, the cylinder has a cross-sectional area A and pressure exerted by system on the piston face is p. The work done by the system on the surroundings for small displacement dx is $dW = pAdx$.

$$W = \int dW = \int_{V_i}^{V_f} pdV$$

i.e. work done in a finite change of volume from V_i to V_f.

- Work done by the system depends on the initial and final states.
- If volume of the system increases, then work is done by the system and it is taken as positive work done.
- If volume of the system decreases, then work is done on the system and it is taken as negative work done.

First Law of Thermodynamics

According to this law, the heat given to a system (ΔQ) is equal to the sum of increase in its internal energy (ΔU) and the work done (ΔW) by the system against the surroundings.

Mathematically, $\Delta Q = \Delta U + \Delta W$

Sign Convention

$\Delta Q = +$ ve when heat supplied
$= -$ ve when heat is ejected
$\Delta U = +$ ve when temperature increases
$= -$ ve when temperature decreases
$\Delta W = +$ ve when work is done by the system (expansion)
$= -$ ve when work is done on system (compression)

First law of thermodynamics is based on the energy conservation.

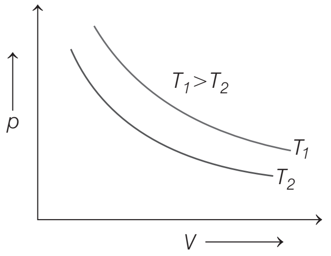
Thermodynamic Processes

A thermodynamic process is the process of change of state of a system involving change of thermodynamic variables, e.g. p, V, T etc. When a system undergo a thermodynamic change, then work done either by system on surrounding or by surroundings on system is called external work.

$$W_{\text{ext}} = \int_{V_1}^{V_2} p\,dV = \text{area under } p\text{-}V \text{ curve.}$$

1. Isothermal Process

It is that process in which temperature remains constant. Here, exchange of heat with the surroundings is allowed.



p-V graph of an isothermal expansion process

As temperature T remains constant in an isothermal process, hence as per Boyle's law

$$p \propto \frac{1}{V} \text{ or } pV = \text{constant}$$

Molar specific heat of a gas under isothermal condition

$$C = \frac{\Delta Q}{m\Delta T} = \frac{\Delta Q}{m\,(0)} = \infty$$

Slope of p-V curve at any point is $\frac{dp}{dV} = -\frac{p}{V}$.

Work done in an isothermal process

$$\Delta W = \int_{V_i}^{V_f} p dV = nRT \ln\left(\frac{V_f}{V_i}\right)$$

where, n = number of moles, R = gas constant

and T = temperature.

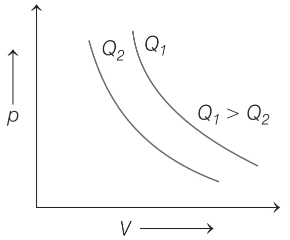
V_f and V_i are final and initial volume of the gas respectively.

As per first law of thermodynamics, since, $\Delta T = 0$, hence, $\Delta U = 0$ for an ideal gas and we have $\Delta Q = \Delta W$.

Thus, heat supplied to the system in an isothermal process is entirely used to do work against external surroundings.

2. Adiabatic Process

It is that process in which there is no exchange of heat of the system with its surroundings. Thus, in an adiabatic process p, V and T change but $\Delta Q = 0$ or entropy remains constant $\left(\Delta S = \frac{\Delta Q}{T} = 0\right)$.



p-V graph for adiabatic process

The equation of state for an adiabatic process is

$$pV^{\gamma} = \text{constant}$$

or $$T\,V^{\gamma - 1} = \text{constant}$$

or $$T^{\gamma} p^{1-\gamma} = \text{constant}$$

- **Molar specific heat** of a gas under adiabatic condition

$$C = \frac{\Delta Q}{m \cdot \Delta T} = \frac{0}{m \cdot \Delta T} = 0$$

- **Slope of an adiabatic curve** at a point is $\frac{dp}{dV} = -\gamma\,\frac{p}{V}$.
- **Work done in an adiabatic process**

$$\Delta W = \int_{V_i}^{V_f} p\,dV = \frac{\mu R}{(\gamma - 1)}\,(T_i - T_f)$$

During an adiabatic expansion $\Delta W = +$ ve, hence, temperature of gas falls, i.e. an adiabatic expansion is always accompanied with cooling.

As per first law of thermodynamics, since, $\Delta Q = 0$ in an adiabatic process hence,

$$\Delta U = -\,\Delta W$$

- **Free expansion** is an adiabatic process in which $\Delta W = 0$.

Hence, in accordance with first law of thermodynamics $\Delta U = 0$ i.e. the final and initial values of the internal energy are equal in free expansion.

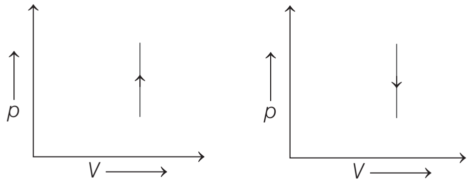
3. Isochoric Process

It is that thermodynamic process in which volume remains constant.

In an isochoric process for a given mass of gas

$$p \propto T \text{ or } \frac{p}{T} = \text{ constant}$$

- **Indicator diagram** for an isochoric process is a straight line parallel to p-axis.



(a) (b)

Graph (a) shows isometric heating graph in which pressure increases, temperature increases, ΔQ is positive and ΔU is positive.

Similarly, Graph (b) shows isometric cooling graph in which pressure decreases, temperature decreases, ΔQ is negative and ΔU is negative.

- **Molar specific heat** of a gas under isochoric condition $C_V = \frac{f}{2}R$, where f is the number of degrees of freedom per molecule.
- **Work done in an isochoric process**

$$\Delta W = \int p\,dV = 0$$

As $\Delta W = 0$ hence, according to first law of thermodynamics, we have

$$(\Delta Q)_V = \Delta U = \mu C_V\,\Delta T = \frac{\mu R}{(\gamma - 1)}\,\Delta T$$

4. Isobaric Process

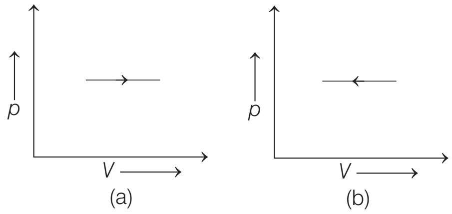
It is that process in which pressure remains constant.

As in an isobaric process for a given mass of gas

$$V \propto T$$

or $$\frac{V}{T} = \text{constant}$$

- **Indicator diagram** for an isobaric process is a straight line parallel to X-axis.



(a) (b)

Graph (a) represent isobaric expansion, graph (b) represent isobaric compression.

- **Work done in an isobaric process**

$$\Delta W = \int p\,dV = p\int_{V_i}^{V_f} dV$$

$$= p(V_f - V_i) = p\Delta V$$

- **Molar specific heat** of a gas under isobaric condition

$$C_p = \left(\frac{f}{2} + 1\right) R = C_V + R$$

Second Law of Thermodynamics

Two most common statements of second law of thermodynamics are given below

Clausius Statement

It is impossible for a self-acting machine, working in a cyclic process to transfer heat from a colder body to a hotter body without the aid of an external agency.

Kelvin-Planck's Statement

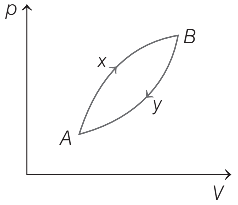
It is impossible to design an engine which extracts heat from a reservoir and fully converts it into work without producing any other effect.

Reversible and Irreversible Processes

A reversible process is one which can be reversed in such a way that all changes taking place in the direct process are exactly repeated in inverse order and in opposite sense, and no changes are left in any of the bodies taking part in the process or in the surroundings. Any process which is not reversible exactly is an irreversible process.

Cyclic Process

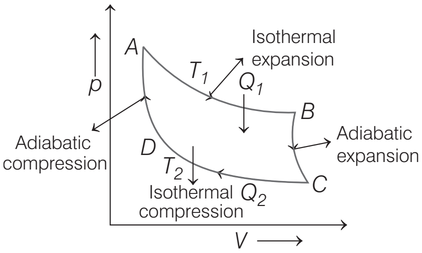
In cyclic process, if the process takes the path AxB, it returns *via* ByA, the initial and final points are same.



p-V graph of cyclic process

Carnot Engine and its Efficiency

Carnot engine is a theoretical, ideal heat engine working in a reversible cyclic process operating between two temperatures T_1 (heat source) and T_2(heat sink). The Carnot's cycle consists of two isothermal processes connected by two adiabatic processes as shown in the figure.



Various process in Carnot cycle

The efficiency of a Carnot's cycle is given by

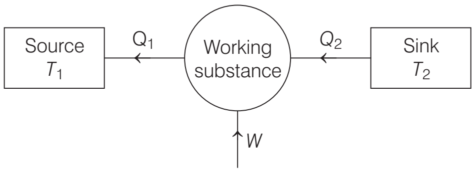
$$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

The efficiency does not depend on the nature or quantity of the working substance.

Refrigerator

A refrigerator or heat pump is basically a heat engine running in reverse direction. It takes heat from colder body (sink) and after doing some work gives the rest heat to the hotter body (source).

An ideal refrigerator can be regarded as Carnot's ideal heat engine working in the reverse direction.



Working of refrigerator

Coefficient of Performance of a Refrigerator

It is defined as the ratio of quantity of heat removed per cycle (Q_2) to the work done (W) on the working substance per cycle to remove this heat.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad \text{or} \quad \beta = \frac{T_2}{T_1 - T_2} = \frac{1-\eta}{\eta}$$

Equation of State of a Perfect Gas

The equation which relates the pressure (P), volume (V) and temperature (T) of the given state of an ideal gas is known as ideal or perfect gas equation.

For 1 mole of gas, $\frac{pV}{T} = R$ (constant)

$$pV = RT$$

where, R is **universal gas constant**. The SI unit of gas constant is J/mol-K. Its value is 8.314 J/mol-K or 8.314×10^7 erg/mol-K or 2 cal/mol-K. The dimensions of R are $[\mathrm{ML^2T^{-1}\theta^{-1}}]$.

Moreover, gas constant $R = \frac{\text{Work}}{\text{Moles} \times \text{Temperature}}$

- The perfect gas equation for 1 molecule of gas is
$$pV = kT$$
- Boltzmann's constant is represented by per mole gas constant

  i.e. $k = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$

  Its dimensions are $[\mathrm{ML^2T^{-2}\theta^{-1}}]$.

Kinetic Theory of Gases

Kinetic theory of gases relates the macroscopic properties of gases (such as pressure, temperature etc.) to the microscopic properties of gas molecules (such as speed, momentum, kinetic energy of molecules etc).

Assumptions of Kinetic Theory of Gases

- Every gas is composed of tiny particles known as molecules. The size of molecules is much smaller than the intermolecular spacing.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- Molecules are in a state of random rapid motion. They collide with each other. There is no loss of energy during collision. Only the direction of motion is changed.
- The time spent in collision between two molecules is negligible in comparison to time between two successive collisions.
- The number of collisions per unit volume in a gas remains constant. No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
- Molecules constantly collide with the walls of container due to which their momentum changes. The change in momentum is transferred to the walls of the container. Consequently, on the walls of container pressure is exerted by gas molecules. The density of gas is constant at all points of the container.

Work Done on Compressing a Gas

Work done $W = p \cdot \Delta V$, where p = pressure of the gas and ΔV = change in volume of the gas.

When two ideal gases having molar masses M_1 and M_2 are mixed, then thermodynamic variables/parameters for mixture would be given by

- M (molar mass) $= \frac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$
- C_V (of the mixture) $= \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$
- C_p (of the mixture) $= \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$
- γ (of the mixture) $= \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$ or is given by

  $\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

  where, symbols have their usual meanings.

Kinetic Energy and Temperature

In ideal gases, the point particles can have only translational motion and thus only translational energy.

- Translational KE of a molecule $= \frac{1}{2} m\overline{c^2} = \frac{3}{2} kT$
- Mean KE per molecule $= \frac{3}{2} kT$
- Mean kinetic energy per gram mole is given by
$$\mathrm{KE_{mole}} = \left(\frac{1}{2} m\overline{c^2}\right) N = \frac{3}{2} kTN = \frac{3}{2} RT$$
- Average kinetic energy of gas $= \frac{3}{2} pV$
- KE per molecule $= \frac{3pV}{2N} = \frac{3RT}{2N}$
- KE per mole $= \frac{3}{2} kT$
- KE per volume $= \frac{3}{2} p$

Concept of Pressure

Pressure p exerted by a perfect gas on the walls of container is given by

$$p = \frac{1}{3} \frac{mN\overline{c^2}}{V} = \frac{1}{3} \frac{M}{V} \overline{c^2}$$

$$pV = \frac{1}{3}(\rho\overline{c^2}) = \frac{1}{3}\left(\frac{2}{2}\rho\overline{c^2}\right) = \frac{2}{3}\left(\frac{1}{2}\rho\overline{c^2}\right) = \frac{2}{3} E$$

Here, m = mass of each molecule, $\bar{c}$ = root mean square velocity of molecules, ρ = density of gas, M = mass of gas enclosed in volume V of container, and E = Total KE of the ideal gas.

Various Speeds of Gas Molecules

- **Root mean square speed** It is defined as the square root of mean of squares of the speed of different molecules

  i.e. $$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots}{N}} = \sqrt{\overline{v^2}}$$

  From the expression of pressure, $p = \frac{1}{3}\rho v_{\text{rms}}^2$

  $$v_{\text{rms}} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3pV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

- **Most probable speed** It is defined as the speed which is possessed by maximum fraction of total number of molecules of the gas.

  $$v_{\text{mp}} = \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

- **Average speed** It is the arithmetic mean of the speeds of molecules in a gas at given temperature.

  $$v_{\text{av}} = \frac{v_1 + v_2 + v_3 + v_4 + \dots}{N}$$

  Average speed, $v_{\text{av}} = \sqrt{\frac{8p}{\pi\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8kT}{\pi m}}$

Degree of Freedom (f)

The term degree of freedom of a system refers to the possible independent motions a system can have

- for monoatomic gas, $(f) = 3$
- for diatomic gas, $(f) = 5$
- for triatomic gas, $(f) = 6$(non-linear)
- for triatomic (linear) gas, $(f) = 7$
- for N-atomic molecule $(f) = 6N - 3$
- for N-atomic linear molecule $(f) = 6N - 5$

Law of Equipartition of Energy

According to law of equipartition of energy for any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom and each degree of freedom is associated with energy $\frac{1}{2}kT$

(where, $k = 1.38 \times 10^{-23}$ J/K and T = absolute temperature of the system).

Specific Heat Capacities of Gases

The specific heat of gas can have many values, but out of them following two values are important

Specific Heat at Constant Volume

The specific heat of a gas at constant volume is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1°C or 1 K when its volume is kept constant i.e.

$$c_V = \frac{(\Delta Q)_V}{m\Delta T}$$

For one mole of gas,

$$C_V = Mc_V = \frac{M\,(\Delta Q)_V}{m\Delta T} = \frac{1}{n}\frac{(\Delta Q)_V}{\Delta T} \qquad \left[\because n = \frac{m}{M}\right]$$

Specific Heat at Constant Pressure

The specific heat of a gas at constant pressure is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1K, when its pressure is kept constant i.e.

$$c_p = \frac{(\Delta Q)_p}{m\Delta T}.$$

For one mole of gas,

$$\therefore \quad C_p = Mc_p = \frac{M\,(\Delta Q)_p}{m\Delta T} \quad \dots\text{(i)}$$

$$= \frac{1}{n}\frac{(\Delta Q)_p}{\Delta T} \qquad [\because n = m/M]$$

Specific heat of a gas at constant pressure is greater than the specific heat at constant volume i.e. $C_p > C_V$.

C_p and C_V are related to each other according to relation,

$$C_p - C_V = \frac{R}{J} \quad \dots\text{(ii)}$$

Eq. (ii) is called Mayer's relation. If C_p and C_V are measured in the units of work and R is also in the units of work (or energy), then Eq. (ii) becomes $C_p - C_V = R$.

Specific Heat in Terms of Degree of Freedom

For a gas at temperature T, the internal energy $U = \frac{f}{2}nRT$.

Change in energy, $\Delta U = \frac{f}{2}nR\Delta T \Rightarrow C_V = \frac{1}{2}fR$

Specific heat at constant pressure, $C_p = \left(\frac{f}{2} + 1\right)R$.

Ratio of C_p and C_V, $\gamma = \frac{C_p}{C_V} = \frac{\left(\frac{f}{2} + 1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$.

Mean Free Path

The distance travelled by a gas molecule between two successive collision is known as free path.

$$\text{Mean free path} = \frac{\text{Total distance covered}}{\text{Number of collisions}}$$

The mean free path of a gas molecule is the average distance between two successive collisions. It is represented by λ.

$$\lambda = \frac{1}{\sqrt{2}\,\pi\,\sigma^2 n}$$

Here, σ = diameter of the molecule and n = number of molecules per unit volume.

Avogadro's Number

According to Avogadro's hypothesis, gram atomic masses of all elements contain the same number of atoms and this number is called Avogadro's number (N_A) and its value is 6.02×10^{23}.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23 \times 10^{-6}$ /°C) **→ AIEEE 2011**

(a) 28.9 cc (b) 2.89 cc
(c) 9.28 cc (d) 49.8 cc

2 C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p - C_V = a$ for hydrogen gas $C_p - C_V = b$ for nitrogen gas. The correct relation between a and b is **→ JEE Main 2017**

(a) $a = b$ (b) $a = 14\,b$
(c) $a = 28\,b$ (d) $a = \frac{1}{14}\,b$

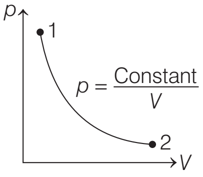
3 A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is (Take, room temperature = 30°C, specific heat of copper = 0.1 cal/g°C)

(a) 885°C (b) 1250°C
(c) 825°C (d) 800°C

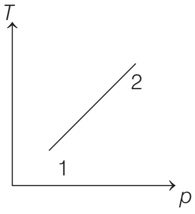
4 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (Take, specific heat of water is 4184 J/kg/K) **→ AIEEE 2011**

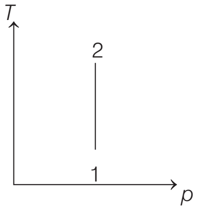
(a) 8.4 kJ (b) 84 kJ
(c) 2.1 kJ (d) 4.2 kJ

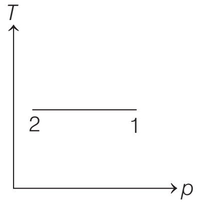
5 Consider p-V diagram for an ideal gas shown in figure.

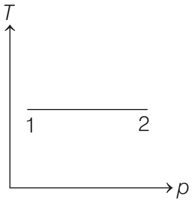


Out of the following diagrams which represents the T-p diagram?

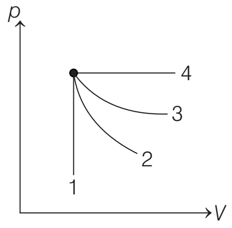
(a)


(b)


(c)


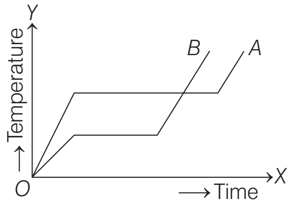
(d)


6 An ideal gas undergoes four different processes from the same initial state. Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1, 2, 3 and 4 which one is adiabatic.



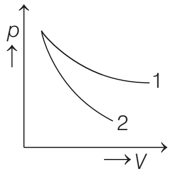
(a) 4 (b) 3 (c) 2 (d) 1

7 Equal masses of two liquids A and B contained in vessels of negligible heat capacity are supplied heat at the same rate. The temperature-time graphs for the two liquids are shown in the figure. If S represents specific heat and L represents latent heat of liquid, then



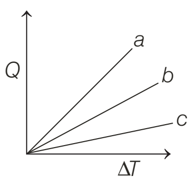
(a) $S_A > S_B$; $L_A < L_B$
(b) $S_A > S_B$; $L_A > L_B$
(c) $S_A < S_B$; $L_A < L_B$
(d) $S_A < S_B$; $L_A > L_B$

8 p-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



(a) He and O_2 (b) O_2 and He (c) He and Ar (d) O_2 and N_2

9 Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic (P) gas. The initial state of all the gases are the same and the scale for the axes coincide, ignoring vibrational degrees of freedom, the lines a, b and c respectively, correspond to

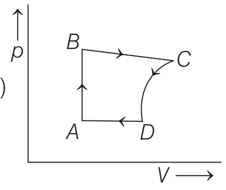


➞ **JEE Main (Online) 2013**

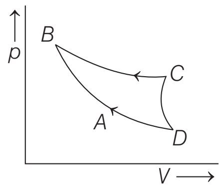
(a) P, M and D (b) M, D and P
(c) P, D and M (d) D, M and P

10 A certain amount of gas is taken through a cyclic process $(A\,B\,C\,D\,A)$ that has two isobars, one isochoric and one isothermal. The cycle can be represented on a p-V indicator diagram as ➞ **JEE Main (Online) 2013**

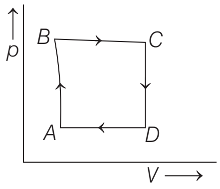
(a)



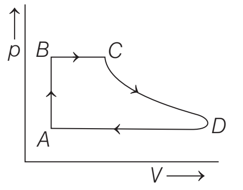
(b)



(c)



(d)



11 An ideal Carnot engine whose efficiency is 40%, receives heat at 500 K. If the efficiency is to be 50%, the intake temperature for the same exhaust temperature is

(a) 600 K (b) 900 K
(c) 700 K (d) 800 K

12 The pressure inside a tyre is 4 atm at 27°C. If the tyre bursts suddenly, its final temperature will be $\left(\text{Given, } r = \frac{7}{5}\right)$

(a) $300(4)^{7/2}$ (b) $300(4)^{2/7}$
(c) $300(2)^{7/2}$ (d) $300(4)^{-2/7}$

13 A refrigerator works between the temperature of melting ice and room temperature (17°C). The amount of energy (in kWh) that must be supplied to freeze 1kg of water at 0°C is

(a) 1.4 (b) 1.8 (c) 0.058 (d) 2.5

14 A gas expands with temperature according to the relation $V = kT^{2/3}$. Calculate the work done when the temperature changes by 60 K ?

(a) 10 R (b) 30 R
(c) 40 R (d) 20 R

15 A Carnot engine takes 3×10^6cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is

(a) 4.2×10^6 J (b) 8.4×10^6 J (c) 16.8×10^6 J (d) 3×10^6 J

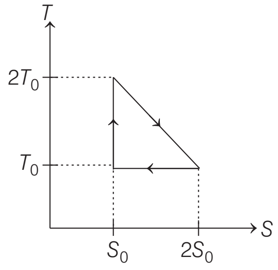
16 A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then, T_1 and T_2 are respectively ➞ **AIEEE 2011**

(a) 372 K and 330 K (b) 330 K and 268 K
(c) 310 K and 248 K (d) 372 K and 310 K

17 A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be ➞ **AIEEE 2012**

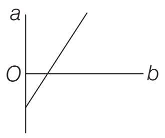
(a) efficiency of Carnot engine cannot be made larger than 50%
(b) 1200 K
(c) 750 K
(d) 600 K

18 The temperature-entropy diagram of a reversible engine is given in the figure. Its efficiency is



(a) 1/4 (b) 1/2
(c) 1/3 (d) 2/3

19 The expansion on of unit mass of a perfect gas at constant pressure is shown below.



(a) a = volume, b = °C temperature
(b) a = volume, b = K temperature
(c) a = °C temperature, b = volume
(d) a = K temperature, b = volume

20 The temperature of an open room of volume $30\,\text{m}^3$ increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be **→ JEE Main 2017**

(a) 1.38×10^{23}
(b) 2.5×10^{25}
(c) -2.5×10^{25}
(d) -1.61×10^{23}

21 Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion, the relation between T and R is **→ JEE Main 2015**

(a) $T \propto e^{-R}$
(b) $T \propto e^{-3R}$
(c) $T \propto \frac{1}{R}$
(d) $T \propto \frac{1}{R^3}$

22 A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by **→ AIEEE 2011**

(a) $\frac{(\gamma - 1)}{2\gamma R} Mv^2 K$
(b) $\frac{\gamma M v^2}{2R} K$
(c) $\frac{(\gamma - 1)}{2R} Mv^2 K$
(d) $\frac{(\gamma - 1)}{2(\gamma + 1)R} Mv^2 K$

23 Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3, respectively. Assuming no loss of energy , the final temperature of the mixture is **→ AIEEE 2011**

(a) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$
(b) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
(c) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$
(d) $\frac{(T_1 + T_2 + T_3)}{3}$

24 The value of molar specific heat at constant volume for 1 mole of polyatomic gas having n number of degrees of freedom at temperature T K is
(here, R = universal gas constant)

(a) $\frac{nR}{2T}$
(b) $\frac{nR}{2}$
(c) $\frac{nRT}{2}$
(d) $2nRT$

25 p-V diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be

(a) $4R$
(b) $3R$
(c) $4R/3$
(d) $2.5R$

Direction (Q. Nos. 26-30) *Each of these questions contains two statements : Statement* I and Statement II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below:*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

26 **Statement I** Work done by a gas in isothermal expansion is more than the work done by the gas in the same expansion adiabatically.

Statement II Temperature remains constant in isothermal expansion, but not in adiabatic expansion.

27 **Statement I** When 1 g of water at 100°C is converted to steam at 100°C, the internal energy of the system does not change.

Statement II From $dU = nC_V\,dT$, if temperature of the system remains constant, then $dU = 0$, i.e. internal energy remains constant.

28 **Statement I** In an isothermal process (quasistatic), the heat exchange between the system and surroundings takes place even though the gas has the same temperature as that of the surrounding.

Statement II There is an infinitesimal difference in temperature between the system and the surroundings.

29 **Statement I** A special type of thermometer (used to measure very high temperatures and calibrated for an ideal black body) measures a value lower than the actual value of the temperature of a red hot iron piece kept in open.

Statement II As the iron piece is kept in open, it loses its heat.

30 **Statement I** The internal energy of a perfect gas is entirely kinetic and depends only on absolute temperature of the gas and not on its pressure or volume.
Statement II A perfect gas is heated keeping pressure constant and later at constant volume. For the same amount of heat the temperature of the gas at constant pressure is lower than that at constant volume.
→ JEE Main (Online) 2013

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways.

(i) Sequentially keeping in contact with 2 reservoirs such that, each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases, body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively, is

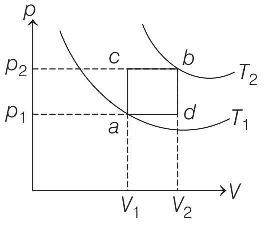
→ **JEE Main 2015**

(a) ln2, 4ln2
(b) ln2, ln2
(c) ln2, 2ln2
(d) 2ln2, 8ln2

2 Consider a collection of a large number of particles each with speed v. The direction of velocity is randomly distributed in the collection. What is the magnitude of the relative velocity between a pairs in the collection?

(a) $\frac{2v}{\pi}$ (b) $\frac{v}{\pi}$
(c) $\frac{8v}{\pi}$ (d) $\frac{4v}{\pi}$

3 An ideal gas (molar specific heat $C_V = 5R/2$) is taken along paths acb, adb and ab, $p_2 = 2p_1, V_2 = 2V_1$. Along ab, $p = kV$, where k is a constant. The various parameters are shown in the figure. Match the Column I with the corresponding options of Column II and mark the correct option from the codes given below.



	Column I		Column II
A.	W_{acb}	1.	$15RT_1/2$
B.	W_{adb}	2.	$-15RT_1/2$
C.	ΔU_{ab}	3.	RT_1
D.	ΔU_{bca}	4.	$2RT_1$

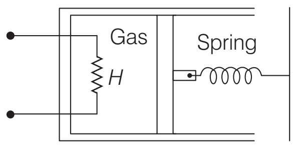
Codes

	A	B	C	D		A	B	C	D
(a)	3	4	1	2	(b)	2	3	4	1
(c)	1	2	3	4	(d)	4	3	1	2

4 Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory $pV = \frac{2}{3}E, E$ is

(a) the total energy per unit volume
(b) only the translational part of energy, because rotational energy is very small compared to the translational energy
(c) only the translational part of the energy, because during collisions with the wall, pressure related to change in linear momentum
(d) the translational part of the energy, because rotational energies of molecules can be of either sign and its average over all the molecules is zero

5 An ideal monoatomic gas is confined in a cylinder by a spring-loaded piston of cross-section $8 \times 10^{-3} \text{m}^2$. Initially, the gas is at 300 K and occupies a volume of $2.4 \times 10^{-3} \text{m}^3$ and the spring is in a relaxed state. The gas is heated by a small heater coil H. The force constant of the spring is 8000 Nm^{-1} and the atmospheric pressure is 1×10^5 Pa. The cylinder and piston are thermally insulated. The piston and the spring are massless and there is no friction between the piston and cylinder. There is no heat loss through heater coil wire and thermal capacity of the heater coil is negligible. With all the above assumptions, if the gas is heated by the heater until the piston moves out slowly by 0.1 m, then the final temperature is



(a) 400 K (b) 800 K
(c) 1200 K (d) 300 K

6 A diatomic ideal gas is used in a car engine as the working substance. If during the adiabatic expansion part of the cycle, volume of the gas increases from V to $32V$, the efficiency of the engine is → **AIEEE 2010**

(a) 0.5 (b) 0.75
(c) 0.99 (d) 0.25

7 The specific heat capacity of a metal at low temperature (T) is given as $C_p(\text{kJK}^{-1}\text{ kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$. A 100 g vessel of this metal is to be cooled from 20 K to 4 K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is **→ AIEEE 2011**

(a) equal to 0.002 kJ
(b) greater than 0.148 kJ
(c) between 0.148 kJ and 0.028 kJ
(d) less than 0.028 kJ

8 A horizontal cylinder with adiabatic walls is closed at both ends and is divided into two parts by a frictionless piston that is also insulating. Initially, the value of pressure and temperature of the ideal gas on each side of the cylinder are V_0, p_0 and T_0, respectively. A heating coil in the right-hand part is used to slowly heat the gas on that side until the pressure in both parts reaches $64p_0/27$. The heat capacity C_V of the gas is independent of temperature and $C_p/C_V = \gamma = 1.5$.

Take, $V_0 = 16\,\text{m}^3$, $T_0 = 324$ K, $p_0 = 3\times10^5$ Pa

Column I represents the physical parameters of the gas, Column II gives their corresponding values, match the Column I with Column II and mark the correct option from the codes given below.

	Column I		Column II
A.	Final left-hand volume (in m^3)	1.	432
B.	Final left-hand temperature (in K)	2.	9
C.	Final right-hand temperature (in K)	3.	1104
D.	Work done (in kJ) on the left-hand gas	4.	3200

Codes

	A	B	C	D		A	B	C	D
(a)	2	1	3	4	(b)	1	2	3	4
(c)	4	1	2	3	(d)	3	4	1	2

9 The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal and rebound classically with a speed of 10^3 m/s, then the pressure on the wall is nearly **→ JEE Main 2018**

(a) $2.35\times10^3\text{ N/m}^2$ (b) $4.70\times10^3\text{ N/m}^2$
(c) $2.35\times10^2\text{ N/m}^2$ (d) $4.70\times10^2\text{ N/m}^2$

10 Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas and (ii) change in its internal energy. **→ JEE Main 2018**

(a) (i) 189 K (ii) 2.7 kJ (b) (i) 195 K (ii) –2.7 kJ
(c) (i) 189 K (ii) –2.7 kJ (d) (i) 195 K (ii) 2.7 kJ

11 An external pressure p is applied on a cube at 0°C, so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by **→ JEE Main 2017**

(a) $\frac{p}{\alpha K}$ (b) $\frac{3\alpha}{pK}$ (c) $3pK\alpha$ (d) $\frac{p}{3\alpha K}$

12 Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma = \frac{C_p}{C_V}\right)$ **→ JEE Main 2015**

(a) $\frac{3\gamma+5}{6}$ (b) $\frac{3\gamma-5}{6}$ (c) $\frac{\gamma+1}{2}$ (d) $\frac{\gamma-1}{2}$

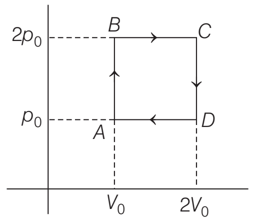
13 A pendulum clock loses 12 s a day, if the temperature is 40°C and gains 4 s in a day, if the temperature is 20°C. The temperature at which the clock will show correct time and the coefficient of linear expansion (α) of the metal of the pendulum shaft are respectively **→ JEE Main 2016**

(a) 25°C, $\alpha = 1.85\times10^{-5}/^\circ\text{C}$ (b) 60°C, $\alpha = 1.85\times10^{-4}/^\circ\text{C}$
(c) 30°C, $\alpha = 1.85\times10^{-3}/^\circ\text{C}$ (d) 55°C, $\alpha = 1.85\times10^{-2}/^\circ\text{C}$

14 An ideal gas undergoes a quasistatic, reversible process in which its molar heat capacity C remains constant. If during this process, the relation of pressure p and volume V is given by pV^n = constant, then n is given by (Here, C_p and C_V are molar specific heat at constant pressure and constant volume, respectively) **→ JEE Main 2016**

(a) $n = \frac{C_p}{C_V}$ (b) $n = \frac{C - C_p}{C - C_V}$ (c) $n = \frac{C_p - C}{C - C_V}$ (d) $n = \frac{C - C_V}{C - C_p}$

15 Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly (assume the gas to be close to ideal gas) **→ AIEEE 2012**

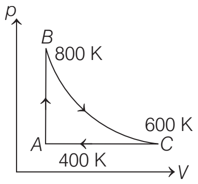


(a) 15.4% (b) 9.1% (c) 10.5%2 (d) 12.5%

16 An ideal gas is taken from the state A (pressure p, volume V) to the state B (pressure $p/2$, volume $2V$), a long straight line path in the p-V diagram. Select the correct statement from the following.

(a) The work done by the gas in the process A to B, exceeds the work that would be done by it, if system were taken along the isothermal
(b) In the T-V diagram, the path AB becomes a part of a hyperbola
(c) In the p-T diagram, the path AB becomes a part of a hyperbola
(d) In going from A to B, the temperature T of the gas first decreases to a minimum value and then increases

17 One mole of diatomic ideal gas undergoes a cyclic process *ABC* as shown in the figure. The process *BC* is adiabatic. The temperatures at *A*, *B* and *C* are 400 K, 800 K and 600 K, respectively. Choose the correct statement. **→ JEE Main 2014**

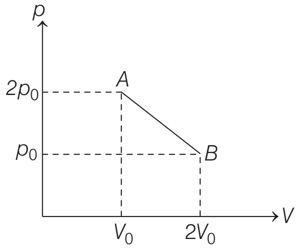


(a) The change in internal energy in whole cyclic process is 250 *R*
(b) The change in internal energy in the process *CA* is 700 *R*
(c) The change in internal energy in the process *AB* is −350 *R*
(d) The change in internal energy in the process *BC* is −500 *R*

18 Two moles of helium are mixed with *n* moles of hydrogen. The root mean square speed of the gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. Then, value of *n* is

(a) 1 (b) 3/2 (c) 2 (d) 3

19 *n* moles of an ideal gas undergoes a process *A* and *B* as shown in the figure. The maximum temperature of the gas during the process will be **→ JEE Main 2016**



(a) $\frac{9}{4}\frac{p_0V_0}{nR}$ (b) $\frac{3}{2}\frac{p_0V_0}{nR}$ (c) $\frac{9}{2}\frac{p_0V_0}{nR}$ (d) $\frac{9p_0V_0}{nR}$

ANSWERS

SESSION 1	**1** (a)	**2** (b)	**3** (a)	**4** (a)	**5** (c)	**6** (c)	**7** (d)	**8** (b)	**9** (c)	**10** (c)
	11 (a)	**12** (d)	**13** (c)	**14** (c)	**15** (b)	**16** (d)	**17** (c)	**18** (c)	**19** (c)	**20** (c)
	21 (c)	**22** (c)	**23** (a)	**24** (b)	**25** (b)	**26** (a)	**27** (a)	**28** (a)	**29** (c)	**30** (b)
SESSION 2	**1** (b)	**2** (b)	**3** (d)	**4** (c)	**5** (b)	**6** (b)	**7** (c)	**8** (a)	**9** (a)	**10** (c)
	11 (d)	**12** (c)	**13** (a)	**14** (b)	**15** (a)	**16** (a)	**17** (d)	**18** (c)	**19** (a)	

Hints and Explanations

SESSION 1

1 Cubical expansion, we get

$$\Delta V = \gamma V\Delta T = 3\alpha V\Delta T$$
$$= 3 \times 23 \times 10^{-6} \times \left(\frac{4}{3}\pi (10)^3\right) \times 100 \quad \left(\because r = \frac{d}{2} = 10\text{cm}\right)$$
$$= 28.9 \text{ cc}$$

2 By Mayer's relation, for 1 g mole of a gas,

$$C_p - C_V = R$$

So, when *n* gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2}; \text{for } H_2 \quad \ldots \text{(i)}$$
$$b = C_p - C_V = \frac{R}{28}; \text{for } N_2 \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$a = 14b$$

3 Heat gained (water + calorimeter)

= Heat lost by copper ball

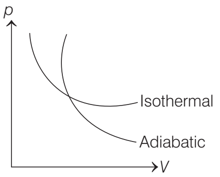
$$\Rightarrow m_w s_w\Delta T + m_c s_c\Delta T = m_B s_B\Delta T$$
$$\Rightarrow 170 \times 1 \times (75 - 30) + 100 \times 0.1 \times (75 - 30) = 100 \times 0.1 \times (T - 75)$$
$$\therefore T = 885°\text{C}$$

4 As, work done = 0

$$\therefore \Delta U = mC\Delta T$$
$$= 100 \times 10^{-3} \times 4184 \times (50 - 30)$$
$$= 8.4 \text{ kJ}$$

5 In the diagram, T is constant and $p_1 > p_2$. This situation is represented by curve (c). In the solution figure, in which $p_1 > p_2$ and straight line parallel to pressure axis represents constant temperature.

6



Slope of *p*-*V* curve

Isothermal process, $\frac{dp}{dV} = -\frac{p}{V}$

Adiabatic process, $\frac{dp}{dV} = -\gamma\frac{p}{V}$

Thus, (c) is correct.

7 As temperature of A rises faster than the temperature of B, therefore specific heat of A is less than that of B, i.e. $S_A < S_B$. Horizontal portions of graphs represent conversion of liquid into vapours. The horizontal portion is larger for liquid A, therefore $L_A > L_B$.

8 As it is clear from the figure,

Slope of curve 2 > Slope of curve 1

$$(\gamma p)_2 > (\gamma p)_1$$

$$\Rightarrow \quad \gamma_2 > \gamma_1$$

$$\Rightarrow \quad \gamma_{\text{He}} > \gamma_{O_2}$$

Adiabatic curve 2 corresponds to helium and adiabatic curve 1 corresponds to oxygen.

9 We know that,

$$Q = C_p \Delta T$$

$\because$ Degree of freedom $\propto C_p$

So, slope is higher for higher degree of freedom.

10 From given figure, in processes BC and DA, pressure of gas is constant, hence these represent isobaric process.

In process CD, volume is constant, therefore it represents isochoric process.

In process AB, temperature is constant, so it represent isothermal process.

11 As, $\quad \eta = 1 - \dfrac{T_2}{T_1}$

$$\Rightarrow \quad \frac{T_2}{T_1} = 1 - \eta$$

$$\Rightarrow \quad \frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$$

$$\Rightarrow \quad T_2 = 300\text{ K}$$

Now, $\dfrac{T_2}{T_1'} = 1 - \eta'$

$$= 1 - \frac{50}{100} = \frac{1}{2}$$

$$\Rightarrow \quad T_1' = 2T_2 = 2 \times 300$$

$$= 600\text{ K}$$

12 In an adiabatic process,

$$p_2^{(1-\gamma)} T_2^{\gamma} = p_1^{(1-\gamma)} T_1^{\gamma}$$

$$T_2 = T_1 \left(\frac{p_1}{p_2}\right)^{(1-\gamma)/\gamma}$$

$$= 300 \left(\frac{4}{1}\right)^{\frac{(1-7/5)}{7/5}} = 300\,(4)^{-2/7}$$

13 $T_2 = 0°\text{C} = 273\text{ K}, \; T_1 = 17°\text{C} = 17 + 273 = 290\text{ K}$

$$\text{Coefficient of performance} = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\frac{80 \times 1000 \times 4.2}{W} = \frac{273}{290 - 273} = \frac{273}{17}$$

$$\therefore \quad W = \frac{80 \times 1000 \times 4.2 \times 17}{273}\text{ J}$$

$$\text{or} \quad W = \frac{33.6 \times 17 \times 10^4}{273 \times 3.6 \times 10^5}\text{ kWh}$$

$$= 0.058\text{ kWh}$$

14 $dW = p\,dV = \dfrac{RT}{V}\,dV \qquad \ldots\text{(i)}$

As, $V = kT^{2/3}$,

$$dV = k\,\frac{2}{3}\,T^{-1/3}\,dT$$

$$\because \quad \frac{dV}{V} = \frac{k\,\frac{2}{3}\,T^{-1/3}\,dT}{kT^{2/3}} = \frac{2}{3}\frac{dT}{T}$$

From Eq. (i), we get

$$W = \int_{T_1}^{T_2} RT\,\frac{dV}{V} = \int_{T_1}^{T_2} RT\,\frac{2}{3}\,\frac{dT}{T}$$

$$W = \frac{2}{3}R\,(T_2 - T_1) = \frac{2}{3}R \times 60 = 40R$$

15 Here, $T_1 = 627°\text{C}$

$$= 627 + 273 = 900\text{ K}$$

$$T_2 = 27°\text{C} = 27 + 273 = 300\text{ K},$$

$$Q_1 = 3 \times 10^6\text{ cal}$$

$$\because \quad \frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{300}{900} = \frac{1}{3} \Rightarrow Q_2 = \frac{Q_1}{3}$$

$$\therefore \quad W = Q_1 - Q_2 = Q_1 - \frac{Q_1}{3}$$

$$= \frac{2}{3}Q_1 = \frac{2}{3} \times 3 \times 10^6\text{ cal}$$

$$W = 2 \times 10^6\text{ cal}$$

$$= 8.4 \times 10^6\text{ J}$$

16 $\because \eta_1 = 1 - \dfrac{T_2}{T_1}$

$$\Rightarrow \quad \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{5}{6} \qquad \ldots\text{(i)}$$

$$\therefore \quad \eta_2 = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \quad \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} \qquad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$T_1 = 372\text{ K and } T_2 = 310\text{ K}$$

17 Efficiency, $\eta = 1 - \dfrac{T_{\text{sink}}}{T_{\text{source}}}$

Now, $\quad 0.4 = 1 - \dfrac{T_{\text{sink}}}{500\text{ K}}$

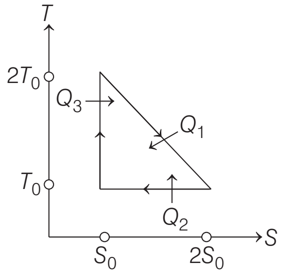
$$\Rightarrow \quad T_{\text{sink}} = 0.6 \times 500\text{ K} = 300\text{ K}$$

Thus, $\quad 0.6 = 1 - \dfrac{300\text{ K}}{T'_{\text{source}}}$

$$\Rightarrow \quad T'_{\text{source}} = \frac{300\text{ K}}{0.4} = 750\text{ K}$$

18 We have,

$$Q_1 = T_0 S_0 + \frac{1}{2} T_0 S_0 = \frac{3}{2} T_0 S_0$$



$$Q_2 = T_0 S_0, Q_3 = 0$$

$$\Rightarrow \quad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

19 In the given graph, line has positive slope with the X-axis and negative intercept on the Y- axis.

So, we can write the equation of lines as

$$y = mx - C \qquad \ldots\text{(i)}$$

According to Charles' law,

$$V_t = \frac{V_0}{273}t + V_0$$

By rewriting this equation, we get

$$t = \left(\frac{273}{V_0}\right)V_t - 273 \qquad \ldots\text{(ii)}$$

By comparing Eqs. (i) and (ii), we can say that temperature is represented on the Y-axis and volume on the X-axis.

20 From $pV = nRT = \dfrac{N}{N_A}RT$

We have,

$$n_f - n_i = \frac{pVN_A}{RT_f} - \frac{pVN_A}{RT_i}$$

$$\Rightarrow \quad n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \cdot \left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -\,2.5 \times 10^{25}$$

$$\therefore \quad \Delta n = -\,2.5 \times 10^{25}$$

21 According to question,

$$p = \frac{1}{3}\left(\frac{U}{V}\right)$$

$$\Rightarrow \quad \frac{nRT}{V} = \frac{1}{3}\left(\frac{U}{V}\right) \qquad [\because pV = nRT]$$

$$\text{or} \quad \frac{nRT}{V} \propto \frac{1}{3}T^4$$

$$\text{or} \quad VT^3 = \text{constant}$$

$$\text{or} \quad \frac{4}{3}\pi R^3 T^3 = \text{constant}$$

$$\text{or} \quad TR = \text{constant} \Rightarrow T \propto \frac{1}{R}$$

22 As no heat is lost.

Loss of kinetic energy

= Gain of internal energy of gas

$$\frac{1}{2}mv^2 = nC_V\Delta T$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{m}{M}\cdot\frac{R}{\gamma-1}\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}K$$

23 $\frac{F}{2}n_1kT_1 + \frac{F}{2}n_2kT_2 + \frac{F}{2}n_3kT_3$

$$= \frac{F}{2}(n_1+n_2+n_3)kT$$

$$\Rightarrow \quad T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1+n_2+n_3}$$

24 According to law of equipartition of energy, average KE per molecule per degree of freedom at temperature T is $\frac{1}{2}kT$. The average KE per molecule of polyatomic gas molecule $= \frac{n}{2}kT$

The average KE per molecule of polyatomic gas

$$(E) = \frac{n}{2}kT \times N = \frac{n}{2}RT$$

$$C_V = \frac{d}{dt}\left(\frac{n}{2}RT\right) = \frac{n}{2}R$$

25 As p-V diagram is a straight line passing through origin, therefore $p \propto V$ or pV^{-1} = constant.

In the process, pV^x = constant, molar heat capacity is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x}$$

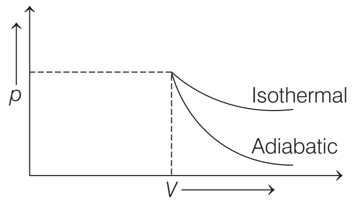
where, $x = -1$ and $\gamma = 1.4$ for diatomic gas

$$C = \frac{R}{1.4-1} + \frac{R}{1-(-1)}$$

$$= \frac{5}{2}R + \frac{R}{2}$$

$$\therefore \quad C = 3R$$

26



The slope of adiabatic curve is several times the slope of an isothermal curve and slope of both is negative. Thus, area under adiabatic curve is smaller than that under isothermal curve.

27 Since, 1 g of water is converted into steam at constant temperature of 100°C, i.e. $dT = 0$.

$\therefore$ Change in internal energy,

$$dU = nC_VdT = 0 \text{ i.e. } U = \text{constant}$$

28 In isothermal process, the heat exchange between system and surrounding at constant temperature, i.e. there is an infinitesimal difference in temperature between the system and the surrounding.

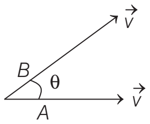
29 Since, the thermometer is caliberated with an ideal black body, the body that emits or absorbs all the radiations falling on it, shows a lower value of temperature. This is because iron is not a black body, i.e. does not absorb/emit all radiation falling on it.

30 The external energy depends upon absolute temperature of gas. Also, Statement II is correct, but both the statements are independently true.

SESSION 2

1 Since, entropy is a state function, therefore change in entropy in both the processes must be same.

2 Consider any two particles having angle θ between directions of their velocities.



Then, $\vec{v}_{\text{rel}} = \vec{v}_B - \vec{v}_A$

i.e. $v_{\text{rel}} = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$

$$= \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$$

So, average v_{rel} over all pairs

$$v_{\text{rel}} = \frac{\int_0^{2\pi} v_{\text{rel}}\,d\theta}{\int_0^{2\pi} d\theta} = \frac{\int_0^{2\pi} 2v\sin\frac{\theta}{2}\,d\theta}{\int_0^{2\pi} d\theta}$$

$$= \frac{2v \times 2[-\cos(\theta/2)]_0^{2\pi}}{2\pi} = \frac{4v}{\pi} > v$$

3 A → 4.; B → 3.; C → 1.; D → 2.

$$W_{acb} = W_{ac} + W_{cb} = 0 + p_2(V_2 - V_1)$$

$$= p_2V_1 = 2p_1V_1 = 2RT_1$$

$$W_{adb} = W_{ad} + W_{db}$$

$$= p_1(V_2 - V_1) + 0 = p_1V_1 = RT_1$$

$$\Delta U_{ab} = U_{ac} + U_{cb}$$

$$= (Q_{ac} - W_{ac}) + (Q_{cb} - W_{cb})$$

$$C_V(T_c - T_1) + C_V(T_2 - T_c) = \frac{5R}{2}(T_2 - T_1)$$

$$C_V = \frac{5R}{2} \text{ (given)}$$

For an ideal gas,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \Rightarrow T_2 = 4T_1$$

$$\Delta U_{ab} = \frac{5R}{2}(4T_1 - T_1) = \frac{15RT_1}{2}$$

For the process,

$$\Delta U_{bca} = -\Delta U_{ab} = \frac{-15RT_1}{2}$$

4 In the relation $pV = \frac{2}{3}E$, E is only the translational part of energy of molecules. This is because during collision of molecules with the walls, pressure exerted relates to change in linear momentum of gas molecules.

5 $V_1 = 2.4 \times 10^{-3}\,\text{m}^3$, $p_1 = p_0 = 10^5\,\text{N m}^{-2}$

and $T_1 = 300\,\text{K}$ (given)

If area of cross-section of piston is A and it moves through distance x, then increment in volume of the gas $= Ax$.

If force constant of a spring is k, then force $F = kx$ and

pressure $= F/A = kx/A$.

$$V_2 = V_1 + Ax$$

$$= 2.4 \times 10^{-3} + 8 \times 10^{-3} \times 0.1$$

$$= 3.2 \times 10^{-3}$$

and $p_2 = p_0 + \frac{kx}{A}$

$$= 10^5 + \frac{8000 \times 0.1}{8 \times 10^{-3}} = 2 \times 10^5$$

From ideal gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\Rightarrow \quad \frac{10^5 \times 2.4 \times 10^{-3}}{300} = \frac{2 \times 10^5 \times 3.2 \times 10^{-3}}{T_2}$$

$$\Rightarrow \quad T_2 = 800\,\text{K}$$

6 The efficiency of cycle,

$$\eta = 1 - \frac{T_2}{T_1}$$

For adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

For diatomic gas, $\gamma = \frac{7}{5}$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_1 = T_2(32)^{\frac{7}{5}-1}$$

$$= T_2(2^5)^{2/5} = T_2 \times 4$$

$$T_1 = 4T_2$$

$$\therefore \quad \eta = \left(1 - \frac{1}{4}\right)$$

$$= \frac{3}{4} = 0.75$$

7 Heat required to change the temperature of vessel by a small amount dT

$$-dQ = mC_p dT$$

Total heat required,

$$-Q = m\int_{20}^{4} 32\left(\frac{T}{400}\right)^3 dT$$

$$= \frac{100\times10^{-3}\times32}{(400)^3}\left[\frac{T^4}{4}\right]_{20}^{4}$$

$$\Rightarrow Q = 0.001996 \text{ kJ}$$

Work done required to maintain the temperature of sink to T_2,

$$W = Q_1 - Q_2 = \frac{Q_1 - Q_2}{Q_2}Q_2$$

$$= \left(\frac{T_1}{T_2} - 1\right)Q_2$$

$$\Rightarrow W = \left(\frac{T_1 - T_2}{T_2}\right)Q_2$$

For $T_2 = 20$ K,

$$W_1 = \frac{300-20}{20}\times 0.001996$$

$$= 0.028 \text{ kJ}$$

For $T_2 = 4$ K,

$$W_2 = \frac{300-4}{4}\times 0.001996$$

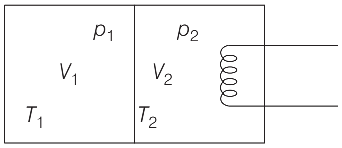
$$= 0.148 \text{ kJ}$$

As temperature is changing from 20 K to 4 K, work done required will be more than W_1, but less than W_2.

8 A → 2; B → 1; C → 3; D → 4.

The compression in the left-hand side is adiabatic

$$p_0V_0^\gamma = p_1V_1^\gamma$$



$$V_1 = V_0\left(\frac{p_0}{p_1}\right)^{2/3} = V_0\left(\frac{27}{64}\right)^{2/3}$$

$$= \frac{9V_0}{16} = 9 \text{ m}^3$$

Also, $\frac{p_0V_0}{T_0} = \frac{p_1V_1}{T_1}$

$$\Rightarrow \quad T_1 = \frac{p_1V_1}{p_0V_0} = \frac{4T_0}{3} = 432 \text{ K}$$

$$\therefore \quad p_2 = \frac{64p_0}{27}$$

and $\frac{p_0V_0}{T_0} = \frac{p_2V_2}{T_2}$

$$\Rightarrow \quad V_2 = 2V_0 - V_1$$

$$= 23 \text{ m}^3$$

$$\Rightarrow \quad T_2 = \frac{92T_0}{27} = 1104 \text{ K}$$

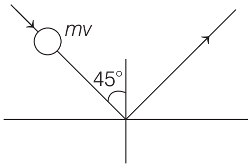
Work done on the left-hand side gas is

$$W = \frac{p_1V_1 - p_0V_0}{\gamma - 1}$$

$$= \frac{\left(\frac{64}{27}\times\frac{9}{16} - 1\right)p_0V_0}{\frac{3}{2} - 1} = \frac{2}{3}p_0V_0$$

$$= 3200 \text{ kJ}$$

9



Momentum imparted due to first collision $= 2mv\sin 45° = \sqrt{2}mv$

$$\left[\because \sin 45° = \frac{1}{\sqrt{2}}\right]$$

$\therefore$ Pressure on surface $= \frac{n\sqrt{2}mv}{\text{Area}}$

$$= \frac{10^{23}\times\sqrt{2}\times3.32\times10^{-27}\times10^3}{(2\times10^{-2})^2}$$

$$p = 2.35\times10^3 \text{ N/m}^2$$

10 For adiabatic process, relation of temperature and volume is,

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$\Rightarrow \quad T_2(2V)^{2/3} = 300(V)^{2/3}$$

$$\left[\gamma = \frac{5}{3} \text{ for monoatomic gases}\right]$$

$$\Rightarrow \quad T_2 = \frac{300}{2^{2/3}} \approx 189 \text{ K}$$

Also, in adiabatic process,

$$\Delta Q = 0, \Delta U = -\Delta W$$

or $$\Delta U = \frac{-nR(\Delta T)}{\gamma - 1}$$

$$= -2\times\frac{3}{2}\times\frac{25}{3}(300-189)$$

$$\approx -2.7 \text{ kJ}$$

11 $K = \frac{p}{(-\Delta V/V)}$

$$\Rightarrow \quad -\frac{\Delta V}{V} = \frac{p}{K}$$

$$\Rightarrow \quad -\Delta V = \frac{pV}{K}$$

Change in volume, $\Delta V = \gamma V\,\Delta T$

where, γ = coefficient of volume expansion.

Again, $\gamma = 3\alpha$

where, α is coefficient of linear expansion.

$$\therefore \quad \Delta V = V(3\alpha)\,\Delta T$$

$$\therefore \quad \frac{pV}{K} = V\,(3\alpha)\,\Delta T$$

$$\therefore \quad \Delta T = \frac{p}{3\alpha K}$$

12 For an adiabatic process,

$$TV^{\gamma-1} = \text{constant.}$$

We know that, average time of collision between molecules,

$$\tau = \frac{1}{n\pi\sqrt{2}\,v_{\text{rms}}\,d^2}$$

where, n = number of molecules per unit volume

and v_{rms} = rms velocity of molecules.

As, $\quad n \propto \frac{1}{V}$

and $\quad v_{\text{rms}} \propto \sqrt{T}$

$$\Rightarrow \quad \tau \propto \frac{V}{\sqrt{T}}$$

Thus, we can write

$$n = K_1V^{-1}$$

and $\quad v_{\text{rms}} = K_2\,T^{1/2}$

where, K_1 and K_2 are constants.

For adiabatic process,

$$TV^{\gamma-1} = \text{constant.}$$

Thus, we can write

$$\tau \propto VT^{-1/2} \propto V\,(V^{1-\gamma})^{-1/2}$$

or $\quad \tau \propto V^{\frac{\gamma+1}{2}}$

13 Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where, l is length of pendulum and g is acceleration due to gravity.

Such as change in time period of a pendulum,

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

When clock losses 12 s, we get

$$\frac{12}{T} = \frac{1}{2}\alpha\,(40-\theta) \quad \text{...(i)}$$

When clock gains 4 s, we get

$$\frac{4}{T} = \frac{1}{2}\alpha\,(\theta-20) \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$3 = \frac{40-\theta}{\theta-20}$$

$$\Rightarrow \quad \theta = 25°\text{C}$$

Substituting the value of θ in Eq. (i), we have

$$\frac{12}{T} = \frac{1}{2}\alpha\,(40-25)$$

$$\Rightarrow \frac{12}{24\times3600} = \frac{1}{2}\alpha\,(15)$$

$$\alpha = \frac{24}{24\times3600\times15}$$

$$\alpha = 1.85\times10^{-5}/°\text{C}$$

14 For polytropic process, specific heat for an ideal gas,

$$C = \frac{R}{1-n} + C_V$$

$$\therefore \quad \frac{R}{1-n} + C_V = C$$

$$\Rightarrow \quad \frac{R}{1-n} = C - C_V$$

$$\Rightarrow \quad \frac{R}{C - C_V} = 1 - n$$

(where, $R = C_p - C_V$)

$$\Rightarrow \quad \frac{C_p - C_V}{C - C_V} = 1 - n$$

$$\Rightarrow \quad n = 1 - \frac{C_p - C_V}{C - C_V}$$

$$\Rightarrow \quad n = \frac{C - C_p}{C - C_V}$$

Thus, number of moles n is given by

$$n = \frac{C - C_p}{C - C_V}$$

15 Efficiency of a process is defined as the ratio of work done to energy supplied. Here,

$$\eta = \frac{\Delta W}{\Delta Q} = \frac{\text{Area under } p\text{-}V \text{ diagram}}{\Delta Q_{AB} + \Delta Q_{BC}}$$

$$\therefore \quad \eta = \frac{p_0V_0}{nC_V\Delta T_1 + nC_p\Delta T_2}$$

$$= \frac{p_0V_0}{\frac{3}{2}nR(T_B - T_A) + \frac{5}{2}nR(T_C - T_B)}$$

$$= \frac{p_0V_0}{\left[\frac{3}{2}(2p_0V_0 - p_0V_0) + \frac{5}{4}(4p_0V_0 - 2p_0V_0)\right]}$$

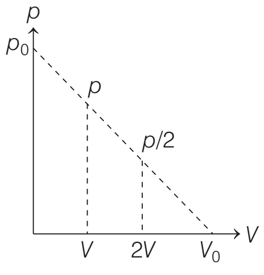
$$= \frac{p_0V_0}{\frac{3}{2}p_0V_0 + \frac{5}{2}\cdot 2p_0V_0}$$

$$= \frac{1}{6.5} = 15.4\%$$

16 Work done $= \frac{1}{2}\left(p + \frac{p}{2}\right)V$

$$= \frac{3}{4}pV = 0.75\,pV$$

Work done during isothermal process



$$= RT \times 2.3026\log_{10}\left(\frac{2V}{V}\right)$$

$$= 0.693\,pV$$

Thus, statement (a) is correct.

17 According to first law of thermodynamics, we get

(i) Change in internal energy from A to B, i.e.

$$\Delta U_{AB} = nC_V(T_B - T_A)$$

$$= 1 \times \frac{5R}{2}(800 - 400)$$

$$= 1000\,R$$

(ii) Change in internal energy from B to C,

$$\Delta U_{BC} = nC_V(T_C - T_B)$$

$$= 1 \times \frac{5R}{2}(600 - 800)$$

$$= -500\,R$$

(iii) $\Delta U_{\text{isothermal}} = 0$

(iv) Change in internal energy from C to A, i.e. ΔU_{CA}

$$\Delta U_{CA} = nC_V(T_A - T_C)$$

$$= 1 \times \frac{5R}{2}(400 - 600)$$

$$= -500\,R$$

18 $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$,

$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$,

$v_{\text{rms}} = \sqrt{2}\,v_{\text{sound}}$

Solving it, we get

$$\sqrt{3} = \sqrt{2\gamma}$$

$\therefore \quad \gamma = \frac{3}{2}$ for the mixture.

As, $$\gamma = \frac{C_p}{C_V} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1 + n_2} \times \frac{n_1 + n_2}{n_1C_{V_1} + n_2C_{V_2}}$$

$$\gamma = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{V_1} + n_2C_{V_2}}$$

For helium,

$$C_{p_1} = \frac{5}{2}R, C_{V_1} = \frac{3}{2}R$$

For hydrogen,

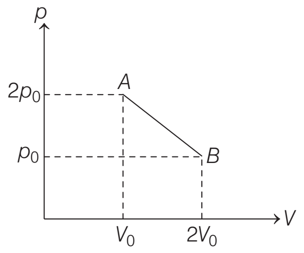
$$C_{p_2} = \frac{7}{2}R, C_{V_2} = \frac{5}{2}R$$

$$\therefore \quad \frac{3}{2} = \frac{2\left(\frac{5}{2}R\right) + n\left(\frac{7}{2}R\right)}{2\left(\frac{3}{2}R\right) + n\left(\frac{5}{2}R\right)}$$

$$= \frac{10 + 7n}{6 + 5n}$$

$$\Rightarrow \quad n = 2$$

19 As, T will be maximum temperature, where product of pV is maximum.



Equation of line AB, we have

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad p - p_0 = \frac{2p_0 - p_0}{V_0 - 2V_0}(V - 2V_0)$$

$$\Rightarrow \quad p - p_0 = \frac{-p_0}{V_0}(V - 2V_0)$$

$$nRT = \frac{-p_0}{V_0}V^2 + 3p_0V$$

$$T = \frac{1}{nR}\left(\frac{-p_0}{V_0}V^2 + 3p_0V\right)$$

For maximum temperature,

$$\frac{\partial T}{\partial V} = 0$$

$$\frac{-p_0}{V_0}(2V) + 3p_0 = 0$$

$$\frac{-p_0}{V_0}(2V) = -3p_0$$

$$\Rightarrow \quad V = \frac{3}{2}V_0$$

(condition for maximum temperature)

Thus, the maximum temperature of the gas during the process will be

$$T_{\max} = \frac{1}{nR}\left(\frac{-p_0}{V_0} \times \frac{9}{4}V_0^2 + 3p_0 \times \frac{3}{2}V_0\right)$$

$$= \frac{1}{nR}\left(-\frac{9}{4}p_0V_0 + \frac{9}{2}p_0V_0\right)$$

$$= \frac{9}{4}\frac{p_0V_0}{nR}$$