UNIT 7 PROPERTIES OF MATTER

Many of the greatest advances that have been made from the beginning of the world to the present time have been made in the earnest desire to turn the knowledge of the properties of matter to some purpose useful to mankind– Lord Kelvin

Learning Objectives

In this unit, the student is exposed to

- inter atomic or intermolecular forces in matter
- stress, strain and elastic modulus
- surface tension
- viscosity
- properties of fluids and their applications

7.1 INTRODUCTION

One of the oldest dams in the world is Kallanai (கல்லணை) located at Trichy. Kallanai was built across river Kaveri for the purpose of irrigation. During heavy floods, the velocity of water is generally very high in river Kaveri. The stability and utility of Kallanai dam reveal the intuitive scientific understanding of Tamils who designed this dam as early as 2^{nd} century AD. The other example known for insightful constructions of the past is the pyramids in Egypt. The flyovers and overbridges are common worldwide today. Heavy vehicles ply over and hence, the bridges are always under stress. Without effective design using suitable materials, the bridges and flyovers will not

Kallanai (கல்லணை)

be stable. The growth of human civilization is due to the understanding of various forms of matter (solid, liquid, and gas).

The study of properties of matter is very essential in selecting a material for a specific application. For example, in technology, the materials used for space applications should be of lightweight but should be strong. Materials used for artificial human organ replacements should be biocompatible. Artificial body fluids are used as tissue substitute for radiotherapy analysis in medicine. Fluids used as lubricants or fuel should possess certain properties. Such salient macroscopic properties are decided by the microscopic phenomena within matter. This unit deals with the properties of solids and fluids and the laws governing the behaviour of matter.

7.2 MICROSCOPIC UNDERSTANDING OF VARIOUS STATES OF MATTER

Even though various forms of matter such as solid food, liquids like water, and the air that we breathe are familiar in the day – to – day lifestyle for the past several thousand years, the microscopic understanding of solids, liquids, and gases was established only in 20th century. In the universe, everything is made up of atoms. If so, why the same materials exist in three states? For example, water exists in three forms as solid ice, liquid water, and gaseous steam. Interestingly ice, water, and steam are made up of same water molecules; two hydrogen atoms and one oxygen atom form a water molecule. Physics helps us to explore this beauty of nature at the microscopic level. The distance between the atoms or molecules determines whether it exists in solid, liquid or gaseous state.

Solids

In solids, atoms or molecules are tightly fixed. In the solid formation, atoms get bound together through various types of bonding. Due to the interaction between the atoms, they position themselves at a particular interatomic distance. This position of atoms in this bound condition is called their mean positions.

Liquids

When the solid is not given any external energy such as heat, it will remain as a solid due to the bonding between atoms. When heated, atoms of the solid receive thermal energy and vibrate about their mean positions. When the solid is heated above its melting point, the heat energy will break the bonding between atoms and eventually the atoms receive enough energy and wander around. Here also the intermolecular (or interatomic) forces are important, but the molecules will have enough energy to move around, which makes the structure mobile.

Gases

When a liquid is heated at constant pressure to its boiling point or when the pressure is reduced at a constant temperature it will convert to a gas. This process of a liquid changing to a gas is called evaporation. The gas molecules have either very weak bonds or no bonds at all. This enables them to move freely and quickly. Hence, the gas will conform to the shape of its container and also will expand to fill the container. The transition from solid to liquid to gaseous states with the variation in external energy is schematically shown in Figure 7.1.

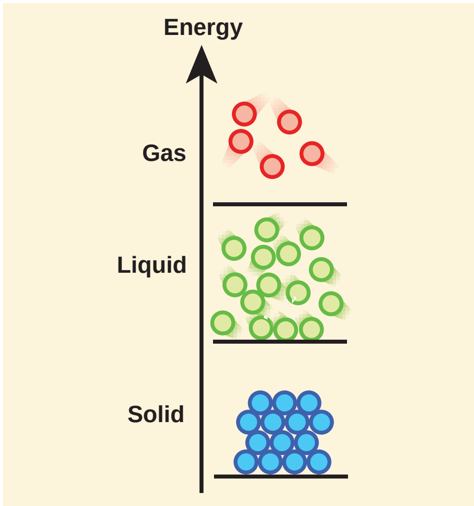


Figure 7.1 Schematic representations of the transition from solid to liquid to gaseous states with a change in external energy

In addition to the three physical states of matter (solid, liquid, and gas), in extreme environments, matter can exist in other states such as plasma, Bose-Einstein condensates. Additional states, such as quark-gluon plasmas are also believed to be possible. A major part of the atomic matter of the universe is hot plasma in the form of rarefied interstellar medium and dense stars.

In the study of Newtonian mechanics (Volume 1), we assumed the objects to be either as point masses or perfect rigid bodies (collection of point masses). Both these are idealized models. In rigid bodies, changes in the shape of the bodies are so small that they are neglected. In real materials, when a force is applied on the objects, there could be some deformations due to the applied force. It is very important to know how materials behave when a deforming force is applied.

7.2.1 Elastic behaviour of materials

In a solid, interatomic forces bind two or more atoms together and the atoms occupy the positions of stable equilibrium. When a deforming force is applied on a body, its atoms are pulled apart or pushed closer. When the deforming force is removed, interatomic forces of attraction or repulsion restore the atoms to their equilibrium positions. **If a body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity**. The force which changes the size or shape of a body is called a deforming force.

Examples: Rubber, metals, steel ropes.

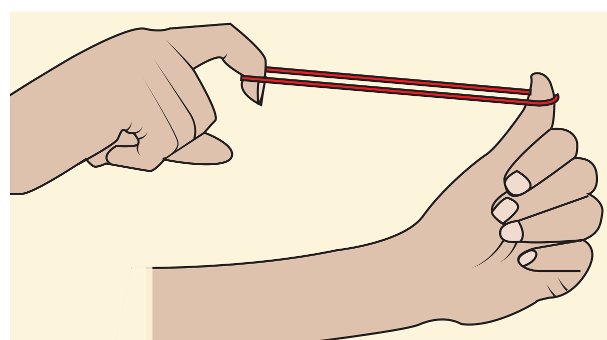


Figure 7.2 Elasticity

Plasticity:

If a body does not regain its original shape and size after removal of the deforming force, it is said to be a plastic body and the property is called plasticity.
Example: Glass

7.2.2 Stress and strain

(a) Stress:

When a force is applied, the size or shape or both may change due to the change in relative positions of atoms or molecules. This deformation may not be noticeable to our naked eyes but it exists in the material itself. When a body is subjected to such a deforming force, internal force is developed in it, called as restoring force. The force per unit area is called as stress.

$$\text{Stress, } \sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} \quad (7.1)$$

The SI unit of stress is N m^{-2} or pascal (Pa) and its dimension is [ML^{-1}T^{-2}]. Stress is a tensor.

(i) Longitudinal stress and shearing stress:

Let us consider a body as shown in Figure 7.3. When many forces act on the system (body), the center of mass (defined in unit 5) remains at rest. However, the body gets deformed due to these forces and so the internal forces appear. Let ΔA be the cross sectional area of the body. The parts of the body on two sides of ΔA exert internal forces $\vec{F}$ and $-\vec{F}$ on each other which is due to deformation. The force can be resolved in two components; F_n normal to the surface ΔA (perpendicular to the surface) and F_t tangential to the surface ΔA (tangent to the surface). The normal stress or longitudinal stress (σ_n) over the area is defined as

$$\sigma_n = \frac{F_n}{\Delta A}$$

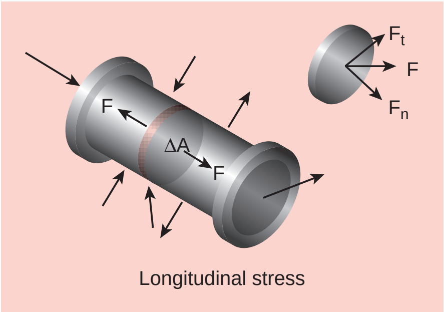


Figure 7.3 Longitudinal stress

Similarly, the tangential stress or shearing stress σ_t over the area is defined as

$$\sigma_t = \frac{F_t}{\Delta A}$$

Longitudinal stress can be classified into two types, tensile stress and compressive stress.

Tensile stress

Internal forces on the two sides of ΔA may pull each other, i.e., it is stretched by equal and opposite forces. Then, the longitudinal stress is called tensile stress.

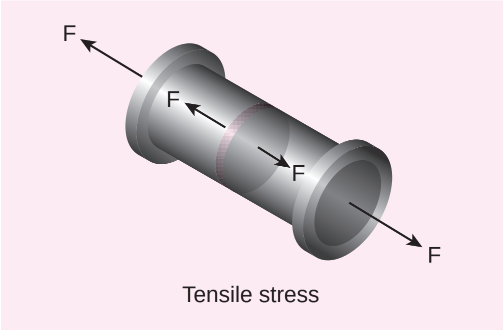


Figure 7.4 Tensile stress

Compressive stress

When forces acting on the two sides of ΔA push each other, ΔA is pushed by equal and opposite forces at the two ends. In this case, ΔA is said to be under compression. Then, the longitudinal stress is called compressive stress.

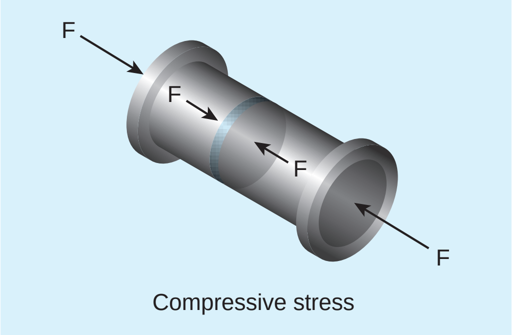


Figure 7.5 Compressive stress

(ii) Volume stress

This happens when a body is acted by forces everywhere on the surface such that the force at any point is normal to the surface and the magnitude of the force on a small surface area is proportional to the area. For instance, when a solid is immersed in a fluid, the pressure at the location of the solid is P, the force on any area ΔA is

$$F = P\,\Delta A$$

Where, F is perpendicular to the area. Thus, force per unit area is called volume stress.

$$\sigma_v = \frac{F}{A}$$

which is the same as the pressure.

(b) Strain:

Strain measures how much an object is stretched or deformed when a force is applied. Strain deals with the fractional change in the size of the object, in other words, strain measures the degree of deformation. As an example, in one dimension, consider a rod of length L when it stretches to a new length ΔL then

$$\text{Strain, } \varepsilon = \frac{\text{Change in size}}{\text{Original size}} = \frac{\Delta L}{L} \qquad (7.2)$$

ε is a dimensionless quantity and has no unit. Strain is classified into three types.

(1) Longitudinal strain

When a rod of length L is pulled by equal and opposite forces, the longitudinal strain is defined as

$$\varepsilon_l = \frac{\text{Increase in length of the rod}}{\text{Original or natural length of the rod}} = \frac{\Delta L}{L} \qquad (7.3)$$

Longitudinal strain can be classified into two types

(i) Tensile strain: If the length is increased from its natural length then it is known as tensile strain.

(ii) Compressive strain: If the length is decreased from its natural length then it is known as compressive strain.

(2) Shearing strain

Consider a cuboid as shown in Figure 7.6. Let us assume that the body remains in translational and rotational equilibrium. Let us apply the tangential force F along AD such that the cuboid deforms as shown in Figure 7.6. Hence, shearing strain or shear is (ε_s)

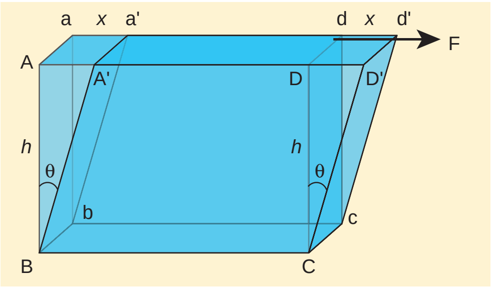


Figure 7.6 Shearing strain

$$\varepsilon_s = \frac{AA'}{BA} = \frac{x}{h} = \tan\theta \qquad (7.4)$$

For small angle, $\tan\theta \approx \theta$

Therefore, shearing strain or shear,

$$\varepsilon_s = \frac{x}{h} = \theta = \text{ Angle of shear}$$

(3) Volume strain

If the body is subjected to a volume stress, the volume will change. Let V be the original volume of the body before stress and $V + \Delta V$ be the change in volume due to stress. The volume strain which measures the fractional change in volume is

$$\text{Volume strain, } \varepsilon_v = \frac{\Delta V}{V} \qquad (7.5)$$

Elastic Limit

The maximum stress within which the body regains its original size and shape after the removal of deforming force is called the elastic limit.

If the deforming force exceeds the elastic limit, the body acquires a permanent deformation. For example, rubber band loses its elasticity if pulled apart too much. It changes its size and becomes misfit to be used again.

7.2.3 Hooke's law and its experimental verification

Hooke's law states that for a small deformation within the elastic limit, the strain produced in a body is directly proportional to the stress that produces it.

It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed point O. A pan and a pointer are attached at the free end of the wire as shown in Figure 7.7 (a).

The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL. It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis. This graph is a straight line passing through the origin as shown in Figure 7.7 (b).

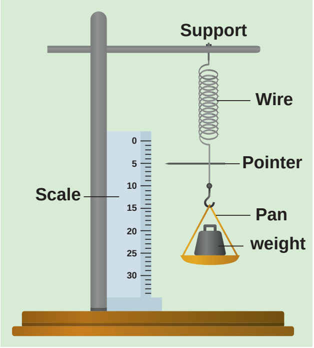


Figure 7.7 (a) Experimental verification of Hooke's law

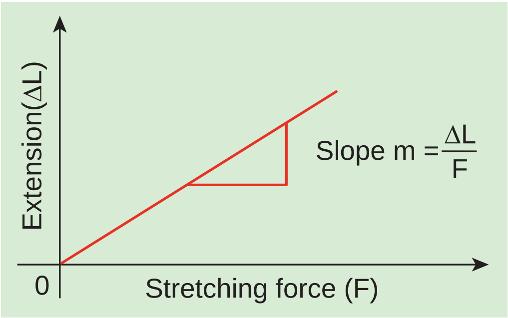


Figure 7.7 (b) Variation of ΔL with F

Therefore,

$$\Delta L = (\text{slope})F$$

Multiplying and dividing by volume,

$$V = A\,L,$$

$$F\,(\text{slope}) = \frac{AL}{AL}\Delta L$$

Rearranging, we get

$$\frac{F}{A} = \left(\frac{L}{A(slope)}\right)\frac{\Delta L}{L}$$

Therefore, $\frac{F}{A} \propto \left(\frac{\Delta L}{L}\right)$

Comparing with equation (7.1) and equation (7.2), we get equation (7.5) as

$$\sigma \propto \varepsilon$$

i.e., the stress is proportional to the strain in the elastic limit.

Stress – Strain profile curve:

The stress versus strain profile is a plot in which stress and strain are noted for each load and a graph is drawn taking strain along the X-axis and stress along the Y-axis. The elastic characteristics of the materials can be analyzed from the stress-strain profile.

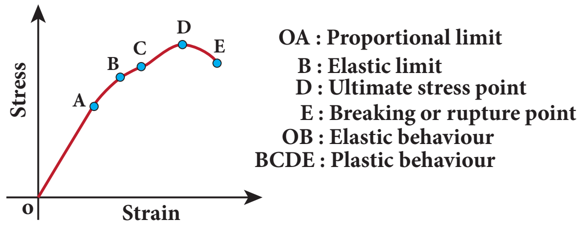


Figure 7.8 Stress-Strain profile

(a) Portion OA:

In this region, stress is very small such that stress is proportional to strain, which means Hooke's law is valid. The point A is called *limit of proportionality* because above this point Hooke's law is not valid. The slope of the line OA gives the Young's modulus of the wire.

(b) Portion AB:

This region is reached if the stress is increased by a very small amount. In this region, stress is not proportional to the strain. But once the stretching force is removed, the wire will regain its original length. This behaviour ends at point B and hence, the point B is known as *yield point* (elastic limit). The elastic behaviour of the material (here wire) in stress-strain curve is OAB.

(c) Portion BC:

If the wire is stretched beyond the point B (elastic limit), stress increases and the wire will not regain its original length after the removal of stretching force.

(d) Portion CD:

With further increase in stress (beyond the point C), the strain increases rapidly and reaches the point D. Beyond D, the strain increases even when the load is removed and breaks (ruptures) at the point E. Therefore, the maximum stress (here D) beyond which the wire breaks is called *breaking stress* or *tensile strength*. The corresponding point D is known as *fracture point*. The region BCDE represents the plastic behaviour of the material of the wire.

7.2.4 Moduli of elasticity

From Hooke's law,

$$\text{stress} \propto \text{strain}$$

$$\frac{\text{stress}}{\text{strain}} = \text{a constant, known as modulus of}$$

elasticity. Its SI unit is Nm^{-2} or pascal and its dimensional formula is $ML^{-1}T^{-2}$.

In this section, we shall define the elastic modulus of a given material. There are three types of elastic modulus.

(a) Young's modulus

(b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.

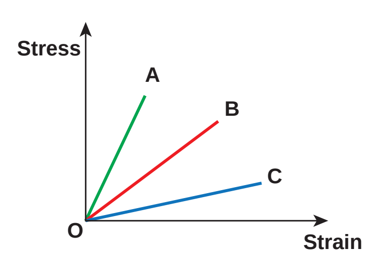
Young modulus of a material

$$= \frac{\text{Tensile stress or compressive stress}}{\text{Tensile strain or compressive strain}}$$

$$Y = \frac{\sigma_t}{\varepsilon_t} \quad \text{or} \quad Y = \frac{\sigma_c}{\varepsilon_c} \qquad (7.6)$$

EXAMPLE 7.1

Within the elastic limit, the stretching strain produced in wires A, B, and C due to stress is shown in the figure. Assume the load applied are the same and discuss the elastic property of the material.



Write down the elastic modulus in ascending order.

Solution

Here, the elastic modulus is Young modulus and due to stretching, stress is tensile stress and strain is tensile strain.

Within the elastic limit, stress is proportional to strain (obey Hooke's law). Therefore, it shows a straight line behaviour. So, Young modulus can be computed by taking slope of these straight lines. Hence, calculating the slope for the straight line, we get

Slope of A > Slope of B > Slope of C

Which implies,

Young modulus of C < Young modulus of B < Young modulus of A

Notice that larger the slope, lesser the strain (fractional change in length). So, the material is much stiffer. Hence, the elasticity of wire A is greater than wire B which is greater than C. From this example, we have understood that Young's modulus measures the resistance of solid to a change in its length.

EXAMPLE 7.2

A wire 10 m long has a cross-sectional area 1.25×10^{-4} m^2. It is subjected to a load of 5 kg. If Young's modulus of the material is 4×10^{10} N m^{-2}, calculate the elongation produced in the wire. Take g = 10 ms^{-2}.

Solution

We know that $\frac{F}{A} = Y \times \frac{\Delta L}{L}$

$$\Delta L = \left(\frac{F}{A}\right)\left(\frac{L}{Y}\right)$$

$$= \left(\frac{50}{1.25\times10^{-4}}\right)\left(\frac{10}{4\times10^{10}}\right) = 10^{-4}\,\text{m}$$

Bulk modulus:

The bulk modulus is defined as the ratio of the volume stress to the volume strain.

Bulk modulus, K =

$$\frac{\text{Normal (Perpendicular) stress or Pressure}}{\text{Volume strain}}$$

The normal stress or pressure is

$$\sigma_n = \frac{F_n}{\Delta A} = \Delta p$$

The volume strain is

$$\varepsilon_v = \frac{\Delta V}{V}$$

Therefore, Bulk modulus is

$$K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta P}{\frac{\Delta V}{V}} \quad (7.7)$$

The negative sign in the equation (7.7) means that when pressure is applied on the body, its volume decreases. Further, the equation (7.7) implies that a material can be easily compressed if it has a small value of bulk modulus. In other words, bulk modulus measures the resistance of solids to change in their volume.

For an example, we know that gases can be easily compressed than solids. This means that gases have small value of bulk modulus compared to solids.

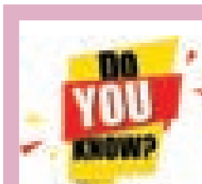
Compressibility

The reciprocal of the bulk modulus is called compressibility. It is defined as the fractional change in volume per unit increase in pressure.

From equation (7.7), we can say that the compressibility

$$C = \frac{1}{K} = -\frac{\varepsilon_v}{\sigma_n} = -\frac{\frac{\Delta V}{V}}{\Delta P} \quad (7.8)$$

Since gases have small value of bulk modulus than solids, their values of compressibility is very high.



After pumping the air in the cycle tyre, usually we press the cycle tyre to check whether it has enough air. What is checked here is essentially the compressibility of air. The tyre should be less compressible for its easy rolling

In fact the rear tyre is less compressible than front tyre for a smooth ride.

EXAMPLE 7.3

A metallic cube of side 100 cm is subjected to a uniform force acting normal to the whole surface of the cube. The pressure is 10^6 pascal. If the volume changes by 1.5×10^{-5} m^3, calculate the bulk modulus of the material.

Solution

By definition, $$K = \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = P\frac{V}{\Delta V}$$

$$K = \frac{10^6 \times 1}{1.5 \times 10^{-5}} = 6.67 \times 10^{10}\,\text{N m}^{-2}$$

Rigidity modulus or shear modulus:

The rigidity modulus is defined as the ratio of the shearing stress to the shearing strain.

$$\eta_R = \frac{\text{Shearing stress}}{\text{Angle of shear or shearing strain}}$$

The shearing stress is

$$\sigma_s = \frac{\text{Tangential force}}{\text{Area over which it is applied}} = \frac{F_t}{\Delta A}$$

The angle of shear or shearing strain

$$\varepsilon_s = \frac{x}{h} = \theta$$

Therefore, Rigidity modulus is

$$\eta_R = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{F_t}{\Delta A \theta} \qquad (7.9)$$

Further, the equation (7.9) implies that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire twisted through an angle θ, a restoring torque (τ) developed is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since the rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small.

For the best understanding, the elastic coefficients of some of the important materials are listed in Table 7.1.

Table 7.1 Elastic coefficient of some materials

Material	Young's modulus (Y) (10^{10}N m^{-2})	Bulk modulus (K) (10^{10} N m^{-2})	Shear modulus (η_R) (10^{10} N m^{-2})
Steel	20.0	15.8	8.0
Aluminium	7.0	7.0	2.5
Copper	12.0	12.0	4.0
Iron	19.0	10	6.4
Glass	7.0	3.6	2.6

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EXAMPLE 7.4

A metal cube of side 0.20 m is subjected to a shearing force of 4000 N. The top surface is displaced through 0.50 cm with respect to the bottom. Calculate the shear modulus of elasticity of the metal.

Solution

Here, L = 0.20 m, F = 4000 N, x = 0.50 cm = 0.005 m and Area A = L^2 = 0.04 m^2

Therefore,

$$\eta_R = \frac{F}{A} \times \frac{L}{x} = \frac{4000}{0.04} \times \frac{0.20}{0.005} = 4 \times 10^6 \text{ N m}^{-2}$$

7.2.5 Poisson's ratio

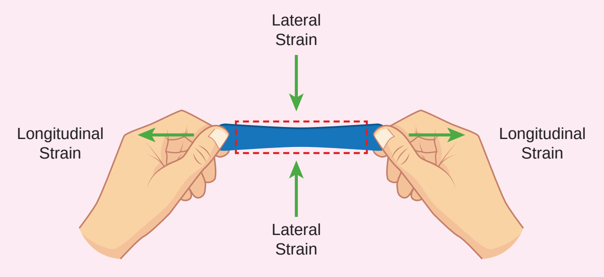

Figure 7.9 Lateral strain versus longitudinal strain

Suppose we stretch a wire, its length increases (elongation) but its diameter decreases (contraction). Similarly, when we stretch a rubber band (elongation), it becomes noticeably thinner (contraction). That is, deformation of the material in one direction produces deformation in another direction. To quantify this, French Physicist S.D. Poisson proposed a ratio, known as Poisson's ratio.

It is defined as the ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \quad (7.10)$$

Consider a wire of length L with diameter D. Due to applied force, wire stretches and let the increase in length be l and decrease in diameter be d. Then

$$\mu = -\frac{\frac{d}{D}}{\frac{l}{L}} = -\frac{L}{l} \times \frac{d}{D}$$

Negative sign indicates the elongation is along longitudinal and the contraction along lateral dimension. Further, notice that it is the ratio between quantities of the same dimension. So, Poisson's ratio has no unit and no dimension (dimensionless number). The Poisson's ratio values of some of the materials are listed in Table 7.2.

Table 7.2 Poisson's ratio of some of the materials

Material	Poisson's ratio
Rubber	0.4999
Gold	0.42 -0.44
Copper	0.33
Stainless steel	0.30-0.31
Steel	0.27-0.30
Cast iron	0.21-0.26
Concrete	0.1-0.2
Glass	0.18-0.3
Foam	0.10-0.50
Cork	0.0

7.2.6 Elastic energy

When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy.

Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension l and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by dl, dW = F dl

The total work done in stretching the wire from 0 to l is

$$W = \int_0^l F\,dl \qquad (7.11)$$

From Young's modulus of elasticity,

$$Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L} \qquad (7.12)$$

Substituting equation (7.12) in equation (7.11), we get

$$W = \int_0^l \frac{YAl}{L} dl$$

Since l is the dummy variable in the integration, we can change l to l' (not in limits). Therefore

$$W = \int_0^l \frac{YAl'}{L} dl' = \frac{YA}{L}\left(\frac{l'^2}{2}\right)\Bigg|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right) l = \frac{1}{2} F l$$

$$W = \frac{1}{2} Fl = \text{Elastic potential energy}$$

The energy per unit volume is called energy density which is is given by,

$$u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2} F l}{A L}$$

$$= \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2} \times \text{Stress} \times \text{Strain} \qquad (7.13)$$

EXAMPLE 7.5

A wire of length 2 m with the area of cross-section 10^{-6} m^2 is used to suspend a load of 980 N. Calculate i) the stress developed in the wire ii) the strain and iii) the energy stored. Given: Y=12 × 10^{10}N m^{-2}.

Solution

(i) stress $= \frac{F}{A} = \frac{980}{10^{-6}} = 98 \times 10^7 \, N m^{-2}$

(ii) strain $= \frac{\text{stress}}{Y} = \frac{98 \times 10^7}{12 \times 10^{10}} = 8.17 \times 10^{-3}$ (no unit)

(iii) Since volume = 2 × $10^{-6} m^3$,

$$\text{Energy} = \frac{1}{2}(\text{stress} \times \text{strain}) \times \text{volume}$$

$$= \frac{1}{2}(98 \times 10^7) \times (8.17 \times 10^{-3}) \times 2 \times 10^{-6} = 8 \text{ J}$$

7.2.7 Applications of elasticity

The mechanical properties of materials play a very vital role in everyday life. The elastic behaviour is one such property which especially decides the structural design of the columns and beams of a building.

As far as the structural engineering is concerned, the amount of stress that the design could withstand is a primary safety factor. A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight. The elastic behaviour or in other words, the bending of beams is a major concern over the stability of the buildings or bridges.

For an example, to reduce the bending of a beam for a given load, one should use the material with a higher value of Young's modulus of elasticity. It is obvious from Table 7.1 that the Young's modulus of steel is greater than aluminium or copper. Iron comes next to steel. This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.

DO YOU KNOW?

Which one is more elastic? Rubber or steel? Steel is more elastic than rubber. If an equal stress is applied to both steel and rubber, the steel produces less strain. So the Young's modulus is higher for steel than rubber. The object which has higher Young's modulus is more elastic.

7.3 FLUIDS

7.3.1 Introduction

Fluids are found everywhere in the world. Earth's surface has about two-thirds of water and one-third of land. Fluids are different from solids. Unlike solid, fluid has no defined shape of its own. As far as fluid is concerned, liquid has fixed volume whereas gas fills the entire volume of the container.

Pressure of a fluid:

Fluid is a substance which begins to flow when an external force is applied on it. It offers a very small resistance to the applied force. If the force acts on a smaller area, then the impact will be more and vice versa. This particular idea decides yet another quantity called '*pressure*'.

Assume that an object is submerged in a fluid (say water) at rest. In this case, the fluid will exert a force on the surface of the object. This force is always normal to the surface of the object. If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.

$$P = \frac{F}{A} \qquad (7.14)$$

Pressure is a scalar quantity. Its S.I. unit and dimensions are $N\,m^{-2}$ or pascal (Pa) and $[ML^{-1}T^{-2}]$ respectively.

Another common unit of pressure is atmosphere which is abbreviated as 'atm'. It is defined as the pressure exerted by the atmosphere at sea level.

i.e., 1 atm = 1.013×10^5 Pa or $N\,m^{-2}$.

Apart from pressure, there are two more parameters such as density and relative density (or specific gravity) which are used to describe the nature of fluids.

Density of a fluid:

The density of a fluid is defined as its mass per unit volume. For a fluid of mass m occupying volume *V*, the density $\rho = \frac{m}{V}$. The dimensions and S.I unit of ρ are $[ML^{-3}]$ and $kg\,m^{-3}$, respectively. It is a positive scalar quantity.

Mostly, a liquid is largely incompressible and hence its density is nearly constant at ambient pressure (i.e. at 1 atm. pressure). In the case of gases, there are variations in densities with reference to pressure.

Relative density or specific gravity:

The relative density of a substance is defined as the ratio of the density of a substance to the density of water at 4 °C. It is a dimensionless positive scalar quantity.

For example, the density of mercury is $13.6 \times 10^3 kg\,m^{-3}$. Its relative density is equal to $\frac{13.6 \times 10^3\,kg\,m^{-3}}{1.0 \times 10^3\,kg\,m^{-3}} = 13.6$.

[The density of water at 4 °C is 1000 kg m^{-3}]

EXAMPLE 7.6

A solid sphere has a radius of 1.5 cm and a mass of 0.038 kg. Calculate the specific gravity or relative density of the sphere.

Solution

Radius of the sphere R = 1.5 cm

mass m = 0.038 kg

Volume of the sphere $V = \frac{4}{3}\pi R^3$

$= \frac{4}{3} \times (3.14) \times (1.5\times10^{-2})^3 = 1.413 \times 10^{-5} m^3$

Therefore, density

$$\rho = \frac{m}{V} = \frac{0.038\,kg}{1.413\times10^{-5}\,m^3} = 2690\ kg\,m^{-3}$$

Hence, the specific gravity of the sphere

$$= \frac{2690}{1000} = 2.69$$

7.3.2 Pressure due to fluid column at rest

A mountaineer climbing the mountain is able to experience a decrease in pressure with altitude. A person jumping into the swimming pool always realizes an increase in pressure with depth below the water surface. In both the cases, the pressure encountered by the mountaineer and diver is usually due to the hydrostatic pressure, because they are due to fluids that are static.

In order to understand the increase in pressure with depth below the water surface, consider a water sample of cross sectional area A in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder respectively as shown in Figure 7.10(a). Let F_1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2 such that $F_1 = P_1 A$ and $F_2 = P_2 A$. Let us assume the mass of the sample to be m and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force (F_1 + mg). In other words, the gravitational force will act downward which is being exactly balanced by the difference between the force $F_2 - F_1$.

$$F_2 - F_1 = mg = F_G \qquad (7.15)$$

Let ρ be the density of the water, then the mass of water available in the sample element is

$$m = \rho V = \rho A (h_2 - h_1)$$
$$V = A (h_2 - h_1)$$

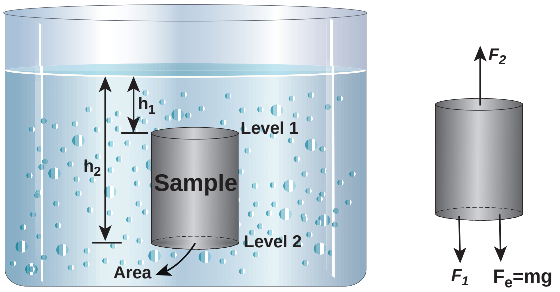

Figure 7.10 (a) A sample of water with base area A in a static fluid with its forces in equilibrium

Hence gravitational force,

$$F_G = \rho A (h_2 - h_1) g$$

On substituting the value of F_G in equation (7.15)

$$F_2 = F_1 + m g$$
$$P_2 A = P_1 A + \rho A (h_2 - h_1) g$$

Cancelling out A on both sides,

$$P_2 = P_1 + \rho (h_2 - h_1) g \qquad (7.16)$$

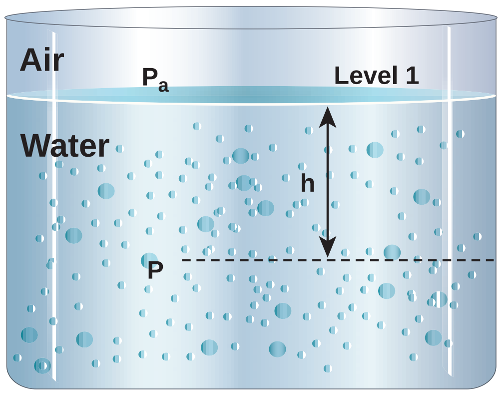

Figure 7.10 (b) The pressure (P) at a depth (h) below the water surface

If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface (as shown in Figure 7.10(b)), then the value of h_1 becomes zero ($h_1 = 0$) and in turn P_1 assumes the value of atmospheric pressure (say P_a). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation (7.16), we get

$$P = P_a + \rho g h \qquad (7.17)$$

Therefore, the pressure at a depth h is greater than the pressure on the surface of the liquid.

If the atmospheric pressure is neglected or ignored, then the pressure at a depth *h* is

$$P = \rho g h \qquad (7.18)$$

For a given liquid, ρ is fixed and g is also constant, then the pressure due to the fluid column is directly proportional to vertical distance or height of the fluid column. This implies that the height of the fluid column is more important to decide the pressure and not the cross sectional or base area or even the shape of the container.

Hydrostatic Paradox

When we talk about liquid at rest, the liquid pressure is the same at all points at the same horizontal level (or same depth). This statement can be demonstrated by an experiment called 'hydrostatic paradox'. Let us consider three vessels of different shapes A, B, and C as shown in Figure 7.11. These vessels are connected at the bottom by a horizontal pipe. When they are filled with a liquid (say water), it occupies the same level even though the vessels hold different amounts of water. It is true because the liquid at the bottom of each section of the vessel experiences the same pressure.

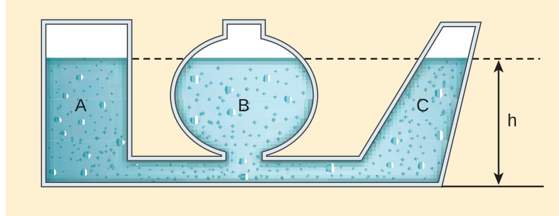

Figure 7.11 Illustration of hydrostatic paradox

DO YOU KNOW?

The atmospheric pressure at a place is the gravitational force exerted by air above that place per unit surface area. It changes with height and weather conditions (i.e. density of air). In fact, the atmospheric pressure decreases with increasing elevation.

The decrease of atmospheric pressure with altitude has an unwelcome consequence in daily life. For example, it takes longer time to cook at higher altitudes. Nose bleeding is another common experience at higher altitude because of larger difference in atmospheric pressure and blood pressure. Its value on the surface of the Earth at sea level is 1atm.

1 atm pressure = 1.013×10^5 pa

ACTIVITY

Take a metal container with an opening. Connect a vacuum pump to the opening. Evacuate the air from inside the container. Why the shape of the metal container gets crumbled?

Inference:

Due to the force of atmospheric pressure acting on its outer surface, the shape of the container crumbles.

ACTIVITY

Take a glass tumbler. Fill it with water to the brim. Slide a cardboard on its rim so that no air remains in between the cardboard and the tumbler. Invert the tumbler gently. The water does not fall down.

Inference:

This is due to the fact that the weight of water in the tumbler is balanced by the upward thrust caused due to the atmospheric pressure acting on the lower surface of the cardboard that is exposed to air.

7.3.3 Pascal's law and its applications

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. Pascal's law states that *if the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.*

Application of Pascal's law

Hydraulic lift

A practical application of Pascal's law is the hydraulic lift which is used to lift a heavy load with a small force. It is a force multiplier. It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure 7.12). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$).

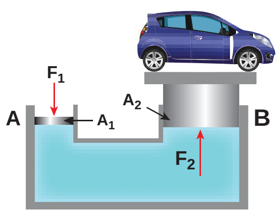

Figure 7.12 Hydraulic lift

Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to $P\left(\text{where, } P = \frac{F_1}{A_1}\right)$. According to Pascal's law, this increased pressure P is transmitted undiminished in all directions. Therefore, a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2$$

$$F_2 = \frac{A_2}{A_1} \times F_1 \qquad (7.19)$$

Therefore by changing the force on the smaller piston A, the force on the piston B can be increased by the factor $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

EXAMPLE 7.7

Two pistons of a hydraulic lift have diameters of 60 cm and 5 cm. What is the force exerted by the larger piston when 50 N is placed on the smaller piston?

Solution

Since, the diameter of the pistons are given, we can calculate the radius of the piston

$$r = \frac{D}{2}$$

Area of smaller piston, $A_1 = \pi\left(\frac{5}{2}\right)^2 = \pi(2.5)^2$

Area of larger piston, $A_2 = \pi\left(\frac{60}{2}\right)^2 = \pi(30)^2$

$$F_2 = \frac{A_2}{A_1} \times F_1 = (50N) \times \left(\frac{30}{2.5}\right)^2 = 7200\,N$$

This means that with the force of 50 N, the force of 7200 N can be lifted.

7.3.4 Buoyancy

When a body is partially or fully immersed in a fluid, it displaces a certain amount of fluid. The displaced fluid exerts an upward force on the body. The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called *upthrust* or *buoyant force* and the phenomenon is called *buoyancy*.

Archimedes principle:

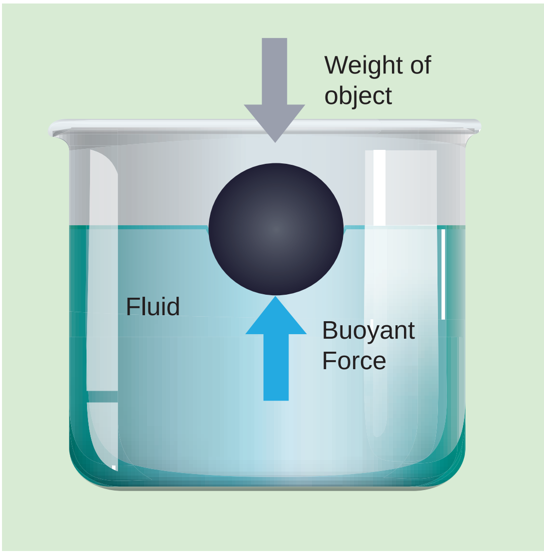

Figure 7.13 Archimedes principle

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

upthrust or buoyant force = weight of liquid displaced.

Law of floatation

It is well-known that boats, ships, and some wooden objects move on the upper part of the water. We say that they float. Floatation can be defined as the tendency of an object to rise up to the upper levels of the fluid or to stay on the surface of the fluid.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body. For example, a wooden object weighs 300 kg (about 3000 N) floats in water displaces 300 kg (about 3000 N) of water.

Note If an object floats, the volume of fluid displaced is equal to the volume of the object submerged and the percentage of the volume of the object submerged is equal to the relative density of an object with respect to the density of the fluid in which it floats.

For example, if an ice cube of density 0.9 g cm^{-3} floats in the fresh water of density 1.0 g cm^{-3} then the percentage volume of an object submerged in fresh water is. $\frac{0.9\,g\,cm^{-3}}{1.0\,g\,cm^{-3}}\times 100\% = 90\%$.

Conversely, if the same ice cube floats in sea water of density 1.3 g cm^{-3}, then the percentage volume of the object submerged in seawater would be $\frac{0.9\,g\,cm^{-3}}{1.3\,g\,cm^{-3}}\times 100\% = 69.23\%$ only.

EXAMPLE 7.8

A cube of wood floating in water supports a 300 g mass at the centre of its top face. When the mass is removed, the cube rises by 3 cm. Determine the volume of the cube.

Solution

Let each side of the cube be l. The volume occupied by 3 cm depth of cube,

$$V=(3\text{cm}) \times l^2 = 3l^2\text{cm}$$

According to the principle of floatation, we have

$$V\rho g = mg \Rightarrow V\rho = m$$

ρ is density of water = 1000 $kg\ m^{-3}$

$$\Rightarrow (3l^2 \times 10^{-2}\text{m}) \times (1000\ \text{kgm}^{-3}) = 300 \times 10^{-3}\text{kg}$$

$$l^2 = \frac{300 \times 10^{-3}}{3 \times 10^{-2} \times 1000} m^2 \Rightarrow l^2 = 100 \times 10^{-4} m^2$$

$$l = 10 \times 10^{-2}\ \text{m} = 10\ \text{cm}$$

Therefore, volume of cube $V = l^3 = 1000\ \text{cm}^3$

DO YOU KNOW?

Submarines can sink or rise in the depth of water by controlling its buoyancy. To achieve this, the submarines have ballast tanks that can be filled with water or air alternatively. When the ballast tanks are filled with air, the overall density of the submarine becomes lesser than the surrounding water and it surfaces (positive buoyancy). If the tanks are flooded with water replacing air, the overall density becomes greater than the surrounding water and submarine begins to sink (negative buoyancy). To keep the submarine at any depth, tanks are filled with air and water (neutral buoyancy).

Examples of floating bodies:

i) A person can swim in sea water more easily than in river water.

ii) Ice floats on water.

iii) The ship is made of steel but its interior is made hollow by giving it a concave shape.

7.4 VISCOSITY

7.4.1 Introduction

In section 7.3, the behaviour of fluids at rest is discussed. Successive discussions will bring out the influence of fluid motion on different properties. A fluid in motion is a complex phenomenon as it possesses potential, kinetic, and gravitational energy besides causing friction viscous forces to come into play. Therefore, it is necessary to consider the case of an ideal liquid to simplify the task. An ideal liquid is incompressible (i.e., bulk modulus is infinity) and in which no shearing forces can be maintained (i.e., the coefficient of viscosity is zero).

Most of the fluids offer resistance towards motion. A frictional force acts at the contact surface when a fluid moves relative to a solid or when two fluids move relative to each other. This resistance to fluid motion is similar to the friction produced when a solid moves on a surface. The internal friction existing between the layers of a moving fluid is viscosity. So, viscosity is defined as the property of a fluid to oppose the relative motion between its layers.

ACTIVITY

Consider three steel balls of the same size, dropped simultaneously in three tall jars each filled with air, water, and oil respectively. It moves easily in air, but not so easily in water. Moving in oil would be even more difficult. There is a relative motion produced between the different layers of the liquid by the falling ball, which causes a viscous force. This frictional force depends on the density of the liquid. This property of a moving fluid to oppose the relative motion between its layers is called viscosity.

Cause of viscosity:

Consider a liquid flowing over a horizontal surface with two neighboring layers. The upper layer tends to accelerate the lower layer and in turn, the lower layer tends to retard the upper layer. As a result, a backward tangential force is set-up. This tends to destroy the relative motion. This accounts for the viscous behaviour of fluids.

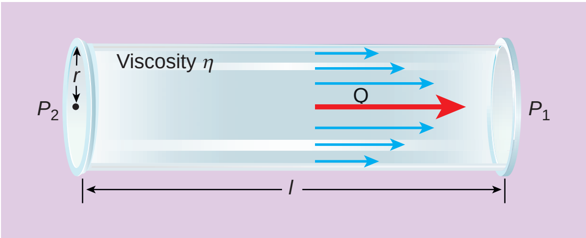

Figure 7.14 Viscosity

Coefficient of viscosity:

Consider a liquid flowing steadily over a horizontal fixed layer (Figure 7.15). The velocities of the layers increase uniformly as we move away from the fixed layer. Consider any two parallel layers A and B. Let v and $v + dv$ be the velocities of the neighboring layers at distances x and $x + dx$ respectively from the fixed layer.

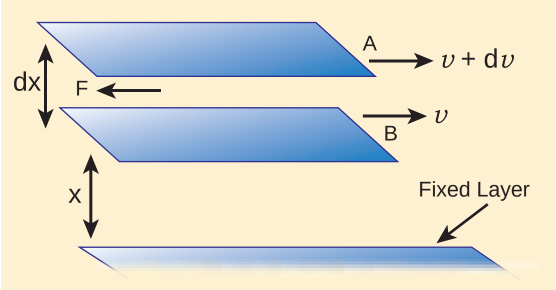

Figure 7.15 Flow of liquid over the horizontal layers

The force of viscosity F acting tangentially between two layers is given by Newton's First law. This force is proportional to (i) area A of the liquid and (ii) the velocity gradient $\frac{dv}{dx}$

$$F \propto A \text{ and } F \propto \frac{dv}{dx}$$

$$\Rightarrow F = -\eta A \frac{dv}{dx} \qquad (7.20)$$

Where the constant of proportionality η is called the coefficient of viscosity of the liquid and the negative sign implies that the force is frictional and it opposes the relative motion. The dimensional formula for coefficient of viscosity is $\left[ML^{-1}T^{-1}\right]$.

Note Viscosity is similar to friction. The kinetic energy of the substance is dissipated as heat energy.

EXAMPLE 7.9

A metal plate of area 2.5×10^{-4} m^2 is placed on a 0.25×10^{-3} m thick layer of castor oil. If a force of 2.5 N is needed to move the plate with a velocity 3×10^{-2} m s^{-1}, calculate the coefficient of viscosity of castor oil.

Given: $A=2.5\times10^{-4}$ m^2, $dx = 0.25\times10^{-3}$m, F=2.5 N and $dv = 3\times10^{-2}$ m s^{-1}

Solution

$$F = -\eta A \frac{dv}{dx}$$

In magnitude, $\eta = \frac{F}{A}\frac{dx}{dv}$

$$= \frac{(2.5N)}{(2.5\times10^{-4}\text{m}^2)}\frac{(0.25\times10^{-3}\text{m})}{(3\times10^{-2}\text{m s}^{-1})}$$

$= 0.083\times10^{3}$ N m^{-2}s

7.4.2 Streamlined flow

The flow of fluids occurs in different ways. It can be a steady or streamlined flow, unsteady or turbulent flow, compressible or incompressible flow or even viscous or non-viscous flow. For example, consider a calm flow of water through a river. Careful observation reveals that the velocity of water at different locations of the river is quite different. It is almost faster at the center and slowest near the banks. However, the velocity of the particle at any point is constant. For better understanding, assume that the velocity of the particle is about 4 meter per second at the center of the river. Hence it will be of the same value for all other particles crossing through this point. In a similar way, if the velocity of the particle flowing near the bank of the river is 0.5 meter per second, then the succeeding particles flowing through it will have the same value.

When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a *streamlined flow*. It is also referred to as steady or laminar flow. The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point as shown in Figure 7.16. It is named so because the flow looks like the flow of a stream or river under ideal conditions.

If we assume a bundle of streamlines having the same velocity over any cross section perpendicular to the direction of flow then such bundle is called a '*tube of flow*'. Thus, it is important to note that any particle in a tube of flow always remains in the tube throughout its motion and cannot mix with liquid in another tube. Always the axis of the tube of flow gives the streamline. The streamlines always represent the trajectories of the fluid particles. **The flow of fluid is streamlined up to a certain velocity called critical velocity.** This means a steady flow can be achieved at low flow speeds below the critical speed.

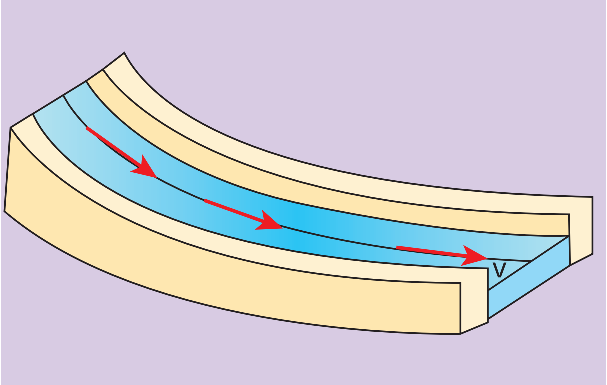

Figure 7.16 Flow is steady velocity at any point in the liquid remains constant

7.4.3 Turbulent flow

When the speed of the moving fluid exceeds the critical speed v_c, the motion becomes turbulent. In this case, the velocity changes both in magnitude and direction from particle to particle and hence the individual particles do not move in a streamlined path. Hence, the path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies (Figure 7.17 (a) and (b)). The flow of water just behind a boat or a ship and the air flow behind a moving bus are a few examples of turbulent flow.

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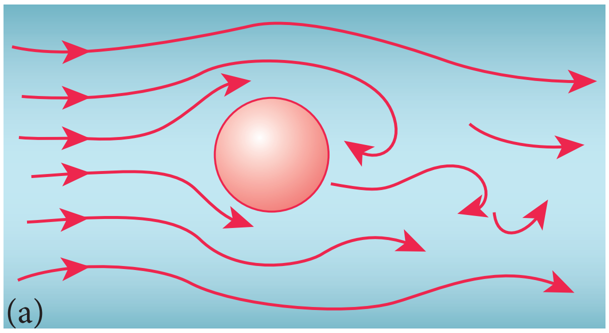

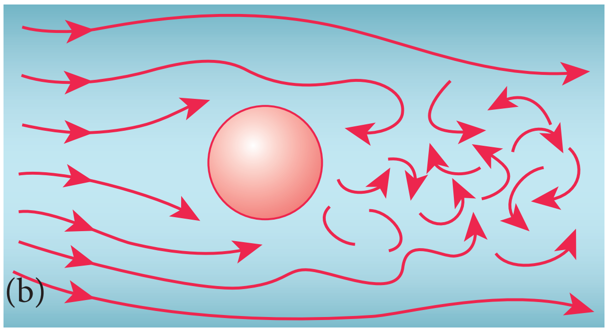

Figure 7.17 (a) turbulent flow around a sphere (when $v = v_c$) (b) turbulent flow around a sphere (when $v > v_c$)

The distinction between the two types of motion can be easily demonstrated by injecting a jet of ink axially in a wide tube through which water flows. When the velocity of the fluid is small, the ink will move in a straight line path. Conversely, when the velocity is increased beyond a certain value, the ink will spread out showing the disorderliness and hence the motion becomes turbulent. The zig-zag motion results in the formation of eddy currents and as a consequence, much energy is dissipated.

7.4.4 Reynold's number

We have learnt that the flow of a fluid becomes steady or laminar when the velocity of flow is less than the critical velocity v_c otherwise, the flow becomes turbulent. Osborne Reynolds (1842-1912) formulated an equation to find out the nature of the flow of fluid, whether it is streamlined or turbulent.

$$R_c = \frac{\rho v D}{\eta} \qquad (7.21)$$

It is a dimensionless number called '*Reynold's number*'. It is denoted by the symbol R_c or K. In the equation, ρ denotes the density of the fluid, v the velocity of the flowing fluid, D is the diameter of the pipe in which the fluid flow, and η is the coefficient of viscosity of the fluid. The value of R_c remains the same in any system of units.

Table 7.3 To understand the flow of liquid, Reynold has estimated the value of R_c as follows

S. No.	Reynold's number	Flow
1	$R_c < 1000$	Streamline
2	$1000 < R_c < 2000$	Unsteady
3	$R_c > 2000$	Turbulent

Hence, Reynold's number R_c is a critical variable which decides whether the flow of a fluid through a cylindrical pipe is streamlined or turbulent.

In fact, the critical value of R_c at which the turbulence sets is found to be the same for geometrically similar flows. For example, when two liquids (say oil and water) of different densities and viscosities flow in pipes of same shapes and sizes, the turbulence sets in at almost the same value of R_c. The above fact leads to the *Law of similarity* which states that when there are two geometrically similar flows, both are essentially equal to each other, as long as they embrace the same Reynold's number. The *Law of similarity* plays a very important role in technological applications. The shape of ships, submarines, racing cars, and airplanes are designed in such a way that their speed can be maximized.

7.4.5 Terminal velocity

To understand terminal velocity, consider a small metallic sphere falling freely from rest through a large column of a viscous fluid.

The forces acting on the sphere are (i) gravitational force of the sphere acting vertically downwards, (ii) upthrust U due to buoyancy and (iii) viscous drag acting upwards (viscous force always acts in a direction opposite to the motion of the sphere).

Initially, the sphere is accelerated in the downward direction so that the upward force is less than the downward force. As the velocity of the sphere increases, the velocity of the viscous force also increases. A stage is reached when the net downward force balances the upward force and hence the resultant force on the sphere becomes zero. It now moves down with a constant velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity v_t. In the Figure 7.18, a graph is drawn with velocity along y- axis and time along x- axis. It is evident from the graph that the sphere is accelerated initially

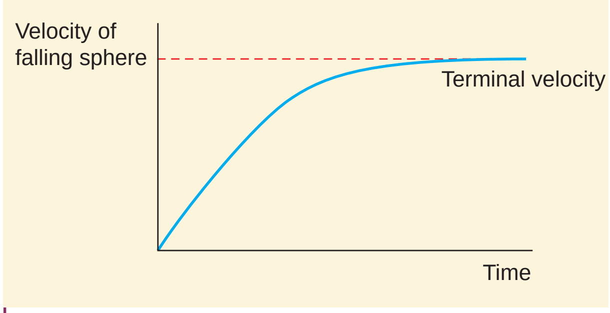

Figure 7.18 Velocity verses time graph

and in course of time it becomes constant, and attains terminal velocity (v_t).

Expression for terminal velocity:

Consider a sphere of radius r which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

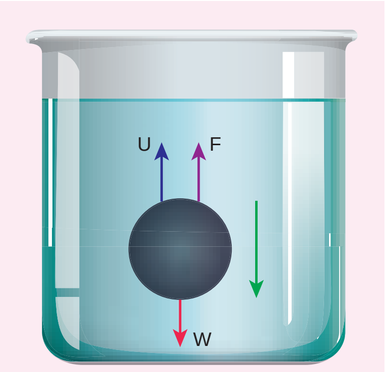

Figure 7.19 Forces acting on the sphere when it falls in a viscous liquid

Gravitational force acting on the sphere,

$F_G = mg = \frac{4}{3}\pi r^3 \rho g$ (downward force)

Up thrust, $U = \frac{4}{3}\pi r^3 \sigma g$ (upward force)

Viscous force, $F = 6\pi\eta r v_t$

At terminal velocity v_t,

The net downward force = upward force

$$F_G = U + F$$

$$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g = 6\pi\eta r v_t$$

$$v_t = \frac{2}{9}\times\frac{r^2(\rho-\sigma)}{\eta}g \Rightarrow v_t \propto r^2 \qquad (7.22)$$

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius.

If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity. That is why air bubbles rise up through water or any fluid. This is also the reason for the clouds in the sky to move in the upward direction.

Point to ponder

i) The terminal speed of a sphere is directly proportional to the square of the radius of the sphere. Hence, larger raindrops fall with greater speed as compared to the smaller raindrops.
ii) If the density of the material of the sphere is less than the density of the medium, then the sphere shall attain terminal velocity in the upward direction. That is why gas bubbles rise up in soda water.

7.4.6 Stoke's law and its applications

When a body falls through a viscous medium, it drags the layer of the fluid immediately in contact with it. This produces a relative motion between the different layers of the liquid. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force F acting on a spherical body of radius r depends directly on

i) radius (r) of the sphere
ii) velocity (v) of the sphere and
iii) coefficient of viscosity η of the liquid

Therefore F $\propto \eta^x r^y v^z \Rightarrow F = k\eta^x r^y v^z$, where k is a dimensionless constant.

Using dimensions, the above equation can be written as

$$\left[MLT^{-2}\right] = k\left[ML^{-1}T^{-1}\right]^x \times \left[L\right]^y \times \left[LT^{-1}\right]^z$$

On solving, we get x=1, y=1, and z=1

Therefore, F=kη rv

Experimentally, Stoke found that the value of k = 6π

$$F = 6\pi\eta\, rv \qquad (7.23)$$

This relation is known as Stoke's law

Practical applications of Stoke's law

Since the raindrops are smaller in size and their terminal velocities are small, remain suspended in air in the form of clouds. As they grow up in size, their terminal velocities increase and they start falling in the form of rain.

This law explains the following:

a) Floatation of clouds
b) Larger raindrops hurt us more than the smaller ones
c) A man coming down with the help of a parachute acquires constant terminal velocity.

7.4.7 Poiseuille's equation

Poiseuille analyzed the steady flow of liquid through a capillary tube. He derived an expression for the volume of the liquid flowing per second through the capillary tube.

As per the theory, the following conditions must be retained while deriving the equation.

- The flow of liquid through the tube is streamlined.
- The tube is horizontal so that gravity does not influence the flow
- The layer in contact with the wall of the tube is at rest
- The pressure is uniform over any cross section of the tube

We can derive Poiseuille's equation using dimensional analysis. Consider a liquid flowing steadily through a horizontal capillary tube. Let $v = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient $\left(\frac{P}{l}\right)$. Then,

$$v \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$v = k\eta^a r^b \left(\frac{P}{l}\right)^c \qquad (7.24)$$

where, k is a dimensionless constant. Therefore,

$$[v] = \frac{\text{volume}}{\text{time}} = [L^3T^{-1}], \left[\frac{dP}{dx}\right] = \frac{\text{Pressure}}{\text{distance}}$$

$$[ML^{-2}T^{-2}], [\eta] = [ML^{-1}T^{-1}] and [r] = [L]$$

Substituting in equation (7.24)

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

$$M^0L^3T^{-1} = M^{a+c}L^{-a+b-2c}T^{-a-2c}$$

So, equating the powers of M, L, and T on both sides, we get

$$a + c = 0, -a + b - 2c = 3, \text{ and } -a - 2c = -1$$

We have three unknowns a, b, and c. We have three equations, on solving, we get

$$a = -1, b = 4, \text{ and } c = 1$$

Therefore, equation (7.24) becomes,

$$v = k\eta^{-1}r^4 \left(\frac{P}{l}\right)^1$$

Experimentally, the value of k is shown to be $\frac{\pi}{8}$, we have

$$v = \frac{\pi r^4 P}{8\eta l} \qquad (7.25)$$

The above equation is known as *Poiseuille's equation* for the flow of liquid through a narrow tube or a capillary tube. This relation holds good for the fluids whose velocities are lesser than the critical velocity (v_c).

7.4.8 Applications of viscosity

The importance of viscosity can be understood from the following examples.

(1) The oil used as a lubricant for heavy machinery parts should have a high viscous coefficient. To select a suitable lubricant, we should know its viscosity and how it varies with temperature [Note: As temperature increases, the viscosity of the liquid decreases]. Also, it helps to choose oils with low viscosity used in car engines (light machinery).

(2) The highly viscous liquid is used to damp the motion of some instruments and is used as brake oil in hydraulic brakes.

(3) Blood circulation through arteries and veins depends upon the viscosity of fluids.

(4) Millikan conducted the oil drop experiment to determine the charge of an electron. He used the knowledge of viscosity to determine the charge.

7.5 SURFACE TENSION

7.5.1 Intermolecular forces

Some liquids do not mix together due to their physical properties such as density, surface tension force, etc. For example, water and kerosene do not mix together. Mercury does not wet the glass but water sticks to it. Water rises up to the leaves through the stem. They are mostly related to the free surfaces of liquids. Liquids have no definite shape but have a definite volume. Hence they acquire a free surface when poured into a container. Therefore, the surfaces have some additional energy, called as surface energy. The phenomenon behind the above fact is called surface tension. Laplace and Gauss developed the theory of surface and motion of a liquid under various situations.

The molecules of a liquid are not rigidly fixed like in a solid. They are free to move about. The force between the like molecules which holds the liquid together is called '*cohesive force*'. When the liquid is in contact with a solid, the molecules of these solid and liquid will experience an attractive force which is called '*adhesive force*'.

These molecular forces are effective only when the distance between the molecules is very small about 10^{-9} m (i.e., 10 Å). The distance through which the influence of these molecular forces can be felt in all directions constitute a range and is called *sphere of influence*. The forces outside this range are rather negligible.

Consider three different molecules A, B, and C in a given liquid as shown in Figure 7.20. Let a molecule 'A' be considered well inside the liquid within the sphere of influence. Since this molecule interacts with all other molecules in all directions, the net force experienced by A is zero. Now consider a molecule 'B' in which three-fourth lies below the liquid surface and one–fourth on the air. Since B has more molecules towards its lower side than the upper side, it experiences a net force in the downward direction. In a similar way, if another molecule 'C' is chosen on the liquid surface (i.e, upper half in air and lower half in liquid), it experiences a maximum downward force due to the availability of more number of liquid molecules on the lower part. Hence it is obvious that all molecules of the liquid that falls within the molecular range inside the liquid interact with the molecule and hence experience a downward force.

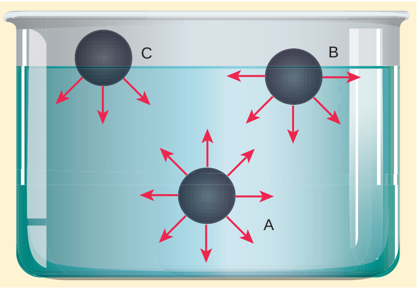

Figure 7.20 Molecules at different levels of liquid

When any molecule is brought towards the surface from the interior of the liquid, work is done against the cohesive force among the molecules of the surface. This work is stored as potential energy in molecules. So the molecules on the surface will have greater potential energy than that of molecules in the interior of the liquid. But for a system to be under stable equilibrium, its potential energy (or surface energy) must be a minimum. Therefore, in order to maintain stable equilibrium, a liquid always tends to have a minimum number of molecules. In other words, the liquid tends to occupy a minimum surface area. This behaviour of the liquid gives rise to surface tension.

Examples for surface tension.

Water bugs and water striders walk on the surface of water (Figure 7.21). The water molecules are pulled inwards and the surface of water acts like a springy or stretched membrane. This balance the weight of water bugs and enables them to walk on the surface of water. We call this phenomenon as surface tension.

Figure 7.21 Water striders can walk on water because of the surface tension of water

The hairs of the painting brush cling together when taken out of water. This is because the water films formed on them tends to contract to a minimum area (Figure 7.22).

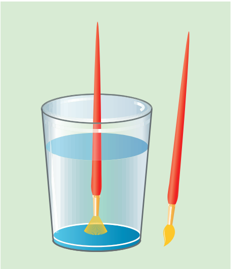

Figure 7.22 Painting brush hairs cling together due to surface tension

ACTIVITY

Needle floats on water surface

Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating.

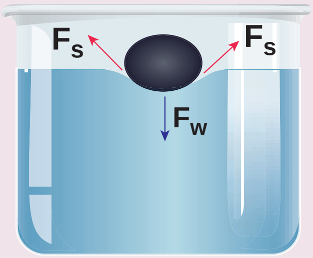

Figure 7.23 Floating needle

The floating needle causes a little depression; the forces F_s due to the surface tension of the curved surface are inclined as shown in Figure 7.23. The vertical components of these two forces support the weight of the needle. Now add liquid soap to the water and stir it. We find that the needle sinks.

ACTIVITY

Take a plastic sheet and cut out a piece in the shape of a boat (Figure 7.24). A tapering and smooth front with a notch at the back is suggested. Put a piece of camphor into the notch of the boat. Gently release the boat on the surface of the water and we find that the boat is propelled forward when the camphor dissolves. The surface tension is lowered, as the camphor dissolves and produces a difference in surface tension in the water nearby the notch. This causes the water to flow away from the back of the boat, which moves the boat forward.

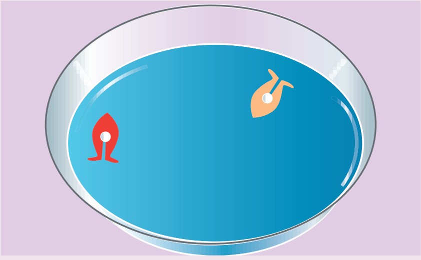

Figure 7.24 Camphor boat

7.5.2 Factors affecting the surface tension of a liquid

Surface tension for a given liquid varies in following situations

(1) ***The presence of any contamination or impurities*** considerably affects the force of surface tension depending upon the degree of contamination.

(2) ***The presence of dissolved substances*** can also affect the value of surface

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tension. For example, a highly soluble substance like sodium chloride (*NaCl*) when dissolved in water (H_20) increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water.

(3) ***Electrification*** affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases.

(4) ***Temperature*** plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature. For a small range of temperature, the surface tension at T_t at $t\,^{o}$C is $T_t = T_0\ (1 - \alpha\, t)$

Where, T_0 is the surface tension at temperature 0°C and α is the temperature coefficient of surface tension. It is to be noted that at the critical temperature, the surface tension is zero as the interface between liquid and vapour disappear. For example, the critical temperature of water is 374°C. Therefore, the surface tension of water is zero at that temperature. van der Wall suggested the important relation between the surface tension and the critical temperature as

$$\mathrm{T}_t = \mathrm{T}_0\left(1 - \frac{t}{t_c}\right)^{\frac{3}{2}}$$

Generalizing the above relation, we get

$$\mathrm{T}_t = \mathrm{T}_0\left(1 - \frac{t}{t_c}\right)^{n}$$

which gives more accurate value. Here, n varies for different liquids and t and t_c denote the temperature and critical temperature in absolute scale (Kelvin scale) respectively.

7.5.3 Surface energy (S.E.) and surface tension (S.T.)

Surface Energy

Consider a sample of liquid in a container. A molecule inside the liquid is being pulled in all direction by other molecules that surround it. However, near the surface, a molecule is pulled down only by the molecules below it and there is a net downward force. As a result, the entire surface of the liquid is being pulled inward. The liquid surface thus tends to have the least surface area. To increase the surface area, some molecules are brought from the interior to the surface. For this reason, work has to be done against the forces of attraction. The amount of work done is stored as potential energy. Thus, the molecules lying on the surface possess greater potential energy than other molecules. This excess energy per unit area of the free surface of the liquid is called '*surface energy*'. In other words, the work done in increasing the surface area per unit area of the liquid against the surface tension force is called the surface energy of the liquid.

$$\text{Surface energy} = \frac{\text{work done in increasing the surface area}}{\text{increase in surface area}}$$

$$= \frac{W}{\Delta A} \qquad (7.26)$$

It is expressed in J m^{-2} or N m^{-1}.

Surface tension

The surface tension of a liquid is defined as the force per unit length of the liquid or the energy per unit area of the surface of a liquid

$$\mathrm{T} = \frac{F}{l} \qquad (7.27)$$

The SI unit and dimensions of T are $N\,m^{-1}$ and $M\,T^{-2}$, respectively.

Relation between surface tension and surface energy:

Consider a rectangular frame of wire ABCD in a soap solution (Figure 7.25). Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inward due to surface tension. Let F be the force due to surface tension, then

$$F = (2T)l$$

here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance Δx to new a position $A'B'$. Since the area increases, some work has to be done against the inward force due to surface tension.

Work done = Force × distance = $(2T\,l)\,(\Delta x)$

Increase in area of the film

$$\Delta A = (2l)\,(\Delta x) = 2l\,\Delta x$$

Therefore,

$$\text{surface energy} = \frac{\text{work done}}{\text{increase in surface area}}$$

$$= \frac{2Tl\Delta x}{2l\Delta x} = T \qquad (7.28)$$

Hence, the surface energy per unit area of a surface is numerically equal to the surface tension.

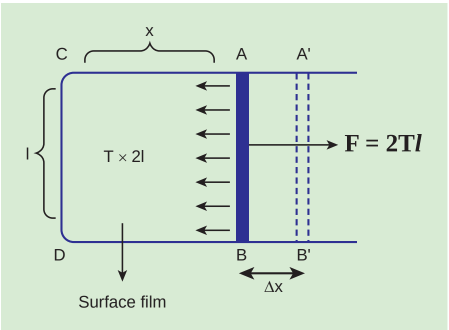


Figure 7.25 A horizontal soap film on a rectangular frame of wire ABCD

Note It should be remembered that a liquid drop has only one free surface. Therefore, the surface area of a spherical drop of radius r is equal to $4\pi r^2$, whereas, a bubble has two free surfaces and hence the surface area of a spherical bubble is equal to $2 \times 4\pi r^2$.

EXAMPLE 7.10

Let $2.4\times10^{-4}\,J$ of work is done to increase the area of a film of soap bubble from $50\,\text{cm}^2$ to $100\,\text{cm}^2$. Calculate the value of surface tension of soap solution.

Solution:

A soap bubble has two free surfaces, therefore increase in surface area $\Delta A = A_2 - A_1 = 2(100-50)\times10^{-4}\text{m}^2 = 100\times10^{-4}\text{m}^2$.

Since, work done $W = T\times\Delta A \Rightarrow T = \dfrac{W}{\Delta A}$

$$= \frac{2.4\times10^{-4}\,\text{J}}{100\times10^{-4}\,\text{m}^2} = 2.4\times10^{-2}\,\text{N}\,\text{m}^{-1}$$

7.5.4 Angle of contact

When the free surface of a liquid comes in contact with a solid, then the surface of the liquid becomes curved at the point of contact. Whenever the liquid surface becomes a curve, then the angle between the two medium (solid-liquid interface) comes in the picture. For an example, when a glass plate is dipped in water with sides vertical as shown in figure, we can observe that the water is drawn up to the plate. In the same manner, instead of water the glass plate is dipped in mercury, the surface is curved but now the curve is depressed as shown in Figure 7.29

The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is known as the *angle of contact between the solid and the liquid*. It is denoted by θ.

Its value is different at interfaces of different pairs of solids and liquids. In fact, it is the factor which decides whether a liquid will spread on the surface of a chosen solid or it will form droplets on it.

Let us consider three interfaces such as liquid-air, solid-air and solid-liquid with reference to the point of contact 'O' and the interfacial surface tension forces T_{sa}, T_{sl} and T_{la} on the respective interfaces as shown in Figure 7.26.

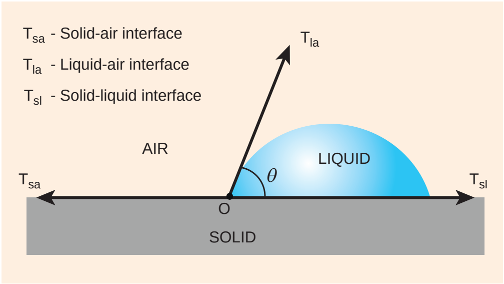


Figure 7.26 Angle of contact of a liquid

Since the liquid is stable under equilibrium, the surface tension forces between the three interfaces must also be in equilibrium. Therefore,

$$T_{sa} = T_{la}\cos\theta + T_{sl} \Rightarrow \cos\theta = \frac{T_{sa} - T_{sl}}{T_{la}} \quad (7.29)$$

From the above equation, there are three different possibilities which can be discussed as follows.

(i) If $T_{sa} > T_{sl}$ and $T_{sa} - T_{sl} > 0$ (water-plastic interface), then the angle of contact θ is acute angle (θ less than 90°) as cosθ is positive.

(ii) If $T_{sa} < T_{sl}$ and $T_{sa} - T_{sl} < 0$ (water-leaf interface), then the angle of contact is obtuse angle (θ less than 180°) as cosθ is negative.

(iii) If $T_{sa} > T_{la} + T_{sl}$, then there will be no equilibrium and liquid will spread over the solid.

Therefore, the concept of *angle of contact* between the solid-liquid interface leads to some practical applications in real life. For example, soaps and detergents are wetting agents. When they are added to an aqueous solution, they will try to minimize the angle of contact and in turn penetrate well in the cloths and remove the dirt. On the other hand, water proofing paints are coated on the outer side of the building so that it will enhance the angle of contact between the water and the painted surface during the rainfall.

7.5.5 Excess of pressure inside a liquid drop, a soap bubble, and an air bubble

As it is discussed earlier, the free surface of a liquid becomes curved when it has contact with a solid. Depending upon the nature of liquid-air or liquid-gas interface, the magnitude of interfacial surface tension varies. In other words, as a consequence of surface tension, the above such interfaces have energy and for a given volume, the surface will have a minimum energy with least area. Due to this reason, the liquid drop becomes spherical (for a smaller radius).

When the free surface of the liquid is curved, there is a difference in pressure between the inner and outer the side of the surface (Figure 7.27).

i) When the liquid surface is plane, the forces due to surface tension (*T, T*) act tangentially to the liquid surface in

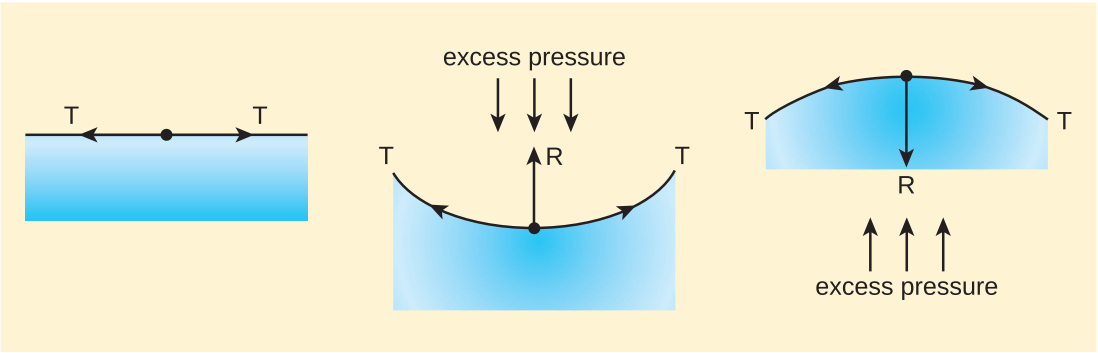


Figure7.27 Excess of pressure across a liquid surface

opposite directions. Hence, the resultant force on the molecule is zero. Therefore, in the case of a plane liquid surface, the pressure on the liquid side is equal to the pressure on the vapour side.

ii) When the liquid surface is curved, every molecule on the liquid surface experiences forces (F_T, F_T) due to surface tension along the tangent to the surface. Resolving these forces into rectangular components, we find that horizontal components cancel out each other while vertical components get added up. Therefore, the resultant force normal to the surface acts on the curved surface of the liquid. Similarly, for a convex surface, the resultant force is directed inwards towards the centre of curvature, whereas the resultant force is directed outwards from the centre of curvature for a concave surface. Thus, for a curved liquid surface in equilibrium, the pressure on its concave side is greater than the pressure on its convex side.

Excess of pressure inside a bubble and a liquid drop:

The small bubbles and liquid drops are spherical because of the forces of surface tension. The fact that a bubble or a liquid drop does not collapse due to the combined effect indicates that the pressure inside a bubble or a drop is greater than that outside it.

1) Excess of pressure inside air bubble in a liquid.

Consider an air bubble of radius R inside a liquid having surface tension T as shown in Figure 7.28 (a). Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is $\Delta P = P_2 - P_1$.

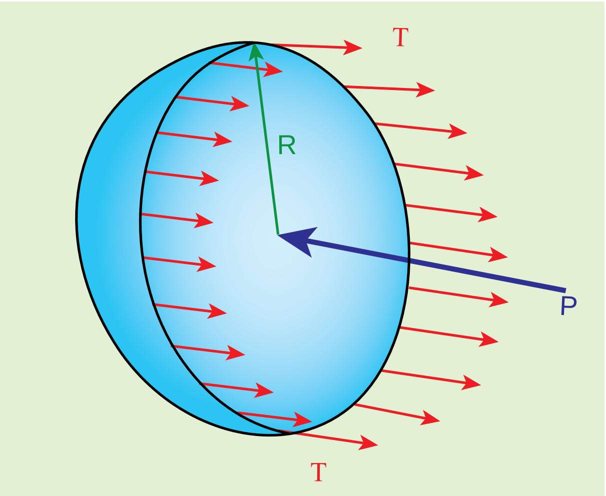


Figure 7.28. (a) Air bubble

In order to find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble. For the hemispherical portion of the bubble, considering the forces acting on it, we get,

i) The force due to surface tension acting towards right around the rim of length $2\pi R$ is $F_T = 2\pi RT$

ii) The force due to outside pressure P_1 is to the right acting across a cross sectional area of πR^2 is $F_{P_1} = P_1\pi R^2$

iii) The force due to pressure P_2 inside the bubble, acting to the left is $F_{P_2} = P_2\pi R^2$.

As the air bubble is in equilibrium under the action of these forces, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 2\pi RT + P_1\pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

$$\text{Excess pressure is } \Delta P = P_2 - P_1 = \frac{2T}{R} \quad (7.30)$$

2) Excess pressure inside a soap bubble

Consider a soap bubble of radius R and the surface tension of the soap bubble be T as shown in Figure 7.28 (b). A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is $2\times 2\pi RT$. The various forces acting on the soap bubble are,

i) Force due to surface tension $F_T = 4\pi RT$ towards right

ii) Force due to outside pressure, $F_{P_1} = P_1\pi R^2$ towards right

iii) Force due to inside pressure, $F_{P_2} = P_2\pi R^2$ towards left

As the bubble is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 4\pi RT + P_1\pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$$

$$\text{Excess pressure is } \Delta P = P_2 - P_1 = \frac{4T}{R} \quad (7.31)$$

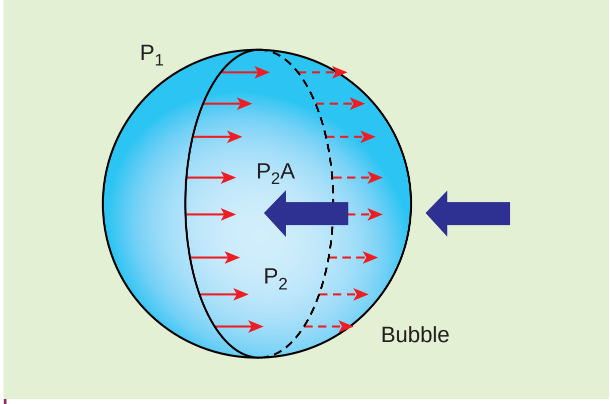


Figure7.28 (b) Soap bubble

3) Excess pressure inside the liquid drop

Consider a liquid drop of radius R and the surface tension of the liquid is T as shown in Figure 7.28 (c).

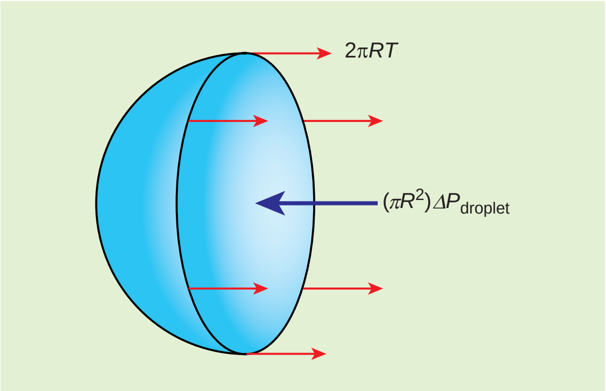


Figure7.28 (c) Liquid drop

The various forces acting on the liquid drop are,

i) Force due to surface tension $F_T = 2\pi RT$ towards right

ii) Force due to outside pressure, $F_{P_1} = P_1\pi R^2$ towards right

iii) Force due to inside pressure, $F_{P_2} = P_2\pi R^2$ towards left

As the drop is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 2\pi RT + P_1\pi R^2$$

$$\Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

$$\text{Excess pressure is } \Delta P = P_2 - P_1 = \frac{2T}{R} \quad (7.32)$$

The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside, the tiny fog droplets are rigid enough to behave like solids.

When an ice-skater skate over the surface of the ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates, the tiny droplets of water act as rigid ball- bearings and help the skaters to run along smoothly.

EXAMPLE 7.11

If excess pressure is balanced by a column of oil (with specific gravity 0.8) 4 *mm* high, where R = 2.0 *cm*, find the surface tension of the soap bubble.

Solution

The excess of pressure inside the soap bubble is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

But $\Delta P = P_2 - P_1 = \rho gh \Rightarrow \rho gh = \frac{4T}{R}$

$\Rightarrow$ Surface tension,

$$T = \frac{\rho ghR}{4} = \frac{(800)(9.8)(4\times10^{-3})(2\times10^{-2})}{4} =$$

$$T = 15.68\times10^{-2}\,N\,m^{-1}$$

7.5.6 Capillarity

The word 'capilla' means hair in Latin. If the tubes were hair thin, then the rise would be very large. It means that the tube having a very small diameter is called a 'capillary tube'. When a glass capillary tube open at both ends is dipped vertically in water, the water in the tube will rise above the level of water in the vessel. In case of mercury, the liquid is depressed in the tube below the level of mercury in the vessel (shown in Figure 7.29). In a liquid whose angle of contact with solid is less than 90°, suffers capillary rise. On the other hand, in a liquid whose angle of contact is greater than 90°, suffers capillary fall (Table 7.4). The rise or fall of a liquid in a narrow tube is called capillarity or capillary action. Depending on the diameter of the capillary tube, liquid rises or falls to different heights.

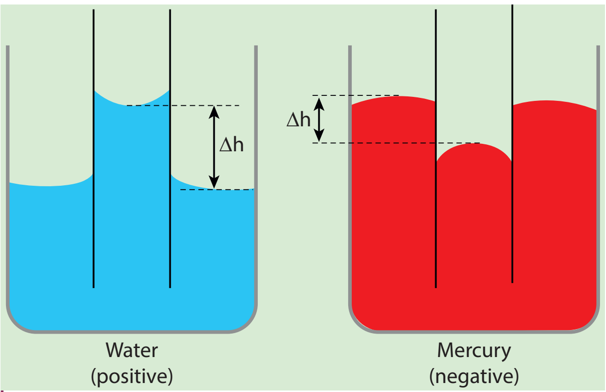


Figure 7.29 Capillary rise or fall

Table 7.4 Capillary rise and fall

Contact angle	Strength of		Degree of wetting	Meniscus	Rise or fall of liquid in the capillary tube
	Cohesive force	Adhesive force			
θ=0 (A)	Weak	Strong	Perfect Wetting	Plane	Neither rises nor is depressed
θ<90 (B)	Weak	Strong	High	Concave	Rise of liquid
θ>90 (C)	Strong	Weak	Low	Convex	Fall of liquid

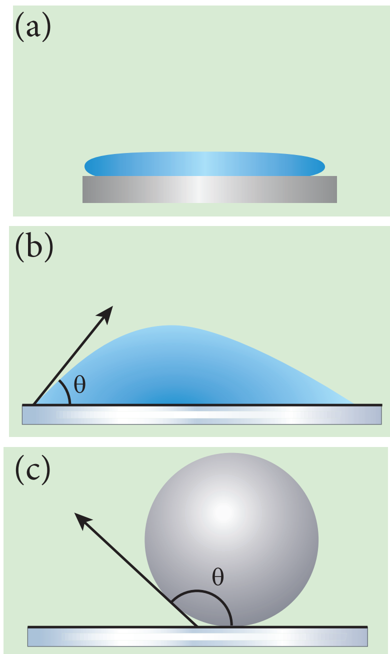


Figure 7.30 (a) water on silver surface (b) glass plate on water (c) glass on mercury

Practical applications of capillarity

- Due to capillary action, oil rises in the cotton within an earthen lamp. Likewise, sap rises from the roots of a plant to its leaves and branches.
- Absorption of ink by a blotting paper
- Capillary action is also essential for the tear fluid from the eye to drain constantly.
- Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for sweat.

7.5.7 Surface Tension by capillary rise method

The pressure difference across a curved liquid-air interface is the basic factor behind the rising up of water in a narrow tube (influence of gravity is ignored). The capillary rise is more dominant in the case of very fine tubes. But this phenomenon is the outcome of the force of surface tension. In order to arrive a relation between the capillary rise (h) and surface tension (T), consider a capillary tube which is held vertically in a beaker containing water; the water rises in the capillary tube to a height h due to surface tension (Figure 7.31).

The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into two components i) Horizontal component $T\sin\theta$ and ii) Vertical component $T\cos\theta$ acting upwards, all along the whole circumference of the meniscus.

Total upward force

$$= (T\cos\theta)\,(2\pi r) = 2\pi r T\cos\theta$$

where θ is the angle of contact, r is the radius of the tube. Let ρ be the density of water and h be the height to which the liquid rises inside the tube. Then,

$$\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube, V}\end{pmatrix} = \begin{pmatrix}\text{volume of the}\\ \text{liquid column of radius r}\\ \text{height h}\end{pmatrix} + \begin{pmatrix}\text{volume of liquid of radius r}\\ \text{and height r - Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$$

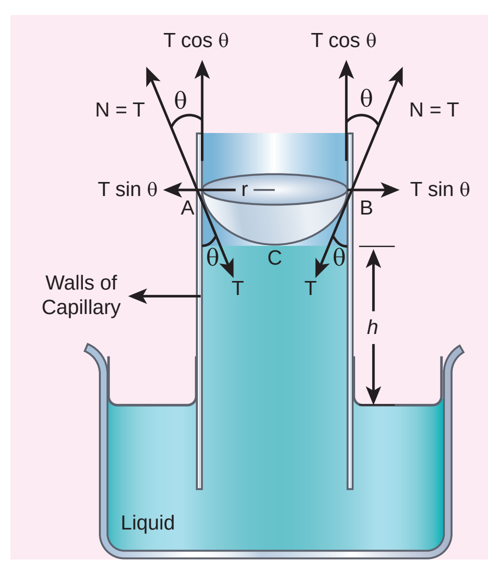


Figure 7.31 Capillary rise by surface tension

$$V = \pi r^2 h + \left(\pi r^2 \times r - \frac{2}{3}\pi r^3\right) \Rightarrow V = \pi r^2 h + \frac{1}{3}\pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore,

$$2\pi r T\cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow T = \frac{r\left(h+\frac{1}{3}r\right)\rho g}{2\cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then $\frac{r}{3}$ can be neglected when it is compared to the height h. Therefore,

$$T = \frac{r\rho gh}{2\cos\theta} \qquad (7.33)$$

Liquid rises through a height h

$$h = \frac{2T\cos\theta}{r\rho g} \Rightarrow h \propto \frac{1}{r} \qquad (7.34)$$

This implies that the capillary rise (h) is inversely proportional to the radius (r) of the tube. i.e, the smaller the radius of the tube greater will be the capillarity.

EXAMPLE 7.12

Water rises in a capillary tube to a height of 2.0cm. How much will the water rise through another capillary tube whose radius is one-third of the first tube?

Solution

From equation (7.34), we have

$$h \propto \frac{1}{r} \Rightarrow hr = \text{constant}$$

Consider two capillary tubes with radius r_1 and r_2 which on placing in a liquid, capillary rises to height h_1 and h_2, respectively. Then,

$$h_1 r_1 = h_2 r_2 = \text{constant}$$

$$\Rightarrow h_2 = \frac{h_1 r_1}{r_2} = \frac{(2\times10^{-2} m)\times r}{\frac{r}{3}} \Rightarrow h_2 = 6\times10^{-2}\text{m}$$

EXAMPLE 7.13

Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 2 mm, made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury T=0.456 N m^{-1}; Density of mercury $\rho = 13.6 \times 10^3$ kg m^{-3}

Solution

Capillary descent, cos140 = cos(90+50) −sin50 = −0.7660

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times(0.456\,N\,m^{-1})(\cos 140^0)}{(2\times10^{-3}m)(13.6\times10^3)(9.8\,m s^{-2})}$$

$$= \frac{2\times0.456\times(-0.7660)}{2\times13.6\times9.8}$$

$$= \frac{-0.6986}{266.56} = -2.62\times10^{-3} m$$

where, negative sign indicates that there is fall of mercury (mercury is depressed) in glass tube.

7.5.8 Applications of surface tension

- Mosquitoes lay their eggs on the surface of water. To reduce the surface tension of water, a small amount of oil is poured. This breaks the elastic film of water surface and eggs are killed by drowning.
- Chemical engineers must finely adjust the surface tension of the liquid, so it forms droplets of designed size and so it adheres to the surface without smearing. This is used in desktop printing, to paint automobiles and decorative items.
- Specks of dirt get removed when detergents are added to hot water while washing clothes because surface tension is reduced.
- A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact.

7.6 BERNOULLI'S THEOREM

7.6.1 Equation of continuity

In order to discuss the mass flow rate through a pipe, it is necessary to assume that the flow of fluid is steady, the flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant with respect to time. Under this condition, the path taken by the fluid particle is a streamline.

Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively as shown in Figure 7.32.

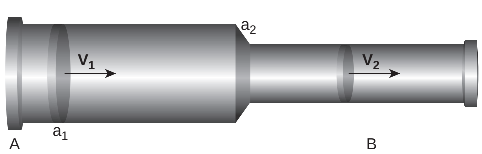


Fig 7.32 A streamlined flow of fluid through a pipe of varying cross sectional area

Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \Delta t)\,\rho$

Let m_2 be the mass of fluid flowing through section B in time Δt, $m_2 = (a_2 v_2 \Delta t)\,\rho$

For an incompressible liquid, mass is conserved $m_1 = m_2$

$$a_1 v_1 \Delta t\, \rho = a_2 v_2 \Delta t\, \rho$$

$$a_1 v_1 = a_2 v_2 \Rightarrow a\,v = \text{constant} \qquad (7.35)$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, $a\,v$ = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

EXAMPLE 7.14

In a normal adult, the average speed of the blood through the aorta (radius $r = 0.8$ cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

Solution:

$$a_1 v_1 = 30\, a_2\, v_2 \Rightarrow \pi r_1^2 v_1 = 30\, \pi r_2^2 v_2$$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$$

$$\Rightarrow v_2 = \frac{1}{30} \times \left(\frac{0.8 \times 10^{-2}\,\text{m}}{0.4 \times 10^{-2}\,\text{m}}\right)^2 \times (0.33\,\text{ms}^{-1})$$

$$v_2 = 0.044\ \text{m s}^{-1}$$

7.6.2 Pressure, kinetic and potential energy of liquids

A liquid in a steady flow can possess three kinds of energy. They are (1) Kinetic energy, (2) Potential energy, and (3) Pressure energy, respectively.

i) **Kinetic energy:** The kinetic energy of a liquid of mass m moving with a velocity v is given by

$$KE = \frac{1}{2} m v^2$$

The kinetic energy per unit mass =

$$\frac{KE}{m} = \frac{\frac{1}{2} m v^2}{m} = \frac{1}{2} v^2$$

Similarly, the kinetic energy per unit volume

$$= \frac{KE}{\text{volume}} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$$

ii) **Potential energy:** The potential energy of a liquid of mass *m* at a height *h* above the ground level is given by

$$PE = mgh$$

The potential energy per unit mass

$$= \frac{PE}{m} = \frac{mgh}{m} = gh$$

Similarly, the potential energy per unit volume $= \frac{PE}{\text{volume}} = \frac{mgh}{V} = \left(\frac{m}{V}\right)gh = \rho gh$

iii) **Pressure energy:** The energy acquired by a fluid by applying pressure on the fluid. We know that

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \Rightarrow \text{Force} = \text{Pressure} \times \text{Area}$$

$$F \times d = (P\,A) \times d = P\,(A \times d)$$

$$\Rightarrow F \times d = W = P\,V = \text{pressure energy}$$

Therefore, pressure energy, $E_p = PV$

The pressure energy per unit mass =

$$\frac{E_P}{m} = \frac{PV}{m} = \frac{P}{\frac{m}{V}} = \frac{P}{\rho}$$

Similarly, the pressure energy per unit volume $= \frac{E_P}{\text{volume}} = \frac{PV}{V} = P$

7.6.3 Bernoulli's theorem and its applications

In 1738, the Swiss scientist Daniel Bernoulli developed a relationship for the flow of fluid through a pipe of varying cross section. He proposed a theorem for the streamline flow of a liquid based on the law of conservation of energy.

Bernoulli's theorem

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant. Mathematically,

$$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant} \qquad (7.36)$$

This is known as Bernoulli's equation.

Proof:

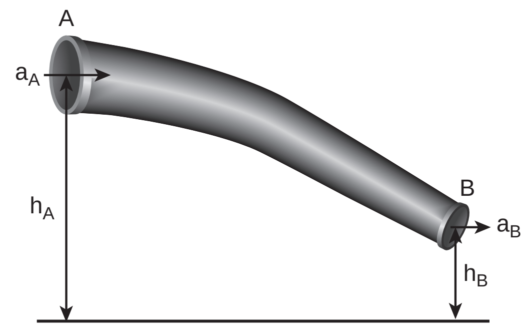


Figure 7.33 Flow of liquid through a pipe AB

Let us consider a flow of liquid through a pipe AB as shown in Figure 7.33. Let *V* be the volume of the liquid when it enters *A* in a time *t* which is equal to the volume of the liquid leaving *B* in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at *A* respectively.

Let the force exerted by the liquid at *A* is

$$F_A = P_A a_A$$

Distance travelled by the liquid in time *t* is

$$d = v_A\, t$$

Therefore, the work done is

$$W = F_A d = P_A a_A v_A\, t$$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at *A*.

Thus, the work done is the pressure energy (at *A*), $W = F_A d = P_A V$

Pressure energy per unit volume at A

$$\frac{\text{Pressure energy}}{\text{volume}}=\frac{P_A V}{V}=P_A$$

Pressure energy per unit mass at A

$$\frac{\text{Pressure energy}}{\text{mass}}=\frac{P_A V}{m}=\frac{P_A}{\frac{m}{V}}=\frac{P_A}{\rho}$$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is

$$E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$$

Potential energy of the liquid at A,

$$PE_A = \text{mg } h_A,$$

Due to the flow of liquid, the kinetic energy of the liquid at A,

$$KE_A = \frac{1}{2}\text{m } v_A^2$$

Therefore, the total energy due to the flow of liquid at A, $E_A = E_{PA} + KE_A + PE_A$

$$E_A = m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + \text{mg } h_A$$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at *B*. Calculating the total energy at E_B, we get

$$E_B = m\frac{P_B}{\rho} + \frac{1}{2}mv_B^2 + \text{mg } h_B$$

From the law of conservation of energy,

$$E_A = E_B$$

$$m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + mgh_A = m\frac{P_B}{\rho} + \frac{1}{2}mv_B^2 + mgh_B$$

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gh_B = \text{constant}$$

Thus, the above equation can be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

The above equation is the consequence of the conservation of energy which is true until there is no loss of energy due to friction. But in practice, some energy is lost due to friction. This arises due to the fact that in a fluid flow, the layers flowing with different velocities exert frictional forces on each other. This loss of energy is generally converted into heat energy. Therefore, Bernoulli's relation is strictly valid for fluids with zero viscosity or non-viscous liquids. Notice that when the liquid flows through a horizontal pipe, then $h = 0 \Rightarrow \frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g}$ = constant.

Applications of Bernoulli's Theorem

(a) Blowing off roofs during wind storm

In olden days, the roofs of the huts or houses were designed with a slope as shown in Figure.7.34. One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.

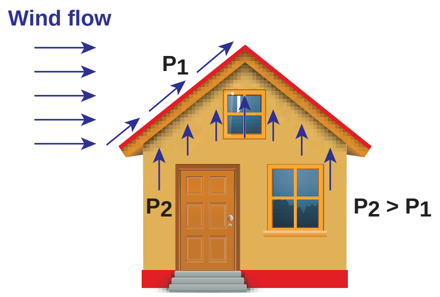


Figure 7.34 Roofs of the huts or houses

During cyclonic condition, the roof is blown off without damaging the other parts of the house. In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1. The pressure under the roof P_2 is greater. Therefore, this pressure difference $(P_2–P_1)$ creates an up thrust and the roof is blown off.

(b) Aerofoil lift

The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the real edge. As the aircraft moves, the air moves faster above the aerofoil than at the bottom as shown in Figure 7.35.

According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an upthrust called the dynamic lift to the aircraft.

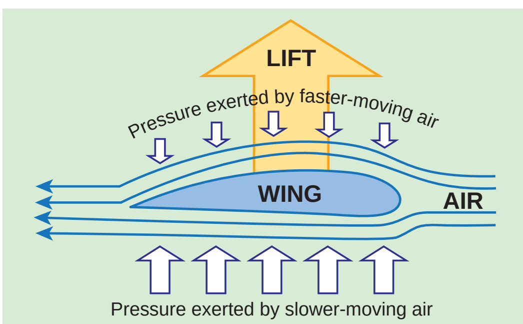


Figure 7.35 Aerofoil lift

(c) Bunsen burner

In this, the gas comes out of the nozzle with high velocity, hence the pressure in the stem decreases. So outside air reaches into the burner through an air vent and the mixture of air and gas gives a blue flame as shown in Figure 7.36.

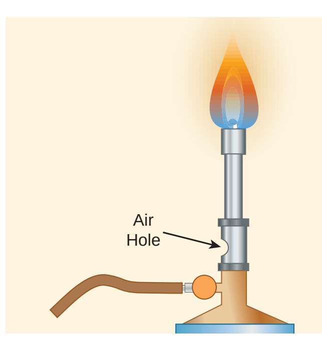


Figure 7.36 Bunsen burner

(d) Venturimeter

This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem. It consists of two wider tubes A and A' (with cross sectional area A) connected by a narrow tube B (with cross sectional area a). A manometer in the form of U-tube is also attached between the wide and narrow tubes as shown in Figure7.37. The manometer contains a liquid of density 'ρ_m'.

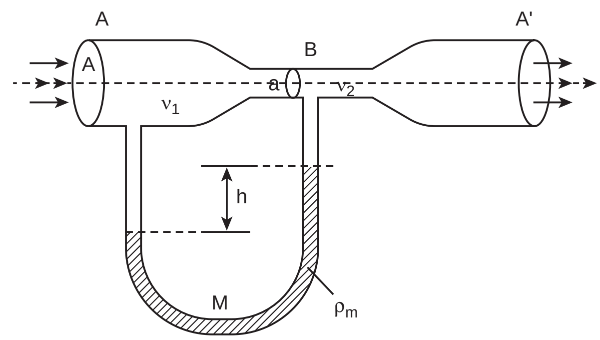


Figure 7.37 A schematic diagram of venturimeter

Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that $Av_1 = a\, v_2$ which means that

$$v_2 = \frac{A}{a} v_1.$$

Using Bernoulli's equation,

$$P_1 + \rho \frac{v_1^2}{2} = P_2 + \rho \frac{v_2^2}{2} = P_2 + \rho \frac{1}{2}\left(\frac{A}{a} v_1\right)^2$$

From the above equation, the pressure difference

$$\Delta P = P_1 - P_2 = \rho \frac{v_1^2}{2} \frac{(A^2 - a^2)}{a^2}$$

Thus, the speed of flow of fluid at the wide end of the tube A

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)} \Rightarrow v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$$

The volume of the liquid flowing out per second is

$$V = Av_1 = A\sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}} = aA\sqrt{\frac{2(\Delta P)}{\rho(A^2 - a^2)}}$$

(e) Other applications

This Bernoulli's concept is mainly used in the design of carburetor of automobiles, filter pumps, atomizers, and sprayers. For example, the carburetor has a very fine channel called nozzle through which the air is allowed to flow in larger speed. In this case, the pressure is lowered at the narrow neck and in turn, the required fuel or petrol is sucked into the chamber so as to provide the correct mixture of air and fuel necessary for ignition process.

ACTIVITY

A bottle is filled with thermocol balls. One end of a flexible tube is kept inside the bottle immersed inside the balls. The free end is rotated and we find the balls sprayed all around. This explains the working of an atomizer or sprayer.

DO YOU KNOW?

A spider web is much stronger than what we think. A single strand of spider silk can stop flying insects which are tens and thousands times its mass. The young's modulus of the spider web is approximately $4.5 \times 10^9\ N m^{-2}$. Compare this value with Young's modulus of wood.

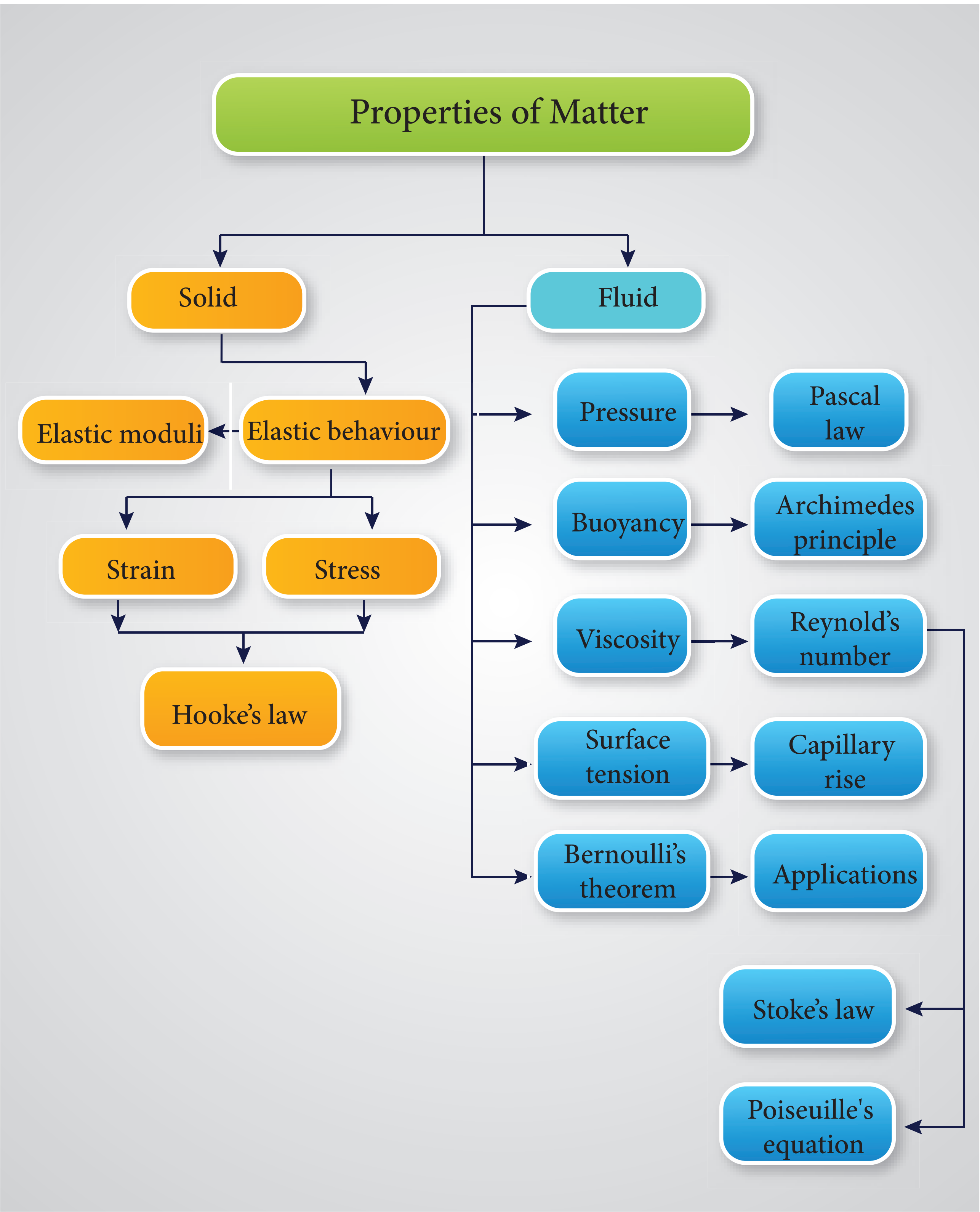
SUMMARY

- The force between the atoms of an element is called inter-atomic force whereas the force between the molecules of a compound is called inter-molecular force.
- *Hooke's law:* within the elastic limit, the stress is directly proportional to strain.
- The force per unit area is known as *stress*. If F is the force applied and A is the area of cross section of the body then the magnitude of *stress* is equal to F/A. Tensile or compressional stress can be expressed using a single term called *longitudinal stress*.
- The ratio of change in length to the original length of a cylinder is ΔL/L, which is known as *longitudinal strain*
- Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called the Young' s modulus of the material of the wire.
- Within the elastic limit, the ratio of volume stress to the volume strain is called the *bulk modulus*.
- Within the elastic limit, the ratio of shear stress to the shear strain is called the *rigidity modulus*.
- Poisson's ratio = lateral strain/longitudinal strain
- The elastic potential energy stored in the wire per unit volume is $U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$, where Y denotes Young's modulus of the material.
- If F is the magnitude of the normal force acting on the surface area A, then the pressure is defined as the '*force acting per unit area*'.
- The total pressure at a depth h below the liquid surface is $P = P_a + \rho gh$, where P_a is the atmospheric pressure which is equal to 1.013×10^5 Pa.
- Pascal's law states that the pressure in a fluid at rest is the same at all points if they are at the same height.
- The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.
- The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.
- When a liquid flows such that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor then the flow of liquid is said to be streamlined flow.
- During the flow of fluid, when the critical velocity is exceeded by the moving fluid, the motion becomes *turbulent*.
- Reynold's number has a significance as it decides which decides whether the flow of fluid through a cylindrical pipe is streamlined or turbulent.

SUMMARY (cont.)

- Stokes formula F= 6πηav, where F is the viscous force acting on a sphere of radius a and v is the terminal velocity of the sphere.
- The surface tension of a liquid is defined as the force of tension acting on a unit length of an imaginary line drawn on the free surface of the liquid, the direction of the force being perpendicular to the line so drawn and acting parallel to the surface.
- The angle between tangents drawn at the point of contact to the liquid surface and solid surface inside the liquid is called the *angle of contact* for a pair of solid and liquid.
- The flow of a fluid is said to be steady if, at any given point, the velocity of each passing fluid particle remains constant with respect to time.
- The equation $a_1 v_1 = a_2 v_2$ is called the equation of continuity for a flow of fluid through a tube and it is due to the conservation of mass in the flow of fluids. It states that the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains constant. i.e., $P/\rho + v^2/2 + gh = \text{constant}$.

CONCEPT MAP



Properties of Matter
Solid
Fluid
Elastic moduli
Elastic behaviour
Strain
Stress
Hooke's law
Pressure
Pascal law
Buoyancy
Archimedes principle
Viscosity
Reynold's number
Surface tension
Capillary rise
Bernoulli's theorem
Applications
Stoke's law
Poiseuille's equation

EVALUATION

I. Multiple Choice Questions

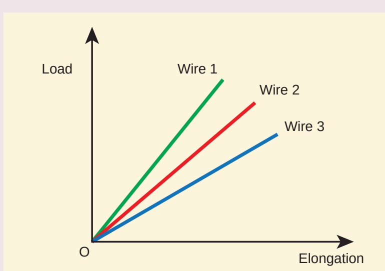
1. Consider two wires X and Y. The radius of wire X is 3 times the radius of Y. If they are stretched by the same load then the stress on Y is

(a) equal to that on X
(b) thrice that on X
(c) nine times that on X
(d) Half that on X

2. If a wire is stretched to double of its original length, then the strain in the wire is

(a) 1 (b) 2
(c) 3 (d) 4

3. The load – elongation graph of three wires of the same material are shown in figure. Which of the following wire is the thickest?



(a) wire 1
(b) wire 2
(c) wire 3
(d) all of them have same thickness

4. For a given material, the rigidity modulus is $\left(\frac{1}{3}\right)^{rd}$ of Young's modulus. Its Poisson's ratio is

(a) 0 (b) 0.25
(c) 0.3 (d) 0.5

5. A small sphere of radius 2cm falls from rest in a viscous liquid. Heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity is proportional to

(NEET model 2018)

(a) 2^2 (b) 2^3
(c) 2^4 (d) 2^5

6. Two wires are made of the same material and have the same volume. The area of cross sections of the first and the second wires are A and 2A respectively. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount?

(NEET model 2018)

(a) 2 F (b) 4 F
(c) 8 F (d) 16 F

7. With an increase in temperature, the viscosity of liquid and gas, respectively will

(a) increase and increase
(b) increase and decrease
(c) decrease and increase
(d) decrease and decrease

8. The Young's modulus for a perfect rigid body is

(a) 0 (b) 1
(c) 0.5 (d) infinity

9. Which of the following is not a scalar?
(a) viscosity
(b) surface tension
(c) pressure
(d) stress

10. If the temperature of the wire is increased, then the Young's modulus will
(a) remain the same
(b) decrease
(c) increase rapidly
(d) increase by very a small amount

11. Copper of fixed volume V is drawn into a wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. If Y represents the Young's modulus, then which of the following graphs is a straight line?

(NEET 2014 model)

(a) Δl versus V
(b) Δl versus Y
(c) Δl versus F
(d) Δl versus $\frac{1}{l}$

12. A certain number of spherical drops of a liquid of radius R coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then
(a) energy $= 4\,V\,T\left(\frac{1}{r}-\frac{1}{R}\right)$ is released
(b) energy $= 3\,V\,T\left(\frac{1}{r}+\frac{1}{R}\right)$ is absorbed
(c) energy $= 3\,V\,T\left(\frac{1}{r}-\frac{1}{R}\right)$ is released
(d) energy is neither released nor absorbed

13. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?
(a) length = 200 cm, diameter = 0.5 mm
(b) length= 200 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm
(d) length= 200 cm, diameter = 3 m

14. The wettability of a surface by a liquid depends primarily on
(a) viscosity
(b) surface tension
(c) density
(d) angle of contact between the surface and the liquid

15. In a horizontal pipe of non-uniform cross section, water flows with a velocity of 1 m s^{-1} at a point where the diameter of the pipe is 20 cm. The velocity of water (1.5 m s^{-1}) at a point where the diameter of the pipe is (in cm)

(a) 8 (b) 16
(c) 24 (d) 32

Answers:

1) c	2) a	3) a	4) d
5) d	6) b	7) c	8) d
9) d	10) b	11) c	12) c
13) a	14) d	15) b	

II. Short Answer Questions

1. Define stress and strain.
2. State Hooke's law of elasticity.
3. Define Poisson's ratio.
4. Explain elasticity using intermolecular forces.
5. Which one of these is more elastic, steel or rubber? Why?

6. A spring balance shows wrong readings after using for a long time. Why?
7. What is the effect of temperature on elasticity?
8. Write down the expression for the elastic potential energy of a stretched wire.
9. State Pascal's law in fluids.
10. State Archimedes principle.
11. What do you mean by upthrust or buoyancy?
12. State the law of floatation.
13. Define coefficient of viscosity of a liquid.
14. Distinguish between streamlined flow and turbulent flow.
15. What is Reynold's number? Give its significance.
16. Define terminal velocity.
17. Write down the expression for the Stoke's force and explain the symbols involved in it.
18. State Bernoulli's theorem.
19. What are the energies possessed by a liquid? Write down their equations.
20. Two streamlines cannot cross each other. Why?
21. Define surface tension of a liquid. Mention its S.I unit and dimension.
22. How is surface tension related to surface energy?
23. Define angle of contact for a given pair of solid and liquid.
24. Distinguish between cohesive and adhesive forces.
25. What are the factors affecting the surface tension of a liquid?
26. What happens to the pressure inside a soap bubble when air is blown into it?
27. What do you mean by capillarity or capillary action?
28. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?
29. State the principle and usage of Venturimeter.

III. Long Answer Questions

1. State Hooke's law and verify it with the help of an experiment.
2. Explain the different types of modulus of elasticity.
3. Derive an expression for the elastic energy stored per unit volume of a wire.
4. Derive an equation for the total pressure at a depth 'h' below the liquid surface.
5. State and prove Pascal's law in fluids.
6. State and prove Archimedes principle.
7. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.
8. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.
9. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.
10. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.
11. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

12. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

13. Describe the construction and working of venturimeter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

IV. Exercises

1. A capillary of diameter dmm is dipped in water such that the water rises to a height of 30mm. If the radius of the capillary is made $\left(\frac{2}{3}\right)$ of its previous value, then compute the height up to which water will rise in the new capillary?

(Answer: 45 mm)

2. A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of 4×10^5 N is applied at the other end. If the rigidity modulus of the cylinder is 6×10^{10} N m^{-2} then, calculate the twist produced in the cylinder. (Answer: 45.60)

3. A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

4. A block of Ag of mass x kg hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block. (Answer: $x = 4$ kg)

5. The reading of pressure meter attached with a closed pipe is 5×10^5 N m^{-2}. On opening the valve of the pipe, the reading of the pressure meter is 4.5×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

(Answer: 10 ms^{-1})

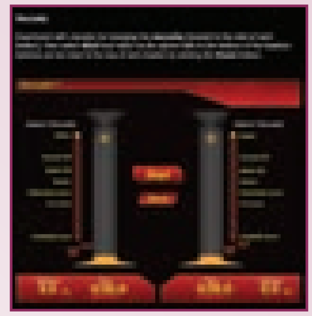
BOOKS FOR REFERENCE

1. Serway and Jewett, Physics for scientist and Engineers with modern physics, Brook/Coole publishers, Eighth edition
2. Paul Tipler and Gene Mosca, Physics for scientist and engineers with modern physics, Sixth edition, W.H.Freeman and Company
3. H.C.Verma, Concepts of physics volume 1 and Volume 2, Bharati Bhawan Publishers

ICT CORNER

Properties of Matter

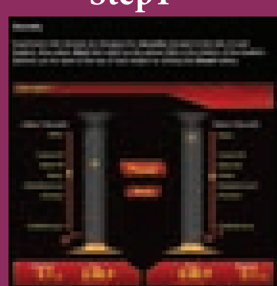
Through this activity you will be able to learn about the Viscosity.



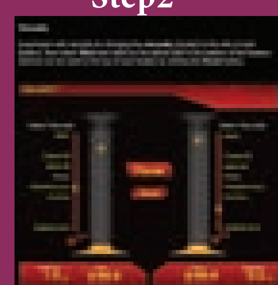
STEPS:

- Use the URL or scan the QR code to open 'Viscosity' activity page.
- In the activity window , 'Select Viscosity' by dragging the ball in the meter in the side.
- Select the 'Start' button , sphere falls from top to bottom of the beaker. We can see the changes in Distance and Time.
- Spheres can be reset to the top of each beaker by clicking the 'Reset' button.

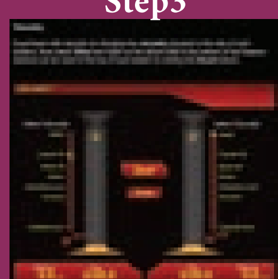
Step1



Step2



Step3



Step4

URL:

http://www.geo.cornell.edu/hawaii/220/PRI/viscosity.html

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.