ELECTROSTATICS

Electricity Charge

Amber, glass, ebonite, sulphur etc. on being rubbed attract light bodies. This property in materials is developed due to electrification by friction. On acquiring this property the material is said to be electrified and this property is called electricity.

On being electrified material acquiring charges, electrified material is called changed material.

Quantization of charge : Charge on a body is always some integral multiple of a smallest unit of charge, which in magnitude is equal to the charge of an electron. That is the charge acquired by body can be $q = \pm \ell, \pm 2\ell, \pm 3\ell$ ...

In general $q = \pm n\ell$.

Coulomb's law : According to this law, the force of attraction or repulsion between two stationary point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between, them. This force acts along the line joining the charges

$$F \propto \frac{q_1 q_2}{r^2} \text{ and } F = K\frac{q_1 q_2}{r^2}$$

Where K is the constant of proportionality.

$$\text{In SI unit } K = 9 \times 10^9 \frac{N \times m^2}{\text{coul}^2}$$

Therefore the force between two point charges placed in vacuum (or air) is

$$F = (9.0 \times 10^9)\ \frac{q_1 q_2}{r^2} \text{ newton}$$

In SI system the charges are in coulombs and distance is in metres, then the force will be in newtons. The above formula can be written as

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

Where ε_0 is permittivity of force space and its value is $8.86 \times 10^{-12}\ C^2N^{-1}m^{-2}$.

Coulomb : Coulomb is the unit of charge. One Coulomb charge is that charge which will repel an equal and similar charge placed in vacuum (or air) at a distance of one metre, with a force 9×10^9 N. It is represented by C.

Also $1C = 3 \times 10^9$ e.s.u. of charge

and $1C = 1/10$ e.m.u of charge.

Electric Field : It is the region surrounding an electric charge, or a group of charges, in which another charge experiences a force of attraction or repulsion.

Electric lines of Force : An electric line of force is that imaginary smooth curve drawn in an electric field along which a free, isolated unit positive charge moves.

Properties of Electric lines of force :

(i) Electric lines of force start from a positive charge and end on a negative charge.

(ii) The tangent drawn at any point on the line of force gives the direction of the force acting on a positive charge at that point.

(iii) No two line of force can intersect each other, because if they do so then at the point in intersection two tangents can be drawn which would mean two directions of the force at that point which is impossible.

(iv) These lines have a tendency to contract in length like a stretched elastic string. This

explains attraction between opposite charges.

(v) These lines have a tendency to separate from each other in the direction perpendicular to their lengths. This explains repulsion between like charges.

(vi) Lines of force are crowded at the places of greater intensity and they are farther apart at places of weaker intensity.

(vii) Lines of force of uniform field are parallel.

(viii) Unit positive charge gives $\frac{4\pi}{K}$ lines in a medium of dielectric constant K.

(ix) Electric lines of force can never enter the conductor, because inside a conductor the intensity of electric field is zero.

Intensity of Electric field (or strength of electric field) : Intensity of electric field at any point in the electric field is defined as the force experienced by a unit positive charge (test charge) placed at that point.

If $\vec{F}$ be the force acting on a test charge $+q_0$ placed at a point in the electric field, then the electric field intensity $\vec{E}$ at that point will be,

$$\vec{E} = \frac{\vec{F}}{q_0}$$

It is a vector quantity. Its direction being the same as that of $\vec{F}$. The SI unit of electric field intensity is newton per coulomb (NC^{-1}).

Intensity of electric field due to a point charge : Electric field intensity at any point distant r from a point charge q is given by,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,\hat{r}$$

The magnitude of the electric field intensity is given by

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$$

Intensity of Electric field due to a number of point charge : Electric field intensity at any point due to n number of point charges is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i^2}\,\hat{r}_i$$

The magnitude of the electric field intensity is given by,

$$E = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i^2}$$

Intensity of Electric Field due to Circular loop of Charge :

Electric field intensity due to a circular loop of charge q and radius 'a' at a distance x from its centre is given by.

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{qx}{(x^2 + a^2)^{3/2}}$$

If $x >> a$, then

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}$$

Electric dipole : An electric dipole is a system of two equal and opposite point charges separated by a very small distance.

Electric dipole moment : Dipole moment of an electric dipole is equal to the product of the magnitude of either charge and the distance between the two charges or the length of the electric dipole.

If two equal and opposite point charges $+q$ and $-q$ are separated by a very small distance '$2a$', then their electric dipole moment,

$$P = q\,(2a)$$

It is a vector quantity. In vector form,

$$\vec{P} = q(2\vec{a})$$

The SI unit of dipole moment is coulomb metre (cm).

Torque on an electric dipole : Torque on an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ is given as

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The magnitude of the torque is given by,

$$\tau = pE \sin\theta$$

Where θ = angle between the directions of $\vec{p}$ and $\vec{E}$.

Potential Energy of an electric dipole : The potential energy of an electric dipole in a uniform electric field E is given by

$$U = pE(\cos\theta_1 - \cos\theta_2)$$

Where θ_1 = initial angle between the electric dipole and the electric field.

θ_2 = final angle between the electric dipole and the electric field.

If $\theta_1 = 90°$ and $\theta_2 = \theta$, then

$$U = -pE\cos\theta$$

$$= -\vec{p}.\vec{E}$$

Intensity of Electric field on axial line of an electric dipole : Electric field intensity due to an electric dipole at a distance r from its centre on axial line is given by,

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

Where, $2a$ = length of the electric dipole, and
p = dipole moment of the electric dipole

For small dipole, $2a << r$, then

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{2p}{r^3}.$$

Intensity of Electric field on equatorial line of an electric dipole : Electric field intensity due to an electric dipole at a distance r from its centre on equatorial line is given by

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$$

Where $2a$ = length of the electric dipole, and
p = dipole moment of the electric dipole

For small dipole, $2a << r$, then

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{p}{r^3}$$

Electrostatic potential : Electrostatic potential at a point in the electric field is defined as the amount of work done in moving a unit positive test charge from infinity to that point against electrostatic force. It is a scalar quantity. Its SI unit is volt (V).

Electric Potential Difference : Electric potential difference two points in the electric field is defined as the amount of work done in moving a unit positive test charge from one point to another point against electric force. It is a scalar quantity. Its SI unit is volt (V). If W_{AB} is the amount of work done in moving a small positive test charge q_0 from point A to point B in the electrostatic field of charge q, then the potential difference between the two points A and B is given by,

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.dr$$

$$= \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Volt : Electric potential difference between two points in the electric field is said to be 1 volt, when 1 joule work is done in moving a positive charge of 1 coulomb from one point to another point against electrostatic force. Hence

$$1 \text{ Volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

or $$1 \text{ V} = \frac{1 \text{ J}}{1 \text{ C}} = 1 \text{ JC}^{-1}$$

Potential at a point :

(i) **due to single point charge :** Electric potential at any point due to a single point charge q distant r from the point is given by,

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$$

(ii) **due to a group of point charge :** Electric potential at any point due to a number of point charges $q_1, q_2, q_3, q_4, ..., q_n$, distant $r_1, r_2, r_3, ... r_n$ from the point, is given by

$$V = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

(iii) **due to continuous charge distribution :** Electric potential at any point due to a number of point charges as well as three types of continuous charge distribution is given by,

$$V = \frac{1}{4\pi\varepsilon_o}\left[\sum_{i=1}^{n}\frac{q_i}{r_i} + \int_L \frac{\lambda dl}{r} + \int_S \frac{\sigma ds}{r} + \int_V \frac{\rho dv}{r}\right]$$

(iv) **due to an electric dipole :** Electric potential at any point distant r from the centre of an electric dipole of dipole moment p and length of dipole '$2a$', is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

where θ is the angle which the line joining the point to the centre of the dipole makes with the length of the dipole.

(a) If the point lies on the axial line of the dipole, then $\theta = 0°$ and hence $\cos\theta = \cos 0° = 1$

$$\therefore\ V = \frac{1}{4\pi\varepsilon_o}\cdot\frac{p}{(r^2 - a^2)}$$

For small dipole, $2a << r$,

$$\therefore\ V = \frac{1}{4\pi\varepsilon_o}\cdot\frac{p}{r^2}$$

(b) If the point lies on the equatorial line of the dipole, then $\theta = 90°$, and hence

$\cos\theta = \cos 90° = 0$

$\therefore V = 0$

Relation between the intensity of electric field and potential gradient : The intensity of electric field at any point in the electric field is negative of the potential gradient at that point.

$$E = \frac{-dv}{dr}$$

Electrostatic potential energy : Electrostatic potential energy of the system of two points charges q_1, q_2, separated by a distance r_{12} is given by,

$$U = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{r_{12}}$$

Electric Flux : Electric flux over an area in an electric field represents the total number of lines of force passing through this area. It is represented by ϕ. It is a scalar quantity. Electric Flux over surface area S is given by

$$\phi = \oint_S \vec{E} . \vec{d}S$$

Gauss's Theorem : This law gives a relation between the electric flux through any closed hypothetical surface and the charge enclosed by the surface. It states that the total electric flux over a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_o}$ times the total charge enclosed inside it. *i.e.,*

$$\phi = \frac{Q}{\varepsilon_o}$$

Where Q is the total charge enclosed and ε_0 is the permittivity of the free space.

Expression for electric field intensity :

According to Gauss's theorem,

$$\phi = \oint_S \vec{E}.d\vec{S} = \frac{Q}{\varepsilon_o}$$

(i) Electric field intensity due to infinitely long straight wire (a line charge)

$$E = \frac{1}{2\pi\varepsilon_o}.\frac{\lambda}{r}$$

where, λ = linear charge density

r = perpendicular distance of the wire from the point of observation.

(ii) Electric field intensity due to a thin infinite plane sheet of charge.

$E = \frac{\sigma}{2\varepsilon_o}$; where σ = surface charge density.

(iii) Electric field intensity between two thin infinite plane parallel sheets of charge

$$E = \frac{\sigma}{\varepsilon_o}$$

where σ = surface charge density

(iv) Electric field intensity due to uniformly charged spherical shell.

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \text{ for } r > R \text{ \{ point outside the shell \}}$$

$E = 0$ for $r < R$ { point inside the shell }

$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$ for $r = R$ { point at the surface of the shell }.

where, $q = 4\pi R^2\sigma$

σ = surface charge density

R = radius of the shell

r = distance of the point of observation from the centre of the shell.

(v) Electric field intensity due to charged solid sphere.

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \text{ for } r > R \text{ \{ point outside the sphere \}}$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{qr}{R^3} \text{ for } r < R \text{ \{ point inside the sphere \}}$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{R^2} \text{ for } r = R \text{ \{ point at the surface of the sphere \}}$$

Where $q = \frac{4}{3}\pi R^3 \rho$

ρ = volume charge density

R = radius of the sphere

r = distance of the point of observation from the centre of the sphere.

Capacitor : A capacitor is a device of storing large quantity of electric charge.

Capacitance : Capacitance of a conductor is defined as the amount of charge required to raise its potential by unity. Hence

$$\text{Capacitance} = \frac{\text{Electric charge}}{\text{Electric potential}}$$

or $$C = \frac{Q}{V}$$

Its SI unit is farad (F)

Farad : Capacitance of a conductor is said to be 1 farad, when 1 coulomb of charge raised its potential by 1 volt. Hence,

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

$$\text{or} \quad 1 \text{ F} = \frac{1 \text{ C}}{1 \text{ V}} = 1 \text{ CV}^{-1}$$

Expressions for Capacitance :

(i) Isolated spherical conductor

$$C = 4\pi\varepsilon_0 r$$

where r = radius of the spherical conductor.

(ii) Parallel plate capacitor

$C = \dfrac{\varepsilon_o A}{d}$ (with air or vacuum in between the plates)

$C = \dfrac{\varepsilon_o K A}{d}$ (with dielectric in between the plates)

where, A = area of each plate of the capacitor

d = distance between the two plates of the capacitor

K = dielectric constant of the dielectric.

(iii) Spherical capacitor

$$C = 4\pi\varepsilon_o \frac{ab}{b-a}$$

Where, a = radius of the inner spherical shell of capacitor

b = radius of the outer spherical shell of the capacitor.

(iv) Cylindrical capacitor :

$$C = \frac{2\pi\varepsilon_o l}{\log_e \frac{b}{a}} = \frac{2\pi\varepsilon_o l}{2.3026 \log_{10} \frac{b}{a}}$$

Where, l = length of the cylindrical capacitor

a = radius of the inner cylindrical shell of the capacitor

b = radius of the outer cylindrical shell of the capacitor

Energy stored in a capacitor : Energy stored in a charged capacitor is given by

$$E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}.\frac{Q^2}{C}$$

Where, C = capacitance of the capacitor

V = potential of the capacitor

Q = charge on the capacitor

Grouping of Capacitors :

(i) *Series combinations* : Equivalent capacitance C of a number of capacitors C_1, C_2, C_3, ... in series is given by,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

(ii) *Parallel combination* : Equivalent capacitance C of a number of capacitors $C_1, C_2, C_3, ...$ in parallel is given by $C = C_1 + C_2 + C_3, + ...$

Equipotential surfaces : Equipotential surfaces are those surfaces at which the potential is same at all points of the surface. These surfaces have following properties :

(i) The work done in moving any charge over these surfaces is zero.

(ii) Equipotential surfaces are always normal to lines of force.

(iii) Two equipotential surfaces never intersect each other.

(iv) Closer the equipotential surfaces, more is the intensity of electric field.