

6. $\lim_{x \rightarrow \infty} a^x \sin\left(\frac{b}{a^x}\right)$, $a, b > 1$ is equal to [1]
 a) b b) $b \log_e a$
 c) $a \log_e b$ d) a
7. The number of terms in the expansion of $(a + b + c)^{10}$ is [1]
 a) 55 b) 21
 c) 11 d) 66
8. The mean and standard deviation of 100 items are 50, 5 and that of 150 items are 40, 6 respectively. [1]
 What is the variance of all 250 items?
 a) 53.3 b) 55.6
 c) 59.3 d) 50.6
9. The sum of an infinite GP is $\frac{80}{9}$ and its common ratio is $-\frac{4}{5}$. The first term of the GP is [1]
 a) 16 b) 8
 c) 20 d) 12
10. If $\frac{\pi}{2} < x < \frac{3\pi}{2}$, then $\sqrt{\frac{1-\sin x}{1+\sin x}}$ is equal to [1]
 a) $\sec x + \tan x$ b) None of these
 c) $\sec x - \tan x$ d) $\tan x - \sec x$
11. The intercept cut off by a line from y-axis is twice than that from x-axis, and the line passes through the point (1, [1]
 2). The equation of the line is
 a) $2x + y = 4$ b) $2x + y + 4 = 0$
 c) $2x - y = 4$ d) $2x - y + 4 = 0$
12. $\lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} =$ [1]
 a) does not exist b) 0
 c) $-\frac{1}{3}$ d) $\frac{1}{3}$
13. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is [1]
 a) $|z|$ b) $\frac{|z|}{2}$
 c) $2|z|$ d) none of these
14. The coordinates of the foot of the perpendicular drawn from the point (2, -3, 4) on the y-axis is: [1]
 a) (2, 0, 4) b) (2, 3, 4)
 c) (-2, -3, -4) d) (0, -3, 0)
15. If the line $3x + 4y - 24 = 0$ intersects the X-axis at the point A and the Y-axis at the point B, then the incentre of [1]
 the triangle OAB, where O is the origin, is
 a) (4, 4) b) (4, 3)
 c) (3, 4) d) (2, 2)
16. If ${}^{18}C_{15} + 2({}^{18}C_{16}) + {}^{17}C_{16} + 1 = {}^nC_3$ then n is equal to: [1]

Find the number of integral terms in the expansion of $(5^{1/2} + 7^{1/8})^{1024}$.

29. Find the coordinates of the foot of perpendicular drawn from the point A (-1,8,4) to the line joining the points B (0,-1,3) and C(2,-3,-1). Hence, find the image of the point A in the line BC. [3]

OR

Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled isosceles triangle.

30. If the letters of the word ASSASSINATION are arranged at random. Find the probability that: No two A's are coming together [3]

31. Find the range of the function $f(x) = \frac{|x+4|}{x+4}$. [3]

Section D

32. Find the mean and standard deviation for the following data: [5]

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	2	4	6	5	5	5	2	8	5

33. A class has 175 students. The following description gives the number of students studying one or more of the subjects in this class: mathematics 100, physics 70, chemistry 46; mathematics and physics 30; mathematics and chemistry 28; physics and chemistry 23; mathematics, physics and chemistry 18. Find [5]

- how many students are enrolled in mathematics alone, physics alone and chemistry alone,
- the number of students who have not offered any of these subjects.

OR

In a survey of 25 students, it was found that 12 have taken physics, 11 have taken chemistry and 15 have taken mathematics; 4 have taken physics and chemistry; 9 have taken physics and mathematics; 5 have taken chemistry and mathematics while 3 have taken all the three subjects. Find the number of students who have taken

- physics only;
- chemistry only;
- mathematics only;
- physics and chemistry but not mathematics;
- physics and mathematics but not chemistry;
- only one of the subjects;
- at least one of the three subjects;
- none of the three subjects.

34. Find the vertex, axis, focus, directrix and length of latusrectum of parabola $y^2 - 8y - x + 19 = 0$. [5]

OR

Find the equation of the hyperbola whose foci are (4, 2) and (8,2) and eccentricity is 2.

35. Differentiate If $y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$ show that $\frac{dy}{dx} = (\sec x \tan x + \sec x)$ [5]

Section E

36. **Read the text carefully and answer the questions:** [4]

One morning a big circus arrived in the Ramleela maidan at Delhi. The arrival of the circus was seen in the morning at 08:00 AM by Gopal. He passed this information on 08:15 to 2 other residents of the city.

Each of these 2 people then informed the other 2 residents at 08:30, and again at 08:45, they reported the arrival of the circus every 2 to other uninformed residents

This chain continued the same way till 12:00 PM. By 12:00 PM enough people were informed about the arrival of the circus.



Using the above information answer the following questions:

- (i) If the customer has a 2-course meal, the number of ways of doing this is:
- | | |
|--------|--------|
| a) 38 | b) 110 |
| c) 200 | d) 329 |
- (ii) If the customer has a 3-course meal, the number of combinations is:
- | | |
|--------|--------|
| a) 300 | b) 200 |
| c) 110 | d) 57 |
- (iii) How many different possible meals do the restaurant offer i.e. The number of possible meals is:
- | | |
|--------|--------|
| a) 300 | b) 329 |
| c) 310 | d) 200 |

OR

A person who eats an appetizer and the main meal has:

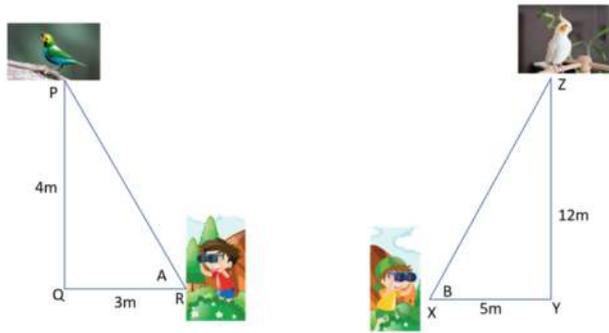
- | | |
|---------------|----------------|
| a) 20 choices | b) 50 choices. |
| c) 40 choices | d) 60 choices |

38. **Read the text carefully and answer the questions:**

[4]

Anand and Sri went for walking. Anand observes a bird on a top tree with an angle of elevation A . The distance between Anand and the tree on the ground is 3 m and height of the tree on which bird is sitting is 4m. At the same time Sri observes another bird on the top of house with angle of elevation B . The distance between Sri and

house on the ground is 5m and height of the house where bird is sitting is 12m.



- (i) Find the value of $\cos A + \sin B$.
- (ii) Find the value of $\sin(A + B)$.

Solution

CBSE SAMPLE PAPER - 02

Class 11 - Mathematics

Section A

1. (c) [1, 6]

Explanation: For $f(x)$ to be real, we must have,

$$x - 1 \geq 0 \text{ and } 6 - x \geq 0$$

$$\Rightarrow x \geq 1 \text{ and } x - 6 \leq 0$$

$$\therefore \text{Domain} = [1, 6]$$

2. (d) $\beta + i\alpha$

Explanation: $(a + bi)^9 = \alpha + i\beta \Rightarrow i(\alpha + i\beta) = i(a + bi)^9$

$$\Leftrightarrow i\alpha - \beta = (ia - b)^9 \dots [i = i^9]$$

Take conjugate

$$-i\alpha - \beta = (-ia - b)^9 = -(ia + b)^9$$

$$\Rightarrow (b + ia)^9 = i\alpha + \beta$$

3. (c) null set

Explanation: $\frac{2x-1}{3} - \frac{3x}{5} + 1 < 0$

$$\Rightarrow 15 \cdot \frac{2x-1}{3} - 15 \cdot \frac{3x}{5} + 15 < 0 \text{ [Multiply the inequality throughout by the L.C.M]}$$

$$\Rightarrow 5(2x - 1) - 3(3x) + 15 < 0$$

$$\Rightarrow 10x - 5 - 9x + 15 < 0$$

$$\Rightarrow x + 10 < 0$$

$$\Rightarrow x < -10, \text{ but given } x \in W$$

Hence the solution set will be null set.

4. (c) $\frac{6}{55}$

Explanation: Total number of ways of selecting 2 different numbers from $\{0, 1, 2, \dots, 10\} = {}^{11}C_2 = 55$

Let two numbers selected be x and y

Then, $x + y = 4m \dots(i)$

and $x - y = 4n \dots(ii)$

$$\Rightarrow 2x = 4(m + n) \text{ and } 2y = 4(m - n)$$

$$\Rightarrow x = 2(m + n) \text{ and } y = 2(m - n)$$

$\therefore x$ and y both are even numbers

x	y
0	4, 8
2	6, 10
4	0, 8
6	2, 10
8	0, 4
10	2, 6

$$\therefore \text{Required probability} = \frac{6}{55}$$

5. (d) $F_2 \cup F_3 \cup F_4 \cup F_1$

Explanation: We know that

Every rectangle, square and rhombus is a parallelogram

But, no trapezium is a parallelogram

$$\text{Thus, } F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$$

6. (a) b

Explanation: $\lim_{x \rightarrow \infty} a^x \sin\left(\frac{b}{a^x}\right)$

$$= \lim_{x \rightarrow \infty} b \left(\frac{\sin \frac{b}{a^x}}{\frac{b}{a^x}} \right)$$

$$\text{Let } \frac{b}{a^x} = y$$

$$x \rightarrow \infty$$

$$\therefore y \rightarrow 0$$

$$= \lim_{y \rightarrow 0} \frac{b \sin y}{y} = b$$

7. (d) 66

Explanation: The number of terms in the expansion of $(a + b + c)^{10} = \frac{(10+1)(10+2)}{2} = \frac{11 \times 12}{2}$
 $= \frac{132}{2} = 66$

8. (b) 55.6

Explanation: Given that, mean of 100 items, $\bar{x}_{100} = 50$

Mean of 150 items, $\bar{x}_{150} = 40$ and standard deviation of 100 items, $\sigma_{100} = 5$

Standard deviation of 150 items, $\sigma_{150} = 6$

Variance of all the 250 items

$$= (\sigma_{250})^2 = (7.456)^2 = 55.59 \approx 55.6$$

9. (a) 16

Explanation: Here, we have $r = \frac{-4}{5}$ and therefore $|r| = \frac{4}{5} < 1$.

$$S_{\infty} = \frac{80}{9} \Rightarrow \frac{a}{(1-r)} = \frac{80}{9} \Rightarrow \frac{a}{\left(1 + \frac{4}{5}\right)} = \frac{80}{9}$$

$$\Rightarrow \frac{5a}{9} = \frac{80}{9} \Rightarrow a = \left(\frac{80}{9} \times \frac{9}{5}\right) = 16$$

\therefore the first term = 16.

10. (d) $\tan x - \sec x$

Explanation: $\sqrt{\frac{1-\sin x}{1+\sin x}}$

$$= \sqrt{\frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}}$$

$$= \sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}$$

$$= \sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}$$

$$= \frac{(1-\sin x)}{-\cos x} \text{ [as, } \frac{\pi}{2} < x < \frac{3\pi}{2}, \text{ so } \cos \theta \text{ will be negative]}$$

$$= -(\sec x - \tan x)$$

$$= -\sec x + \tan x$$

11. (a) $2x + y = 4$

Explanation: Suppose the line make intercept 'a' on x-axis. Then, it makes intercept '2a' on y-axis.

Thus, the equation of the line is given by $\frac{x}{a} + \frac{y}{2a} = 1$

It passes through (1, 2), so, we get

$$\frac{1}{a} + \frac{2}{2a} = 1 \text{ or } a = 2$$

Thus, the required equation of the line is given by $\frac{x}{2} + \frac{y}{4} = 1$ or $2x + y = 4$

12. (c) $-\frac{1}{3}$

Explanation: Equation is in the form of $\frac{0}{0}$

Using L' Hospital rule we get $\frac{-\frac{1}{2\sqrt{5+x}}}{\frac{1}{2\sqrt{5-x}}}$

Substituting $x = 4$ we get $\frac{-1}{3}$

13. (b) $\frac{|z|}{2}$

Explanation: Given $f(z) = \frac{7-z}{1-z^2}$ where $z = 1 + 2i$

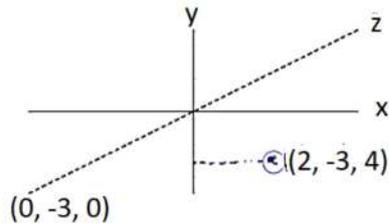
$$\Rightarrow |z| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\begin{aligned}
\Rightarrow f(z) &= \frac{7-z}{1-z^2} \\
&= \frac{7-(1+2i)}{1-(1+2i)^2} \\
&= \frac{7-(1+2i)}{1-(1+4i+4i^2)} \\
&= \frac{6-2i}{4-4i} \\
&= \frac{3-i}{2-2i} \\
&= \frac{3-i}{2-2i} \times \frac{2+2i}{2+2i} \\
&= \frac{6+6i-2i-2i^2}{4-4i} \\
&= \frac{6+4i+2}{4-4i} \\
&= \frac{8+4i}{8} \\
&= 1 + \frac{1}{2}i \\
\Rightarrow |f(z)| &= \sqrt{1^2 + \left(\frac{1}{2}\right)^2} \\
&= \sqrt{1 + \frac{1}{4}} \\
&= \frac{\sqrt{5}}{2} \\
&= \frac{|z|}{2}
\end{aligned}$$

14. (d) (0, -3, 0)

Explanation:

for y-axis ..x = 0, y = ?, z = 0



15. (d) (2, 2)

Explanation: Given equation of line is

$$3x + 4y - 24 = 0$$

For intersection with X-axis put y = 0

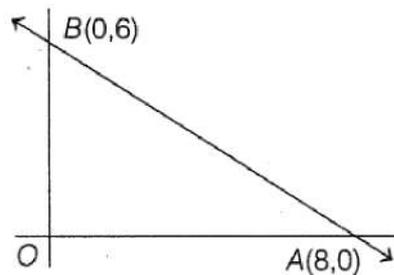
$$\Rightarrow 3x - 24 = 0$$

$$\Rightarrow x = 8$$

For intersection with Y-axis, put x = 0

$$\Rightarrow 4y - 24 = 0 \Rightarrow y = 6$$

\therefore A(8, 0) and B(0, 6)



$$\text{Let } AB = c = \sqrt{8^2 + 6^2} = 10$$

$$OB = a = 6$$

$$\text{and } OA = b = 8$$

Also, let incentre is (h, k). then

$$h = \frac{ax_1 + bx_2 + cx_3}{a+b+c} \quad (\text{here, } x_1 = 8, x_2 = 0, x_3 = 0)$$

$$= \frac{6 \times 8 + 8 \times 0 + 10 \times 0}{6+8+10} = \frac{48}{24} = 2$$

$$\text{and } k = \frac{ay_1 + by_2 + cy_3}{a+b+c} \quad (\text{here, } y_1 = 0, y_2 = 6, y_3 = 0)$$

$$= \frac{6 \times 0 + 8 \times 6 + 10 \times 0}{6 + 8 + 10} = \frac{48}{24} = 2$$

∴ Incentre is (2, 2)

16. (a) 20

Explanation: ${}^{18}C_{15} + 2({}^{18}C_{16}) + {}^{17}C_{16} + 1 = {}^nC_3$

$${}^{18}C_{15} + {}^{18}C_{16} + {}^{18}C_{16} + {}^{17}C_{16} + 1 = {}^nC_3 \quad [∵ {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r]$$

$${}^{19}C_{16} + {}^{18}C_{16} + {}^{17}C_{16} + {}^{17}C_{17} = {}^nC_3$$

$${}^{19}C_{16} + {}^{18}C_{16} + {}^{18}C_{17} = {}^nC_3$$

$${}^{19}C_{16} + {}^{19}C_{17} = {}^nC_3$$

$${}^{20}C_{17} = {}^nC_3$$

$${}^{20}C_3 = {}^nC_3$$

$$n = 20$$

17. (b) 1

Explanation: We know that,

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$$\therefore \frac{|x|}{x} = \begin{cases} \frac{x}{x}, & \text{if } x \geq 0 \\ \frac{-x}{x}, & \text{if } x < 0 \end{cases} = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$$

Now, for all $x \geq 0$ (however, x may large be).

$$\frac{|x|}{x} = 1$$

$$\therefore \lim_{x \rightarrow \infty} \frac{|x|}{x} = 1$$

18. (d) $(-\infty, -7) \cup (7, \infty)$

Explanation: $|x| > 7$

$$\Rightarrow -7 > x > 7$$

$$\Rightarrow x < -7 \text{ or } x > 7 \quad [∵ |x| > a \Leftrightarrow x < -a \text{ or } x > a]$$

$$\Rightarrow x \in (-\infty, -7) \text{ or } x \in (7, \infty)$$

$$\Rightarrow x \in (-\infty, -7) \cup (7, \infty)$$

19. (c) A is true but R is false.

Explanation: Assertion:

$$\sin(-690^\circ) \cos(-300^\circ) + \cos(-750^\circ) \sin(-240^\circ) - \sin(360 \times 2 - 30) \cos(360 - 60) + \cos(360 \times 2 + 30) [-\sin(180 + 60)]$$

$$+ \sin 30 \times \cos 60 + \cos 30 [-\sin(-60)]$$

$$\sin 30 \times \cos 60 - \cos 30 [+ \sin 60]$$

$$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$\frac{1}{4} + \frac{3}{4} \Rightarrow \frac{4}{4} \Rightarrow 1$$

Reason: The value of cos is positive in fourth quadrant.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: $\sum_{r=0}^{100} 500-r C_3 + 400 C_4$

$$= 500 C_3 + 499 C_3 + \dots + 401 C_3 + 400 C_3 + 400 C_4$$

$$= 500 C_3 + 499 C_3 + \dots + 401 C_3 + 401 C_4$$

$$= 500 C_3 + 499 C_3 + \dots + 402 C_4$$

Similarly,

$$500 C_3 + 500 C_4$$

$$501 C_4 = \text{RHS}$$

21. To prove: $\frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A+B)}{\sin(A-B)}$

$$\begin{aligned} \text{L.H.S} &= \frac{\tan A + \tan B}{\tan A - \tan B} \\ &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} \\ &= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}} \\ &= \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B} \\ &= \frac{\sin(A+B)}{\sin(A-B)} \\ &= \text{R.H.S} \end{aligned}$$

Hence proved.

22. $A \cup (A \cap B)$ [\therefore union is distributive over intersection]

Therefore, we get,

$$\begin{aligned} &= (A \cup A) \cap (A \cup B) \quad [\therefore A \cup A = A] \\ &= A \cap (A \cup B) \\ &= A \end{aligned}$$

23. Let p denote the probability of drawing a white ball from an urn containing 5 white, 7 red and 8 black balls. Then, we have,

$$p = \frac{{}^5C_1}{{}^{20}C_1} = \frac{5}{20} = \frac{1}{4} \quad \text{so, } q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

Let X be a random variable denoting the number of white balls in 4 draws with replacement. Then, X is a binomial variate with parameters $n = 4$ $p = \frac{1}{4}$ such that

$$P(X = r) = \text{Probability that } r \text{ balls are white} = {}^4C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{4-r}, r = 0, 1, 2, 3, 4$$

Now,

$$\begin{aligned} \text{Probability that all are white} &= P(X = 4) \\ &= {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-4} = \left(\frac{1}{4}\right)^4 \quad [\text{Using (i)}] \end{aligned}$$

OR

We know that,

$$\begin{aligned} P(B) + P(B') &= 1 \\ \Rightarrow 0.45 + P(B') &= 1 \\ \Rightarrow P(B') &= 1 - 0.45 \\ \Rightarrow P(B') &= 0.55 \end{aligned}$$

24. Suppose $(t, 0)$ be a point on the x-axis.

It is given that the perpendicular distance of the line $\frac{x}{a} + \frac{y}{b} = 1$ from a point is a .

$$\begin{aligned} \therefore \left| \frac{\frac{t}{a} + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| &= a \\ \Rightarrow a^2 \left(\frac{1}{a^2} + \frac{1}{b^2} \right) &= \frac{t^2}{a^2} + 1 - \frac{2t}{a} \\ \Rightarrow 1 + \frac{a^2}{b^2} &= \frac{t^2}{a^2} + 1 - \frac{2t}{a} \\ \Rightarrow \frac{a^2}{b^2} &= \frac{t^2}{a^2} - \frac{2t}{a} \\ \Rightarrow b^2 t^2 - 2ab^2 t - a^4 &= 0 \\ \Rightarrow t &= \frac{2ab^2 \pm 2\sqrt{a^2 b^4 + b^2 a^4}}{2b^2} \\ \Rightarrow t &= \frac{a}{b} (b \pm \sqrt{a^2 + b^2}) \end{aligned}$$

Therefore, the required points on the x-axis are

$$\left(\frac{a}{b} (b - \sqrt{a^2 + b^2}), 0 \right) \text{ and } \left(\frac{a}{b} (b + \sqrt{a^2 + b^2}), 0 \right)$$

25. Given $A = \{3, 5, 7\}$, $B = \{2, 6, 10\}$ and $R = \{(x, y) : x \text{ and } y \text{ are relatively prime}\}$

According to the condition, x and y are prime numbers.

$$\Rightarrow R = \{(3, 2), (5, 2), (7, 2)\}$$

$$R^{-1} = \{ (2, 3), (2, 5), (2, 7) \}$$

Section C

26. Let $\sqrt{-5 + 12i} = (x + iy) \dots(i)$

On squaring both sides of (i), we get

$$-5 + 12i = (x + iy)^2 \Rightarrow -5 + 12i = (x^2 - y^2) + i(2xy) \dots(ii)$$

On comparing real parts and imaginary parts on both sides of (ii), we get

$$x^2 - y^2 = -5 \text{ and } 2xy = 12$$

$$\Rightarrow x^2 - y^2 = -5 \text{ and } xy = 6$$

$$\Rightarrow (x^2 + y^2) = \sqrt{(x^2 - y^2)^2 + 4x^2y^2} = \sqrt{(-5)^2 + 4 \times 36} = \sqrt{169} = 13$$

$$\Rightarrow x^2 - y^2 = -5 \dots(iii) \text{ and } x^2 + y^2 = 13 \dots(iv)$$

add equation (iii) and (iv) and subtract equation (iii) and (iv) we get,

$$\Rightarrow 2x^2 = 8 \text{ and } 2y^2 = 18$$

$$\Rightarrow x^2 = 4 \text{ and } y^2 = 9$$

$$\Rightarrow x = \pm 2 \text{ and } y = \pm 3$$

Since $xy > 0$, so x and y are of the same sign.

$$\therefore (x = 2 \text{ and } y = 3) \text{ or } (x = -2 \text{ and } y = -3).$$

$$\text{Hence, } \sqrt{-5 + 12i} = (2 + 3i) \text{ or } (-2 - 3i).$$

27. Here $x + \frac{x}{2} + \frac{x}{3} < 11$

$$\Rightarrow \frac{6x + 3x + 2x}{6} < 11$$

$$\Rightarrow \frac{11x}{6} < 11$$

Multiplying both sides by 6, we have

$$11x < 66$$

Dividing both sides by 11, we have

$$x < 6$$

Thus the solution set is $(-\infty, 6)$

OR

Given that,

$$\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, x > 0$$

$$\Rightarrow 4 \leq 3(x+1) < 6 \text{ [multiply by } (x+1)]$$

$$\Rightarrow 4 \leq 3x + 3 < 6$$

$$\text{now, } 3x + 3 \geq 4 \text{ and } 3x + 3 < 6$$

$$\Rightarrow 3x \geq 1 \text{ and } 3x < 3$$

$$\Rightarrow x \geq \frac{1}{3} \text{ and } x < 1$$

$$\Rightarrow \frac{1}{3} \leq x < 1$$

28. Here $(x + a)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + \dots + {}^n C_n a^n$
 $= P + Q \dots (i)$

where $P = {}^n C_0 x^n + {}^n C_3 x^{n-3} a^3 + \dots$

$Q = {}^n C_1 x^{n-1} a + {}^n C_3 x^{n-3} a^3 + \dots$

Also $(x - a)^n = {}^n C_0 x^n - {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + \dots + (-1)^n {}^n C_n a^n \dots (ii)$
 $= P - Q$

(i) Squaring and adding (i) and (ii) we have

$$(x + a)^{2n} + (x - a)^{2n} = (P + Q)^2 + (P - Q)^2$$

$$= P^2 + Q^2 + 2PQ + P^2 + Q^2 - 2PQ$$

$$= 2P^2 + 2Q^2 = 2(P^2 + Q^2)$$

(ii) Multiplying (i) and (ii) we have

$$(x + a)^n (x - a)^n = (P + Q)(P - Q)$$

$$(x^2 - a^2)^n = P^2 - Q^2$$

OR

The general term T_{r+1} in the expansion of $(5^{1/2} + 7^{1/8})^{1024}$ is given by

$$T_{r+1} = {}^{1024}C_r (5^{1/2})^{1024-r} (7^{1/8})^r$$

$$\Rightarrow T_{r+1} = {}^{1024}C_r 5^{512-r/2} 7^{r/8}$$

$$\Rightarrow T_{r+1} = \{ {}^{1024}C_r 5^{512-r} \} \times 5^{r/2} \times 7^{r/8}$$

$$\Rightarrow T_{r+1} = \{ {}^{1024}C_r 5^{512-r} \} \times (5^4 \times 7)^{r/8}$$

Clearly, T_{r+1} will be an integer, if

$r/8$ is an integer such that $0 < r \leq 1024$

$\Rightarrow r$ is a multiple of 8 satisfying $0 < r \leq 1024$

$\Rightarrow r = 8, 16, 24, \dots, 1024$

$\Rightarrow r$ can assume 128 values.

Hence, there are 128 integral terms in the expansion of $(5^{1/2} + 7^{1/8})^{1024}$.

29. The equation of a line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$ is

$$\vec{r} = (0\hat{i} - \hat{j} + 3\hat{k}) + \lambda[(2-0)\hat{i} + (-3+1)\hat{j} + (-1-3)\hat{k}]$$

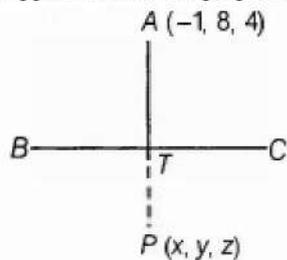
$$\Rightarrow \vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 2\hat{j} - 4\hat{k})$$

$$\Rightarrow \vec{r} = (2\lambda)\hat{i} + (-2\lambda - 1)\hat{j} + (-4\lambda + 3)\hat{k}$$

Any point on line BC is to the form

$$(2\lambda, -2\lambda - 1, -4\lambda + 3)$$

Suppose foot of the perpendicular drawn from point A to the line BC be $T(2\lambda, -2\lambda - 1, -4\lambda + 3)$



Direction Ratios of line AT is $(2\lambda + 1, -2\lambda - 1 - 8, -4\lambda - 1)$

Direction Ratios of BC is $(2-0, -3+1, -1-3) = (2, -2, -4)$

Since, AT is perpendicular to BC,

$$\therefore 2 \times (2\lambda + 1) + (-2 \times (-2\lambda - 9)) + (-4)(-4\lambda - 1) = 0 \quad [\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$$

$$\Rightarrow 4\lambda + 2 + 4\lambda + 18 + 16\lambda + 4 = 0$$

$$\lambda = -1$$

\therefore Coordinates of foot of perpendicular is

$$T(2 \times (-1), -2 \times (-1) - 1, -4 \times (-1) + 3)$$

$$= T(-2, 1, 7)$$

Suppose $P(x, y, z)$ be the image of a point A with respect to the line BC. So, point T is the mid-point of AP.

\therefore Coordinates of T = Coordinates of mid-point of AP

$$\Rightarrow (-2, 1, 7) = \left(\frac{x-1}{2}, \frac{y+8}{2}, \frac{z+4}{2} \right)$$

Equating the corresponding coordinates,

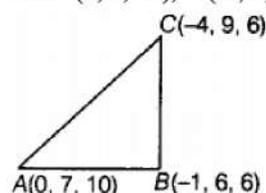
$$\Rightarrow -2 = \frac{x-1}{2}, 1 = \frac{y+8}{2} \text{ and } 7 = \frac{z+4}{2}$$

$$\Rightarrow x = -3, y = -6 \text{ and } z = 10$$

Coordinates of the foot of perpendicular is $T(-2, 1, 7)$ and image of the point A is $P(-3, -6, 10)$.

OR

Let $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$ be the given points. We have,



$$\text{Now, } AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} \quad [\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}]$$

$$= \sqrt{1 + 1 + 16} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

and $AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$

$$= \sqrt{16+4+16}$$

$$\therefore AC = \sqrt{36} = 6 \dots\dots (i)$$

Now, $AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 18 + 18 = 36$

$$\therefore AB^2 + BC^2 = AC^2 \text{ [from Eq. (i)]}$$

Also, $AB = BC = 3\sqrt{2}$

Hence, ABC is a right isosceles triangle.

30. Since we have to get the probability that no two A's are coming together,

So, first we arrange the alphabets except A's

S		S		S		S		I		N		T		I		O		N	
---	--	---	--	---	--	---	--	---	--	---	--	---	--	---	--	---	--	---	--

$$\therefore \text{Number of ways of arranging all alphabets except A's} = \frac{10!}{4!2!2!}$$

As we know that there are 11 vacant places between these alphabets.

Total A's in the word ASSASSINATION are 3

$$\therefore 3 \text{ A's can be placed in 11 place in } {}^{11}C_3 \text{ ways}$$

$$= \frac{11!}{3!(11-3)!} = \frac{11!}{3!8!}$$

$$\therefore \text{Total number of words when no two A's together}$$

$$= \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$$

$$\therefore \text{Required Probability} = \frac{\frac{11!}{3!8!} \times \frac{10!}{4!2!2!}}{\frac{13!}{3!4!2!2!}}$$

$$= \frac{11!}{3!8!} \times \frac{10!}{4!2!2!} \times \frac{3!4!2!2!}{13!}$$

$$= \frac{11! \times 10 \times 9 \times 8!}{8! \times 13 \times 12 \times 11!}$$

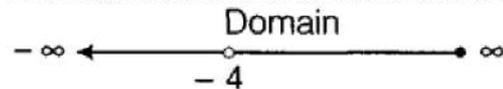
$$= \frac{10 \times 9}{13 \times 12}$$

$$= \frac{15}{26}$$

31. Given, $f(x) = \frac{|x+4|}{x+4}$

Clearly, for f to be defined, the denominator $x + 4 \neq 0$ i.e., $x \neq -4$.

\therefore the domain of f is the set of all real numbers excluding -4.



Now, consider the two cases

When $x + 4 > 0$ i.e., $x > -4$

Then, $|x + 4| = x + 4,$

$$\Rightarrow f(x) = \frac{|x+4|}{x+4} = \frac{x+4}{x+4} = 1 \text{ for all } x > -4$$

When $x + 4 < 0$ i.e., $x < -4$

Then, $|x + 4| = -(x + 4)$

$$\therefore f(x) = \frac{|x+4|}{x+4} = \frac{-(x+4)}{(x+4)} = -1 \text{ for all } x < -4$$

$$f(x) = \begin{cases} 1, & \text{if } x > -4 \\ -1, & \text{if } x < -4 \end{cases}$$

\therefore The range of f is $\{-1, 1\}$.

Section D

32. We make the table from the given data:

Class marks	Mid value (x_i)	$d_i = x_i - a = x_i - 45$	f_i	$f_i d_i$	d_i^2	$f_i d_i^2$
0-10	5	-40	3	-120	1600	4800
10-20	15	-30	2	-60	900	1800
20-30	25	-20	4	-80	400	1600
30-40	35	-10	6	-60	100	600

40-50	45	0	5	0	0	0
50-60	55	10	5	50	100	500
60-70	65	20	5	100	400	2000
70-80	75	30	2	60	900	1800
80-90	85	40	8	320	1600	12800
90-100	95	50	5	250	2500	12500
			$\sum f_i = 45$	$\sum f_i d_i = 460$		$\sum f_i d_i^2 = 38400$

Let $a = 45$.

$$\therefore \text{Mean} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 45 + \frac{460}{45}$$

$$= 45 + 10.22 = 55.22$$

$$\therefore \text{Standard deviation} = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$$

$$= \sqrt{\frac{38400}{45} - (10.22)^2}$$

$$= \sqrt{853.33 - 104.45}$$

$$= \sqrt{748.88}$$

$$= 27.36$$

33. Here, it is given that

Number of students in class = 175

Number of students enrolled in Mathematics = 100 and we have,

Number of students enrolled in Physics = 70

Number of students enrolled in Chemistry = 46

Number of students enrolled in Mathematics and Physics = 30

Number of students enrolled in Physics and Chemistry = 23

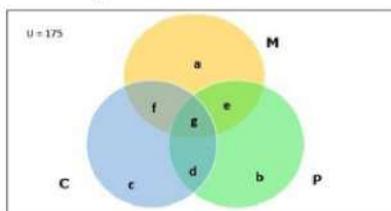
Number of students enrolled in Mathematics and Physics = 28

Number of students enrolled in all three subjects = 18

We have to find:

i. Number of students enrolled in Mathematics alone, Physics alone and Chemistry alone

Venn diagram:



Number of students enrolled in Mathematics = 100 = $n(M)$

Number of students enrolled in Physics = 70 = $n(P)$

Number of students enrolled in Chemistry = 46 = $n(C)$

Number of students enrolled in Mathematics and Physics = 30 = $n(M \cap P)$

Number of students enrolled in Mathematics and Chemistry = 28 = $n(M \cap C)$

Number of students enrolled in Physics and Chemistry = 23 = $n(P \cap C)$

Number of students enrolled in all the three subjects = 18 = $n(M \cap P \cap C) = g$

Therefore, we have,

$$n(M \cap P) = e + g$$

$$30 = e + 18$$

$$e = 30 - 18 = 12$$

$$n(M \cap C) = f + g$$

$$28 = f + 18$$

$$f = 28 - 18 = 10$$

$$n(P \cap C) = d + g$$

$$23 = d + 18$$

$$d = 23 - 18 = 5$$

a = Number of students enrolled only in Mathematics

b = Number of students enrolled only in Physics

c = Number of students enrolled only in Chemistry

Therefore, we have,

$$M = a + e + f + g$$

$$100 = a + 12 + 10 + 18$$

$$a = 100 - 40$$

$$a = 60$$

Thus, Number of students enrolled only in Mathematics = 60

$$P = b + e + d + g$$

$$70 = b + 12 + 5 + 18$$

$$b = 70 - 35$$

$$b = 35$$

Thus, Number of students enrolled only in Physics = 35

$$C = c + f + d + g$$

$$46 = c + 10 + 5 + 18$$

$$c = 46 - 33$$

$$c = 13$$

Thus, Number of students enrolled only in Chemistry = 13

ii. Number of students who have not offered any of these subjects

Number of students who have not offered any of these subjects

$$= 175 - \{n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)\}$$

$$= 175 - (100 + 70 + 46 - 30 - 28 - 23 + 18)$$

$$= 175 - 153 = 22$$

Thus, Number of required students who have not offered any of these subjects = 22

OR

Let P, C and M be the sets of students who have taken physics, chemistry and mathematics respectively. Let a, b, c, d, e, f and g denote the number of students in the respective regions, as shown in the adjoining Venn diagram.

As per data given, we have

$$a + b + c + d = 12,$$

$$b + c + e + f = 11,$$

$$c + d + f + g = 15,$$

$$b + c = 6,$$

$$c + d = 9,$$

$$c + f = 5,$$

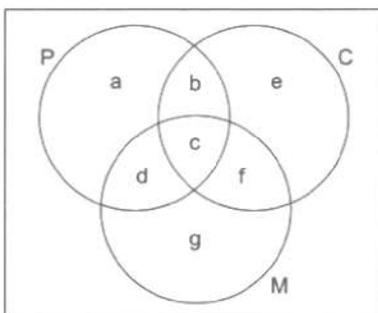
$$c = 3.$$

From these equations, we get

$$c = 3, f = 2, d = 6, b = 1.$$

$$\text{Now, } c + d + f + g = 15 \Rightarrow 3 + 6 + 2 + g = 15 \Rightarrow g = 4;$$

$$b + c + e + f = 11 \Rightarrow 1 + 3 + e + 2 = 11 \Rightarrow e = 5;$$



$$a + b + c + d = 12 \Rightarrow 0 + 1 + 3 + 6 = 12 \Rightarrow 0 = 2;$$

$\therefore a = 2, b = 1, c = 3, d = 6, e = 5, f = 2$ and $g = 4$.

So, we have:

i. Number of students who offered physics only = $0 = 2$.

ii. Number of students who offered chemistry only = $e = 5$.

iii. Number of students who offered mathematics only $g = 4$.

iv. Number of students who offered physics and chemistry but not mathematics = $b = 1$.

v. Number of students who offered physics and mathematics but not chemistry = $d = 6$.

vi. Number of students who offered only one of the given subjects = $(0 + e + g) = (2 + 5 + 4) = 11$.

vii. Number of students who offered at least one of the given subjects = $(a + b + c + d + e + f + g) = (2 + 1 + 3 + 6 + 5 + 2 + 4) = 23$.

viii. Number of students who offered none of the three given subjects = $(25 - 23) = 2$

34. Given equation is

$$y^2 - 8y - x + 19 = 0$$

$$\Rightarrow y^2 - 8y + 16 = x - 19 + 16$$

$$\Rightarrow (y - 4)^2 = x - 3 \dots(i)$$

Let $y - 4 = Y$ and $x - 3 = X$

Then, Eq. (i) becomes,

$$Y^2 = X \dots(ii)$$

Now, from Eq. (ii), coordinates of vertex are,

$$X = 0 \text{ and } Y = 0$$

$$\Rightarrow x - 3 = 0 \text{ and } y - 4 = 0$$

$$\Rightarrow x = 3 \text{ and } y = 4$$

On comparing Eq. (ii) with $Y^2 = 4aX$, we get

$$4a = 1 \Rightarrow a = \frac{1}{4}$$

Coordinates of focus of parabola (ii) are,

$$X = a, Y = 0$$

$$\Rightarrow x - 3 = \frac{1}{4}, y - 4 = 0$$

$$\Rightarrow x = \frac{1}{4} + 3, y = 4 \Rightarrow x = \frac{13}{4}, y = 4$$

Equation of directrix of parabola (ii) is,

$$X = -a$$

$$\Rightarrow x - 3 = -\frac{1}{4}$$

$$\Rightarrow x = -\frac{1}{4} + 3 \Rightarrow x = \frac{11}{4}$$

$$\text{Length of latusrectum} = |4a| = \left|4 \cdot \frac{1}{4}\right| = 1$$

Hence, for given parabola vertex = $(3, 4)$, axis, $y = 4$, focus = $\left(\frac{13}{4}, 4\right)$, directrix, $x = \frac{11}{4}$ and the length of latusrectum = 1.

OR

The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{4+8}{2}, \frac{2+2}{2}\right)$ i.e., $(6, 2)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity.

Then, the equation of the hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \dots(i)$$

Now, the distance between two foci = $2ae$

$$\Rightarrow \sqrt{(8-4)^2 + (2-2)^2} = 2ae \text{ [}\because \text{ foci} = (4, 2) \text{ and } (8, 2)\text{]}$$

$$\Rightarrow \sqrt{(4)^2} = 2ae$$

$$\Rightarrow 2ae = 4$$

$$\Rightarrow 2 \times a \times 2 = 4 \text{ [}\because e = 2\text{]}$$

$$\Rightarrow a = \frac{4}{4} = 1$$

$$\Rightarrow a^2 = 1$$

Now,

$$b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow b^2 = 1(2^2 - 1) [\because e = 2]$$

$$\Rightarrow b^2 = 4 - 1$$

$$\Rightarrow b^2 = 3$$

Putting $a^2 = 1$ and $b^2 = 3$ in equation (i), we get

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x-6)^2 - (y-2)^2}{3} = 1$$

$$\Rightarrow 3(x-6)^2 - (y-2)^2 = 3$$

$$\Rightarrow 3[x^2 + 36 - 12x] - [y^2 + 4 - 4y] = 3$$

$$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y = 3$$

$$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$$

This is the equation of the required hyperbola.

35. We have to show that $\frac{dy}{dx} = (\sec x \tan x + \sec x)$

where, it is given that

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$$

$$y = \sqrt{\frac{\frac{1}{\cos x} - \frac{\sin x}{1}}{\frac{1}{\cos x} + \frac{\sin x}{1}}} = \sqrt{\frac{1 - \sin x}{1 + \sin x}}$$

$$u = 1 - \sin x, v = 1 + \sin x, x = \frac{1 - \sin x}{1 + \sin x}$$

$$\text{if } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + \sin x) \times (-\cos x) - (1 - \sin x) \times (\cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1 + \sin x)^2}$$

$$= \frac{-2 \cos x}{(1 + \sin x)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[-\frac{\cos x}{1} \times \left(\frac{1 - \sin x}{1} \right)^{-\frac{1}{2}} \right] \times \left[\frac{1}{(1 + \sin x)^{2 - \frac{1}{2}}} \right]$$

$$= \left[\cos x \times (1 + \sin x)^{-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left(\frac{1 + \sin x}{1 + \sin x} \right)^{\frac{3}{2}}$$

Multiplying and dividing by $(1 + \sin x)^{\frac{3}{2}}$

$$= \left[\cos x \times (1 + \sin x)^{\frac{2}{2} - \frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{2}{2}} \times \left(\frac{1}{1 + \sin x} \right)^{\frac{3}{2}}$$

$$= \left[\cos x \times (1 + \sin x)^{\frac{2}{2} - \frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{2}{2}} \times (1 + \sin x)^{-\frac{2}{2}}$$

$$= [\cos x \times (1 + \sin x)^1] \times (1 - \sin^2 x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1 + \sin x)^1] \times (\cos^2 x)^{-\frac{3}{2}}$$

$$= [\cos x \times (1 + \sin x)^1] \times (\cos x)^{-3}$$

$$= [(1 + \sin x)^1] \times (\cos x)^{-3+1}$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \times \frac{1 + \sin x}{\cos^2 x}$$

$$= \sec x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x} \right)$$

$$= \sec x (\sec x + \tan x)$$

Hence proved

Section E

36. Read the text carefully and answer the questions:

One morning a big circus arrived in the Ramleela maidan at Delhi. The arrival of the circus was seen in the morning at 08:00 AM by Gopal. He passed this information on 08:15 to 2 other residents of the city.

Each of these 2 people then informed the other 2 residents at 08:30, and again at 08:45, they reported the arrival of the circus every 2 to other uninformed residents

This chain continued the same way till 12:00 PM. By 12:00 PM enough people were informed about the arrival of the circus.



(i) (a) 131017

Explanation: 131017

(ii) (c) 511

Explanation: 511

(iii) (a) 7936

Explanation: 7936

OR

(a) 32,64,128

Explanation: 32,64,128

37. Read the text carefully and answer the questions:

A restaurant offers 5 choices of appetizer, 10 choices of the main meal, and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all the three courses. Assuming all choices are available.



Using the above information answer the following questions:

(i) (b) 110

Explanation: 110

(ii) (b) 200

Explanation: 200

(iii) (b) 329

Explanation: 329

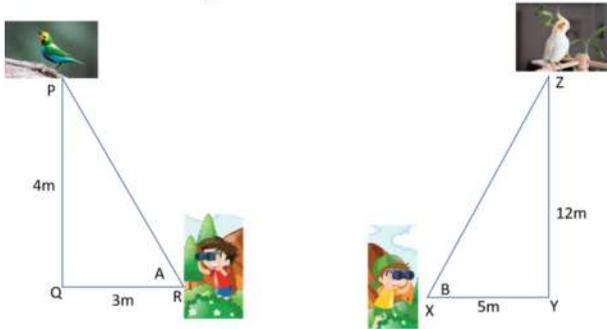
OR

(b) 50 choices.

Explanation: 50 choices.

38. Read the text carefully and answer the questions:

Anand and Sri went for walking. Anand observes a bird on a top tree with an angle of elevation A . The distance between Anand and the tree on the ground is 3 m and height of the tree on which bird is sitting is 4 m. At the same time Sri observes another bird on the top of house with angle of elevation B . The distance between Sri and house on the ground is 5 m and height of the house where bird is sitting is 12 m.



(i) from fig $PR = \sqrt{16 + 9} = \sqrt{25} = 5$ m
and $XZ = \sqrt{144 + 25} = \sqrt{169} = 13$ m
 $\Rightarrow \sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$
 $\therefore \cos A = \sqrt{1 - \sin^2 A}$ [$\because A$ lies in 1st quadrant]
 $= \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}}$
 $\Rightarrow \cos A = \sqrt{\frac{16}{25}} = \frac{4}{5}$
and $\cos B = \frac{5}{13}$, $0 < B < \frac{\pi}{2}$
 $\therefore \sin B = \sqrt{1 - \cos^2 B}$ [$\because B$ lies in 1st quadrant]
 $= \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}}$
 $\Rightarrow \sin B = \sqrt{\frac{144}{169}} = \frac{12}{13}$
 $\therefore \cos A + \sin B = \frac{4}{5} + \frac{12}{13} = \frac{39+60}{65} = \frac{99}{65}$

(ii) from fig $PR = \sqrt{16 + 9} = \sqrt{25} = 5$ m
and $XZ = \sqrt{144 + 25} = \sqrt{169} = 13$ m
 $\Rightarrow \sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$
and $\cos A = \frac{4}{5}$ and $\sin B = \frac{12}{13}$
 $\sin(A + B) = \sin A \cos B + \cos A \sin B$
 $= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$