Chapter 4

Two Port Networks

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Open circuit or impedance(z) parameters
- Equivalent circuit of z-parameters
- y-parameters or short circuit admittance parameters
- Hybrid parameters
- g-parameters or inverse hybrid parameters
- Transmission or abcd parameters
- inverse transmission parameters
- Inter connection of networks
- Terminated two-port network
- Network functions
- Network graphs
- Twigs and links
- Incidence matrix and formulation of KCL
- Tie-set matrix and branch currents

Introduction

A pair of terminals through which a current may enter or leave a network is known as a port.

The current entering one terminal leaves through the other terminals so that the net current in the port equals zer o.

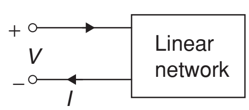


Figure 1 One-port network

A two-port networks is an electrical network with two separate ports for input and output.

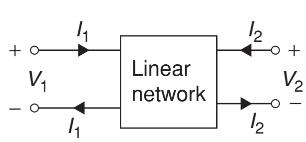


Figure 2 Two-port network

To characterize a two-port network required that we relate the terminal quantities V_1, V_2, I_1 and I_2.

Classification of Networks

1. **Linear circuits:**
   It is the circuit whose parameters remain constant with change in applied voltage or current ($V \alpha I$ ohm's Law)
   Example: Resistance, inductance and capacitance
2. **Non–linear circuits:**
   It is a circuit whose parameters changed with voltage or current.
   Example: Diodes, transistor … etc
   Non linear circuits do not obey ohm's Law.
3. **Unilateral circuits Bilateral circuit:**
   When the direction of current is changed, the characteristic or properties of the circuit may change. This circuit is called unilateral circuits.
   Example: Diode, transistor, UJT … etc
   Otherwise, it is called bilateral circuit.
   Example: *R*, *L*, C circuits.

Active and Passive Elements

If a circuit element has the capability of enhancing the energy level of a signal passing through it, it is called an active element.
Example: Transistors, op-amp, vaccum tubes … etc. Otherwise it is called passive elements.
Example: Resistors, inductors, thermistors capacitors etc are passive elements.

Lumped and distributed network

Physically separable network elements are like *R*, *L* and *C* are known as lumped elements.

A transmission line on a cable in the other hand is an example of distributed parameter network. They are not physically separable. If the network is fabricated with its elements in lumped form, it is called a lumped network and if in distributed form it is called distributed network.

Recurrent and Non-recurrent networks

When a large circuit consists of similar networks connected one after another, the network is called as recurrent network or cascaded network. It is also called as ladder network. Otherwise a single network is called non-recurrent network.

Symmetrical and asymmetrical network

If the network looks the same from both the ports then it is said to be symmetrical. Otherwise it is called asymmetric network.

Examples:

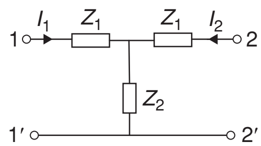


Figure 3 Symmetrical network

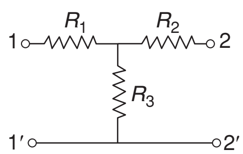


Figure 4 Symmetrical network

Reciprocal and Non-reciprocal networks

If the network obeys the reciprocity theorem then it is called reciprocal network. Otherwise it is called non-reciprocal network.

All the passive networks are always reciprocal and all the active networks are always non-reciprocal.

Network Configuration

T-section

When a network section looks like a '*T*'. It is known as T-section.

Examples:

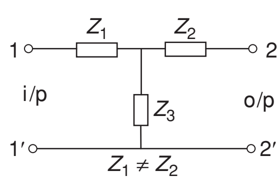


Figure 5 Un-symmetrical T-section

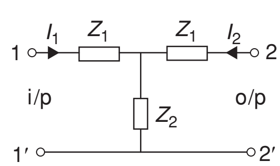


Figure 6 Symmetrical and unbalanced T-section

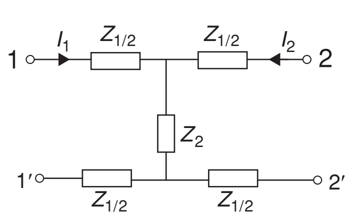


Figure 7 Balanced symmetrical T-section

π-section

Examples:

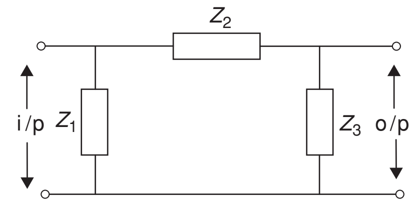


Figure 8 Unbalanced asymmetrical π-section ($Z_1 \neq Z_3$)

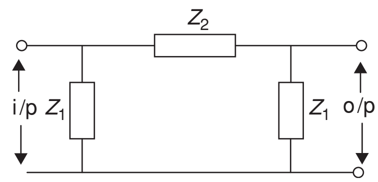


Figure 9 Unbalanced symmetrical π-section

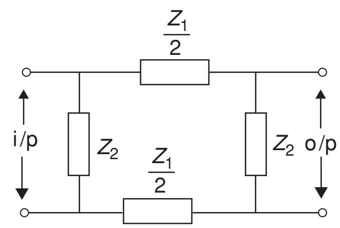
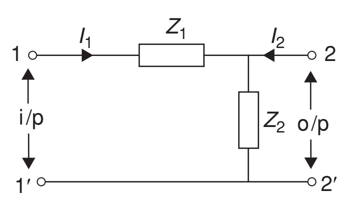


Figure 10 Balanced symmetrical π-section

L-section

When the network section looks like '*L*' the configuration is termed as L-section.



Lattice Section

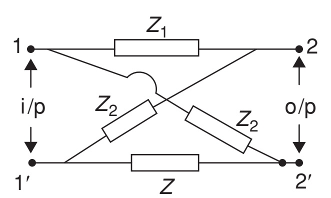
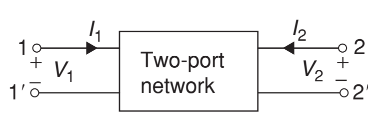


Figure 11 Symmetrical lattice section

Two-port netwoks



A two-port network has two pairs of accessible terminals; one pair represents the input and the other represents the output. Both the currents I_1 and I_2 enter the network and the polarities of the voltages are shown in the figure. There are four variable V_1, V_2, I_1 and I_2 of these four variables, two can be taken as independent variables, the remaining two will be dependent variables.

Open Circuit or Impedance (*Z*) Parameters

Here the two voltages V_1 and V_2 are functions of I_1 and I_2

$V_1 = f(I_1, I_2)$

$V_2 = f(I_1, I_2)$

$[V] = [Z] \cdot [I]$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (2)$$

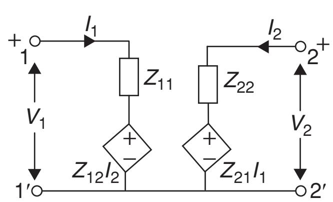
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I1=0}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \end{bmatrix}$$

From (1) and (2), the network can be drawn as shown in figure

Equivalent Circuit of *Z*-Parameters



Condition of reciprocity and symmetry

Network must be reciprocal when ratio of response at port 2 to the excitation at port 1, is same as ratio of response at port 1 to port 2, then the network is called reciprocal.

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{V_1}{I_2} \text{ at } I_1 = 0$$

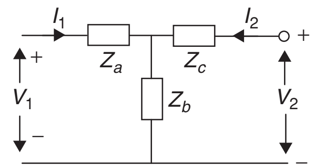
Condition for reciprocal:

$$Z_{12} = Z_{21}$$

Condition for symmetrical network:

$$\boxed{Z_{11} = Z_{22}}$$

Example 1:



Find the *Z*-parameters for the circuit shown in figure

Solution: $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Let $I_2 = 0$

$V_1 = I_1 (Z_a + Z_b)$

$$Z_{11} = \frac{V_1}{I_1} = Z_a + Z_b$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$V_2 = I_1 Z_b$

$Z_{21} = Z_b$

Let $I_1 = 0$

$V_2 = I_2 (Z_c + Z_b)$

$Z_{22} = Z_c + Z_b$

$V_1 = I_2 Z_b$

$Z_{12} = Z_b$

$$\Rightarrow \quad [Z] = \begin{bmatrix} Z_a + Z_b Z_b \\ Z_b Z_b + Z_c \end{bmatrix}$$

Example 2: The following readings are obtained experimentally for an unknown two-port network:

	V_1	V_2	I_1	I_2
o/p open	80 V	60 V	10 A	0
i/p open	50 V	40 V	0	5 A

The *Z*-parameters are

(A) $\begin{bmatrix} 8 & 6 \\ 10 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} 8 & 10 \\ 6 & 8 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & 10 \\ 8 & 4 \end{bmatrix}$

(D) None of the above

Solution: (B)
We know $[V] = [Z]\,[I]$

If $I_2 = 0$; $Z_{11} = \dfrac{V_1}{I_1}$ and $Z_{21} = \dfrac{V_2}{I_1}$

So $Z_{11} = \dfrac{80}{10} = 8\,\Omega$

$Z_{21} = \dfrac{60}{10} = 6\,\Omega$

If $I_1 = 0$.

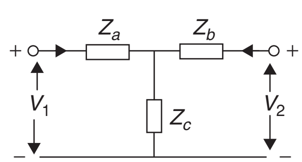
$$Z_{22} = \frac{V_2}{I_2} \text{ and } Z_{12} = \frac{V_1}{I_2}$$

$$Z_{22} = \frac{40}{5} = 8\,\Omega \text{ and } Z_{12} = \frac{50}{5} = 10\,\Omega$$

$$\therefore\quad [Z] = \begin{bmatrix} 8 & 10 \\ 6 & 8 \end{bmatrix}$$

$\therefore Z_{11} = Z_{22} = 8\,\Omega \Rightarrow$ Symmetrical network
$Z_{12} \neq Z_{21} \Rightarrow$ Non-reciprocal network

Example 3:



If $Z_a = 2\,\angle 0°$, $Z_b = 5\,\angle{-90°}$, and $Z_c = 3\,\angle 90°$. Then the above T-network, Z-parameters are

(A) Symmetrical and Reciprocal
(B) Symmetrical and Non-reciprocal
(C) Asymmetrical and Reciprocal
(D) Asymmetrical and Non-Reciprocal

Solution: (C)
Apply KVL, the loop equations are
$V_1 = 2I_1 + 3\,\angle 90°(I_1 + I_2)$
$V_1 = (2 + j3)\,I_1 + j3I_2$ (3)
$V_2 = 5\,\angle{-90°}I_2 + 3\,\angle 90°\,(I_1 + I_2)$
$V_2 = j3\,I_1 + (3j - 5j)\,I_2$
$V_2 = j3I_1 - 2jI_2$ (4)
From equation (3) and (4)

$$[Z] = \begin{bmatrix} 2+j3 & j3 \\ j3 & -2j \end{bmatrix} = \begin{bmatrix} 3.6\angle 56° & 3\angle 90° \\ 3\angle 90° & 2\angle -90° \end{bmatrix}$$

$Z_{11} \neq Z_{22} \Rightarrow$ unsymmetrical
$Z_{12} = Z_{21} \Rightarrow$ Reciprocal network.

Y-Parameters or Short Circuit Admittance Parameters

In a two port network, the input currents I_1 and I_2 can be expressed in terms of input and output voltages V_1 and V_2 respectively as $[I] = [Y]\,[V]$
where $[Y]$ is the admittance matrix.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

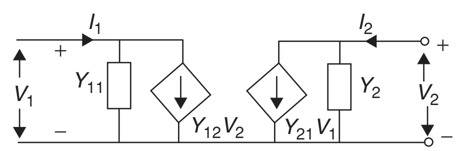
Here, $I_1 = f(V_1, V_2)$
$I_2 = f(V_1, V_2)$
$I_1 = Y_{11}V_1 + Y_{12}V_2$ (5)
$I_2 = Y_{21}V_1 + Y_{22}V_2$ (6)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

From (5) and (6), the circuit can be drawn as shown in the figure.

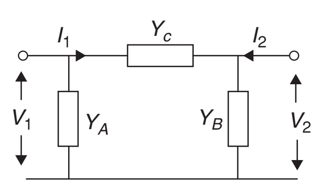
Equivalent Circuit of Y-Parameters



Condition for reciprocity and symmetrical

1. If $Y_{11} = Y_{22} \Rightarrow$ Symmetrical otherwise asymmetrical network
2. If $Y_{12} = Y_{21} \Rightarrow$ Reciprocal network. (Or) passive network otherwise Non-reciprocal or active network.

Example 4: Find the *Y*-parameters of the following π-circuit shown in below figure.



Solution: Using KCL at node a
$I_1 = V_1\,Y_A + (V_1 - V_2)\,Y_c$
$I_1 = (Y_A + Y_C)\,V_1 - (Y_C)\,V_2$ (7)
Apply KCL at node *b*
$I_2 = V_2.\,Y_B + (V_2 - V_1)\,Y_C$
$I_2 = -Y_c\,V_1 + (Y_B + Y_C)\,V_2$ (8)
From equation (7) and (8)

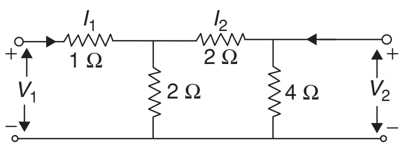
$$[Y] = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix}$$

Where,

$$Y = \frac{1}{R}, Y_L = \frac{1}{SL}$$

And $Y_C = SC$

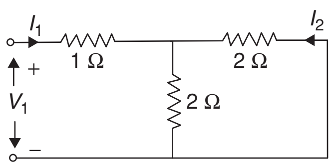
Example 5: Find the Y-parameters for the network shown in figure



Solution: $I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

With $V_2 = 0$



$V_1 = I_1 \times (1 + 2 \parallel 2)$

$V_1 = I_1 (1 + 1)$

$$\frac{I_1}{V_1} = \frac{1}{2} = 0.5 \text{ mho}$$

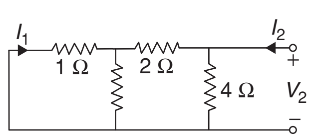
$$-I_2 = I_1 \times \frac{2}{2+2}$$

$$-I_2 = I_1 \times \frac{2}{4}$$

$$-I_2 = \frac{V_1}{2} \times \frac{1}{2}$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{-1}{4} \text{ mho}$$

With $V_1 = 0$



$V_2 = I_2 [(1 \| 2 + 2) \parallel 4]$

$$V_2 = I_2 \left[\left(\frac{1 \times 2}{1+2} + 2 \right) \parallel 4 \right]$$

$$= \left[\left(\frac{2}{3} + 2 \right) \parallel 4 \right]$$

$$V_2 = I_2 \left[\frac{\frac{8}{3} \times 4}{\frac{8}{3} + 4} \right] = \frac{\frac{8}{3}}{\frac{8+12}{3}} = \frac{32}{20} = \frac{8}{5}$$

$$\frac{I_2}{V_2} = Y_{22} = \frac{5}{8} \text{ mho}$$

$$I_2' = I_2 \times \frac{4}{4 + \frac{8}{3}}$$

$$I_2' = I_2 \times \frac{12}{20}$$

$$-I_1 = I_2' \times \frac{2}{2+1}$$

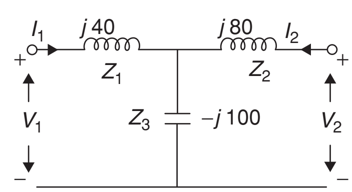
$$-I_1 = I_2 \times \frac{12}{20} \times \frac{2}{3}$$

$$-I_1 = I_2 \times \frac{2}{5}$$

$$-I_1 = \frac{5}{8} V_2 \times \frac{2}{5}$$

$$\frac{I_1}{V_2} = Y_{12} = \frac{-1}{4} \text{ mho}$$

Example 6. Find Y-parameters of network shown in figure.



Solution:

$Y = [Z]^{-1}$

$$[Z] = \begin{bmatrix} -j60 & -j100 \\ -j100 & -j20 \end{bmatrix} \Rightarrow \begin{bmatrix} j60 & j100 \\ j100 & j20 \end{bmatrix}$$

$[Y] = [Z^{-1}]$

$$= \frac{1}{|Z|}\{\text{adj}Z\} = \frac{1}{8800}\begin{bmatrix} -j20 & j100 \\ j100 & -j60 \end{bmatrix}$$

$$= \begin{bmatrix} -j2.27\times10^{-3}\ \Omega & j11.36\times10^{-3}\ \Omega \\ j11.36\times10^{-3}\ \Omega & -j6.8\times10^{-3}\ \Omega \end{bmatrix}$$

$$Y_{11} = -j2.27\times10^{-3}\ \Omega$$
$$Y_{12} = -j11.36\times10^{-3}\ \Omega$$
$$Y_{21} = -j11.36\times10^{-3}\ \Omega$$
$$Y_{22} = -j6.8\times10^{-3}\ \Omega$$

Hybrid Parameters

The Z and Y-parameters of a two-port network do not always exist.

A two port network can be represented using the h-parameters. The describing equations for h-parameters are

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

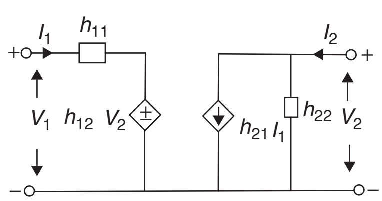
The value of the parameters are determined as

$$h_{11} = \frac{\frac{V_1}{I_1}}{V_2} = 0;\ h_{12} = \frac{\frac{V_1}{V_2}}{I_1} = 0$$

$$h_{21} = \frac{\frac{I_2}{I_1}}{V_2} = 0;\ h_{22} = \frac{\frac{I_2}{V_2}}{I_1} = 0$$

Where

$h_{11} \Rightarrow$ short circuit input impedance
$h_{12} \Rightarrow$ open circuit reverse voltage gain
$h_{21} \Rightarrow$ short circuit forward current gain
$h_{22} \Rightarrow$ open circuit o/p admittance



Condition of reciprocity

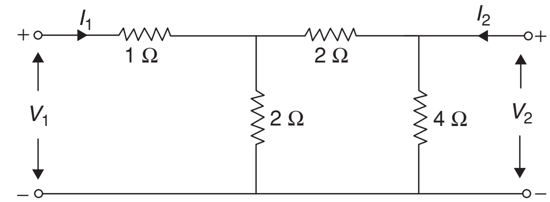
$$\boxed{h_{12} = -h_{21}}$$

Condition of symmetry

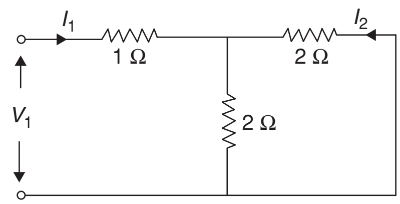
$$|h| = 1$$

$$\text{i.e., } h_{11}\cdot h_{22} - h_{12}\cdot h_{21} = 1$$

Example 7 Find the h-parameters of the network shown in figure



With $V_2 = 0$



$$\frac{V_1}{I_1} = h_{11} = 1 + 2\,||\,2$$
$$= 1 + 1 = 2\ \Omega$$
$$-I_2 = I_1 \times \frac{2}{2+2}$$
$$-I_2 = I_1 \times \frac{1}{2}$$
$$h_{21} = \frac{I_2}{I_1} = \frac{-1}{2}$$

With $I_1 = 0$

$$V_1 = V_2 \times \frac{2}{2+2}$$
$$\frac{V_1}{V_2} = h_{12} = \frac{1}{2}$$
$$V_2 = I_2 \times 4||(2+2)$$
$$V_2 = I_2 \times 2$$
$$\frac{I_2}{V_2} = h_{22} = \frac{1}{2}$$

G-Parameters or Inverse Hybrid Parameters

These are represented by

$I_1 = g_{11} V_1 + g_{12} I_2$
$V_2 = g_{21} V_1 + g_{22} I_2$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

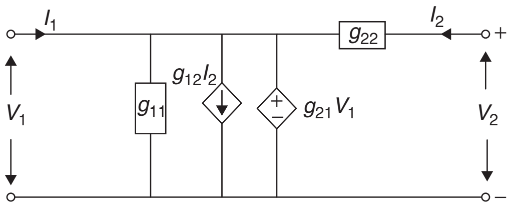


Figure 12 *G*-parameter equivalent circuit

The *G*-Parameters can be defined as

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2 = 0}; \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_i = 0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2 = 0}; \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1 = 0}$$

Condition for reciprocity

$$g_{12} = -\,g_{21}$$

Condition for symmetry

$$|g| = 1$$

$$\text{i.e., } g_{11} \cdot g_{22} - g_{12} \cdot g_{21} = 1$$

Transmission or ABCD Parameters

The transmission parameters express the required source variables V_1 and I_1 in terms of the existing destination variables V_2 and I_2. They are called ABCD or *T*-parameters and are defined by

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The transmission parameters are determined as

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}; \qquad \text{(No units)}$$

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2 = 0} \qquad (\Omega)$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0} \qquad \text{(mho)}$$

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2 = 0} \qquad \text{(No units)}$$

Thus, the transmission parameters are called specifically

A = Open circuit voltage ratio

B = Negative short-circuit transfer impedance

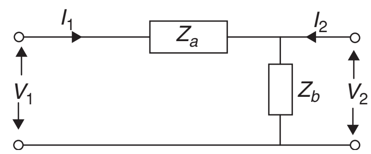
C = Open circuit transfer admittance

D = Negative short circuit current ratio

Condition for reciprocity and symmetry

$AD - BC = 1 \Rightarrow$ Reciprocal

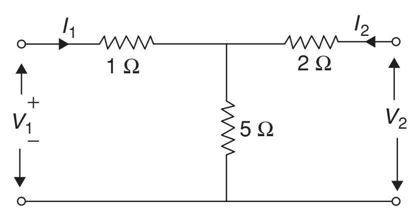
And Symmetrical $A = D$, if



Above circuit *T*-parameters definedas

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{Z_a}{Z_b} & Z_a \\ \dfrac{1}{Z_b} & 1 \end{bmatrix}$$

Example 8:



Find the transmission parameters for the circuit shown in figure

Solution:

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

With $I_2 = 0$

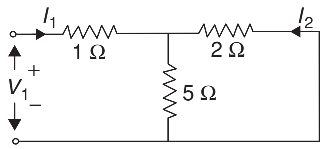
$$V_2 = V_1 \times \frac{5}{5+1}$$

$$A = \frac{V_1}{V_2} = \frac{6}{5}$$

$$V_2 = I_1 \times 5$$

$$C = \frac{I_1}{V_2} = \frac{1}{5}$$

With $V_2 = 0$



$$-I_2 = I_1 \times \frac{5}{5+2}$$

$$V_1 = I_1 \times \left[1 + \frac{5\times 2}{5+2}\right] = I_1\left[1+\frac{10}{7}\right]$$

$$V_1 = \frac{-7}{5} I_2 \times \frac{17}{10}$$

$$B = \frac{-V_1}{I_2} = \frac{-119}{50}$$

$$D = \frac{-I_1}{I_2}$$

$$D = \frac{7}{5}.$$

Inverse Transmission Parameters

$V_2 = A'\,V_1 - B'\,I_1$
$I_2 = C'\,V_1 - D'\,I_1$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

The inverse transmission parameters can be defined as

$$A' = \frac{\frac{V_2}{V_1}}{I_1} = 0$$

Forward voltage ratio with sending end open circuit is:

$$C' = \frac{I_2}{V_1}\bigg|_{I_1=0}$$

Transfer admittance with sending end open circuit is:

$$B' = \frac{V_2}{-I_1}\bigg| V_1 = 0$$

Transfer impedance with sending end short circuit:

$$D = \frac{I_2}{-I_1}\bigg| V_1 = 0$$

$A' = D' \Rightarrow$ Symmetrical
$A'D' - B'C' = 1 \Rightarrow$ Reciprocal

Relationships between parameters

1. $[Y] = [Z]^{-1}$
2. $[g] = [h]^{-1}$
3. $[t] \neq [T]^{-1}$

Inter Connection of Networks

A large complex network may be divided into sub networks for the purpose of analysis and design. The sub networks are modelled as two port networks, inter connected to form the original network. The inter connection can be in series, in parallel, or in cascade. The inter connected network can be described by any of the six parameters sets. For example, when the networks are in series, their undivided Z-parameters add up to give the Z-parameter of the larger network.

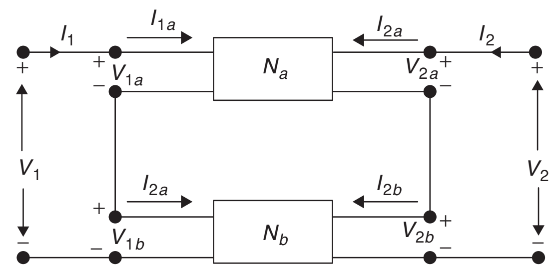


Figure 13 Series connection of two-port networks

$$[Z] = [Z_a] + [Z_b]$$

1. Two-port n/w are in parallel when their port voltages are equal and the port currents of the larger network are the sums of the individual port elements. The parallel connection of two two-port networks is shown in figure.

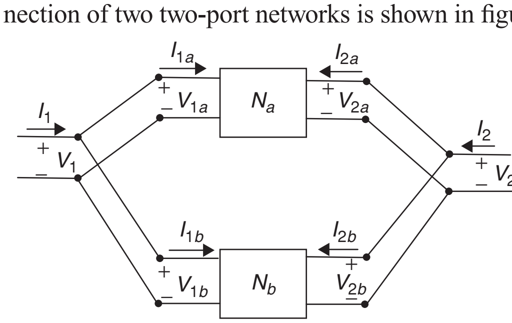


Figure 14 Parallel connection of two two-port networks

From the above figure,
$I_{1a} = Y_{11a}\,V_{1a} + Y_{12a}\,V_{2a}$
$I_{2a} = Y_{22a}\,V_{1a} + V_{22a}\,V_{2a}$
And
$I_{1b} = Y_{11b}\,V_{1b} + Y_{12b}\,V_{2b}$
$I_{2b} = Y_{21b}\,V_{1b} + V_{22b}\,.\,V_{2b}$
But from the figure
$V_1 = V_{1a} = V_{1b}$
$V_2 = V_{2a} = V_{2b}$
And
$I_1 = I_{1a} + I_{2b}$
$I_2 = I_{2a} + I_{2b}$

Thus, the Y-parameters of the overall network are

$$[Y] = [Y_a] + [Y_b]$$

2. Two networks are said to be cascaded when the O/P of one is the I/p of the other.

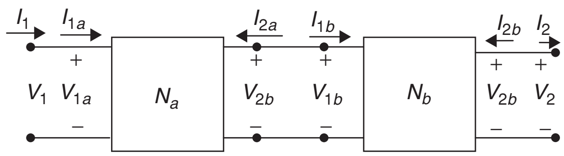
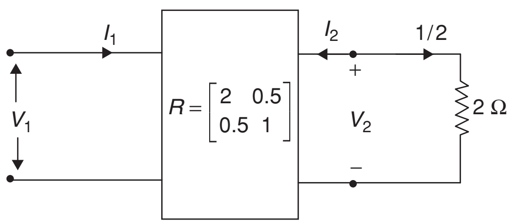


Figure 15 Cascade connection of two-port networks

$$[T] = [T_a]\,[T_b]$$

Example 7: Determine the value of V_1.



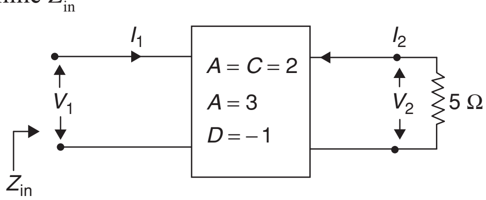
(A) 6 V (B) 5.5 V
(C) 5.75 V (D) None of the above

Solution: (C)
Form the given data
$I_2 = -1/2$ A
$V_2 = 1/2 \times 2 = 1$ V
$V_1 = 2I_1 + 0.5I_2$ (9)
$V_2 = 0.5I_1 + 1I_2$ (10)
Sub I_2 and V_2 in equation (10)

$$1 = 0.5I_1 \quad 0.5 \quad \Rightarrow \quad I_1 = \frac{1.5}{0.5} = 3 \text{ A}$$

$V_1 = 2 \times 3 + 0.5\,(-0.5)$
$= 6 - 0.25$
$= 5.75$ V

Example 8: (C)
Determine Z_{in}



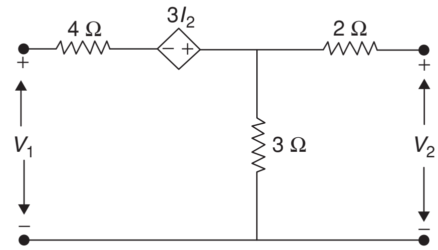
(A) −12/9 (B) 12/13
(C) 13/9 (D) 13/11

Solution: (C)

$$Z_{in} = \frac{AZ_L + B}{CZ_L + D}$$

$$= \frac{2 \times 5 + 3}{2 \times 5 - 1} = \frac{13}{9}.$$

Example 9:



The Z-parameters of the two-port network are

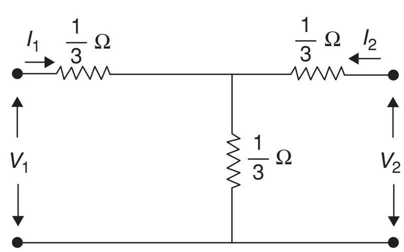
(A) $\begin{bmatrix} 7 & 0 \\ -3 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 3 \\ 0 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 3 \\ 7 & 0 \end{bmatrix}$

Solution: (C)
Applying KVL to the input loop
$V_1 = 4I_1 - 3I_2 + 3(I_1 + I_2)$
$V_1 = 7I_1 + 0.I_2$ (11)
Apply KVL to the O/P loop
$V_2 = 2I_2 + 3(I_1 + I_2)$
$V_2 = 3I_1 + 5I_2$ (12)
From equation (11) and (12)

$$[Z] = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$$

Example 10: A two-port network, shown in fig

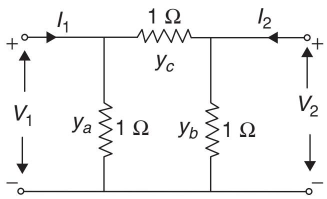


The admittance parameters, Y_{11}, Y_{12}, Y_{21} and Y_{22} are

(A) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -0.5 \\ \frac{-1}{3} & 2 \end{bmatrix}$

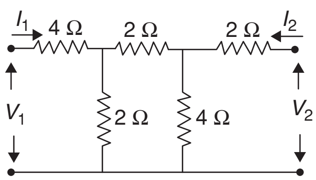
Solution: (A)
Convert star to delta connection



$$[Y] = \begin{bmatrix} y_a + y_c & -y_c \\ -y_c & -y_b + y_c \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Example 11:



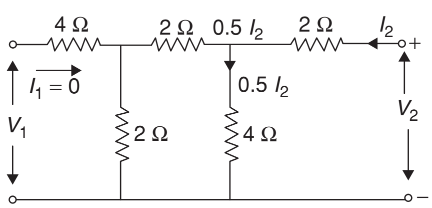
For the two-port network shown in figure the value of h_{12} is given by

(A) 0.125 (B) 0.167
(C) 0.625 (D) 0.25

Solution: (D)
H-parameters can be defined by

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$
$$h_{12} = \frac{V_1}{V_2} \text{ at } I_1 = 0$$

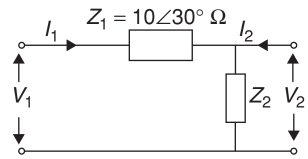
So $I_1 = 0$
The circuit becomes



$$V_2 = 2I_2 + 4 \cdot \frac{I_2}{2}$$
$$V_2 = 4I_2$$
$$I_2 = \frac{V_2}{4}$$
$$V_1 = \frac{I_2}{2} \times 2 = I_2$$
$$V_1 = \frac{V_2}{4}$$

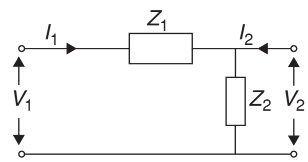
$$\Rightarrow \quad h_{12} = \frac{V_1}{V_2} = \frac{1}{4} = 0.25$$

Example 12: Two networks are connected in cascade as shown in the figure with the usual notations the equivalent A, B, C and D constants are obtained. Given that $C = 0.025\angle 45°$, the value of Z_2 is



(A) 10 ∠30° Ω (B) 40 ∠−45° Ω
(C) 1 Ω (D) 0 Ω

Solution: (B)
We know



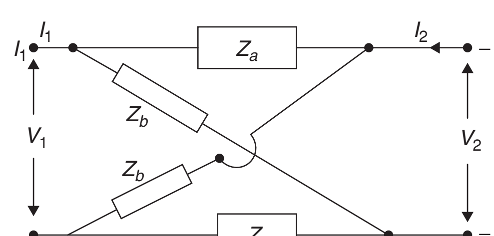
$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

Given $C = 0.025 \angle 45°$

So $C = \frac{1}{Z_2}$

$$Z_2 = \frac{1}{C} = \frac{1}{0.025} \angle \quad 45°$$
$$= 40 \angle -45°.$$

Example 13: Find the Z-parameters of the two-port circuit of figure shown below.



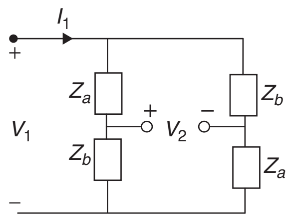
Solution:

$$Z_{11} = \frac{V_1}{I_1} \text{ at } I_2 = 0$$

Z_a and Z_b having same current

So $V_1 = (Z_a + Z_b) \parallel (Z_a + Z_b)$
$= 1/2\,(Z_a + Z_b)\ \Omega$
The circuit is symmetric so $Z_{11} = Z_{22} = 1/2[Z_a + Z_b]$
Similarly

$$Z_{21} = \frac{V_2}{I_1} \text{ at } = 0$$



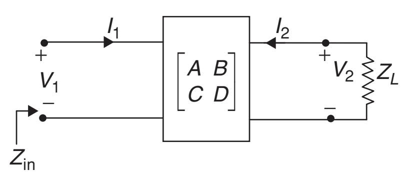
$$V_2 = Z_b\left(\frac{I_1}{2}\right) - \frac{Z_a}{2}\cdot I_1$$

$$\frac{V_2}{I_1} = \frac{1}{2}[Z_b - Z_a]$$

∴ For a symmetrical lattice network

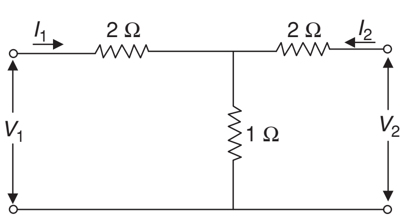
$$[Z] = \begin{bmatrix} \dfrac{Z_a + Z_b}{2} & \dfrac{Z_b - Z_a}{2} \\ \dfrac{Z_b - Z_a}{2} & \dfrac{Z_b + Z_a}{2} \end{bmatrix}$$

Note:



Then $Z_{in} = \dfrac{AZ_L + B}{CZ_L + D}$

Example 14:

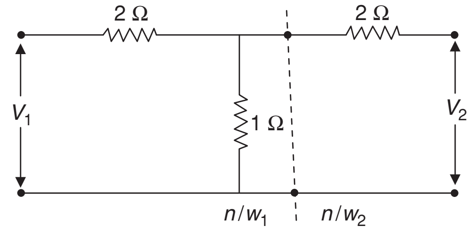


The T-parameters of the network are

(A) $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$

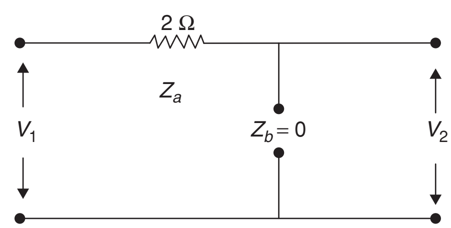
(C) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 7 & 2 \\ 7 & 3 \end{bmatrix}$

Solution: (B)



$$[T_1] = \begin{bmatrix} 1 + \dfrac{2}{1} & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$



$$[T_2] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

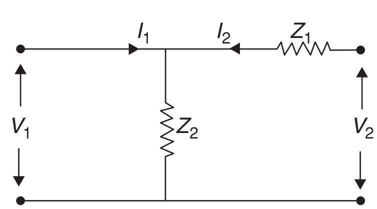
$$[T] = [T_1]\,[T_2]$$

$$= \begin{bmatrix} \rightarrow & \\ 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \downarrow & 1 & 2 \\ & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 \\ 1 & 3 \end{bmatrix}$$

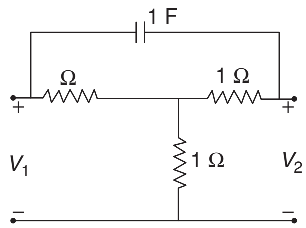
∴ $A = D \Rightarrow$ Symmetrical
And $AD - BC = 1 \Rightarrow$ Reciprocal

Note:

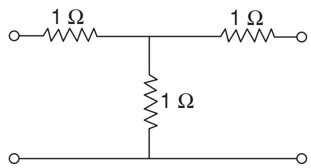


$$[T] = \begin{bmatrix} 1 & Z_1 \\ \dfrac{1}{Z_2} & 1 + \dfrac{Z_1}{Z_2} \end{bmatrix}$$

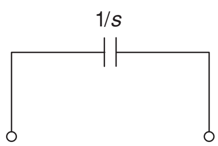
Example 15: Determine Y-parameters for the following networks



Solution:



(a)



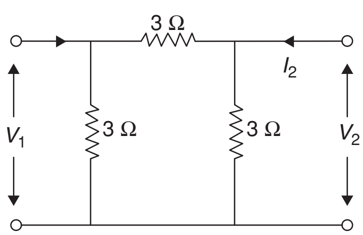
(b)

Figure (a) and figure (b) are in parallel, so

$[y] = [y_1] + [y_2]$

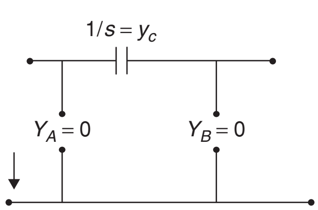
From the figure (a)

Y-Δ transformation



$$[Y_1] = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \Omega$$

From figure (b)



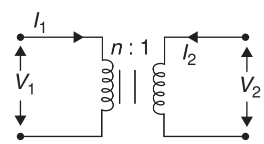
$$[Y_2] = \begin{bmatrix} \frac{1}{s} & \frac{-1}{s} \\ \frac{-1}{s} & \frac{1}{s} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} + \begin{bmatrix} \frac{1}{s} & \frac{-1}{s} \\ \frac{-1}{s} & \frac{1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3}+\frac{1}{s} & -\left(\frac{1}{3}+\frac{1}{s}\right) \\ -\left(\frac{1}{3}+\frac{1}{s}\right) & \frac{2}{3}+\frac{1}{s} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2s+3}{3s} & \frac{-(s+3)}{3s} \\ \frac{-(s+3)}{3s} & \frac{2s+3}{3s} \end{bmatrix}$$

Example 16: The $ABCD$ parameters of an ideal n:1 transformer shown in figure are $\begin{bmatrix} n & 0 \\ 0 & x \end{bmatrix}$. The value of '$X$' will be



(A) n (B) $1/n$ (C) n^2 (D) $1/n^2$

Solution: (B)

For the given ideal transformer

$$\frac{V_1}{V_2} = \frac{n}{1} = \frac{-I_2}{I_1}$$

$V_1 = nV_2 - 0.I_2$ (13)

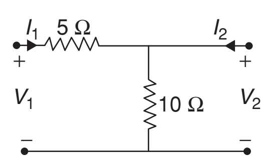
$I_1 = 0.V_2 - 1/n \,.\, I_2$ (14)

From equation (13) and (14)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$\Rightarrow X = 1/n$$

Example 17: The h-parameters of the circuit shown in figure are



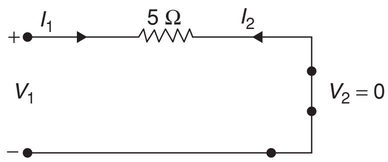
(A) $\begin{bmatrix} 5 & 1 \\ -1 & 0.1 \end{bmatrix}$ (B) $\begin{bmatrix} 0.2 & -1 \\ 1 & 10 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & -1 \\ 0.1 & 1 \end{bmatrix}$ (D) None of the above

Solution:
h-parameters are defined by
$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

From $V_2 = 0 \Rightarrow h_{11} = \frac{V_1}{I_1}\,\Omega$

$$h_{21} = \frac{I_2}{I_1}$$

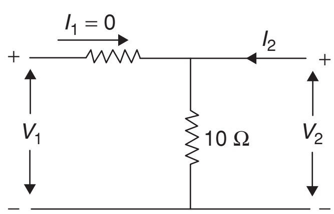


$$\frac{V_1}{I_1} = 5\,\Omega = h_{11}$$

$$I_1 = -I_2$$

$$\frac{I_2}{I_1} = h_{21} = -1$$

For $I_1 = 0$



$\therefore \quad V_1 = V_2$

$$h_{12} = \frac{V_1}{V_2} = 1$$

$$h_{22} = \frac{I_2}{V_2}$$

$$V_2 = 10 I_2$$

$$\frac{I_2}{V_2} = \frac{1}{10} = 0.1\,\Omega^{-1}$$

Relation between Z and Y parameters

$$Y_{11} = \frac{Z_{22}}{\Delta z};\ Y_{12} = \frac{-Z_{12}}{\Delta z}$$

$$Y_{21} = \frac{Z_{21}}{\Delta z};\ Y_{22} = \frac{+Z_{11}}{\Delta z}$$

Where $\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}$

Similarly

$$Z_{11} = \frac{Y_{22}}{\Delta y};\ Z_{12} = \frac{-Y_{12}}{\Delta y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta y};\ Z_{12} = \frac{Y_{11}}{\Delta y}$$

ABCD Parameters in Terms of Z-Parameters and Y-Parameters

$$A = \frac{Z_{11}}{Z_{21}} = \frac{-Y_{22}}{Y_{21}}$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{-1}{Y_{21}}$$

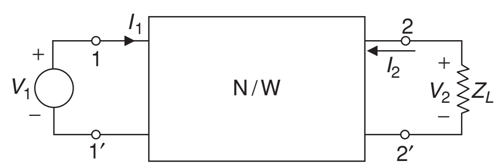
$$C = \frac{1}{Z_{21}} = \frac{-\Delta y}{Y_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{-Y_{11}}{Y_{21}}$$

Terminated Two-port Network

Driving point impedance at the input port of a load terminated network.

Figure shows a two-port network connected to an ideal generator at the input port and to a load impedance at the output port.



$V_2 = -I_2 Z_L$
$V_1 = Z_{11} I_1 + Z_{12} I_2$
$V_2 = Z_{21} I_1 + Z_{22} I_2$
$-I_2 Z_L = Z_{21} I_1 + Z_{22} I_2$

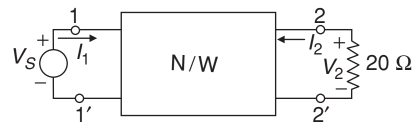
$$I_2 = \frac{-I_1 Z_{21}}{Z_L + Z_{22}}$$

$$V_1 = Z_{11} I_1 - \frac{Z_{12} Z_{21} I_1}{Z_L + Z_{22}}$$

$$V_1 = I_1 \left[Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}} \right]$$

$$\frac{V_1}{I_1} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}$$

Example 18:



The Z-parameter of a two port network shown in figure are $Z_{11} = Z_{22} = 10\ \Omega$, $Z_{12} = Z_{21} = 4\ \Omega$. If the source voltage is 20 V, determine I_1, V_2, I_2 and input impedance

Solution:

$$V_1 = V_s = 20\text{ V}$$

$$V_1 = I_1\left(Z_{11} - \frac{Z_{12}Z_{21}}{Z_L + Z_{22}}\right)$$

$$\therefore\quad 20 = I_1\left(10 - \frac{4\times 4}{20+10}\right)$$

$$I_1 = 2.11\text{ A}$$

$$I_2 = -I_1\frac{Z_{21}}{Z_L + Z_{22}}$$

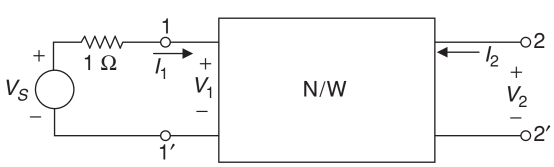
$$= -2.11\times\frac{4}{20+10} = -0.281\text{ A}$$

$$V_2 = -I_2 \times 20 = 0.281 \times 20 = 5.626\text{ V}$$

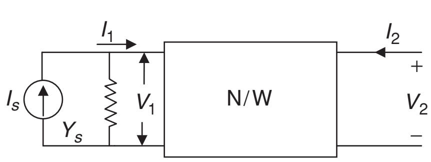
$$\text{Input impedance } = \frac{V_1}{I_1} = \frac{20}{2.11}$$

$$= 9.478\ \Omega$$

Example 19: The Y-parameter of the two-port network shown in figure are $Y_{11} = Y_{22} = 6$ mho; $Y_{12} = Y_{21} = 4$ mho. Determine the driving point admittance at port $2 - 2'$ if the source voltage is 100 V and has an impedance of 1 Ω.



Solution:



$$I_1 = I_s - V_1Y_s$$
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$
$$I_s - V_1Y_s = Y_{11}V_1 + Y_{12}V_2$$
$$-V_1(Y_s + Y_{11}) = Y_{12}V_2 - I_s$$

$$-V_1 = \frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}}$$

$$I_2 = -Y_{21}\left(\frac{Y_{12}V_2 - I_s}{Y_s + Y_{11}}\right) + Y_{22}V_2$$

$$I_2 = -\frac{-Y_{21}Y_{12}V_2}{Y_s + Y_{11}} + Y_{22}V_2$$

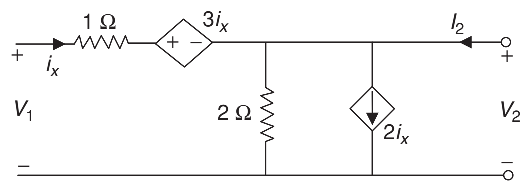
$$\frac{I_2}{V_2} = \frac{Y_{22}Y_5 + Y_{22}Y_{11} - Y_{21}Y_{12}}{Y_s + Y_{11}}$$

$$= \frac{6\times 1 + 6\times 6 - 4\times 4}{1+6}$$

$$= 3.714\text{ mho}$$

Driving point impedance at port $2 - 2' = \dfrac{1}{3.714}\ \Omega$

Example 20:

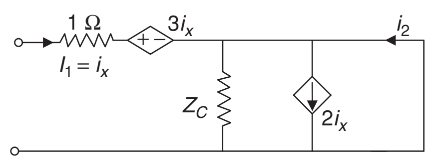


Find the H-parameters for the network shown in figure?

Solution:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

For $V_2 = 0$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$

$$V_1 = I_1 \times 1 + 3I_1$$

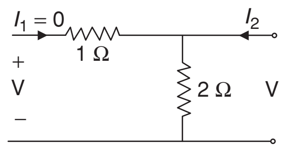
$$\frac{V_1}{I_1} = 4 = h_{11}$$

By KCL,

$$I_2 = 2I_1 - I_1$$
$$I_2 = I_1$$

$$\frac{I_2}{I_1} = 1 = h_{21}$$

For $I_1 = 0$



$V_2 = V_1$

$$\frac{V_1}{V_2} = h_{12} = 1$$

$V_2 = I_2 \times 2$

$$\frac{I_2}{V_2} = \frac{1}{2} = 0.5 = h_{22}$$

Network Functions

For a one-port network, the driving point impedance of the network is defined as

$$Z(s) = \frac{V(s)}{I(s)}$$

Similarly the driving point admittance

$$Y(s) = \frac{I(s)}{V(s)}$$

For a two-port network without internal sources, the driving point impedance at port $1 - 1'$ is

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

At port $2 - 2'$, it is

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

Similarly $Y_{11}(s) = \dfrac{I_1(s)}{V_1(s)}; Y_{22}(s) = \dfrac{I_2(s)}{V_2(s)}$

$Y_{11}(s)$ and $Y_{22}(s)$ are transfer admittances. Voltage transfer ratio

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

And $G_{12}(s) = \dfrac{V_1(s)}{V_2(s)}$

Current transfer ratio

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}$$

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

Transfer impedance

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

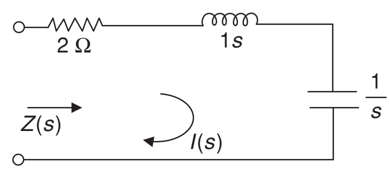
$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

Transfer admittance

$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

Example 21:



For the network shown in figure, obtain the driving point impedance.

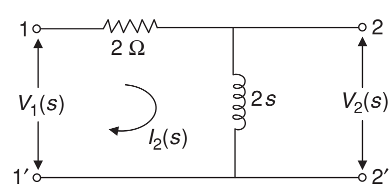
Solution:

$$Z(s) = \frac{v(s)}{I(s)}$$

$$= 2 + s + \frac{1}{s}$$

$$Z(s) = \frac{2s + s^2 + 1}{s} = \frac{s^2 + 2s + 1}{s}$$

Example 22: For the network shown in figure, obtain the transfer functions $G_{21}(s)$ and $Z_{21}(s)$ and the driving point impedance $Z_{11}(s)$.



Solution: $V_1(s) = I_1(s)\,[2 + 2s]$

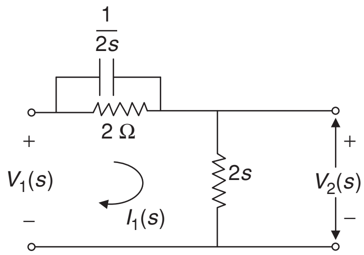
$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} = 2(s+1)$$

$$V_2(s) = I_1(s) \times 2s$$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{2s}{2s+2} = \frac{s}{s+1}$$

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = 2s$$

Example 23:



For the network shown in figure. Find $G_{21}(s)$. $Z_{21}(s)$ and $Z_{11}(s)$

Solution:

$$\frac{V_1(s)}{I_1(s)} = Z_{11}(s) = \frac{2\times\frac{1}{2s}}{2+\frac{1}{2s}} + 2$$

$$= \frac{2}{4s+1} + 2$$

$$Z_{11}(s) = \frac{8s+4}{4s+1}$$

$$V_2(s) = \frac{V_1(s)\cdot 2}{2+\frac{2}{4s+1}}$$

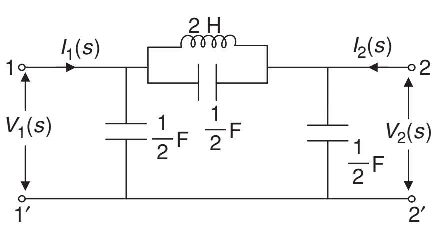
$$\frac{V_2(s)}{V_1(s)} = G_{21}(s) = \frac{2(4s+1)}{8s+4}$$

$$= \frac{8s+2}{8s+4}$$

$$V_2(s) = I_1(s).2$$

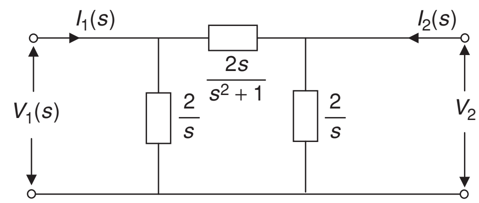
$$\frac{V_2(s)}{I_1(s)} = Z_{21}(s) = 2$$

Example 24: For the network shown in figure, determine the transfer function $G_{21}(s)$ and $Z_{21}(s)$



Solution:

Transform the circuit into s-domain



$$V_2(s) = V_1(s)\cdot\frac{\frac{2}{s}}{\frac{2}{s}+\frac{2s}{s^2+1}}$$

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{2}{s}}{2\frac{(s^2+1)+2s^2}{(s^2+1)s}}$$

$$= \frac{2(s^2+1)}{4s^2+2}$$

$$G_{21}(s) = \frac{s^2+1}{2s^2+1}$$

$$V_2(s) = I_1(s) = \frac{\frac{2}{s}}{\frac{4}{s}+\frac{2s}{s^2+1}}\times\frac{2}{s}$$

$$\frac{V_2(s)}{I_1(s)} = Z_{21}(s)$$

$$= \frac{2(s^2+1)}{s(3s^2+2)}$$

Network Graphs

The solution of a linear network problem required the formation of a set of equations describing the response of the network first and then the manipulation of the co-efficient matrix so produced.

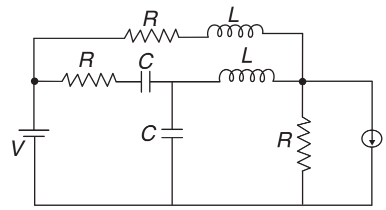
Networks topology deals with concepts involving inter connections in the networks, rather than the actual nature of the elements.

Graph

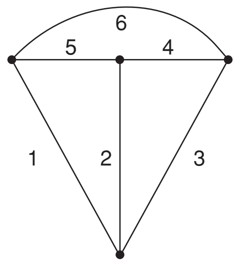
The connection of the network topology shown by replacing all physical elements by lines is called a graph.

While constructing a graph from the given network all passive elements and the ideal voltage sources are replaced by short circuit, all the ideal current sources are replaced by open circuit.

Example:
A network and it related graph is shown in figure a and figure b



(a)



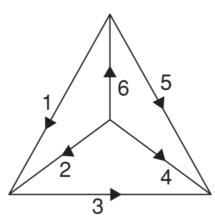
(b)

Directed graph

A graph in which each branch is assigned a direction is called a directed or oriented graph.

Complete graph (or) standard graph

For a standard graph, between any pair of nodes only one branch is connected for all combination.

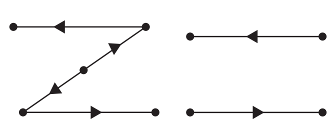


Example: The no. of edges in a complete graph with n-nodes in ${}^{n}C_2 \Rightarrow \dfrac{n(n-1)}{2} = b.$

Connected graph

In a connected graph all the nodes are connected by at least one branch, otherwise it is said to be unconnected.

Example:



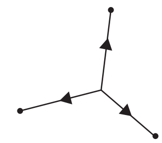
(i) (ii)

Connected graph

Sub graph

It is a graph with less no. of branches as compared with the original graph.

Example:



Sub graph

Planar and non-planar graphs

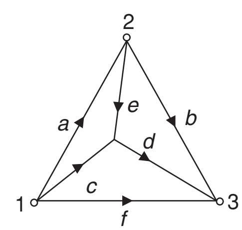
A graph is said to be planar if it can be drawn on a plane surface such that no two branches cross each other.

A non-planar graph can not be drawn on a plane surface without a crossover.

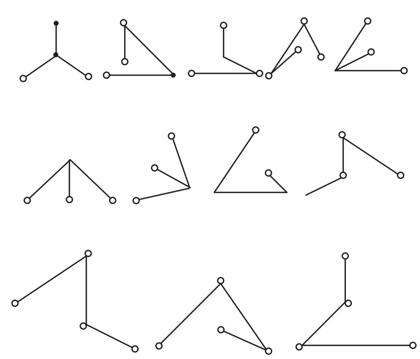
Tree and co-tree

A tree is a connected of a network which consists of all the nodes of the original graph but no closed paths. The number of nodes in the graphs is equal to the number of nodes in the tree.

Example 25: For the given graph shown in figure, draw the number of possible trees

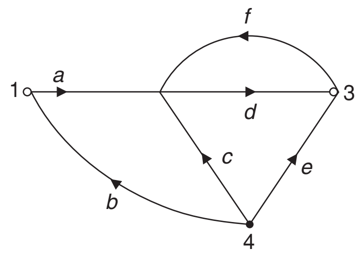


Solution:
There are four nodes. The possible trees are shown in the figure

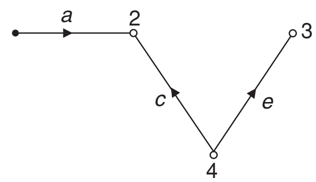


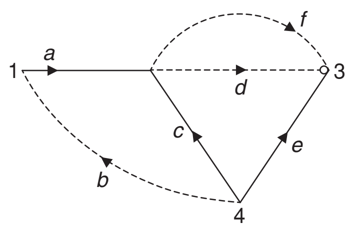
Twigs and Links

The branches of a tree are called its 'twigs'. For a given branch, the complementary set of branches of the tree is called the co-tree of the graph. The branches of co-tree are called links, i.e., those elements of the connected graph that are not included in the tree links and form a sub graph.



For the graph shown in the figure the tree branches are "ace" as shown in figure

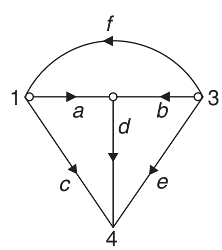




The set of branches (b, d, f) represented by dotted lines form a co-tree of the graphs. These branches are called links of this tree.

For a network with 'b' branches and 'n' nodes, the number of twigs for a selected tree is $(n - 1)$ and the number of links 'l' with respect to this tree is $b - n + 1$. The number twigs is called the rank of the tree

Incidence matrix (A)



For the oriented graph shown in the figure, the incidence matrix is

↓ Nodes Branches →

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} a & b & c & d & e & f \\ 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix}_{n \times b}$$

In matrix A with 'n' rows and b columns an entry a_{ij} in the ith row and jth column has the following values

$a_{ij} = 1$, if the jth branch is incident to and oriented away from the ith node

$aij = 1$, if the jth branch is incident and oriented towards the ith node

$a_{ij} = 0$, if the jth branch is not incident to the ith node

Note: Consider incoming branches are "–ve" sign and outgoing branches are "+ve" sign.

Example 26: Draw the graph corresponding to the given incidence matrix

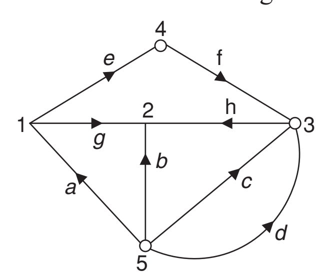
$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & +1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ +1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution:

There are five rows and eight columns which indicate that there are five nodes and eight branches

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & +1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ +1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The graph can be drawn as shown in figure

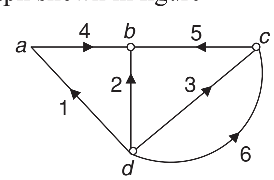


Incidence Matrix and Formulation of KCL

$A_I I = 0$

Where A_I is the Incidence matrix and I represents branch current vectors $I_1, I_2 \ldots$

Consider the graph shown in figure



It has four nodes a, b, c and d. Let node 'd' be taken as reference node. Let the branch currents be $i_1, i_2, \ldots, i_6$

Applying KCL at nodes a, b, and c

$$-i_1 + i_4 = 0$$
$$-i_2 + i_5 - i_4 = 0$$
$$-i_3 - i_5 - i_6 = 0$$

In matrix form, $\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1- & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$A_I I_b = 0$ … (KCL)

Relation Between Twigs and Links

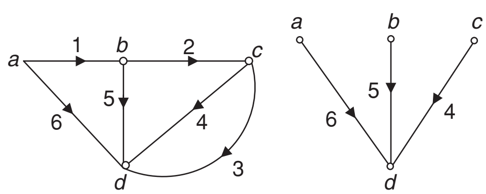
The number of twigs on a tree is always one less than the number of nodes.
i.e., twigs = $(n - 1)$
Let n = number of nodes
Also, if 'ℓ' represents the total number of links, while 'b' the total number of branches
$L = b - (n - 1)$
$L = b - n + 1$

Tie-set Matrix

For a given tree of a graph, addition of each link between any two nodes forms a loop called the fundamental loop. In a loop there exists a closed path and a circulating current, which is called the link current.

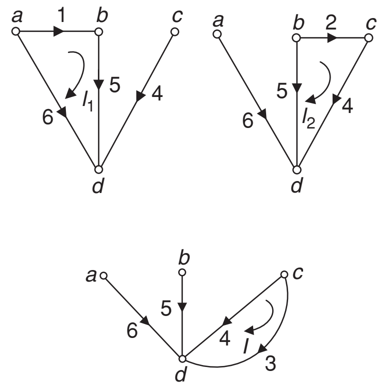
The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is called a tie set.

Consider a connected graph shown in figure (a). It has four nodes and six branches. One of its trees is arbitrarily chosen and is shown in figure



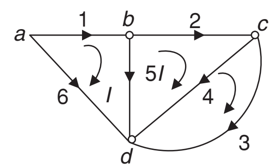
(a) (b)

No. of nodes $n = 4$
No. of branches $b = 6$
No. of tree branches or twigs = $n - 1$
No. of link branches $I = b - (n - 1) = 3$
Let i_1, i_2 …, i_6 be the branch currents with directions as shown in figure 'a'. When a link is added to the tree, a closed circuit is formed. The closed loops are as shown in the figures.



By convention a fundamental loop is given the same orientation as its defining link, i.e., the link current. I_1 coincides with the branch current i_1 direction in ab. Similarly I_2 coincides with the branch current direction in bc and I_3 coincides with the direction of cd.

Tie-set



Consider the above figure. Kirchoff's voltage law can be applied to the fundamental loops to get a set of linearly independent equation.

These are three fundamental loops I_1, I_2 and I_3 corresponding to the link branches 1, 2 and 3 respectively. If V_1, V_2, …V_6 are the branch voltages, the KVL equations for the three fundamental loops are

$$V_1 + V_5 - V_6 = 0$$
$$V_2 + V_4 - V_5 = 0$$
$$V_3 - V_4 = 0$$

The above equation can be written in matrix form.

loop ↓ 1, 2, 3, 4   branches→

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

i.e., $B\,V_b = 0$ … (KVL)
Where B is an $I \times b$ matrix called the tie-set matrix or fundamental loop matrix and V_b is a column vector of branch voltages.

Tie-Set Matrix and Branch Currents

$$[I_b] = [B^T]\,[I_L]$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$[I_b] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}; [I_L] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The branch currents are

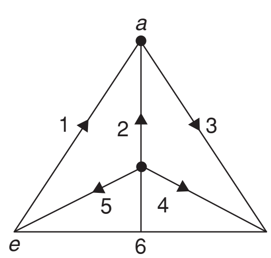
$i_1 = I_1 \quad i_2 = I_2 \quad i_3 = I_3$

$i_4 = I_2 - I_3 \quad i_5 = I_1 - I_2 \quad i_6 = -I_1$

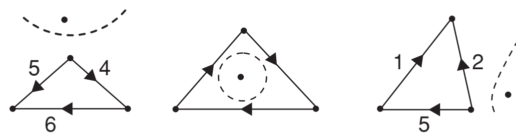
Cut-set

It is a set of branches of a connected graph (G), where in the removal of all the branches of the set causes remaining graph to have two unconnected sub-graphs, i.e., the cut set is a minimal set of branches of the graph, remove of which divides the graph in to two sub graphs.

Example:



Standard graph



C(1, 2, 3) C(2, 4, 5) C(3, 4, 6)

Note:

$$aij = \begin{cases} +1\text{: If branch } j \text{ leaves node } (i) \\ -1\text{: If branch } j \text{ enters node } (i) \\ 0\text{: If branch } j \text{ is not incident on } (i) \end{cases}$$

Properties of a Tree in a graph

1. It consists of all the nodes of the graph.
2. If the graph has N no. of nodes the tree will have (N − 1) branches.
3. There will be no closed path in the tree.

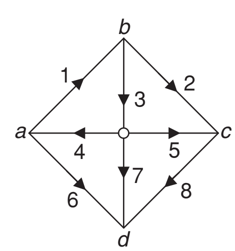
Note:

1. Rank of a graph = (n − 1)
   Where $n \Rightarrow$ number of nodes
2. The number of trees for a given standard graph = $(n)^{n-2}$
3. Total number of KCL equations equal to $n - 1$
4. Number of fundamental tie sets for a graph equal to number of Links
   i.e., $L = (b - n + 1)$
5. Rank of tie-set matrix = $(b - n + 1)$ = Links
6. Number of nodal equations in a given graph equal to $(n - 1) \Rightarrow$ f-cut sets.
7. Number of mesh equations = f-loops = $b - n + 1$

Fundamental cut-sets

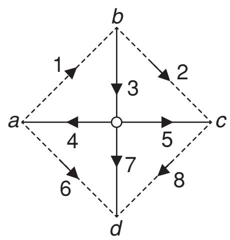
The fundamental cut-sets are defined for a given tree of the graph. From a connected graph, first a tree is selected, and then a twig is selected. Removing this twig from the tree separates the tree into two parts, All the links which go from one part of the disconnected tree to the other, together with the twig of the selected tree will constitute a cut-set. This cut-set is called a fundamental cut-set of the graph.

Example 27: Obtain the fundamental cut-set matrix Q_f for the network shown in figure

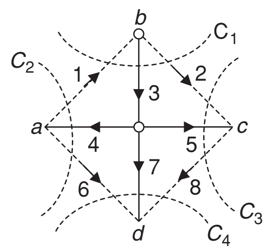


Solution:

A selected tree of the graph is shown is the figure



The twigs of the tree are {3, 4, 5, 7}. The remaining branches 1, 2, 6 and 8 are the links, corresponding to the selected tree.



The fundamental cut-set matrix is formed as fundamental

$$\begin{array}{c} \text{Cutset} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{branches}\rightarrow \\ \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

The branch voltages in terms of twig voltages are

$V_1 = -V_3 - V_4 = -V_{t3} - V_{t4}$

$V_2 = -V_3 - V_5 = +V_{t3} - V_{t5}$

$V_3 = V_{t3}$

$V_4 = V_{t4}$

$V_5 = V_{t5}$

$V_6 = V_7 - V_4 = V_{t7} - V_{t4}$

$V_7 = V_{t7}$

$V_8 = V_7 - V_5 = V_{t7} - V_{t5}$

In matrix form

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_{t3} \\ V_{t4} \\ V_{t5} \\ V_{t7} \end{bmatrix}$$

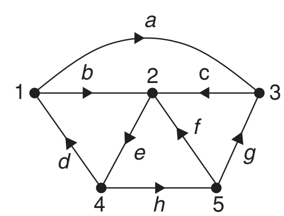
Example 28: A standard graph consists of 55 branches, the number of f-cut sets, Tie-sets, f-cut sets matrices and Tie -set matrices are

Solution:
Given $b = 55$

$$55 = \frac{n(n-1)}{2}$$

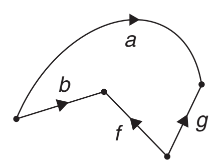
$n(n - 1) = 11$
$n = 11$
$L = b - n + 1 = 55 - 11 + 1 = 45$
f-loops or Tie-sets = links = 45
f-cut set matrices = Tie-set matrices
f-loop matrices = $(n)^{n-2}$
$= (11)^9$

Example 29: Identify which of the following is NOT a tree of the graph, shown in figure

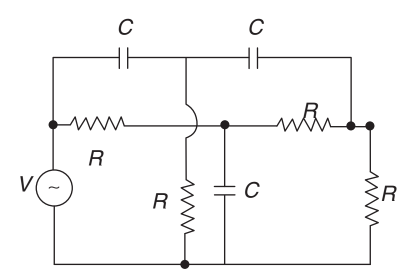


(A) *b c g h* (B) *d e f g*
(C) *a b f g* (D) *a e g h*

Solution: (C)
Tree is a connected graph, with out forming a closed path. From the given options (C) is not satisfied. Above statement

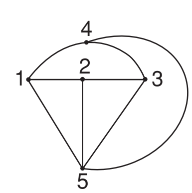


Example 30: Minimum no. of equation required to analyze the circuit shown in figure is



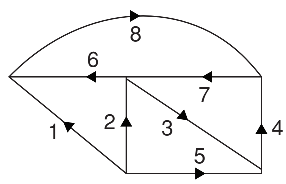
(A) 3 (B) 4
(C) 6 (D) 7

Solution: (B)



No. of nodes = 5
Nodal equations = $N - 1 = 4$
Number of mesh equations = $L = b - n + 1$
$= 8 - 5 + 1 = 4$
Minimum number of equations = min (nodal, Mesh equations)

Example 31: Match List I with List II for the co-tree branches, 1, 2, 3, and 8, of the graph shown in the figure and select the correct answer using the codes given below the lists.

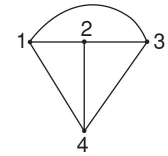


	List I		List II
p.	Twigs	1.	4, 5, 6, 7
q.	Links	2.	1, 2, 3, 8
r.	Fundamental cut set	3.	1, 2, 3, 4
s.	Fundamental loop	4.	6, 7, 8

(A) $p - 1, q - 2, r - 3\ s - 4$
(B) $p - 3, q - 2, r - 1, s - 4$
(C) $p - 1, q - 4, r - 3, s - 2$
(D) $p - 3, q - 4, r - 1, s - 2$

Solution: (A)
Total no. of branches = twigs + Links
From the given data
Links ⇒ 1, 2, 3, 8
∴ Twigs = 4., 5, 6, 7 = $n - 1 = 4$
Given nodes = 5
Fundamental cut set having at a time only one tree branch
i.e., 1, 2, 3, 4 → twig
∴ f-loops ⇒ having at a time only one link
⇒ 6, 7, 8 → Link

Example 32: What is the total number of trees for the graph shown below?



(A) 4
(B) 8
(C) 12
(D) 16

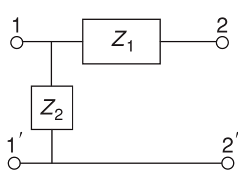
Solution: (D)
For a standard graph total number of trees = $(n)^{n-2}$
Where $n = 4$
∴ no. of trees = $(4)^{4-2} = 4^2 = 16$

Exercises

Practice Problems I

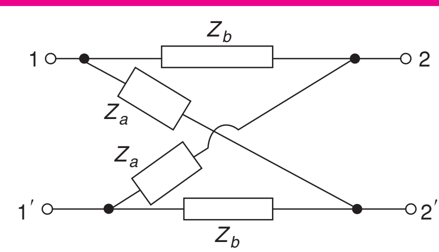
Directions for questions 1 to 31: Select the correct alternative from the given choices.

1. The z-parameter of the network shown in the figure is



(A) $\begin{bmatrix} z_1 + z_2 & z_1 \\ z_2 & z_1 + z_2 \end{bmatrix}$
(B) $\begin{bmatrix} z_1 & z_2 \\ z_1 + z_2 & z_1 - z_2 \end{bmatrix}$
(C) $\begin{bmatrix} z_2 & z_2 \\ z_2 & z_1 + z_2 \end{bmatrix}$
(D) $\begin{bmatrix} z_1 & z_1 \\ z_1 & z_1 + z_2 \end{bmatrix}$

2. For the lattice circuit shown in figure, $Z_a = j2\ \Omega$ and $Z_b = 2\ \Omega$. The values of the open circuit impedance parameters $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ are



(A) $\begin{bmatrix} 1-j & 1+j \\ 1-j & 1+j \end{bmatrix}$
(B) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$
(C) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$
(D) $\begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$

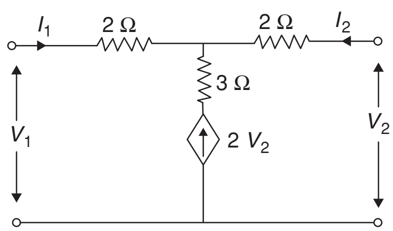
3. A two port network is represented by $ABCD$ parameters given by $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

If port-2 is terminated by R_L, the input impedance seen at port-1 is given by ______.

(A) $\dfrac{A+BR_L}{C+DR_L}$ (B) $\dfrac{AR_L+C}{BR_L+D}$

(C) $\dfrac{DR_L+A}{BR_L+C}$ (D) $\dfrac{B+AR_L}{D+CR_L}$

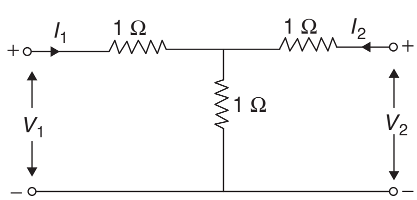
4. The admittance parameter of the network shown in the figure is ______.



(A) $\begin{bmatrix} \frac{1}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{-1}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{-3}{4} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{4} & \frac{5}{4} \\ \frac{-1}{4} & \frac{-3}{4} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & \frac{-5}{4} \\ \frac{-1}{4} & \frac{-3}{4} \end{bmatrix}$

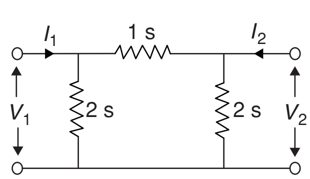
5. The transmission parameter $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ of the two port network shown in the figure is ______.



(A) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

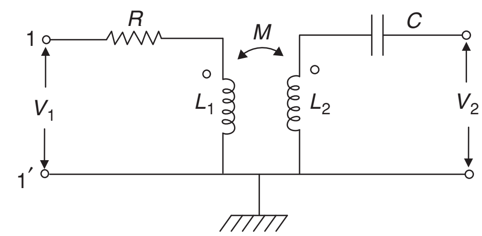
6. If two identical sections of the network shown in the figure are connected in parallel, the Y-parameter of the resulting network is given by ______.



(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$

7.

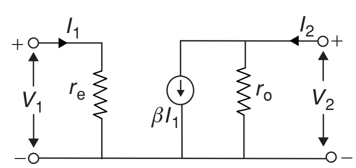


The element Z_{22} of the two port network shown in the above figure is

(A) $R + sL_1$ (B) $\dfrac{1}{Cs} + sL_2$

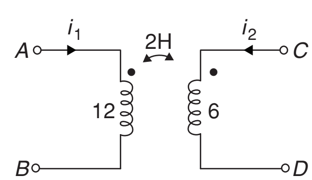
(C) $\dfrac{1}{Cs} + sL_1$ (D) sL_2

8. In the two-port network shown in the figure below, Z_{12} and Z_{21} respectively are

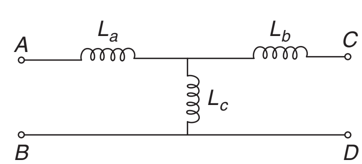


(A) r_e and βr_o (B) 0 and $-\beta r_o$
(C) 0 and βr_o (D) r_e and $-\beta r_o$

9. A linear transformer and its T equivalent circuit are shown in figure (a) and figure (b) respectively. The values of L_a, L_b and L_c respectively are ______.



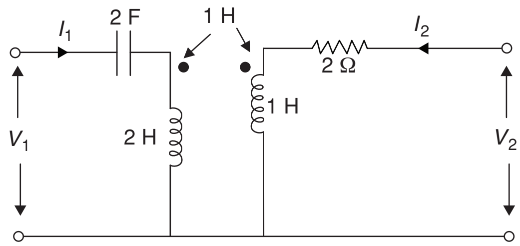
(a)



(a)

(A) 10 H, 4H, –2 H (B) 14 H, 8H, –2H
(C) 10 H, 4H, 2H (D) 14 H, 8H, +2H

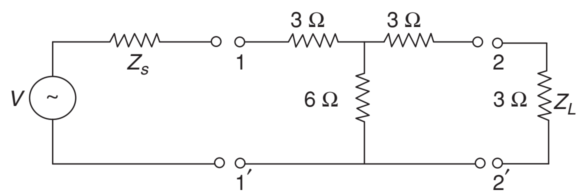
10. For the network shown below the 'Z' parameter will be



(A) $\begin{bmatrix} \frac{1}{2s}+2s & 2+s \\ 2+s & s \end{bmatrix}$ (B) $\begin{bmatrix} s & \frac{1}{2s}+2s \\ 2+s & s \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{2s}+2s & s \\ s & 2+s \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{2s}-s & s \\ s & 2-s \end{bmatrix}$

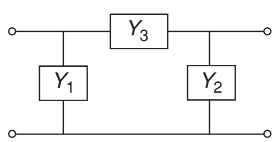
11. An impedance match is desired at the 1–1′ port of the two-port network shown in the given figure. The match will be obtained when Z_S equals



(A) 6 Ω (B) 3 Ω

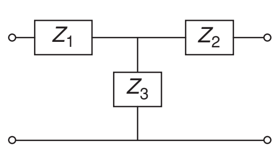
(B) $\frac{3}{2}\,\Omega$ (D) $\frac{2}{3}\,\Omega$

12. The admittance parameter of the 2-port network shown in the figure are $Y_{11}=10\ \Omega$, $Y_{12}=Y_{21}=6\ \Omega$ and $Y_{22}=8\ \Omega$ The values of y_1, y_2, and y_3 will be respectively



(A) 2, 4 and 6 (B) 4, 2 and –6
(C) 2, 4 and –6 (D) 16, 14 and –6

13.



To construct a High pass filter as in the above circuit

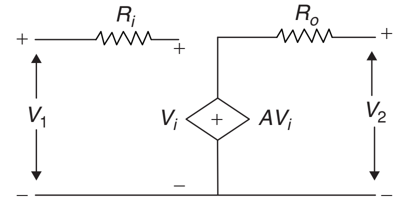
(A) z_1, z_2 are capacitors and z_3 inductor

(B) z_1 z_2 are resistors and z_3 capacitor

(C) z_1 z_2 are inductors z_3 capacitor

(D) z_1 z_2 are resistors z_3 inductor.

14. For the given equivalent circuit find input impedance, output impedance and output voltage



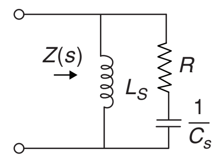
(A) 0, α, α (B) α, R_0, Av_i
(C) R_i, R_o, Av_i (D) R_i, R_0, A

15. Find the driving point admittance of the network shown. Given

$R = 1\ \text{M}\Omega$

$C = 10\ \mu\text{F}$

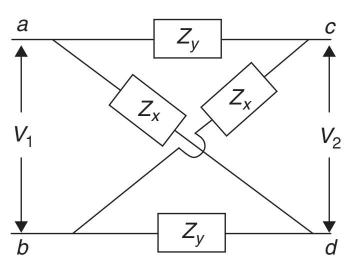
$L = 1000$ H



(A) $\dfrac{1\times10^3}{1+10s+10^4 s^2}$ (B) $\dfrac{(1+10s)10^5 s}{s^2+10^3 s+10^2}$

(C) $\dfrac{10s}{10^{-3}s^2+10s+1}$ (D) $\dfrac{10}{10^{-3}s^2+10s+1}$

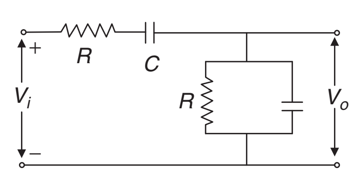
16. Circuit below shows a lattice circuit $z_x = 4j\ \Omega$ and $z_y = 4\ \Omega$ find the values of open circuit impedance $Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$



(A) $\begin{bmatrix} 2+2j & 2-2j \\ -2+2j & -2-2j \end{bmatrix}$ (B) $\begin{bmatrix} 2+2j & -2+2j \\ -2+2j & 2+2j \end{bmatrix}$

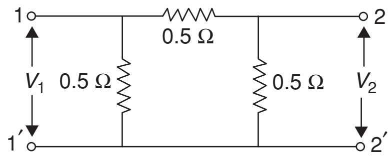
(C) $\begin{bmatrix} -2+2j & 2-2j \\ 2-2j & 2+2j \end{bmatrix}$ (D) $\begin{bmatrix} -2-2j & 2-2j \\ 2-2j & -2-2j \end{bmatrix}$

17. The RC circuit shown in the figure is



(A) A low- pass filter (B) A high- pass filter
(C) A band- pass filter (D) A band- reject filter

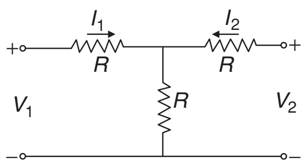
18. For the two-port network shown below, the short circuit admittance parameter matrix is



(A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

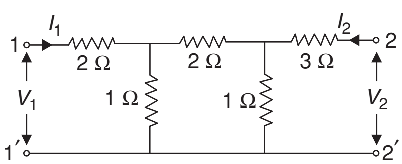
19. A two port network is shown in the figure. The parameter h_{21} for this network can be given by



(A) $\frac{-1}{2}$ (B) $+\frac{1}{2}$

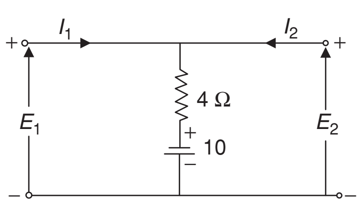
(C) $\frac{-3}{2}$ (D) $+\frac{3}{2}$

20. The impedance parameters Z_{11} and Z_{12} of the two-port network in the figure are



(A) $Z_{11} = 2.75\ \Omega$ and $Z_{12} = 0.25\ \Omega$
(B) $Z_{11} = 3\ \Omega$ and $Z_{12} = 0.5\ \Omega$
(C) $Z_{11} = 3\ \Omega$ and $Z_{12} = 0.25\ \Omega$
(D) $Z_{11} = 2.25\ \Omega$ and $Z_{12} = 0.5\ \Omega$

21. The Z-parameters Z_{11} and Z_{21} for the two-port network in the figure,



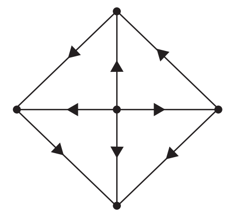
(A) $Z_{11} = \frac{-6}{11}\ \Omega;\ Z_{21} = \frac{16}{11}\ \Omega$

(B) $Z_{11} = \frac{6}{11}\ \Omega;\ Z_{21} = \frac{4}{11}\ \Omega$

(C) $Z_{11} = \frac{6}{11}\ \Omega;\ Z_{21} = \frac{-16}{11}\ \Omega$

(D) $Z_{11} = \frac{4}{11}\ \Omega;\ Z_{21} = \frac{4}{11}\ \Omega$

22. For the graph shown in the figure, the order of the tie set matrix is

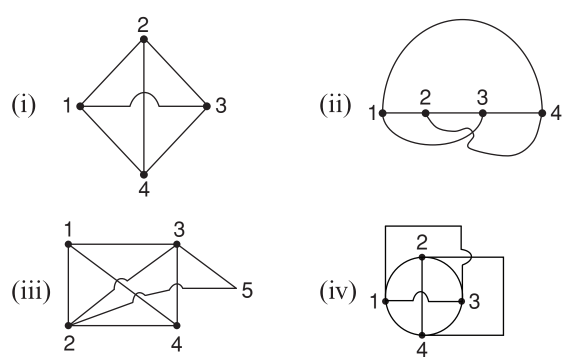


(A) 4×4 (B) 4×8
(C) 8×4 (D) 8×8

23. If V_b, Q, V_t represents branch voltage matrix, cut set matrix, and the twig voltage matrix then the relation between them is given by
(A) $V_b = Q\,V_t^T$ (B) $V_b = Q^T\,V_t^T$
(C) $V_b = V_t\,Q$ (D) $V_b = Q^T\,V_t$

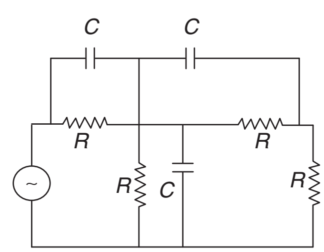
24. A planar graph has 5 nodes and 9 branches. The number of meshes in the dual graph is
(A) 5 (B) 4
(C) 14 (D) none

25. From the graph given below, which of them is non planer?



(A) i and ii (B) ii and iii
(C) iii only (D) iv only

26. The minimum number of equations required to analyze the circuit shown in the figure is



(A) 3 (B) 4
(C) 6 (D) 7

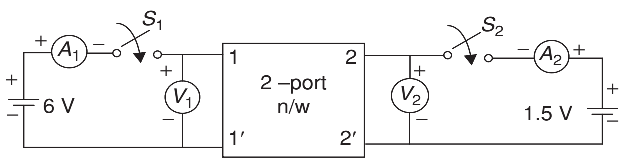
27. A two-part network shown below is excited by external dc sources. The voltages and the current are measured with voltmeters V_1, V_2 and ammeters A_1, A_2 as indicated. Under following switch conditions the readings obtained are:

(i) S_1-Open, S_2-closed

$A_1 = 0\text{A}$, $V_1 = 4.5$ V, $V_2 = 1.5$ V, $A_2 = 1$ V

(ii) S_1-closed, S_2-open

$A_1 = 4$ A, $V_1 = 6$ V, $V_2 = 6$ V, $A_2 = 0\text{A}$



The Z-parameter matrix for this network is

(A) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ (B) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ (D) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

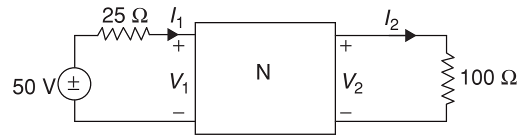
28. From the above questions data the H-parameter matrix for this network is

(A) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & -1 \\ +3 & 0.67 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

29. In the circuit shown below, the network N is described by the following Y matrix $[Y] = \begin{bmatrix} 0.1\,\Omega & -0.01\,\Omega \\ 0.01\,\Omega & 0.1\,\Omega \end{bmatrix}$

The voltage gain $\dfrac{V_2}{V_1}$ is

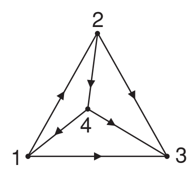


(A) 1/11 (B) −1/11
(C) −1/99 (D) 1/90

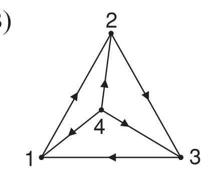
30. The incidence matrix of a graph is as given below

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$ the graph is

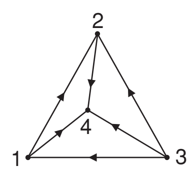
(A)



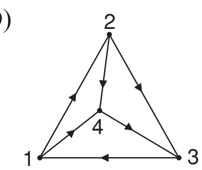
(B)



(C)



(D)



31. The incidence matrix of a graph is as given below

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The number of possible trees is
(A) 40 (B) 70
(C) 50 (D) 240

Practice Problems 2

Directions for questions 1 to 23: Select the correct alternative from the given choices.

1. Which parameters are used in the analysis of transistors?
(A) Z-parameters
(B) Y-parameters
(C) h-parameters
(D) Transmission parameters

2. If a transmission line is represented by a two port network whose parameters are A, B, C, D, then the sending and voltage end current are given by______.

(A) $V_S = AV_r + BI_r$
$I_S = CV_r + DI_r$
(B) $V_S = AV_r + CI_r$
$I_S = BV_r + DI_s$
(C) $V_S = AV_r - BI_r$
$I_S = CV_r - DI_r$
(D) $V_S = AV_r - CI_r$
$I_S = BV_r - DI_r$

3. A two-port network is reciprocal if and only if
(A) $Z_{11} = Z_{22}$. (B) $Y_{12} = Y_{21.}$
(C) $BC - AD = -1$. (D) $h_{12} = h_{21.}$

4. A two-port network is symmetrical if

(A) $z_{11} = z_{22}$ (B) $z_{11}z_{22} - z_{12}z_{21} = 1$

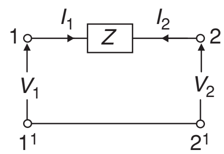
(C) $h_{11}h_{22} - h_{12}h_{21} = 1$ (D) Both A and C

5. A two-port network is reciprocal if

(a) $Z_{12} = Z_{21}$ (B) $A = D$

(C) $Y_{11} = Y_{22}$ (D) $BC - AD = 1$

6. For the two port network shown in the figure which of the following statements is true.



(A) It has Z-parameter.
(B) It has no Z-parameter.
(C) It has no Y-parameter.
(D) It has no transmission parameter.

7. Two two-port networks have Z-parameters

$$[Z]_x = \begin{bmatrix} Z_{11x} & Z_{12x} \\ Z_{21x} & Z_{22x} \end{bmatrix} \text{ and } [Z]_y = \begin{bmatrix} Z_{11y} & Z_{12y} \\ Z_{21y} & Z_{22y} \end{bmatrix}$$

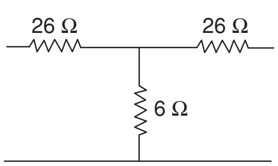
Then the open circuit transfer impedance of the cascaded network is

(A) $Z_{12x} + Z_{12y}$ (B) $Z_{21x} + Z_{21y}$

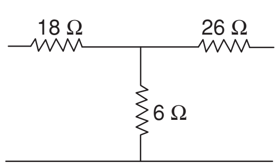
(C) $\dfrac{Z_{21x}Z_{21y}}{Z_{11x} + Z_{22y}}$ (D) $\dfrac{Z_{12x}Z_{12y}}{Z_{12x} + Z_{12y}}$

8. A two-port network is represented by $V_1 = 32I_1 + 6I_2$ and $V_2 = 6I_1 + 24I_2$. Which one of the following networks is represented by these equations.

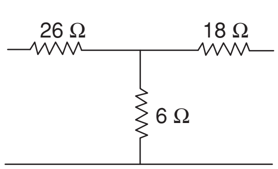
(A)



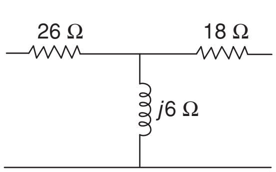
(B)



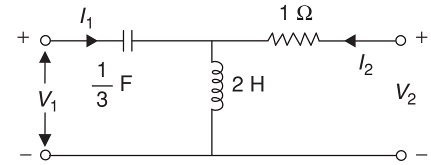
(C)



(D)



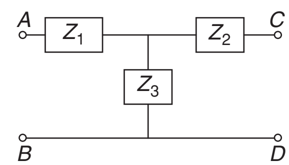
9. Z-matrix for the network shown in the given figure is



(A) $\begin{bmatrix} \frac{1}{3s} & 2s \\ 2s & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2s+\frac{3}{s} & -2s \\ -2s & 2s+1 \end{bmatrix}$

(C) $\begin{bmatrix} 2s+\frac{3}{s} & 2s \\ 2s & 2s+1 \end{bmatrix}$ (D) $\begin{bmatrix} 2s & 2s+\frac{3}{s} \\ 2s+\frac{3}{s} & 2s \end{bmatrix}$

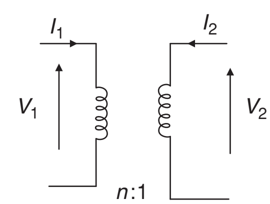
10. Find values of Z_1, Z_2, Z_3 in the network shown



$Z_{11} = 16\ \Omega$, $Z_{12} = 10\ \Omega$, $Z_{22} = 15\ \Omega$, $Z_{21} = 10\ \Omega$

(A) $Z_1 = 10\ \Omega$, $Z_2 = 5\ \Omega$, $Z_{3=}\ 6\ \Omega$
(B) $Z_1 = 26\ \Omega$, $Z_2 = 25\ \Omega$, $Z_3 = 10\ \Omega$
(C) $Z_1 = 6\ \Omega$, $Z_2 = 5\ \Omega$, $Z_3 = 10\ \Omega$
(D) $Z_1 = 25\ \Omega$, $Z_2 = 26\ \Omega$, $Z_3 = 10\ \Omega$

11. The $ABCD$ parameters of an ideal $n : 1$ transformer shown in the figure are $\begin{bmatrix} n & o \\ o & x \end{bmatrix}$. The value of x will be



(A) n (B) $\dfrac{1}{n}$

(C) n^2 (D) $\dfrac{1}{n^2}$

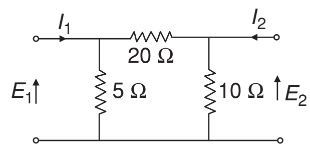
12. The short-circuit admittance matrix of a two port network is

$$\begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

The two-port network is

(A) Non-reciprocal and passive
(B) Non-reciprocal and active
(C) Reciprocal and passive
(D) Reciprocal and active

13. The admittance parameter Y_{12} in the two-port network shown in the figure is

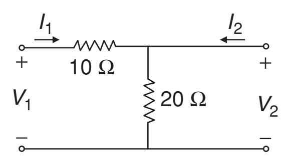


(A) −0.2 mho (B) 0.1 mho
(C) −0.05 mho (D) 0.05 mho

14. For a two-port network to be reciprocal
(A) $Z_{11} = Z_{22}$ (B) $Y_{21} = Y_{22}$
(C) $h_{21} = -h_{12}$ (D) $AD - BC = 0$

15. Which parameters are widely used in transmission line theory?
(A) *Z*-parameters (B) *Y*-parameters
(C) *ABCD* parameters (D) *H*-parameters

16. The *H*-parameters of the circuit shown in the figure are:



(A) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

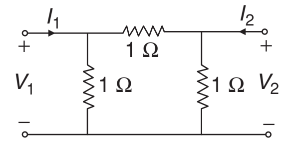
(C) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

17. The impedance matrices of two, two-port networks are given by $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 15 & 5 \\ 5 & 25 \end{bmatrix}$. If the two networks are connected in series, the impedance matrix of the combination is

(A) $\begin{bmatrix} 3 & 5 \\ 2 & 25 \end{bmatrix}$ (B) $\begin{bmatrix} 18 & 7 \\ 7 & 28 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 2 \\ 5 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 7 \\ 25 & 3 \end{bmatrix}$

18.



The *Y*-parameters for the network is

(A) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

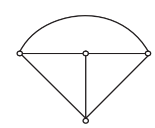
(C) $\begin{bmatrix} 0.5 & 1 \\ 1 & 0.5 \end{bmatrix}$ (D) $\begin{bmatrix} 0.5 & -1 \\ -1 & 0.5 \end{bmatrix}$

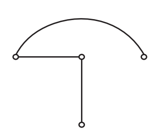
19. If the graph of an electrical network has 'N' nodes and 'B' branches. The number of links 'L' is given by
(A) $N - B + 1$ (B) $B - N + 1$
(C) $N + B$ (D) $B - N$

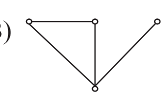
20. A connected network of $N > 2$ nodes has at the most one branch directly connecting any pair of nodes. The graph of the network
(A) Must have at least 'N' branches for one or more closed paths to exist.
(B) Can have an unlimited number of branches.
(C) Can only have at the most N branches.
(D) Can have a minimum number of branches not decided by N.

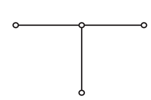
21. If B is tie-set matrix and 'I_L' Loop current matrix then branch current matrix I_b is given by
(A) $I_b = BI_L$ (B) $I_b = I_L B^T$
(C) $I_b = B^T I_L$ (D) $I_b = B^T I_L^{\,T}$

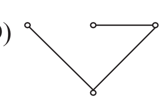
22. Consider the network graph shown in the figure. Which one of the following is NOT a 'tree' of this graph?



(A)


(B)


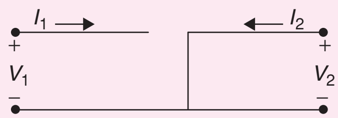
(C)


(D)


23. A network has 7 nodes and 5 independent loops. The number of branches in the network is
(A) 13 (B) 12
(C) 11 (D) 10

Previous Years' Questions

1. The parameter type and the matrix representation of the relevant two port parameters that describe the circuit shown are **[2006]**



(A) Z-parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

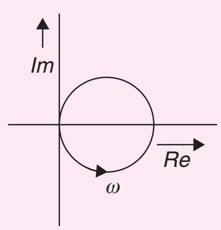
(B) H-parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) H-parameters, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

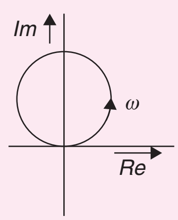
(D) Z-parameters, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. The R-L-C series circuit shown is supplied from a variable frequency voltage source. The admittance-locus of the R-L-C network at terminals AB for increasing frequency ω is **[2007]**

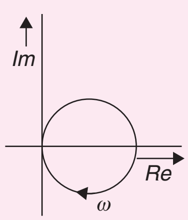
(A)



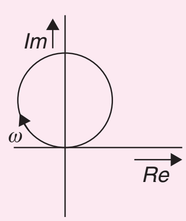
(B)



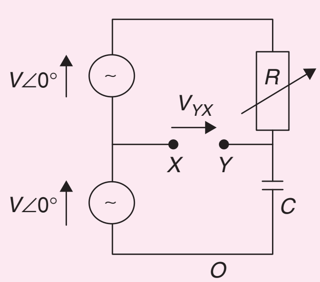
(C)



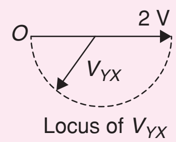
(D)



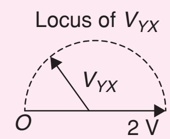
3. In the figure given below all phasors are with reference to the potential at point 'O'. The locus of voltage phasor V_{YX} as R is varied from zero to infinity is shown by **[2007]**



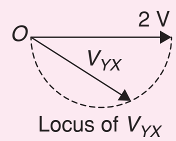
(A)



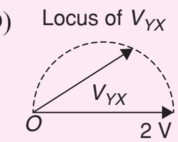
(B)



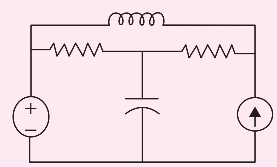
(C)



(D)

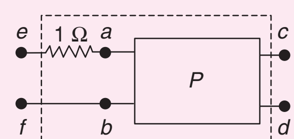


4. The number of chords in the graph of the given circuit will be **[2008]**



(A) 3 (B) 4
(C) 5 (D) 6

5. The two-port network P shown in the figure has ports 1 and 2, denoted by terminals (a, b) and $(c, 0)$, respectively. It has an impedance matrix Z with parameters denoted by z_{ij}. A 1 Ω resistor is connected in series with the network at port 1 as shown in the figure. The impedance matrix of the modified two-port network (shown as a dashed box) is **[2010]**



(A) $\begin{pmatrix} z_{11}+1 & z_{12}+1 \\ z_{21} & z_{22}+1 \end{pmatrix}$

(B) $\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21} & z_{22}+1 \end{pmatrix}$

(C) $\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$

(D) $\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21}+1 & z_{22} \end{pmatrix}$

Common Data for Questions 6 and 7: With 10 V dc connected at port A in the linear non reciprocal two-part network shown below, the following were observed:

(i) 1 Ω connected at port B draws a current of 3 A.
(ii) 2.5 Ω connected at port B draws a current of 2 A.

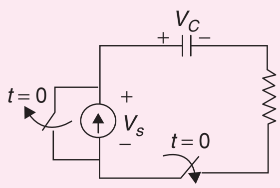
6. For the same network, with 6 V dc connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V DC is connected to port *A*, the open circuit voltage at port *B* is **[2012]**

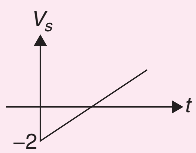
(A) 6 V (B) 7 V
(C) 8 V (D) 9 V

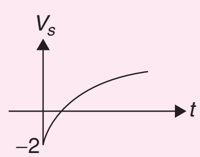
7. With 10 V dc connected at port A, the current drawn by 7 Ω connected at port *B* is **[2012]**

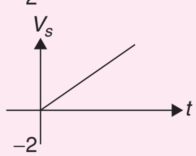
(A) 3/7 A (B) 5/7 A
(C) 1 A (D) 9/7 A

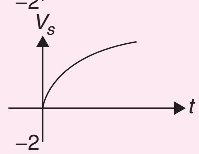
8. A combination of 1 μF capacitor with an initial voltage $v_c(0) = -2$ V in series with a 100 Ω resistor is connected to a 20 mA ideal dc current source by operating both switches at $t = 0$ s as shown. Which of the following graphs shown in the options approximates the voltage v_s across the current source over the next few seconds? **[2014]**



(A)


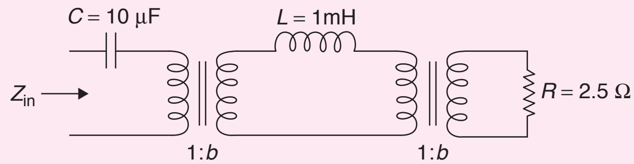
(B)


(C)


(D)


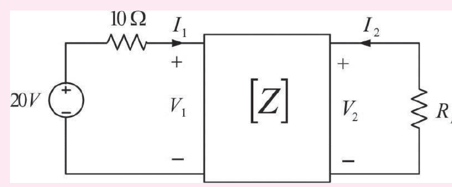
9. In a linear two-port network, when 10 V is applied to Port 1, a current of 4 A flows through Port 2 when it is short-circuited. When 5 V is applied to Port 1, a current of 1.25 A flows through a 1 | resistance connected across Port 2. When 3 V is applied to Port 1, the current (in Ampere) through a 2 | resistance connected across Port 2 is ______. **[2015]**

10. Find the transformer ratios a and b such that the impedance (Z_{in}) is resistive and equals 2.5 | when the network is excited with a sine wave voltage of angular frequency of 5000 rad/s. **[2015]**



(A) $a = 0.5, b = 2.0$
(B) $a = 2.0, b = 0.5$
(C) $a = 1.0, b = 1.0$
(D) a = 4.0, b = 0.5

11. The *Z*-parameters of the two port network shown in the figure are $Z_{11} = 40\Omega$, $Z_{12} = 60\Omega$, $Z_{21} = 80\Omega$, and $Z_{22} = 100\Omega$. The average power delivered to $R_L = 20\Omega$, in watts, is __________. **[2016]**



Answer Keys

Exercises

Practice Problems I

1. C	**2.** D	**3.** D	**4.** D	**5.** C	**6.** C	**7.** B	**8.** B	**9.** C	**10.** C
11. A	**12.** D	**13.** C	**14.** B	**15.** B	**16.** B	**17.** C	**18.** A	**19.** A	**20.** A
21. C	**22.** B	**23.** D	**24.** A	**25.** D	**26.** B	**27.** C	**28.** A	**29.** B	**30.** D
31. A									

Practice Problems I

1. C	**2.** A	**3.** C	**4.** D	**5.** A	**6.** B	**7.** C	**8.** C	**9.** C	**10.** C
11. B	**12.** B	**13.** C	**14.** C	**15.** C	**16.** D	**17.** B	**18.** B	**19.** B	**20.** D
21. C	**22.** B	**23.** C							

Previous Years' Questions

1. C	**2.** A	**3.** A	**4.** A	**5.** C	**6.** B	**7.** C	**8.** C	**9.** 0.5454
10. B	**11.** 35.55							