

Mathematics

The Triangle and Its Properties



NCERT

Exercises

(Questions-Solutions)

Exercise 6.1

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1. In $\triangle PQR$, D is the mid-point of QR .

PM is

PD is

Is $QM = MR$?

Sol. \overline{PM} is the altitude.
 PD is the median.
No! $QM \neq MR$.

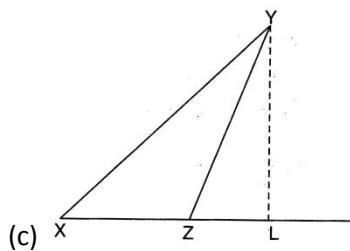
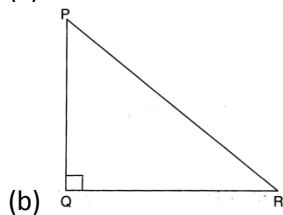
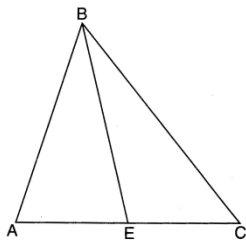
2. Draw rough sketches for the following :

(a) In $\triangle ABC$, BE is a median.

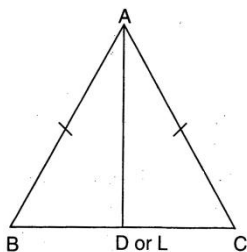
(b) In $\triangle PQR$, PQ and PR are altitudes of the triangle.

(c) In $\triangle XYZ$, YL is an altitude in the exterior of the triangle.

Sol. (a)



3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.



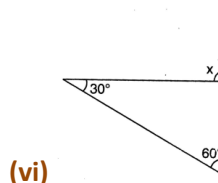
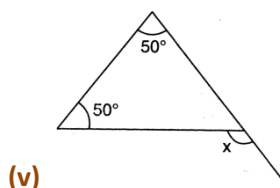
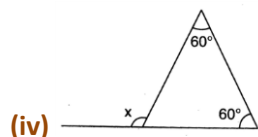
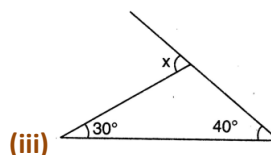
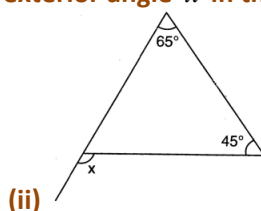
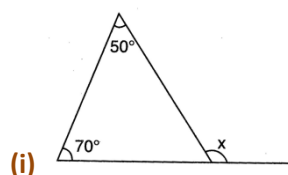
Sol.

AD is the median.
AL is the altitude.

Exercise 6.2

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1. Find the value of the unknown exterior angle x in the following diagrams:



Sol.

$$x = 50^\circ + 70^\circ$$

The exterior angle of a triangle is equal to the sum of its two interior opposite angles

$$\Rightarrow x = 120^\circ$$

(ii) $x = 45^\circ + 65^\circ$

The exterior angle of a triangle is equal to the sum of its two interior opposite angles

$$\Rightarrow x = 110^\circ$$

(iii) $x = 30^\circ + 40^\circ$

The exterior angle of a triangle is equal to the sum of its two interior opposite angles

$$\Rightarrow x = 70^\circ$$

(iv) $x = 60^\circ + 60^\circ$

The exterior angle of a triangle is equal to the sum of its two interior opposite angles

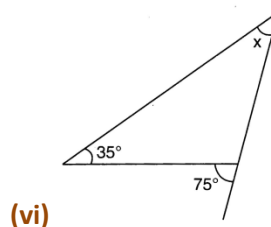
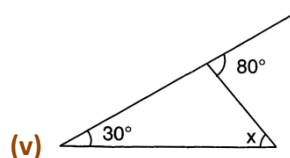
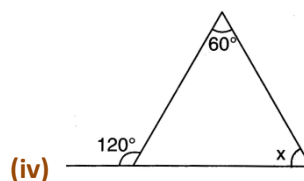
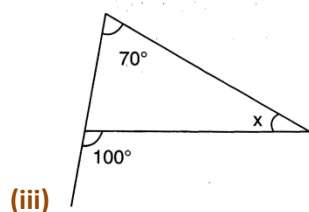
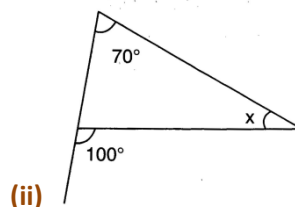
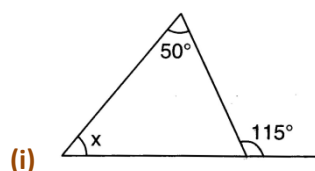
$$\Rightarrow x = 120^\circ$$

(v) $x = 50^\circ + 50^\circ$

- \Rightarrow The exterior angle of a triangle is equal to the sum of its two interior opposite angles
 $x = 100^\circ$
 (vi) $x = 30^\circ + 60^\circ$
 The exterior angle of a triangle is equal to the sum of its two interior opposite angles
 $\Rightarrow x = 90^\circ$

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2. Find the value of the unknown interior angle x in the following figures:



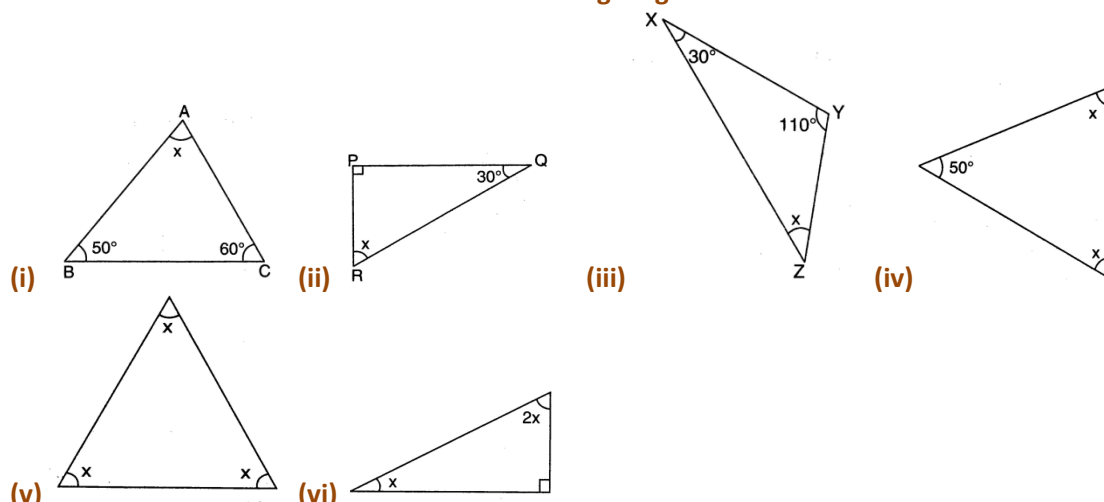
- Sol.**
- (i) $115^\circ = x + 50^\circ$
 The exterior angle of a triangle is equal to the sum of its two interior opposite angles
 $\Rightarrow x = 115^\circ - 50^\circ$
 $\Rightarrow x = 65^\circ$
- (ii) $100^\circ = x + 70^\circ$
 The exterior angle of a triangle is equal to the sum of its two interior opposite angles
 $\Rightarrow x = 100^\circ - 70^\circ$
 $x = 30^\circ$
- (iii) $125^\circ = x + 90^\circ$
 The exterior angle of a triangle is equal to the sum of its two interior opposite angles
 $\Rightarrow x = 125^\circ - 90^\circ$
 $\Rightarrow x = 35^\circ$

- (iv) $120^\circ = x + 60^\circ$
 |The exterior angle of a triangle is equal to
 |the sum of its two interior opposite angles
 $\Rightarrow x = 120^\circ - 60^\circ$
 $\Rightarrow x = 60^\circ$
- (v) $80^\circ = x + 30^\circ$
 |The exterior angle of a triangle is equal to
 |the sum of its two interior opposite angles
 $\Rightarrow x = 80^\circ - 30^\circ$
 $\Rightarrow x = 50^\circ$
- (vi) $75^\circ = x + 35^\circ$
 |The exterior angle of a triangle is equal to
 |the sum of its two interior opposite angles
 $\Rightarrow x = 75^\circ - 35^\circ$
 $\Rightarrow x = 40^\circ$

Exercise 6.3

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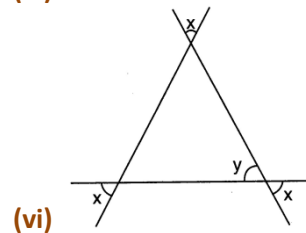
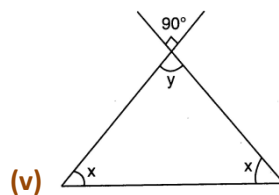
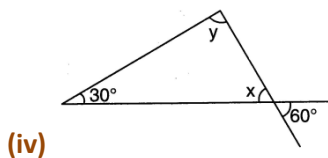
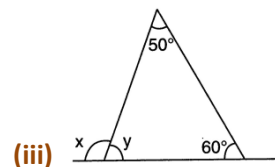
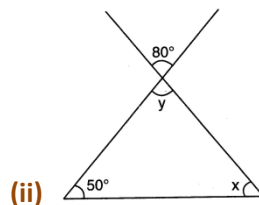
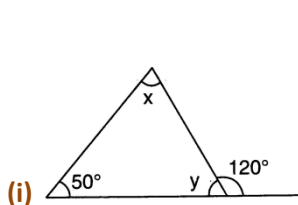
1. Find the value of the unknown x in the following diagrams:



- Sol.** (i) $x + 50^\circ + 60^\circ = 180^\circ$
 |By the angle – sum property of a triangle
 $\Rightarrow x + 110^\circ = 180^\circ$
 $\Rightarrow x = 180^\circ - 110^\circ$
 $\Rightarrow x = 70^\circ$
- (ii) $x + 90^\circ + 30^\circ = 180^\circ$
 |By the angle – sum property of a triangle
 $\Rightarrow x + 120^\circ = 180^\circ$

$$\begin{aligned}
 &\Rightarrow x = 180^\circ - 120^\circ \\
 &\Rightarrow x = 60^\circ \\
 \text{(iii)} \quad &x + 30^\circ + 110^\circ = 180^\circ \\
 &\quad \text{[By the angle – sum property of a triangle]} \\
 &\Rightarrow x + 140^\circ = 180^\circ \\
 &\Rightarrow x = 180^\circ - 140^\circ \\
 &\Rightarrow x = 40^\circ \\
 \text{(iv)} \quad &x + x + 50^\circ = 180^\circ \\
 &\quad \text{[By the angle – sum property of a triangle]} \\
 &\Rightarrow 2x + 50^\circ = 180^\circ \\
 &\Rightarrow 2x = 180^\circ - 50^\circ \\
 &\Rightarrow 2x = 130^\circ \\
 &\Rightarrow x = \frac{130^\circ}{2} \\
 &\Rightarrow x = 65^\circ \\
 \text{(v)} \quad &x + x + x = 180^\circ \\
 &\quad \text{[By the angle - sum property of a triangle]} \\
 &\Rightarrow 3x = 180^\circ \\
 &\Rightarrow x = \frac{180^\circ}{3} \\
 &\Rightarrow x = 60^\circ \\
 \text{(vi)} \quad &x + 2x + 90^\circ = 180^\circ \\
 &\quad \text{[By the angle - sum property of a triangle]} \\
 &\Rightarrow 3x + 90^\circ = 180^\circ \\
 &\Rightarrow 3x = 180^\circ - 90^\circ \\
 &\Rightarrow 3x = 90^\circ \\
 &\Rightarrow x = \frac{90^\circ}{3} \\
 &\Rightarrow x = 30^\circ
 \end{aligned}$$

2. Find the values of the unknowns x and y in the following diagrams:



Sol. (i) $50^\circ + x = 120^\circ$

|By exterior – angle property of a triangle ...(1)

$$\Rightarrow x = 120^\circ - 50^\circ$$

$$\Rightarrow x = 70^\circ \quad \dots (2)$$

Again, $x + y + 50^\circ = 180^\circ$

$$\Rightarrow x + y + 50^\circ = 180^\circ$$

|By angle – sum property of a triangle ...(4)

$$\Rightarrow x + y = 180^\circ - 50^\circ$$

$$\Rightarrow x + y = 130^\circ$$

$$\Rightarrow 70^\circ + y = 130^\circ \quad | \text{Using (2)}$$

$$\Rightarrow y = 130^\circ - 70^\circ$$

$$\Rightarrow y = 60^\circ \quad \dots (5)$$

(ii) $y = 80^\circ$

|Vertically opposite angles are equal ...(1)

$$\Rightarrow x + 50^\circ + y = 180^\circ$$

|By angle – sum property of a triangle

$$\Rightarrow x + y = 180^\circ - 50^\circ$$

$$\Rightarrow x + y = 130^\circ$$

$$\Rightarrow x + 80^\circ = 130^\circ \quad | \text{Using(1)}$$

$$\Rightarrow x = 130^\circ - 80^\circ$$

$$\Rightarrow x = 50^\circ$$

(iii) $x = 50^\circ + 60^\circ$

|By exterior-angle property of a triangle

$$\Rightarrow x = 110^\circ$$

$$\Rightarrow x = 110$$

$$y + 50^\circ + 60^\circ = 180^\circ$$

|By angle-sum property of a triangle

$$\Rightarrow y + 110^\circ = 180^\circ$$

$$\Rightarrow y = 180^\circ - 110^\circ$$

$$\Rightarrow y = 70^\circ$$

(iv) $x = 60^\circ \quad \dots(1)$

|Vertically opposite angles are equal

$$x + 30^\circ + y = 180^\circ$$

|By angles-sum property of triangle

$$x + y = 180^\circ - 30^\circ$$

$$\Rightarrow x + y = 150^\circ$$

$$\Rightarrow 60^\circ + y = 150^\circ \quad | \text{Using (1)}$$

$$\Rightarrow y = 150^\circ - 60^\circ$$

$$\begin{aligned}
&\Rightarrow y = 90^\circ \\
\text{(v)} \quad &y = 90^\circ \quad \dots (1) \\
&\quad | \text{Vertically opposite angles are equal} \\
&\quad x + x + y = 180^\circ \quad | \text{By angle-sum property of a triangle} \\
&\Rightarrow 2x + y = 180^\circ \\
&\Rightarrow 2x + 90^\circ = 180^\circ \quad | \text{Using (1)} \\
&\Rightarrow 2x = 180^\circ - 90^\circ \\
&\Rightarrow 2x = 90^\circ \\
&\Rightarrow x = \frac{90^\circ}{2} \\
&\Rightarrow x = 45^\circ \\
\text{(vi)} \quad &x = y \quad \dots (1) \\
&\quad x + x + y = 180^\circ \quad | \text{Vertically opposite angles are equal} \\
&\Rightarrow 2x + y = 180^\circ \\
&\quad | \text{By angle-sum property of a triangle} \\
&\Rightarrow 2x + x = 180^\circ \quad | \text{Using (1)} \\
&\Rightarrow 3x = 180^\circ \\
&\Rightarrow 3x = \frac{180^\circ}{3} \\
&\Rightarrow x = 60^\circ \quad \dots (2) \\
&\Rightarrow y = 60^\circ \quad | \text{Using (1)}
\end{aligned}$$

Exercise 6.4

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1. Is it possible to have a triangle with the following sides?

- (i) 2 cm, 3 cm, 5 cm
- (ii) 3 cm, 6 cm, 7 cm
- (iii) 6 cm, 3 cm, 2 cm.

Sol. (i) **2 cm, 3 cm, 5 cm**

We have $2 + 3 = 5$

\Rightarrow Sum of the lengths of two sides = Length of the third side

This is impossible since the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

(ii) **3 cm, 6 cm, 7 cm**

We see that $3 + 6 > 7$

$$6 + 7 > 3$$

$$7 + 3 > 6$$

Therefore, it is possible to have a triangle with side lengths 3 cm, 6 cm, 7 cm.

(iii) **6 cm, 3 cm, 2 cm**

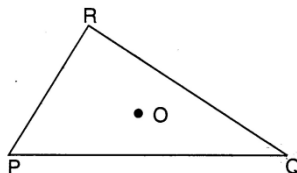
We see that $6 + 3 = 9 > 2$

$$3 + 2 = 5 \not> 6$$

$$2 + 6 = 8 > 3$$

Therefore, it is not possible to have a triangle with side lengths 6 cm, 3 cm, 2 cm .

2. Take any point O in the interior of a triangle PQR. Is



(i) $OP + OQ > PQ$?

(ii) $OQ + OR > QR$?

(iii) $OR + OP > RP$?

Sol. (i) Yes! $OP + OQ > PQ$... (1)

Sum of the lengths of any two sides of a triangle
is greater than the length of the third side ... (2)

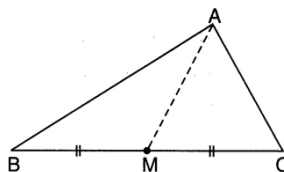
(ii) Yes! $OQ + OR > QR$

Sum of the length of any two side of a triangle is
greater than the length of the third side ... (3)

(iii) Yes! $OR + OP > RP$

Sum of the lengths of any two sides of a triangle is
greater than the length of the third side

3. AM is a median of a triangle ABC.



Is $AB + BC + CA > 2AM$?

(Consider the sides of triangles $\triangle ABM$ and $\triangle AMC$.)

Sol. In $\triangle ABM$,

$$AB + BM > AM$$

Sum of the length of any two side of a triangle
is greater than the length of the third side

In $\triangle ACM$,

$$CA + CM > AM \quad \dots (2)$$

Sum of the lengths of any two sides of a triangle
is greater than the length of the third side

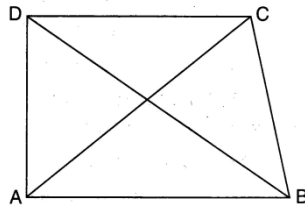
Sum (1) and (2),

$$(AB + BM) + (CA + CM) > AM + AM$$

$$\Rightarrow AB + (BM + CM) + CA > 2AM$$

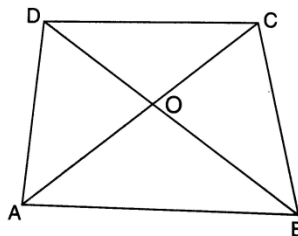
$$\Rightarrow AB + BC + CA > 2AM$$

4. ABCD is a quadrilateral, Is $AB + BC + CD + DA > AC + BD$?



Sol. In $\triangle ABC$,
 $AB + BC > AC$... (1)
 Sum of the length of any two sides of a triangle
 is greater than the length of the third side
 In $\triangle ACD$,
 $CD + DA > AC$... (2)
 Sum of the lengths of any two sides of a triangle
 is greater than the length of the third side
 Adding (1) and (2),
 $AB + BC + CD + DA > 2AC$... (3)
 In $\triangle ABD$,
 $AB + DA > BD$... (4)
 Sum of the lengths of any two sides of a triangle
 is greater than the length of the third side
 In $\triangle BCD$,
 $BC + CD > BD$... (5)
 Sum of the lengths of any two sides of a triangle is
 greater than the length of the third side
 Adding (4) and (5),
 $AB + BC + CD + DA > 2BD$... (6)
 Adding (3) and (6),
 $2[AB + BC + CD + DA] > 2(AC + BD)$
 $\Rightarrow AB + BC + CD + DA > AC + BD$

5. ABCD is a quadrilateral. Is
 $AB + BC + CD + DA < 2(AC + BD)$?



Sol. In $\triangle OAB$,
 $OA + OB > AB$... (1)
 Sum of the length of any two sides of a triangle is
 greater than the length of the third side.
 In $\triangle OBC$, $OB + OC > BC$... (2)

Sum of the lengths of any two sides of a triangle
is greater than the length of the third side

$$\text{In } \triangle OCA, OC + OA > CA \quad \dots (3)$$

Sum of the lengths of any two sides of a triangle is
greater than the length of the third side

$$\text{In } \triangle OAD, OA + OD > AD \quad \dots (4)$$

Sum of the lengths of any two sides of a triangle is
greater than the length of the third side

Adding (1), (2), (3) and (4),

$$2(OA + OB + OC + OD) > AB + BC + CD + DA$$

$$\Rightarrow AB + BC + CD + DA < 2(OA + OB + OC + OD)$$

$$\Rightarrow AB + BC + CD + DA < 2(OA + OC + OB + OD)$$

$$\Rightarrow AB + BC + CD + DA < 2(AC + BD).$$

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6. The lengths of two sides of a triangle are 12 cm and 15 cm between what two measures should the length of the third side fall?

Sol. Let x cm be the length of the third side.

\therefore Sum of the lengths of any two sides of a triangle is greater than the length of the third side.

\therefore We should have

$$12 + 15 > x \Rightarrow 27 > x \Rightarrow x < 27$$

$$15 + x > 12 \Rightarrow x > 12 - 15 \Rightarrow x > -3$$

$$x + 12 > 15 \Rightarrow x > 15 - 12 \Rightarrow x > 3$$

$$x > -3 \text{ and } x > 3 \Rightarrow x > 3$$

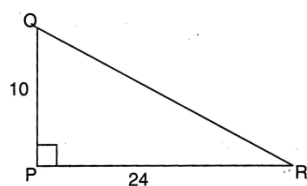
\therefore The length of the third side should be any length between 3 cm and 27 cm.

Exercise 6.5

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1. PQR is a triangle right-angled at P. If $PQ = 10$ cm and $PR = 24$ cm, find QR.

Sol.



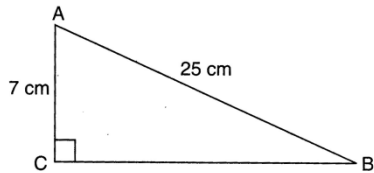
$$QR^2 = 10^2 + 24^2$$

By Pythagoras Property

$$\Rightarrow = 100 + 576 = 676$$

$$\Rightarrow QR = 26 \text{ cm}$$

2. **ABC is a triangle right-angled at C. If $AB = 25$ cm and $AC = 7$ cm, find BC**
Sol.



$$AC^2 + BC^2 = AB^2$$

| By Pythagoras Property

$$\Rightarrow 7^2 + BC^2 = 25^2$$

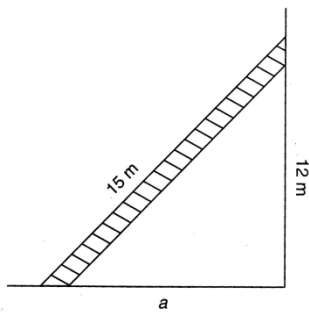
$$\Rightarrow 49 + BC^2 = 625$$

$$\Rightarrow BC^2 = 625 - 49$$

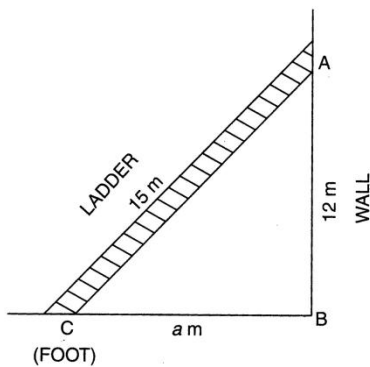
$$\Rightarrow BC^2 = 576$$

$$\Rightarrow BC = 24 \text{ cm.}$$

3. **A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance**
a. Find the distance of the foot of the ladder from the wall.



Sol.



Let the distance of the foot of the ladder from the wall be a m. Then,

$$a^2 + 12^2 = 15^2$$

| By Pythagoras Property

$$\Rightarrow a^2 + 144 = 225$$

$$\Rightarrow a^2 = 225 - 144$$

$$\Rightarrow a = 81$$

$$\Rightarrow a = 9$$

Hence, the distance of the foot of the ladder from the wall is 9 m.

4. Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm.

(ii) 2 cm, 2 cm, 5 cm.

(iii) 1.5 cm, 2 cm, 2.5 cm.

In the case of right-angled triangles, identify the right angles.

Sol. (i) **2.5 cm, 6.5 cm, 6 cm**

We see that

$(2.5)^2 + 6^2 = 6.25 + 36 = 42.25 = (6.5)^2$ Therefore, the given lengths can be the sides of a right triangle. Also, the angle between the lengths, 2.5 cm and 6 cm is a right angle.

(ii) **2 cm, 2 cm, 5 cm**

$\therefore 2 + 2 = 4 \not> 5$

\therefore The given lengths cannot be the sides of a triangle

The sum of the lengths of any two sides of a triangle is greater than the third side

(iii) **1.5 cm, 2 cm, 2.5 cm**

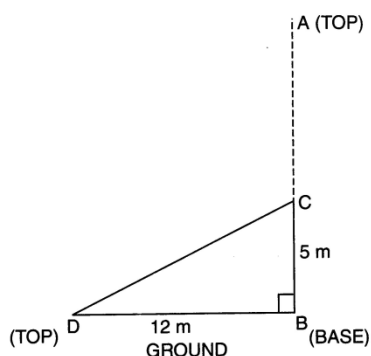
We find that

$$1.5^2 + 2^2 = 2.25 + 4 = 6.25 = 2.5^2$$

Therefore, the given lengths can be the sides of a right triangle. Also, the angle between the lengths 1.5 cm and 2 cm is a right angle.

5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Sol.



$$AC = CD$$

| Given

In right angled triangle DBC,

$$DC^2 = BC^2 + BD^2$$

| by Pythagoras Property

$$= 5^2 + 12^2$$

$$= 25 + 144$$

$$= 169$$

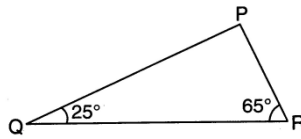
$$\Rightarrow DC = 13$$

$$\Rightarrow AC = 13$$

$$\Rightarrow AB = AC + BC = 13 + 5 = 18$$

Therefore, the original height of the tree = 18 m.

6. Angles Q and R of a ΔPQR are 25° and 65° . Write which of the following is true:



(i) $PQ^2 + QR^2 = RP^2$

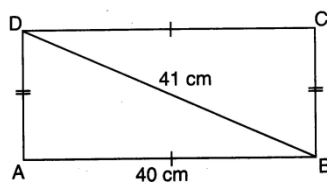
(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$

Sol. (ii) $PQ^2 + RP^2 = QR^2$ is true.

7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Sol.



In right-angled triangle DAB,

$$AB^2 + AD^2 = BD^2$$

$$\Rightarrow 40^2 + AD^2 = 41^2$$

$$\Rightarrow AD^2 = 41^2 - 40^2$$

$$\Rightarrow AD^2 = 1681 - 1600$$

$$\Rightarrow AD^2 = 81$$

$$\Rightarrow AD = 9$$

\therefore Perimeter of the rectangle

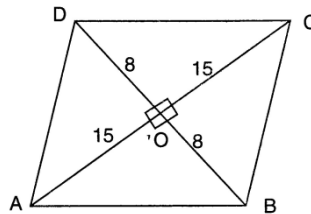
$$= 2(AB + AD)$$

$$= 2(40 + 9)$$

$$= 2(49) = 98 \text{ cm}$$

Hence, the perimeter of the rectangle is 98 cm.

8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.



Sol. Let ABCD be a rhombus whose diagonals BD and AC are of lengths 16 cm and 30 cm respectively. Let the diagonals BD and AC intersect each other at O.

Since the diagonals of a rhombus bisect each other at right angles. Therefore

$$BO = OD = 8 \text{ cm},$$

$$AO = OC = 15 \text{ cm},$$

$$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ \text{ In right-angled triangle AOB.}$$

$$AB^2 = OA^2 + OB^2$$

[By Pythagoras Property]

$$\Rightarrow AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = 15^2 + 8^2$$

$$\Rightarrow AB^2 = 225 + 64$$

$$\Rightarrow AB^2 = 289$$

$$\Rightarrow AB = 17 \text{ cm}$$

Therefore, perimeter of the rhombus ABCD

$$= 4 \text{ side}$$

$$= 4 \text{ AB}$$

$$= 4 \times 17 \text{ cm}$$

$$= 68 \text{ cm}$$

Hence, the perimeter of the rhombus is 68 cm.