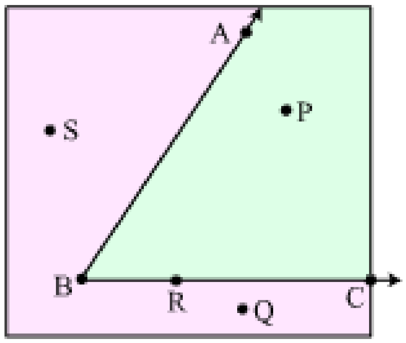
Angles and Lines

- The given figure represents ∠ABC with some points in its region.

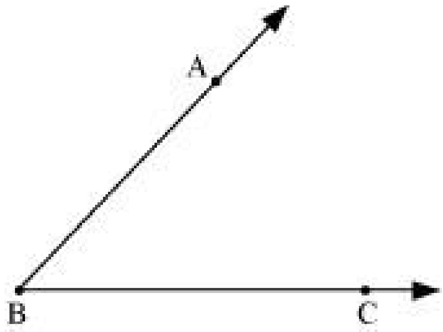


- The region of the angle shaded by green colour lies between the two arms of the angle. This region is called the **interior region of the angle**. Every point in this region is said to lie in the **interior** of the angle. Here, point P is in the interior.

- The region of the angle shaded by pink colour lies outside the two arms of the angle.This region is called the **exterior region of the angle**. Every point in this region is said to lie in the **exterior** of the angle. Here, point Q and S are in the exterior.

- The boundary of ∠ABC is formed by its arms $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Every point lying on the arms is said to lie on the **boundary of the angle**. Here, points A, B, C and R lie on the boundary of the angle.

- **Angle:** An angle is made up of two rays starting from a common end point.



In this figure, rays $\overline{BA}$ and $\overline{BC}$ have one common end point, that is, B. The rays $\overline{BA}$ and $\overline{BC}$ are called the arms or sides of the angle. The common end point B is the vertex of the angle.
We can name the above angle as ∠ABC or ∠CBA.

- One complete turn of the hand of a clock is one revolution. The angle of one revolution is called a **complete angle.**

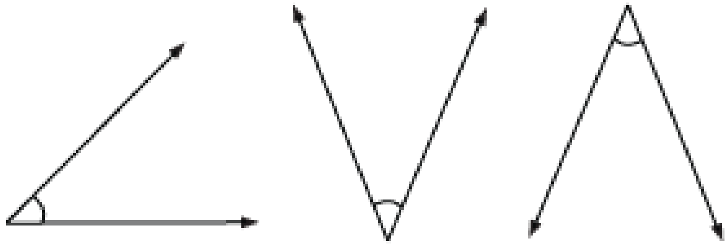
- A right angle is $\left(\frac{1}{4}\right)^{\text{th}}$ of a revolution and a straight angle is $\left(\frac{1}{2}\right)^{\text{th}}$ of a revolution.

Right angle

Straight angle

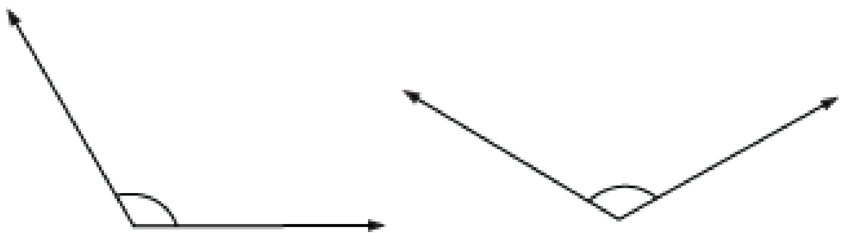
- 1 complete angle = 2 straight angles = 4 right angles
- 1 straight angle = 2 right angles
- If an angle measures less than a right angle then it is known as an **acute angle.**

  The following angles are acute:



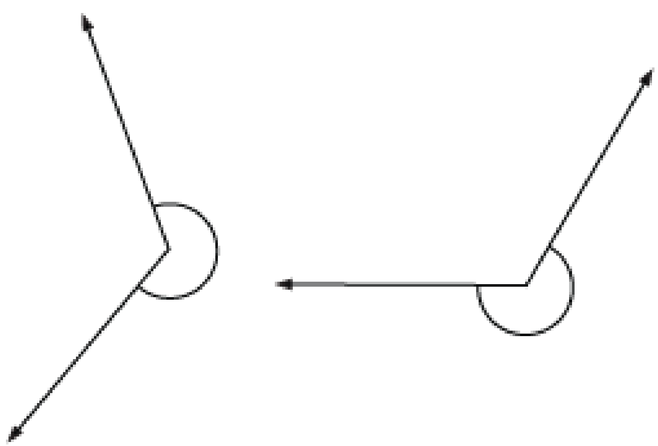
- If an angle measures more than a right angle but less than a straight angle, then it is an **obtuse angle.**

  The following angles are obtuse:



- If an angle measures more than a straight angle, then it is known as a **reflex angle.**

  The following angles are reflex:

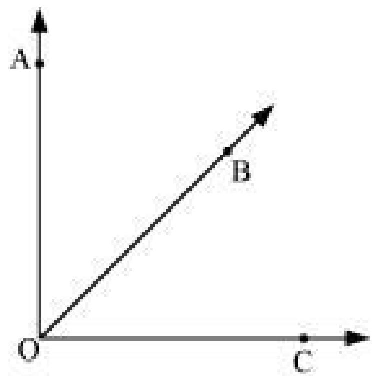


- We use a protractor to measure an angle.
- One complete revolution is divided into 360 equal parts. Each part is called a **degree.** Thus, the unit of angle is degree ($^\circ$).
- Right angle measures 90°, complete angle measures 360°, and straight angle measures 180°.
- Acute angle is less than 90°, obtuse angle is more than 90° but less than 180°, and reflex angle is more than 180° but less then 360°.

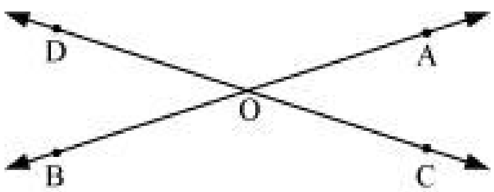
- A pair of angles are called adjacent angles, if:
  - they have a common vertex

- they have a common arm
- the non-common arms are on either side of the common arm

  For example, ∠AOB and ∠BOC are adjacent angles as they have a common vertex O, common arm OB, and non-common arms OA and OC lie on either side of OB.

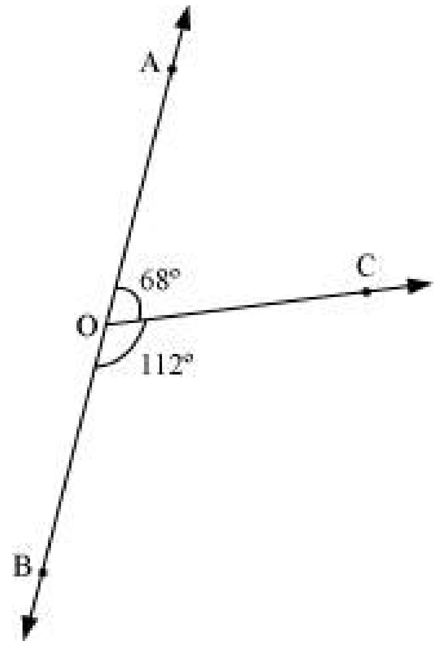


- When two lines intersect, the vertically opposite angles so formed are equal.



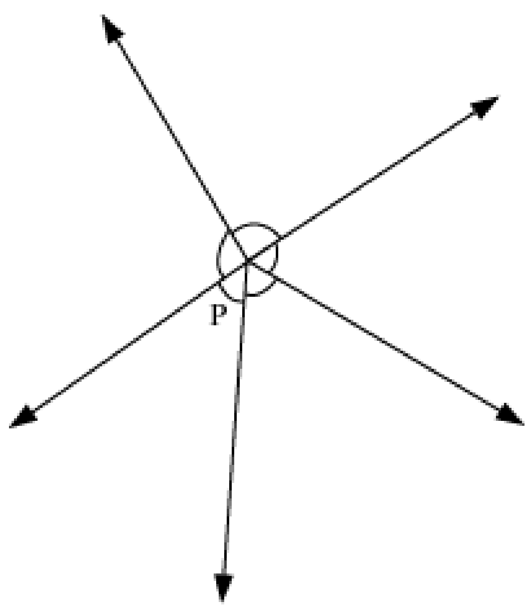
Here, ∠AOC = ∠BOD and ∠AOD = ∠BOC.

- A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays.
- The sum of the measures of the adjacent angles is 180°.



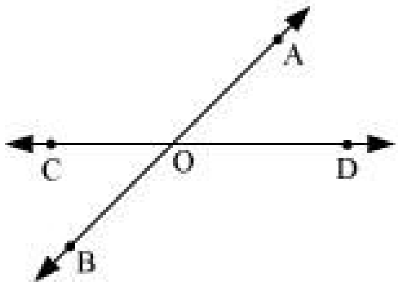
Here, ∠AOC and ∠BOC form a linear pair as ∠AOC + ∠BOC = 180°.

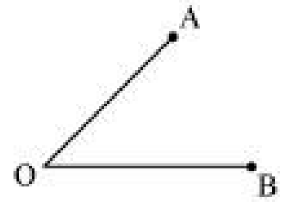
- The sum of angles around a point is equal to 360°.



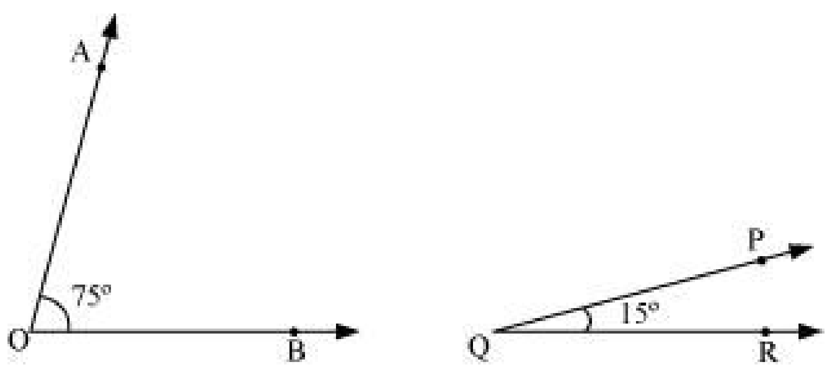
In this figure, five angles have a common vertex, which is point P. In other words, the five angles make a complete turn and therefore the sum of these five angles will be equal to 360°. This is true no matter how many angles make a complete turn.

- An angle is made when two lines or line segments meet. For example:



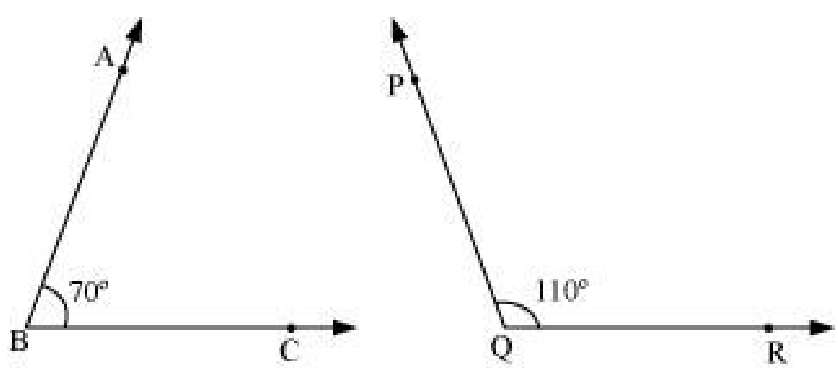


- When the sum of the measures of two angles is 90°, the angles are called **complementary angles**.



Here, ∠AOB and ∠PQR are complementary as (∠AOB + ∠PQR) = 75° + 15° = 90°.

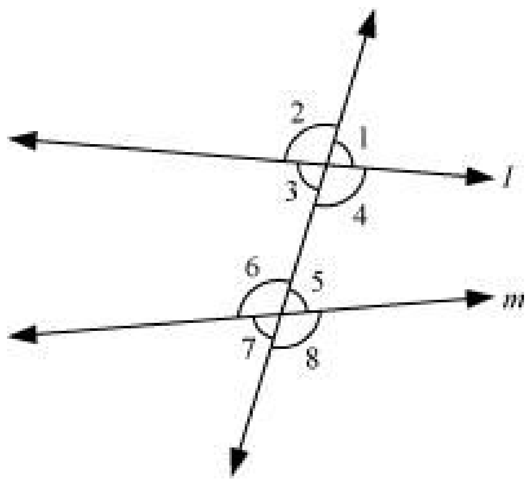
- When the sum of the measures of two angles is 180°, the angles are called **supplementary angles**.



Here, ∠ABC and ∠PQR are supplementary as (∠ABC + ∠PQR) = 110° + 70° = 180°.

- **Corresponding angles**

When a transversal intersects two lines *l* and *m*, the **c**orresponding angles so formed at the intersection points are named as follows:



∠1 and ∠5, ∠2 and ∠6

∠3 and ∠7, ∠4 and ∠8

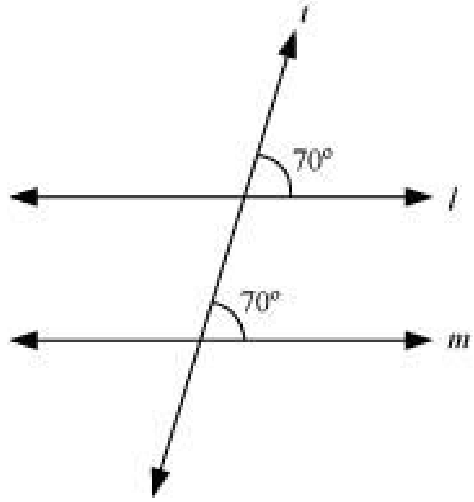
- **Corresponding angles axiom**

If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
In the above figure, if lines *l* and *m* become parallel then we will have following pair of equal angles:

∠1 =∠5, ∠2 =∠6, ∠3 =∠7 and ∠4 = ∠8

- **Converse of corresponding angles axiom**

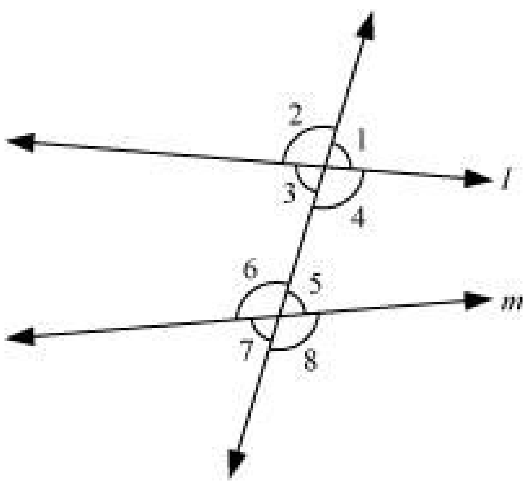
If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.



In the figure, the corresponding angles are equal. Therefore, the lines *l* and *m* are parallel to each other.

- **Alternate angles**

When a transversal intersects two lines *l* and *m*, the alternate angles so formed at the intersection points are named as follows:

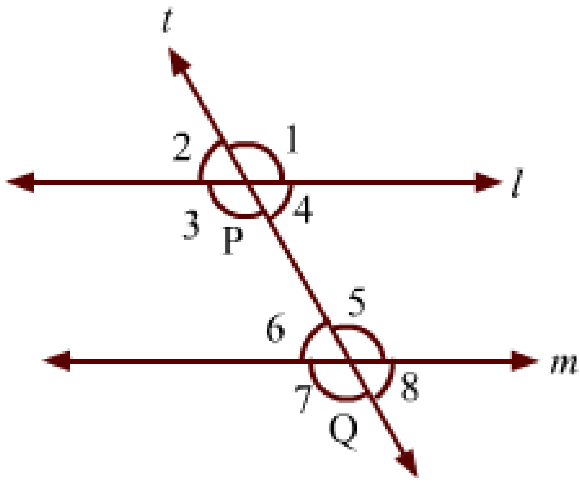


Alternate interior angles
$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

Alternate exterior angles
$\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$

- **Alternate angles axiom**

If a transversal intersects two parallel lines, then the angles in each pair of alternate angles are equal.

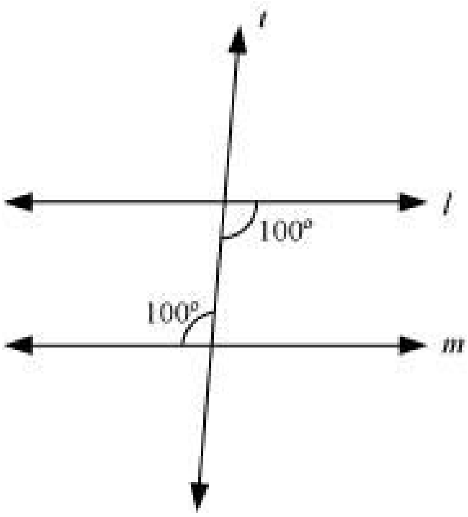


In the above figure, lines l and m are parallel. So, by using the alternate angles axiom, we can say that:

$\angle 1 = \angle 7$, $\angle 2 = \angle 8$, $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$

- **Converse of alternate angles axiom**

  If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.

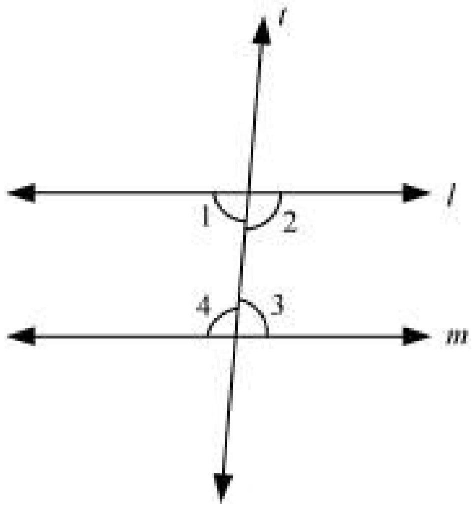


In the above figure, alternate interior angles are equal (100°) and thus, lines l and m are parallel.

- **Property of interior angles on the same side of a transversal:**

  If a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

For example,



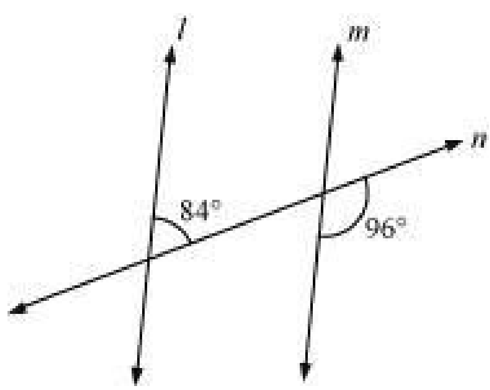
In the given figure, if lines *l* and *m* are parallel to each other then $\angle 1 + \angle 4 = 180°$ and $\angle 2 + \angle 3 = 180°$.

- **Converse of the property of interior angles on the same side of a transversal:**

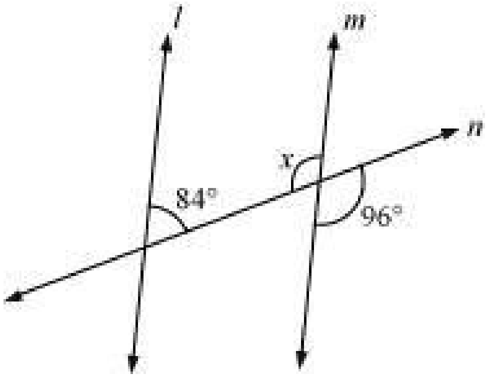
  If a transversal intersects two lines such that the interior angles on the same side of the transversal are supplementary, then the lines intersected by the transversal are parallel.

  Example:

  In the given figure, decide whether *l* is parallel to *m* or not.



Solution:



$\angle x = 96°$ (Vertically opposite angles)

$\angle x + 84 = 96° + 84° = 180°$

i.e., Sum of the interior angles on the same side of the transversal is supplementary. Therefore, $l \| m$.

• When a line intersects two distinct lines at different points, then this line is known as transversal and the portion of the **transversal** lying between these two distinct lines is known as **intercept**.

In the given figure, MN is the intercept made by lines *a* and *b* on transversal *x*.

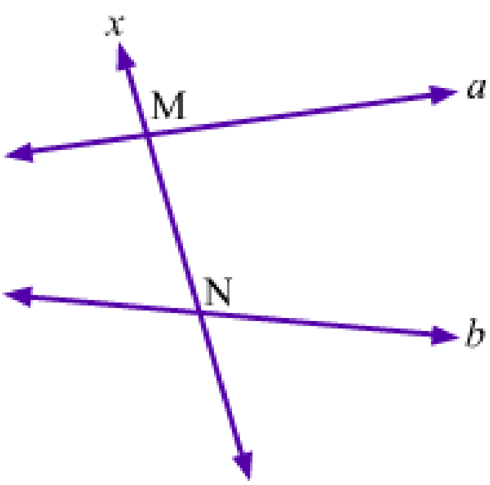


Fig. (1)

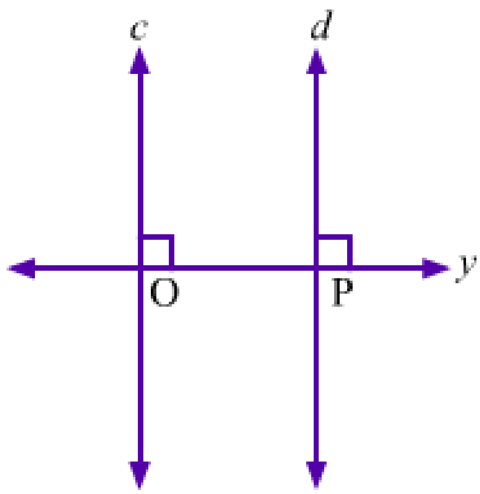


Fig. (2)

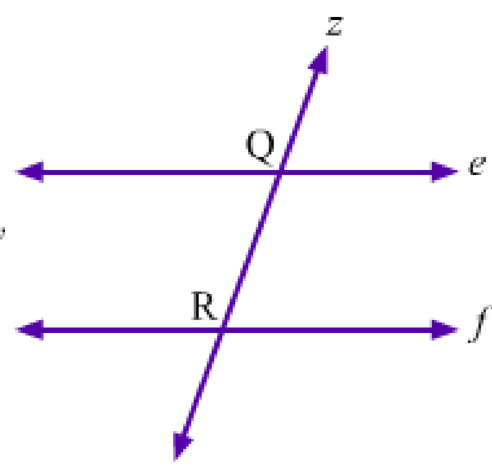
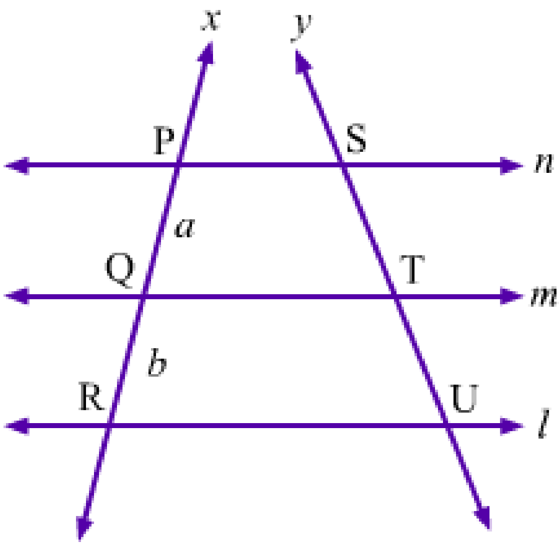


Fig. (3)

• The lengths of the intercepts made by three parallel lines on one transversal are in the same ratio as the lengths of the corresponding intercepts made by the same lines on any other transversal.

In the given figure, $\frac{PQ}{QR} = \frac{ST}{TU} = \frac{a}{b}$

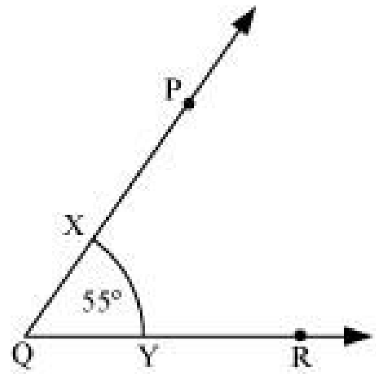


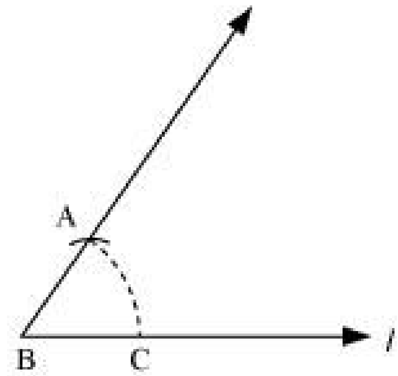
• If three parallel lines form congruent intercepts on one transversal, then the intercepts formed by them on the other transversals are also congruent.

- **Steps for the construction of copy of a given angle:**

  Given ∠PQR = 55°.

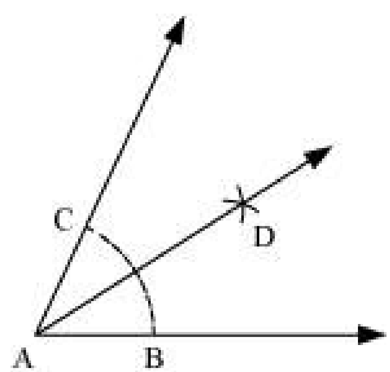
  1. Draw a line *l* and mark a point B on it.
  2. Place the compass at Q and draw an arc to cut the rays QP and QR at points X and Y respectively.
  3. Use the same compass setting to draw an arc with B as the centre, cutting *l* at C.
  4. Set your compass to length XY.
  5. Place the compass pointer at C and draw the arc (with the same setting) that cuts the arc drawn earlier at A.
  6. Join B with A and extend it.



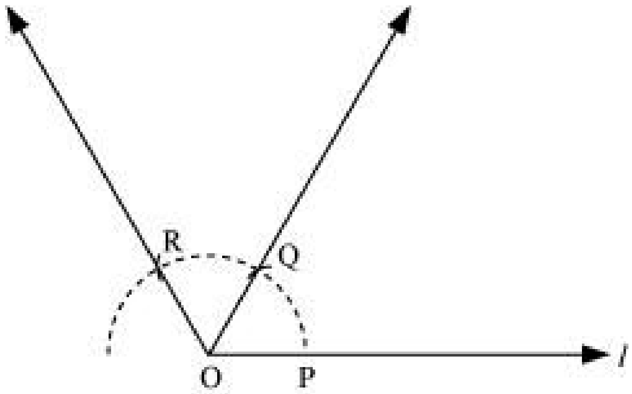


Now, ∠ABC = ∠PQR = 55°

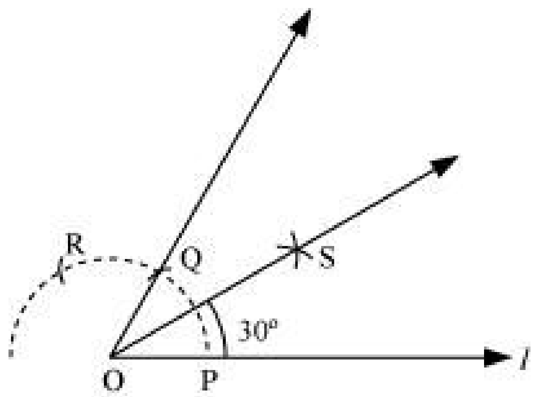
- **Steps of construction for the bisector of a given angle (say 60°):**
    1. Draw ∠A such that ∠A = 60°
    2. With A as the centre, draw an arc that cuts both the rays of ∠A at B and C.
    3. With B and C as centres and radius more than $\frac{1}{2}$ BC, draw two arcs that intersect each other at D.
    4. Join AD. AD is the bisector of ∠A.



- The steps for the construction of angles of measures 60° and 120° are as follows:
    1. Draw a line *l* and mark a point O on it.
    2. Place the pointer of the compass at O and draw an arc of convenient radius that cuts *l* at P.
    3. With the same radius, draw an arc with centre P that cuts the previous arc at Q.
    4. Similarly, with the same radius, draw an arc with centre Q that cuts the arc at R.
    5. Join OQ and OR to get ∠QOP = 60° and ∠ROP = 120°.



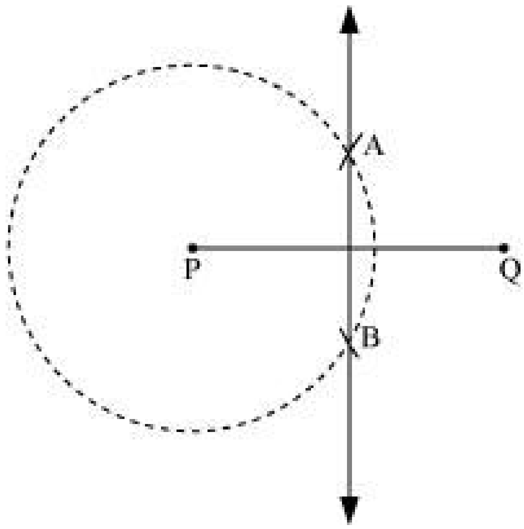
- Now, 30° is nothing but half of angle 60°. Therefore, 30°angle can be obtained by drawing the bisector of ∠QOP.



Here, ∠SOP = 30°.

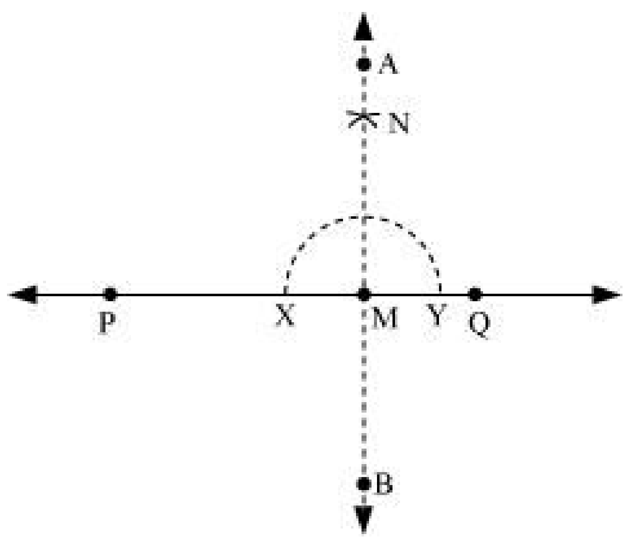
Similarly, we can draw other angles of measures 45°, 90°, 135°, and 150° using the above method.

- **Steps of construction for the perpendicular bisector of a line segment $\overline{PQ}$ where $\overline{PQ}$= 9.4 cm:**
    1. Draw a line segment $\overline{PQ}$ whose length is 9.4 cm.
    2. With P as the centre and radius more than half of $\overline{PQ}$, draw a circle using compass.
    3. With the same radius and Q as the centre, draw two arcs that cut the previous circle at points A and B. Join AB to get the perpendicular bisector of $\overline{PQ}$.



- **Steps to construct perpendicular to a line PQ through a point M on it:**

  1. Draw a line $\overleftrightarrow{PQ}$ and mark a point M on it.
  2. With M as the centre and a convenient radius, construct an arc intersecting $\overleftrightarrow{PQ}$ at two points i.e., X and Y. With X and Y as centres and radius greater than MX, construct two arcs that cut each other at N.
  3. Draw a line through points M and N and name this line as $\overleftrightarrow{AB}$. Now, $\overleftrightarrow{AB} \perp \overleftrightarrow{PQ}$.



- **Steps to construct perpendicular to a line AB through a point M not on it:**

  1. Draw line $\overleftrightarrow{AB}$. Mark a point M outside it.

2. With M as the centre, draw an arc that intersects $\overleftrightarrow{AB}$ at two points i.e., X and Y.
3. Using the same radius and with X and Y as centres, construct two arcs such that they intersect at N on the other side of the line.
4. Join $\overleftrightarrow{MN}$ to get $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$.