Ampere's Law. For this consider circle of radius OQ = r (amperean loop 1) as shown figure 4.6 which is perpendicular to the wire as a closed loop. Such a circle and line elements ($\vec{dl}$) over its circumference are shown in Figure 4.6.

Suppose the magnetic field of all such element is $\vec{B}$. Using this fact in the equation representing Ampere's Law.

$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I$, we get

$\oint B.dl \ cos\theta = \mu_0 I$

As $\vec{B}$ and $\vec{dl}$ are in the same direction at every element,

$cos\theta = cos0 = 1$

$\therefore \oint B.dl = \mu_0 I$

As B is constant

$B \oint dl = \mu_0 I$

Here $\oint dl = dl$ circumference of the circle with radius $r = 2r$

$\therefore B(2\pi r) = \mu_0 I$

$$\therefore B = \frac{\mu_0}{2\pi} \frac{I}{R} \qquad (4.5.2)$$

Here current is positive as per our sign convention.

from equation (4.5.2)

$B \propto \frac{1}{r}$ (outside the conductor)

Magnetic Field Inside the conductor : Now as shown in the figure 4.6 radius of the wire is a and we want to find magnetic field at a perpendicular distance r_1 from its axis inside the wire that is $r_1 < a$. Consider circle with radius r_1 as amperean loop 2 as shown in figure 4.6 (which is around the axis inside the wire). If current enclosed by this loop is I_e then

$$I_e = \left(\frac{I}{\pi a^2}\right)\pi r_1^2 = I\frac{r_1^2}{a^2}$$

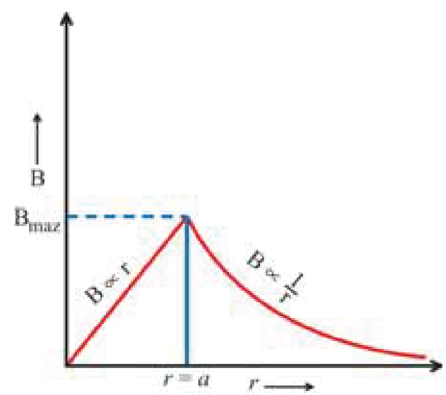


Figure 4.6 Magnetic Field B at distance *r* from the Centre of the Wire

Using Ampere's Law

$$B(2\pi r_1) = \mu_0 \frac{r_1^2}{a^2} I$$

$$\therefore B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r_1 \qquad (4.5.3)$$

Now representing r_1 by r that is for $r < a$ (for magnetic field inside the conductor)

$B \propto r$

Hence in the form of common symbol r the above facts can be represented as follows

(i) If $r > a$, then $B \propto \frac{1}{r}$

(ii) If $r < a$, then $B \propto r$

(iii) At $r = a$ B is maximum.

These facts are shown in the from of plot of $B \rightarrow r$ in figure 4.7.

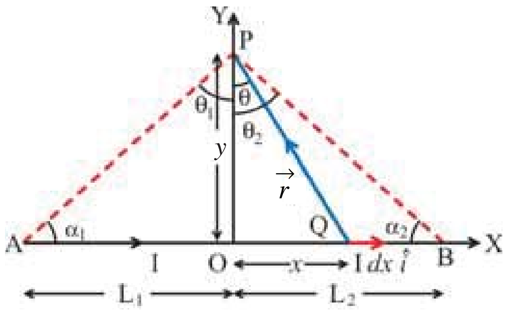


Figure 4.8 Magnetic Field Due to Current Carrying Conductor of Finite Length

For Conductor of Finite Length : To find the magnetic field $\vec{B}$ produced due to the conductor with finite length carrying current consider figure 4.8.

In this case following formula for $\vec{B}$ can be obtained using Biot–Savart Law.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{y}\left[\sin\theta_1 + \sin\theta_2\right]\hat{k} \qquad (4.5.4)$$

Where y is perpendicular distance of the given point P from the wire, θ_1 and θ_2 are the angles subtended with the perpendicular drawn on the wire from the given point by the lines joining given point and the ends of the wire (See Figure 4.8)

(2) Solenoid : As shown in the Figure 4.9 two identical rings carrying same current are placed closed to each other co-axially.

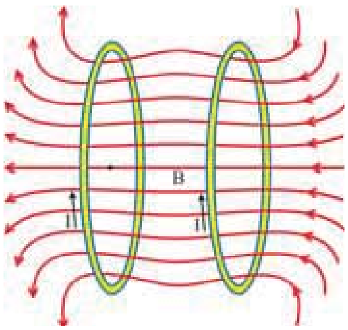


Figure 4.9

It is obvious from the Figure that the magnetic field produce due to the rings are in the same direction on their common axes. Moreover the lines close to the axis are almost parallel to the axis and in the same direction. Thus if a number of such rings (in principle of infinite number) are kept very close to each other and current is passed in the same direction, it is found that inside the region covered by the rings, the field lines are arranged at equal distance from each other obout the axis i.e. magnetic field is uniform. But the magnetic field due to two consecutive rings are in mutually opposite directions outside the rings, so they multiply each other. Hence, magnetic field in the outer region near the rings is zero. Solenoid is a device in which this situation is realized.

A helical coil consisting of closely wound turns of insulated conducting wire is called a solenoid

In practice long and short solenoids are used. **When length of a solenoid is very large as compared to its radius, the solenoid is called long solenoid.**

To find magnetic field inside a long solenoid using Ampere's Circuital Law.

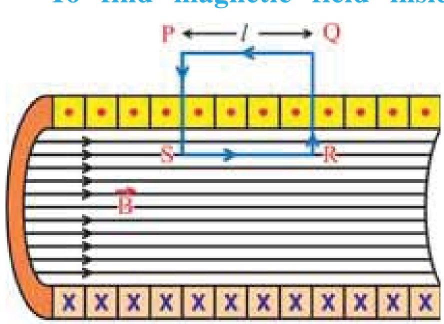


Figure 4.10 Solenoid

Figure 4.10 shows a cross-section of a long solenoid taken with plane of the page of the book. Symbol (X) shows the direction of currents going inside the plane of the page and symbol (•) shows the directions of the current coming out of the plane of the page.

Suppose we want to find the magnetic field at point S lying inside the solenoid. Considering a rectangular loop of length l, PQRS as shown in the Figure 4.10 as Amperean loop, we will take line integral $\vec{B}$ over the loop.

$$\therefore \oint \vec{B}\cdot\vec{dl} = \int_P^S \vec{B}\cdot\vec{dl} + \int_S^R \vec{B}\cdot\vec{dl} + \int_R^Q \vec{B}\cdot\vec{dl} + \int_Q^P \vec{B}\cdot\vec{dl}$$

From the figure 4.10 it is clear that the magnetic field on part PQ of the loop will be zero as it is lying outside the solenoid and hence $\int_Q^P \vec{B} \cdot \vec{dl} = 0$

Moreover, some part of sides QR and SP is outside the solenoid and the part which is inside is perpendicular to the magnetic field, therefore $\int_R^Q \vec{B} \cdot \vec{dl} = \int_P^S \vec{B} \cdot \vec{dl} = 0.$

$$\therefore \oint \vec{B} . \vec{dl} = \int_S^R B dl \cos 0^o = B \int_S^R dl = Bl \quad (4.5.5)$$

Now suppose that the number turns per unit length of the solenoid is n. Therefore, the number of turns passing through the Amperean loop is nl. Current passing through each turn is I, so total current passing through the loop is $\Sigma I = nlI$.

From Ampere's Circuital Law

$$\oint \vec{B} . \vec{dl} = \mu_0 nlI$$

$$\therefore Bl = \mu_0 nlI \quad \text{(from equation 4.5.5)}$$

$$\therefore B = \mu_0 nI \quad (4.5.6)$$

This method can be used only for a long solenoid because only in case of a long solenoid, all the points inside the solenoid can be considered equivalent and magnetic field inside the solenoid as uniform. In the region outside the solenoid in the vicinity of it is zero. This method should not be used for a solenoid of finite length.

For Solenoid of Finite Length : For solenoid of finite length magnetic field inside of it can be determined using Biot–Savart's Law. For this consider figure 4.11. Formula for the magnetic field inside the solenoid of finite length is as under.

$$B = \frac{\mu_0 nI}{2}(\sin\alpha_1 + \sin\alpha_2) \quad (4.5.7)$$

Here α_1 and α_2 are the angles subtended by two ends of the solenoid with normal drawn at point P respectively.

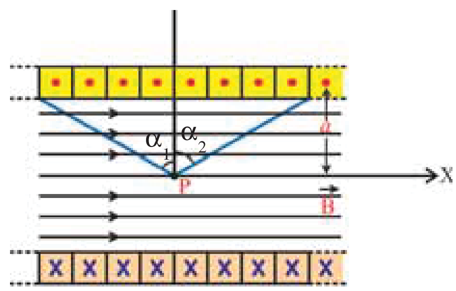


Figure 4.11 Solenoid of Finite Length

Toroid : If a solenoid is bent in the form of a circle and its two ends are joined with each other then the device is called a toroid.

A toroid can also be prepared by closely winding an insulated conducting wire arround non-conducting hollow ring. (In short, the shape of a toroid is the same as that of an inflated tube, also called doughnut shape.) The magnetic field produced inside the toroid carrying electric current can be obtained using Ampere's Circuital Law.

Suppose we want to find the magnetic field at a point P inside the toroid which is at a distance r from its centre as shown in the figure 4.12. If we consider a circle of radius r with its centre at O as an Amperean loop from the symmetry it is clear that the magnitude of the magnetic field at every point on the loop is same and directed towards the tangent to the circle. Therefore,

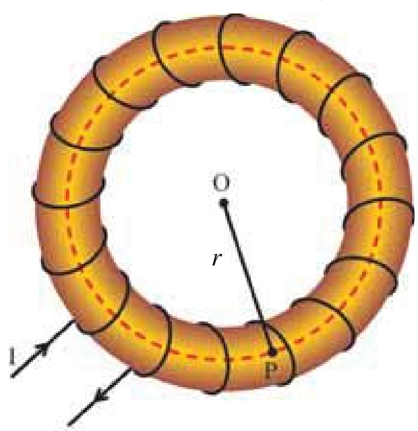


Figure 4.12 Toroid

$$\oint \vec{B} \cdot \vec{dl} = \oint B dl = B \oint dl = B(2\pi r) \qquad (4.5.8)$$

If the total number of turns is N and current passing is I, the total current passing through the said loop must be NI. From Ampere's Circuital Law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 NI \qquad (4.5.9)$$

Comparing equations (4.5.8) and (4.5.9)

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 nI}{2\pi r} = \mu_0 NI \qquad (4.5.10)$$

Here, $n = \frac{N}{2\pi r}$ the number of turns per unit length of the toroid. This is the equation of magnetic field produced inside the toroid. This magnetic field is uniform at each point inside the toroid.

In an ideal toroid, the turns are completely circular. In such a toroid magnetic field the inside the toroid is uniform and outside the toroid is zero. But in the toroid used in practice, the will is helical and hence, a small magnetic field also exist outside the toroid.

For nuclear fusion, the device Tokamak is used for the confinement of plasma. Toroid is an important component of Tokamak.

4.6 Force on a Current Carrying Wire Placed in a Magnetic Field

Within week of the publicity of the news of Oersted's observation scientist Ampere made another observation. In this observation he showed that **"Two parallel wires placed near each other exert an attractive force if they are carrying currents in the same direction, and exert a repulsive force if they are carrying currents in the opposite directions."**

We have seen that magnetic field is created around the wire carrying electric current. Now, if another wire carrying current is placed in its neighbourhood (i.e. second wire carrying current is placed in the magnetic field produced by the current in the first wire) then the force acts on the other wire due to magnetic field produced by current in the first wire. In the same manner the first wire is lying in the magnetic field produced by the current in the other wire. Hence the force acts on the first wire due to the magnetic field produced by the current in the other wire. This is the magnetic force between two wires.

This interaction can schematically be represented as follows.

$$\left.\begin{matrix}\text{Current in the}\\ \text{first wire}\end{matrix}\right\} \begin{matrix}\rightarrow\\ \leftarrow\end{matrix} \text{Magneticfield} \left\{\begin{matrix}\text{Current in the}\\ \text{second wire}\end{matrix}\right.$$

Thus in other words the force acts between the two wires (carrying current) is due to magnetic field.

To find this force acting between two wires, one must know, the force acting on a wire currying a current placed in magnetic field. The Law giving this force was established by Ampere through the experimental studies is as under :

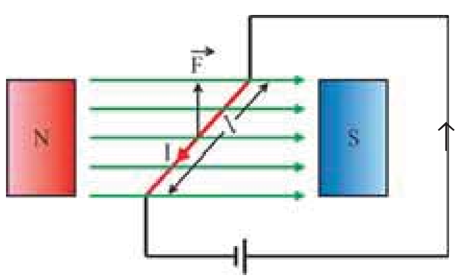


Figure 4.13

The force acting on a current element $I\vec{dl}$ due to the magnetic induction $\vec{B}$ is given by

$$\vec{dF} = I\vec{dl} \times \vec{B} \qquad (4.6.1)$$

If a straight wire of length l carrying current I is placed in uniform magnetic field $\vec{B}$, the force acting on the wire can be given by

$$\vec{F} = I\vec{l} \times \vec{B} \quad (4.6.2)$$

Such an arrangement is shown in the Figure 4.13.

The direction of force can be determined using the right hand rule for vector product.

4.6.1 The Force between Two parallel Current Carrying Wires

Consider two very long conducting wires placed parallel to each other along X-axis, separated by a distance y and carrying currents I_1 and I_2 in the same direction (See figure 4.14)

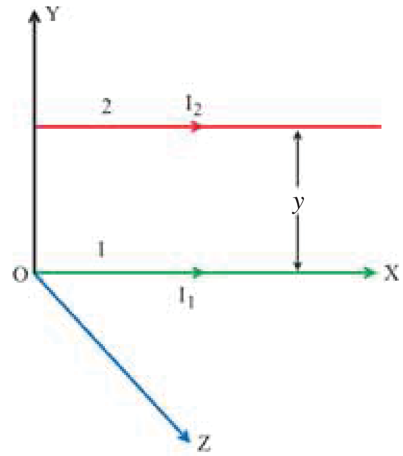


Figure 4.14

Magnetic field at a distance y from first conductor carrying current I_1 is

$$\vec{B}_1 = \frac{\mu_0}{2\pi} \cdot \frac{I_1}{y} \hat{k} \quad (4.6.3)$$

The strength of this field is same at all points on the second wire carrying current I_2 and directed along Z-axis. Therefore, the force acting on the second wire over its length l will be

$$\vec{F}_2 = I_2 \vec{l} \times \vec{B}_1 \quad \text{(From equation 4.6.2)}$$

substituting value of B_1 from equation (4.4.3) in the above equation

$$\vec{F}_2 = I_1 I_2 \frac{\mu_0}{2\pi y} l\hat{i} \times \hat{k} \quad \text{(As current } I_2 \text{ being along the X-axis)}$$

$$\therefore \vec{F}_2 = -\frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{y} \hat{j} \quad (4.6.4)$$

Above equation shows that the force $\vec{F}_2$ acts along negative Y-direction.

Now the force $\vec{F}_1$ acting on the first wire carrying current I_1 can be obtained in the same manner which is as under :

$$\vec{F}_1 = \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{y} \hat{j} \quad (4.6.5)$$

The above equation shows that the force F_1 acting on the first wire is in positive y direction.

Thus $\vec{F}_1 = -\vec{F}_2 \quad (4.6.6)$

This fact shows that force acting between indicates attraction takes place between them.

If the currents are flowing in the mutually opposite directions in the two wires then repulsion is produced between them.

From equation (4.6.6) it is obvious that here also Newton's third Law is obeyed.

Definition of Ampere :

In equation (4.6.4) if we take

$I_1 = I_2 = 1A$, $y = 1$ m and $l = 1$m

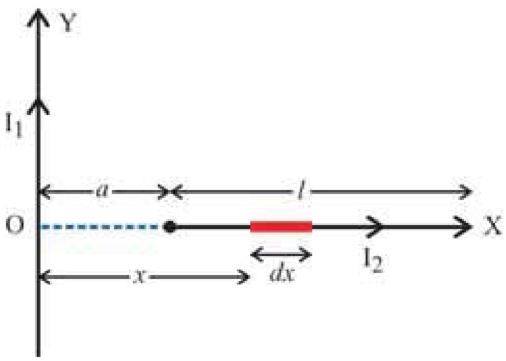
$$|\vec{F}_2| = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N} \quad (4.6.7)$$

Using this fact definition of SI unit of 1 ampere current is given as under :

"When the magnetic force acting per metre length in two infinitely long wires placed parallel to each other at a distance of 1 meter in vacuum, carrying identical current is 2×10^{-7} N, the current passing through each wire is 1 ampere."

Illustration 5 : As shown in the figure very long conducting wire carrying current I_1 is arranged in y direction. Another conducting wire of length l carrying current I_2 is placed on X-axis at a distance from this wire. Find the torque acting on this wire with respect to point O.

Solution : The force acting on a current element $I_2 dx$ located at a distance x from O is,



$$d\vec{F} = I_2 dx\,\hat{i} \times \vec{B}$$

where, $\vec{B} = \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$

(the magnetic field due to a very long conductor)

$$\therefore\ d\vec{F} = I_2 dx\,\hat{i} \times \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j}$$

$\therefore$ The torque acting on this element with respect to O is,

$$d\vec{\tau} = x\hat{i} \times d\vec{F} = x\hat{i} \times \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j} = \frac{\mu_0 I_1 I_2}{2\pi} dx\,\hat{k}$$

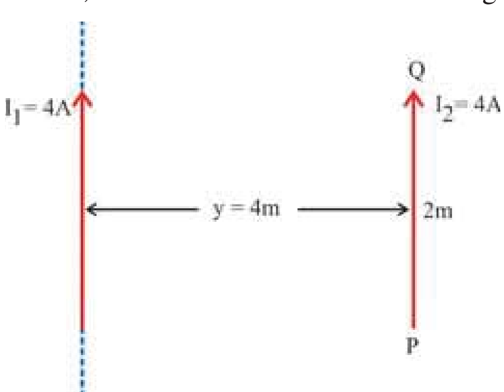
Total torque acting on this coil can be obtained by taking integration of this equation between $x = a$ to $x = a + l$,

$$\therefore\ \vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi}\int_a^{a+l} dx\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi}[x]_a^{a+l}\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi}[a + l - a]\hat{k}$$

$$\therefore\ \vec{\tau} = \frac{\mu_0 I_1 I_2 l}{2\pi}\hat{k}$$

Illustration 6 : As shown in the figure, a straight wire PQ of length 2 m carrying 4A current is placed parallel to a very long wire at a distance of 2m. Find the force acting on wire PQ if the current passing through the long wire is also 4A.

Solution : According to Newton's 3rd Law of motion, the force exerted by the smaller wire on the longer wire is the same as the force exerted by the long wire on the smaller one. Hence, we will find the force acting on the smaller wire.



Suppose magnetic field on the smaller wire due to the longer wire is $\vec{B}$

$$\therefore\ \vec{B} = \frac{\mu_0 I_1}{2\pi y}\hat{n} \quad (1)$$

where $\hat{n}$ is the unit vector in the direction of $\vec{B}$. (1)

Now force on the longer wire is,

$$\vec{F} = I_2\vec{l} \times \vec{B}$$

$\therefore\ |\vec{F}| = I_4 l B\ (\because\ \vec{l} \perp \vec{B})$

Using the equation (1),

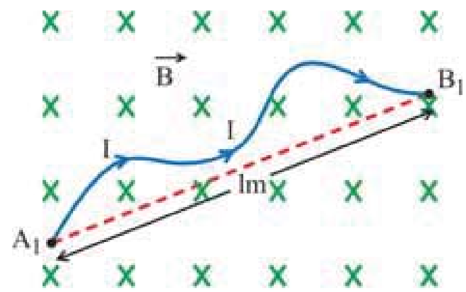
$$\therefore\ |\vec{F}| = \frac{I_2 l \mu_0 I_1}{2\pi y}$$

$$= \frac{4\times2\times4\times3.14\times10^{-7}\times4}{2\times3.14\times4}$$

$\therefore\ |\vec{F}| = 16 \times 10^{-7}$ N

This force produced here is attractive.

Illustration 7 : A wire carrying electric current I is placed on the plane of paper. A magnetic field of induction $\vec{B}$ is applied in a direction going into the plane of paper normally. Find the force acting on the wire.



A straight line joining A_1 and B_1, which is not a part of the wire, of length 1 m is shown in the figure.

Solution : The force acting on a current element $I\vec{dl}$ due to the magnetic field $\vec{B}$ is,

$$d\vec{F} = I\,\vec{dl} \times \vec{B}$$

$\therefore$ The total force acting on the wire is,

$\vec{F} = \int I\cdot\vec{dl} \times \vec{B}$ (Here integration is taken over the whole length of the wire.) Here, n is the number of (free) charge carriers per unit volume of the conductor.

$$\therefore\ \vec{F} = I[\int_l \vec{dl}\,] \times \vec{B}$$

But, $\int \vec{dl} = \overrightarrow{A_1B_1} = 1\hat{n}\ (\because\ A_1B_1 = 1\text{m})$

where, $\hat{n} = \overrightarrow{A_1B_1}$ the unit vector in the dirction of

$$\therefore\ \vec{F} = I\hat{n} \times \vec{B} \Rightarrow |\vec{F}| = IB$$

4.7 Force on an Electric Charge Moving in a Magnetic Field and Lorentz Force

In Chapter-3 we studied that the current I flowing through a cross section A of a conductor is

$I = nAv_d q$

Here q = Charge on the positively charged particle.

n = number of (free) charge carrier per unit volume of the conductor

v_d = drift velocity

$\therefore\ I\vec{dl} = qnAv_d\vec{dl} = qnA\vec{v_d}\,dl$ ($\because\ v_d$ and dl are in the same direction)

When this conductor is placed in a magnetic field of intensity $\vec{B}$, the force acting on current element $I\vec{dl}$ is given by

$$\vec{dF} = I\vec{dl} \times \vec{B}$$

$$\therefore \quad \vec{dF} = qnAdl(\vec{v_d} \times \vec{B}) \qquad (4.7.1)$$

But $nAdl$ = total number of charged particle in current element

∴ the magnetic force acting on a single particle of charge q will be given by

$$\vec{F_m} = \frac{\vec{dF}}{nAdl} = \frac{qnAdl(\vec{v_d} \times \vec{B})}{nAdl}$$

$$\therefore \quad \vec{F_m} = q(\vec{v_d} \times \vec{B}) \qquad (4.7.2)$$

$|\vec{F_m}| = Bqv_d \sin\theta$. This shows (i) if charge is stationary this force is zero (ii) moreover if charge is moving paralel or anti-parallel to the magnetic field then also this force is zero.

Now, if this electric charge q is moving in the electric field of intensity $\vec{E}$ over and above the magnetic field $\vec{B}$, the force $\vec{F_e} = \vec{E} \cdot q$ due to electric field acts on the charge q. In this circumstanes total force acting on the charge.

$$\vec{F} = \vec{F_e} + \vec{F_m}$$

$$\therefore \quad \vec{F} = q[\vec{E} + (\vec{v_d} \times \vec{B})] \qquad (4.7.3)$$

$$\therefore \quad |\vec{F_m}| = Bqv_d \sin\theta$$

the force obtained by this equation is called Lorentz Force.

The magnetic force acting on a charge moving through the magnetic field is perpendicular to the velocity of the particle, work done by the force is zero and hence its kinetic energy remains constant. Only direction of velocity goes on changing at every instant.

The magnitude of the magnetic force depends on the velocity of the particle, hence such a force is called velocity dependent force.

Illustration 8 : A particle having 2 C charge passes through magnetic field of $4\hat{k}$ T and some uniform electric field with velocity $25\hat{j}$. If the Lorentz force acting on it is $400\hat{i}$ N find the electric field in this region.

Solution : Lorentz force

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

Here, $q = 2$ C, $\vec{v} = 25\hat{j}$ m s^{-1}, B = $4\hat{k}$ T, $\vec{F} = 400\hat{i}$

$$\therefore \quad 400\hat{i} = 2 \quad [\vec{E} + (25)(4)(\hat{j} \times \hat{k})]$$

$$= 2\vec{E} + 200\hat{i}$$

$$\therefore \quad 2\vec{E} = 200\hat{i}$$

$$\therefore \quad \vec{E} = 100\hat{i} \text{ V m}^{-1}$$

Illustration 9 : In copper there are 8×10^{28} free (conducting) electrons per cubic meter. A current copper wire having length 1 m and cross-sectional area 8×10^{-6} m^2 is placed perpendicularly in the magnetic field of 4×10^{-3} T. The force acting on this wire is $8\text{-}0 \times 10^{-2}$ N. Find the drift velocity of the free electron.

Solution : Magnetic force acting on the wire is given by the formula $\vec{F} = I\vec{l} \times \vec{B}$. Here wire perpendicular to the magnetic field. $|\vec{F}| = IlB$ where $F = 8.0 \times 10^{-2}$, $B = 4.0 \times 10^{-3}$T and l = 1m

$$\therefore\ I = \frac{F}{Bl} = \frac{8\times10^{-2}}{4\times10^{-3}\times1} = 20\ A.$$

Now $I = Av_d n.e$

n = No. of electrons in the unit volume = 8×10^{28}

A = 8×10^{-6} m^2 and $e = 1.6 \times 10^{-19}$ C

$$\therefore\ v_d = \frac{I}{n.A.e}$$

$$= \frac{20}{8\times10^{28}\times8\times10^{-6}\times1.6\times10^{-19}} = 1.953 \times 10^{-4}$$

$$\approx 2 \times 10^{-4}\ m\ s^{-1}$$

Illustration 10 : Write the equation of magnetic force acting on a particle moving through a magnetic field. Using it obtain Newton's equation of motion and show that kinetic energy of the particle remains constant with time.

Solution : $\vec{F}_m = q(\vec{v} \times \vec{B})$

$$\therefore\ m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$$

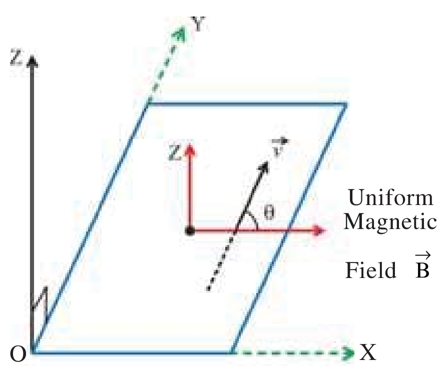
Taking dot product $\vec{v}$ with on both the sides,

$$m\vec{v}\cdot\frac{d\vec{v}}{dt} = q\vec{v}(\vec{v} \times \vec{B})$$

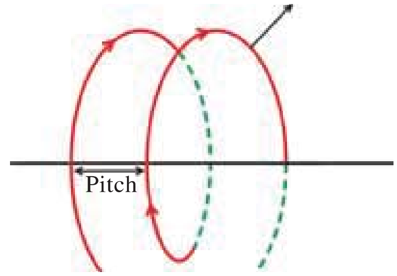
$$\therefore\ m\frac{1}{2}\frac{d}{dt}(\vec{v} \cdot \vec{v}) = 0\ (\because\ .\vec{v}\ \text{and}\ \vec{v} \times \vec{B}\ \text{are mutually perpendicular})$$

$$\therefore\ \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \Rightarrow \frac{1}{2}mv^2 = \text{constant}$$

Illustration 11 : Suppose a particle of mass m and charge q is incident on XZ plane with velocity v in a direction making angle θ with a uniform magnetic field applied along X-axis according to figure (a). Show that motion of this particle is helical and find the pitch of the path.



(a)



(b)

Solution : Considering two components of velocity in XZ plane,

$v_z = v \sin\theta$ and $v_x = v \cos\theta$

As v_x component is in the direction of magnetic field, $qv_x\hat{i} \times B\hat{i} = 0$. Since this force is zero, the particle will continue to move with constant velocity $v_x = v \cos\theta$ along X axis.

Now the force due to v_z component $= qv_z\hat{k} \times B\hat{i} = qv_zB\hat{j}$. This force acts perpendicularly to v_z hence the particle will perform circular motion on YZ plane with linear velocity v_z.

Now the centripetal force needed for circular motion is,

$$\frac{mv_z^2}{r} = qv_zB$$

$$\therefore\ r = \frac{mv_z}{qB} = \frac{mv\sin\theta}{qB}$$

Radius of the circular path of the particle can be determined using above equation, period, $T = \frac{2\pi r}{v_z}$.

$$\therefore\ T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{qB}$$

The particle covers a distance of v_xT during the time interval equal to its period along X axis.

$$\therefore \text{ distance travelled along X direction } = \frac{2\pi m v_x}{qB} = \frac{2\pi mv\cos\theta}{qB}$$

It is clear from this discussion that the particle moves on a helical path whose axis is along X direction. Here, distance v_xT is called the pitch of the helix (See figure (b)).

4.8 Cyclotron

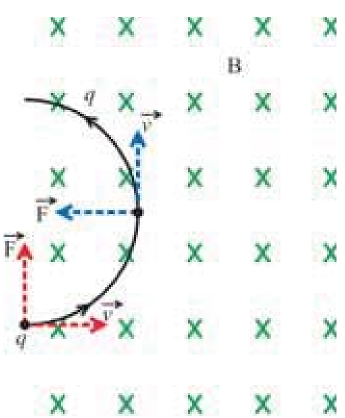


Figure 4.15
Motion of Charged Particle Entering Normally in the Magnetic Field

In the study of nuclear structure very high energy particles are required to be Bombarded on the Nucleus. For this purpose the charged particles are to be accelerated. To do so E.O. Lawrence and M. S. Livingston developed an instrument called cyclotron.

In this instrument the force on a charged particle moving perpendicularly inside a magnetic field is being used. Hence to understand its working we have to study the motion of a charged particle moving perpendicularly inside a magnetic field.

Consider a particle with charge q, moving with velocity $\vec{v}$ in the magnetic field of induction $\vec{B}$ as shown in the figure 4.15.

Here the magnetic field $\vec{B}$ perpendicularly entering into the plane of paper and the electron is maving in the plane of paper.

According to equation (4.7.2), the magnetic force on this particle is $\vec{F} = q(\vec{v} \times \vec{B})$

The value of this force is $qvB\sin\theta$ and the direction is normal to the plane formed by $\vec{v}$ and $\vec{B}$. Here, since the particle is moving perpendicular to the magnetic field the value of this force is qvB. It is clear that in this condition the path of the particle will be circular. Since this force is normal to its velocity at every moment, the value of velocity will not change, only its direction will be continuously changing. As a result it will perform circular motion. The necessary centripetal force for this motion is the magnetic force Bqv.

$\therefore \; qvB = \frac{mv^2}{r}$

Where m = mass of particle and r = radius of circular path.

$\therefore \; r = \frac{mv}{qB} = \frac{p}{qB}$ (4.8.1)

This equation shows that the radius of the circular path of the particle is proportional to the momentum of particle $p = mv$. If the momentum increases the radius of the circular path of the particle also increases.

Here for the circular motion we can write $v = rw_c$. w_c is the angular frequency of the particle which is called the cyclotron frequency. Substituting this value in equation (4.8.1), we get

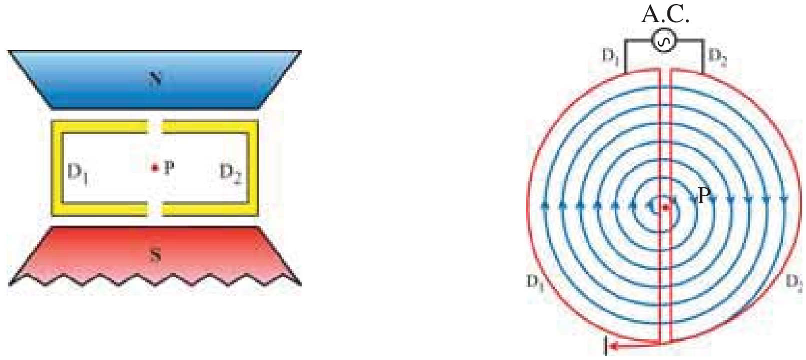
$r = \frac{m(w_C r)}{qB}$

$\therefore \; w_C = \frac{qB}{m}$ (4.8.2)

$\therefore \; f_C = \frac{qB}{2\pi m}$ (4.8.3)

This f_C is called cyclotron frequency.

Here, it is clear that the angular frequency of the particle w_C does not depend on its momentum. Hence on increasing the linear momentum of the particle, the radius of its cercular path definitely increases but the frequency w_C does not change. This fact is used in the design of a cyclotron.



(a) Side View (b) Top View

Figure 4.16 Schematic Diagram of Cyclotorn

Construction : Two hollow matallic boxes of D-shape are kept in front of each other with their diameters facing each other and with a small gap between them as shown in the figure 4.16. Two strong electromagnets are kept in such a way that a uniform magnetic field is developed in the space enveloped by the two boxes. These two boxes are called Dees as they are D-shaped. An A.C. of high frequency is applied between the two Dees. This device is then kept in an evacuated chamber in order to avoid the possible collision of charged particle with the air molecules.

Working : Suppose a charged particle is released from the centre P of the gap between the Dees of time $t = 0$. Exactly at the same time suppose one of the Dees is at negative potential. **If the particle is positively charged,** it gets attracted towards this Dee. Now as a uniform magnetic field is existing in the space between the Dees, the charged particle performs circular motion in the gap and enters the magnetic field in the Dees perpendicularly with a certain momentum. Now there is no electric field in the Dees, hence the particle moves on a circular path of radius depending on its momentum and comes out of the Dee after completing a half circle.

Now, if the opposite Dee becomes negative at the moment at which the particle emerges from one Dee the particle gains momentum due to electric field while passing through the gap before entering the other Dee. It moves in the other Dee on a circular path of larger radius. When this particle emerges out from the second Dee, if the opposit Dee acquires negative potential, the particle gets even more momentum and moves on a circular path of even greater radius in this Dee.

If this process is repeated the radius of circular path goes on increasing but the frequency w_c remains constant. To make this possible the frequency of A.C. voltage (f_{AC}) should be equal to the frequency of revolution f_C. (Here $w_C = 2\pi f_C$). This is nothing but resonance.

In this manner the charged particle goes on gaining energy which becomes maximum on reaching the circumference of the Dee.

For bambarding this charged particle on some target it should be brought out of the Dee. For this when the particle is on the edge, it is brought out of the Dee by deflecting with the help of another magnetic field and allowed to hit the nuclei of the atoms of target.

Here, we have discussed about accelerating positively charged particle (e.g. proton, positive ions), such accelerated particles are used in the study of nuclear reactions, preparation of artificial radioactive substances, treatment of cancer and ion implantation in solids.

Limitations : According to the theory of relativity as the velocity of particle approaches that of light, its mass goes on increasing. In this situation the condition of resonance ($f_{AC} = f_C$) is not satisfied.

To accelerate very light particles like electron, the frequency of A.C. is required to be very high (of the order of GHz)

Moreover, the size of Dees is also large. It is difficult to maintain a uniform magnetic field over a large region. Hence accelerators like synchrotron are developed.

4.9 Torque Acting on a Rectangular Current Carrying Coil Kept in Uniform Magnetic Field

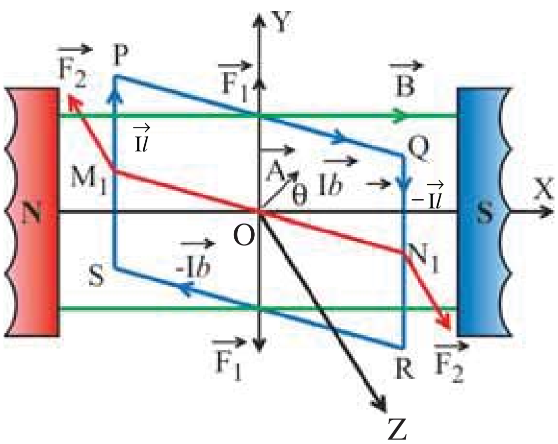


Figure 4.17

Consider a rectangular coil of length QR = l and width PQ = b carrying Current I as shown in figure 4.11. Here, direction of the magnetic field $\vec{B}$ is taken along X-axis.

$\therefore \vec{B} = B\hat{i}$

The force acting on the element constituted by side PQ of the coil = $I\vec{b}$.

Therefore force acting on this element $\vec{F_1} = I\vec{b} \times \vec{B}$. (Positive Y-direction). Similarly the force acting on the element formed by side RS is $\vec{F_1}' = I\vec{b} \times \vec{B}$ (negative Y-direction).

Here, forces $\vec{F_1}$ and $\vec{F_1}'$ are equal in magnitude, opposite in direction and collinear hence, they cancel each other.

Now consider the element (QR)I = $-Il\hat{j}$. The force acting on it

$$\vec{F_2} = -Il\hat{j} \times \vec{B}\hat{i} = -IlB(\hat{j} \times \hat{i}) = IlB\hat{k} \qquad (4.9.1)$$

is along positive Z-direction.

Similarly the force acting on the element (SP) I = $Il\hat{j}$ is

$$\vec{F_2}' = Il\hat{j} \times \vec{B}\hat{i} = -IlB\hat{k} \qquad (4.9.2)$$

is in negative Z-direction.

Equations (4.9.1) and (4.9.2) show that $|\vec{F}_2| = |\vec{F}_2\,'|$

It is also clear from the figure 4.17 that they are opposite in direction. But they are non-collinear. So they constitute a torque (couple)

Viewing the coil from above (in negative Y−direction), $\vec{F}_2$, $\vec{F}_2\,'$, X-axis and vector $\vec{A}$ appear as shown in figure 4.18. Here $\vec{A}$ is the vector representing the area of the plane of the coil which makes an angle θ with X-axis.

Thus,

Torque acting on coil = (magnitude of a force) (Perpendicular distance between two forces)

The perpendicular distance between two forces is (See Figure 4.18)

$$M'N' = 2\frac{b}{2}cos(\frac{\pi}{2} - \theta) = bsin\theta \qquad (4.9.3)$$

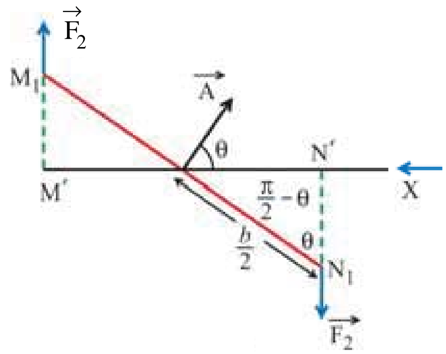


Figure 4.18 Torque Acting on Rectangular Coil

$$\therefore \text{ Torque } |\vec{\tau}| = |\vec{F}_2| (M'N') = (IlB)(bsin\theta) \qquad (4.9.4)$$

$$\therefore |\vec{\tau}| = IABsin\theta$$

Where lb = A is the area of the coil.

For coil having N turns,

$$|\vec{\tau}| = NIAB\ sin\theta \qquad (4.9.5)$$

Taking area A of the coil in the vector form, equation (4.9.5) can be written in the vector form as

$$\vec{\tau} = NI\vec{A} \times \vec{B} \qquad (4.9.6)$$

The vector quantity $NI\vec{A}$ is called "magnetic moment" linked with the coil and denoted by ($\vec{\mu}$)

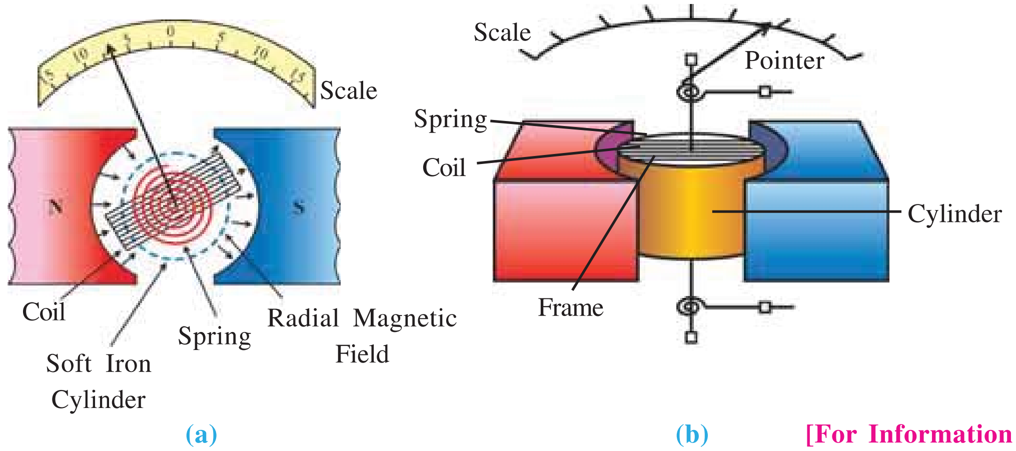
$$\therefore \vec{\tau} = \vec{\mu} \times \vec{B} \qquad (4.9.7)$$

equation (4.9.7) is valid for any shape of the coil.

Direction of $\vec{\mu}$ can be determined using right hand screw rule. Keep a right hand screw perpendicular to the plane of the coil and rotate it in the direction of current, the direction in which screw advances shifts gives the direction of $\vec{\mu}$.

4.10 Galvanometer

Galvanometer is a device used to detect and measure small electric currents.



[For Information Only]

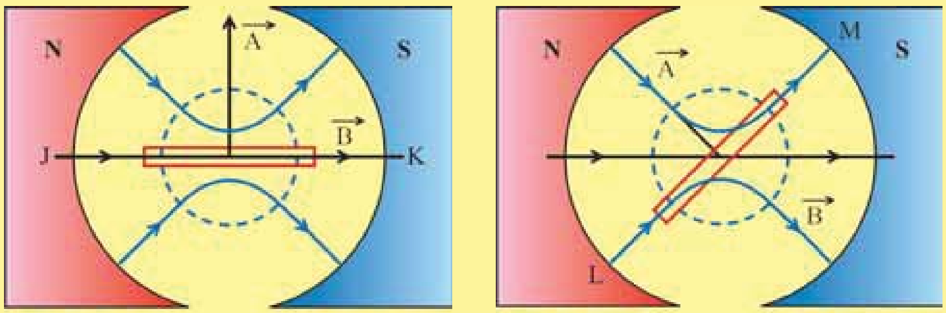
Figure 4.19 Construction of Galvanometer

In galvanometer, a coil of thin insulated copper wire is wound on a light rectangular (non-magnetic) frame. The frame is pivoted between two almost frictionless pivots and placed between two cylindrical poles of a permanent magnet so that it can freely move in the region between the poles. A small soft iron cylindrical core is placed at the axis of the coil (free from coil) so that uniform radial magnetic field is produced. When current is passed through the coil a torque acts on it and deflected. The steady deflection coil is indicated by a pointer attached with it. Knowing the position of the pointer on the scale current can be known.

Principle and Working : If the area vector of the coil marks an angle θ with the magnetic field, from equation (4.9.5) torque acting on the coil.

$\tau = NIAB\sin\theta$ (where N = number of turns in the coil) (4.10.1)

(For Information Only : In the present case magnetic field is radial)



Position (1)   Position (2)

Figure 4.20

Figure 4.10 represent figure 4.20 the radially uniform magnetic field obtained in presence of a cylinder of soft iron. For convenience only a few magnetic field lines are shown here. When the coil is in position 1, the line JK is the only effective line. In this case the angle between $\vec{A}$ and $\vec{B}$ is 90^o.

Similarly for position 2 of the coil, the line LM becomes effective. In this case also the angle between $\vec{A}$ and $\vec{B}$ is 90^o. Thus for any position of the coil the angle between $\vec{A}$ and $\vec{B}$ is 90^o.

Due to the raidal field, the angle between $\vec{A}$ and $\vec{B}$ will always be 90^o.

$\therefore \tau = NIAB$ (4.10.2)

which is called deflecting torque. (The torque due to which the coil is deflected.)

Due to the deflection of the coil, the restoring torque is produced in the springs which is directly proportional to the deflection of the coil.

$\therefore \tau$ (restoring) $= k\phi$ (4.10.3)

Here k = effective torsional constant of the springs.

If the coil becomes steady after a deflection ϕ,

Deflecting torque = Restoring torque. $NIAB = k\phi$

$$\therefore I = \left[\frac{k}{NAB}\right]\phi \quad (4.10.4)$$

$$\therefore I \propto \phi \quad (4.10.5)$$

The scale of a galvanometer can be appropriately calibrated to measure I by knowing ϕ.

From equation (4.10.5)

$$\frac{\phi}{I} = \frac{NAB}{k} \quad (4.10.6)$$

Where $\frac{\phi}{I}$ is called current sensitivety(s) of the galvanometer.

Thus, deflection produced per unit current is called current sensitivity of the galvanometer one of the ways to increase the current sensitivity of the galvanometer is to use stronger magnetic field $\vec{B}$.

To measure very weak currents of the order of 10^{-11} A, the galvanometer with coil suspended by an elastic fibre between magnetic poles are used.

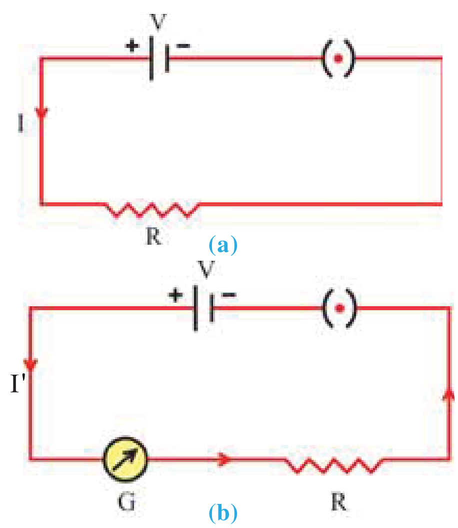
4.10.1 Measurement of Electric Current and Potential Difference

We often need to measure the parameters related to a circuit component like the electric current passing through it and the potential difference across its two ends. The instruments to measure these quantities are called an ammeter and a voltmeter respectively. The basic instrument to measure electric current or the voltage is the galvanometer.

4.10.1 (a) Ammeter : A galvanometer has to be joined in series with the component through which the electric current is to be measured. If the potential difference between the two ends of a component is to be measured, the galvanometer has to be joined in parallel between these two ends.

In practice if a galvanometer is directly used as a current–meter, two difficulties arise.

(1) To measure the electric current passing through a component of a circuit, the current–meter is to be joined in series with that component. As for example, we want to measure current passing through the resistance R in a circuit shown in the figure 4.21(a). For this purpose, current meter is joined in series with resistance R, as shown in the figure 4.21(b). In such a connection the resistance G of the galvanometer is added in the circuit. As the total resistance of the circuit is changed the value of current to be measured itself is changed. Thus the true value of current is not obtained. This fact indicates that the resistance of current meter should be as small as possible (in principle zero)



(a)
(b)
Figure 4.21

(2) Moreover, the moving coil galvanometers are very sensitive. Even when a small fraction of one ampere current (of the order of 10^{-6} A) passes through it, it shows full scale deflection.

The electric current, for which the galvanometer shows full scale deflection, is called the current capacity of galvanometer (I_G). If the galvanometer is used to measure a current greater than its range (current capacity), it is likely to be damaged.

Moreover due to larger current passing through thin copper wire of its coil, large quantity of heat is produced according to I^2Rt and hence it is likely to be burnt.

In order to remove the above mentioned difficulties a resistance of proper small value is joined in parallel to the coil of galvanometer. This resistance is called a **Shunt**. As the value of shunt is very much smaller than the resistance of galvanometer (G), most of the current passes through the shunt and the galvanometer is protected against the damage.

Moreover the shunt and the resistance of galvanometer being in parallel their equivalent resistance becomes even smaller than the value of shunt. Thus after joining the shunt the resistance of the current meter becomes very small. Hence both of the above mentioned difficulties are removed.

Known currents are passed through the instrument prepared after joining the shunt and its scale is calibrated in ampere, milliampere or microampere.

The instrument thus prepared is called ammeter, milliammetre or microammeter respectively. For this purpose the proper value of shunt is obtained as follows :

Formula for shunt : Suppose a galvanometer having resistance G and current capacity I_G is to be converted into an ammeter which can measure a maximum current I. For this the value of required shunt is suppose S. Here the shunt should be so chosen that out of current I, only I_G current passes through the galvanometer and the remaining $I_S = I - I_G$ current passes through the shunt. This situation is shown in the figure 4.22.

Using Krichoff's first Law, at junction A,

$$I = I_G + I_S \quad (4.10.7)$$

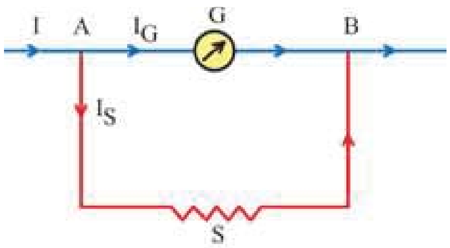


Figure 4.22

Using Kirchoff's second Law on ASBGA path,

$$-I_G G + I_S S = 0$$

$$\therefore S = \frac{GI_G}{I_S}$$

From equation 4.10.7, $I_S = I - I_G$

$$\therefore S = \frac{GI_G}{I-I_G} \quad (4.10.8)$$

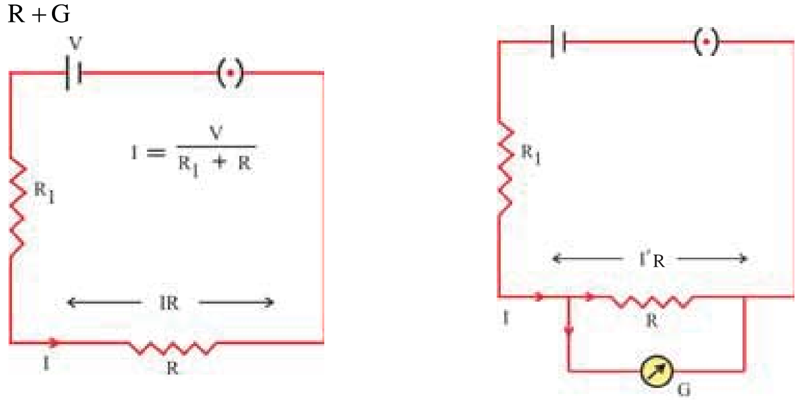
This is the formula for the required shunt. It is clear from this that in order to make the range of ammeter higher and higher the value of the required shunt is smaller and smaller.

To make the range of ammeter n times, the required shunt will be $S = \frac{G}{n-1}$, which you may varify for yourself.

4.10.1 (b) Voltmeter : The instrument to measure the potential difference (also called valtage) between the two ends of component in a circuit, is called voltmeter. For this purpose the voltmeter is joined in parallel to that component.

Suppose the voltage across the two ends of the resistance R shown in the figure 4.23(a) is to be measured. For this if a galvanometer with resistance G and current capacity I_G is used, we find the following difficulties. On joining the galvanometer as shown in the figure 4.23(b), the total resistance of circuit becomes

$$R' = R_1 + \frac{RG}{R+G} \quad (4.10.9)$$



(a) **Figure 4.23** (b)

As a result, after joining the galvanometer, the resistance of circuit change and the current passing through R also changes. Thus value of potential difference = IR (which is to be measured), between two ends of the resistance R, also changes.

If the value of G is very high, then in R + G; neglecting R as compared to G,

$$R' = R_1 + \frac{RG}{R+G} \approx R_1 + R \quad (4.10.10)$$

In this condition the resistance of the circuit is not appreciably changed and since value of G is greater, most of the current passes through R and hence the value of IR is almost maintained.

The above discussion shows that the resistance of the instrument measuring the electric potential difference should be as great as possible (in principle infinite). Thus by joining a proper greater resistance in series with the galvanometer, it can be converted into a voltmeter. Here since the resistane is very large, the current passing through the galvanometer is very small and it is not likely to be damaged.

The maximum voltage that can be measured with a galvanometer ($I_G G$) is called its (voltage capacity).

Formula for Series Resistance : Suppose the resistance of a galvanometer is G and its current capacity is I_G. Hence its voltage capacity will be $I_G G$. This galvanometer is to be converted into a voltmeter which can measure a maximum potential difference of V volt. For this the required series resistance is suppose R_S. In figure 4.24 if the potential difference between A and B is V, then by joining the galvanometer and R_S between these points, the galvanometer shows full scale deflection that is the current passing through it will be I_G. From the Figure,

$$I_G G + I_G R_S = V$$

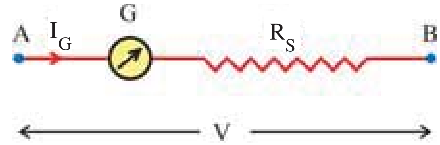


Figure 4.24

$$\therefore \ R_S = \frac{V}{I_G} - G \qquad (4.10.11)$$

By joining a resistance given by the above formula in series with the given galvanometer, and then by properly calibrating the scale of galvanometer, the voltmeter is prepared. From equation 4.10.11, it is clear that in order to make the range of voltmeter greater and greater the larger and larger value of series resistance (R_S) should be taken.

In order to make the voltage capacity of voltmeter, *n* times, the required series resistance will be $R_S = (n - 1)G$; which you may varify.

By dividing both the sides of equation 4.10.6 by the resistance of voltmeter R.

$$\frac{\phi}{IR} = \frac{NAB}{k}\frac{1}{R}$$

$$\therefore \ \frac{\phi}{V} = \frac{NAB}{kR} \qquad (4.10.12)$$

Here, $\frac{\phi}{V}$ is called the voltage sensitivity (S_V) of voltmeter.

Illustration 12 : There are 21 marks (zero to 20) on the dial of a galvanometer, that is there are 20 divisious. On passing 10 μA current through it, it shows a deflection of 1 division. Its resistance is 20 Ω (a). How can it be converted into an ammeter which can measure 1 A current ? (b) How can the original galvanometer be converted into a voltmeter which can measure a potential difference of 1 V ? Also find the effective resistance of both of the above mentioned meters.

Solution : (a) When a current of 10 μA passes through the galvanometer, its pointer shows a deflection of 1 division. There are 20 divisions in this galvanometer.

∴ The maximum current which can be measured by it (current capacity) $I_G = 10 \times 10^{-6} \times 20 = 200 \times 10^{-6}$A.

For ammeter, the required shunt to be joined in parallel to galvanometer is

$$S = \frac{GI_G}{I - I_G} \qquad I_G = 200 \times 10^{-6} A = 2 \times 10^{-4} A$$

$$= \frac{20 \times 200 \times 10^{-6}}{(10000 \times 10^{-4}) - (2 \times 10^{-4})} \qquad G = 20\ \Omega$$

$$= \frac{20 \times 2 \times 10^{-4}}{(10000 \times 10^{-4}) - (2 \times 10^{-4})} \qquad I = 1\ A = 10000 \times 10^{-4} A$$

$$= \frac{40}{9998} \approx 0.004\ \Omega$$

Thus to convert this galvanometer into an ammeter which can measure 1 A current, a shunt of 0.004 Ω should be joined.

The effective resistance of this ammeter will be $G' = \frac{GS}{G+S} = \frac{20 \times 0.004}{20 + 0.004} \approx 0.004\ \Omega$.

(b) For Voltmeter : In order to convert the galvanometer into a voltmeter, the required series resistance is

$$R_S = \frac{V}{I_G} - G \qquad \text{Here, } V = 1 \text{ volt}$$

$$I^G = 2 \times 10^{-4} A$$

$$G = 20\ \Omega$$

$$= \frac{1}{2 \times 10^{-4}} - 20$$

$$= 0.5 \times 10^4 - 20$$

$$= 5000 - 20$$

$$= 4980\ \Omega$$

In order to convert this galvanometer into a voltmeter which can measure 1 volt, a series resistance of 4920 should be joined with it.

The effective resistance of this voltmeter will be $R'_S = R_S + G = 4980 + 20 = 5000\ \Omega$. ($\therefore$ R_S and G are in series)

SUMMARY

1. **Oersted's Observation :** "When electric current is passed through a conducting wire kept parallel to and below the magnetic needle, the magnetic needle is deflected."

2. **Biot-Savart's Law :** The magnetic field due to a current element $I\vec{dl}$ at a point with position vector $\vec{r}$ with respect to it, is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \hat{r}}{r^2}$$

Since such elements are continuously distributed in the entire conducting wire, the magnetic field due to such a wire can be written in the form of a line integral as

$$\vec{B} = \int \vec{dB} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$\text{or } \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \vec{r}}{r^3}$$

Here, the line integral is on the entire circuit made up with the conducting wire.

3. The magnetic field due to a circular coil (ring) of N turns, radius a and carrying current I at a point on its axis at a distance x from its centre is

$$B(x) = \frac{\mu_0 N I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

For magnetic field at the centre of the coil (ring),

taking $x = 0$, B(centre) $= \frac{\mu_0 N I}{2a}$

For a point very much away from the centre,
taking $x >> a$;

$$B(x) = \frac{\mu_0 N I a^2}{2x^3}$$

4. **Ampere's Circuital Law :** "The line integral of magnetic field on a closed curve (loop) in a magnetic field, is equal to the product of the algebraic sum of the electric currents enclosed by that closed curve and the permeability of vacuum."
In the form of an equation this Law can be written as under :

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I.$$

5. If current I is passed through a very long straight wire, the magnetic field at a point at normal distance r from the wire is,

$$B = \frac{\mu_0 I}{2\pi r}$$

6. The magnetic field at a point on the axis of a very long solenoid carrying current is B $= \mu_0 n I$
Where n = number of turns per unit length of solenoid.

7. The force on a conducting wire of length l and carrying current I placed in a magnetic field $\vec{B}$, is $\vec{F} = I \vec{l} \times \vec{B}$
The direction of this force can be found by the right hand screw rule for the vector product.

8. The force between two very long parallel current carrying conductors is $F = \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{y}$,
Where y = perpendicular distance between two wires. If the currents in the wires are in mutually opposite directions, the force is repulsive and if the currents are in the same direction, the force is attractive.

9. The magnetic force on a charge q, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is $\vec{F}_m = q(\vec{v} \times \vec{B})$

The force on the charge q in an electric field $\vec{E}$ is $\vec{F}_e = q\vec{E}$

The force on the charge in the region where both the fields are present simultaneously, is $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$, which is called the Lorentz force.

10. Cyclotron is the instrument to accelerate the charged particles. The radius of the circular path of the charged particle moving in it, is

$r = \frac{mv}{Bq}$ which is dependent on its momentum.

The angular frequency w of this particle is called the cyclotron frequency (w_e)

$w_C = \frac{qB}{m}$ or $f_C = \frac{qB}{2\pi m}$ ... $(\because\ w_C = 2\pi f_C)$

11. The torque acting on a current carrying coil suspended in a uniform magnetic field is $\vec{\tau} = NI\vec{A} \times \vec{B}$

$\vec{\mu} = NI\vec{A}$ is called the magnetic moment of the coil.

$\therefore\ \vec{\tau} = \vec{\mu} \times \vec{B}$

12. For measuring very small electric currents galvanometer is used. In a moving and pivoted coil galvanometer, $\tau = NIAB$. Due to this the coil is diflected and springs attached with it are twisted. Hence restoring torque is produced. The restoring torque is $\tau = k\phi$.
In equilibrium condition.
$k\phi = NIBA$

$\therefore\ I = \frac{k}{NBA}\phi \qquad \therefore\ I \propto \phi$

13. The small resistance joined in parallel to a galvanometer to convert it into an ammeter is called a shunt. Its formula is $S = \frac{G I_G}{I - I_G}$.

To convert a galvanometer into a voltmeter a resistance of a high value is joined in series with it. The formula to find this series resistance Rs is $R_S = \frac{V}{I_G} - G$.

EXERCISE

For the following statements choose the correct option from the given options :

1. Two concentric rings are kept in the same plane. Number of turns in both the rings is 20. Their radii are 40 cm and 80 cm and they carry electric currents of 0.4 A and 0.6 A respectively, in mutually opposite directions. The magnitude of the magnetic field produced at their centre is ...... T.

(A) $4\mu_0$ (B) $2\mu_0$ (C) $\frac{10}{4}\mu_0$ (D) $\frac{5}{4}\mu_0$

2. A particle of mass m has an electric charge q. This particle is accelerated through a potential difference V and then entered normally in a uniform magnetic field B. It performs a circular motion of radius R. The ratio of its charge to the mass $\left(\frac{q}{m}\right)$ is = ... [$\left(\frac{q}{m}\right)$ is also called specific charge.]

(A) $\frac{2V}{B^2R^2}$ (B) $\frac{V}{2BR}$ (C) $\frac{VB}{2R}$ (D) $\frac{mV}{BR}$

3. A proton, a deuteron ion and an α-particle of equal kinetic energy perform circular motion normal to a uniform magnetic field B. If the radii of their paths are r_p, r_d and r_α respectively then..... [Here, $q_d = q_p$, $m_d = 2m_p$]

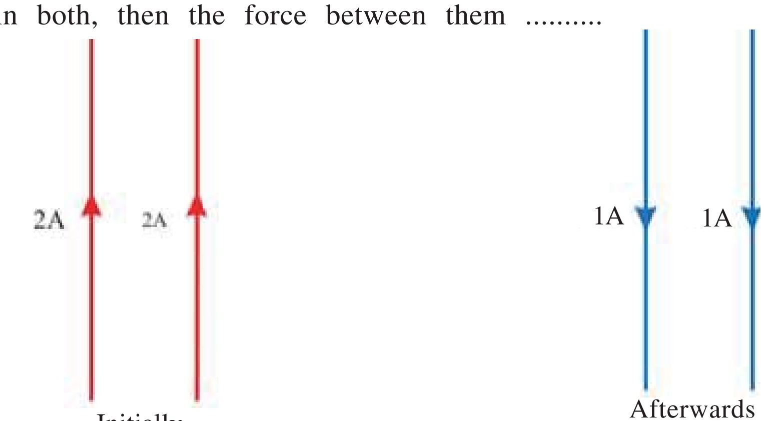
(A) $r_\alpha = r_p < r_d$ (B) $r_\alpha = r_d > r_p$

(C) $r_\alpha > r_d > r_p$ (D) $r_\alpha = r_d = r_p$

4. An electron performs circular motion of radius r, perpendicular to a uniform magnetic field B. The kinetic energy gained by this electron in half the revolution is ....

(A) $\frac{1}{2}mv^2$ (B) $\frac{1}{4}mv^2$ (C) zero (D) $\pi rBev$

5. As shown in the figure two very long straight wires are kept parallel to each other and 2A current is passed through them in the same direction. In this condition the force between them is F. Now if the current in both of them is made 1 A and directions are reversed in both, then the force between them ....

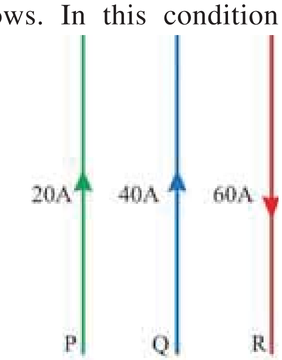


(a) (b)

(A) will be $\frac{F}{4}$ and attractive (B) will be $\frac{F}{2}$ and repulsive

(C) will be $\frac{F}{2}$ and attractive (D) will be $\frac{F}{4}$ and repulsive.

6. As shown in the figure 20A, 40A and 60A currents are passing through very long straight wires P, Q and R respectively in the directions shown by the arrows. In this condition the direction of the resultant force on wire Q is



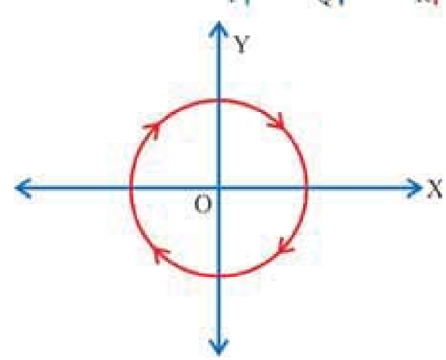
(A) towards left of wire Q

(B) towards right of wire Q

(C) normal to the plane of paper

(D) in the direction of current passing through Q.

7. As shown in the figure a circular conducting wire carries current I. It lies in XY-plane with centre at O. The tendency of this circular loop is to



(A) contract

(B) expand

(C) move towards positive X-direction

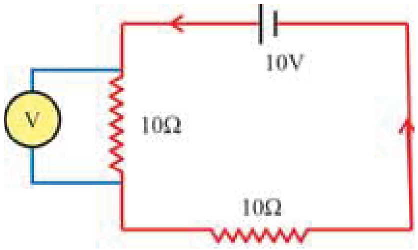
(D) move towards negative X-direction.

8. At a place an electric field and a magnetic field are in the downward direction. There an electron moves in the downward direction. Hence this electron ....
(A) will bend towards left (B) will bend towards right
(C) will gain velocity (D) will lose velocity.

9. Two parallel long thin wires, each carrying current I are kept at a separation r from each other. Hence the magnitude of force per unit length of one wire due to the other wire is ....

(A) $\frac{\mu_0 I^2}{r^2}$ (B) $\frac{\mu_0 I^2}{2\pi r}$ (C) $\frac{\mu_0 I}{2\pi r}$ (D) $\frac{\mu_0 I}{2\pi r^2}$

10. A voltmeter of a very high resistance is joined in the circuit as shown in the figure. The voltage shown by this voltmeter will be ....



(A) 5 V (B) 10 V
(C) 2.5 V (D) 7.5 V

11. A particle of charge q and mass m moves on a circular path of radius r in a plane inside and normal to a uniform magnetic field B. The time taken by this particle to complete one revolution is ....

(A) $\frac{2\pi mq}{B}$ (B) $\frac{2\pi q^2 B}{m}$ (C) $\frac{2\pi qB}{m}$ (D) $\frac{2\pi m}{Bq}$

12. A long wire carries a steady current. When it is bent in a circular form, the magnetic field at its centre is B. Now if this wire is bent in a circular loop of n turns, what is the magnetic field at its centre ?
(A) nB (B) n^2B (C) $2n$B (D) $2n^2$B

13. A conducting wire of 1 m length is used to form a circular loop. If it carries a current of 1 ampere, its magnetic moment will be .... Am^2.

(A) 2π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4\pi}$

14. When a charged particle moves in a magnetic field its kinetic energy ....
(A) remains constant (B) can increase
(C) can decrease (D) can increase or decrease

15. At each of the two ends of a rod of length $2r$, a particle of mass m and charge q is attached. If this rod is rotated about its centre with angular speed ω, the ratio of its magnetic dipole moment to the total angular momentum of this particle is ....

(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$ (C) $\frac{2q}{m}$ (D) $\frac{q}{\pi m}$

16. There are 100 turns per cm length in a very long solenoid. It carries a current of 5 A. The magnetic field at its centre on the axis is .... T.
(A) 3.14×10^{-2} (B) 6.28×10^{-2} (C) 9.42×10^{-2} (D) 12.56×10^{-2}

17. Two very long conducting parallel wires are separated by a distance d from each other and equal currents are passed through them in mutually opposite directions. A particle of charge q passes through a point, at a distance $\frac{d}{2}$ from both wires, with velocity v perpendicularly to the plane formed by the wires. The resultant magnetic force acting on this particle is ....

(A) $\frac{\mu_0 I q v}{2\pi d}$ (B) $\frac{\mu_0 I q v}{\pi d}$ (C) $\frac{2\mu_0 I q v}{\pi d}$ (D) zero

18. A very long solenoid of length L has n layers. There are N turns in each layer. Diameter of the solenoid is D and it carries current I. The magnetic field at the centre of the solenoid is ....

(A) directly propotional to D (B) inversely proportional to D.
(C) independent of D (D) directly proportional to L.

19. The angular speed of the charged particle is independent of ....

(A) its mass (B) its linear speed
(C) charge of particle (D) magnetic field.

20. A charged particle gains energy due to ....

(A) electric field (B) magnetic field
(C) both these fields (D) none of these fields.

21. A charged particle is moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The magnetic force acting on it will be maximum when ....

(A) $\vec{v}$ and $\vec{B}$ are in same direction

(B) $\vec{v}$ and $\vec{B}$ are in opposite direction

(C) $\vec{v}$ and $\vec{B}$ are mutually perpendicular

(D) $\vec{v}$ and $\vec{B}$ moke an angle of 45^{o} with each other

22. Equal currents are passing through two very long and straight parallel wires in mutually opposite directions. They will ....

(A) attract each other (B) repel each other
(C) lean towards each other (D) neither attract nor repel each other.

23. A charged particle is moving in a uniform magnetic field. Then ....

(A) its momentum changes but kinetic energy does not change
(B) its momentum and kinetic energy both change
(C) neither the momentum nor kinetic energy changes.
(D) Kinetic energy charges but the momentum does not change.

24. If the speed of a changed particle moving through a magnetic field is increased, then the radius of curvature of its trajectory will ....

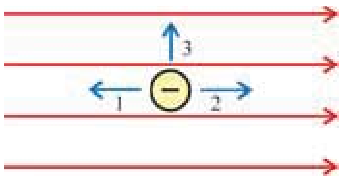
(A) decrease (B) increase (C) not change (D) become half

ANSWERS

1. (C)	2. (A)	3. (A)	4. (C)	5. (A)	6. (A)
7. (B)	8. (D)	9. (B)	10. (A)	11. (D)	12. (B)
13. (D)	14. (A)	15. (A)	16. (B)	17. (D)	18. (C)
19. (B)	20. (A)	21. (C)	22. (A)	23. (A)	24. (B)

Answer the following questions in brief :

1. State the observation made by Oersted.
2. Write the statement of Biot–Savart's Law.
3. Give the formula showing Ampere's Circuital Law.
4. State the Law giving the direction of magnetic field due to a straight conductor carrying current.
5. What is the magnitude of the magnetic field in the region near the outside of the solenoid.
6. State the direction of magnetic field due to current in a toroid.
7. State Ampere's observation after the observation made by Oersted.
8. Does the angular frequency of particle depend on its momentum in cyclotron ? Yes or No ?
9. Can a neutron be accelerated using cyclotron ? Why ?
10. State the functions of electric field and magnetic field in a cyclotron.
11. State two limitations of cyclotorn.
12. What should be the resistances of an ideal ammeter and an ideal voltmeter ?
13. What is meant by current sensitivity of a galvanometer ?
14. What should be done to increase the voltage capacity of a voltmeter.
15. If the radius of the ring and the current through it both are doubled, what change would occur in the magnetic field at its centre ?
16. Give the magnitude of the magnetic force on the electron for the three cases of its motion shown in the Figure.



Answer the following questions :

1. Write Biot–Savart's Law and explain it.
2. Write the formula for the magnetic field at a point on the axis of a current carrying circular ring and explain with a suitable diagram the right hand rule to find the direction of this magnetic field.
3. State and explain Ampere's Circuital Law.
4. Using Ampere's Circuital Law, obtain the magnitude of magnitic field at a perpendicular disance r due to very long staright conductor carrying current I.
5. Using Ampere's circuital Law obtain the formula for the magnitude of magnetic field due to current in a toroid.
6. Obtain the formula for the force of attraction between two parallal wires carrying currents in the same direction.
7. Obtain the formula for the Lorentz force on a moving electric charge
8. Explain the working of cyclotron and obtain the formula for the cyclotron frequency w_C.
9. With a suitable diagram explain the construction of galvanometer.
10. What should be d one to convert a galvanometer into an ammeter. Obtain the formula for the shunt.
11. Derive an expression for the magnetic field at a point on the axis of a current carrying circular ring.
12. Obtain the formula for the magnetic field produced inside a very long current carrying solenoid usiing Ampere's Circuital Law.
13. Obtain the formula for the torque acting on a rectangular coil carrying current, suspended in a uniform magnetic field.

Solve the following examples :

1. Distance between two very long parallel wires is 0.2 m. Electric currents of 4 A in one wire and 6A in the other wire are passing in the same direction. Find the position of a point on the perpendicular line joining the two wires at which the magnetic field intensity is zero.

   [**Ans :** 80 mm away from the wire with 4A current and between the two wires]

2. A very long wire is held vertical in a direction perpendicular to the horizontal component of Earth's magnetic field. Find the value of current to be passed through this wire so that the resultant magnetic field at a point 10 cm away from this wire becomes zero. What will be the magnetic induction at a point 10 cm away from the wire on the opposite side of this point ? Horizontal component of Earth's magnetic field H = 0.36×10^{-4}T, $\mu_0 = 4\pi \times 10^{-7}$T *m*/A. [**Ans. :** 18 A, 0.72×10^{-4} T]

3. When a galvanometer with a shunt is joined in an electrical circuit 2% of the total current passes through the galvanometer. Resistance of galvanometer is G. Find the value of shunt. [**Ans. :** $\frac{G}{49}$]

4. Two particles of masses M_1 and M_2 and having the equal electric charge are accelerated through equal potential difference and then move inside a uniform magnetic field, normal to it. If the radii of their circular paths are R_1 and R_2 respectively find the ratio of their masses. [**Ans :** $\frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^2$]

5. A circular coil having N turns is made from a wire L meter long. If a current of I ampere is passed through this coil suspended in a uniform magnetic field of B tesla, find the maximum torque that can act on this coil. [**Ans. :** $\frac{IL^2B}{4\pi N}$ N m]

6. A proton and a deuteronion having the same kinetic energies enter a region of uniform magnetic field perpendicularly. Deuteron's mass is twice that of proton. Calculate the ratio of the radii of their circular paths. [**Ans. :** $\frac{r_d}{r_p} = \sqrt{2}$]

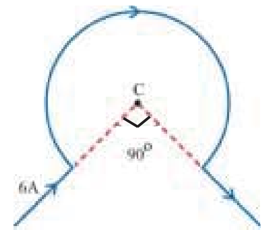
7. A rectangular coil of 120 turns and an area of 10×10^{-4} m^2 is suspended in a radial magnetic field of 45×10^{-4} T. If a current of 0.2 mA through the coil gives it a deflection of 36^o find the effective torsional constant for the spring system holding the coil. [**Ans. :** 17.2×10^{-8} N m/rad]

8. Two rings X and Y are placed in such a way that their axes are along the X and the Y axes respectively and their centres are at the origin. Both the rings X and Y have the same radii of 3.14 cm. If the current through X and Y rings are 0.6 A and 0.8 A respecively, find the value of the resultant magnetic field at the origin. $\mu_0 = 4\pi \times 10^{-7}$SI. [**Ans. :** 2×10^{-5}T]

9. Two parallel very long straight wires carrying currents of 20 A and 30 A respectively are at a separation of 3 m between them. If the currents are in the same direction, find the attractive force between them per unit length. [**Ans. :** 4×10^{-5}N m^{-1}]

10. A very long straight wire carries a current of 5 A. An electron moves with a velocity of 10^6 m s^{-1} remaining parallel to the wire at a distance of 10 cm from wire in a direction opposite to that of electric current. Find the force on this electron. (Here the mass of electron is taken as constant) $e = -1.6 \times 10^{-19}$C, $\mu_0 = 4\pi \times 10^{-7}$SI. [**Ans. :** 16×10^{-19}N]

11. A current of 6 A passes through the wire shown in the Figure. Find the magnitude of magnetic field at point C. The radius is 0.02 m $\mu_0 = 4\pi \times 10^{-7}$T m A^{-1}. [**Ans. :** 1.41×10^{-4}T]



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5 MAGNETISM AND MATTER

5.1 Introduction

The word magnet is derived from the name of an island in Greece called Magnesia, where magnetic ore deposits were found as early as 800 BC. Shepherds on this island complained that the nails of their shoes were getting stuck to the ground. The tip of their staff were also getting stuck to chunks of magnetite while they pastured their flocks. Greeks observed that the stone of magnetite (Fe_3O_4) attracts the pieces of iron.

The chinese were the first to use magnetic needles for navigation on ships. Caravans used the magnetic needles to navigate across the Gobi desert. Magnetism is much older than the genesis of life and the subsequent evolution of human beings on earth. It exists everywhere in the entire universe. The earth's magnetism predates human evolution.

In 1269 a Frenchman named Pierre-de Maricourt mapped out the directions of magnetic lines on the surface of a spherical natural magnet by using magnetic needle. He observed that the directions of magnetic lines formed on the sphere were passing through two points diametrically opposite to each other, which he called the poles of the magnet. Afterwards other experiments also showed that every magnet, regardless of its shape and size, has two poles called north and south poles. Some commonly known facts regarding magnetism are as follows :

(1) The Earth behaves as a magnet with the magnetic field pointing approximately from geographic south to north direction.

(2) When a bar magnet is suspended from its mid–point such that it can rotate freely in horizontal plane, then it continues to rotate (oscillate) until it aligns in the north–south direction. The end of the magnet pointing towards the north is called the magnetic **North pole** of the magnet, and the end pointing towards the south pole is called the magnetic **South pole** of the magnet.

(3) Like magnetic poles repel each other, and the unlike poles attract each other.

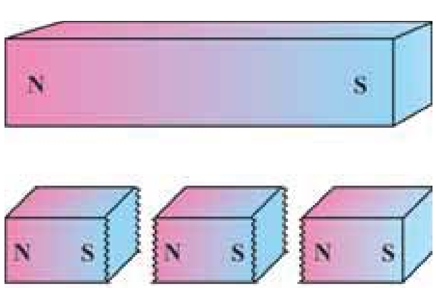


Figure 5.1 Magnet and its Fragments Behaving as Independent Magnets

(4) The positive and negative charges in **electric dipole** may be separated and can exist independently, called **electric monopoles.** The magnet with two poles may be regarded as a **magnetic dipole.** But the magnetic poles are always found in pairs. The north and south magnetic poles cannot be separated by splitting the magnet into two parts. Even if the bar magnet is broken into two or more parts, then also each fragment of the magnet behaves as an independent magnet with north and south magnetic poles with somewhat weaker magnetic field (See figure 5.1). Thus an independent magnetic monopole does not exist. The search for magnetic monopoles is going on.

(5) Magnets can be prepared from iron and its alloys.

In this chapter you will learn the equivalence between magnetic field of a bar magnet and a solenoid, the magnetic dipole moment of a current carrying loop and the dipole moment of orbiting electron in an atom.

The magnetic field strength produced by a magnetic dipole at a point on its equator and at a point along its axis is calculated. The magnetic field of the earth, geomagnetic elements, as well as, para, dia and ferro–magnetic materials are also discussed with suitable examples in this chapter. At the end of this chapter, the applications of permanent magnets and electromagnets are explained in brief.

5.2 The Bar Magnet

The great scientist Albert Einstein got a magnet as a gift when he was a child. He was much fascinated by it and used to play with it. When the magnet attracted iron nails, pins etc., he wondered how the magnet could attract the things without touching them.

Figure 5.2 shows the arrangement of iron filings sprinkled on a plane paper, which is kept on a bar magnet. When the paper is tapped twice or thrice, the iron filings rearrange in a systematic pattern representing the magnetic field lines. Similar picture of magnetic field lines can be formed if the bar magnet is replaced by a short solenoid, through which a DC current passes.

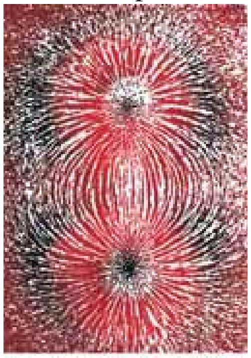
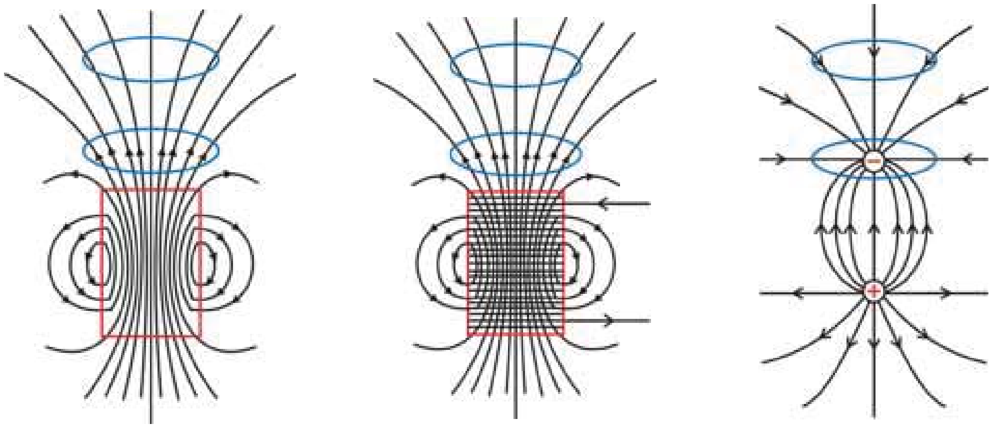


Figure 5.2 Systematic Arrangement of Iron Filings Representing Magnetic Field Lines of a Bar Magnet



(a) Bar Magnet **(b) Solenoid** **(c) Electric-Dipole**

Figure 5.3 Magnetic and Electric Field Lines (Only for Information)

Figure 5.3 shows the magnetic field lines due to a bar magnet and a short solenoid. Electric field lines due to an electric dipole are also shown for comparison.

Following conclusions can be made from the study of figure 5.3 :

(1) **The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. The magnetic field lines emerge out from the magnetic north pole, reach the magnetic south pole and then passing through the magnet, reach the north pole to complete the loop.** In the electric dipole, these field lines begin from a positive charge and end on the negative charge or escape to infinity.

It is impossible to have a static arrangement of electric charges, whose electric field lines form closed loops. This is a typical property of the static electric field.

(2) The tangent to a magnetic field line at a point through which it passes, indicates the direction of magnetic field $\vec{B}$ at that point.

For example, a compass needle may be used to trace out the magnetic field lines of a bar magnet by putting it at different positions surrounding the bar magnet.

(3) The magnitude of magnetic field in the region surrounding a magnet can be represented by the number of magnetic field lines passing normally through a unit area in that region. In figures 5.3 (a) and 5.3 (b) the magnitude of magnetic field B is larger around region (i) than in region (ii).

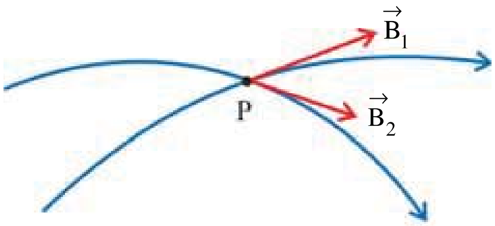


Figure 5.3 (d)

(4) **The magnetic field lines do not intersect with each other.** If they intersect at a point, then the tangents to the lines at the point of intersection would represent two different directions of the magnetic field at that point, which is impossible. (See figure 5.3(d))

If the magnetic field lines intersect at point P, the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ point in different directions.

5.3 Current Loop as a Magnet and its Magnetic Moment

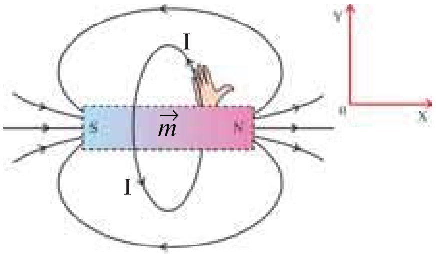


Figure 5.4 Magnetic Field Produced by a Current Loop Like that of a Bar Magnet of Magnetic Dipole Moment $\vec{m}$

In Chapter-4 you studied that, a loop of area A and carrying current I behaves as a magnet, with magnetic depole moment

$$m = IA \qquad (5.3.1)$$

The direction of magnetic moment $\vec{m}$ of the loop can be found using right hand rule as shown in Figure (5.4)

$$\text{Thus, } \vec{m} = I\vec{A} \qquad (5.3.2)$$

If there are N turns in the loop, then

$$\vec{m} = NI\vec{A} \qquad (5.3.3)$$

For the points on the axis of the loop of radius a, far from its centre ($x >> a$), the magnetic field (Chapter-4) is given by

$$B(x) = \frac{\mu_0 I a^2}{2x^3} \qquad (5.3.4)$$

$$= \frac{\mu_0}{2\pi}\frac{I\pi a^2}{x^3} = \frac{\mu_0}{2\pi}\frac{IA}{x^3} \qquad (A = \pi a^2 = \text{area of the loop})$$

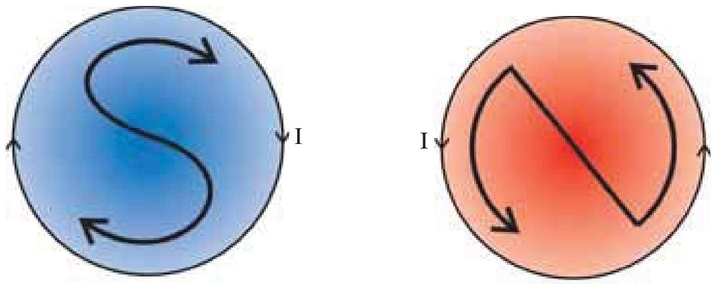
$$\therefore \ B(x) = \frac{\mu_0}{2\pi}\frac{m}{x^3} \qquad (5.3.5)$$

Since B(x) and m have same direction,

$$\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3} \qquad (5.3.6)$$

which is the axial magnetic field in terms of magnetic dipole moment $\vec{m}$ of the loop at $x >> a$. Equation (5.3.6) is equally applicable for a (short) bar magnet of magnetic dipole moment $\vec{m}$.

5.3.1 Direction of Magnetic Pole in a Current Carrying Loop :



(a) Magnetic South Pole (b) Magnetic North Pole

Figure 5.5

Figure 5.5(a) shows the current I flowing in clockwise direction in a circular loop lying in the plane of the page. According to right hand rule, the side of the loop towards us behaves as a magnetic south pole whereas the opposite side of the loop behave as a magnetic north pole. The symbolic notation S indicates magnetic south pole pointing towards us.

Simliarly, if the current flows in anticlockwise direction in the loop, the side of the loop towards us behaves as a magnetic north pole and opposite side as a magnetic south pole (See Figure 5.5(b)). The symbolic notation N indicates the magnetic north pole pointing outwards.

5.4 Magnetic moment of an electron rotating around the nucleus of an atom :

Dear students, now you know that a magnetic field is produced by the motion of charged particles or by an electric current. Any material is made up of atoms, and in these atoms definite number of electrons (depending on the nature of the element), move in various possible orbits. Such motion of electrons in orbits can be considered as an electric current around a closed path, with magnetic moment IA (I = electric current, and A = area enclosed by the orbit). The magnetic dipole moment of an atom of any given element, depends upon the distribution of electrons in various orbits and on their spins.

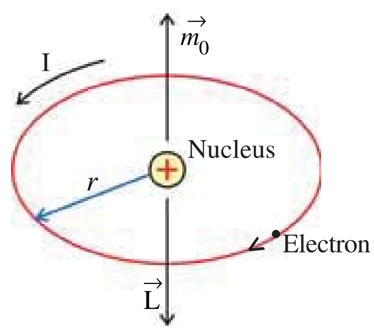


Figure 5.6 Non-zero Magnetic Moment of Atom

As shown in figure 5.6, consider an electron moving with constant speed v in a circular orbit of radius r about the nucleus. If the electron travels a distance $2\pi r$ (circumference of the circle) in time T, then its orbital speed is $v = \frac{2\pi r}{T}$. Thus the current I associated with this orbiting electron of charge e is, $I = \frac{e}{T}$.

Here, $T = \frac{2\pi}{\omega}$, and $\omega = \frac{v}{r}$

$$\therefore\ I = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

The orbital magnetic moment associated with this orbital current loop is

$$m_0 = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{1}{2} evr \quad (5.4.1)$$

where $A = \pi r^2$ = area enclosed by the circular orbit.

For this electron, the orbital angular momentum is $L = m_e vr$. Hence, the orbital magnetic moment of the electron can be represented as

$$m_0 = \left(\frac{e}{2m_e}\right)(m_e vr) = \left(\frac{e}{2m_e}\right)L \quad (5.4.2)$$

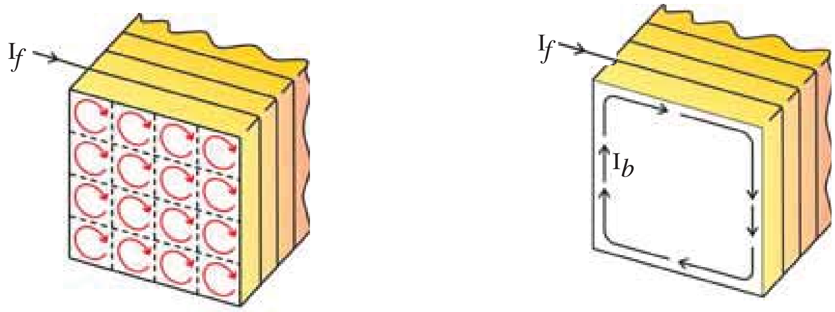
Equation (5.4.2) shows that the magnetic moment of the electron is proportional to its orbital angular momentum L. But since the charge of electron is negative, the vectors $\vec{m_0}$ and $\vec{L}$ point in opposite directions, perpendicular to the plane of the orbit.

$$\therefore \quad \vec{m_0} = -\left(\frac{e}{2m_e}\right)\vec{L} \qquad (5.4.3)$$

The ratio $\frac{e}{2m_e}$ is a constant called the gyro–magnetic ratio, and its value is 8.8×10^{10} C kg^{-1}.

5.5 Magnetism in Matter

In general, the magnets are prepared from iron (Fe). The atoms of iron normally possess magnetic dipole moment, but an ordinary piece of iron does not behave as a magnet .



(a) Atomic Current Loops Due to Current I_f (Magnetized Iron)

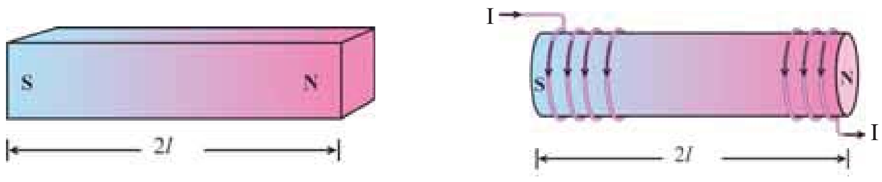
(b) Overall Bound Current Loop Equivalent to Loops in Part (a)

Figure 5.7

The same iron piece can be converted into a magnet, if it is kept in a strong magnetic field for some time and then the applied magnetic field is removed. As shown in figure 5.7 a wire is wound on a piece of iron. If $I_f = 0$, then the magnetic dipole moments of current loops of atoms are randomly oriented. Thus the resultant magnetic moment of the iron piece becomes zero and the iron piece does not behave as a magnet.

When sufficient current I_f passes through the wire, a strong magnetic field is generated in the iron piece, due to which the elemental atomic currents redistribute in the iron piece. Thus a resultant bound current I_b is generated in the iron piece (See Figure 5.7(b)). When the current I_f is slowly reduced to zero, all of the elemental atomic currents do not return to original state even though the external magnetic field becomes zero. This way the iron piece sustains magnetic field.

5.6 Equivalence between a Bar Magnet and a Solenoid



(a) (b)

Figure 5.8 A Bar Magnet and a Solenoid

Figure 5.8 shows a bar magnet and a solenoid. If the pole strength of bar magnet is p_b (even though such individual poles do not exist), and the distance between two poles is $2l$ then according to definition, the magnetic dipole moment of bar magent is

$$m_b = 2lp_b \quad (5.6.1)$$

$$\therefore \; p_b = \frac{m_b}{2l} \quad (5.6.2)$$

The suffix b here indicates that the magnetic moment is due to bar magnet.

Note : Only for infomation : The poles (p_b) of the bar magnet are not on the end faces of the bar magnet, but are situated inside, in such a way that the distance between the two poles (magnetic length) is $2l_m$, which is slightly less than the geometric length $2l$ of the bar magnet. For practical purposes the magnetic length $2l_m = \frac{5}{6} \times 2l$, is taken as geometric length $2l$, in this book.

In a solenoid of cross sectional area A, carrying current I, each turn can be treated as a closed current loop, and hence a magnetic dipole moment IA can be associated with each turn. As the magnetic dipole moment of every turn is in the same direction, the magnetic dipole moment of the solenoid is a vector sum of dipole moments of all turns. If there are total N turns in length $2l$ of the solenoid, then its magnetic moment is

$$m_s = \text{NIA} \quad (5.6.3)$$

From equations (5.6.1) and (5.6.3), we can define equivalent pole strength of solenoid as

$$p_s = \frac{m_s}{2l} = \frac{\text{NIA}}{2l} = n\text{IA} \quad (5.6.4)$$

where $n = \frac{\text{N}}{2l}$ = number of turns per unit length of solenoid.

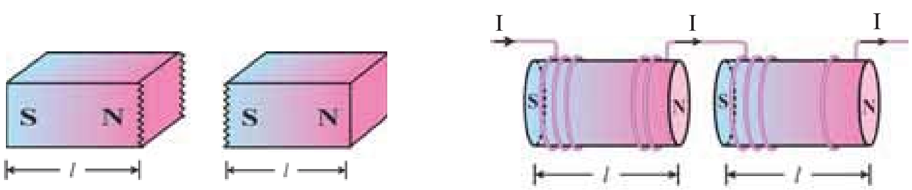
From equation (5.6.4), the unit of pole strength is A m.

As mentioned in the article (5.3) the magnetic field along the axis of dipole moment $\vec{m}$ is

$$\vec{\text{B}}(x) = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{x^3} \quad (5.6.5)$$

Hence, the magnetic field produced by a bar magnet or a solenoid can be calculated by replacing $\vec{m}$ by $\vec{m}_b$ or $\vec{m}_s$, respectively, in equation (5.6.5).

What happens if bar magnet is broken ?



(a) (b)

Figure 5.9 Broken Bar Magnet and a Solenoid

If the solenoid of figure 5.8 is broken into two equal pieces as shown in figure 5.9.(b), then the pole strength of each piece of solenoid remains same as nIA, since the number of turns per unit length (n) remains same. By analogy we can say that the pole strength of each piece of bar magnet also remains same.

In both cases, the magnetic length becomes half of the original length. Hence the magnetic dipole moment also becomes half.

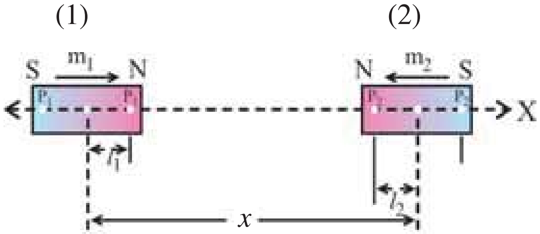
5.6.1 The Electrostatic Analogue : Comparing equatians (5.6.1) and (5.6.5) with corresponding equations for electric charge (chapter 1), it can be observed that the magnetic field at large distances due to a bar magnet or current loop of magnetic moment $\vec{m}$ can be obtained directly from the equations of electric field due to an electric dipole of dipole moment $p = 2aq$, by making following replacements.

$$\vec{E} \rightarrow \vec{B}, \quad \vec{p} \rightarrow \vec{m}, \quad \frac{1}{4\pi\varepsilon_0} \rightarrow \frac{\mu_0}{4\pi}$$

Table 5.1 Analogy between Electric and Magnetic Dipoles

Quantity	Electrostatics	Magnetics
Constant	$\frac{1}{4\pi\varepsilon_0}$ q (charge)	$\frac{\mu_0}{4\pi}$ p (pole strength)
Dipole moment	$\vec{p} = q(2\vec{a})$	$\vec{m} = p(2\vec{l})$
Equatorial Field	$\vec{E}(y) = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(y^2+a^2)^{\frac{3}{2}}}$	$\vec{B}(y) = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{(y^2+l^2)^{\frac{3}{2}}}$
$y >> a$ $y >> l$	$= -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$	$= -\frac{\mu_0}{4\pi}\frac{\vec{m}}{y^3}$
Axial Field	$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}z}{(z^2-a^2)^2}$	$\vec{B}(z) = \frac{\mu_0}{4\pi}\frac{2\vec{m}z}{(z^2-l^2)^2}$
$z >> a$ $z >> l$	$= \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{z^3}$	$= \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}$
Force	$\vec{F} = q\vec{E}$	$\vec{F} = p\vec{B}$
Torque (in External Field)	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{m} \times \vec{B}$
Energy (in External Field)	$U = -\vec{p}\cdot\vec{E}$	$U = -\vec{m}\cdot\vec{B}$

Illustration 1 : Find the force between two small bar magnets of magnetic moments $\vec{m}_1$ and $\vec{m}_2$ lying on the same axis, as shown in the Figure. (p_1 and p_2 are the pole strength of magnets (1) and (2) respectively)



$x >> l_1$, $x >> l_2$

Solution : To find the force on magnet (2) due to magnet (1), calculate the magnetic field due to magnet (1) at the poles of magnet (2). The axial magnetic field at the north pole of magnet (2) due to magnetic moment m_1 is (from the geometry of Figure)

$$B_N = \frac{\mu_0}{4\pi} \frac{2m_1}{(x-l_2)^3} \quad (1)$$

Similarly, the axial magnetic field at the south pole of magnet (2) is

$$B_S = \frac{\mu_0}{4\pi} \cdot \frac{2m_1}{(x+l_2)^3} \quad (2)$$

The repulsive force F_N acting on the north pole of magnet (2) having pole strength p_2 is (like $F = qE$ in electrostatics)

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi} \frac{2p_2 m_1}{(x-l_2)^3} \quad (3)$$

which is acting away from magnet (1)

Similarly, the attractive force F_S acting on the south pole of magnet (2) is

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi} \frac{2p_2 m_1}{(x+l_2)^3} \quad (4)$$

which is acting towards magnet (1)

Hence the resultant force on magnet (2) is

$$F = F_N - F_S$$

$$= \frac{\mu_0}{4\pi} \cdot 2p_2 m_1 \left[\frac{1}{(x-l_2)^3} - \frac{1}{(x+l_2)^3} \right] = \frac{\mu_0}{2\pi} p_2 m_1 \left[\frac{(x+l_2)^3 - (x-l_2)^3}{\{(x-l_2)(x+l_2)\}^3} \right]$$

$$= \frac{\mu_0}{2\pi} p_2 m_1 \left[\frac{6x^2 l_2}{(x^2 - l_2^2)^3} \right]$$

[Because $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$ and $l_2^3 << x^2 l_2$ in numerator]

$$\therefore F = \frac{\mu_0 m_1}{2\pi} \cdot \frac{2l_2 P_2 \cdot 3x^2}{x^6} \quad \text{(Since } l_2^2 << x^2 \text{, and hence } l_2^2 \text{ can be neglected)}$$

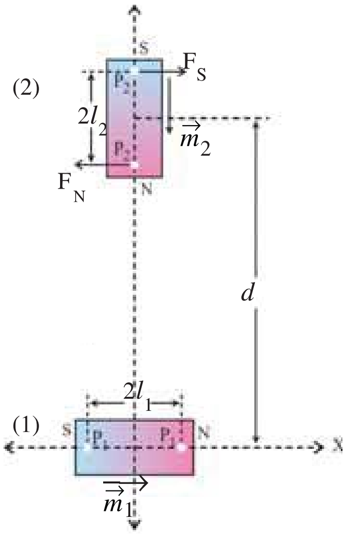
$$\therefore F = \frac{3\mu_0 m_1 m_2}{2\pi x^4} \quad (5)$$

Where $m_2 = 2l_2 p_2$ = magnetic moment of magnet (2)

This resultant force is repulsive for the magnet positions shown in Figure, and acts on magnet (2) in a direction away from magnet (1).

[What will be the resultant force between the two bar magnets, if the direction of one of the magnets is reversed ? Think !]

Illustration 2 : Find the torque on small bar magnet (2) due to small bar magnet (1), when they are placed perpendicular to each other as shown in Figure. ($l_1 << d$, $l_2 << d$)



Solution : From the geometry of Figure, it is seen that both the poles of magnet (2) are lying on the equatorial line of magnet (1).

The magnetic field B_N produced by the small bar magnet (1) at distance $(d - l_2)$ on its equatorial plane is

$$B_N = \frac{\mu_0}{4\pi} \frac{m_1}{(d-l_2)^3} \quad (1)$$

Similarly the magnetic field B_S produced by the magnet (1) at south pole of magnet (2), lying at a distance $(d + l_2)$ on its equatorial plane is

$$B_S = \frac{\mu_0}{4\pi} \frac{m_1}{(d+l_2)^3} \quad (2)$$

Thus as shown in figure the forces F_N and F_S acting on the north and south poles of magnet (2) having pole strength p_2 are

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi} \frac{m_1 p_2}{(d-l_2)^3} \quad (3)$$

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi} \frac{m_1 p_2}{(d+l_2)^3} \quad (4)$$

As $l_1 << d$ and $l_2 << d$, l_1 and l_2 can be neglected in comparison with d in equations (3) and (4).

$$\therefore \; F_S = F_N = \frac{\mu_0}{4\pi} \frac{m_1 p_2}{d^3} \quad (5)$$

As the non-colinear forces F_S and F_N are acting on magnet (2) in opposite direction, they form a couple. Hence the torque due to these forces is

$$\vec{\tau} = 2\vec{l_2} \times \vec{F}_N = 2\vec{l_2} \times \vec{F}_S \quad (\because \vec{\tau} = \vec{r} \times \vec{F})$$

Since $\vec{F}_N \perp \vec{l_2}$ and $\vec{F}_S \perp \vec{l_2}$, the magnitude of the torque with respect to centre of magnet (2)

$$\tau = 2F_N l_2 = \frac{\mu_0}{4\pi} \frac{m_1 2 l_2 p_2}{d^3} = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{d^3} \quad (6)$$

where $2l_2 p_2 = m_2$ = magnetic moment of magnet (2).

5.7 Torque Acting on a Magnetic Dipole (Bar Magnet) in a Uniform Magnetic Field

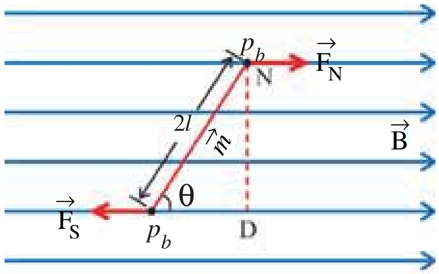


Figure 5.10 Torque Acting on a Magnetic Dipole of Magnetic Moment $\vec{m}$ in Uniform Magnetic Field $\vec{B}$

In Chapter-4 we have studied that the torque acting on a rectangular coil of magnetic moment $\vec{m}$, placed in a uniform magnetic field $\vec{B}$ is

$$\left.\begin{aligned} \vec{\tau} &= \vec{m} \times \vec{B} \\ \therefore \; \tau &= mB\sin\theta \end{aligned}\right\} \quad (5.7.1)$$

Where θ is the angle between $\vec{m}$ and $\vec{B}$ (sometimes magnetic moment is also represented by symbol $\vec{\mu}$).

This fact can be observed by placing a bar magnet or magnetic needle of magnetic dipole moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ (See figure 5.10). In terms of pole strength, the magnetic field $\vec{B}$ can be considered equivalent to the force acting on unit pole strength. The magnetic field exerts equal and opposite forces $\vec{F}_N$ and $\vec{F}_S$ on the north and south poles. But since these forces do not lie on a straight line, they form a couple. Perpendicular distance between these two forces is ND. Under the influence of this couple, the magnetic dipole rotates to a new position making angle θ with the direction of magnetic field $\vec{B}$.

If the angle θ (in radian) in equation (5.7.1) is small, then $\sin\theta \approx \theta$.

$$\therefore \ \tau = mB\theta \tag{5.7.2}$$

This torque, in the figure, is trying to rotate the dipole in a clockwise direction. Now if we try to rotate the dipole in anticlockwise direction further by a small angle θ with respect to this equilibrium position, then the torque represented by equation (5.7.1) will act in opposite direction. Thus we may write this restoring torque with negative sign as

$$\tau = -mB\theta \tag{5.7.3}$$

According to Newton's second law of motion (for rotational motion)

$$I_m \frac{d^2\theta}{dt^2} = -mB\theta \tag{5.7.4}$$

Where I_m is the moment of inertia of the magnetic dipole with respect to an axis perpendicular to the plane of figure and passing through the centre of the dipole.

$$\therefore \frac{d^2\theta}{dt^2} = -\frac{mB}{I_m}\theta = -\omega^2\theta \tag{5.7.5}$$

Equation (5.7.5) is similar to the differential equation for angular simple harmonic motion. Hence the angular frequency

$$\omega = \sqrt{\frac{mB}{I_m}} \tag{5.7.6}$$

$$\therefore \ T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_m}{mB}} \tag{5.7.7}$$

$$\text{which gives } B = \frac{4\pi^2 I_m}{mT^2} \tag{5.7.8}$$

The potential energy of the magnetic dipole in the external field $\vec{B}$ is given by

$$U_B = \int \tau d\theta = \int mB\sin\theta d\theta = mB\int \sin\theta d\theta$$

$$\therefore \ U_B = -mB\cos\theta = -\vec{m}\cdot\vec{B} \tag{5.7.9}$$

In equation (5.7.9) we have taken the constant of integration to be zero by considering the potential energy to be zero at $\theta = 90^\circ$, i.e. when the magnetic dipole is perpendicular to the field.

At $\theta = 0^\circ$, $U_B = -mB\cos 0^\circ = -mB$,

which is the minimum value of potential energy representing most stable position of the magnetic dipole.

At $\theta = 180^\circ$, $U_B = -mB\cos 180^\circ = mB$,

which is the maximum value of potential energy representing most unstable position of the magnetic dipole.

Illustration 3 : A magnetic needle placed in uniform magnetic field has magnetic moment 6.7×10^{-2} A m^2, and moment of inertia of 15×10^{-6} kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?

Solution : The periodic time of oscillation is, $T = \frac{6.70}{10} = 0.67$ s, and

$$B = \frac{4\pi^2 I_m}{mT^2} = \frac{4\times(3.14)^2\times15\times10^{-6}}{6.7\times10^{-2}\times(0.67)^2} = 0.02 \text{ T}$$

Illustration 4 : A short bar magnet is placed in an external magnetic field of 600 G. When its axis makes an angle of 30° with the external field, it experiences a torque of 0.012 N *m*.

(a) What is the magnetic moment of the magnet ?

(b) What is the work done in moving it from its most stable to most unstable position ?

(c) The bar magnet is replaced by a solenoid of cross-sectional area 2×10^{-4} m^2 and 1000 turns, but having the same magnetic moment. Determine the current flowing through the solenoid.

Solution : $B = 600$ G $= 600 \times 10^{-4}$T, $\theta = 30°$, $\tau = 0.012$ N *m*, N = 1000,

$A = 2 \times 10^{-4}$ m^2

(a) From equation (5.7.1)

$\tau = mB\sin\theta$

$\therefore$ $0.012 = m \times 600 \times 10^{-4} \times \sin 30°$

$\therefore$ $m = 0.40$ A m^2 (since $\sin 30° = \frac{1}{2}$)

(b) From equation (5.7.9), the most stable position is at $\theta = 0°$ and the most unstable position is at $\theta = 180°$. Hence the work done,

$W = U_B(\theta = 180°) - U_B(\theta = 0°) = mB - (-mB) = 2mB$

$= 2 \times 0.40 \times 600 \times 10^{-4} = 0.048$ J

(c) From equation (5.6.3)

$m_S = NIA$

But $m_S = m = 0.40$ A m^2, from part (a).

$\therefore$ $0.40 = 1000 \times I \times 2 \times 10^{-4}$

$\therefore$ $I = 2$ A

5.8. Gauss's Law for Magnetic Field

From Figure (5.3-*a*) and (5.3-*b*) we can see that, for any closed surface like (i) or (ii), the number of magnetic field lines entering the closed surface is equal to the number of field lines leaving the surface. Since the magnetic field lines always form closed loops, the magnetic flux, associated with any closed surface is always zero.

$$\therefore \oint_{\text{closed surface}} \vec{B}\cdot d\vec{a} = 0 \qquad (5.8.1)$$

where $\vec{B}$ is the magnetic field and $d\vec{a}$ is an infinitesimal area vector of the closed surface. **"The net magnetic flux passing through any closed surface is zero."** This statement is called Gauss's law for magnetic field.

According to the Gauss's law for electric field

$$\oint \vec{E} \cdot \vec{da} = 0 \;=\; \frac{\Sigma q}{\varepsilon_0} \tag{5.8.2}$$

In equation (5.8.2) if $\sum q = 0$, then

$$\oint \vec{E} \cdot \vec{da} = 0 \tag{5.8.3}$$

Comparing this equation with equation (5.8.1), we can write that the Gauss law for magnetic fields indicate that there does not exist any net magnetic monopole (magnetic charge ?) that is enclosed by the closed surface. The unit of magnetic flux is Weber (Wb).

1 Wb = 1 T m^2 = 1 NmA^{-1}

5.9. The Magnetism of Earth and Magnetic Elements

We all are aware of the fact that the Earth has its own magnetic field. The magnetic field on the surface of Earth is of the order of 10^{-5} T (T = tesla).

The magnetic field on the Earth resembles that of a (hypothetical) magnetic dipole as shown in figure 5.11.

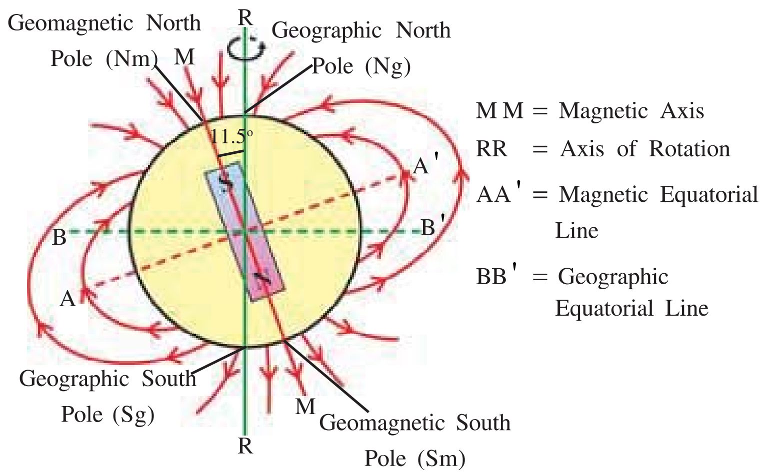


Figure 5.11 Magnetic Field of Earth

The magnitude of magnetic moment $\vec{m}$ of this (hypothetical) dipole is of the order of 8.0×10^{22} J T^{-1}. The axis MM of the dipole moment $\vec{m}$ does not coincide with the axis of rotation RR of the Earth, but is tilted by about 11.5°. The dipole axis MM intersects the Earth's geomagnetic north pole somewhere in north Canada, and the geomagnetic south pole in Antarctica. The magnetic field lines emerge out in the southern hemisphere and enter in the northern hemisphere. The actual south pole of earth's magnetic dipole is lying in the direction in which the north pole of magnetic needle, capable of rotating freely in the horizontal plane, remains stationary. Generally, we call this direction on earth as "Earth's magnetic north." The geomagnetic poles of Earth are located approximately 2000 km away from the geographic poles.

The geographic and geomagnetic equators intersect each other at longitude 6° west and 174° east. In India, Thumba near Trivandrum is on the magnetic equator, and hence it has been selected as the rocket launching station.

Each place on earth has a particular latitude and longitude which can be obtained from a good book of horoscope or map. The longitude circle passing through any place determines its geographic North-South direction. An imaginary vertical plane at a place on the Earth containing the longitude circle and the geographic axis of the Earth is called the **geographic meridian** (See figure 5.12).

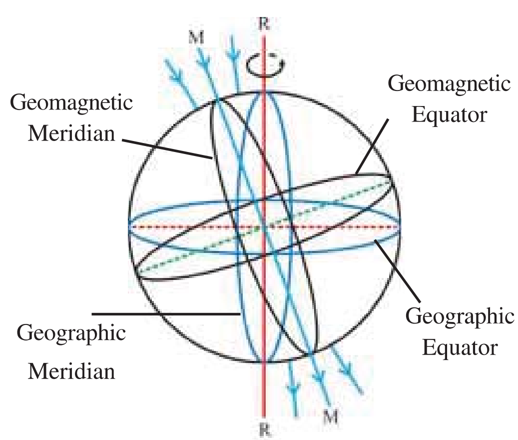


Figure 5.12 Geographic and Geomagnetic, Equator and Meridian of the Earth

Further, the magnetic field lines of geomagnetic dipole are also passing through every place on Earth. **Hence an imaginary vertical plane at a place on the Earth, passing through the magnetic axis and containing magnetic field lines is called magnetic meridian at that place.**

Illustration 5 : The Earth's magnetic field at some place on magnetic equator of Earth is 0.4 G. Estimate the magnetic dipole moment of the Earth. Consider the radius of Earth at that place to be 6.4×10^6 *m*. ($\frac{\mu_0}{4\pi} = 10^{-7}$T m A^{-1}, and 1 G = 10^{-4} T)

Solution : The magnitude of equatorial magnetic field, according to equation (5.6.6) is

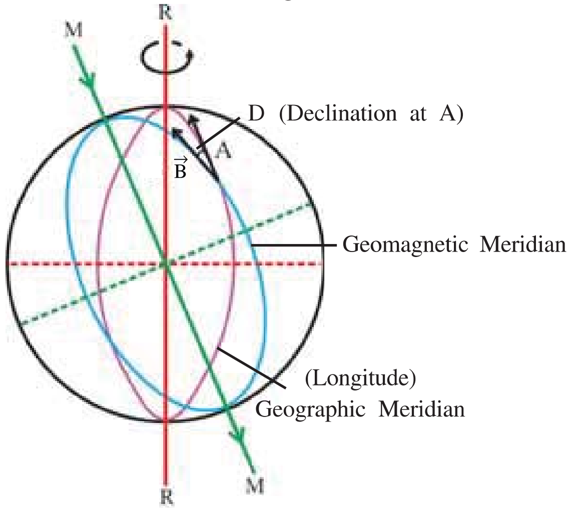
$$B_E = \frac{\mu_0 m}{4\pi y^3}$$

But $B_E = 0.4$ G $= 4 \times 10^{-5}$ T

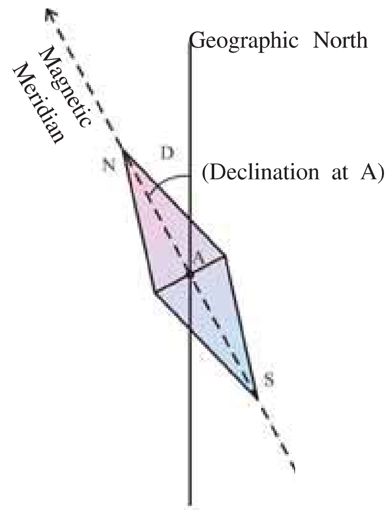
$$\therefore\ m = \frac{4\pi y^3 B_E}{\mu_0} = \frac{B_E y^3}{\left(\mu_0/4\pi\right)} = \frac{4\times10^{-5}\times(6.4\times10^6)^3}{10^{-7}} = 1.05 \times 10^{23} \text{ A m}^2$$

5.9.1. Geomagnetic Elements : In order to describe the magnetic field of Earth scientifically, certain magnetic parameters are defined, called geo-magnetic elements.

Magnetic Declination : The angle between the magnetic meridian and the geographic meridian at a place on surface of Earth is called magnetic declination at that place. Thus, the angle between the true geographic north and the magnetic north at any place on the surface of Earth is the magnetic declination (D) or simply declination at that place.



(a) Declination at Point A on the Surface of Earth



(b) Magnetic Needle Placed in Horizontal Plane at Point A

Figure 5.13

As shown in figure (5.13-*a*) consider point A on the surface of Earth. At this point, the direction of true geographic north is determined from tangent to A the longitude circle of geographic meridian. A magnetic needle free to rotate in horizontal plane aligns along the magnetic meridian at point A. The north pole of the needle points towards the geomagnetic north pole (tangent to the magnetic meridian at A). The angle between the geographic meridian and magnetic meridian at point A indicates the declination at the point A.

The declination is larger at higher latitudes and smaller near the equator. The declination is small in India, it being 0°58' west at Bombay, and 0°41' east at Delhi. Thus, at both these places the magnetic needle shows true north quite accurately.

Magnetic dip angle or inclination : Magnetic dip angle or inclination is the angle ϕ (up or down) that the magnetic field of Earth makes with the horizontal at a place in magnetic meridian.

Magnetic field lines are not locally horizontal at all places on Earth. At a place near north Canada, magnetic field lines point vertically downwards, whereas at a place on the magnetic equator, these field lines are horizontal. At the magnetic equator, dip angle is zero. As we move towards magnetic pole, the dip angle increases and becomes 90° at magnetic poles.

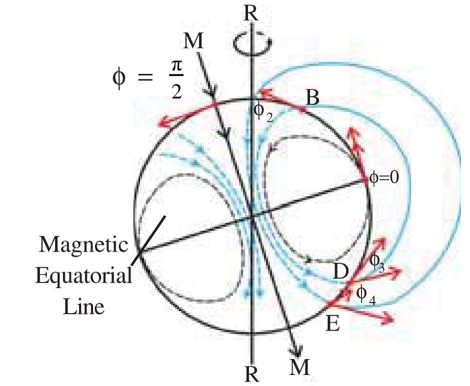


Figure 5.14 Magnetic Dip Angle at Different Places on the Earth's Surface in Magnetic Meridian [Only for Information]

Horizontal Component and Veritical Component of Earth's Magnetic field

Figure 5.15 shows the Earth's magnetic field ($\vec{B}$), angle of declination (D) and the angle of dip (ϕ) at a place (P).

The magnetic field $\vec{B}$ at point P is resolved into horizontal component $\vec{B}_H$ pointing towards geomagnetic north pole, and vertical component $\vec{B}_V$ pointing towards the centre of Earth. The angle made by $\vec{B}_H$ with geographic meridian is the angle of declination (D), whereas the angle between $\vec{B}_H$ and $\vec{B}$ is the angle of dip or inclination (ϕ).

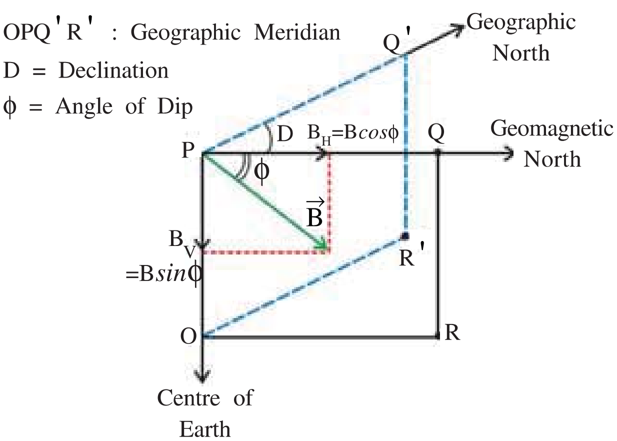


Figure 5.15 Components of Earth's Magnetic Field $\vec{B}$

The declination D, the angle of dip ϕ, and the horizontal component of Earth's field $\vec{B}_H$ are known as geomagnetic elements or the elements of Earth's magnetic field.

For the magnetic meridian OPQR of figure (5.15), we have

$$B_V = B\sin\phi \quad (5.9.1)$$

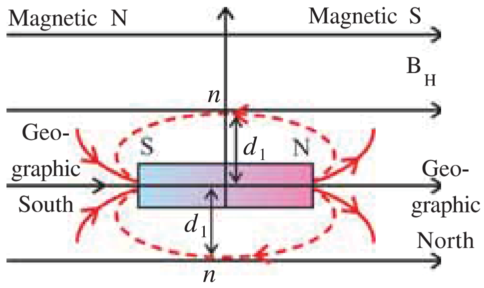
$$B_H = B\cos\phi \quad (5.9.2)$$

$$\therefore \tan\phi = \frac{B_V}{B_H} \quad (5.9.3)$$

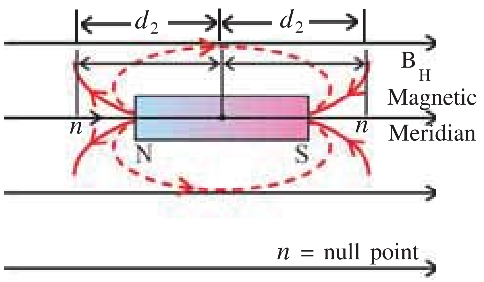
$$\text{and } B = \sqrt{B_V^2 + B_H^2} \quad (5.9.4)$$

Illustration 6 : A short bar magnet with magnetic dipole moment 1.6 A m^2 is kept in magnetic meridian in such a way that its north pole is in north direction. In this case, the null (neutral) point is found at a distance of 20 cm from the centre of the magnet. Find the horizontal component of the Earth's magnetic field.

Next, the magnet is kept in such a way that its magnetic north pole is in south direction. Find the positions of neutral (null) points in this case.



(a)



(b)

Solution : From the figure (a) one can observe that on the magnetic equator of the magnet, horizontal field lines of the earth's magnetic field and the magnetic field lines due to the magnet are in mutually opposite directions. Hence in this case, one finds two points on magnetic equator of the magnet at equal distance from the magnet (one above and one below) in such a way that at these points the above mentioned two magnetic fields are equal in magnitude and opposite in directions. At such points the resultant magnetic field is zero. Such points are called **neutral** or **null points.**

Here, $m = 1.6$ A m^2

Let, the distance of neutral points from the centre of the magnet is

$d_1 = 20$ cm $= 0.2$ m

Now the magnetic field due to a short bar magnet on its equatorial plane B_1 must equal B_H.

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m}{d_1^3} = B_H$$

$$\therefore B_H = \frac{10^{-7} \times 1.6}{(0.2)^3} = 2 \times 10^{-5} \text{ T}$$

However if the bar magnet is kept as in part (b) of the Figure, then it is clear that on the magnetic axis, B_H and the magnetic field due to the magnet are in mutually oposite directions. In this case the neutral points are on the axis. Let d_2 be the distance of such points from the centre of magnet, then B_2, the magnetic field on axis, must be equal to B_H,

$$\therefore B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{d_2^3} = B_H$$

$$\therefore d_2^3 = \frac{10^{-7} \cdot 2m}{B_H} = \frac{10^{-7} \times 2 \times 1.6}{2 \times 10^{-5}} = 16 \times 10^{-3}$$

$$\therefore d_2 = 2.52 \times 10^{-1} \ m = 2.52 \text{ cm}$$

Illustration 7 : A magnet is hung horizontally in the magnetic meridian by a wire without any twist. If the supporting wire is given a twist of 180° at the top, the magnet rotates by 30°. Now if another magnet is used, then a twist of 270° at the supporting end of wire also produces a rotation of the magnet by 30°. Compare the magnetic dipole moments of the two magnets.

Solution : If resultant twist in the wire = δ,

$$\delta_1 = 180° - 30° = 150° = 150 \times \frac{\pi}{180} \text{ rad}$$

and $\delta_2 = 270° - 30° = 240° = 240 \times \frac{\pi}{180}$ rad

If the twist–constant for the wire is k then

Rotating torque, $\tau_1 = k\delta_1$ and $\tau_2 = k\delta_2$

Here α is the angle made by the magnetic dipole moment with the magnetic meridian.

$\tau_1' = m_1 B_H \sin\alpha$

Since the second magnet is also rotated by the same angle.

$\tau_2' = m_2 B_H \sin\alpha$

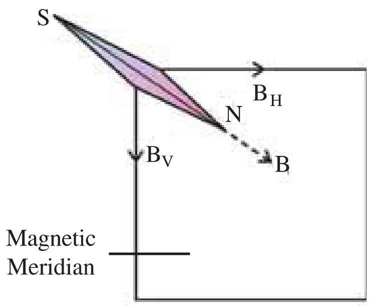
At equilibrium $\tau_1 = \tau_1'$ and $\tau_2 = \tau_2'$

$\therefore \frac{\tau_1'}{\tau_2'} = \frac{\tau_1}{\tau_2}$

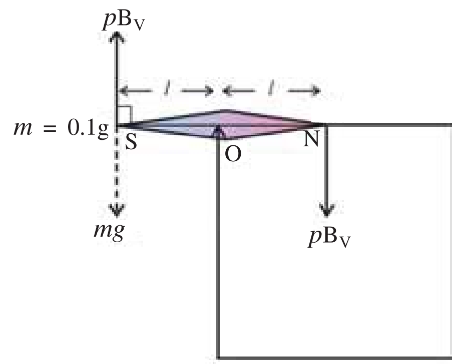
$\therefore \frac{m_1}{m_2} = \frac{\delta_1}{\delta_2} = \frac{150}{240} = \frac{5}{8}$

Illustration 8 : A magnetic needle is hung by an untwisted wire, so that it can rotate freely in the magnetic meridian. In order to keep it in the horizontal position, a weight of 0.1g is kept on one end of the needle. If the magnetic pole strength of this needle is 10 A *m*, find the value of the vertical component of the earth's magnetic field. ($g = 9.8$ m s^{-2})

Solution :



(a) Normal Position



(b) Position after inserting weight

Figure (a) shows the position of the magnetic needle in the magnetic meridian without any weight. In figure (b), a mass m is kept on the S–pole of the needle.

The vector sum of torques due to all forces must be zero for the equilibrium of the needle in horizontal direction.

$\therefore -pB_v(l) - pB_v(l) + m.g(l) = 0$

[The torque producing rotations in clockwise direction is taken as negative.]

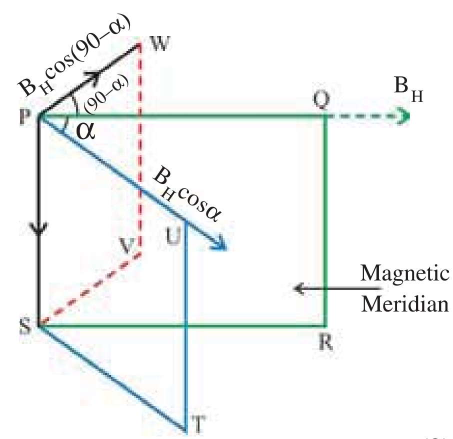
$\therefore 2pB_v = mg$

$\therefore B_v = \frac{mg}{2P} = \frac{10^{-4} \times 9.8}{2 \times 10}$ | $m = 0.1$ g $= 10^{-4}$ kg,

$\therefore B_v = 4.9 \times 10^{-5}$ T | $p = 10$ A m

Illustration 9 : As shown in figure, plane PSTU forms an angle of α and plane PSVW makes an angle of $(90° - \alpha)$ with the magnetic meridian, respectively. The value of magnetic dip angle in plane PSTU is ϕ_1 and its value in plane PSVW is ϕ_2. If the actual dip angle at the place is ϕ, show that,

$cot^2\phi = cot^2\phi_1 + cot^2\phi_2$



Solution : $tan\phi = \frac{B_V}{B_H}$ (1)

In plane PSTU horizontal component is $B_H cos\alpha$

$\therefore tan\phi_1 = \frac{B_V}{B_H cos\alpha} \Rightarrow cos\alpha = \frac{tan\phi}{tan\phi_1} = tan\phi \cdot cot\phi_1$

(from equation (1)) (2)

Similarly for plane PSVW

$sin\alpha = tan\phi \cdot cot\phi_2$ (3)

Squaring and summing the equations (2) and (3)

$cos^2\alpha + sin^2\alpha = 1 = tan^2\phi\,(cot^2\phi_1 + cot^2\phi_2)$

$\therefore cot^2\phi = cot^2\phi_1 + cot^2\phi_2$

5.10 Magnetization and Magnetic Intensity

Consider a solenoid of N turns having length l. When a current I_f is passed through it, the magnetic field produced inside the solenoid (with air or vacuum) is

$$B_0 = \mu_0 n I_f \qquad (5.10.1)$$

Where $n = \frac{N}{l}$ = number of turns per unit length of solenoid

This current I_f is called **free current**. If we denote the free current per unit length by i_f then

$$i_f = nI_f \qquad (5.10.2)$$

$$\therefore\ B_0 = \mu_0 i_f \qquad (5.10.3)$$

Now a material whose magnetic properties are to be studied is placed inside the solenoid. Let l be the length of the material, and A be its cross-sectional area. The magnetic field B_0, present inside the solenoid due to magnetizing current i_f, magnitizes the material such that it acquires some magnetic moment, say $\vec{m}$. This magnetic moment $\vec{m}$ of the material can be considered to be produced due to an equivalent surface current loop carrying current I_b. This current is called **bound current.** The dipole moment of this current loop is

$$\vec{m} = I_b \vec{A} \qquad (5.10.4)$$

where A = area of cross-section of the material = area of current loop.

The net magnetic moment per unit volume of the material is called magnetization M of the material. Thus

$$M = \frac{m}{V} = \frac{I_b A}{lA} = \frac{I_b}{l} = i_b \qquad (5.10.5)$$

Here $i_b = \frac{I_b}{l}$ = bound current per unit length of the core material.

The unit of M is A $m^2\ m^{-3}$ = A m^{-1}. Here M is a vector quantity. Its direction is along $\vec{m}$.

Thus the total magnetic field inside the magnetic core material placed inside the solenoid is due to both currents i_f and i_b.

$$\therefore\ B = \mu_0 (i_f + i_b) \qquad (5.10.6)$$

Using equation (5.10.5) in (5.10.6)

$$B = \mu_0 (i_f + M) \qquad (5.10.7)$$

$$\therefore\ \frac{B}{\mu_0} - M = i_f \qquad (5.10.8)$$

Here, $\frac{B}{\mu_0} - M$ is defined as magnetic intensity H, and its value is equal to magnetizing current, i_f. Hence

$$\frac{B}{\mu_0} - M = H = i_f \qquad (5.10.9)$$

$$B = \mu_0 (H + M) \qquad (5.10.10)$$

Thus, the magnetic field B induced in a substance, depends on H and M. Further, it is observed that, if H is not too much strong, then the magnetization M induced in the substance is proportional to magnetic intensity H.

$\therefore\ M = \chi_m H$ (5.10.11)

Here χ_m is a constant, called magnetic susceptibility of the material of the substance. It is a dimensionless quantity. Its value depends on the type of material and its temperature. It is a measure of how a magnetic material responds to external magnetic field. The magnetic susceptibility of some of the substances is listed in table (5.2) for information only.

Table 5.2 Magnetic Susceptibility of some Elements at 300 K (for information only)

Dimagnetic Substance	χ_m	Paramagnetic Substance	χ_m
Bismuth	-1.66×10^{-5}	Aluminium	2.3×10^{-5}
Copper	-9.8×10^{-6}	Calcium	1.9×10^{-5}
Dimond	-2.2×10^{-5}	Choromium	2.7×10^{-4}
Gold	-3.6×10^{-5}	Lithium	2.1×10^{-5}
Lead	-1.7×10^{-5}	Magnesium	1.2×10^{-5}
Mercury	-2.9×10^{-5}	Niobim	2.6×10^{-5}
Nitrogen (STP)	-5.0×10^{-9}	Oxygen (STP)	2.1×10^{-6}
Silver	-2.6×10^{-5}	Platinum	2.9×10^{-4}
Silicon	-4.2×10^{-6}	Tungsten	6.8×10^{-5}

The interpretation of equation (5.10.6) shows that, without putting magnetic material in solenoid, if the same magnetic field $[B = \mu_0 (i_f + i_b)]$ is required to be produced, then over and above the current I_f, an additional current I_m must be passed through the solenoid, such that the additional magnetizing current per unit length $nI_m = i_b$ is produced.

The substances for which χ_m is positive are called paramagnetic, for which $\vec{M}$ and $\vec{H}$ are in the same direction. The substances for which χ_m is negative are called diamagnetic, for which $\vec{M}$ and $\vec{H}$ are in opposite direction.

Substituting (5.10.11) in (5.10.10),

$B = \mu_0 [H + \chi_m H] = \mu_0 (1 + \chi_m) H = \mu H$ (5.10.12)

Where $\mu = \mu_0 (1 + \chi_m)$ is called permeability (magnetic permeability) of the material. $\frac{\mu}{\mu_0}$ is called relative permeability of the material, denoted by μ_r.

$\therefore\ \mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$ (5.10.13) which gives,

$$B = \mu_0 \mu_r H \quad (5.10.14)$$

Note : The vacuum cannot be magnetized. Hence for vacuum M = 0. Thus from equation (5.10.10), for vacuum $B = \mu_0 H$.

Illustration 10 : A solenoid has a core of material with relative permeability of 400. The current passing through the wire of solenoid is 2A. If the number of turns per cm are 10, calculate the magnitude of

(a) H, (b) B, (c) χ_m, (d) M, and (e) the additional magnetizing current I_m. (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}).

Solution : Here $\mu_r = 400$, I = 2 A, $n = 10 \frac{\text{turns}}{\text{cm}} = 1000 \frac{\text{turns}}{\text{m}}$

(a) Magnetic intensity $H = i_f = nI = 1000 \times 2 = 2000$ A m^{-1}

(b) Magnetic field $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 400 \times 2000 = 1.0$ T

(c) Magnetic susceptibility of the core material is

$$\chi_m = \mu_r - 1 = 400 - 1 = 399$$

(d) Magnetization

$$M = \chi_m H = 399 \times 2000 = 7.98 \times 10^5 \approx 8 \times 10^5 \text{ A m}^{-1}$$

(e) The additional magnetizing current I_m is obtained from $M = nI_m = i_b$ as

$$I_m = \frac{M}{n} = \frac{8 \times 10^5}{1000} = 800 \text{ A}$$

Illustration 11 : The region inside a current carrying torodial winding is filled with tungsten of susceptibility 6.8×10^{-5}. What is the percentage increase in the magnetic field in the presence of the material with respect to the magnetic field without it ?

Solution : The magnetic field in the current carrying torodial winding without tungsten is

$$B_0 = \mu_0 H$$

The magnetic field in the same current carrying torodial winding with tungsten is

$$B = \mu H$$

$$\therefore \quad \frac{B - B_0}{B_0} = \frac{\mu - \mu_0}{\mu_0}$$

But $\mu = \mu_0 (1 + \chi_m) \Rightarrow \frac{\mu}{\mu_0} = 1 + \chi_m \Rightarrow \frac{\mu}{\mu_0} - 1 = \chi_m \Rightarrow \frac{\mu - \mu_0}{\mu_0} = \chi_m$

Hence, $\frac{B - B_0}{B_0} = \chi_m$

$\therefore$ Percentage increase in the magnetic field in presence of tungsten is

$$\frac{B - B_0}{B_0} \times 100 = (6.8 \times 10^{-5}) \times 100 = 6.8 \times 10^{-3} \ \%$$

5.11 Magnetic Properties of Materials : Dia, Para and Ferro Magnetism

We know that each electron in an atom possess an orbital magnetic dipole moment and a spin magnetic dipole moment, that add vectorially. This type of resultant magnetic moment of each electron in an atom add vectorially, and the resultant dipole moment of each atom in the sample of a material add vectorially. If the resultant of all these dipole moments produces a magnetic field, then the material is said to be magnetic material.

The behaviour of a material in presence of an external magnetic field classifies the material as diamagnetic, paramagnetic or ferromagnetic. The classification of dia, para and ferro magnetic materials in terms of their susceptibility, relative permeability, and a small positive number ε (this 'ε' should be not be taken as permitivity of the medium) used to quantity paramagnetic material are briefly represented in Table 5.3.

Table 5.3

Diamagnetic	Parmagnetic	Ferromagnetic
$-1 \leq \chi_m < 0$	$0 < \chi_m < \varepsilon$	$\chi_m >> 1$
$0 \leq \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

5.11.1 Diamagnetic Materials : The atoms/molecules of gold, silver, copper, silicon, water and bismuth etc. do not possess permanent magnetic dipole moments. The orbital motion of the electrons and their spins are such that their total magnetic dipole moment is zero. Such materials are called diamagnetic materials.

When the diamagnetic material is placed in an external magnetic field, a net magnetic moment in a direction opposite to that of the external magnetic field is induced in each atom. Due to this, each atom of diamagnetic material experiences repulsion.

Figure 5.16 shows a bar of diamagnetic material placed in an external magnetic field $\vec{B}$. The field lines are repelled by induced magnetic field (weak) in the material, and the resultant field inside the material is reduced.

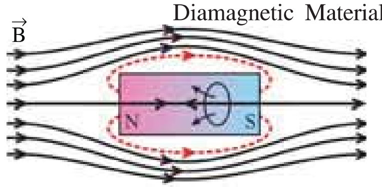


Figure 5.16 Diamagnetic Material in External Magnetic Field

As shown in figure 5.17, when the bar of diamagnetic material is placed in a non-uniform magneic field, the induced magnetic south pole is in the strong magnetic field, and the induced north pole is in the weak magnetic field. Hence, the magnetic force on the induced S-pole ($\vec{F}_S$ acting towards left) is more than the force on induced N-pole ($\vec{F}_N$ towards right). As a result the bar of diamagnetic material experiences a resultant force towards the region of weaker magnetic field. The magnetic susceptibility χ_m of diamagnetic materials is negative.

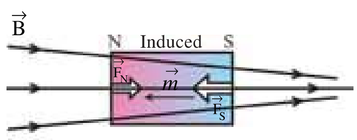


Figure 5.17 Force Acting on Diamagnetic Material Placed in Non-uniform Magnetic Field

For superconductors $\chi_m = -1$ and $\mu_r = 0$. When superconductors are placed in an external magnetic field, the field lines are completely expelled. The phenomenon of perfect diamagnetism in superconductors is called the **Meissner effect,** after the name of its discoverer. Superconducting magnets can be used for running magnetically leviated superfast trains.

5.11.2 Paramagnetism : In paramagnetic material, the atoms/molecules possess permanent magnetic dipole moments. Normally, the molecules are arranged such that, their magnetic dipole moments are randomly oriented. Hence the resultant magnetic moment of the material is zero (See figure 5.18).

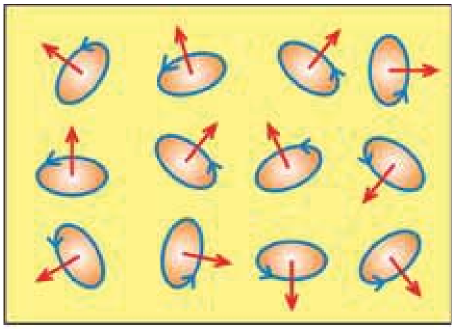


Figure 5.18 Normal Dipole Distribution in Paramagnetic Material

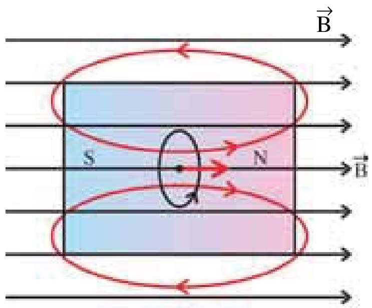


Figure 5.19 Magnetic Dipole Moment of One Dipole Shown Aligned with $\vec{B}$

When the paramagnetic material is placed in external magnetic field $\vec{B}$, these tiny dipoles try to align in the direction of $\vec{B}$. However, due to thermal agitation, all dipoles could not attain 100% alignment in the direction of $\vec{B}$.

Figure 5.19 shows the magnetic field due to the magnetic dipole aligned with $\vec{B}$. The field lines get concentrated inside the material (see figure 5.20)

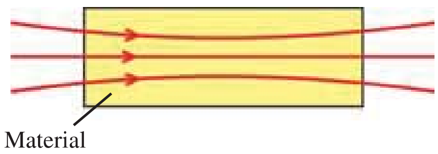


Figure 5.20 Magnetic Field Lines in Paramagnetic Material

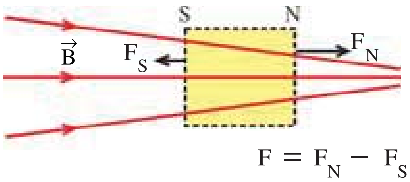


Figure 5.21 Paramagnetic Material in Non-uniform Magnetic Field

When a bar of paramagnetic material is placed in non-uniform magnetic field (See figure 5.21) the resultant north pole of the magnetized material feels strong magnetic field, whereas the south pole experiences comparatively weak magnetic field. As a result of which the resultant force $(F_N - F_S)$ acts towards the stronger magnetic field (towards right) on the bar of paramagnetic material. In practice, this force is very weak.

Aluminium, sodium, calcium, oxygen at STP and copper chloride are few examples of paramagnetic materials. The magnetic susceptibility χ_m of paramagnetic materials is positive.

In 1895 Pierre Curie observed that the magnetization M of a paramagnetic material is directly proportional to the external magnetic field $\vec{B}$ and inversely proportional to its absolute temperature T, called Curie's law,

$$M = C\frac{B}{T} \tag{5.11.1}$$

Where C = Curie's constant

From equation (5.11.1)

$$M = C\frac{B}{\mu_0}\frac{\mu_0}{T} = CH\frac{\mu_0}{T}$$

$$\therefore \frac{M}{H} = \chi_m = C\frac{\mu_0}{T} \tag{5.11.2}$$

$$\therefore \ \mu_r - 1 = C\frac{\mu_0}{T} \tag{5.11.3}$$

As we increase the applied external magnetic field or decrease the temperature of the paramagnetic material, or both, then alignment of atomic magnetic moments increase. Thus magnetization M increases. When magnetic moments of all atoms are aligned parallel to the external magnetic field, M, μ_r and χ_m become maximum. This situation is called **satuaration magnetization**. Curie's law is not obeyed after this state. If there are N atoms in volume V of the sample, each with magnetic moment $\vec{m}$, then at saturation magnetization

$$\vec{M}_{max} = \frac{N\vec{m}}{V} \tag{5.11.4}$$

5.11.3 Ferromagnetism : The atoms of iron, cobalt, nickel and their alloys possess permanent magnetic dipole moments due to spin of electrons in outermost orbits. The atoms of such materials are arranged in such a way that over a region called **domain,** the magnetic moment of the atoms are aligned in the same direction. In unmagnetized sample, such domains having a net magnetization are randomly oriented so that the effective magnetic moment is zero (See figure 5.22).

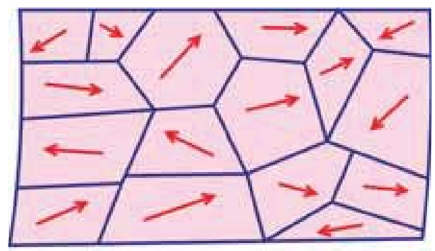


Figure 5.22 Random Arrangement of Domains

The explanation about the formation of such domains requires quantum mechanics which is beyond the scope of this book. The typical domain size is about 1 mm and the domain contains about 10^{11} atoms. The boundaries between the adjacent domains, having different orientations of magnetic moment, are called **domain walls.**

Hysteresis : The effect of an external magnetic field on ferromagnetic material is quite interesting. To understand this, consider an unmagnetized ferromagnetic material having initial magnetic field B = 0. Suppose this material is placed in a solenoid of n turns per unit length as shown in figure 5.8 (b). On passing a current through the solenoid, the magnetic field is generated, which induces magnetic moment inside the rod. Knowing the volume of the rod, we can evaluate M, the magnetic moment per unit volume. We already know that

$$\frac{B}{\mu_0} - M = i_f = H \text{ (See Equation (5.10.9))}$$

where, i_f = current passing through unit length of the solenoid.

From the values of H and M, we can evaluate B and study its variation with i_f (hence the variation of H). The graph of B versus. H can be drawn as shown in figure 5.23.

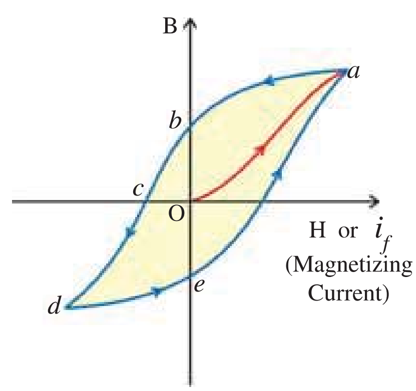


Figure 5.23 Hysteresis Loop

At the point 0 in the graph, the substance is in its normal condition, without any resultant magnetic field. As H (or i_f) is increased, B increases, but this increase is not linear. Near point *a*, B is maximized, which is the saturation magnetization condition of the rod.

One can explain the curve O*a* as follows : Starting from O, as long as the value of H is small, most of the atoms, due to strong bonding with their neighbours, do not respond to the external magnetic field. But the atoms near the domain boundary are in precarious situation. Hence the domain boundaries, instead of remaining sharp, start shifting. In this situation one domain of the two adjacent domains, increases in size and the other one reduces in size. If we still keep on increasing the value of H, ultimately only one domain survives in the substance and the **saturation magnetization** is acquired near point *a* on the graph.

This process is not reversible. At this stage, if the current in the solenoid is reduced, we do not get back the earlier domain constituation, and when H = 0, we do not get B = 0. This means that when H is made zero, the substance retains certain magnetic moment, hence the curve *ab* represents the effect of reducing H.

The value of B, when H = 0, is called **retentivity** or **remanence**. Now, if the current is increased in reverse direction, then we reach at point *c* in the graph, the value of H for which B = 0 is called **coercivity**. At this point, the magnetic moments of the domains are again in random directions but according to some different domain structure.

If we keep on increasing the current in the reverse direction, B goes on increasing in the reverse direction and saturation magnetization is again acquired, but in opposite direction. After reaching *d*, if the current is reduced, the substance follows the curve *de* and again by reversing the current direction and increasing its value, we obtain the curve *ea*. This process is called **hysteresis cycle.** The area enclosed by the B-H curve represents the heat energy (in joules) lost in the sample per unit volume per cycle.

Hard ferromagnetic substances : The substances with large retentivity are called hard ferromagnetic substances. These are used in producing permanent magnets. Obviously, the hysteresis cycle for such substances is broad (See figure 5.24 (a)). Alnico (an alloy of Al, Ni, Co and Cu) is a hard ferromagnetic meterial. Hence permanent magnets are made using Alnico.

Soft Ferromagnetic Substances : The substances with small retentivity, which means the materials with narrow hysteresis cycle (See figure 5.24 (b)), are called soft ferromagnetic substances. For example soft iron; such materials are used for making electromagnets.

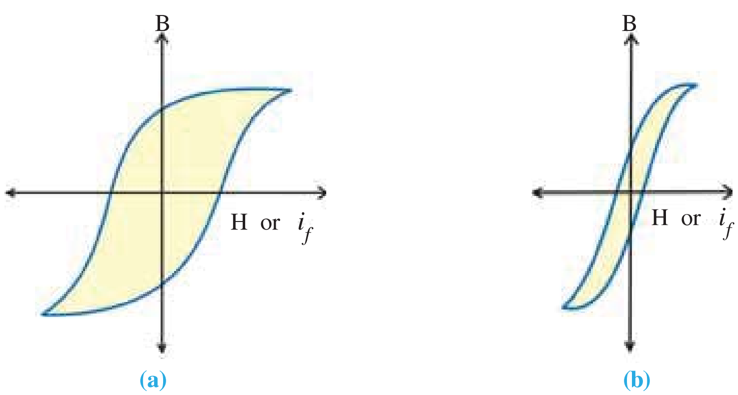


Figure 5.24 Hysteresis Loops for (a) Hard and (b) Soft Ferromagnetic Materials

Effect of Temperature : As the temperature of ferromagnetic substance is increased, the domain structure starts getting distorted. At a certain temperature depending upon the material, it is totally broken up. Each and every atomic magnetic moment attains independence from one another and the substance gets converted to a paramagnetic material.

The temperature at which a ferromagnetic substance is converted into a paramagnetic substance is called **Curie temperature T_C** of that substance. The relation between the magnetic susceptibility of the substance in the acquired paramagnetic form and the temperature T is given by

$$\chi_m = \frac{C_1}{T - T_C}, \ (T > T_C) \tag{5.11.1}$$

where, C_1 is a constant.

Finally, note that the ferromagnetic material is attracted towards the strong field region whenever it is kept in a non-uniform magnetic field.

The hysteresis loop shows that the magnetization of a ferromagnetic material depends on the history (the previous state) of the material as well as on the magnitude of applied field H. The shape and size of the hysteresis loop depends on the properties of ferromagnetic material as well as on the maximum value of applied magnetic field H.

5.12 Permanent Magnets and Electromagnets

The ferromagnetic materials which retain magnetism for a longer period of time at room temperature, are called permanent magnets. These materials have higher retentivity.

Before 400 years, the iron rods were fixed in north-south direction and hammered repeatedly to prepare magnets. Further, if one end of a magnet is continuously rubbed on a fixed steel rod only in one direction, then it acquires permanent magnetism.When a current is passed through a solenoid containing a steel rod, then the rod gets magnetized. Due to hysteresis, the rod retains magnetism even after the current is switched off. The materials like steel, hard alloys, and alnico have high retentivity and high coercivity, and hence are used to prepare permanent magnets.

Soft iron has large permeability and small retentivity, and hence is used to prepare electromagnets. For this purpose, a rod of soft iron is placed in a solenoid as a core, as shown in Figure 5.7(b). On passing a current through the solenoid, the magnetic field associated with the solenoid increases by a thousand fold. When the current through the solenoid is switched off, the associated magnetic field effectively becomes zero.

Electromagnets are used in electric bells and loudspeakers. Giant electromagnets are used in cranes to lift heavy loads made of iron or loads packed in iron containers (boggies).

In certain applications, an AC current is passed through the solenoid containing ferromagnetic material, for example in transformer cores and telephone diaphragms. The hysteresis loop of such materials must be narrow to reduce dissipation of energy in the form of heat.

Illustration 12 : A magnet has coercivity of 3×10^3 A m^{-1}. It is kept in a 10 cm long solenoid with a total of 50 turns. How much current has to be passed through the solenoid to demagnetize it ?

Solution : The value of H for which magnetization is zero is called coercivity.

For a solenoid $H = nI$

Here, $H = 3 \times 10^3$, $n = \frac{N}{l} = \frac{50}{0.1} = 500$

$$\therefore I = \frac{H}{n} = \frac{3 \times 10^3}{5 \times 10^2} = 6 \text{ A}$$

Illustration 13 : There are 2.0×10^{24} molecular dipoles in a paramagnetic salt. Each has dipole moment 1.5×10^{-23} A m^2 (or J T^{-1}). This salt kept in a uniform magnetic field 0.84 T is cooled to a temperature of 4.2 K. In this case the magnetization acquired is 15% of the saturation magnetization. What must be the dipole moment of this sample in magnetic field 0.98 T and at temperature of 2.8 K ? (Assume the applicability of the Curie's law).

Solution : Dipole moment of every molecular dipole = 1.5×10^{-23} A m^2

There are 2.0×10^{24} dipoles in the sample.

$\therefore$ Maximum (saturation) magnetization = $1.5 \times 10^{-23} \times 2.0 \times 10^{24} = 30$ A m^2

But at 4.2 K, sample has 15 % of saturation magnetization

$\therefore$ $m_1 = 30 \times 0.15 = 4.5$ A m^2

Now according to Curie's law, if m_1 is the dipole moment at T_1 and m_2 the dipole moment at T_2 then

$$\frac{m_1}{m_2} = \frac{B_1}{T_1} \times \frac{T_2}{B_2} \quad \left(\text{from } m \propto \frac{B}{T}\right)$$

Here B_1 and B_2 are applied magnetic fields

$$\therefore\ m_2 = m_1 \times \frac{T_1}{T_2} \times \frac{B_2}{B_1}$$

Here, $m_1 = 4.5$ A m^2, $T_1 = 4.2$ K, $T_2 = 2.8$ K, $B_1 = 0.84$ T and $B_2 = 0.98$ T

$$\therefore\ m_2 = \frac{4.5 \times 4.2 \times 0.98}{2.8 \times 0.84} = 7.87 \text{ A m}^2$$

SUMMARY

1. The north and south magnetic poles cannot be separated by splitting the magnet into two or more pieces. The independent magnetic monopoles does not exist.
2. The magnetic field lines do not intersect at a point.
3. The magnetic field lines of a magnet form continuous closed loops. The magnetic field lines emerge out from the magnetic north pole, reach the magnetic south pole and then passing through the magnet, reach the north pole to complete the loop.
4. The magnetic moment of a current loop of area A, carrying current I is given by $m = IA$. If there are N turns of a loop, then $m = NIA$

   If there are N turns of a loop, then $m = NIA$
5. The axial magnetic field of a current loop is given by $\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3}$
6. The orbital magnetic moment of an electron in an atom is given by $m_0 = \frac{1}{2}evr$
7. When a bar magnet is divided into two equal pieces, the pole strength p_b of each piece remains the same, but the magnetic dipole moment of each piece becomes half of the original value.
8. When a magnet of magnetic moment $\vec{m}$ is placed in external magnetic field $\vec{B}$, the torque acting on it is given by $\vec{\tau} = \vec{m} \times \vec{B}$ or $\tau = mB\sin\theta$ and has potential energy $U_B = -\vec{m}\cdot\vec{B}$
9. The Gauss's law for magnetic field is $\oint_{\text{closed surface}} \vec{B}\cdot\vec{da} = 0$ which states that "the net magnetic flux passing through any closed surface is zero.
10. **Magnetic Meridian :** An imaginary vertical plane at a place on the Earth, passing through the magnetic axis is called magnetic meridian at that place.

11. The angle between the magnetic meridian and the geographic meridian at a place on the surface of Earth is called the magnetic declination (D) at that place.

12. **Magnetic dip or inclination(ϕ) :** It is the angle (up or down) that the magnetic field of Earth makes with the horizontal at a place in magnetic meridian.
$\phi = 0^o$ at magnetic equator and $\phi = 90^o$ at geomagnetic poles.

13. The net magnetic moment per unit volume of the material is called magnetization of the material, represented by $\vec{M} = \frac{\vec{m}}{V}$.

14. The magnetic susceptibility χ_m of a material is a measure of how a magnetic material responds to external magnetic field. It is dimensionless quantity.

15. When a diamagnetic material is placed in non-uniform magnetic field, it experiences a resultant force towards the region of weak magnetic field. The magnetic susceptibility χ_m of diamagnetic material is negative.

16. When a paramagnetic material is placed in non-uniform magnetic field, it experiences a (weak) force towards strong magnetic field. The magnetic susceptibility χ_m of paramagnetic material is positive.

17. According to Curie's law, the magnetization M of a paramagnetic material is given by $M = C\frac{B}{T}$.
When magnetic moments of all atoms are aligned with external magnetic field M, χ_m and μ_r become maximum, called saturation magnetization. Curie's law is not obeyed after saturation magnetization.

18. The atoms of ferromagnetic material possess permanent magnetic dipole moment due to spin of electrons in outermost orbits. These atoms are arranged in such a way that over a region called domain, the magnetic moments of such atoms are aligned in the same direction. In unmagnetized sample, such domains having a net magnetization are randomly oriented so that the effective magnetic moment is zero.

19. The temperature at which a ferromagnetic substance is converted into a paramagnetic substance is called Curie temperature T_C of that substance. The relation between the magnetic susceptibility of the substance in the acquired form and the temperature T is
$\chi_m = \frac{C_1}{T-T_C}$, $(T > T_C)$, where C_1 = constant

20. Permanent magnets have higher retentivity and high coercivity.

21. Soft iron used to prepare electromagnets have large permeability and small retentivity.

EXERCISE

For the following statements choose the correct option from the given options :

1. A magnet of magnetic dipole moment 5.0 A m^2 is lying in a uniform magnetic field of 7×10^{-4} T such that its dipole moment vector makes an angle of 30^o with the field. The work done in increasing this angle from 30^o to 45^o is about .... J.
(A) 5.56×10^{-4} (B) 24.74×10^{-4} (C) 30.3×10^{-4} (D) 5.50×10^{-3}

2. A bar magnet is oscillating in Earth's magnetic field with periodic time T. If a similar magnet with the same mass and dimensions has magnetic dipole moment, which is 4 times that of this magnet, then its periodic time will be ....
(A) $\frac{T}{2}$ (B) 2T (C) T (D) 4T

3. A circular loop carrying current I is replaced by a bar magnet of equivalent magnetic dipole moment. The point on the loop is lying ....

(A) on equatorial plane of magnet

(B) on axis of the magnet

(C) A and B both

(D) except equatorial plane or axis of bar magnet

4. When a current carrying loop is replaced by an equivalent magnetic dipole

(A) the distance l between the poles is fixed.

(B) the pole strength p of each pole is fixed.

(C) the dipole moment is reversed.

(D) the product pl is fixed.

5. Let r be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at r is always proportional to

(A) $\frac{1}{r^2}$ (B) $\frac{1}{r^3}$

(C) $\frac{1}{r}$ (D) not necessarily $\frac{1}{r^3}$ at all points

6. Magnetic meridian is a plane ....

(A) perpendicular to magnetic axis of Earth.

(B) perpendicular to geographic axis of Earth.

(C) passing through the magnetic axis of Earth.

(D) passing through the geographic axis.

7. At geomagnetic pole, a magnetic needle allowed to rotate in horizontal plane will

(A) stay in north-south direction only (B) stay in any position

(C) stay in east-west direction only (D) become rigid showing no movement

8. The horizontal and vertical components of magnetic field of Earth are same at some place on the surface of Earth. The magnetic dip angle at this place will be ....

(A) 30° (B) 45° (C) 0° (D) 90°

9. Inside a bar magnet, the magnetic field lines

(A) are not present

(B) are parallel to the cross-sectional area of the magnet

(C) are in the direction from N-pole to S-pole

(D) are in the direction from S-pole to N-pole

10. In non-uniform magnetic field, a diamagnetic substance experiences a resultant force

(A) from the region of strong magnetic field to the region of weak magnetic field.

(B) perpendicular to the magnetic field.

(C) from the region of weak magnetic field to the region of strong magnetic field.

(D) which is zero.

11. A straight steel wire of length l has magnetic moment m. If the wire is bent in the form of a semicircle, the new value of the magnetic dipole moment is

(A) m (B) $\frac{2m}{\pi}$ (C) $\frac{m}{2}$ (D) $\frac{m}{\pi}$

12. At a place on Earth, the horizontal component of Earth's magnetic field is $\sqrt{3}$ times its vertical component. The angle of dip at this place is ....

(A) 0 (B) $\frac{\pi}{2}$ rad (C) $\frac{\pi}{3}$ rad (D) $\frac{\pi}{6}$ rad

13. A place, where the vertical component of Earth's magnetic field is zero has the angle of dip equal to

(A) 0^o (B) 45^o (C) 60^o (D) 90^o

14. A place where the horizontal component of Earth's magnetic field is zero lies at

(A) geographic equator (B) geomagnetic equator

(C) one of the geographic poles (D) one of the geomagnetic poles

15. When a paramagnetic substance is brought near a north pole or a south pole of a bar magnet, it

(A) experiences repulsion (B) experiences attraction

(C) does not experience attraction or repulsion

(D) experiences attraction or repulsion depending upon which pole is brought near to it.

16. A magnetic needle kept on horizontal surface oscillates in Earth's magnetic field. If the temperature of this needle is raised beyond the Curie temperature of the material of the needle, then ....

(A) the periodic time of oscillation will decrease.

(B) the periodic time of oscillation will increase.

(C) the periodic time of the oscillation will not change.

(D) the needle will stop oscillating.

17. A bar magnet of length l, pole strength 'p' and magnetic moment '$\vec{m}$' is split $\frac{l}{2}$ into two equal pieces each of length. The magnetic moment and pole strength of each piece is respectively .... and ....

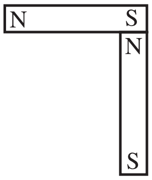
(A) $\vec{m}, \frac{p}{2}$ (B) $\frac{\vec{m}}{2}, p$ (C) $\frac{\vec{m}}{2}, \frac{p}{2}$ (D) $\vec{m}, p$

18. Magnetization for vacuum is ....
(A) negative (B) positive (C) infinite (D) zero

19. A bar magnet of magnetic moment $\vec{m}$ is placed in uniform magnetic field $\vec{B}$ such that $\vec{m} \parallel \vec{B}$. In this position, the torque and force acting on it are .... and .... respectively.
(A) 0, 0 (B) $\vec{m} \times \vec{B}$, mB (C) $\vec{m} \cdot \vec{B}$, mB (D) $\vec{m} \cdot \vec{B}$, $\vec{m} \times \vec{B}$

20. Relative permeability of a substance is 0.075. Its magnetic susceptibility is
(A) 0.925 (B) −0.925 (C) 1.075 (D) −1.075

21. Two similar magnets of magnetic moment m are arranged as shown in figure. The magnetic dipole moment of this combination is



(A) $2m$ (B) $\sqrt{2}\,m$ (C) $\frac{m}{\sqrt{2}}$ (D) $\frac{m}{2}$

22. A magnetic needle kept non-parallel to the magnetic field in a non-uniform magnetic field experiences
(A) a force but not a torque. (B) a torque but not a force
(C) both a force and a torque. (D) neither a force nor a torque

23. A steamer would like to move in the direction making an angle of 10^{o} south with the west. The magnetic declination at that place is 17^{o} west from the north. The steamer should move in a direction ....
(A) making an angle of 83^{o} west with the north pole of Earth.
(B) making an angle of 83^{o} east with the north pole of Earth.
(C) making an angle of 27^{o} west with the south pole of Earth.
(D) making an angle of 27^{o} east with the south pole of Earth.

24. A toroid wound with 100 turns/m of wire carries a current of 3 A. The core of toroid is made of iron having relative magnetic permeability of $\mu_r = 5000$ under given conditions. The magnetic field inside the iron is .... (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(A) 0.15 T (B) 0.47 T (C) 1.5×10^{-2} T (D) 1.88 T

ANSWERS

1. (A)	2. (A)	3. (A)	4. (D)	5. (D)	6. (C)
7. (B)	8. (B)	9. (D)	10. (A)	11. (B)	12. (D)
13. (A)	14. (D)	15. (B)	16. (D)	17. (B)	18. (D)
19. (A)	20. (B)	21. (B)	22. (C)	23. (A)	24. (D)

Answer the following questions in brief :

1. What happens if a bar magnet is cut into two pieces transverse to its length/along its length ?
2. Does a current carrying toroid have a north pole and a south pole ?
3. Which phase / phases of matter cannot be ferromagnetic in character ?
4. Magnetic properties of which materials are affected by temperature ?
5. What should be retentivity and coercivity of permanent magnet ?
6. What happens to a ferromagnetic material when its temperature increases above Curie temperature ?
7. What is the unit of magnetic intensity ?
8. What does the hysteresis loop represent ?

9. What are the applications of electromagnet ?
10. What could be the equation for Gauss's law of magnetism, if a monopole of polestrength p is enclosed by a surface ?
11. What happens when a paramagnetic material is placed in a non-uniform magnetic field ?
12. What is the unit of magnetic susceptibility ?
13. What is the declination for Delhi ?
14. Mention the names of diamagnetic materials.
15. Which property of soft iron makes it useful for preparing electromagnet ?

Answer the following questions :

1. Obtain an expression for axial magnetic field of a current loop in terms of its magnetic moment.
2. Explain symbolic notation for detecting north and south pole of magnetic field in a current carrying loop.
3. Obtain an expression for orbital magnetic moment of an electron rotating about the nucleus in an atom.
4. Explain in brief, the Gauss's law for magnetic fields.
5. What is a geographic meridian and a geomagnetic meridian ? What is the angle between them ?
6. Give definition of magnetic declination. How does the declination vary with latitude ? Where is it minimum ?
7. Give definition of magnetic dip. What is the dip angle at magnetic equator ? What happens to dip angle as we move towards magnetic pole from the magnetic equator ?
8. What happens when a diamagnetic material is placed in non-uniform magnetic field ? Explain with necessary Figure.
9. Discuss Curie's law for paramagnetic materials.
10. Discuss why the soft iron is suitable for preparing electromagnets.

Solve the following examples :

1. A toroidal core with 3000 turns has inner and outer radii of 11 cm and 12 cm, respectively. When a current of 0.70 A is passed, the magnetic field produced in the core is 2.5 T. Find the relative permeability of the core. ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

   [**Ans. :** 685]

2. A paramagnetic gas has 2.0×10^{26} atoms/m^3 with the atomic magnetic dipole moment of 1.5×10^{-23} A m^2 each. The gas is at 27^o C. (i) Find the maximum magnetization intensity of this sample. (ii) If the gas in this problem is kept in a uniform magnetic field of 3 T, is it possible to achieve saturation magnetization ? Why ?

   [**Hint :** Thermal energy of an atom of gas is $\frac{3}{2}k_B T$, and

   Maximum potential energy of the atom = mB.

   Find the ratio of thermal energy to the maximum potential energy and give answer.]

   ($k_B = 1.38 \times 10^{-23}$ J K^{-1}) [**Ans. :** 3.0×10^3 A m^{-1}, No]

3. Two small and similar bar magnets have magnetic dipole moment of 1.0 A m^2 each. They are kept in a plane in such a way that their axes are perpendicular to each other. A line drawn through the axis of one magnet passes through the centre of other magnet. If the distance between their centers is 2 m, find the magnitude of magnetic field at the mid point of the line joining their centers.

   [**Ans. :** $\sqrt{5} \times 10^{-7}$ T]

4. A magnetic pole of bar magnet with pole-strength of 100 A m is 20 cm away from the centre of a bar magnet. Bar magnet has pole-strength of 200 A m and has a length of 5 cm. If the magnetic pole is on the axis of the bar magnet, find the force on the magnetic pole.

[**Ans. :** 2.5×10^{-2} N]

5. The work done for rotating a magnet with magnetic dipole moment m, by 90^o from its magnetic meridian is n times the work done to rotate it by 60^o. Find the value of n.

[**Ans. :** 2]

6. A magnet makes an angle of 45^o with the horizontal in a plane making an angle of 30^o with the magnetic meridian. Find the true value of the dip angle at the place.

[**Ans. :** tan^{-1} (0.866)]

7. An electron in an atom is revolving round the nucleus in a circular orbit of radius 5.3×10^{-11} m, with a speed of 2×10^6 m s^{-1}. Find the resultant orbital magnetic moment and angular momentum of the electron. Take charge of electron = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31}kg. [**Ans. :** 8.48×10^{-24} Am^2, and 9.65×10^{-35} N m s]

8. The magnetic field from a current carrying loop of diameter 1 cm is 10^{-4} T at 10 cm from the centre, along the axis of the loop.
(a) Find the magnetic moment of the loop.
(b) Find the magnetic field at 10 cm from the centre, along the equator of the loop.

Take $\frac{\mu_0}{4\pi} = 10^{-7}$ T m A^{-1} [**Ans. :** (a) 0.5 A m^2, (b) 5×10^{-5} T]

9. A magnet in the form of a cylindrical rod has a length of 5 cm and a diameter of 2 cm. It has a uniform magnetization of 5×10^3 A m^{-1}. Find its net magnetic dipole moment.

[**Ans. :** 7.85×10^{-2} J T^{-1}]

10. An ionized gas consists of 5×10^{21} electrons/m^3 and the same number of ions/m^3. If the average electron kinetic energy is 6×10^{-20} J, and an average ion kinetic energy is 8×10^{-21} J, calculate the magnetization of the gas when a magnetic field of 1.0 T is applied to the gas. [**Ans. :** 340 J T^{-1} m^{-3}]

11. A closely wound solenoid of 6 cm, having 10 turns/cm and area of cross-section 3×10^{-4} m^2 carries a current of 1.0 A. Find the magnetic moment and the pole strength of the solenoid.

[**Ans. :** Magnetic moment of solenoid along its axis = 1.8×10^{-2} A m^2, pole strength of the solenoid = 0.3 A m]

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6 Ray Optics

6.1 Introduction

Light is the agency which stimulates our sense of vision or sight. All curious questions regarding light: its nature, its generation, its interaction with matter, its speed and propagation through medium, etc., are described and explained in a branch of physics called optics. Developments in optics can be classified into three branches:

(1) Ray (Geometric) optics, (2) Wave optics and (3) Quantum optics

Since the wavelength of visible electromagnetic waves (400 nm to 800 nm) is too small compared to objects around us, light can be considered to travel from one point to another along a straight line. This is called rectilinear propagation of light. The path of the light propagation is called a ray, which is never diverging or converging. A bundle of such rays is called deam of light.

The optical phenomena like reflection, refraction and dispersion can be explained by the ray optics. The ray optics is based mainly on the following three assumptions.

(1) Rectilinear propagation of light

(2) Independence of light rays (i.e., they do not disturb one another when they intersect).

(3) Reversibility of path (i.e., they retrace exactly the same path on reversing their direction of propagation).

In the present chapter, we shall study reflection, refraction and dispersion phenomena using ray optics. Optical instruments like microscope and telescope are also studied at the end of the chapter.

6.2 Reflection by Spherical Mirrors

For studying reflection of light by spherical mirrors, we shall revise certain points as under :

The laws of reflection

(1) In the case of reflection of light, the angle of incidence and angle of reflection are equal.

(2) Incident ray, reflected ray and normal drawn at the point of incidence lie in the same plane. While the incident ray and the reflected ray are on either side of the normal.

These laws are valid at every point on any reflecting surface, whether plane or curved.

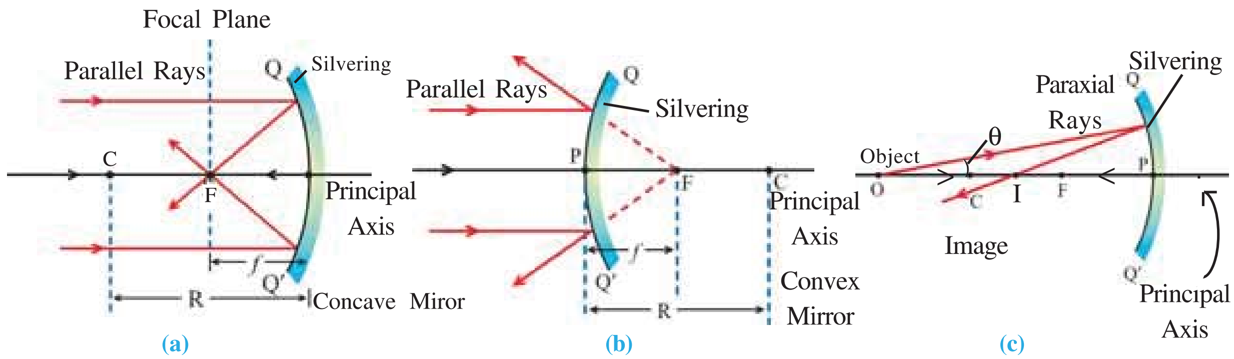


Figure 6.1 **Image Formation by Curved Mirrors**

Some useful terms used to study reflection of light by curved mirrors are as follows :

Pole : The centre of the reflecting surface of a curved mirror is called its pole (P)

Principal Axis : The imaginary line passing through the pole and the centre of curvature ($\overleftrightarrow{CP}$) is called the principal axis of the mirror.

Radius of Curvature : The radius of the spherical shell from which mirrors are made is called the radius of curvature (R) of the curved mirrors. It is the distance between C and P.

Centre of Curvature : The centre of the spherical shell from which mirrors are made is called the centre of curvature (C) of the mirror.

Aperture : The diameter of the reflecting surface (QQ') is called the aperture of the mirror.

Principal Focus : The point where the rays parallel to the principal axis meet for concave mirror or appear to meet for convex mirror on reflection is called the principal focus of the mirror.

Focal Plane : A plane passing through the principal focus and normal to the principal axis is called the focal plane of the mirror.

Focal Length : The distance between the pole and the principal focus of a mirror is called its focal length (f).

Paraxial Rays : Rays close to the principal axis are called Paraxial Rays. We shall study lens and mirrors in reference to Paraxial Rays only.

Sign Convention : In order to specify the position of the object and the image, we require a reference point and sign convention. We adopt Cartesian sign convention as follows.

(1) All the distances are measured from the pole of the mirror on the principal axis.

(2) Distances measured in the direction of the incident ray are taken positive, while those measured in the opposite direction are taken negative.

(3) Height above the principal axis is taken positive, while that below the principal axis is taken negative.

6.3 Relation between Focal Length and Radius of Curvature

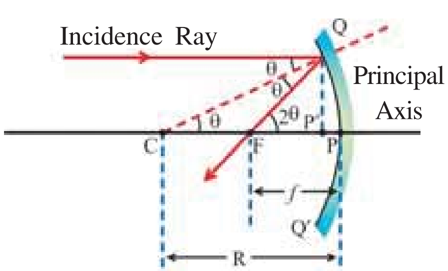


Figure 6.2 **Relation between Focal Length and Radius of Curvature**

In figure 6.2, a ray paraxial and parallel to the principal axis is shown to incident at point Q of a concave mirror of small aperture. The reflected ray passes through the principal focus. Normal drawn to the surface at point Q passes through centre of curvature. $\therefore$ CQ = CP. If the angle of incidence is θ, then the angle of reflection $\angle CQF = \theta = \angle QCF$.

From the geometry of the figure, exterior angle,

$\angle QFP = \theta + \theta = 2\theta$

Since the incident ray is paraxial and the aperture of the mirror is small, points P and P' are very close to each other. i.e., CP' ≈ CP = R

and FP' ≈ FP = f

In ΔQFP, $\sin 2\theta \approx 2\theta = \frac{QP'}{FP'} = \frac{QP}{FP}$

$$\therefore\ 2\theta = \frac{QP}{f} \Rightarrow \theta = \frac{QP}{2f} \qquad (6.3.1)$$

Similarly, from ΔCQP', $\sin\theta \approx \theta = \frac{QP'}{CP'} \approx \frac{QP}{CP}$

$$\therefore\ \theta = \frac{QP}{R} \qquad (6.3.2)$$

From equations (6.3.1) and (6.3.2) $R = 2f$ or $f = \frac{R}{2}$ (6.3.3)

Equation (6.3.3) is also true for a convex mirror. In the case of plane mirror, R is infinite, and therefore its focal length is also infinite.

6.4 Spherical Mirror Formula

Now we shall derive the relation between the object distance (u) image distance (v) and focal length (f) for a concave mirror. As shown in figure 6.3, consider a point object O on the principal axis at a distance u from the pole. Let the aperture of the mirror be small. Let the incident ray OQ makes a small angle (α) with the principal axis and gets reflected as QI. Another ray from object O moving along the axis is incident at P, and gets reflected in the direction PC. Both reflected rays meet at point I and forms the point like image.

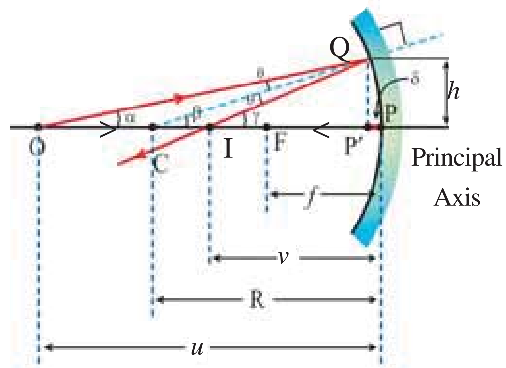


Figure 6.3 **Image of a Point Object Due to Concave Mirror**

Since the aperture of the mirror is small, distance PP' = δ is very small and can be neglected. Hence regions OPQ and IQP can be approximated by ΔOQP' and ΔIQP', respectively.

According to the laws of reflection, angle of incidence, ∠OQC = angle of reflection, ∠CQI = θ. Let CQ and IQ make angle β and γ, respectively, with the principal axis.

In ΔOCQ, exterior angle $\beta = \alpha + \theta$

In ΔCQI, exterior angle $\gamma = \beta + \theta$

Eliminating θ from above equations,

$$\alpha + \gamma = 2\beta \qquad (6.4.1)$$

Using the figure, in ΔOQP',

$$\alpha \text{ (rad)} = \frac{\text{arc QP}}{\text{OP}},$$

$$\beta \text{ (rad)} = \frac{\text{arc QP}}{\text{CP}} \quad \text{and}$$

$$\gamma \text{ (rad)} = \frac{\text{arc QP}}{\text{IP}}$$

Using these values in equation (6.4.1) we have,

$$\frac{\text{arc QP}}{\text{OP}} + \frac{\text{arc QP}}{\text{IP}} = 2\frac{\text{arc QP}}{\text{CP}}$$

$$\frac{1}{\text{OP}} + \frac{1}{\text{IP}} = \frac{2}{\text{CP}}$$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{2}{\text{R}} \text{ or } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (6.4.2)$$

Equation (6.4.2) represents the numerical relationship between object distance, image distance and focal length (or radius of curvature). While using it for calculating any of these physical quantities, we must apply sign convention. In the present case, $u \to -u$, $v \to -v$ and f (or R) $\to$ $-f$ (or $-$R)

Equation (6.4.2) is the **Gauss' equation** for curved mirrors. It is also valid for convex mirror.

6.5 Lateral Magnification

The ratio of the height of the image (h') to the height of the object (h) is called the **transverse** or **lateral** magnification (m).

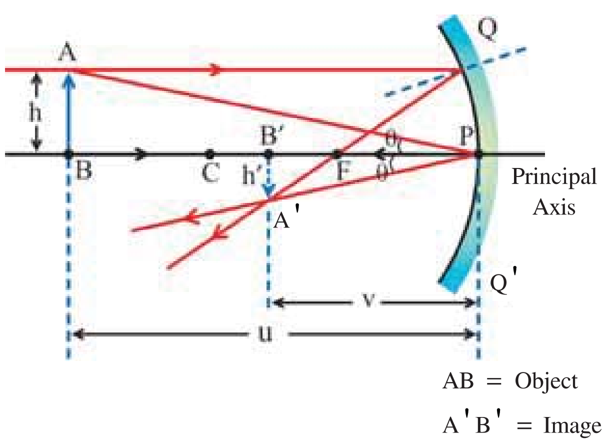


Figure 6.4 Image of an Extended Object

$$\text{i.e., } m = \frac{h'}{h} \quad (6.5.1)$$

For right angled triangles ABP and A′B′P,

$$\tan\theta = \frac{\text{AB}}{\text{BP}} = \frac{\text{A'B'}}{\text{B'P}} \quad (6.5.2)$$

But AB = h, A′B′ = $-h'$, PB = $-u$ and B′P = $-v$ (using sign convention), equation (6.5.2) becomes,

$$\frac{h}{-u} = \frac{-h'}{-v}$$

$$\therefore \quad \frac{h'}{h} = \frac{-v}{u} \qquad (6.5.3)$$

Combining equations (6.5.1) and (6.5.3)

$$m = \frac{-v}{u} \qquad (6.5.4)$$

Equation (6.5.4) is also true for convex mirror.

Illustration 1 : An object lies on the principal axis of a concave mirror with radius of curvature 160 cm. Its image appears erect at a distance 70 cm from it. Determine the position of the object and also the magnification.

Solution : The mirror equation is

$$\frac{2}{R} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \quad \frac{1}{u} = \frac{2}{R} - \frac{1}{v} = \frac{2}{-160} - \frac{1}{70} \text{ (using sign convention)}$$

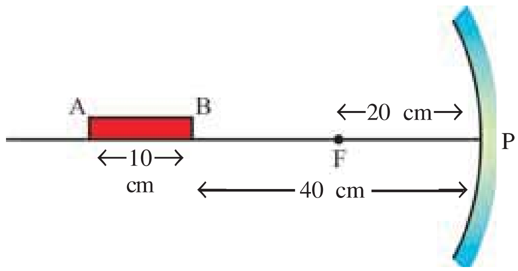
$$\therefore \quad u = -37 \text{ cm} = \frac{-15}{560}$$

i.e., The object is at a distance 37 cm in front of the mirror.

$$\text{Lateral magnification, } m = -\frac{v}{u} = -\frac{70}{-37} = 1.89$$

Illustration 2 : As shown in the figure, a thin rod AB of length 10 cm is placed on the principal axis of a concave mirror such that it's end B is at a distance of 40 cm from the mirror. If the focal length of the mirror is 20 cm, find the length of the image of the rod.

Solution : f = 20 cm and the end B is at distance 40 cm = $2f$ = R. Thus the image of B is formed at B only.



Now for end A,

u = −50 cm, f = − 20 cm, v = ?

In $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, putting these values

$$-\frac{1}{50} + \frac{1}{v} = -\frac{1}{20}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{50} - \frac{1}{20} = \frac{20-50}{20\times 50} = -\frac{30}{1000}$$

$$\therefore \quad v = -\frac{100}{3} = -33.3 \text{ cm}$$

This image A' is on the same side as the object.

Now, length of the image = 40 − 33.3 = 6.70 cm

Illustration 3 : Derive the formula for lateral magnification, $m = \frac{f}{f-u}$ for spherical mirrors; where f = focal length and u = object distance.

Solution : $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf}$

$$\therefore \quad v = \frac{fu}{u-f} \Rightarrow \frac{v}{u} = \frac{f}{u-f}$$

and $m = -\frac{v}{u} = \frac{f}{f-u}$

Note : For a plane mirror $f \to \infty$ $\therefore$ $m = 1$ (Magnitude)

6.6 Refraction of Light

When a ray of light enters obliquely from one transparent medium to another transparent medium its direction changes at the surface separating two media. This phenomenon is known as refraction.

For information only :

• When the characteristics of a medium are same at all points, it is said to be homogeneous. When the characteristics are same in all directions it is said to be isotropic.

• If a medium is not homogenous then a light ray continuously gets refracted and its path is curved.

• If the medium is not isotropic light ray refracts by different amount in different directions.

Laws of Refraction :

(1) The incident ray, refracted ray and the normal drawn to the point of incidence are in the same plane.

(2) "The ratio of the sine of the angle of incidence to the sine of the angle of refraction for the given two media is constant." This constant is called **relative refractive index** of the two media. This statement is known as the Snell's law.

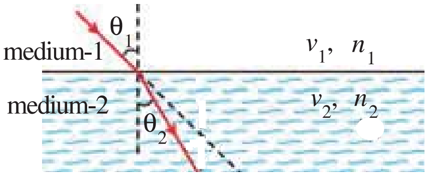


Figure 6.4 (a)

If θ_1 is the angle of incidence (in medium-1) and θ_2 is the angle of refraction (in medium-2) then,

$$\frac{\sin\theta_1}{\sin\theta_2} = n_{21}, \quad (6.6.1)$$

where n_{21} is known as the refractive index of medium-2 with respect to medium-1.

n_{21} depends on the type of media, their temperature and the wavelength of light.

Relative refractive index may also be defined in terms of speed of light in two media.

$$n_{21} = \frac{v_1}{v_2}, \quad (6.6.2)$$

where v_1 = speed of light in medium-1

and v_2 = speed of light in medium-2.

Similarly, refractive index of a medium with respect to vacuum (or in practice air) is

$$n = \frac{c}{v}. \quad (6.6.3)$$

Here, n is known as absolute refractive index. Now,

$$n_{21} = \frac{v_1}{v_2} = \frac{c}{v_2} \times \frac{v_1}{c} = \frac{n_2}{n_1} \quad (6.6.4)$$

$\therefore$ equation 6.6.1 becomes,

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

or $n_1\sin\theta_1 = n_2\sin\theta_2$ (6.6.5)

This equation (6.6.5) is known as general form of Snell's law.

For given media, if $n_2 > n_1 \Rightarrow \sin\theta_1 > \sin\theta_2$

$\therefore \theta_1 > \theta_2$

When a light ray enters from optically rarer medium to optically denser medium, angle of refraction is smaller than the angle of incidence, and the ray bends towards the normal.

If $n_2 < n_1 \Rightarrow \sin\theta_1 < \sin\theta_2$

$\therefore \theta_1 < \theta_2$

When a light ray enters from optically denser medium to optically rarer medium, angle of refraction is greater than the angle of incidence, and the ray bends away from the normal.

The medium with greater refractive index is called **optically denser** medium and the one with smaller refractive index is called optically rarer medium. This optical density is different from the mass density.

Refraction Through Compound Slab :

As shown in figure 6.5, if light passes through a compound slab, refractive index of medium-3 with respect to medium-1 can be written as

$$n_{31} = \frac{v_1}{v_3}$$

$$= \frac{v_2}{v_3} \times \frac{v_1}{v_2} = n_{32} \times n_{21} \qquad (6.6.6)$$

Also, $n_1\sin\theta_1 = n_2\sin\theta_2 = n_3\sin\theta_3 \qquad (6.6.7)$

and $n_{21} = \frac{v_1}{v_2} = \frac{1}{\left(\frac{v_2}{v_1}\right)} = \frac{1}{n_{12}}$

$\therefore n_{21} \times n_{12} = 1 \qquad (6.6.8)$

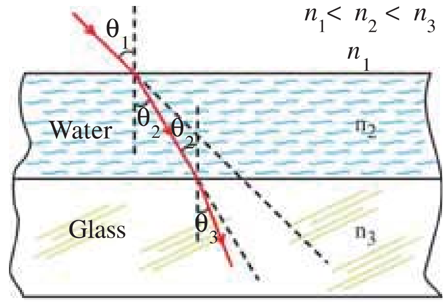


Figure 6.5 Refraction Through Compound Slab

For information only : The visibility of a transparent medium is due to the difference in its refractive index from that of the surrounding medium.

6.6.1 Lateral Shift : As shown in the figure 6.6, light rays undergo refraction twice, once from top (AB) and then from bottom (CD) surfaces of a given homogeneous medium. The emergent ray is parallel to PQR'S' ray. Here, PQR'S' is the path of light ray in absense of the other medium.

Since the emergent ray is parallel to the incident ray but shifted sidways by distance RN. This RN distance is called **lateral shift** (x). We can now calculate this lateral shift as follows :

Let n_1 and n_2 be the refractive indices of the rarer and denser medium, respectively. Also, $n_1 < n_2$. From the figure, $\angle RQN = \theta_1 - \theta_2$, RN = x.

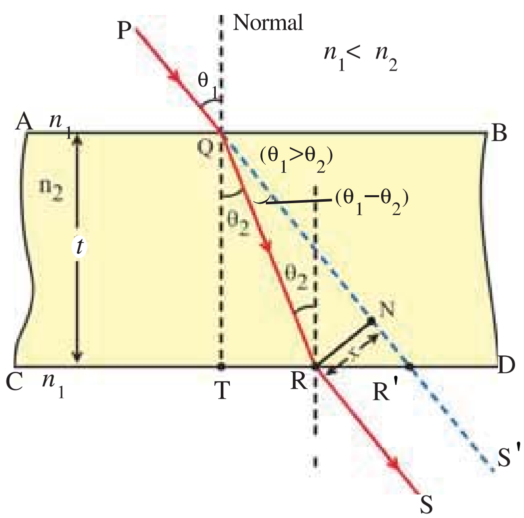


Figure 6.6 Lateral Shift Due to Rectangular Slab

From ΔQRN, $\sin(\theta_1 - \theta_2) = \frac{RN}{QR} = \frac{x}{QR}$ (6.6.9)

In ΔQTR, $\cos\theta_2 = \frac{QT}{QR}$

$\therefore QR = \frac{QT}{\cos\theta_1} = \frac{t}{\cos\theta_2}$

$\therefore$ from equation (6.6.9), $\sin(\theta_1 - \theta_2) = \frac{x}{\left(\frac{t}{\cos\theta_2}\right)}$

$$\therefore x = \frac{t\cdot\sin(\theta_1-\theta_2)}{\cos\theta_2} \qquad (6.6.10)$$

Since angle of incidence θ_1 is very small, θ_2 will also be small.

$\therefore \sin(\theta_1 - \theta_2) \approx (\theta_1 - \theta_2)$ & $\cos\theta_2 \approx 1$

$$\therefore x = \frac{t\cdot(\theta_1-\theta_2)}{1}$$

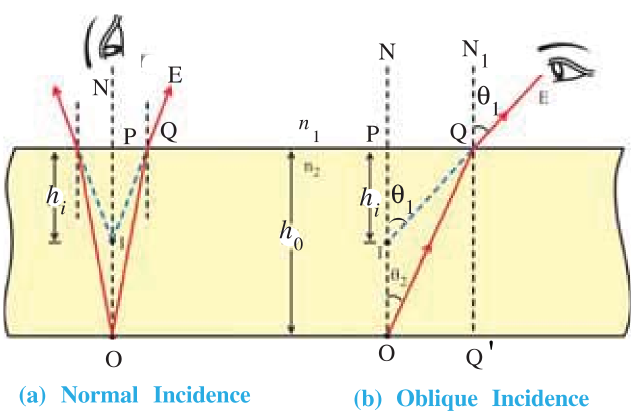
$$x = t\cdot\theta_1\left(1-\frac{\theta_2}{\theta_1}\right) \qquad (6.6.11)$$

But according to Snell's law, $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \approx \frac{\theta_1}{\theta_2}$

$\therefore$ From equation (6.6.11),

$$x = t\cdot\theta_1\left(1-\frac{n_1}{n_2}\right)$$

6.6.2 Real Depth and Virtual Depth : Another manifestation of lateral shift is the apparent depth or hight seen through transparent medium.



(a) Normal Incidence (b) Oblique Incidence

Figure 6.7

As shown in figure 6.7, an object O is kept at depth h_0 in a denser medium (e.g. water) with refractive index n_2. In figure 6.7(b) Ray OQ on refraction moves along QE at the interface. If EQ is extended in the denser medium it meets the normal PN at point I.

So the observer sees the image of object O at position I. Here, PO = h_0 = real depth of an object.

PI = h_i = virtual depth of an image.

From figure 6.7(a) even when $\theta_1 = 0$, $h_0 \neq h_i$ (You will see this as a case of equation (6.8.10)).

But as θ_1 increases h_i becomes smaller compared to h_0. Also, the object appears curved when viewed obliquely through the refracting medium.

6.6.3 Real Height and Virtual Height :

Suppose an observer (e.g., fish) is inside a denser medium (e.g., water). It sees the eye (E) of a person at point E' instead of E. i.e., object is appeared lifted up (figure 6.8).

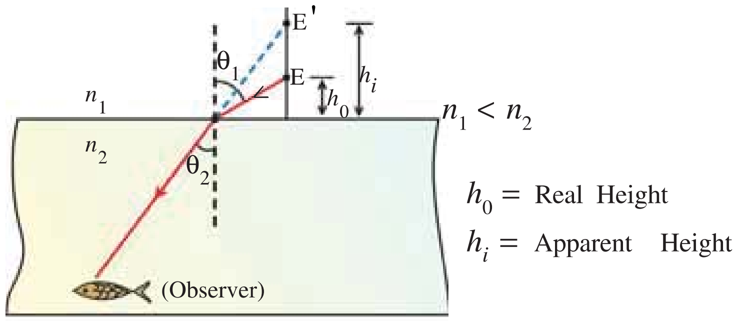


Figure 6.8 Virtual Height

Illustration 4 : Assuming that the angle of incidence at a refractive surface is sufficiently small, derive the relation between real depth, apparent depth and refractive index.

Solution : In figure 6.7, refractive index of denser medium = n_2 and refractive index of rarer medium = n_1. Real depth of the object O is PO = h_O. Depth of the image, i.e., apparent depth of the object = PI = h_i

Applying Snell's law at point Q,

$n_2 \sin\theta_2 = n_1 \sin\theta_1$

For nearly normal incidence, θ_1 and θ_2 are very small.

$\therefore \sin\theta \approx \theta \approx \tan\theta$

$n_2 \tan\theta_2 = n_1 \tan\theta_1$

But, $\tan\theta_2 = \frac{PQ}{PO} = \frac{PQ}{h_O}$ and $\tan\theta_1 = \frac{PQ}{PI} = \frac{PQ}{h_i}$

Using this in equation (1) $n_2\left(\frac{PQ}{h_O}\right) = n_1\left(\frac{PQ}{h_i}\right)$

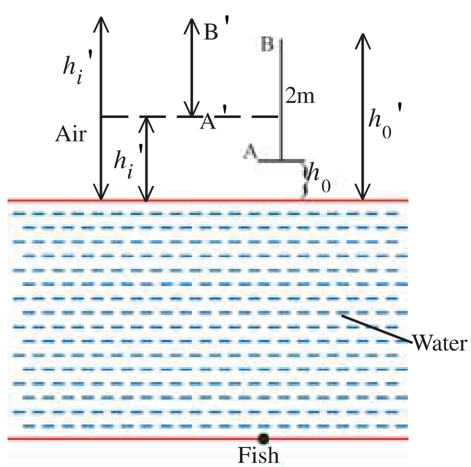
$$\therefore \frac{n_2}{n_1} = \frac{h_O}{h_i} \Rightarrow \frac{h_i}{h_O} = \frac{n_1}{n_2} = \frac{n(\text{rarer})}{n(\text{denser})}$$

Note : It can be proved that if an object kept in a rarer medium, at height h_O from the interface, is viewed normally from the denser medium and it appears to be at height h_i ($h_i > h_O$), then

$$\frac{h_i}{h_O} = \frac{n(\text{denser})}{n(\text{rarer})}$$

Illustration 5 : A swimmer is diving in a swimming pool vertically down with a velocity of 2 m s^{-1}. What will be the velocity as seen by a stationary fish at the bottom of the pool, right below the diver ? Refractive index of water is 1.33.

Solution : In the figure, vertical distance 2m is shown by AB. The height of A from the surface of water is h_O. Suppose it's apparent height is h_i ($h_i > h_O$).



$$\therefore \frac{h_i}{h_O} = \frac{n(\text{water})}{n(\text{air})}$$

$$\therefore h_i = h_O \times 1.33 \qquad (1)$$

Now the real height of B, $h_O' = (h_O + 2)$m

$\therefore$ if it's apparent height is h'_i, then

$$\frac{h_i'}{h_O'} = \frac{n(\text{water})}{n(\text{air})} = 1.33$$

$\therefore\ h'_i = h_O' \times 1.33$

$= (h_O + 2) \times 1.33$ (2)

From equations (1) and (2), the apparent distance, seen by the fish

$= h_i' - h_i = (h_O + 2) \times 1.33 - h_O \times 1.33$

$= 2 \times 1.33 = 2.66$ m

So the fish will see the swimmer falling with a speed of 2.66 m s^{-1}.

6.7 Total Internal Reflection

When light ray enters from one transparent medium to another, it is partially reflected and partially transmitted at an interface. This is true even if light is incident normally to a surface separating two media. In this case, intensity of reflected light is given by

$$I_r = I_0\left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2 \quad (6.7.1)$$

where, I_O = intensity of incident light

I_r = intensity of reflected light.

n_1 = refractive index of the medium–1

n_2 = refractive index of the medium–2

For air (n_2 = 1.0) and glass (n_1 = 1.5), I_r is 4% of the incident intensity. It is to be noted that equation 6.7.1 is true for normal incidence only. For other cases, I_r also depends on the angle of incidence.

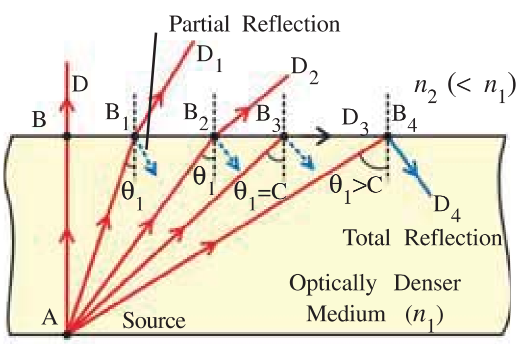


Figure 6.9 Total Internal Reflection

Here, A is a point object (or a light source) in a denser medium. Ray AB, AB_1, AB_2, ... get partially reflected and partially transmitted at points B, B_1, B_2 ... at the interface. It is observed that as the angle of incidence increases (going from B $\rightarrow B_1 \rightarrow B_2 \rightarrow$ ....) the angle of reflection ray also increases. It happens that at particular angle of incidence, refracted ray moves parallel to the surface separating two media. For this particular case, angle of refraction is 90°.

The angle of incidence for which the angle of refraction is 90° is called the critical angle (C) of the denser medium with respect to the rarer medium.

In this situation the interface appears bright. Using Snell's law for the critical angle of incidence,

$n_1\sin\theta_1 = n_2\sin\theta_2$

when $\theta_1 = C$, $\theta_2 = 90°$

$\therefore\ n_1\sin C = n_2$

$\therefore\ \sin C = \frac{n_2}{n_1}$

If rarer medium is air, i.e. $n_2 = 1$

$\therefore\ \sin C = \frac{1}{n_1} = \frac{1}{n}$ (Let $n_1 = n$)

$$\text{or } C = \sin^{-1}\left(\frac{1}{n}\right) \quad (6.7.2)$$

At the critical angle, the reflected ray is known as the **critical ray**.

Now if the angle of incidence is increased slightly more than the critical angle, the intensity of reflected light immediately increases, and the incident ray gets completely (i.e. 100%) reflected back into the denser medium. This is called total internal reflection. It is true for any of incidence greater than the critical angle. In this situation, the surface separating the two media behaves like a perfect mirror. It is to be noted that the total internal reflection obeys the laws of reflection.

For Information Only :

When total internal reflection is studied with respect to electromagnetic waves, it is found that a very small portion of incident light enters into the rarer medium upto a distance equals few wavelengths. Though, its intensity is diminutive. This in quantum mechanics is called **tunneling** effect.

Illustration 6 : As shown in figure, a ray of light is incident at angle of 30^o on a medium at $y = 0$ and proceeds ahead in the medium. The refractive index of this medium varied with distance y as given by,

$n(y) = 1.6 + \frac{0.2}{(y+1)^2}$ where y is in cm. What is the angle formed by the ray with the normal at a very large depth ?

Solution : Suppose the angle is θ at distance y in the medium.

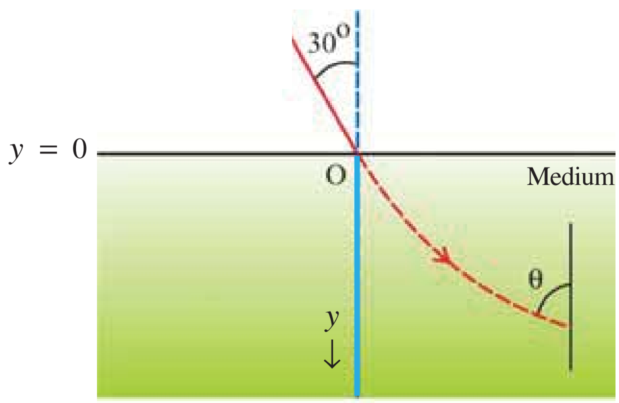
Appyling Snell's law at this point,

$n(y)\sin\theta = C$, where C = constant (1)

This formula is true for all the points.

Applying it to point O,

$n(0)\sin30^o = C$



But, $n(0) = 1.6 + \frac{0.2}{(0+1)^2} = 1.8$

$\therefore\ 1.8 \times \frac{1}{2} = C$

$\therefore\ C = 0.9$

Putting this value in (1), $n(y)\sin\theta = 0.9$

$$\therefore \left\{1.6+\frac{0.2}{(y+1)^2}\right\} sin\theta = 0.9 \quad \therefore \sin\theta = \frac{0.9}{1.6+\frac{0.2}{(y+1)^2}}.$$

When y is very large, taking $y \to \infty$; we get $\sin\theta = \frac{0.9}{1.6}$

$\therefore\ \theta = 34^o\ 14'$

6.7.1 Uses of Total Internal Reflection :

(1) The refractive index of diamond is 2.42 and its critical angle is 24.41^o. Thus, with proper cutting of its faces, whatever the angle at which light enters into the diamond, it undergoes many total internal reflections. Hence it looks bright from the inside, and we call the diamond is sparkling.

(2) For a glass with refractive index 1.50 has a critical angle for an air-glass interface,

$C = \sin^{-1}\left(\frac{1}{1.50}\right) \approx 42^o$

This angle is slightly less than 45°, which makes possible to use prisms with angles 45°–45°–90° as totally reflecting surface. (See figure 6.10).

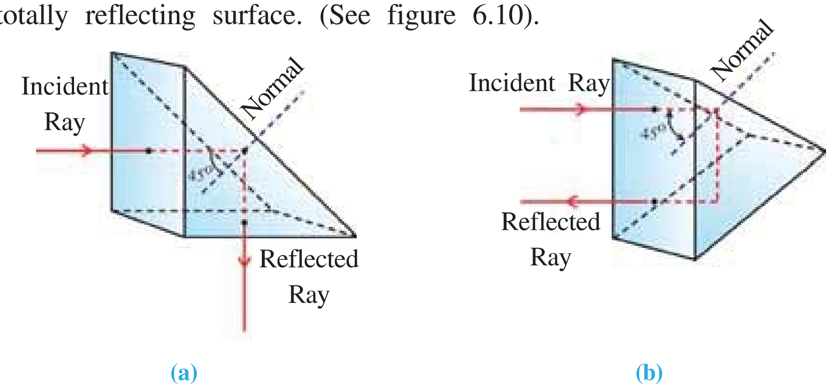


Figure 6.10 Totally Reflecting Prisms

The advantages of totally reflecting prisms over metallic reflectors are (1) superior reflection and (2) the reflecting properties are permanent and not affected by tarnishing.

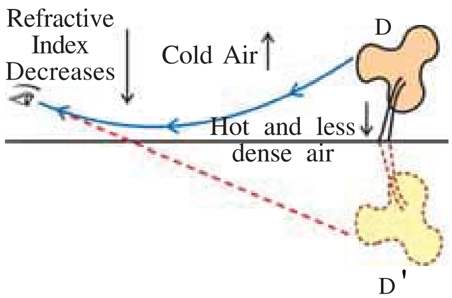


Figure 6.11 Mirage Formation

(3) Mirage : In summer, due to heat, the air in contact with the ground becomes hot while above it is cooler. Thus, air in contact with the ground is rarer and air above is denser. i.e., its refractive index increases as one moves upwards. As shown in the figure 6.11, a ray going from the top of the tree (D) to the ground is travelling continuously from a denser medium to a rarer medium. As it comes closer to the ground its angle of refraction increases and finally it undergoes total internal reflection, and enters into the eye of an observer. Thus, the image of D appears at D' to an observer, giving a feeling of image in a water surface. This phenomenon is called a mirage.

(4) Optical Fibres : The phenomenon of total internal reflection is used in optical fibres. They are made of glass or fused quartz of about 10 to 100 μm in diameter. They are in the form of long and thin fibres. The outer coating of the fibres **(cladding)** has a lower refractive index (n_1) than the core (material) of fibre (n_2). Here, $n_2 > n_1$.

In absense of the cladding layer, due to dust particles, oil or other impurities, some leakage of light may take place. In 1 m distance, in fact, light gets reflected thousands of times. Thus, if leakage occurs, light cannot travel far. Such leakage is prevented using cladding.

Fused quartz is usually used for making optical fibres because of its high transparency.

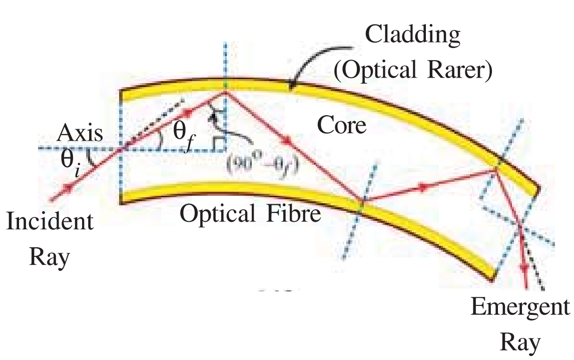


Figure 6.12 Schematic Diagram of Optical Fibre

In figure 6.12, a ray is incident at an angle θ_i to the axis of a fibre from air. θ_f is the angle of refraction. The refracted ray makes an angle θ_f with the axis of the fibre. As shown in the figure, this ray is incident on the wall of the fibre at an angle $(90^o - \theta_f)$. It is clear that if angle $(90^o - \theta_f)$ is greater than the critical angle for fibre cladding interface, the ray will undergo a total internal reflection. In short, the greater the value of $(90^o - \theta_f)$ the greater is the chance for total internal reflection. That is, a small value of θ_f is preferable. This also suggests that smaller the value of θ_i the greater are the chances of total internal reflection. Thus, for a given fibre the value of θ_i should not be greater than some particular value.

The above condition for total internal reflection can also be discussed in terms of the refractive index of the material of the fibre.

We have seen that the value $(90^o - \theta_f)$ should be greater than the critical angle. Thus, the smaller the value of critical angle, the more are the chances of total intermal reflection.

Now, $\sin C = \frac{1}{n}$ relation shows that n should be large in order to have small value of C. Thus, the material of an optical fiber should have value of n more than some minimum value. In our discussion we have taken the medium outside the optical fiber as air.

6.8 Refraction at a Spherically Curved Surface

Images can be formed by reflection as well as by refraction. Here we study the refraction at a spherical surface, i.e., at a spherical interface between two transparent media having different refractive indices. In the following discussion, we shall study refraction of paraxial rays at a spherically curved surface. This will help us to understand the image formation by lenses, though lens has two refracting surfaces. We will follow cartesian sign convention in our discussion, and the spherical surface as a very small part of the sphere.

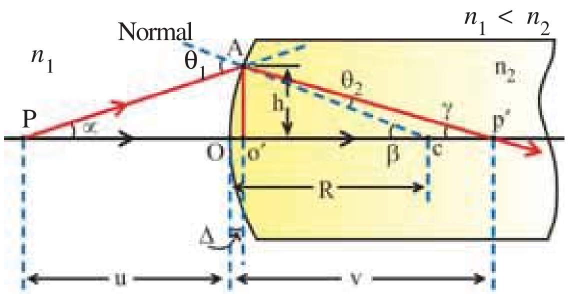


Figure 6.14 Refraction Due to Conve Curved Surface

As shown in the figure, O is the centre of the refracting surface, C is the centre of curvature, OC is the radius of curvature. A point object P is kept at a distance u from O on the principal axis.

To form image after refraction, consider two rays PO and PA from point object P.

For ray PO, angle of incidence is zero. Therefore, according to Snell's law this ray will move along OCP' without bending.

Ray PA is incident at point A on the surface. AC is the normal to the surface at point A. θ_1 is the angle of incidence. Suppose the refractive index (n_1) of the medium-1 is less than the refractive index (n_2) of the medium-2. As a result, the refracted ray bends towards the normal and moves along AP'. Let α, β and γ be the angles made by PA, CA and P'A respectively with principal axis.

Both refracted rays OP' and AP' meet at point P', and forms point like image of an object P. Here, θ_2 is the angle of refraction.

Applying Snell's law at point A,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (6.8.1)$$

Since we are considering paraxial rays, θ_1 and θ_2 are small (measured in radian)

$$\therefore\ n_1\theta_1 = n_2\theta_2 \qquad (6.8.2)$$

From figure, θ_1 is the exterior angle of ΔPAC.

$$\therefore\ \theta_1 = \alpha + \beta \qquad (6.8.3)$$

Similarly, angle β is exterior to ΔCP'A.

$\therefore\ \beta = \theta_2 + \gamma$

$\therefore\ \theta_2 = \beta - \gamma$ (6.8.4)

Using (6.8.3) and (6.8.4) in equation (6.8.2),

$n_1(\alpha + \beta) = n_2(\beta - \gamma)$

$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$ (6.8.5)

From right angled triangle O'P'A, $\tan\gamma \approx \gamma = \frac{h}{v-\Delta}$ (6.8.6)

where v = image distance

From right angled ΔO'CA, $\tan\beta \approx \beta = \frac{h}{R-\Delta}$ (6.8.7)

And from ΔPAO', $\tan\alpha \approx \alpha = \frac{h}{-u+\Delta}$, (6.8.8)

where $u \to -u$, object distance, as per the sign convention.

The curved surface considered here is a very small part of the sphere from which it is cut. Thus, Δ is negligible compared to R, u and v.

$\therefore\ \gamma \approx \frac{h}{v},\ \beta = \frac{h}{R}$ and $\alpha = \frac{h}{-u}$ (6.8.9)

Combining equations (6.8.5) and (6.8.9), $n_1\left(\frac{h}{-u}\right) + n_2\left(\frac{h}{v}\right) = (n_2 - n_1).\frac{h}{R}$

$\therefore\ \frac{-n_1}{u} + \frac{n_2}{v} = \frac{(n_2-n_1)}{R}$ (6.8.10)

Equation (6.8.10) is valid for concave surface also. Equation (6.8.10) is the general equation which relates object distance, image distance and radius of curvature of the curved surface. This equation is derived for the ray travelling **from rarer medium (with refrective index n_1) to the denser medium (with refrective index n_2)**. In the similar way when the ray travels **from denser medium (with refrective index n_2) to the rarer medium (with refrective index n_1)**, we can derive the following equation using Snell's law.

$\frac{-n_2}{u} + \frac{n_1}{v} = \frac{(n_1-n_2)}{R}$ (6.8.11)

Case : If surface is plane (plane glass slab).

i.e. R = ∞. Therefore equation (6.8.10) becomes $\frac{+n_1}{u} = \frac{n_2}{v}$

or $\frac{v}{u} = \frac{n_2}{n_1} = \frac{h'}{h}$ (see the topic of magnification).

Whether the image is real or virtual is decided by the sign convention. If image distance is positive, i.e., image is formed on the right of point O, it is real or otherwise.

6.9 Spherical Lenses

In general, a lens is an image forming device, having two bounded refracting surfaces. Of the two surfaces at least one surface is curved. For example, the following figure depicts different types of lenses.

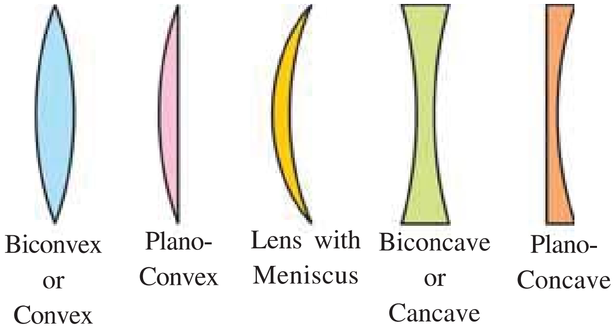


Figure 6.15 Different Type of Lenses

Since spherical surfaces are easy to construct, we first consider image formation by a spherical lens or crystal ball as strategic example.

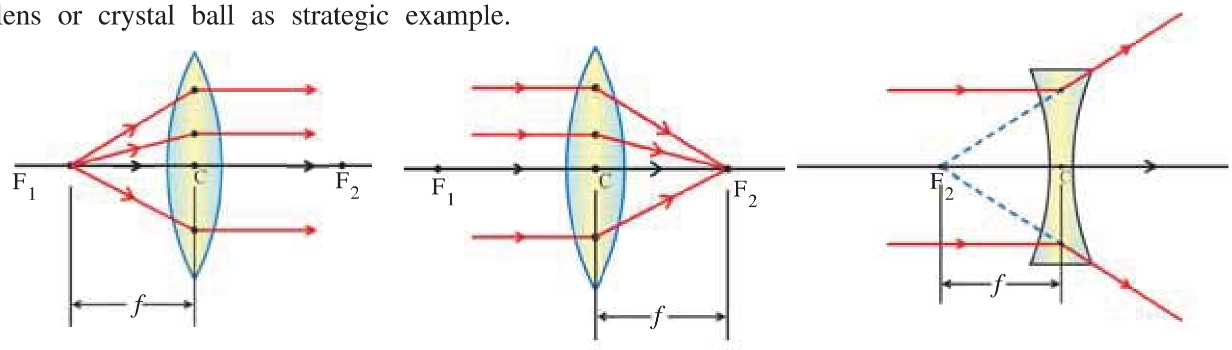


Figure 6.16 Focus of Thin Lens

If a point object is placed on the principal axis of a convex lens such that the rays refracted are parallel to the axis figure (a), then the position of the point object is called the **first principal focus** (F_1) of the lens.

If the object is situated at infinite (figures (b) and (c)), refracted rays meet (or appear to meet) for convex (or concave) lens to a point (F_2), then the position of this point is known as **second principal focus** (F_2).

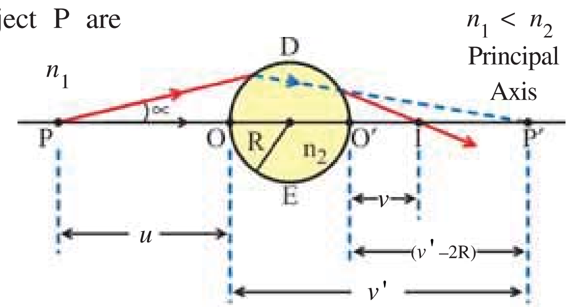
The geometrical centre of the medium of the lens is called its **optical centre** (C).

Distance of principal focus from the optical centre (C) is known as focal length (f) of the lens.

As per the sign convention, f is positive for convex lens and negative for concave lens.

Illustration 8 : Obtain the expression for image distance in terms of the radius of curvature for crystal.

Solution : Here, the rays coming from point object P are refracted twice at surfaces DOE and DO'E, respectively, before forming the final image. But for the sake of understanding, we consider both the refraction separately. Using the formula for spherical surface (equation 6.7.10) at both the surfaces we can determine the position of the (final) image.



At the surface DOE,

$$\frac{-n_1}{(-u)} + \frac{n_2}{v'} = \frac{(n_2 - n_1)}{R} \qquad (1)$$

(We have used Cartesian sign convention.)

Let $u > R$. In this case, v' will be large and positive. That is, image of P due to spherical surface DOE will form at point P' on the right and far from the ball.

Now, for surface DO'E image P' will behave as virtual object. Therefore at the surface DO'E,

$$-\frac{n_2}{(v' - 2R)} + \frac{n_1}{v} = \left(\frac{n_1 - n_2}{R}\right) \qquad (2)$$

Since v' is very large, $(v' - 2R)$ is positive. This gives v to be positive, i.e., the final point image will form on the right of the surface DO'E.

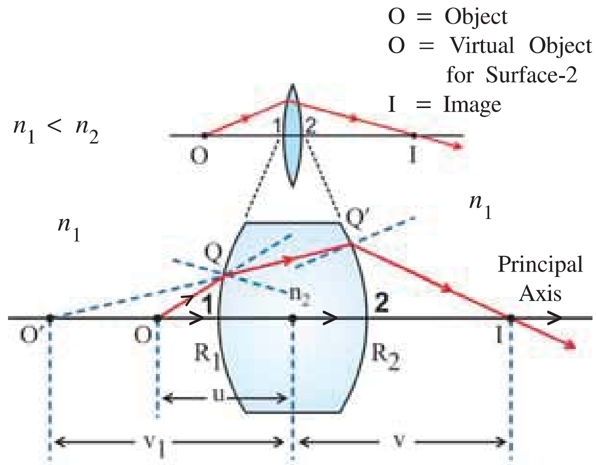


Figure 6.17 Image Formation Due to Thin Lens

6.9.1 Thin Lens : The lens for which the distance between the two refracting surfaces is negligible as compared to the object distance, the image distance and radius of curvature is called a thin lens. In general, radii of curvatures of the two refracting surfaces need not be equal. Being thin lens, the distance can be measured from either surface or even from the centre of the lens.

To obtain the relation between object distance, image distance and radii of curvatures for thin lens consider the following case as shown in the figure 6.17.

To understand, how final image due to thin lens is formed, assume that the two refracting surfaces are separated. Thus, the final image (I) is assumed to be formed due to two refractions at curved surface-1 and then due to surface-2, respectively.

The object O is in the medium having refractive index n_1. The incident ray OQ is refracted at surface-1 into the denser medium with refractive index n_2. (Here $n_2 > n_1$). The image is formed at O'. For the refraction at surface-1 using equation (6.8.10), we can write,

$$\frac{-n_1}{u} + \frac{n_2}{v_1} = \frac{(n_2 - n_1)}{R_1} \qquad (6.9.1)$$

Here, u = object distance and v_1 = image distance.

This image O' serves as virtual object for surface-2. For surface-2 the ray QQ' travelling from denser medium is refracted into rarer medium and meets the axial ray from O at I. Thus I is the final image. For refraction at surface-2, using equation (6.8.11), we can write,

$$\frac{-n_2}{v_1} + \frac{n_1}{v} = \frac{(n_1 - n_2)}{R_2} = \frac{(n_2 - n_1)}{-R_2} \qquad (6.9.2)$$

Here, v_1 = object distance for surface-2 and v = image distance.

Adding equations (6.9.1) and (6.9.2), we have

$$\frac{-n_1}{u} + \frac{n_1}{v} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore\ -\frac{1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.3)$$

Equation (6.9.3) is the disired equation. While using it in practice, proper sign convention should be employed.

6.9.2 Lens-Maker's Formula :

If medium on both sides of a lens is same, and object is at infinite (i.e., $u = \infty$) then $v = f$. From equation (6.9.4)

$$\frac{1}{f} - \frac{1}{\infty} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore\ \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.4)$$

Equation (6.9.4) is known as **lens-maker's** formula. It is named so because it enables one to calculate focal length and radii of curvatures of the lens.

When equations (6.9.3) and (6.9.4) are compared, we have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (6.9.5)

This equation is known as **Gauss'** formula for a lens.

From equation (6.9.4), if the lens turns around, i.e. R_1 and R_2 get interchanged, then also for proper change in sign, f will be found to be same. Therefore, for a thin lens the focal length is independent of the order of the surfaces. If medium-1 is air (i.e., $n_1 = 1$) and let refractive index of medium-2 be $n_2 = n$, equation (6.9.4) becomes

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.6)$$

For information only : Most general form of lens–maker's formula is,

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \left(\frac{n-1}{n}\right)\cdot\frac{t}{R_1 \cdot R_2};$$

where 't' is the thickness of the lens. For thin lens t is negligible and equation (6.9.6) can be recovered. Above equation also suggests that for thick lens, i.e., t is large, and R_1 and R_2 are small, second term contributes significantly. Thus, for thick lens f is small, i.e., **thick lens converges or diverges strongly.**

6.9.3 Newton's Formula : As we have observed that lens-maker's formula relates radii of curvatures and refractive index of the lens to its focal length. We can also derive an expression relating focal length to image and object distances, which we call **lens user's** formula or **Newton's** formula.

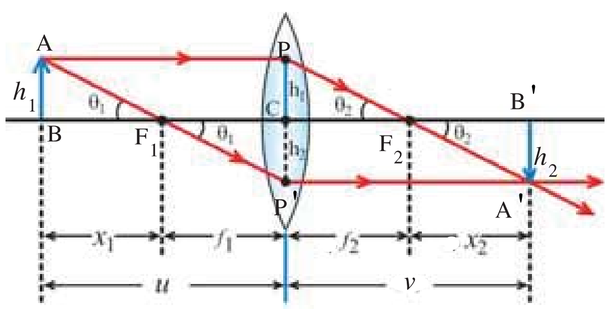


Figure 6.18 Extra Focal Distances of a Convex Lens

On the left of the lens, ΔABF_1 and $\Delta CF_1P'$ are similar triangles. Therefore,

$$\frac{h_1}{x_1} = \frac{h_2}{f_1} \text{ (writing only magnitude)} \qquad (6.9.7)$$

Similarly, for right of the lens,

$$\frac{h_1}{f_2} = \frac{h_2}{x_2} \qquad (6.9.8)$$

Writing combinedly for the ratio $\frac{h_1}{h_2}$,

$$\frac{h_1}{h_2} = \frac{x_1}{f_1} = \frac{f_2}{x_2} \qquad (6.9.9)$$

$$\therefore x_1 \cdot x_2 = f_1 \cdot f_2 \qquad (6.9.10)$$

Equation (6.9.10) is known as the Newton's lens formula. **Here, x_1 and x_2 are known as extra focal object distance and extra focal image distance.** Since these distances are measured from focii rather than from the lens, Newton's formula can be used equally for thin and thick lenses.

When $f_1 = f_2 = f$ (say), equation (6.9.10) becomes

$$x_1 \cdot x_2 = f^2 \qquad (6.9.11)$$

6.9.4 Conjugate Points and Conjugate Distances

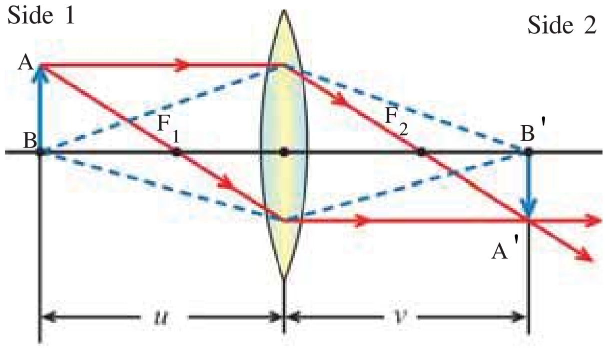


Figure 6.19 Conjugate Points and Distances

As shown in the figure 6.19, all the rays from point A and B are brought to focus at poins A' and B', respectively. Thus, A'B' is the image of an object AB. The principle of reversibility for light rays permits interchange in positions of image and an object. That is, if A'B' is an object, AB becomes the image. Thus, object and image are **conjugate.** Points A and A', and B and B' are called **conjugate points.**

Now, by keeping image distance as the object distance image will form at the object distance. That is, image and object distances are **conjugate distances.**

6.10 Magnification

Convex lenses are used properly to form a magnified image.

$$\text{Magnification, } m = \frac{\text{size of the image}}{\text{size of the object}} \tag{6.10.1}$$

Since for three-dimensional object the image will also three dimensional, correspondingly we have three types of magnifications. Lateral magnification, longitudinal magnification and angular magnification. We discuss only the lateral magnification below.

Lateral Magnification

Lateral magnification is also called as transverse magnification. It is defined as the ratio of height of an image (h_2) to that of the object (h_1) from the figure 6.18,

$$|m| = \frac{h_2}{h_1} \tag{6.10.2}$$

According to Cartesian sign convention, height measured above the principal axis is taken positive and below the principal axis it is negative. Hence, the lateral magnification is positive for erect image and negative for a inverted image. Also, from the figure 6.18,

$$\frac{h_1}{u} = \frac{h_2}{v} \text{ (only magnitude)}$$

$$\therefore\ m = \frac{h_2}{h_1} = \frac{v}{u} \tag{6.10.3}$$

From equation (6.9.10),

$$m = \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \tag{6.10.4}$$

6.11 Power of a Lens

It is defined as the converging or diverging capacity of a lens. General form of lens-maker's formula suggests that the thicker the lens, smaller is the focal length and higher is the convergence or divergence. Thus, converging or diverging ability of a lens is inversely proportional to its focal length.

$$\therefore\ \text{Power of a lens, } P = \frac{1}{f} \tag{6.11.1}$$

For convex lens power is positive, while for the concave lens it is negative.

Its SI unit is m^{-1} or diopter (D).

i.e., 1D = 1 m^{-1}

When an optician prescribes lens of + 2.0 D, it means a convex lens of focal length = $\frac{1}{2}$ = 0.5 m.