Sample Paper-04
Mathematics
Class – XI

Time allowed: 3 hours Maximum Marks: 100

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Compute $(1+2i)i-\frac{3+2i}{1-i}$
2. Write the domain and range of the function $\cos^{-1}x$
3. Find the sign of y if $y=\sin(\cos^{-1}x)$
4. Find $\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)$
5. Write the coordinates of the point of intersections of the parabola represented by $y^2=4ax$ and its latus rectum
6. Find x and y if $(x+7,8)=(10,x+y)$

Section B

7. Solve $\sin^2 x+\sin^2 2x=1$
8. Find the value of $i^{30}+i^{40}+i^{60}$
9. Determine whether the points $(0,0)$ and $(5,5)$ lie on different sides of the straight line $x+y-8=0$ or on the same side of the straight line.
10. Prove that $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$
11. Prove by mathematical induction that $11^{n+2}+12^{2n+1}$ *is divisible by* 133 for all positive integer values of n

12. A, B, C are 3 sets and U is the universal set such that

$n(U)=800, n(A)=200, n(B)=300, n(A\cap B)=100$ Find $n(A'\cap B')$

13. If α, β are the roots of the equation $x^2-bx+c=0$ find the value of $\alpha^2+\beta^2$

14. If P be the sum of the odd terms and Q the sum of the even terms in the expansion of $(x+a)^n$, prove that $P^2-Q^2=(x^2-a^2)^n$

15. Solve the inequality $\frac{x^2-3x+6}{3+4x}<0$

16. Prove that $\cot(A+15)-\tan(A-15)=\frac{4\cos 2A}{1+2\sin 2A}$

17. Find the domain of the function $f(x)=\sqrt{4-x^2}$

18. Evaluate $\frac{1}{2+\cos\theta+\sin\theta}$ if $\tan\frac{\theta}{2}=2$

19. Find the limit $\lim_{x\to 0}\frac{\sin 5x}{x+x^3}$

Section C

20. Differentiate $\log_{10} x$ with respect to x

21. How many 6 digits numbers can be formed with the digits 1, 2, 3, 4, 5, 6, 7 if the 10^{th}, unit's places are always even and repetition is not allowed.

22. Shift the origin to a suitable point so that the equation $x^2+y^2-4x+6y=36$ representing a circle is

transformed in to an equation of a circle with centre at origin in the new coordinate axes.

23. The mean and variance of 7 observations are 8 and 19 respectively. If 5 of the observations are 2, 4,12,14,11. Find the remaining observations.

24. Prove that $\frac{1}{\log_a b}, \frac{1}{\log_{2a} b}, \frac{1}{\log_{4a} b}$ form an AP

25. On the average one person dies out of every 10 accidents find the probability that at least 4 will be safe out of 5 accidents.

26. In the expansion $(1+x)^{40}$, the coefficients of T_{2r+1} and T_{r+2} are equal, find r

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Mathematics
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ANSWERS

Section A

1. **Solution:**

$$\frac{3+2i}{1-i}=\frac{3+2i}{1-i}.\frac{1+i}{1-i}$$
$$=\frac{3+2i+3i+2i^2}{1-i^2}=\frac{1}{2}+\frac{5}{2}i$$
$$(1+2i)i-\frac{3+2i}{1-i}=(-2+i)-(\frac{1}{2}+\frac{5}{2}i)=-\frac{5}{2}-\frac{3}{2}i$$

2. **Solution:**

Domain= $[-1,1]$ Range= $[0,\pi]$

3. **Solution :**

$0\le\cos^{-1}x\le\pi$

sin in this interval is positive and hence y is positive

4. **Solution:**

$$\sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)=\sin^{-1}\left(sin\left(\pi-\frac{\pi}{7}\right)\right)$$
$$=\sin^{-1}\left(sin\left(\frac{\pi}{7}\right)\right)$$
$$=-\frac{\pi}{2}\le\frac{\pi}{7}\le\frac{\pi}{2}$$
$$=\frac{\pi}{7}$$

5. S**olution:** $(a,2a)$,$(a,-2a)$

6. **Solution:**

$x+7=10$

$x=3$

$x+y=8$

$y=5$

Section B

7. **Solution:**

$$\frac{1-\cos 2x}{2}+\frac{1-\cos 4x}{2}=1$$

cos2x + cos4x = 0

2 cos3x cos x = 0

Cos3x = 0

$$x=\frac{\pi}{6}+\frac{\pi}{3}n$$

$$Cosx=0$$

$$x=\frac{\pi}{2}+\pi k=\frac{\pi}{6}+\frac{\pi}{3}n \quad n \text{ is integer}$$

8. **Solution:**

$$i^{30}+i^{40}+i^{60}=(i^4)^7.i^2+(i^4)^{10}+(i^4)^{15}$$

$$i^4=1=-1+1+1=1$$

9. **Solution:**

Substituting the points (0, 0) and (5, 5) on the given line

x + y – 8 = 0

0 + 0 – 8 = -8

5 + 5 – 8 = 2

Since the signs of the resulting numbers are different the given points lie on opposite sides of the given line.

10. **Solution :**

$$\tan^{-1} x = A$$

$$\tan A = x$$

$$\cot^{-1} x = B$$

$$\cot B = x$$

$$\tan(\frac{\pi}{2}-B)=x$$

$$\tan^{-1} x=\frac{\pi}{2}-B$$

$$\tan^{-1} x = A$$

$$A=\frac{\pi}{2}-B$$

$$A+B=\frac{\pi}{2}$$

$$\tan^{-1} x+\cot^{-1} x=\frac{\pi}{2}$$

11.**Solution:**

$11^{n+2}+12^{2n+1}$ is divisible by 133

n = 1

$11^3+12^3=(11+12)(11^2-11.12+12^2)$

=23.133

Let it be true for k

$11^{k+2}+12^{2k+1}$ is divisible by 133

For k = k + 1

$11^{k+3}+12^{2k+3}=11.11^{k+2}+12^2.12^{2k+1}$

$=11.11^{k+2}+144.12^{2k+1}$

$=11.11^{k+2}+133.12^{2k+1}+11.12^{2k+1}$

$=11.11^{k+2}+11.12^{2k+1}+133.12^{2k+1}$

Is divisible by 133 since $11^{k+2}+12^{2k+1}$ is divisible by 133

12.**Solution:**

$n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$

$=n(U)-[n(A)+n(B)-n(A\cap B)]$

$800-[200+300-100]$

$=400$

13.**Solution:** $\alpha+\beta=b$

$\alpha\beta=c$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=b^2-2c$

14.**Solution:**

$(x+a)^n=P+Q$

$(x-a)^n=P-Q$

$(P+Q)(P-Q)=(x+a)(x-a)$

$P^2-Q^2=(x^2-a^2)^n$

15.**Solution:**

Discriminant of numerator = 9 – 24 < and

Coefficient of x^2 is positive. Hence Numerator is always positive

Hence dividing by the numerator on both sides of

The equality does not change the sign of the inequality

Hence we need only consider $\frac{1}{3x+4}<0$

$$x<\frac{-4}{3}$$

$$x\in(-\infty,-\frac{4}{3})$$

16.**Solution:**

$$\cot(A+15)-\tan(A-15)=\frac{\cos(A+15)}{\sin(A+15)}-\frac{\sin(A-15)}{\cos(A-15)}$$

$$=\frac{\cos(A+15)\cos(A-15)-\sin(A+15)\sin(A-15)}{\sin(A+15)\cos(A-15)}$$

$$=\frac{\cos 2A}{\frac{1}{2}(\sin 2A+\frac{1}{2})}$$

$$=\frac{2\cos 2A}{\sin 2A+\frac{1}{2}}$$

$$=\frac{4\cos 2A}{1+2\sin 2A}$$

17.**Solution:**

$$4-x^2\geq 0$$

$$x^2-4\leq 0$$

$$\textit{Domain of } x\in[-2,2]$$

$$y^2=4-x^2$$

$$x^2=4-y^2$$

$$x=\sqrt{4-y^2}$$

$$4-y^2\geq 0$$

$$y^2-4\leq 0$$

$$y\in[-2,2]$$

Also for all values of $x\in[-2,2]$

$$y=\sqrt{4-x^2}\geq 0$$

$$\textit{Range } y\in[0,2]$$

18.**Solution:**

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\cos\theta = \frac{1-4}{1+4} = \frac{-3}{5}$$

$$\sin\theta = \frac{2\tan\theta 2}{1+\tan^2\frac{\theta}{2}} = \frac{2.2}{1+4} = \frac{4}{5}$$

$$\frac{1}{2+\cos\theta+\sin\theta} = \frac{1}{2-\frac{3}{5}+\frac{4}{5}} = \frac{11}{5}$$

19.**Solution:**

$$\lim_{x\to 0}\frac{\sin 5x}{x+x^3} = \lim_{x\to 0}\frac{5\sin 5x}{5x(1+x^2)}$$

$$= \lim_{x\to 0}\frac{5\sin 5x}{5x}\lim_{x\to 0}\frac{1}{(1+x^2)}$$

$$= 5.1.1$$

$$= 5$$

Section C

20.**Solution :**

$$y = \log_{10} x$$

$$x = 10^y$$

$$\log_e x = y\log_e 10$$

$$y = \frac{\log_e x}{\log_e 10}$$

$$\frac{dy}{dx} = \left(\frac{1}{\log_e 10}\right)\frac{1}{x}$$

21.**Solution:**

There are 3 even numbers 2, 4, 6

So the units place, 10th places can be filled in $3p_2$ ways

Remaining 5 digits can be used to fill 4 places in $5p_4$ ways.

Hence the total numbers satisfying the above condition is $3p_2 \times 5p_4 = 720$

22.**Solution:**

Let the origin be shifted to (h,k)

$$x = x' + h$$
$$y = y' + k$$

Then

$$(x'+h)^2 + (y'+k)^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + 2hx' + h^2 + y'^2 + 2ky' + k^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + y'^2 + x'(2h-4) + y'(2k+6) + h^2 + k^2 - 4h + 6k - 36 = 0$$

2h – 4 = 0

h = 2

2k + 6 = 0

k = -3

$$x'^2 + y'^2 + 2^2 + (-3)^2 - 8 - 18 - 36 = 0$$
$$x'^2 + y'^2 + 13 - 62 = 0$$
$$x'^2 + y'^2 = 49$$

23.**Solution:**

$$\frac{2+4+12+14+11+x+y}{7} = 8$$
$$43 + x + y = 56$$
$$x + y = 13$$

$$\frac{2^2+4^2+12^2+14^2+11^2+x^2+y^2}{7} - (mean)^2 = 19$$
$$\frac{4+16+144+196+121+x^2+y^2}{7} - 64 = 19$$
$$\frac{481+x^2+y^2}{7} = 83$$

$$481 + x^2 + y^2 = 581$$
$$x^2 + y^2 = 100$$
$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

$$169 + (x-y)^2 = 200$$
$$(x-y)^2 = 31$$
$$x - y = 5.57$$
$$x + y = 13$$
$$x = 9.285$$
$$y = 3.715$$

24.**Solution:**

$$\frac{1}{\log_a b} = \log_b a$$

$$\frac{1}{\log_{2a} b} = \log_b 2a$$

$$\frac{1}{\log_{4a} b} = \log_b 4a$$

$$\frac{\log_b a + \log_b 4a}{2} = \frac{\log_b (2a)^2}{2}$$

$$= 2\frac{\log_b 2a}{2}$$

$$= \log_b 2a$$

Thus, $\frac{1}{\log_{2a} b}$ is, the, AM, between $\frac{1}{\log_a b}, \frac{1}{\log_{4a} b}$

25.**Solution:**

Probability of surviving = $\frac{9}{10}$

Required to find out the probability of 4 are safe or 5 are safe

Probability of 5 is safe = $\left(\frac{9}{10}\right)^5$

Probability of 4 is safe = ${}^5C_4\left(\frac{9}{10}\right)^4 \frac{1}{10}$

Required Probability = $\left(\frac{9}{10}\right)^5 + 5\left(\frac{9}{10}\right)^4 \frac{1}{10} = \frac{45927}{5000}$

26.**Solution:**

$$T_{2r+1} = {}^{40}C_{2r}$$

$$T_{r+2} = {}^{40}C_{r+1}$$

$${}^{40}C_{2r} = {}^{40}C_{r+1}$$

$$2r + r + 1 = 40$$

$$3r = 39$$

$$r = 13$$