3. Number System

3.1 Types Of Numbers

Following are different types of numbers which we generally use in our calculations.

Natural Numbers: The numbers 1,2,3,4.... are called natural numbers or positive integers.

Whole Numbers: The numbers 0,1,2,3.... are called whole numbers. Whole numbers include ‘0’. Every natural number is a whole number but the converse is not true.

Integers: The numbers .... -3, -2, -1, 0, 1, 2, 3,....are called integers. Every whole number is an integer but the converse is not true.

Negative Integers: The numbers -1, -2, -3, .. are called negative integers.

Rational Numbers: Any number which is a positive or negative integer or fraction, or zero is called a rational number. A rational number is one which can be expressed in the following format $\Rightarrow \frac{a}{b}$, where b $\neq$ 0 and a & b are positive or negative integers. Every integer is a rational number but the converse is not true.

Positive Fractions: The numbers $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}$... are called positive fractions.

Negative Fractions: The numbers $-\frac{6}{8}$, $-\frac{7}{19}$, $-\frac{12}{47}$... are called negative fractions.

Note: Between any two different rational numbers a & b there always exists a rational number calculated by taking the average of a and b i.e. $\frac{a+b}{2}$

Irrational Numbers: A non-terminating non-recurring decimal number is known as an irrational number. These numbers cannot be expressed in the form of a proper fraction a/b where $b \neq 0$.

e.g. $\sqrt{2}$, $\sqrt{5}$, π, etc.

Even Numbers: The numbers which are divisible by 2 are called even numbers e.g. -4, 0, 2, 16 etc.

Odd Numbers: The numbers which are not divisible by 2 are odd numbers e.g. -7, -15, 5, 9 etc.

Prime Numbers: Those numbers, which are divisible only by themselves and 1, are called prime numbers. In other words, a number, which has exactly two factors –

1 and itself, is called a prime number. e.g. 2, 3, 5, 7, etc.

2 is the only even prime number.

There are 25 prime numbers up to 100. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 & 97. These should be learnt by heart.

Composite Number: A number, which has more than two factors, is called a composite number. e.g. 9, 10, 15, 16, ...

A composite number can be expressed as the product of prime numbers in a unique way. e.g. $100 = 2^2 \times 5^2$

1 is neither a composite number nor a prime number.

Real Numbers: The above sets of natural numbers, integers, whole numbers, rational numbers and irrational numbers constitute the set of real numbers. Every real number can be represented by a point on a number line.

Perfect Numbers: If the sum of all the factors of a number excluding the number itself happens to be equal to the number, then the number is called as perfect number. 6 is the first perfect number. The factors of 6 are 1, 2, 3 & 6. Leaving 6, the sum of other factors of 6 are equal to 6. The next three perfect numbers after 6 are 28, 496 and 8128.

3.2 Operations On Odd & Even Numbers

(i) Addition or subtraction of any two odd numbers will always result in an even number or zero. E.g. 1 + 3 = 4; 5 - 3 = 2.

(ii) Addition or subtraction of any two even numbers will always result in an even number or zero. E.g. 2 + 4 = 6; 12 - 4 = 8.

(iii) Addition or subtraction of an odd number from an even number will result in an odd number. E.g. 4 + 3 = 7; 10 - 3 = 7.

(iv) Addition or subtraction of an even number from an odd number will result in an odd number. E.g. $3 + 4 = 7$; $5 - 2 = 3$.

(v) Multiplication of two odd numbers will result in an odd number. E.g. $3 \times 3 = 9$.

(vi) Multiplication of two even numbers will result in an even number. E.g. $2 \times 4 = 8$.

(vii) Multiplication of an odd number by an even number or vice versa will result in an even number. E.g. $3 \times 2 = 6$.

(viii) An odd number raised to an odd or an even power is always odd.

(ix) An even number raised to an odd or an even power is always even.

(x) The standard form of writing a number is $m \times 10^n$ where m lies between 1 and 10 and n is an integer. e.g. 0.89713 $\Rightarrow$ $8.9713/10^1 . \Rightarrow 8.9713 \times 10^{-1}$.

(xi) If n is odd. $n(n^2 - 1)$ is divisible by 24. e.g. take $n = 5 \Rightarrow 5(5^2 - 1) = 120$, which is divisible by 24.

(xii) If n is odd prime number except 3, then $n^2 - 1$ is divisible by 24.

(xiii) If n is odd. $2^n + 1$ is divisible by 3.

(xiv) If n is even. $2^n - 1$ is divisible by 3.

(xv) If n is odd, $2^{2n} + 1$ is divisible by 5.

(xvi) If n is even, $2^{2n} - 1$ is divisible by 5.

(xvii) If n is odd, $5^{2n} + 1$ is divisible by 13.

(xviii) If n is even, $5^{2n} - 1$ is divisible by 13

3.3 Tests Of Divisibility

Divisibility rules are very helpful while doing calculations. Following are some important divisibility rules which you should learn by heart.

(i) **By 2** - A number is divisible by 2 when its units place is 0 or divisible by 2. e.g. 120, 138.

(ii) **By 3** – A number is divisible by 3 when the sum of the digits of the number is divisible by 3. e.g. 15834 is divisible by 3 as the sum of its digits is 21 which is divisible by 3. Note that if n is odd, then $2^n + 1$ is divisible by 3 and if n is even, then $2^n - 1$ is divisible by 3.

(iii) **By 4** - A number is divisible by 4 when the last two digits of the number are 0s or are divisible by 4. As 100 is divisible by 4, it is sufficient if the divisibility test is restricted to the last two digits. e.g. 145896, 128, 18400.

(iv) **By 5** - A number is divisible by 5, if its unit's digit is 5 or 0. e.g. 895, 100.

(v) **By 6** - A number is divisible by 6, if it is divisible by both 2 and by 3. i.e. the number should be an even number and the sum of its digits should be divisible by 3.

(vi) **By 8** - A number is divisible by 8, if the last three digits of the number are 0s or are divisible by 8. As 1000 is divisible by 8, it is sufficient if the divisibility test is restricted to the last three digits e.g. 135128, 45000

(vii) **By 9** - A number is divisible by 9, if the sum of its digits is divisible by 9. e.g. 810, 92754.

(viii) **By 11** - A number is divisible by 11, if the difference between the sum of the digits at odd places and sum of the digits at even places of the number is either 0 or a multiple of 11.

e.g.121, 65967. In the first case 1+1-2 = 0. In the second case 6+9+7 = 22 and 5+6 = 11 and the difference is 11. Therefore, both these numbers are divisible by 11.

(ix) **By 12** - A number is divisible by 12, if it is both divisible by 3 and by 4. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 4. e.g. 144, 8136.

(x) **By 15 –** A number is divisible by 15, if it is divisible by both 5 and 3.

(xi) **By 25** – 2358975 is divisible by 25 if the last two digits of 2358975 are divisible by 25 or the last two digits are 0.

(xii) **By 75** - A number is divisible by 75, if it is both divisible by 3 and by 25. i.e. the sum of the digits should be divisible by 3 and the last two digits should be divisible by 25.

(xiii) **By 125 -** A number is divisible by 125, if its last three right hand digits are divisible by 125 or the last three digits are 0s. e.g. 1254375, 12000

3.4 Properties Of The Numbers

Following are some properties of the numbers which are helpful in solving the questions in the examination.

(i) The sum of 5 successive whole numbers is always divisible by 5.

(ii) The product of 3 consecutive natural numbers is divisible by 6.

(iii) The product of 3 consecutive natural numbers, the first of which is an even number is divisible by 24.

(iv) The sum of a two-digit number and a number formed by reversing its digits is divisible by 11. E.g. 28 + 82 = 110, which is divisible by 11. At the same time, the difference between those numbers will be divisible by 9. e.g. 82– 28 = 54, which is divisible by 9.

(v) $\Sigma n = \frac{n(n+1)}{2}$, Σn is the sum of first n natural numbers.

(vi) $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$, Σn^2 is the sum of first n perfect squares.

(vii) $\Sigma n^3 = \frac{n^2(n+1)^2}{4} = (\Sigma n)^2$, Σn^3 is the sum of first n perfect cubes.

(viii) $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - ... + y^{n-1})$ when n is odd. Therefore, when n is odd, $x^n + y^n$ is divisible by $x + y$.
e.g. $3^3 + 2^3 = 35$ and is divisible by 5.

(ix) $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}.y + ... y^{n-1})$ when n is even. Therefore, when n is even, $x^n - y^n$ is divisible by $x + y$.
e.g. $7^2 - 3^2 = 40$, which is divisible by 10.

(x) $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}.y + ... + y^{n-1})$ for both odd and even n. Therefore, $x^n - y^n$ is divisible by $x - y$.
e.g . $9^4 - 2^4 = 6545$ which is divisible by 7.

3.5 LCM & HCF

- In case of HCF, if some remainders are given, then first those remainders are subtracted from the numbers given and then their HCF is calculated.

- Sometimes in case of HCF questions, the same required remainder is given and the remainder is not given. In such questions, the answer is the HCF of the difference of the numbers taken in pairs.

- In case of LCM, if a single remainder is given, then firstly the LCM is calculated and then that single reminder is added in that.

- In case of LCM, if for different numbers different remainders are given, then the difference between the number and its respective remainder will be equal. In that case, firstly the LCM is calculated, then that common difference between the number and its respective remainder is subtracted from that.

LCM and HCF of Fractions

LCM of fractions = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$;

e.g. LCM of $\frac{3}{4}$ and $\frac{1}{2}$ = $\frac{3\ (\text{LCM of numerators})}{2\ (\text{HCF of denominators})}$

HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

e.g. HCF of $\frac{3}{4}$ and $\frac{1}{2}$ = $\frac{1\text{(HCF of numerators)}}{4\text{(LCM of denominators)}}$

Note that the product of the two fractions is

always equal to the product of LCM and HCF of the two fractions. The product of the two fractions = $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

The product of the LCM and HCF = $\frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$.

3.6 Fractions

Numbers of the form 3/4, 4/5, are called fractions. A fraction can be written as p/q where $q \neq 0$.

- If the numerator and the denominator of a fraction are multiplied / divided by the same number then the value of the fraction does not change.

- For any positive proper fraction p/q ($p<q$), the value of the fraction increases when both the denominator and numerator are added by the same positive number.
  e.g $\frac{3}{4}$= 0.75, $\frac{3+1}{4+1}$= $\frac{4}{5}$ = 0.8.
- For any positive proper fraction p/q ($p<q$), the value of the fraction decreases when both the numerator and denominator are subtracted by the same positive number.
  e.g $\frac{3}{4}$= 0.75, $\frac{3-1}{4-1}$ = $\frac{2}{3}$= 0.67
- For any positive improper fraction p/q ($p>q$), the value of the fraction decreases when both the numerator and the denominator are added to the same positive. E.g. $\frac{5}{4}$= 1.25, adding 1 to the numerator and the denominator, we get $\frac{5+1}{4+1}$= $\frac{6}{5}$= 1.2, which is less than 1.25.
- For any positive improper fraction p/q ($p>q$), the value of the fraction

increases when both the numerator and denominator are subtracted by the same positive number. E.g. $\frac{5}{4}$ = 1.25, by subtracting 1 from both the numerator and denominator we get, $\frac{5-1}{4-1}$ = $\frac{4}{3}$ = 1.33 > 1.25.

Types Of Fractions:

Common Fractions: Fractions such as 3/4, 32/43 etc are called common or vulgar fractions.

Decimal Fractions: Fractions whose denominators are 10, 100, 1000…. are called decimal fractions.

Proper Fraction: A fraction whose numerator is less than its denominator is known as a proper fraction e.g. 3/4

Improper Fraction: A fraction whose numerator is greater than its denominator is known as an improper fraction. e.g. 4/3

Mixed Fractions: Fractions which consist of an integral part and a fractional part are called mixed fractions. All improper fractions can be expressed as mixed fractions and vice versa. e.g. $1\frac{3}{4}$.

Recurring Decimals: A decimal in which a set of figures is repeated continually is called a recurring or periodic or a circulating decimal.

e.g. $1/7$ = 0.142857... the dots indicate that the figure between 1 and 7 will repeat continuously.

3.7 Indices

The expression $a^5 = a \times a \times a \times a \times a$

Similarly for any positive integer n, $a^n = a \times a \times a \times \ldots..$ n times.

In a^n, a is called the base and n is called the index.

Law Of Indices

Let m and n be positive integers, then

(i) $a^m \times a^n = a^{m+n}$
(ii) $(a^m)^n = a^{mn}$
(iii) $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$
(iv) $(ab)^m = a^m \times b^m$
(v) $a^0 = 1$, where $a \neq 0$
(vi) $a^{-n} = \frac{1}{a^n}$
(vii) $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ i.e. the q^{th} root of 'a' raised to the power of 'p'.
(viii) In particular, $a^{\frac{1}{q}} = \sqrt[q]{a}$

3.8 Remainder Theory

Questions from the **number system appear** regularly in almost all competitive exams. Within the number system, the questions on remainders are found to be the most tricky. This will help you learn the different types of remainder questions and the various approaches which you can apply to solve these.

The basic remainder formula is:

Dividend = Divisor × Quotient + Remainder
If remainder = 0, then the number is perfectly divisible by the divisor and the divisor is a factor of the number e.g. when 8 divides 40, the remainder is 0, it can be said that 8 is a factor of 40.
There are few important results relating to numbers. Those will be covered one by one in the following examples.

(i) **Formulas Based Concepts For Remainder:**

$(a^n + b^n)$ is divisible by $(a + b)$, when n is odd.
$(a^n - b^n)$ is divisible by $(a + b)$, when n is even.

$(a^n - b^n)$ is always divisible by $(a - b)$, for every n.

(ii) Concept Of Negative Remainder:

By definition,a remainder cannot be negative. But in certain cases, you can assume *that* for your convenience. But a negative remainder in real sense means that you need to add the divisor in the negative remainder to find the real remainder.
NOTE: Whenever you are getting a negative number as the remainder, make it positive by adding the divisor to the negative remainder.

(iii) Cyclicity In Remainders:

Cyclicity is the property of remainders, due to which they start repeating themselves after a certain point.

(iv) Role Of Euler's Number In Remainders:

Euler's Remainder theorem states that, for co-prime numbers M and N, Remainder $[M^{E(N)} / N] = 1$, i.e. number M raised to Euler number of N will leave a remainder 1 when divided by N. Always check whether the numbers are co-primes or not as Euler's theorem is applicable only for co-prime numbers.