UNIT–IV : VECTORS AND THREE-DIMENSIONAL GEOMETRY

CHAPTER 10

Term-II

VECTORS

Syllabus

- ***Vectors and Scalars, Magnitude and direction of a vector. Direction cosines and direction ratios of a vector , Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.***

STAND ALONE MCQs (1 Mark each)

Q. 1. If θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

(A) $0 < \theta < \frac{\pi}{2}$ (B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$ (D) $0 \leq \theta \leq \pi$

Ans. Option (B) is correct.

Explanation: Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

It is known that, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$\therefore \quad \vec{a}.\vec{b} \geq 0$

$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \quad \cos\theta \geq 0$

$\left[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}\right]$

$\Rightarrow \quad 0 \leq \theta \leq \frac{\pi}{2}$.

Q. 2. Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{2\pi}{3}$

Ans. Option (D) is correct.

Explanation: Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ be the angle between them.

Then, $|\vec{a}+\vec{b}| = |\vec{b}| = 1$.

Now, $\vec{a}+\vec{b}$ is a unit vector if

$|\vec{a}+\vec{b}| = 1$

$\Rightarrow \quad (\vec{a}+\vec{b})^2 = 1$

$\Rightarrow \quad (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$

$\Rightarrow \quad \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}\vec{b} = 1$

$\Rightarrow \quad |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2 = 1$

$\Rightarrow \quad 1^2+2|\vec{a}||\vec{b}|\cos\theta+1^2 = 1$

$\Rightarrow \quad \cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \theta = \frac{2\pi}{3}$

So that, $|\vec{a}+\vec{b}|$ is a unit vector if $\theta = \frac{2\pi}{3}$.

Q. 3. The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ is

(A) 0 (B) −1

(C) 1 (D) 3

Ans. Option (C) is correct.

Explanation :

$$\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j}) = \hat{i}.\hat{i}+\hat{j}.(-\hat{j})+\hat{k}.\hat{k}$$
$$= 1-\hat{j}.\hat{j}+1$$
$$= 1-1+1$$
$$= 1$$

Q. 4. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) π

Ans. Option (B) is correct.

Explanation: Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$$
$$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$
$$\Rightarrow \quad \cos\theta = \sin\theta$$
$$\left[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}\right]$$
$$\Rightarrow \quad \tan\theta = 1$$
$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

So that, $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to $\frac{\pi}{4}$.

Q. 5. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans. Option (B) is correct.

Explanation :

It is given that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$.

We know that $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a}\times\vec{b}$ is a unit vector if

$$|\vec{a}\times\vec{b}| = 1$$
$$\Rightarrow \quad \left||\vec{a}||\vec{b}|\sin\theta\hat{n}\right| = 1$$
$$\Rightarrow \quad |\vec{a}||\vec{b}||\sin\theta| = 1$$
$$\Rightarrow \quad 3\times\frac{\sqrt{2}}{3}\times\sin\theta = 1$$
$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

So that, $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

Q. 6. Area of a rectangle having vertices *A, B, C* and *D* with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) 4

Ans. Option (C) is correct.

Explanation : The position vectors of vertices *A, B, C* and *D* of rectangle *ABCD* are given as :

$$\overline{OA} = -\hat{i}+-\hat{j}+\hat{k}$$
$$\overline{OB} = \hat{i}+\frac{1}{2}\hat{j}+4\hat{k},$$
$$\overline{OC} = \hat{i}-\frac{1}{2}\hat{j}+4\hat{k},$$
$$\overline{OD} = -\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$$

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of the given rectangle are given as :

$$\overrightarrow{AB} = (1+1)\hat{i}+\left(\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k}$$
$$= 2\hat{i}$$
$$\overrightarrow{BC} = (1-1)\hat{i}+\left(-\frac{1}{2}-\frac{1}{2}\right)\hat{j}+(4-4)\hat{k}$$
$$= -\hat{j}$$
$$\therefore \quad \overrightarrow{AB}\times\overrightarrow{BC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 0 & -1 & 0\end{vmatrix}$$
$$= \hat{k}(-2)$$
$$= -2\hat{k}$$
$$\Rightarrow \quad |\overrightarrow{AB}\times\overrightarrow{BC}| = 2$$

Now, it is known that the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a}\times\vec{b}|$.

Therefore, the area of the given rectangle is $|\overrightarrow{AB} \times \overrightarrow{BC}| = 2$sq. units.

Q. 7. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(A) $\lambda = 1$ **(B)** $\lambda = -1$

(C) $a = |\lambda|$ **(D)** $a = \frac{1}{|\lambda|}$

Ans. Option (D) is correct.

Explanation :

Vector $\lambda\vec{a}$ is a unit vector if

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \quad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \quad [|\vec{a}| = a]$$

Therefore, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$.

Q. 8. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect :

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Ans. Option (D) is correct.

Explanation :

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have

$\vec{b} = \lambda\vec{a}$ (For some scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then,

$$\vec{b} = \lambda\vec{a}$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

So that, the respective components of $\vec{a}$ and $\vec{b}$ are proportional. However, vectors $\vec{a}$ and $\vec{b}$ can have different directions.

Q. 9. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

(A) $\hat{i} - 2\hat{j} + 2\hat{k}$ **(B)** $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ **(D)** $9(\hat{i} - 2\hat{j} + 2\hat{k})$

Ans. Option (C) is correct.

Explanation :

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

∴ Vector in the direction of $\vec{a}$ with magnitude 9

$$= 9\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

Q. 10. The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 :1 is

(A) $\frac{3\vec{a} - 2\vec{b}}{2}$ **(B)** $\frac{7\vec{a} - 8\vec{b}}{4}$

(C) $\frac{3\vec{a}}{4}$ **(D)** $\frac{5\vec{a}}{4}$

Ans. Option (D) is correct.

Explanation :

Let the position vector of the R divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$.

∴ Position vector, $R = \frac{3(\vec{a} + \vec{b}) + 1(2\vec{a} - 3\vec{b})}{3 + 1}$

Since, the position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ration $m : n$ internally, is given by $\frac{m\vec{q} + n\vec{p}}{m + n}$.

$$\therefore R = \frac{5\vec{a}}{4}$$

Q. 11. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is :

(A) $-\hat{i} + 12\hat{j} + 4\hat{k}$ **(B)** $5\hat{i} + 2\hat{j} - 4\hat{k}$

(C) $-5\hat{i} + 2\hat{j} + 4\hat{k}$ **(D)** $\hat{i} + \hat{j} + \hat{k}$

Ans. Option (C) is correct.

Explanation:

Required vector $= (-3 - 2)\hat{i} + (7 - 5)\hat{j} + (4 - 0)\hat{k}$

$= -5\hat{i} + 2\hat{j} + 4\hat{k}$

Q. 12. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b} = 2\sqrt{3}$ is :

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{2}$

Ans. Option (B) is correct.

Explanation :

Here, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$ and $\vec{a}.\vec{b} = 2\sqrt{3}$ [Given]

We know that,

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\Rightarrow \quad 2\sqrt{3} = \sqrt{3}.4.\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$$

$$\therefore \quad \theta = \frac{\pi}{3}$$

Q. 13. Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal.

(A) 0 (B) 1

(D) $\frac{3}{2}$ (D) $\frac{-5}{2}$

Ans. Option (D) is correct.

Explanation :

Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal, *i.e.*, $\vec{a}.\vec{b} = 0$

$$\therefore (2\hat{i} + \lambda\hat{j} + \hat{k})\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \quad 2 + 2\lambda + 3 = 0$$

$$\therefore \quad \lambda = \frac{-5}{2}$$

Q. 14. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) $\frac{2}{5}$

Ans. Option (A) is correct.

Explanation:

Let $\vec{a} = 3\hat{i} - 6\hat{j} + \hat{k}$

and $\vec{b} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$

Since, $\vec{a} \parallel \vec{b}$

$$\Rightarrow \quad \frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{2}{3}$$

Q. 15. The vectors from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then the area of triangle OAB is

(A) 340 (B) $\sqrt{25}$

(C) $\sqrt{229}$ (D) $\frac{1}{2}\sqrt{229}$

Ans. Option (D) is correct.

Explanation :

$$\text{Area of } \Delta OAB = \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}|$$

$$= \frac{1}{2}|(2\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})|$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|\left[\hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6)\right]\right|$$

$$= \frac{1}{2}|-9i + 2\hat{j} + 12\hat{k}|$$

$$\text{Area of } \Delta OAB = \frac{1}{2}\sqrt{(81 + 4 + 144)}$$

$$= \frac{1}{2}\sqrt{229}$$

Q. 16. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(A) $\vec{a}^2$ (B) $3\vec{a}^2$

(C) $4\vec{a}^2$ (D) $2\vec{a}^2$

Ans. Option (D) is correct.

Explanation :

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\therefore \quad \vec{a}^2 = x^2 + y^2 + z^2$$

$$\therefore \quad \vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}[0] - \hat{j}[-z] + \hat{k}[-y]$$

$$= z\hat{j} - y\hat{k}$$

$$\therefore \quad (\vec{a} \times \hat{i})^2 = (z\hat{j} - y\hat{k})(z\hat{j} - y\hat{k})$$

$$= y^2 + z^2$$

Similarly, $(\vec{a} \times \hat{j})^2 = x^2 + z^2$ and $(\vec{a} \times \hat{k})^2 = x^2 + z^2$

$$(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y$$

$$= 2(x^2 + y^2 + z^2)$$

$$= 2\vec{a}^2$$

Q. 17. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is

(A) 5 (B) 10

(C) 14 (D) 16

Ans. Option (D) is correct.

Explanation :

Here, $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$ [Given]

$$\therefore \quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$12 = 10 \times 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{12}{20}$$

$$= \frac{3}{5}$$

$$\Rightarrow \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}}$$

$$\sin\theta = \pm\frac{4}{5}$$

$$\therefore \quad |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$$

$$= 10 \times 2 \times \frac{4}{5}$$

$$= 16$$

Q. 18. The vectors $\lambda\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+\lambda\hat{j}-\hat{k}$ and $2\hat{i}-\hat{j}+\lambda\hat{k}$ are coplanar, if

(A) $\lambda = -2$ **(B)** $\lambda = 0$

(C) $\lambda = 1$ **(D)** $\lambda = \;\; 1$

Ans. Option (A) is correct.

Explanation : Let $\vec{a} = \lambda\hat{i}+\hat{j}+2\hat{k}$, $\vec{b} = \hat{i}+\lambda\hat{j}-\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+\lambda\hat{k}$

For $\vec{a}, \vec{b}$ and $\vec{c}$ to be coplanar,

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(\lambda^2-1)-1(\lambda+2)+2(-1-2\lambda) = 0$$

$$\Rightarrow \quad \lambda^3-\lambda-\lambda-2-2-4\lambda = 0$$

$$\Rightarrow \quad \lambda^3-6\lambda-4 = 0$$

$$\Rightarrow \quad (\lambda+2)(\lambda^2-2\lambda-2) = 0$$

$$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \frac{2\pm\sqrt{12}}{2}$$

$$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \frac{2\pm2\sqrt{3}}{2}$$

$$= 1\pm\sqrt{3}$$

Q. 19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(A) 1 **(B)** 3

(C) $\frac{-3}{2}$ **(D)** None of these

Ans. Option (C) is correct.

Explanation :

We have, $\vec{a}+\vec{b}+\vec{c} = 0$ and , $\vec{b}^2 = 1$, $\vec{c}^2 = 1$

$$\therefore \quad (\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$\Rightarrow \quad \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+ \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}^2 = 0$$

$$\Rightarrow \quad \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a},\ \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{b} \text{ and } \vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{c}]$$

$$\Rightarrow \quad 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\frac{3}{2}$$

Q. 20. The projection vector of $\vec{a}$ and $\vec{b}$ is

(A) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\vec{b}$ **(B)** $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ **(D)** $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\hat{b}$

Ans. Option (A) is correct.

Explanation: Projection vector of $\vec{a}$ on $\vec{b}$ is given by,

$$= \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\vec{b}$$

$$= \left(\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\right)\cdot\vec{b}$$

Q. 21. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|\vec{c}| = 5$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(A) 0 **(B)** 1

(C) –19 **(D)** 38

Ans. Option (C) is correct.

Explanation :

Here, $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $\vec{a}^2 = 4$, $\vec{b}^2 = 9$, $\vec{c}^2 = 25$

$$\therefore \quad (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = \vec{0}$$

$$\Rightarrow \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+ \vec{c}\cdot\vec{b}+\vec{c}^2 = \vec{0}$$

$$\Rightarrow \quad \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$$

$$\Rightarrow \quad 4+9+25+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \frac{-38}{2}$$

$$= -19$$

Q. 22. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{a}|$ is

(A) [0, 8] **(B)** [–12, 8]

(C) [0, 12] **(D)** [8, 12]

Ans. Option (C) is correct.

Explanation:

We have, $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$

$$\therefore \quad |\lambda\vec{a}| = |\lambda||\vec{a}|$$

$$= \lambda|4|$$

$$\Rightarrow \quad |\lambda\vec{a}| = |-3|4$$

$$= 12,$$

at $\lambda = -3$

$|\lambda\vec{a}| = |0|4 = 0,$

at $\lambda = 0$

and $|\lambda\vec{a}| = |2|$

$4 = 8,$

at $\lambda = 2$

So, the range of $|\lambda\vec{a}|$ is [0,12].

Q. 23. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(A) one **(B)** two

(C) three **(D)** infinite

Ans. Option (B) is correct.

Explanation: The number of vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is c (say)

i.e., $\vec{c} = \pm(\vec{a} \times \vec{b})$.

So, there will be two vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

Q. 24. Which of the following statement is true.

(A) $\vec{a}$ and $\overrightarrow{-a}$ are collinear

(B) Two collinear vectors are always equal in magnitude

(C) Two vectors having same magnitude are collinear

(D) Two collinear vectors having the same magnitude are equal

Ans. Option (A) is correct.

Explanation :

(A) True

Vectors $\vec{a}$ and $-\vec{a}$ are parallel to the same line.

(B) False

Collinear vectors are those vectors that are parallel to the same line.

(C) False

It is not necessary for two vectors having the same magnitude to be parallel to the same line.

(D) False

Two vectors are said to be equal if they have the same magnitude and direction, regardless of the positions of their initial points.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector be $2\hat{i} + 5\hat{j} + 3\hat{k}$.

Reason (R): The displacement vector of the particle that moves from point $P(2, 3, 5)$ to point $Q(3, 4, 5)$ is $\hat{i} + \hat{j}$.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.

The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector be $3\hat{i} + 2\hat{j} + 5\hat{k}$.

Reason (R) is correct.

The displacement vector of the particle that moves from point $P(2, 3, 5)$ to point $Q(3, 4, 5)$

$= (3-2)\hat{i} + (4-3)\hat{j} + (5-5)\hat{k}$

$= \hat{i} + \hat{j}$

Q. 2. Assertion (A): The direction of cosines of vector $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ are $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, -\frac{5}{\sqrt{45}}$.

Reason (R): A vector having zero magnitude and arbitrary direction is called 'zero vector' or 'null vector'.

Ans. Option (B) is correct.

Explanation: Assertion (A) is correct.

Direction cosines of $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ are :

$$\frac{2}{\sqrt{2^2 + 4^2 + (-5)^2}}, \frac{4}{\sqrt{2^2 + 4^2 + (-5)^2}}, \frac{-5}{\sqrt{2^2 + 4^2 + (-5)^2}}$$

Or, $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}$

Q. 3. Assertion (A): The vectors which can undergo parallel displacement without changing its magnitude and direction are called free vectors.

Reason (R): $\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b}+\vec{a}.\vec{c}$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.
> Reason (R) is the distributive property of dot product.

Q. 4. Assertion (A): The area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a}\times\vec{b}|$.

Reason (R): If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $|\vec{a}\times\vec{b}|$.

Ans. Option (C) is correct.

> ***Explanation:*** If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $\frac{1}{2}|\vec{a}\times\vec{b}|$.

Q. 5. Assertion (A): For any two vectors $\vec{a}$ and $\vec{b}$, we always have $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$.

Reason (R): The given inequality holds trivially when either $\vec{a} = 0$ or $\vec{b} = 0$ *i.e.*, in such a case

$$|\vec{a}+\vec{b}| = 0 = |\vec{a}|+|\vec{b}|.$$

Then consider

So, let us check it for $|\vec{a}| \ne 0 \ne |\vec{b}|$.

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}$$

or $$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}|\,|\vec{b}|\cos\theta$$

For $\cos\theta \le 1$, we have :

$$2|\vec{a}|\,|\vec{b}|\cos\theta \le 2|\vec{a}|\,|\vec{b}|$$

or $$|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}|\,|\vec{b}|\cos\theta \le |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}|\,|\vec{b}|$$

or $$|\vec{a}+\vec{b}|^2 \le (|\vec{a}|+|\vec{b}|)^2$$

or $$|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Q. 6. Assertion (A): The position vector of a point say $P(x, y, z)$ is $\overrightarrow{OP} = \vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ and its magnitude is $|\vec{r}| = \sqrt{x^2+y^2+z^2}$.

Reason (R): If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, then coefficient of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{r}$ *i.e.*, x, y, z are called the direction ratios of vector $\vec{r}$.

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters $P_1(6, 8, 4)$, $P_2(21, 8, 4)$, $P_3(21, 16, 10)$ and $P_4(6, 16, 10)$ **[CBSE QB-2021]**

Q. 1. What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1? (where PV stands for position vector)

(A) 0, 0, 15 : 0, 8, 6 **(B)** 15, 0, 0 : 0, 8, 6
(C) 0, 8, 6 : 0, 0, 15 **(D)** 15, 0, 0 : 6, 8, 8

Ans. Option (B) is correct.

> ***Explanation:***
>
> $$\vec{A} = PV \text{ of } P_2 - PV \text{ of } P_1$$
> $$= 21\hat{i}+8\hat{j}+4\hat{k}-(6\hat{i}+8\hat{k}+4\hat{k})$$
> $$= 15\hat{i}+0\hat{j}+0\hat{k}$$
> $$\vec{B} = PV \text{ of } P_4 - PV \text{ of } P_1$$
> $$= 6\hat{i}+16\hat{j}+10\hat{k}-(6\hat{i}+8\hat{j}+4\hat{k})$$
> $$= (0\hat{i}+8\hat{j}+6\hat{k})$$
> $\therefore$ $A(15,0,0)$ and $B(0,8,6)$

Q. 2. Write the vector in standard notation with $\hat{i}$, $\hat{j}$ and $\hat{k}$ (where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the three axes).

(A) $15\hat{i}+0\hat{j}+0\hat{k}, 0\hat{i}+8\hat{j}+6\hat{k}$

(B) $0\hat{i}+6\hat{j}+8\hat{k}, 15\hat{i}+0\hat{j}+0\hat{k}$

(C) $0\hat{i}+0\hat{j}+0\hat{k}, 0\hat{i}+8\hat{j}+6\hat{k}$

(D) $15\hat{i}+0\hat{j}+0\hat{k}, 6\hat{i}+8\hat{i}+0\hat{k}$

Ans. Option (A) is correct.

Q. 3. What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?

(A) 9, 10 (B) 15, 5

(C) 15, 10 (D) 10, 20

Ans. Option (C) is correct.

Explanation:

$$|\vec{A}| = \sqrt{(15)^2+0^2+0^2}$$
$$= 15 \text{ units}$$
$$|\vec{B}| = \sqrt{0^2+8^2+6^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

Q. 4. What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

(A) –90, 90 (B) 120, 18

(C) –90, 100 (D) –90, 120

Ans. Option (D) is correct.

Explanation:

$$\vec{N} = \vec{A}\times\vec{B}$$
$$N = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$$
$$= -15(6\hat{j}-8\hat{k})$$
$$= -90\hat{j}+120\hat{k};$$

Q. 5. What is the magnitude of $\vec{N}$ and in what units?

(A) 100 (B) 150

(C) 50 (D) 90

Ans. Option (B) is correct.

Explanation:

$$\vec{N} - -90\hat{j}+120\hat{k}$$
$$|\vec{N}| = \sqrt{(90)^2-(120)^2}$$
$$= \sqrt{8100+14400}$$
$$= \sqrt{22500}$$
$$= 150 \text{ units}$$

II. Read the following text and answer the following questions on the basis of the same:

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three non zero vectors.

[CBSE QB 2021]

Q. 1. If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then

(A) $\vec{a} \perp \vec{b}$ (B) $\vec{a} \parallel \vec{b}$

(C) $\vec{a} = \vec{b}$ (D) None of these

Ans. Option (A) is correct.

Q. 2. If $\vec{a} = \hat{i}-2\hat{j}$, $\vec{b} = 2\hat{i}+\hat{j}+3\hat{k}$ then evaluate $(2\vec{a}+\hat{b})\cdot[(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})]$

(A) 0 (B) 4

(C) 3 (D) 2

Ans. Option (A) is correct.

Explanation:

$$\vec{a}+\vec{b} = 3\hat{i}-\hat{j}+3\hat{k}$$
$$\vec{a}-2\vec{b} = -3\hat{i}-4\hat{j}-6\hat{k}$$
$$(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 3 \\ -3 & -4 & -6 \end{vmatrix}$$
$$= (6+12)\hat{i}-(-18+9)\hat{j}+(-12-3)\hat{k}$$
$$= 18\hat{i}+9\hat{j}-15\hat{k}$$
$$(2\vec{a}+\vec{b}) = 2(\hat{i}-2\hat{j})+(2\hat{i}+\hat{j}+3\hat{k})$$
$$= 4\hat{i}-3\hat{j}+3\hat{k}$$
$$(2\vec{a}+\vec{b}).[(\vec{a}+\vec{b})\times(\vec{a}-2\vec{b})]$$
$$= (4\hat{i}+3\hat{j}+3\hat{k})(18\hat{i}+9\hat{j}-15\hat{j})$$
$$- 72-27-45$$
$$= 0$$

Q. 3. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them then $|\vec{a}-\vec{b}|$ is

(A) $\sin\frac{\theta}{2}$ (B) $2\sin\frac{\theta}{2}$

(C) $2\cos\frac{\theta}{2}$ (D) $\cos\frac{\theta}{2}$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} |\vec{a}-\vec{b}|^2 &= (\vec{a}-\vec{b}).(\vec{a}-\vec{b}) \\ &= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}-\vec{b}\cos\theta \\ &= 1+1-2(1)(1)\cos\theta \end{aligned}$$

$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2-2\cos\theta$$

$$= 2(1-\cos\theta)$$

$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 4\sin^2\frac{\theta}{2}$$

$$|\vec{a}-\vec{b}|^2 = 2\sin\frac{\theta}{2}$$

Q. 4. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then $\vec{a}$ =

(A) $2(\vec{b}\times\vec{c})$ **(B)** $-2(\vec{b}\times\vec{c})$

(C) $\pm 2(\vec{b}\times\vec{c})$ **(D)** $2(\vec{b}\pm\vec{c})$

Ans. Option (C) is correct.

Q. 5. The area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diagonals is

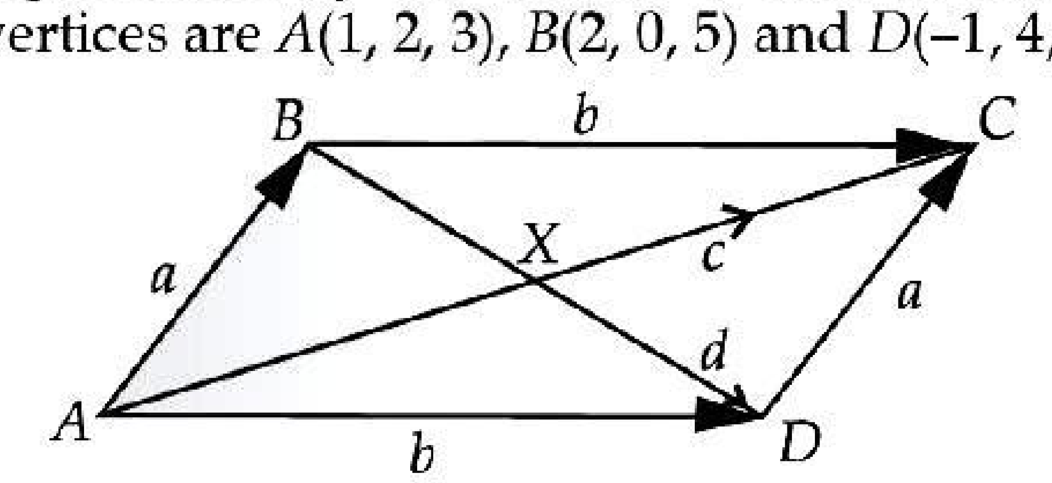
(A) 70 **(B)** 35

(C) $\frac{\sqrt{70}}{2}$ **(D)** $\sqrt{70}$

Ans. Option (C) is correct.

III. Read the following text and answer the following questions on the basis of the same:

ABCD is a parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$. Three of its vertices are $A(1, 2, 3)$, $B(2, 0, 5)$ and $D(-1, 4, 1)$.



Q. 1. The vector $\vec{a}$ = _______.

(A) $2\hat{i}-\hat{j}+\hat{k}$ **(B)** $\hat{i}-2\hat{j}-2\hat{k}$

(C) $\hat{i}-2\hat{j}+2\hat{k}$ **(D)** $2\hat{i}-\hat{j}+2\hat{k}$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \vec{a} &= \overrightarrow{AB} \\ &= \overrightarrow{OB}-\overrightarrow{OA} \\ &= (2\hat{i}+0\hat{j}+5\hat{k})-(\hat{i}+2\hat{j}+3\hat{k}) \\ &= \hat{i}-2\hat{j}+2\hat{k} \end{aligned}$$

Q. 2. The length of side *CD* is _______ units.

(A) 3 **(B)** 4

(C) 5 **(D)** 6

Ans. Option (A) is correct.

Explanation:

$$\begin{aligned} |\overrightarrow{CD}| &= |\vec{a}| \\ &= \sqrt{1+4+4} \\ &= 3 \text{ units} \end{aligned}$$

Q. 3. The diagonal $\vec{c}$ = _______.

(A) $\hat{i}+3\hat{j}+5\hat{k}$ **(B)** $\hat{i}+\hat{j}-\hat{k}$

(C) $-\hat{i}$ **(D)** $-\hat{j}$

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \vec{a} &= \hat{i}-2\hat{j}+2\hat{k} \\ \vec{b} &= -2\hat{i}+2\hat{j}-2\hat{k} \\ \vec{c} &= \vec{a}+\vec{b} \\ &= -\hat{i} \end{aligned}$$

Q. 4. The diagonal $\vec{d}$ = _______.

(A) $-\hat{i}$ **(B)** $-3\hat{i}+4\hat{j}-4\hat{k}$

(C) $\hat{i}+2\hat{j}-\hat{k}$ **(D)** $\hat{i}$

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} \vec{d} &= \vec{b}-\vec{a} \\ &= -3\hat{i}+4\hat{j}-4\hat{k} \end{aligned}$$

Q. 5. Area of ABCD = _______ sq. units.

(A) 8 **(B)** 4

(C) $2\sqrt{2}$ **(D)** 16

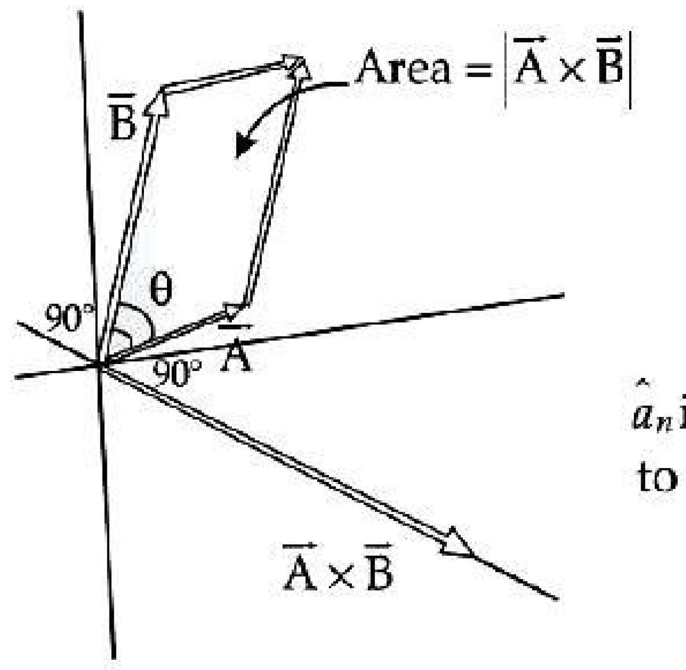
Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \vec{a}\times\vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ -2 & 2 & -2 \end{vmatrix} \\ &= -2\hat{j}-2\hat{k} \\ \text{Area} &= |\vec{a}\times\vec{b}| \\ &= \sqrt{4+4} \\ &= 2\sqrt{2} \text{ sq. units} \end{aligned}$$

IV. Read the following text and answer the following questions on the basis of the same:

The cross product of two vectors gives a vector perpendicular to the plane containing the vectors.



$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{a}_n$$

$\hat{a}_n$ is a unit vector perpendicular to the plane defined by $\vec{A}$ and $\vec{B}$.

Q. 1. $\hat{i}\times\hat{j}$ = _______.

(A) $\hat{k}$ (B) $-\hat{k}$

(C) 0 (D) $\vec{0}$

Ans. Option (A) is correct.

> ***Explanation:*** $\hat{i}\times\hat{j} = \hat{k}$

Q. 2. $\hat{i}\cdot(\hat{j}\times\hat{k})$ = _______.

(A) $\hat{k}$ (B) 0

(C) 1 (D) –1

Ans. Option (C) is correct.

> ***Explanation:***
> $$\hat{i}\cdot(\hat{j}\times\hat{k}) = \hat{i}\cdot\hat{i} = 1$$

Q. 3. The angle between $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ is _______.

(A) 0 (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans. Option (B) is correct.

> ***Explanation:*** $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$
> The angle between $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ is π.

Q. 4. If $\vec{a}$ and $\vec{b}$ are non-zero vectors, then $\vec{a}\cdot(\vec{a}\times\vec{b})$ = _______.

(A) 0 (B) $\vec{0}$

(C) $\vec{a}$ (D) $\vec{b}$

Ans. Option (A) is correct.

> ***Explanation:***
> $$\vec{a}\cdot(\vec{a}\times\vec{b}) = [\vec{a}, \vec{a}, \vec{b}] = 0$$

Q. 5. Area of parallelogram whose adjacent sides are represented by the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j}$ is _______ sq. units.

(A) 3 (B) $\sqrt{10}$

(C) $\sqrt{11}$ (D) $2\sqrt{3}$

Ans. Option (C) is correct.

> ***Explanation:***
> $$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -1 & 0 \end{vmatrix} = -\hat{i} + 3\hat{j} + \hat{k}$$
> $$\text{Area} = |\vec{a}\times\vec{b}| = \sqrt{1+9+1} = \sqrt{11} \text{ sq. units}$$