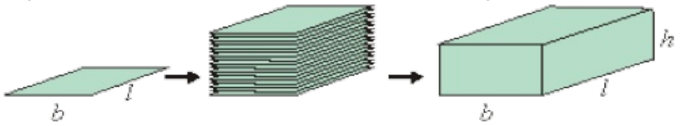
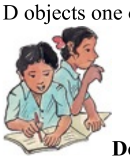
13.VISUALISING 3-D IN 2-D

13.0 Introduction

We are living in a 3-dimensional space. Some of the objects around us are in 3 dimensional shape. We can differntiate 2-D shapes from 3-D shapes by observing them.Look at a poster on the wall. The surface is of rectangular shape . How many measurements does it have ? It has 2 mesurements.i.e length and breadth. Look at the book. What is the shape of the book ? It is in cuboid shape. It has 3 measurements. Along with length and breadth it has one more measurement i.e. height.



A triangle, square, rectangle are plane figures of 2-dimensions. While a cube, cuboid are solid objects with 3 dimensions. By arranging 2-D objects one on another it occupies some space and become a 3-D object as in adjacent fig. It has volume also.

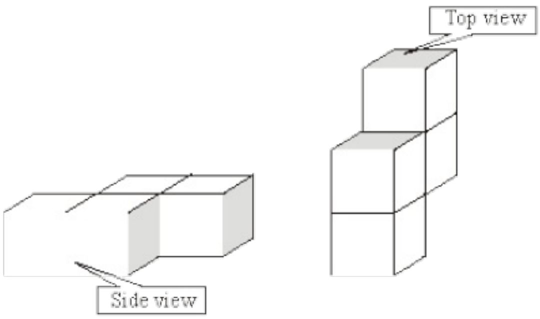


Do This

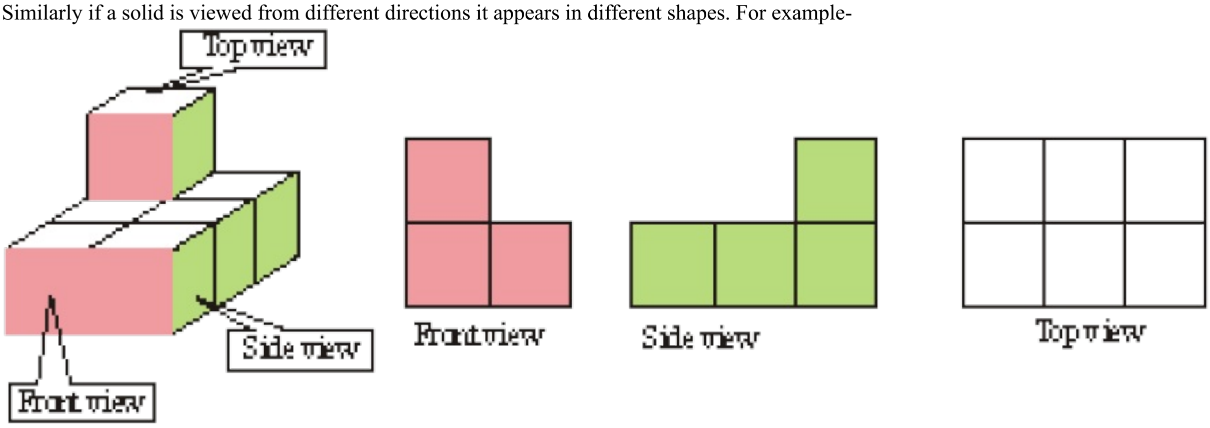
1. Name some 3-Dimensional, objects.
2. Give some examples of 2-Dimensional objects.
3. Draw a kite in your note book . Is it 2-D or 3-D object?
4. Identify some objects which are in cube or cuboid shape.
5. How many dimensions that a circle and sphere have?

13.1 3-D Objects made with cubes

Observe the following solid shapes. Both are formed by arranging four unit cubes.
If we observe them from different positions, it seems to be different. But the object is same.



Similarly if a solid is viewed from different directions it appears in different shapes. For example-



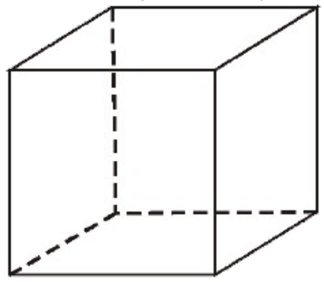
Think and Discuss

How to find area and perimeter of top view and bottom view of the above figure?

13.2 Representation of 3-D figures on 2-D

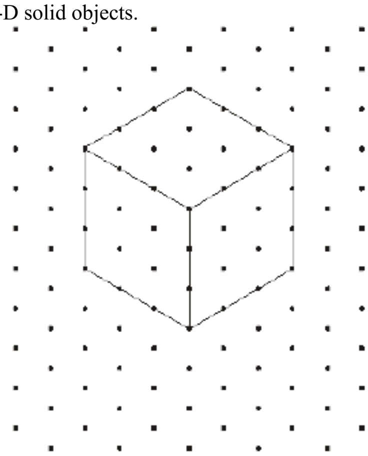
We use to draw 3-D figures on the paper, which is a 2-D. Actually we are able to represent only two dimensions on the plane paper, third dimension is only our imagination.

We have practice of showing 3-D cube object as in adjacent figure.

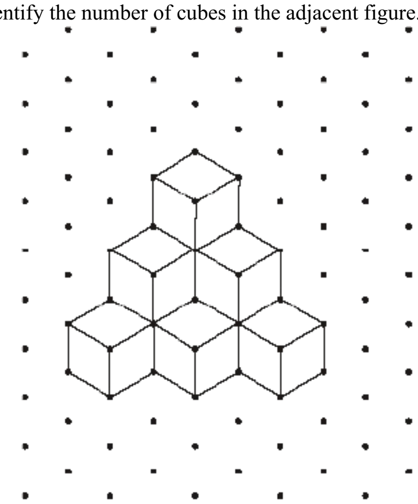


All edges of the cube are equal in length. But in the adjacent figure they are not equal. It has been drawn according to our view.

In order to over come this problem we use isometric dots paper, in which we can represent length, bredth and height with exact measurment of 3-D solid objects.



Example 1 : Identify the number of cubes in the adjacent figure.

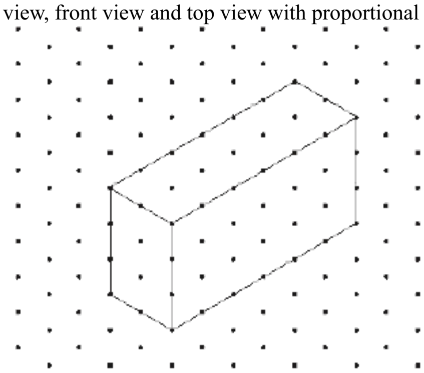


Solution : There are three layers of cubes.

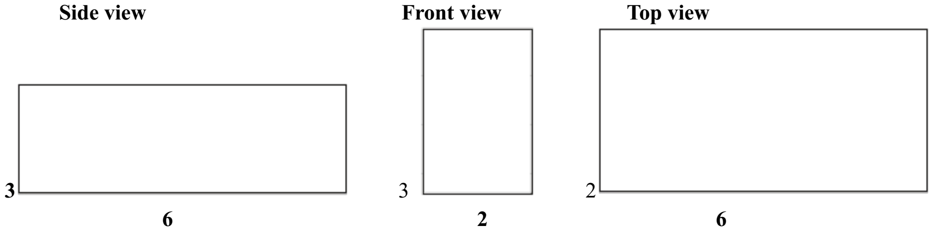
In the top layer, there is only one cube. In the second layer, there are 3 cubes (1 is hidden). In the lower layer, there are 6 cubes (3 are hidden). So total number of cubes = 1 + 3 + 6 = 10 cubes.

Example 2 : Find the the measurements of cuboid in the adjacent figure. Considering the distance between every two consecutive dots to be 1 unit.

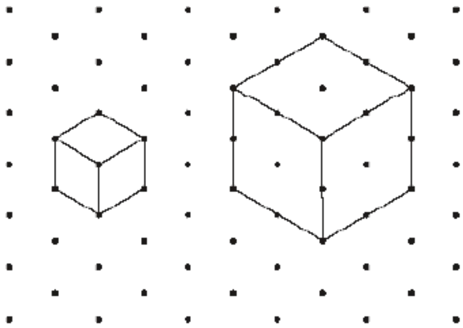
Also draw a side view, front view and top view with proportional measurements.



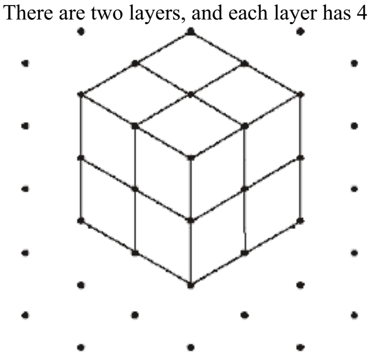
Solution : Length of the cuboid $l = 6$ units
Breadth of the cuboid $b = 2$ units
Height of the cuboid $h = 3$ units.



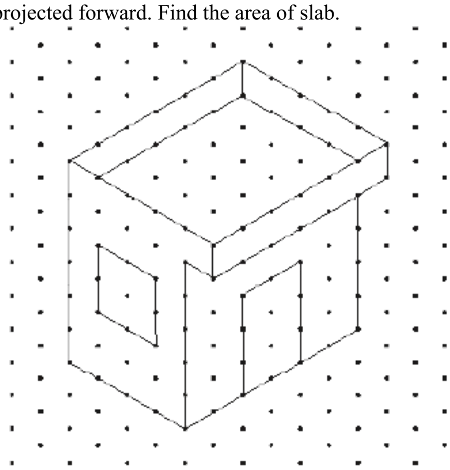
Example 3 : Look at the adjacent figure. Find the number of unit cubes in cube A and cube B and find the ratio.



Solution : There is only one unit cube in A. In figure B, by drawing parallel lines to all side, let us divide it into unit cubes and count. There are two layers, and each layer has 4 unit cubes. So number of unit cubes in B = 8 Ratio of unit cube in A and B = 1 : 8.



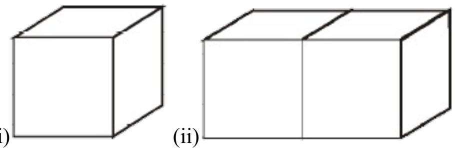
Example 4 : A house design given on isometric dot sheet in adjacent figure. Measure length, breadth and height of the house. Slab is projected forward. Find the area of slab.



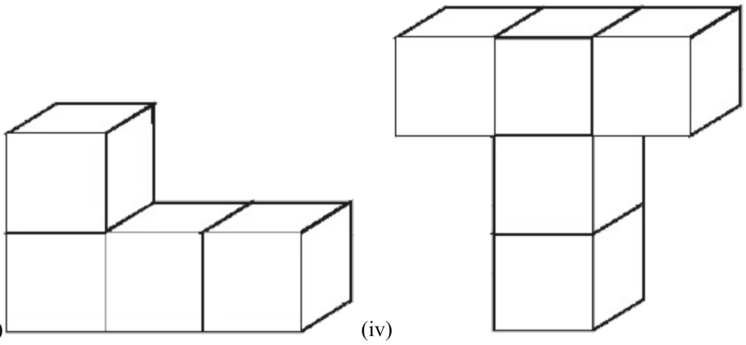
Solution : Length of the house = 6 units
Breadth of the house = 4 units
Height of the house = 5 units
Slab is projected forward for 1 unit
Dimensions of slab = 5×6 unit
Area of the slab = $5 \times 6 = 30$ sq. units.

Exercise - 13.1

1. Draw the following 3-D figures on isometric dot sheet.

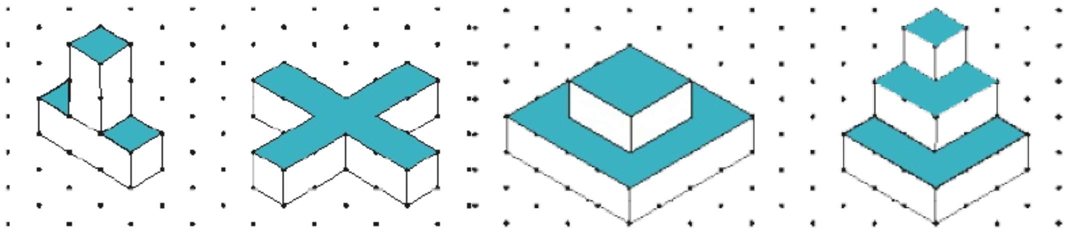


(i) (ii)



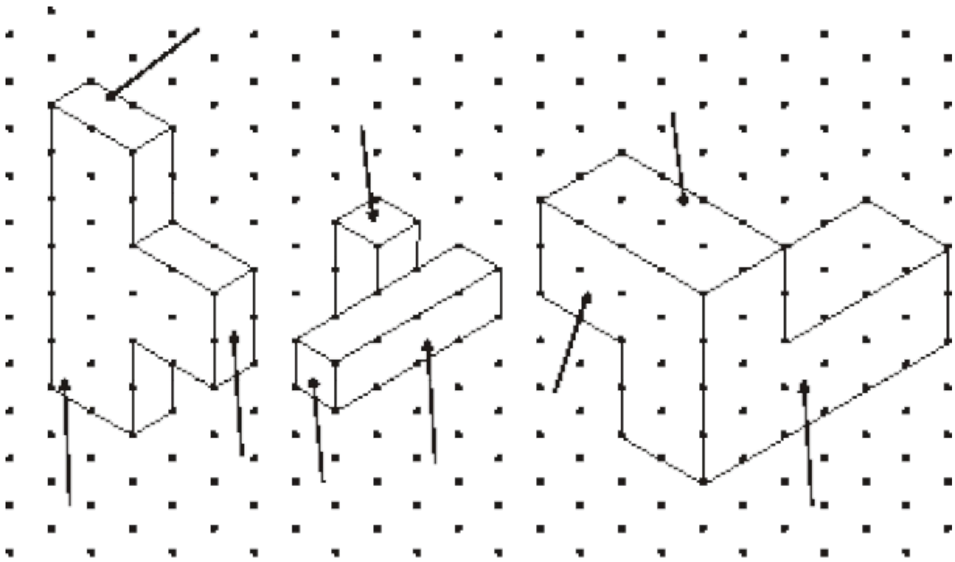
(iii) (iv)

2. Draw a cuboid on the isometric dot sheet with the measurements 5 units × 3 units × 2 units.
3. Find the number of unit cubes in the following 3-D figures.



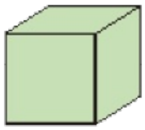
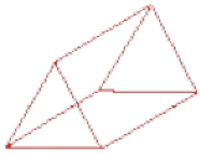
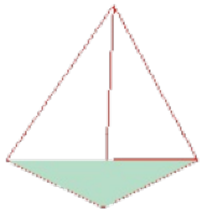
(i) (ii) (iii) (iv)

4. Find the areas of the shaded regions of the 3-D figures given in question number 3.
5. Consider the distance between two consecutive dots to be 1 cm and draw the front view, side view and top view of the following 3-D figures.



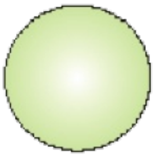
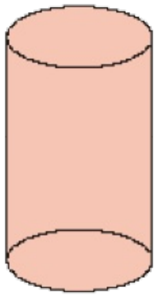
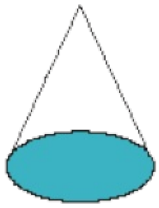
13.3 Various Geometrical Solids

In our surroundings we see various solid objects . Among them some solid objects have curved faces and some solid objects have flat faces. The 3-D objects like box , book, dice , have flat faces. The 3-D objects like ball , pipe etc have curved surfaces. Based on this property we can classify 3-D shapes as polyhedra and non-polyhedra.
Observe the following objects.

Are there any curved faces for above solids? No, all these have only flat surfaces. This type of solid objects with all polygonal faces are called polyhedra (singular is polyhedron)

Now observe these figures.

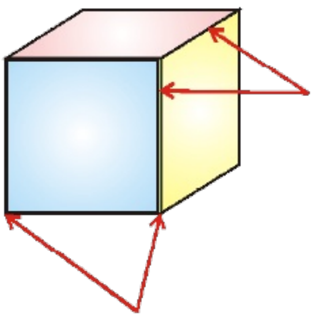
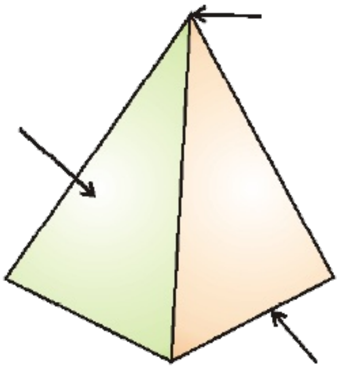
These objects have curved faces. This type of solid objects are called non-polyhedra.

Try This

1. Name three things which are the examples of polyhedron.
2. Name three things which are the examples of non-polyhedron.

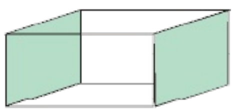
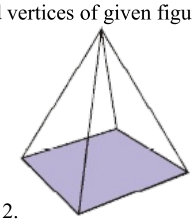
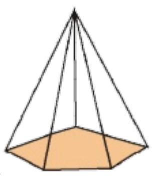
13.4 Faces, Edges and Vertices of 3D-Objects

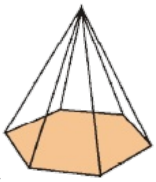
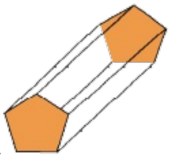
Observe the walls, windows, doors, floor, top, corners etc of our living room and tables, boxes etc. Their faces are flat faces. The flat faces meet at its edges . Two or more edges meet at corners. Each of the corner is called vertex. Take a cube and observe it where the faces meet? Where the edges meet?

Do These

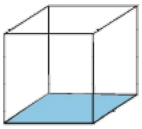
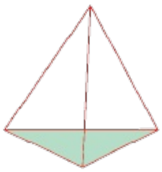
Identify the faces , edges and vertices of given figures .

1.  2.  3. 

4.  5. 

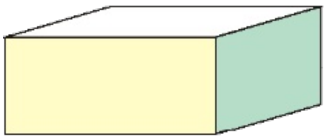
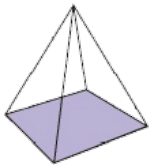
13.5 Regular Polyhedron

Observe the faces, edges and vertices in the following shapes.

Cube Triangular Pyramid (Tetrahedron)

In each of the above two objects all their faces are congruent. All their edges are equal and vertices are formed by equal number of edges. Such type of solid objects are called regular polyhedra. Now observe these figures.

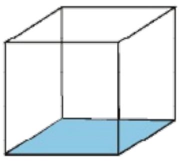
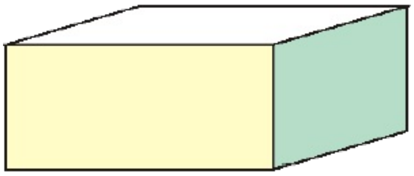
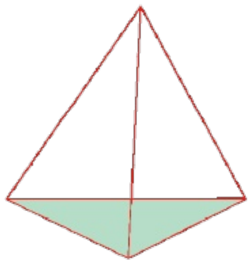
Cubiod Square Pyramid

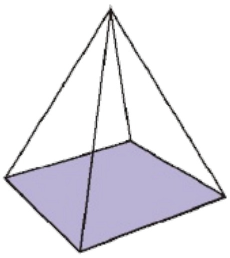
Cuboid is a non -regular polyhedra because all its faces are not congruent and in the square pyramid the one vertex formed by 4 edges and other vertices formed by 3 edges. More over all the faces in pyramid or not congruent. It is also not a regular polyhedra. These type of objects are non- regular polyhedra.

Thus polyhedra can be classified into regular polyhedra and non-regular polyhedra.

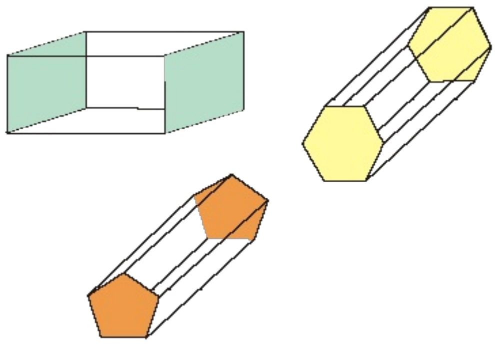
13.4.1 Prism and Pyramid

Now observe the following objects

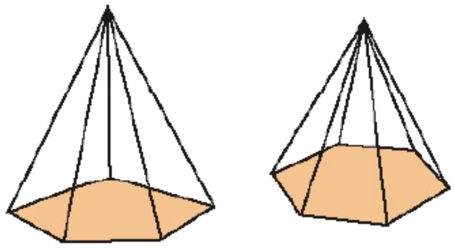
  



The objects in first box have same faces at top and bottom. The objects in the second box have base but the top is a common vertex. Let us observe some more objects like this.



(a)



(b)

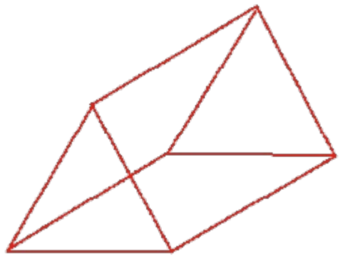
In fig (a) each object has two parallel and congruent polygonal faces, and the lateral faces are rectangles (or Parallelograms). In fig (b) The base is a polygon and lateral faces are triangles ,they meet at a common vertex.

The solid object with two parallel and congruent polygonal faces and lateral faces as rectangles or parallelograms is called a **prism**.

A solid object whose base is a polygon and its lateral faces are triangular faces is called **pyramid**”.

A prism or pyramid is named after its shape of parallel and congruent polygonal faces or the base.

A. Triangular Prism



What is the shape of two congruent and parallel faces in the adjacent figure? And what is the shape of its lateral faces ?

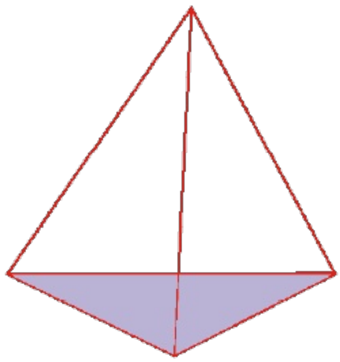
Its two congruent and parallel faces are triangular and its lateral faces are parallelograms.

This is known as triangular prism.

If the base is a square , it is called square prism.

If the base is a pentagon , it is called pentagonal prism.

B. Triangular Pyramid



A pyramid whose base is a triangle is called triangular pyramid .

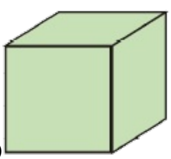
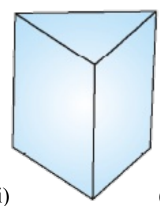
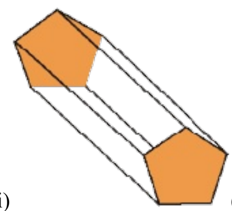
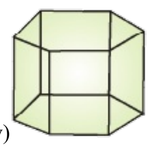
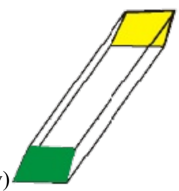
It is known as tetrahedron. (Tetra-means having four faces)

If the base of a pyramid is square , it is called as square pyramid.

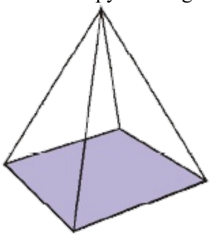
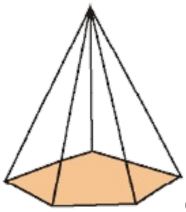
If the base of a pyramid is a pentagon , it is called as pentagonal pyramid.

Do This

1. Write the names of the prisms given below:

(i)  (ii)  (iii)  (iv)  (v) 

2. Write the names of the pyramids given below:


(i)

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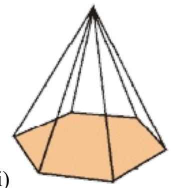
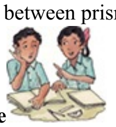

(iii)

3. Fill the table :

Number of sides of base of Prism / pyramid	Name of the prism	Name of the pyramid
3 sides		
4 sides		
5 sides		
6 sides		
8 sides		

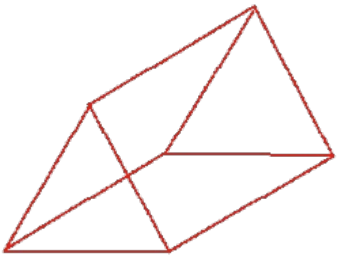
4. Explain the difference between prism and pyramid.



Think Discuss and write

If the number of sides of a polygonal base of a regular pyramid are infinitely increased what would be the shape of the pyramid?

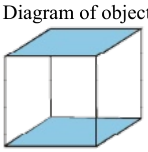
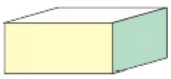
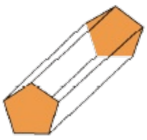
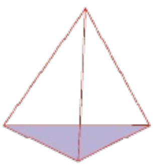
13.6 Number of Edges , Faces and Vertices of polyhedrons

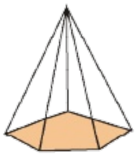


Let us count the number of faces, edges and vertices of a polyhedon .

number of faces = 5 faces
number of edges = 9 edges
number of vertices = 6 vertices

Observe and complete the table.

Diagram of object	Name of the object	Number of Faces(F)	Number of Vertices (V)	Number of Edges (E)	F+V	E+2
	Cube	6	8	12	6 + 8 = 14	12 + 2 = 14
	Cuboid					
	Pentagonal Prism					
	Tetra hedron					



Pentagonal Pyramid

By observing the last two columns of the above table. We can conclude that

$F + V = E + 2$ for all polyhedra.

The relation was first observed by the mathematician Leonhard Euler (pronounced as Oiler).

Leonhard Euler (1707-1783)

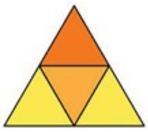
He stated that $\mathbf{F + V = E + 2}$ . . This relation ship is called " **Euler's relation**" for polyhedra.

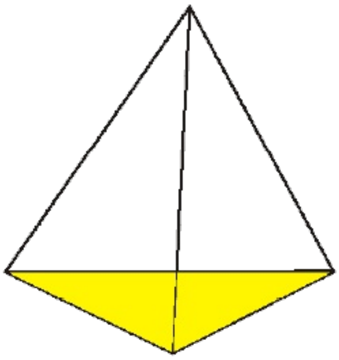
13.7 Net Diagrams

A net is a sort of skeleton - outline in 2-D, which, when folded the net results in 3-D shape.

We can make prisms, pyramids by using net diagrams. Observe the activity given below to make a triangular prism.

Take a piece of paper and cut into a triangle. Mark the vertices as O_1, O_2, O_3 and identify the mid points of sides as A, B, C.



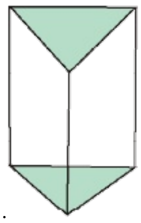
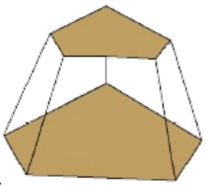
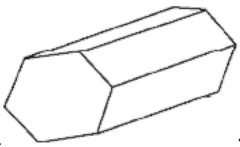
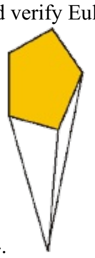


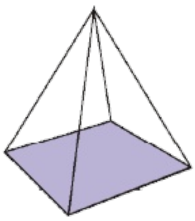
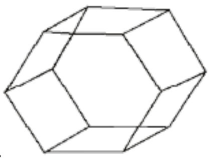
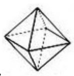
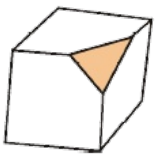
Fold the paper along dotted lines AB, BC, CA and raise the folds till the points O_1, O_2, O_3 meet (say O). By this AO_1 coincides with AO_2, BO_1 with BO_3 and CO_2 with CO_3.

The object so formed is a pyramid. The diagram O_1, O_2, O_3 is a net diagram of the pyramid.

Exercise - 13.2

1. Count the number of faces , vertices , and edges of given polyhedra and verify Euler's formula.

1.  2.  3.  4. 

5.  6.  7.  8. 

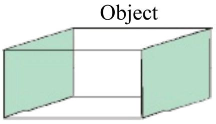
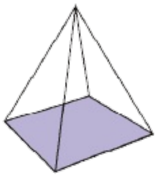
2. Is a square prism and cube are same? explain.
3. Can a polyhedra have 3 triangular faces only? explain.
4. Can a polyhedra have 4 triangular faces only? explain.
5. Complete the table by using Euler's formula.

F	8	5	?
V	6	?	12

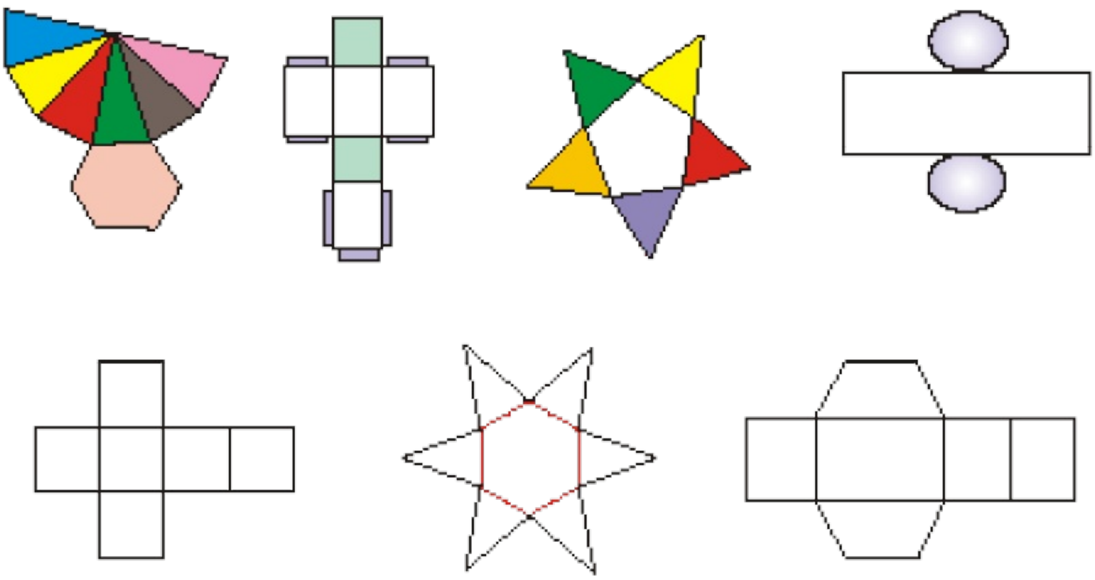
E	?	9	30

6. Can a polyhedra have 10 faces , 20 edges and 15 vertices ?

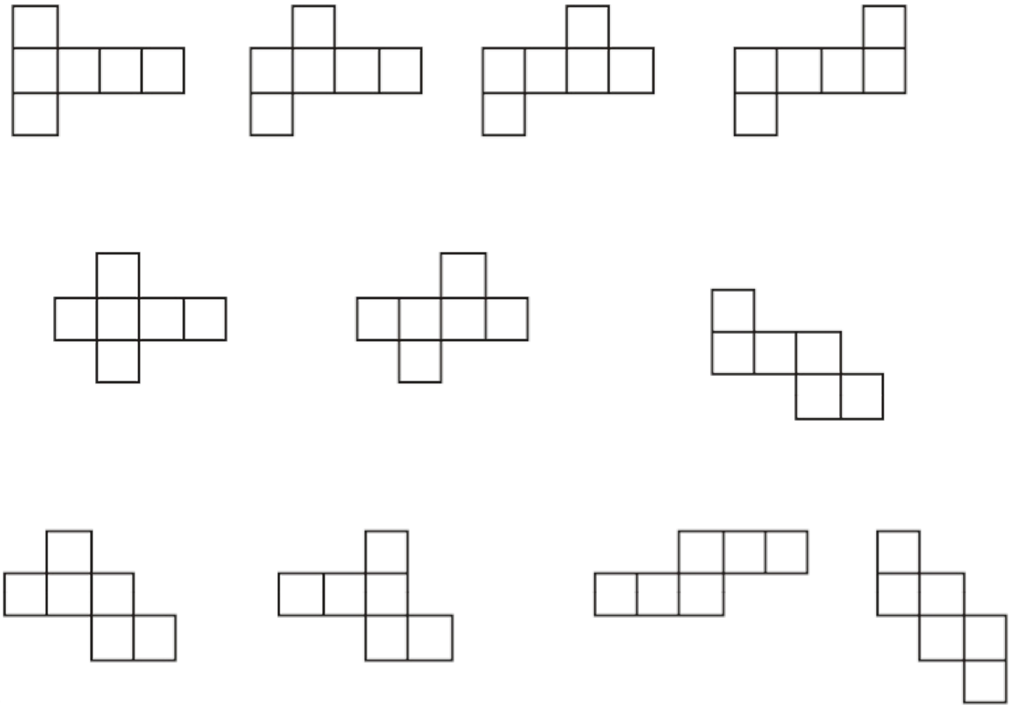
7. Complete the following table

Object	No. of vertices	No. ofedges
		
		
		

8. Name the 3-D objects or shapes that can be formed from the following nets.



9. Draw the following diagram on the check ruled book and find out which of the following diagrams makes cube ?

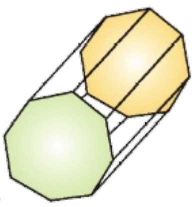


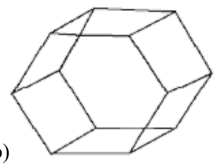
(i)

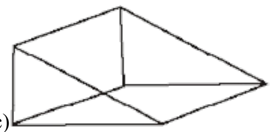
(ii). Answer the following questions.

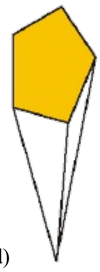
(a) Name the polyhedron which has four vertices, four faces?
(b) Name the solid object which has no vertex?
(c) Name the polyhedron which has 12 edges?
(d) Name the solid object which has one surface?
(e) How a cube is different from cuboid?
(f) Which two shapes have same number of edges , vertices and faces?
(g) Name the polyhedron which has 5 vertices and 5 faces?

(iii). Write the names of the objects given below .


(a)


(b)


(c)

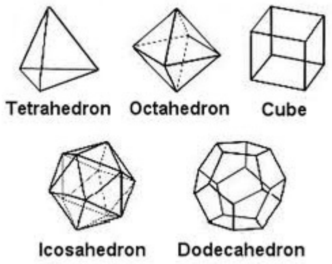

(d)

What we have discussed

1. How to draw 3-D objects on 2-D isometric dot paper.
2. Three different views of 3-D shapes, top view, side view and front view.
3. Polyhedron : Solid objects having flat surfaces.
4. Prism : The polyhedra have top and base as same polygon and other faces are rectangular (parallelogram).
5. Pyramids : Polyhedron which have a polygon as base and a vertex, rest of the faces are triangles.
6. 3-D objects could be make by using 2-D nets.
7. Euler's formula for polyhedra : E + 2 = F + V.

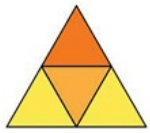
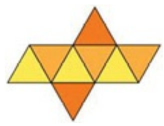
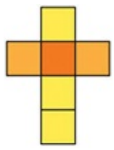
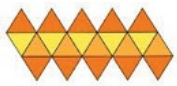
Do you Know?

There are only five regular polyhedra, all of them are complex, often referred as **Platonic solids** as a tribute to Plato



Cube is the only polyhedron to completely fill the space.

Net diagrams of Platonic Solids

Polyhedron Name	**Face of polygons**	**Net diagram**
Tetrahedron	4 Triangles	
Octahedron	8 Triangles	
Cube	6 Squares	
Icosahedron	20 Triangles	
Dodecahedron	12 Pentagons