☆ Time Domain Analysis:

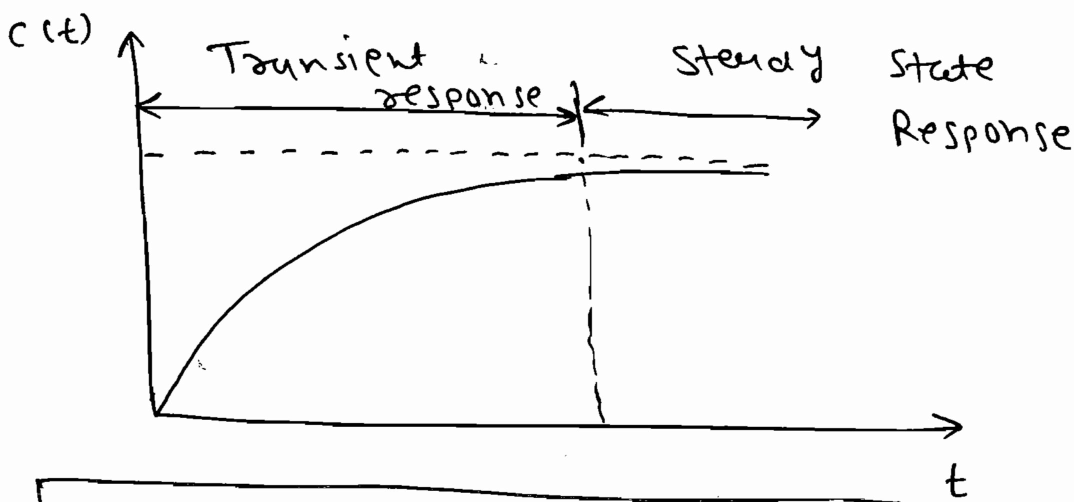
→ **Purpose:** To evaluate the performance of the system w.r.t. to the time.

* Time - Response:-

→ If the response of the system varies with respect to the time then it is called as time response.

→ The time response is nothing but the sum of transient response and steady state response.

⇒



$$\boxed{TR\,(c(t)) = C_{tr}(t) + C_{ss}(t)}$$

⇒ Find the transient and steady state terms in the given time response.

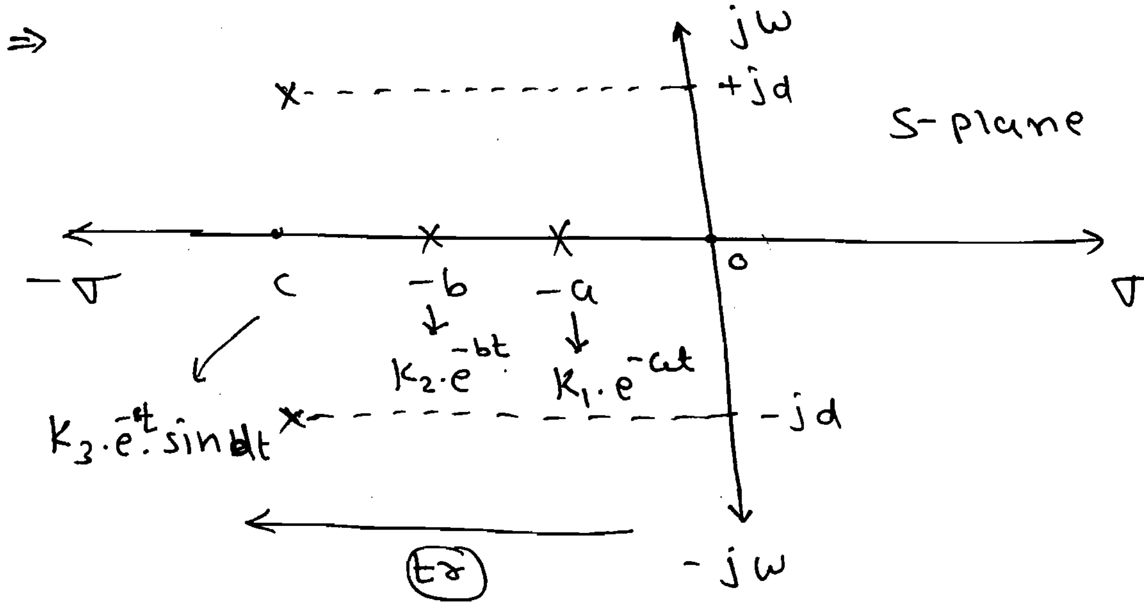
$$C(t) = \underbrace{10 + 2\sin 2t + 3\cos 3t}_{S.S} + \underbrace{4t\cdot e^{-4t} + 5\cdot e^{-5t}\cdot \sin 5t + 6t\cdot e^{-6t}\cdot \cos 6t}_{tr}$$

⇒ **Transient term:**

⇒ It is part of the system that becomes 0 as t becomes very large.

i.e. $$\boxed{\lim_{t \to \infty} C_{tr}(t) = 0}$$

⇒



⇒ The term which consist exponentially decay always gives the transient terms.

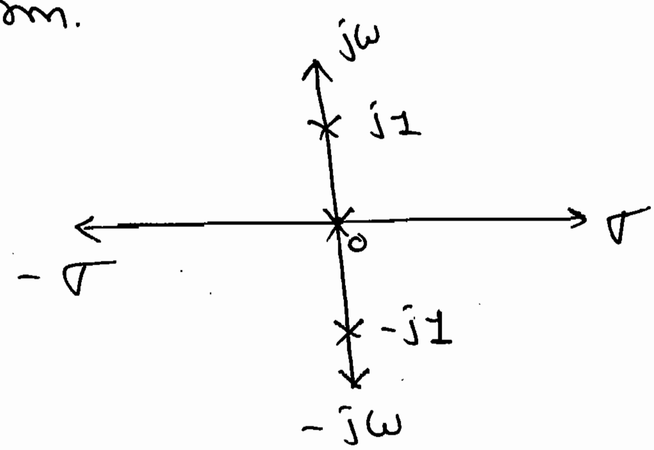
⇒ The poles which lies left hand side of the s-plane gives the transient terms.

⇒ Steady State Response:

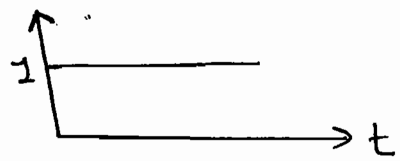
⇒ It is the part of the response that remains after the transient becomes the zero.

→ The pole which lies on the imaginary axis gives the steady state term.

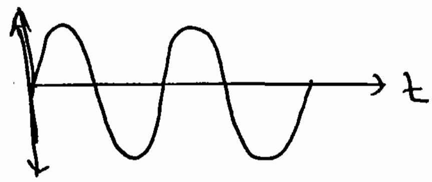
⇒



→ $\frac{1}{s} \Rightarrow u(t)$

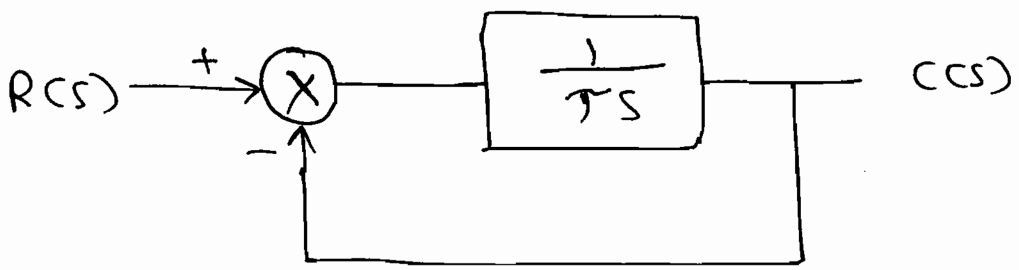


→ $\frac{1}{s^2+1} \rightarrow \sin t$



* Time Response to the first order system.

⇒



→ $G(s) = \frac{1}{\tau s}$, $H(s) = 1$.

∴ Type - 1 & order - 1.

⇒ Practical ckt for the first order system is RC, RL ckt of LPF.

→ Impulse response:

→ $r(t) = \delta(t)$.

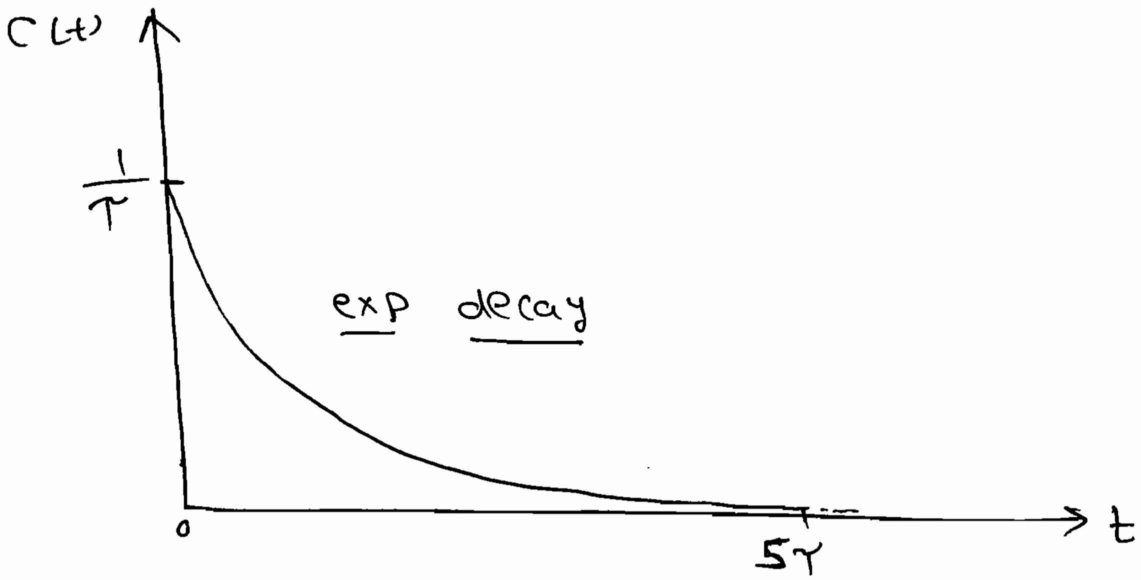
⇒ $R(s) = 1$.

→ $\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$.

→ $C(s) = \frac{1}{\tau s + 1}$.

$= \frac{1}{\tau} \cdot \frac{1}{s + \frac{1}{\tau}}$.

→ $\boxed{c(t) = \frac{1}{\tau} \cdot e^{-t/\tau}}$ → Transient term.

⇒ 

→ The impulse response consist the transient term. Transient term consist the

the System parameters.

→ Hence, the impulse response is called system response (or) Natural response (or) ~~forced~~ free forced response.

* **error:**

→ error is nothing but the deviation of the output from the input.

i.e. $e(t) = r(t) - c(t)$.

* **Steady state error (e_{ss}):**

→ The error at $t \to \infty$.

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t).}$$

$$\rightarrow e_{ss} = \lim_{t \to \infty} r(t) - c(t).$$

$$= \lim_{t \to \infty} \delta(t) - \frac{1}{\tau} \cdot e^{-t/\tau}.$$

→ The impulse ~~response~~ not consist the any steady state terms. Hence we can not defined the steady state errors. (or) The impulse input not exist at $t \to \infty$. Hence we can not compare the o/p with i/p at $t \to \infty$.

$\Rightarrow$ **Unit Response:**

$\Rightarrow$ $r(t) = u(t) \Rightarrow R(s) = \frac{1}{s}$.

$$\frac{C(s)}{R(s)} = \frac{1}{(\tau s + 1)}.$$

$$\therefore C(s) = \frac{1}{s(1+s\tau)}.$$

$$1 = \frac{A}{s} + \frac{B}{(1+s\tau)}.$$

$$\therefore 1 = A(1+s\tau) + Bs.$$

$s \to 0$. $A = 1$

$s \to -1/\tau$ $B = -\tau$.

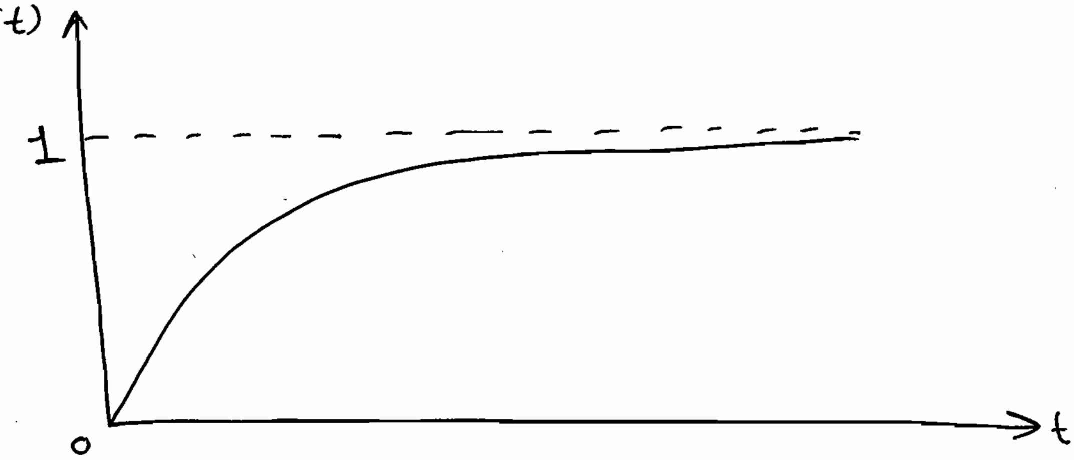
$$\therefore C(s) = \frac{1}{s} - \frac{\tau}{1+s\tau} = \frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}.$$

$$\therefore \boxed{c(t) = \left(1 - e^{-t/\tau}\right)u(t).}$$

SS $\qquad$ tr

$\rightarrow$ In the response, the steady state term because of the input in the response and the transient term because of the system.

$\Rightarrow$



$$\Rightarrow \quad e_{ss} = \lim_{t\to\infty} r(t) - c(t)$$

$$= \lim_{t\to\infty} 1 - 1 + e^{-t/\tau}$$

$$\therefore \boxed{e_{ss} = 0}$$

$\Rightarrow$ <u>Unit Ramp Response:</u>

$$\Rightarrow r(t) = t \quad \Rightarrow \quad R(s) = \frac{1}{s^2}.$$

$$\therefore \quad C(s) = \frac{1}{s^2} \cdot \frac{1}{(s\tau+1)}.$$

$$\rightarrow \frac{1}{s^2(s\tau+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s\tau+1)}.$$

$$\rightarrow \quad 1 = As(s\tau+1) + B(s\tau+1) + Cs^2.$$

$\rightarrow s \rightarrow 0$, $\rightarrow s \rightarrow -\frac{1}{\tau}$

$$\therefore \boxed{1 = B} \qquad 1 = \frac{C}{\tau^2} \Rightarrow \boxed{C = \tau^2}.$$

$s \rightarrow 1$

$$\therefore 1 = A(\tau+1) + B(\tau+1) + C.$$

$$\Rightarrow \cancel{1} = A(\tau+1) + \tau + \cancel{1} + \tau^2.$$

$\therefore \quad A(\tau + \cancel{1}) = -\tau(1+\cancel{\tau})$

$\boxed{A = -\tau}$

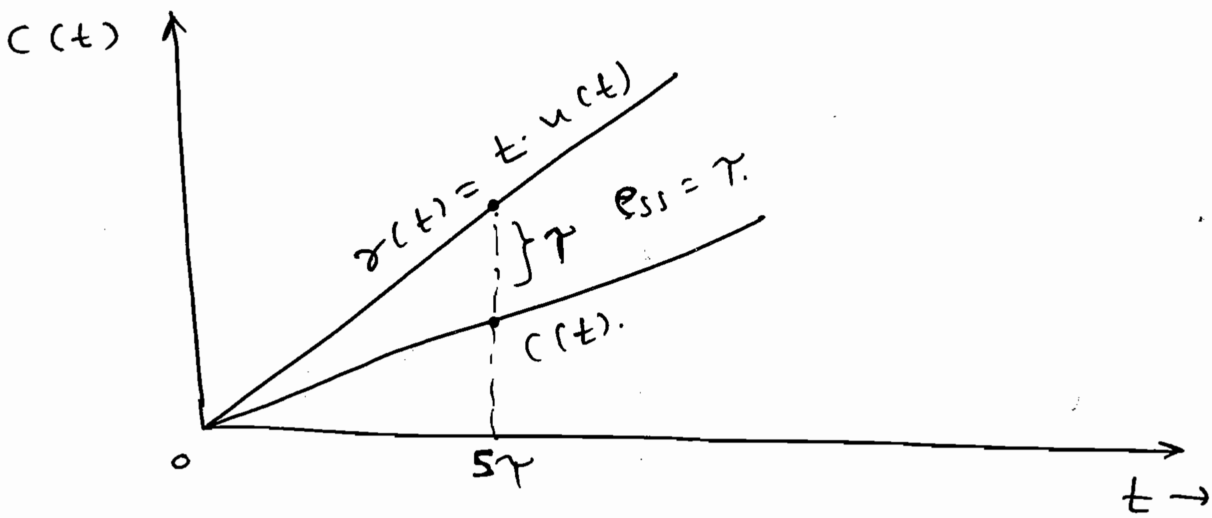
$$\therefore \quad C(s) = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{\tau s+1}.$$

$$\therefore \quad \boxed{c(t) = t - \tau + \tau \cdot e^{-t/\tau}}$$

$$\rightarrow e_{ss} = \lim_{t\to\infty} r(t) - c(t).$$

$$= \lim_{t\to\infty} \tau - e^{-t/\tau} = \tau.$$

$$\therefore \quad \boxed{e_{ss} = \tau}$$



$\Rightarrow$ <u>Unit</u> <u>Parabolic</u> <u>Response:</u>

$\Rightarrow \quad r(t) = t^2/2 \cdot u(t).$

$\Rightarrow \quad R(s) = \dfrac{1}{s^3}.$

$\rightarrow \quad C(s) = \dfrac{1}{s^3} \cdot \dfrac{1}{(s\tau+1)}.$

$$\therefore \frac{1}{s^3(s\tau+1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s^1} + \frac{D}{s\tau+1}.$$

$$\therefore 1 = A(s\tau+1) + Bs(s\tau+1) + Cs^2(s\tau+1) + Ds^3.$$

$\rightarrow s \rightarrow 0$

$\Rightarrow \boxed{1 = A}$

$\rightarrow s \rightarrow -\frac{1}{\tau}.$

$1 = -\frac{D}{\tau^3} \Rightarrow \boxed{D = -\tau^3}$

$\rightarrow$ Co-efficient of s.

$A\tau + B = 0. \Rightarrow \boxed{\cancel{B = -\frac{1}{\tau}}}$ $\boxed{B = -\tau}$

$\rightarrow$ Co-efficient of s^2.

$B\tau + C\tau = 0.$

$C = -B\tau$

$\boxed{C = \tau^2}$

$$\therefore C(s) = \frac{1}{s^3} - \frac{\tau}{s^2} + \frac{\tau^2}{s} - \frac{\tau^3}{s\tau+1}.$$

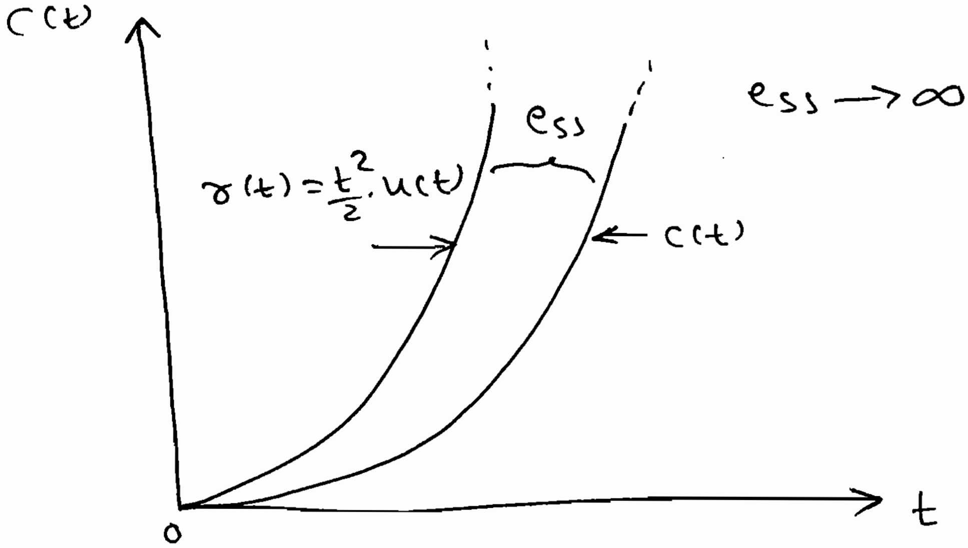
$$\therefore \boxed{c(t) = \frac{t^2}{2} - t\tau + \tau^2 - \tau^2 \cdot e^{-t/\tau}}$$

$$e_{ss} = \lim_{t\to\infty} r(t) - c(t)$$

$$= \lim_{t\to\infty} t\tau - \tau^2 + e^{-t/\tau}.$$

$$\boxed{e_{ss} = \infty.}$$

⇒

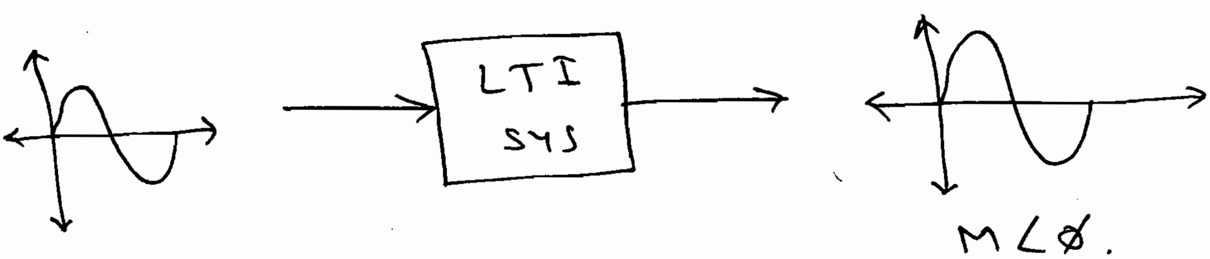


* Sinusoidal Response:

⇒ For any LTI system if input is sinusoidal the o/p also sinusoidal but difference in magnitude and phase.

⇒ The standard form of i/p and o/ps are as follows:

⇒



⇒ $r(t) = A \sin(\omega t \pm \theta) \rightarrow c(t) = A \times M \sin(\omega t \pm \theta \pm \phi)$.

$r(t) = A \cos(\omega t \pm \theta) \longrightarrow c(t) = A \times M \cos(\omega t \pm \theta \pm \phi)$.

$\boxed{Q}$ The CLTF of a LTI system $\frac{C(s)}{R(s)} = \left(\frac{1}{s+1}\right)$. for i/p $r(t) = \sin(t)$ the steady state o/p is ?

Sol^n: $r(t) = \sin(t)$.

$$\Rightarrow \quad \omega = 1 \text{ rad/sec.}$$

$$\frac{C(s)}{R(s)} = H(s) = \frac{1}{s+1}.$$

$$\Rightarrow H(j\omega) = \frac{1}{1+j\omega} = \frac{1}{1+j} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\therefore \boxed{c(t) = \frac{1}{\sqrt{2}} \cdot \sin(t - 45^\circ).}$$

$\boxed{a}$ $\dfrac{C(s)}{R(s)} = \dfrac{s+1}{s+2}$, $\quad r(t) = 10\cos(2t + 45^\circ)$.

find $c(t) = ?$

Sol^n: $r(t) = 10\cos(2t + 45^\circ)$

$$\Rightarrow \quad \omega = 2 \text{ rad/sec.}$$

$$\therefore H(s) = \frac{s+1}{s+2} = \frac{1+j\omega}{2+j\omega} = \frac{1+2j}{2+2j}$$

$$\therefore H(j\omega) = \frac{\sqrt{5}\ \tan^{-1}(2)}{\sqrt{8}\ \tan^{-1}(1)} = \sqrt{\frac{5}{8}} \angle \left(\tan^{-1}(2) - 45^\circ\right)$$

$$\therefore H(j\omega) = \sqrt{\frac{5}{8}} \angle 18.43^\circ.$$

$$\therefore c(t) = 10 \times \sqrt{5/8} \cdot (2t + 45^\circ + 18.43^\circ).$$

$$\therefore \boxed{c(t) = 8\cos(2t + 63.43^\circ).}$$

$\boxed{Q}$ A System $\frac{Y(s)}{X(s)} = \frac{s}{(s+p)}$ as an o/p

10.9

$y(t) = 1.\cos(2t - \pi/3)$. When

$x(t) = p.\cos(2t - \pi/2)$ then the

System parameter p is.

Soln:

$\omega = 2$ rad/sec.

$$\therefore H(s) = \frac{j\omega}{j\omega + p} = \frac{j2}{p+2j} = \frac{2\angle 90^\circ}{\sqrt{p^2+4}\angle \tan^{-1}\left(\frac{2}{p}\right)}$$

$$\therefore H(j\omega) = \frac{2}{\sqrt{p^2+4}} \times \angle 90^\circ - \tan^{-1}\left(\frac{2}{p}\right).$$

Now, $y(t) = x(t).h(t)$.

$$\therefore 1.\cos\left(2t - \frac{\pi}{3}\right) = \frac{2.p}{\sqrt{p^2+4}}.\cos\left(90^\circ + 2t - \frac{\pi}{2} - \tan^{-1}\left(\frac{2}{p}\right)\right)$$

compare, mag. and phase,

$$\frac{2.p}{\sqrt{p^2+4}} = 1$$

$$\therefore -\frac{\pi}{3} = -\tan^{-1}\left(\frac{2}{p}\right)$$

$$\therefore 2p = \sqrt{4+p^2}$$

$$\therefore \tan^{-1}\left(\frac{2}{p}\right) = \pi/3.$$

$$4p^2 = 4 + p^2$$

$$\frac{2}{p} = \tan(\pi/3).$$

$$3p^2 = 4$$

$$\therefore \frac{2}{p} = \sqrt{3}$$

$$\therefore p^2 = \frac{4}{3}$$

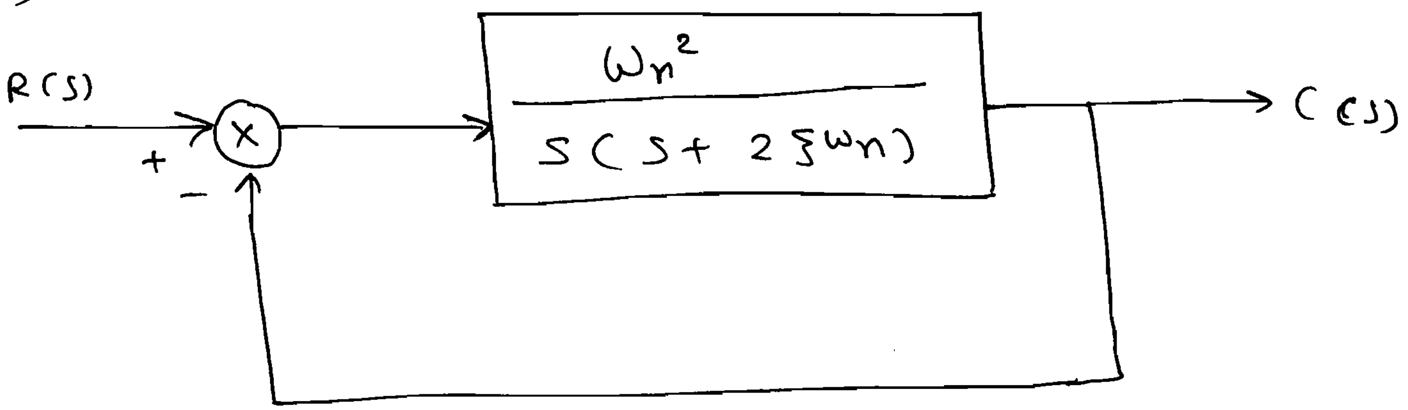
$$\Rightarrow \boxed{p = \cancel{\sqrt{3}/2}}$$

$$\boxed{p = \pm 2/\sqrt{3}}$$

$$p = 2/\sqrt{3}.$$

* Time Response to the second order system :-

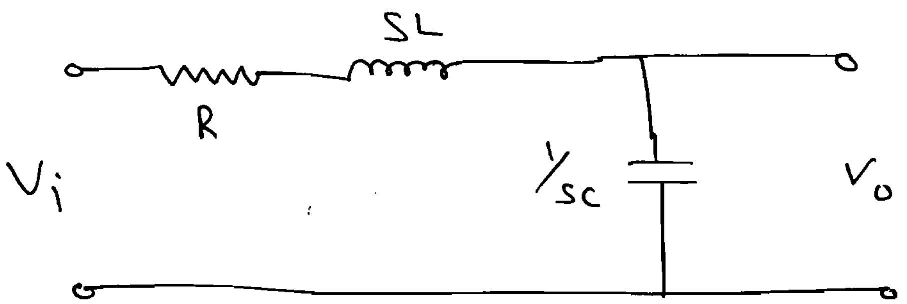
⇒



⇒ $$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}}$$ H.B.

⇒ Type-1, order-2.

⇒ The Practical ckt to the second order system is R-L-C ckt of LPF.



⇒ $$\frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{SC}}{R + SL + \frac{1}{SC}}$$

$$\frac{V_o(S)}{V_i(S)} = \frac{1}{S^2LC + SCR + 1}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}.$$

$$\Rightarrow \omega_n^2 = \sqrt{\frac{1}{LC}}$$

$$\Rightarrow \omega_n = \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = \frac{R}{L}.$$

$$\therefore 2\xi = \frac{R}{L} \times \sqrt{LC}$$

$$\therefore \boxed{\xi = \frac{R}{2} \times \sqrt{\frac{C}{L}}.}$$

$\Rightarrow$ ω_n: is called Natural freq. of oscillation (or) Sustained oscillation (or) Undamped oscillation.

$$\Rightarrow \boxed{\alpha = \frac{1}{2\xi} = \frac{1}{R} \times \sqrt{\frac{L}{C}} = \text{damping ratio.}}$$

$\Rightarrow$ It gives the ratio of energy loosed to energy stored.

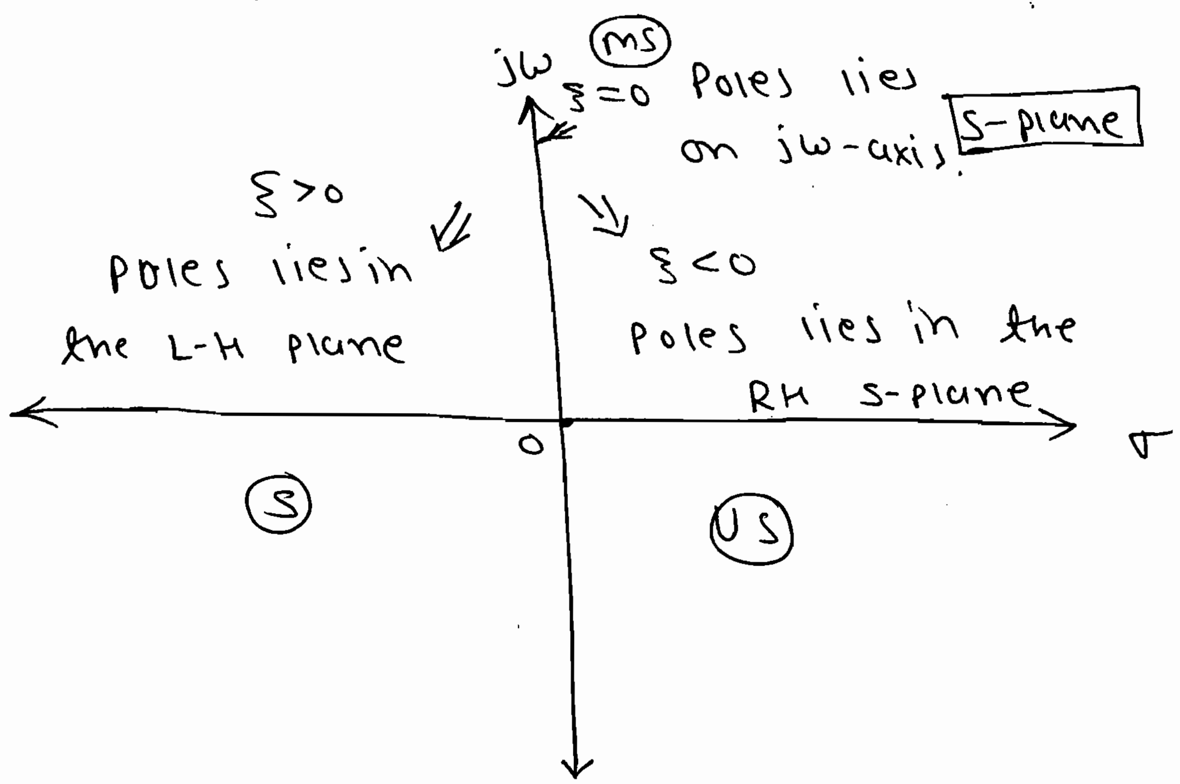
$$\Rightarrow \boxed{\xi\omega_n = \text{damping factor (or) actual damping.}}$$

2) The second order stable for $\xi > 0$.

$\Rightarrow$ The second order system response completely depends on ξ.

⇒ The second order system is stable for all the +ve values of $\xi > 0$. because the poles lies in the LH-plane.

⇒



* **Impulse Response:**

⇒ $r(t) = \delta(t)$.

⇒ $R(s) = 1$.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}.$$

$$\therefore C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}.$$

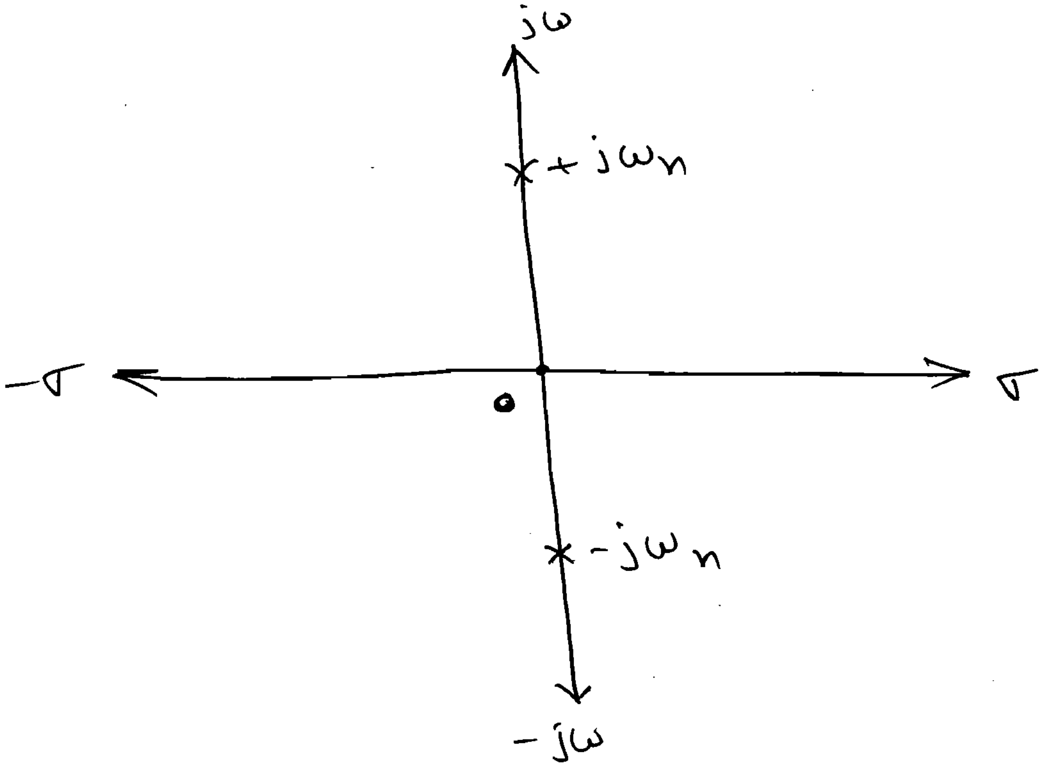
Case - I: $\xi = 0$ ⇒ Undamped.

$$\therefore C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}.$$

∴ Poles: $s^2 + \omega_n^2 = 0$

$$\boxed{s = \pm j\omega_n}$$

=>

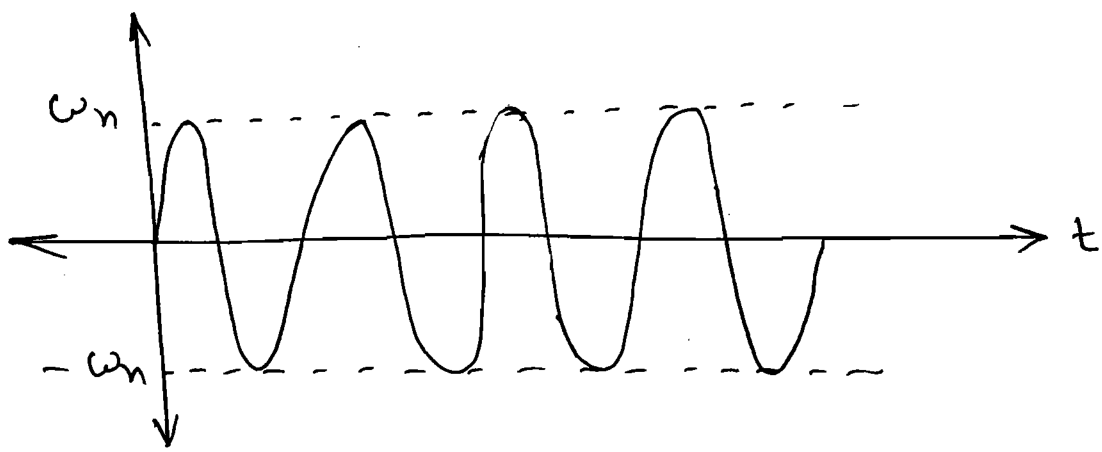


=> By Real part:

$\tau = 1/0 = \infty$.

=> Non-repeated poles on the jω axis hence the system is Marginally stable.

By ILT: $c(t) = \omega_n \cdot \sin\omega_n t$.



Constant Amp. and freq. of oscillation.

undamped oscⁿ / Natural oscⁿ / Sustained oscⁿ.

→ When $\xi = 0$, the second order system response is constant amplitude and freq. of oscillation which are called undamped oscillation.

→ Any system which produced the undamped oscillation is called undamped system and the system becomes marginal stable.

→ The second order system nature completely depends on ξ. for example if $\xi = 0$ the second order system nature is constant amplitude and freq. of oscilln around the input which never bechanged by changing the input signal hence when $\xi = 0$ the second order system is called undamped system irrespective of all the inputs.

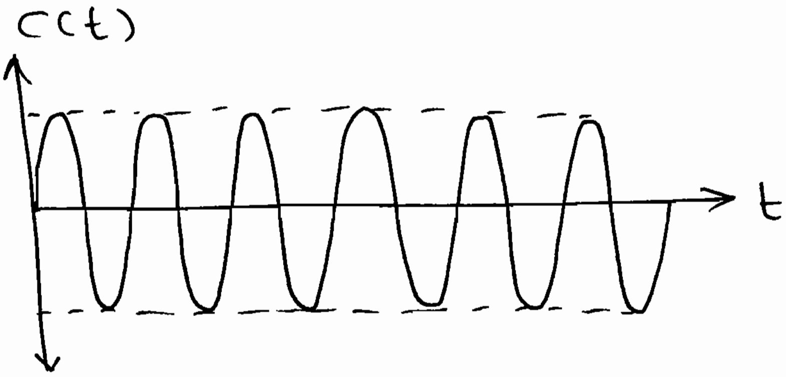
⇒ Similarly when $\xi > 0$ & $\xi < 1$ the system is called under damped system.

$\Rightarrow$ $\xi = 1 \rightarrow$ Critical damped system.

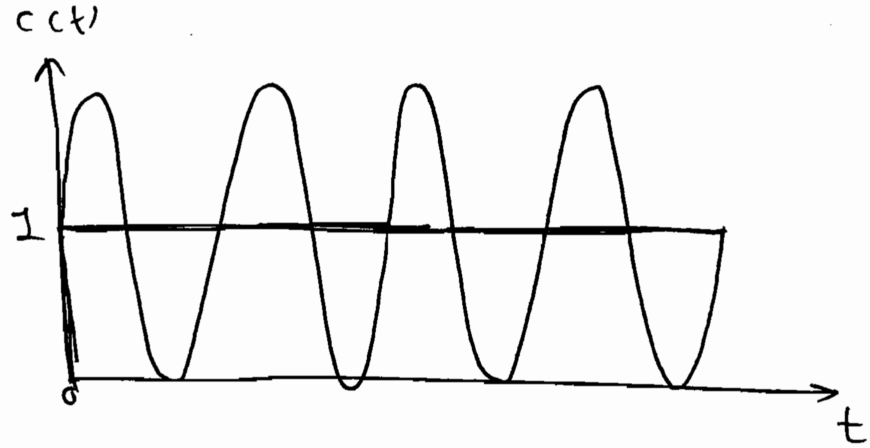
$\Rightarrow$ $\xi > 1 \rightarrow$ Overdamped system.

$\Rightarrow$ $\xi = 0$.

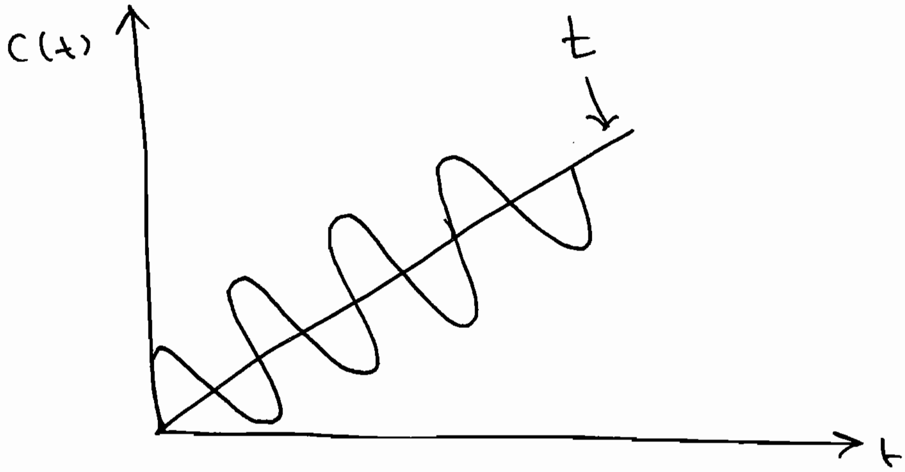
(i) Impulse



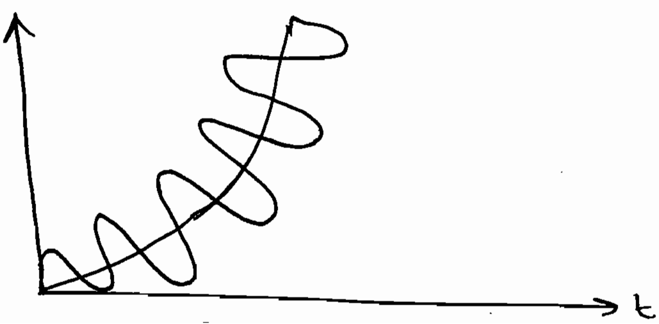
(ii) Unit step.



(iii) Unit Ramp:



(iv) Unit Parabolic:



$\Rightarrow$ when $\xi = 0$ we can not find the steady state error because the system is marginally stable.

H.B.

$\Rightarrow$ The steady state errors are calculate to only closed loop stable system.

Case - (ii) : Underdamped system $(0 \leq \xi < 1)$.

$\Rightarrow$ $S_1, S_2 = ?$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

$$\therefore \quad S_1, S_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$S_1 : \left(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1}\right).$$

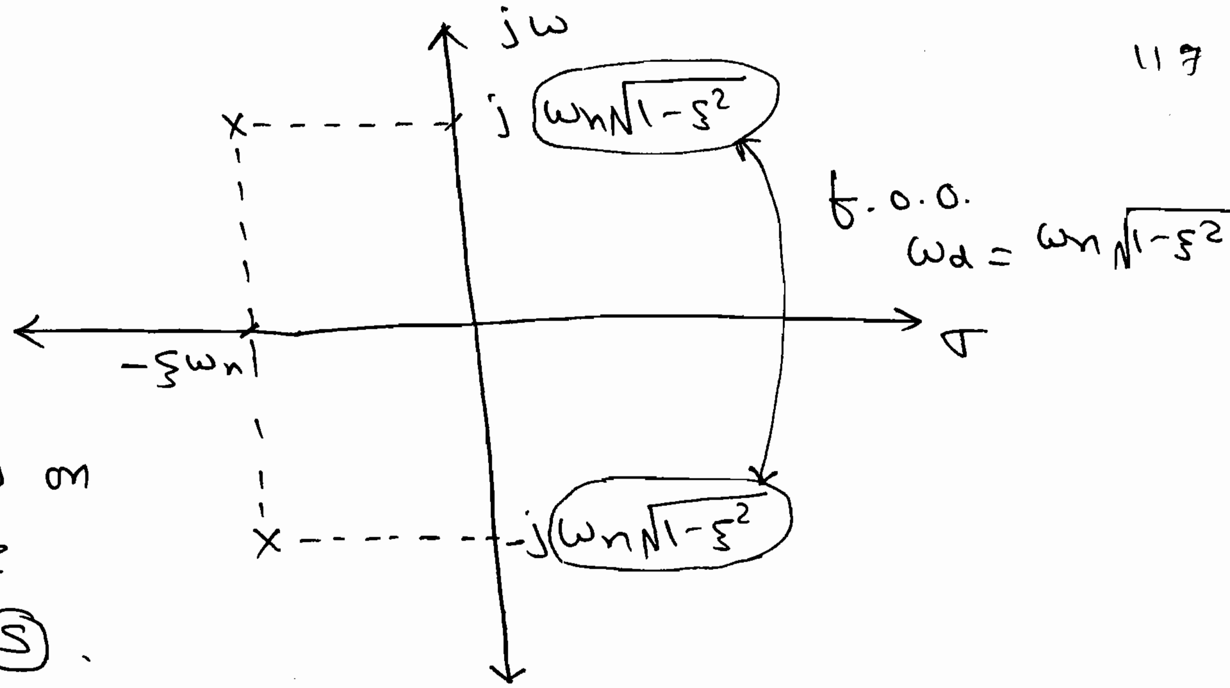
$$S_2 : \left(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1}\right).$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\omega_n^2}{\left(s+\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)}.$$

for $0 < \xi < 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{\left(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right)\left(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2}\right)}.$$

⇒



Poles lies on left side i.e. (S).

→ Real Part:

$$\boxed{\tau = \frac{1}{\xi\omega_n}}$$

(H.B.)

⇒ Time constant $\tau = \dfrac{1}{\xi\omega_n}$.

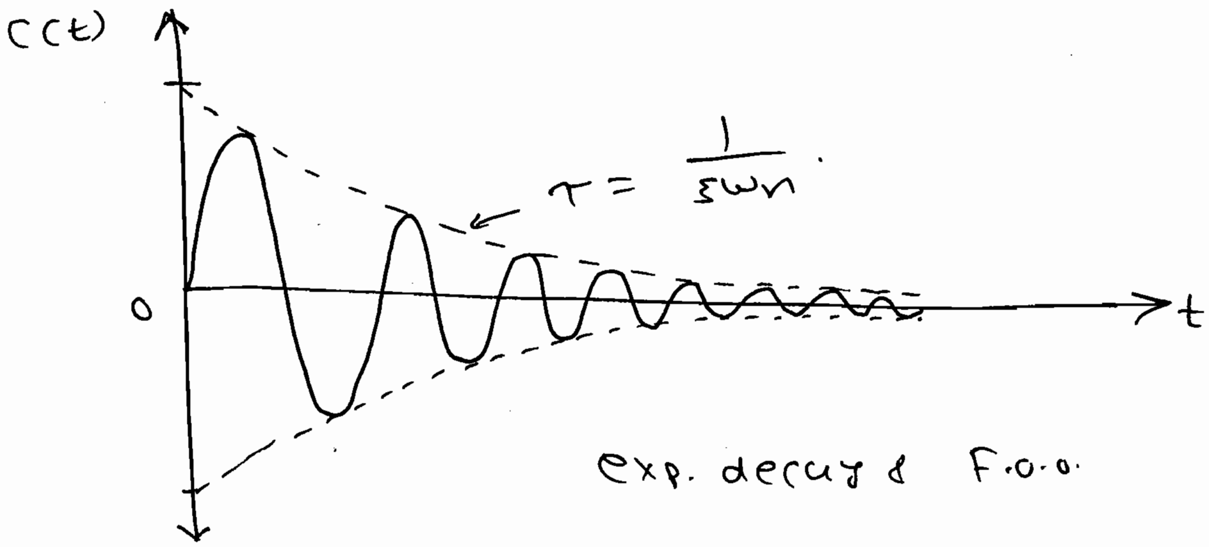
F.O.O $= \omega_d = \omega_n\sqrt{1-\xi^2}$ rad/sec.

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2}$$

$$\therefore C(s) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot \frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2}$$

$$\therefore \boxed{c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot \underbrace{e^{-\omega_n\xi t} \cdot \sin\omega_n\sqrt{1-\xi^2}\,.\,t}_{\text{exp. decay \& F.O.O.}}}$$

$\Rightarrow$



Damped oscillation} Underdamped sys.

$$\therefore \boxed{\omega_d = \omega_n\sqrt{1-\xi^2} \text{ rad/sec.}}$$

$\Rightarrow$ When $\xi > 0 \Rightarrow 0 < \xi < 1$, the poles lies in the left of s-plane which are complex conjugate. The system stable. The system response is exponential decay freq. of oscillations.

$\Rightarrow$ Any system which produced the damped oscillation is called Underdamped system.

$\Rightarrow$ Case-(iii): $\xi = 1$. Critical damped:

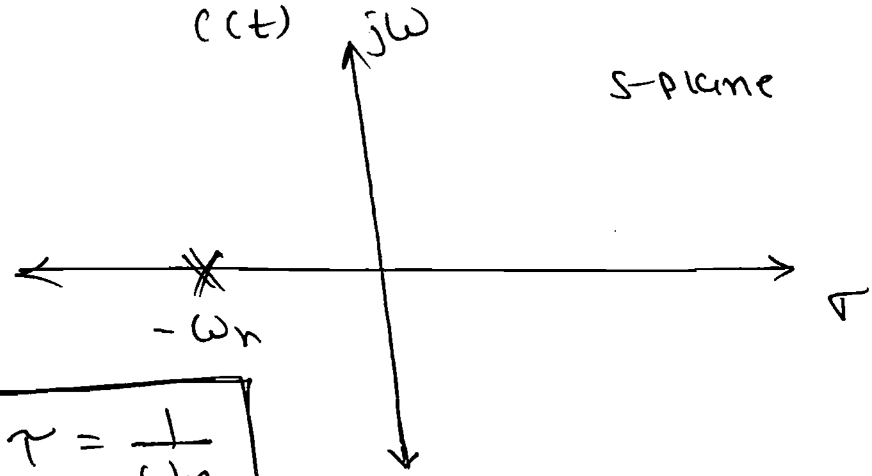
$$\Rightarrow \frac{C(s)}{\cancel{R(s)}} = \frac{\omega_n^2}{s^2 + 2\omega_n + \omega_n^2}.$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2}.$$

$$s = -\omega_n, -\omega_n.$$

$$\therefore \boxed{C(t) = \omega_n^2 \cdot t \cdot e^{-\omega_n t}}.$$

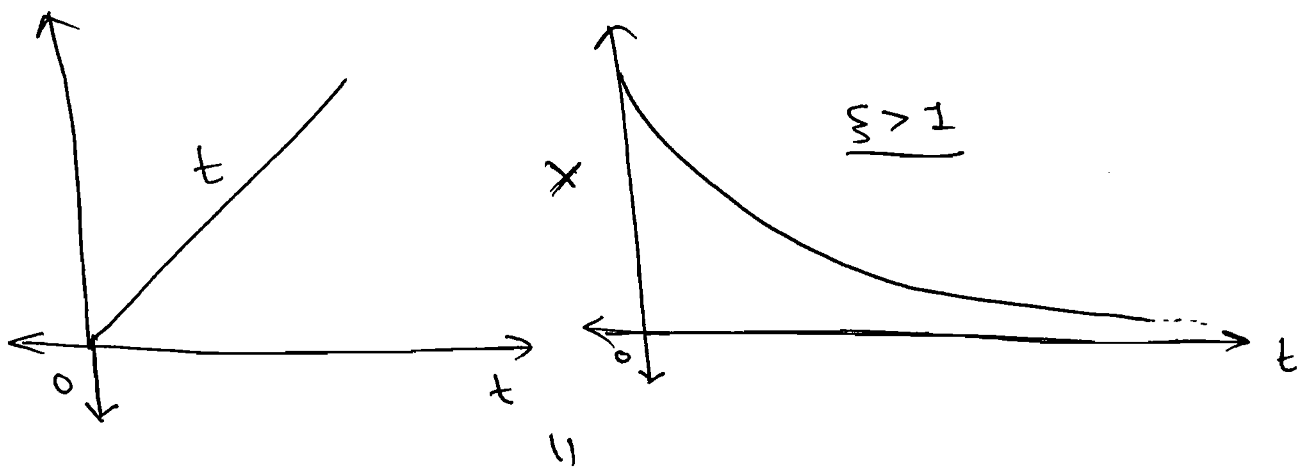
$\Rightarrow$

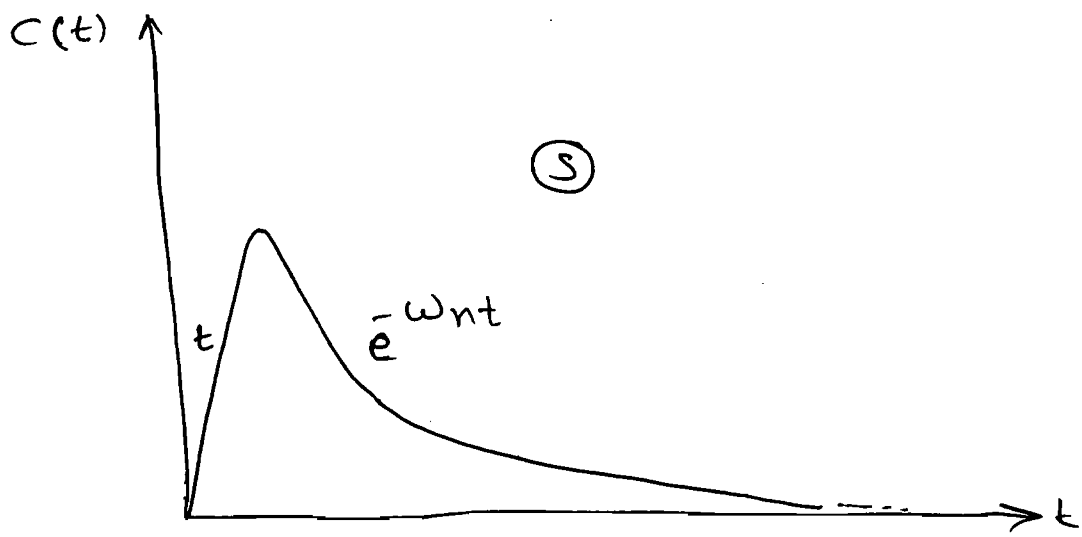


(S)

$$\boxed{\tau = \frac{1}{\omega_n}}$$

F.O.O. = 0.





(S)

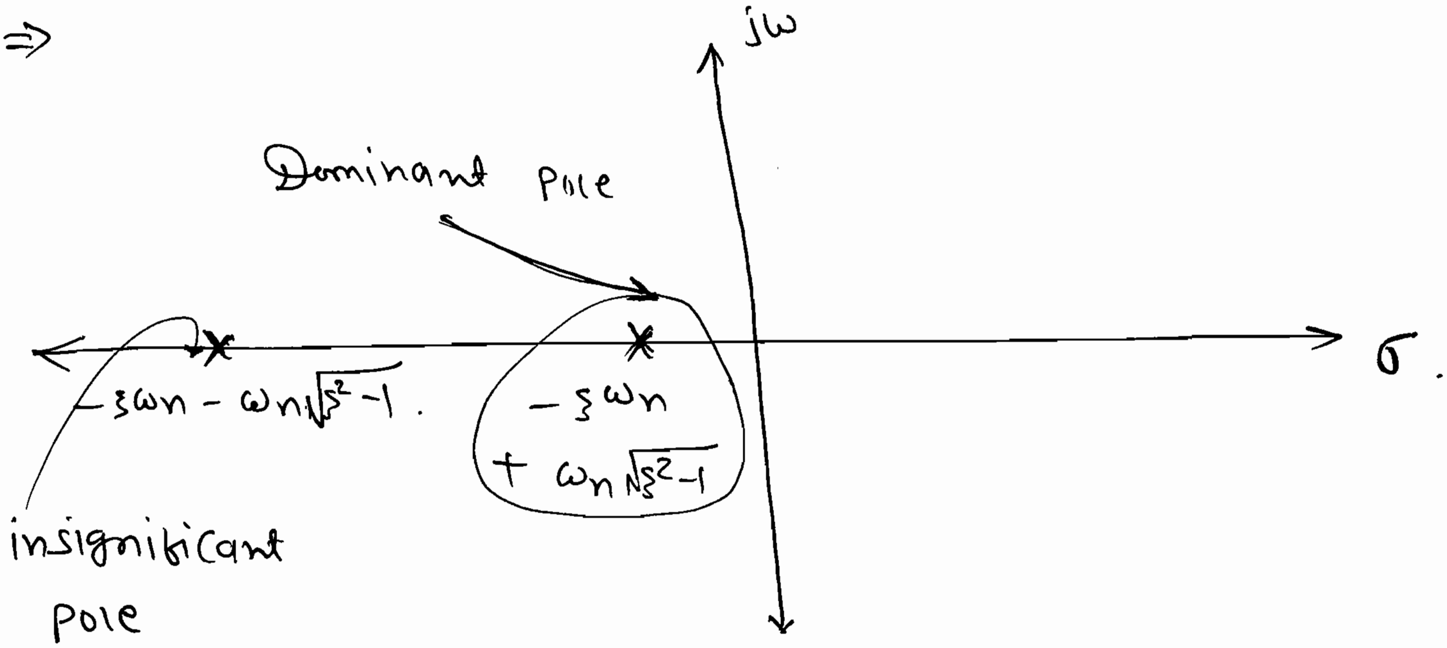
⇒ when $\xi = 1$ both the poles are lies on the -ve real axis at the same location the system is stable. The system response is called as critical damped system because it generates critically one damped oscillations.

⇒ The value of Resistance used to get the critical damped nature is called critical Resistance.

Case-(iv): $\xi > 1$: Overdamped system:

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}.$$

⇒



By Real Part.

$$\boxed{\tau = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}.}$$

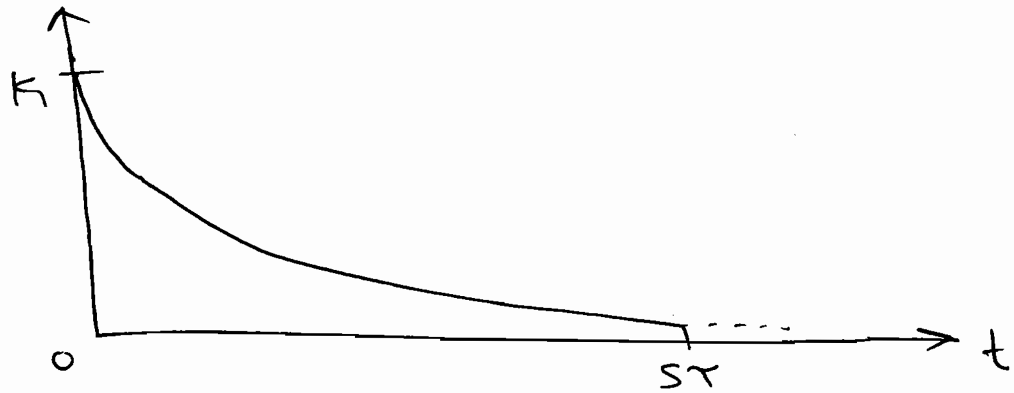
$\boxed{F.O.O = 0}$

$$\Rightarrow \quad C(s) = \frac{\omega_n^2}{\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)\left(s+\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)}$$

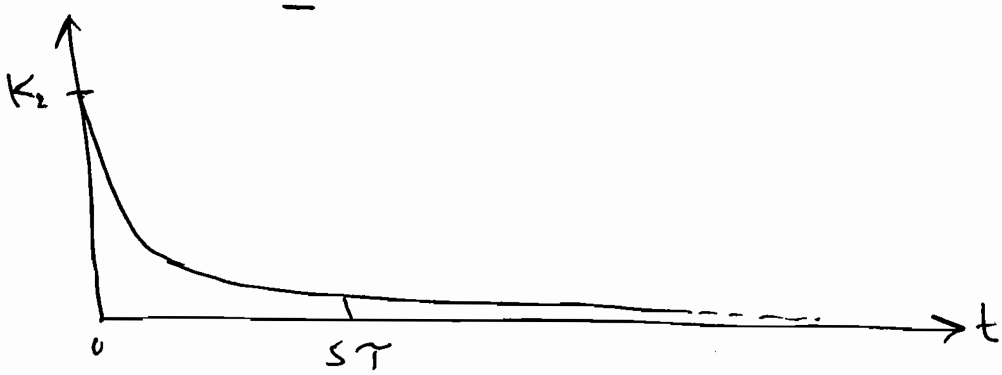
$$\therefore C(s) = \frac{K_1}{\left(s+\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)} - \frac{K_2}{\left(s+\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)}$$

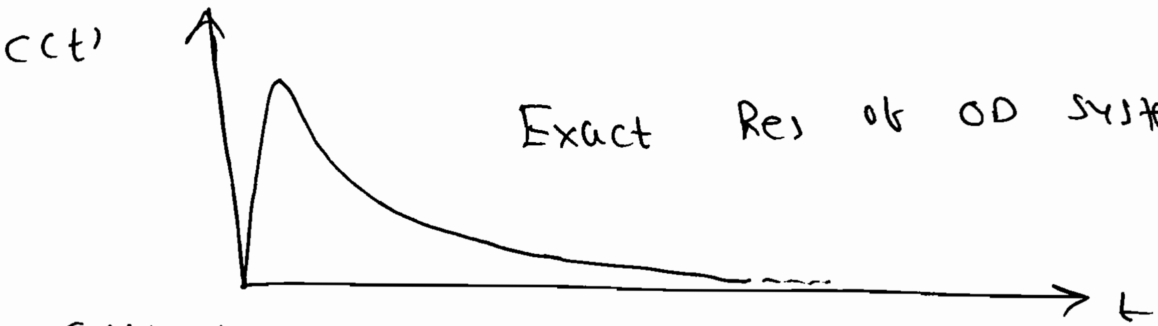
$$C(t) = \underbrace{K_1 \cdot e^{-\left(\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right)t}}_{\text{D.P. } (\tau\uparrow)} - \underbrace{K_2 \cdot e^{-\left(\xi\omega_n + \omega_n\sqrt{\xi^2-1}\right)t}}_{(\text{I.P.})\ (\tau\downarrow)}.$$

$\Rightarrow$

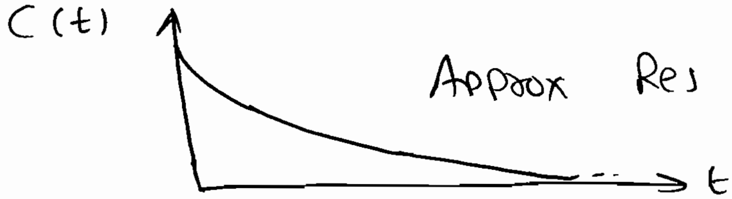


$-$





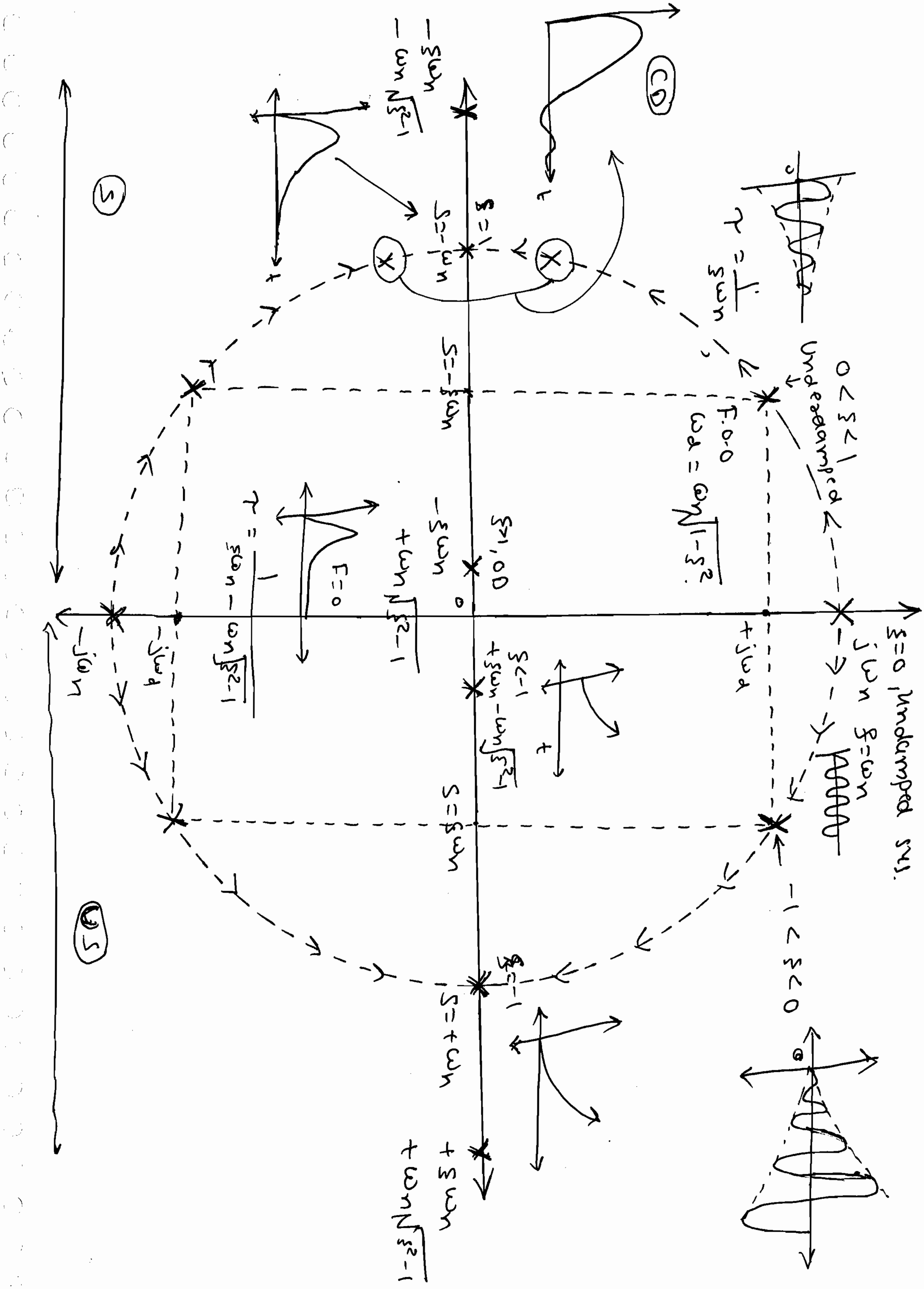
Exact Res of OD system



Approx Res of OD system.

⇒ When $\xi > 1$ both the poles lies in the left of S-plane at different location the system is stable. The system response is called over damped system because the system response elements are over comes the damped oscillation.

* Summary:



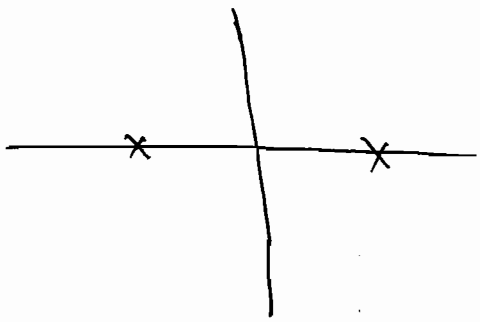
ξ=0 Undamped sys.
$j\omega_n$
$0<\xi<1$ Underdamped
F.O.O
$\omega_d = \omega_n\sqrt{1-\xi^2}$
$+j\omega_d$
$-j\omega_d$
$-j\omega_n$
$-1<\xi<0$
$\tau = \frac{1}{\xi\omega_n}$
$\xi=1$
$s=-\omega_n$
CD
$s=-\xi\omega_n$
$\xi>1$, OD
$-\xi\omega_n + \omega_n\sqrt{\xi^2-1}$
$-\xi\omega_n - \omega_n\sqrt{\xi^2-1}$
$\xi<-1$
$s=\xi\omega_n$
$s=+\omega_n$
$+\xi\omega_n + \omega_n\sqrt{\xi^2-1}$
$-\xi\omega_n - \omega_n\sqrt{\xi^2-1}$
F=0
S
US

* Conclusion:

⇒ When ξ increases from -1 to $+1$, the second order system poles path is a circle with a radius of ω_n.

⇒ Radial distance of Complex Pole is "ω_n".

⇒ The value of ξ to the given pole location is ___.



(a) $\xi=1$

(b) $\xi=-1$

(c) $|\xi|=1$

(d) none

⇒ When ξ increases from 0 to 1, the poles moves towards the left and near to the real-axis. In this, case

1. Time constant ↓
2. Satelling time. ↓
3. ω_d ↓
4. as ω_d ↓ the time speicification t_d, t_r, t_p ↑ and the system becomes more relatively stable.

→ when ξ increases from 1 to ∞ then one pole moves towards to the origin on the real axis. In this case.

① $\tau \uparrow$

② $t_s \uparrow$

③ damped oscillation become 0.

i.e. $\omega_d = 0$.

④ the relative stability of the system decreases.

⇒ <u>Order of the time constant.</u>

$$\underbrace{\tau_{undamped}}_{MS} > \underbrace{\tau_{overdamped} > \tau_{underdamped} > \tau_{critical\ damped}}_{S}$$

$$\infty \qquad \left(\frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}\right) \qquad \left(\frac{1}{\xi\omega_n}\right) \qquad \left(\frac{1}{\omega_n}\right)$$

largest τ (Slow response & sluggish system)

med τ

Smallest τ

* Unit Step response:

$\Rightarrow$ $r(t) = 1 . u(t).$

$R(s) = 1/s .$

$$\Rightarrow \quad C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}.$$

Case-(i): $\xi = 0$: Un~~der~~damped System.

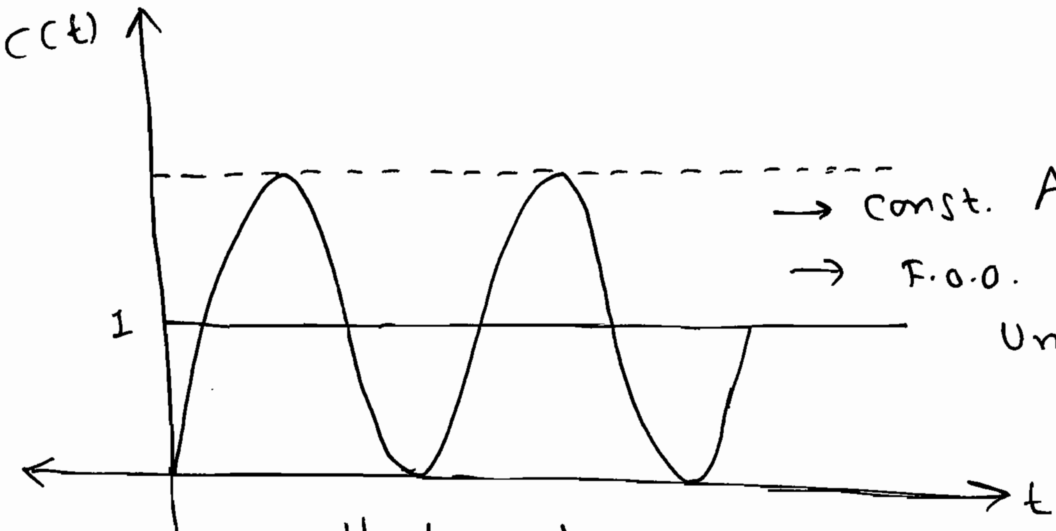
$$\rightarrow \quad C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)}.$$

$$C(t) = \frac{A}{s} + \frac{Bs+C}{s^2+\omega_n^2}.$$

$$= \frac{1}{s} + \frac{s(-1)+0}{s^2+\omega_n^2}.$$

$$\therefore \quad C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}.$$

$$\therefore \quad \boxed{c(t) = 1 - \cos(\omega_n t).}$$



→ const. Amp.

→ F.O.O. around i/p.

Undamped sys.

Undamped system

Case-(iii) : $\xi > 0,\ \xi < 1$ $[0 < \xi < 1]$:

Underdamped system.

$\Rightarrow$

$$C(s) = \frac{\omega_n^2}{s\left(s + \xi\omega_n - j\omega_n\sqrt{1-\xi^2}\right)\left(s + \xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right)}$$

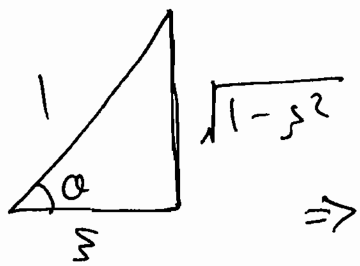
$\Rightarrow$ ILT to the above eqn is,

$\Rightarrow$

$\therefore$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left[\omega_n\sqrt{1-\xi^2}\cdot t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]}$$

$\Rightarrow$ $\tan\theta = \dfrac{\sqrt{1-\xi^2}}{\xi}$



$\Rightarrow \cos\theta = \xi$

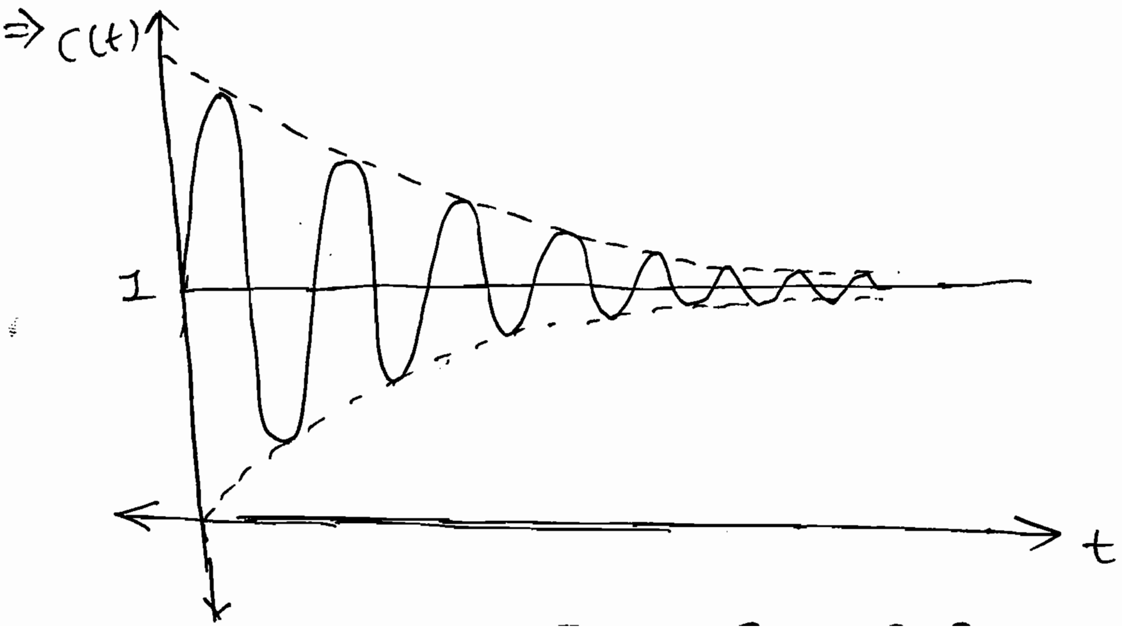
$\theta = \cos^{-1}(\xi)$.

$\Rightarrow$

$\therefore$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin\left(\omega_d t + \cancel{\tan^{-1}}\ \cos^{-1}(\xi)\right)}$$

$\Rightarrow$



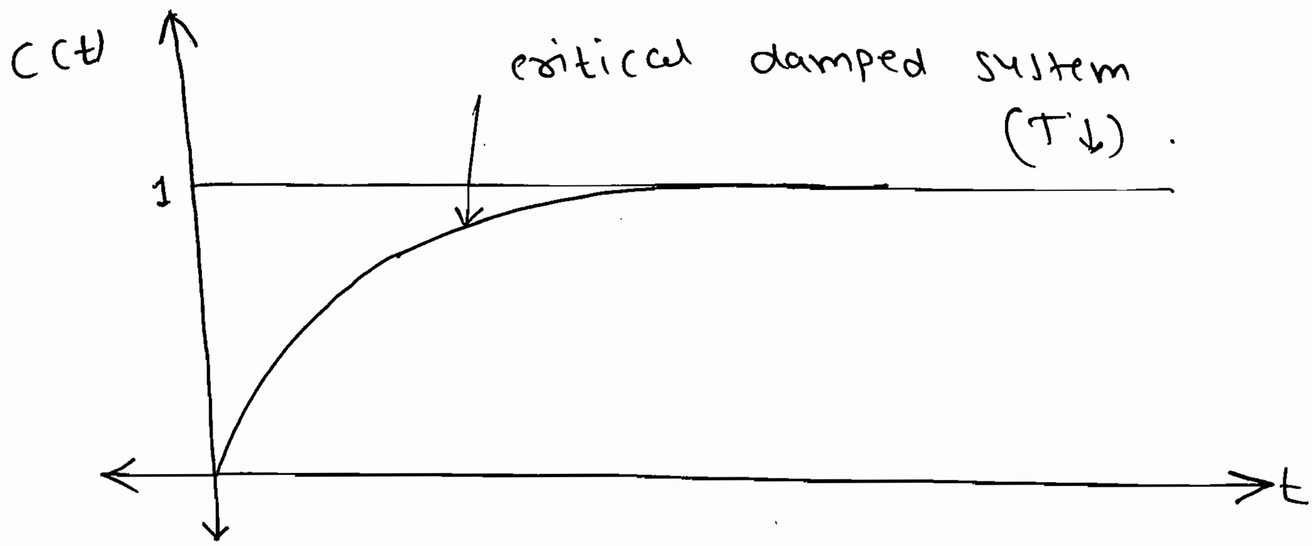
Case - III: $\xi = 1 \Rightarrow$ Critical damped.

$$\Rightarrow C(s) = \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s+\omega_n)^2}.$$

$$= \frac{A}{s} + \frac{B}{(s+\omega_n)} + \frac{C}{(s+\omega_n)^2}.$$

$$C(s) = \frac{1}{s} - \frac{\omega_n}{(s+\omega_n)^2} - \frac{1}{(s+\omega_n)}.$$

$$\therefore \quad c(t) = \left(1 - \omega_n(t \cdot e^{-\omega_n t}) - e^{-\omega_n t}\right).$$



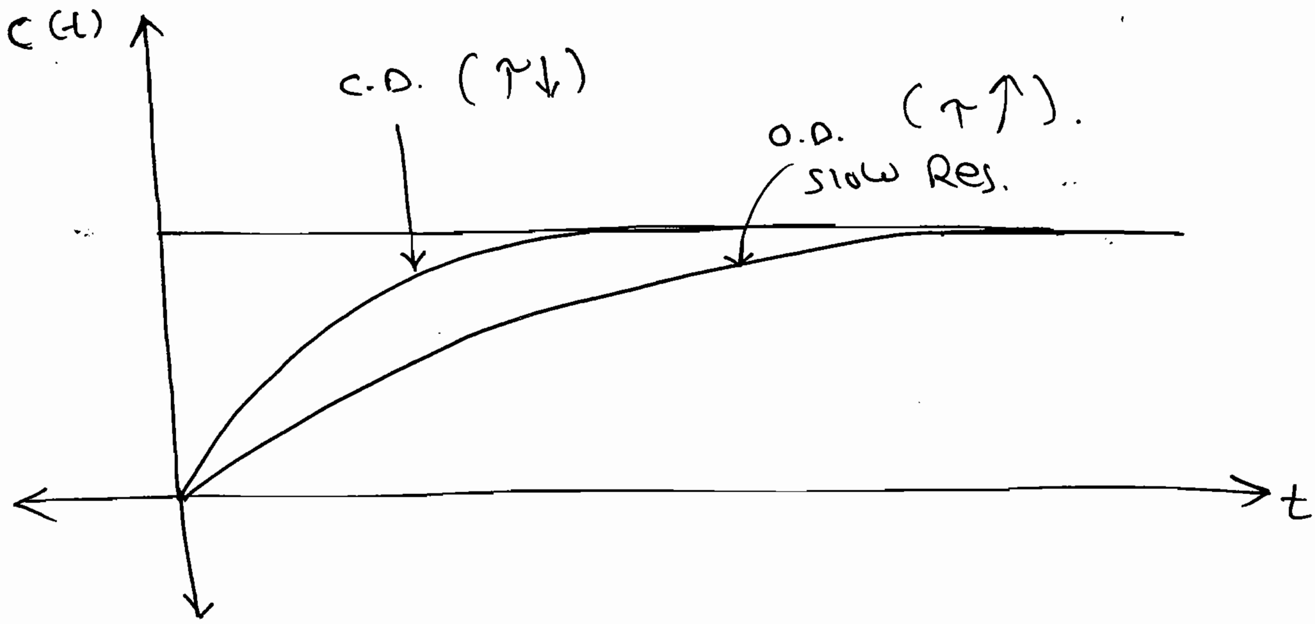
Case - IV: $\xi > 1$ overdamped.

$$\Rightarrow C(s) = \frac{\omega_n^2}{s(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})}.$$

$$= \frac{1}{s} - \frac{K_1}{(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})} - \frac{K_2}{s + \xi\omega_n + (\omega_n\sqrt{\xi^2-1})}$$

$$\Rightarrow C(t) = 1 - K_1 \cdot e^{-(\zeta\omega_n - \omega_n\sqrt{\zeta^2-1})t} - K_2 \cdot e^{-(\zeta\omega_n + \omega_n\sqrt{\zeta^2-1})t}$$

$$= 1 - \left[K_1 \cdot e^{-(\zeta\omega_n - \omega_n\sqrt{\zeta^2-1})t} + K_2 \cdot e^{-(\zeta\omega_n + \omega_n\sqrt{\zeta^2-1})t}\right].$$



☆ Time Domain Specification:

⇒ For time domain specification select a unit step input and underdamped system.

	tr.	ss.	S	
Impulse	✓	×	✓	← Practically not exist.
step	✓	✓	✓	→ Practically exist
Ramp	✓	✓	×	Unbounded I/p's
Parabolic	✓	✓	×.	

⇒ For system behaviour impulse is used.

⇒ For analysis w.r.t. time step is used.