CBSE Board
Class X Mathematics
Board Paper - 2012

Time: 3 hours Total Marks: 90

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **34** questions divided into **four sections** A, B, C, and D.
3. **Section A** contains of **10** questions of 1 mark each, which are multiple choice type question, **Section B** contains of **8** questions of 2 marks each, **Section C** contains of **10** questions of 3 marks each and **Section D** contains of **6** questions of 4 marks each.
4. Question numbers **1 to 8** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
5. There is no overall choice. However, internal choice has been provided in **one** question of **2 marks**, **three** questions of **3 marks** each and **two** questions of 4 **marks** each. You have to attempt only one of the alternatives in all such questions.
6. Use of calculator is **not** permitted.

SECTION – A

1. The length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is

   (A) 45^{o}

   (B) 30^{o}

   (C) 60^{o}

   (D) 90^{o}

2. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is

   (A) 34

   (B) 26

   (C) 17

   (D) 14

3. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

(A) 1 : 2

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

4. Two dice are thrown together. The probability of getting the same number on both dice is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{12}$

5. The coordinates of the point P dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 are

(A) (2, 4)

(B) 3, 5)

(C) (4, 2)

(D) 5, 3)

6. If the coordinates of the one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5), then the coordinates of the other end of the diameter are:

(A) (-6, 7)

(B) (6, -7)

(C) (6, 7)

(D) (-6, -7)

7. The sum of first 20 odd natural number is

(A) 100

(B) 210

(C) 400

(D) 420

8. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals

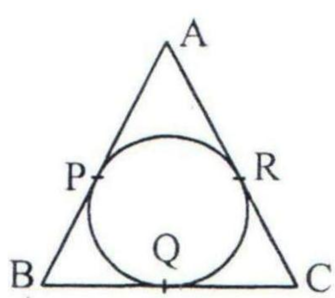
(A) 3

(B) $-\frac{7}{2}$

(C) 6

(D) -3

9. In Fig., the sides AB, BC and CA of a triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then the length of BC (in cm) is



(A) 11

(B) 10

(C) 14

(D) 15

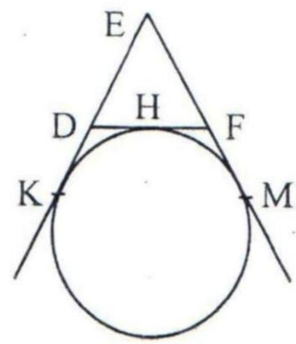
10. In Fig., a circle touches the side DF of ΔEDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of ΔEDF (in cm) is:

(A) 18

(B) 13.5

(C) 12

(D) 9



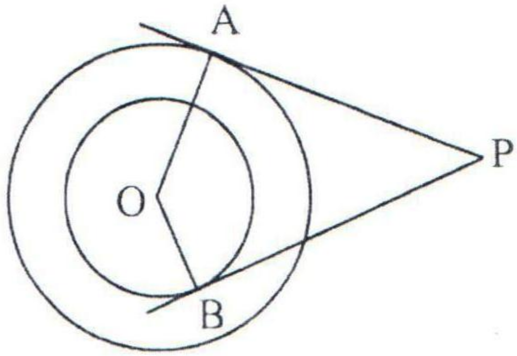
SECTION – B

11. If a point A(0, 2) is equidistant from the points B(3, p) and C(p, 5) then find the value of p.

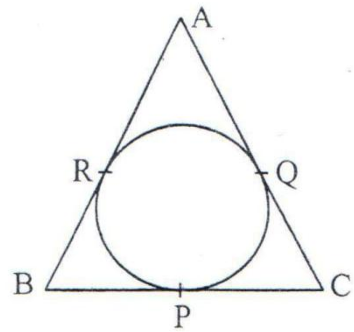
12. A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

13. The volume of a hemisphere is $2425\frac{1}{2}$ cm^3. Find its curved surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$

14. Tangents PA and PB are drawn from an external point P to two concentric circle with centre O and radii 8 cm and 5 cm respectively, as shown in Fig., If AP = 15 cm, then find the length of BP.

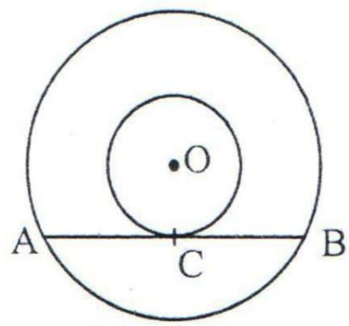


15. In fig., an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.

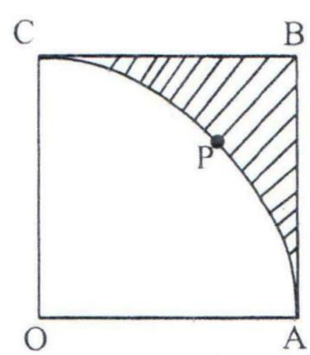


OR

In fig., the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



16. In fig., OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



17. Find the sum of all three digit natural numbers, which are multiples of 7.

18. Find the values (s) of k so that the quadratic equation $3x^2 - 2kx + 12 = 0$ has equal roots.

SECTION – C

19. A point P divides the line segment joining the points A(3, -5) and B(-4, 8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line x + y = 0, then find the value of K.

20. If the vertices of a triangle are (1, -3), (4, p) and (-9, 7) and its area is 15 sq. units, find the value (s) of p.

21. Prove that the parallelogram circumscribing a circle is a rhombus.

OR

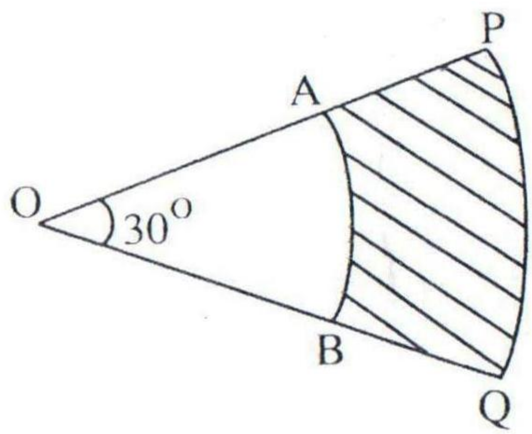
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

22. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

23. In fig., PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If $\angle POQ = 30^o$, then the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



24. Solve for x: $4x^2 - 4ax + (a^2 - b^2) = 0$

Or

Solve for x: $3x^2 - \sqrt{6}x + 2 = 0$

25. A kite is flying at a height of 45 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^o. Find the length of the string assuming that there is slack in the string.

26. Draw a triangle ABC with side BC = 6 cm, $\angle C = 30^o$ and $\angle A = 105^o$. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.

27. The 16^{th} term of an AP is 1 more than twice its 8^{th} term. If the 12^{th} term of the AP is 47, then find its nth term.

28. A card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) the queen of diamonds.

SECTION – D

29. A bucket is in the form of a frustum of a cone and its can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket. $\left[\text{Use } \pi = \frac{22}{7}\right]$

30. The angle of elevation of the top of a hill at the foot of a tower is 60^o and the angle of depression from the top of the tower of the foot of the hill is 30^o. If the tower is 50 m high, find the height of the hill.

31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

32. A shopkeeper buys some books for Rs. 80. If he had bought 4 more books for the same amount, each book would have cost Rs. 1 less. Find the number of books he bought.

OR

The sum of two number is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

33. Sum of the first 20 terms of an AP is -240, and its first term is 7. Find its 24^{th} term.

34. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$.

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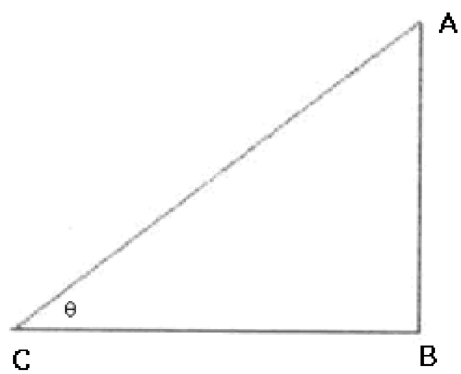
Section A

1. Correct answer: B



Let AB be the tower and BC be its shadow. Let θ be the angle of elevation of the sun.

According to the given information,

BC = $\sqrt{3}$ AB ... (1)

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{AB}{\sqrt{3}AB} = \frac{1}{\sqrt{3}} \quad \text{[Using (1)]}$$

We know that tan 30° = $\frac{1}{\sqrt{3}}$

$\therefore \theta = 30°$

Hence, the angle of elevation of the sun is 30°.

2. Correct answer: B

Diameters of two circles are given as 10 cm and 24 cm.

Radius of one circle = r_1 = 5 cm, Radius of other circle = r_2 = 12 cm

According to the given information,

Area of the larger circle = $\pi(r_1)^2 + \pi(r_2)^2$

$$= \pi(5)^2 + \pi(12)^2$$
$$= \pi(25 + 144)$$
$$= 169\pi$$
$$= \pi(13)^2$$

$\therefore$ Radius of larger circle = 13 cm

Hence, the diameter of larger circle = 26 cm

3. Correct answer: C

Let the original radius and the height of the cylinder be r and h respectively.

Volume of the original cylinder = $\pi r^2 h$

Radius of the new cylinder = $\frac{r}{2}$

Height of the new cylinder = h

Volume of the new cylinder = $\pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$

Required ratio = $\frac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}} = \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{1}{4} = 1:4$

4. Correct answer: C

When two dice are thrown together, the total number of outcomes is 36.

Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$\therefore$ Required probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{36} = \frac{1}{6}$

5. Correct answer: B

It is given that the point P divides AB in the ratio 2: 1.

Using section formula, the coordinates of the point P are

$$\left(\frac{1\times1+2\times4}{2+1},\frac{1\times3+2\times6}{2+1}\right)=\left(\frac{1+8}{3},\frac{3+12}{3}\right)=(3,5)$$

Hence the coordinates of the point P are (3, 5).

6. Correct answer: A

Let the coordinates of the other end of the diameter be (x, y).

We know that the centre is the mid-point of the diameter. So, O(-2, 5) is the mid-point of the diameter AB. The coordinates of the point A and B are (2, 3) and (x, y) respectively.

Using mid-point formula, we have,

$$-2=\frac{2+x}{2}\Rightarrow -4=2+x\Rightarrow x=-6$$
$$5=\frac{3+y}{2}\Rightarrow 10=3+y\Rightarrow y=7$$

Hence, the coordinates of the other end of the diameter are (-6, 7).

7. Correct answer: C

The first 20 odd numbers are 1, 3, 5, ... 39

This is an AP with first term 1 and the common difference 2.

Sum of 20 terms = S_{20}

$$S_{20}=\frac{20}{2}\left[2(1)+(20-1)(2)\right]=10\left[2+38\right]=400$$

Thus, the sum of first 20 odd natural numbers is 400.

8. Correct answer: A

It is given that 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$.

Therefore, $y = 1$ will satisfy both the equations.

$\therefore a(1)^2 + a(1) + 3 = 0$

$\Rightarrow a + a + 3 = 0$

$\Rightarrow 2a + 3 = 0$

$\Rightarrow a = \frac{-3}{2}$

Also, $(1)^2 + (1) + b = 0$

$\Rightarrow 1 + 1 + b = 0 \Rightarrow b = -2$

$\therefore ab = \frac{-3}{2} \times -2 = 3$

9. Correct answer: B

It is known that the lengths of tangents drawn from a point outside a circle are equal in length.

Therefore, we have:

AP = AR ... (1) (Tangents drawn from point A)

BP = BQ ... (2) (Tangents drawn from point B)

CQ = CR ... (3) (Tangents drawn from point C)

Using the above equations,

AR = 4 cm (AP = 4 cm, given)

BQ = 3 cm (BP = 3 cm, given)

AC = 11 cm $\Rightarrow$ RC = 11 cm – 4 cm = 7 cm

$\Rightarrow$ CQ = 7 cm

Hence, BC = BQ + CQ = 3 cm + 7 cm = 10 cm

10. Correct answer: A

It is known that the tangents from an external point to the circle are equal.

$\therefore$EK = EM, DK = DH and FM = FH ... (1)

Perimeter of ΔEDF = ED + DF + FE

= (EK – DK) + (DH + HF) + (EM – FM)

= (EK – DH) + (DH + HF) + (EM – FH) [Using (1)]

= EK + EM

= 2 EK = 2 (9 cm) = 18 cm

Hence, the perimeter of ΔEDF is 18 cm.

SECTION B

11. It is given that the point A (0, 2) is equidistant from the points B(3, p) and C(p, 5).

So, AB = AC $\Rightarrow$ $AB^2 = AC^2$

Using distance formula, we have

$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$
$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$
$\Rightarrow 4 - 4p = 0$
$\Rightarrow 4p = 4$
$\Rightarrow p = 1$

Hence, the value of p = 1.

12. Total number of outcomes is 50.

Favourable outcomes = {12, 24, 36, 48}

$\therefore$Required probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{4}{50} = \dfrac{2}{25}$

13.

Given volume of a hemisphere = $2425\frac{1}{2}\ cm^3 = \frac{4851}{2}\ cm^3$

Now, let r be the radius of the hemisphere

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

$\therefore \quad \frac{2}{3}\pi r^3 = \frac{4851}{2}$

$\Rightarrow \quad \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$

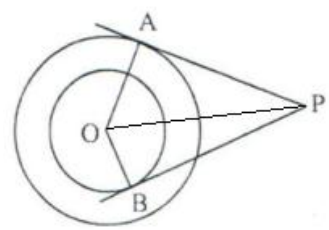
$\Rightarrow \quad r^3 = \frac{4851}{2} \times \frac{3}{2} \times \frac{7}{22} = \left(\frac{21}{2}\right)^3$

$\therefore \quad r = \frac{21}{2}$ cm

So, Curved surface area of the hemisphere $= 2\pi r^2$

$= \cancel{2} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{21}^{3}}{\cancel{2}} \times \frac{21}{\cancel{2}} = 693$ sq.cm

14. Given: Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii OA = 8 cm, OB = 5 cm respectively. Also, AP = 15 cm



Construction: We join the points O and P.

Solution: $OA \perp AP$; $OB \perp BP$

[Using the property that radius is perpendicular to the tangent at the point of contact of a circle]

In right angled triangle OAP,

$OP^2 = OA^2 + AP^2$ [Using Pythagoras Theorem]

$= (8)^2 + (15)^2 = 64 + 225 = 289$

$\therefore$ OP = 17 cm

In right angled triangle OBP,

$OP^2 = OB^2 + BP^2$

$\Rightarrow BP^2 = OP^2 - OB^2 = (17)^2 - (5)^2 = 289 - 25 = 264$

$\therefore \quad BP = \sqrt{264} = 2\sqrt{66}$ cm

15. Given: ABC is an isosceles triangle, where AB = AC, circumscribing a circle.

To prove: The point of contact P bisects the base BC. i.e. BP = PC

Proof: It can be observed that

BP and BR; CP and CQ; AR and AQ are pairs of tangents drawn to the circle from the external points B, C and A respectively.

Since the tangents drawn from an external point to a circle, then

BP = BR --- (i)

CP = CQ --- (ii)

AR = AQ --- (iii)

Given that AB = AC

⇒ AR + BR = AQ + CQ

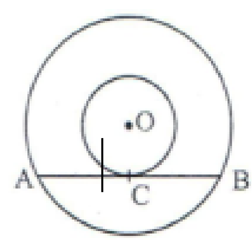
⇒ BR = CQ [from (iii)]

⇒ BP = CP [from (i) and (ii)]

∴ P bisects BC.

OR

Given: The chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C.



To prove: AC = CB

Construction: Let us join OC.

Proof: In the smaller circle, AB is a tangent to the circle at the point of contact C.

∴ OC ⊥ AB ------ (i)

(Using the property that the radius of a circle is perpendicular to the tangent at the point of contact)

For the larger circle, AB is a chord and from (i) we have OC ⊥ AB

∴ OC bisects AB

(Using the property that the perpendicular drawn from the centre to a chord of a circle bisects the chord)

∴ AC = CB

16. Given, OABC is a square of side 7 cm

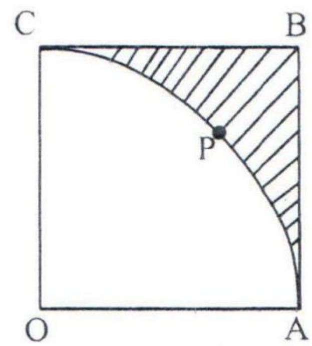
i.e. OA = AB = BC = OC = 7cm

∴ Area of square OABC = $(\text{side})^2 = 7^2 = 49$ sq.cm

Given, OAPC is a quadrant of a circle with centre O.

∴ Radius of the sector = OA = OC = 7 cm.

Sector angle = 90^o



$$\therefore \quad \text{Area of quadrant OAPC} = \frac{90^o}{360^o} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2} \text{ sq.cm} = 38.5 \text{ sq.cm}$$

$$\therefore \quad \text{Area of shaded portion} = \text{Area of Square OABC} - \text{Area of quadrant OAPC}$$

$$= (49 - 38.5) \text{ sq. cm} = 10.5 \text{ sq.cm}$$

17. First three- digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore the series is 105, 112, 119,…

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly, 999 – 5 = 994 is the maximum possible three – digit number divisible by 7.

The series is as follows:

105, 112, 119, …., 994

Here a = 105, d = 7

Let 994 be the nth term of this A.P.

$$a_n = a + (n-1)d$$

$\Rightarrow \quad 994 = 105 + (n-1)7$

$\Rightarrow \quad (n-1)7 = 889$

$\Rightarrow \quad (n-1) = 127$

$\Rightarrow \quad n = 128$

So, there are 128 terms in the A.P.

$$\therefore \quad \text{Sum} = \frac{n}{2}\{\text{first term} + \text{last term}\}$$
$$= \frac{128}{2}\{a_1 + a_{128}\}$$
$$= 64\{105 + 994\} = (64)(1099) = 70336$$

18. Given quadratic equation is $3x^2 - 2kx + 12 = 0$

Here a = 3, b = -2k and c = 12

The quadratic equation will have equal roots if $\Delta = 0$

$\therefore \quad b^2 - 4ac = 0$

Putting the values of a, b and c we get

$$(-2k)^2 - 4(3)(12) = 0$$

$\Rightarrow \quad 4k^2 - 144 = 0$

$\Rightarrow \quad 4k^2 = 144$

$\Rightarrow \quad k^2 = \frac{144}{4} = 36$

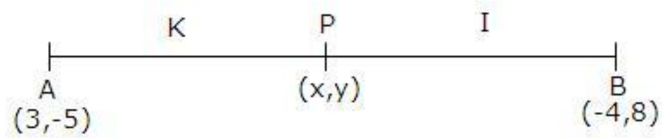
Considering square root on both sides,

$$k = \sqrt{36} = \pm 6$$

Therefore, the required values of k are 6 and -6.

SECTION – C

19.



Let the co-ordinates of point P be (x, y)

Then using the section formula co-ordinates of P are.

$x = \frac{-4K+3}{K+1}$ $\qquad$ $y = \frac{8K-5}{K+1}$

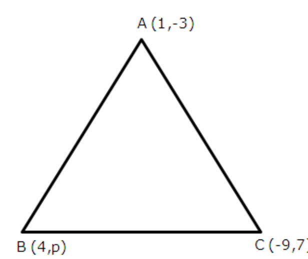
Since P lies on x+y=0

$\therefore \quad \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$

$\Rightarrow 4K - 2 = 0 \Rightarrow k = \frac{2}{4} \Rightarrow K = \frac{1}{2}$

Hence the value of $K = \frac{1}{2}$.

20. The area of a triangle, whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is



$$\Delta = \frac{1}{2}| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

Substituting the given coordinates

Area of $\Delta = \frac{1}{2}| 1(p-7) + 4(7+3) + (-9)(-3-p) |$

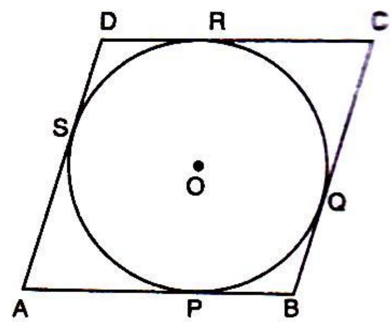
$\Rightarrow \frac{1}{2}|(p-7) + 40 + 27 + 9p| = 15$

$\Rightarrow 10p + 60 = \pm 30$

$\Rightarrow 10p = -30 \quad$ or $\quad 10p = -90$

$\Rightarrow p = -3. \quad$ or $\quad p = -9$

21. Let ABCD be a parallelogram such that its sides touching a circle with centre O. We know that the tangents to a circle from an exterior point are equal in length.



$\therefore$ AP = AS [From A] ...(i)

BP = BQ [From B] ...(ii)

CR = CQ [From C] ...(iii)

and, DR = DS [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ 2AB = 2BC [$\because$ ABCD is a parallelogram $\therefore$ AB=CD and BC = AD]

$\Rightarrow$ AB=BC

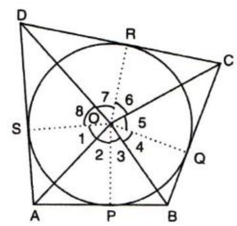
Thus, AB = BC = CD = AD

Hence, ABCD is a rhombus.

OR

A circle with centre O touches the sides AB, BC, CD, and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

TO PROVE $\angle AOB + \angle COD = 180^o$ and, $\angle AOD + \angle BOC = 180^o$



CONSTRUCTION Join OP, OQ, OR and OS.

PROOF Since the two tangents drawn from a external point to a circle subtend equal angles at the centre.

$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$$\left[\begin{array}{l}\text{Sum of all the angles} \\ \text{subtended at a point is } 360^o\end{array}\right]$$

$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^o$ and $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^o$

$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^o$ and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^o$

$$\left[\begin{array}{l}\because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD \\ \angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC\end{array}\right]$$

$\Rightarrow \angle AOB + \angle COD = 180^o$

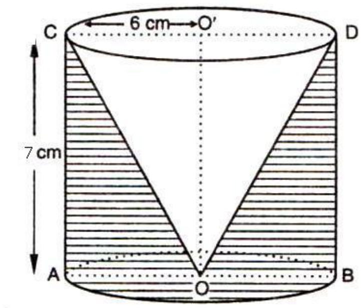
and $\angle AOD + \angle BOC = 180^o$

Hence Proved

22. Given: radius of cyl=radius of cone=r=6cm

Height of the cylinder=height of the cone=h=7cm

Slant height of the cone= l



$\sqrt{7^2+6^2}=\sqrt{85}$ cm

Total surface area of the remaining solid

=curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

$(2\pi rh+\pi r^2+\pi rl)$

$=2x\frac{22}{7}\times6\times7\ +\frac{22}{7}\times6^2+\frac{22}{7}\times6\times\sqrt{85}$

$=264+\frac{792}{7}+\frac{132}{7}\sqrt{85}$

$=377.1+\frac{132}{7}\sqrt{85}$ cm^2

OR

Volume of the conical heap=volume of the sand emptied from the bucket.

Volume of the conical heap=

$\frac{1}{3}\pi r^2h=\frac{1}{3}\pi r^2x24\,cm^2$ (height of the cone is 24)----------(1)

Volume of the sand in the bucket=πr^2h

$=\pi(18)^2 32cm^2$ ------(2)

Equating 1 and 2

$\frac{1}{3}\pi r^2x24=\pi(18)^2 32$

$\Rightarrow r^2=\frac{(18)^2x32x3}{24}$

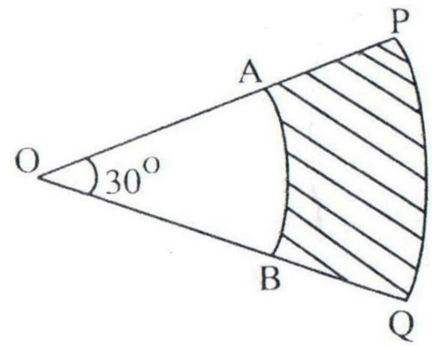
$\Rightarrow r=36cm$

23. Area of the shaded region= Area of sector POQ - Area of sector AOB

Area of Shaded region $= \left(\frac{\theta}{360}\pi R^2 - \frac{\theta}{360}\pi r^2\right)$

$$= \frac{30}{360} \times \frac{22}{7} \times (7^2 - 3.5^2)$$

$$= \frac{77}{8} \text{cm}^2$$



24.

$$4x^2 - 4ax + (a^2 - b^2) = 0$$

$$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$$

$$\Rightarrow \left[(2x)^2 - 2.2x.a + a^2\right] - b^2 = 0$$

$$\Rightarrow \left[(2x - a)^2\right] - b^2 = 0$$

$$\Rightarrow \left[(2x - a) - b\right]\left[(2x - a) + b\right] = 0$$

$$\Rightarrow \left[(2x - a) - b\right] = 0 \quad \text{or} \quad \left[(2x - a) + b\right] = 0$$

$$\Rightarrow x = \frac{a + b}{2}; x = \frac{a - b}{2}$$

OR

$$3x^2 - 2\sqrt{6}\,x + 2 = 0$$

$$\Rightarrow \quad 3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$$

$$\Rightarrow \quad \sqrt{3}\,x\left[\sqrt{3}\,x - \sqrt{2}\right] - \sqrt{2}\left[\sqrt{3}\,x - \sqrt{2}\right] = 0$$

$$\Rightarrow \quad \left(\sqrt{3}\,x - \sqrt{2}\right)\left(\sqrt{3}\,x - \sqrt{2}\right) = 0$$

$$\Rightarrow \quad \left(\sqrt{3}\,x - \sqrt{2}\right)^2 = 0$$

$$\therefore \quad \sqrt{3}\,x - \sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{3}\,x = \sqrt{2}$$

$$\Rightarrow \quad x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \times \sqrt{3}}{\left(\sqrt{3}\right)^2} = \frac{\sqrt{6}}{3}$$

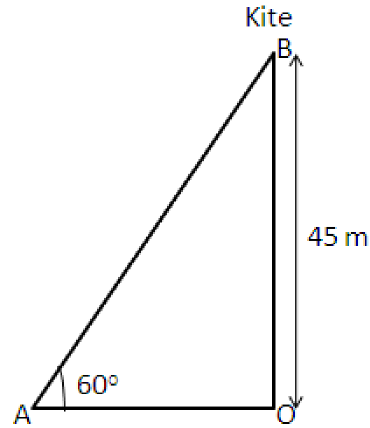
25. Given: Position of kite is B.

Height of kite above ground= 45 m

Angle of inclination = 60^o

Required length of string = AB

In right angled triangle AOB,



$$\sin A = \frac{OB}{AB} \Rightarrow \sin 60^o = \frac{45}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{45}{AB}$$

$$\Rightarrow AB = \frac{45 \times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3}\ m$$

Hence, the length of the string is $30\sqrt{3}$ m

26. It is given that $\angle A = 105°$, $\angle C = 30°$.

Using angle sum property of triangle, we get, $\angle B = 45°$

The steps of construction are as follows:

1. Draw a line segment BC = 6 cm.

2. At B, draw a ray making an angle of 45° with BC.

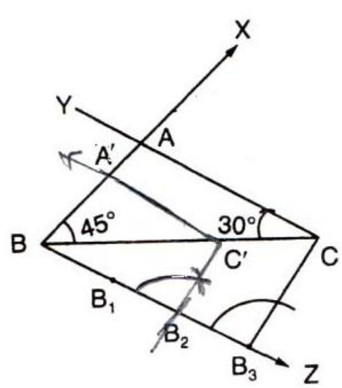
3. At C, draw a ray making an angle of 30° with BC. Let the two rays meet at point A.

4. Below BC, make an acute $\angle CBX$. Along BX mark off three points B_1, B_2, B_3, such that $BB_1 = B_1B_2 = B_2B_3$. Join B_3C.

7. From B2, draw B2C′ || B3C.

8. From C′, draw C′A′ || CA, meeting BA at the point A′.

Then A′BC′ is the required triangle.



27. Let a and d respectively be the first term and the common difference of the AP.

We know that the n^{th} term of an AP is given by $a_n = a + (n - 1)d$

According to the given information,

$a_{16} = 1 + 2\, a_8$

$\Rightarrow a + (16 - 1)d = 1 + 2[a + (8 - 1)d]$

$\Rightarrow a + 15d = 1 + 2a + 14d$

$\Rightarrow a + 15d = 1 + 2a + 14d$

$\Rightarrow -a + d = 1 \qquad \dots (1)$

Also, it is given that, $a_{12} = 47$

$\Rightarrow a + (12 - 1)d = 47$

$\Rightarrow a + 11d = 47 \qquad \dots (2)$

Adding (1) and (2), we have:

$12d = 48$

$\Rightarrow d = 4$

From (1),

$-a + 4 = 1 \Rightarrow a = 3$

Hence, $a_n = a + (n - 1)d = 3 + (n - 1)(4) = 3 + 4n - 4 = 4n - 1$

Hence, the n^{th} term of the AP is $4n - 1$.

28. Total number of outcomes = 52

(i) Probability of getting a red king

Here the number of favourable outcomes = 2

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{2}{52} = \frac{1}{26}$$

(ii) Probability of getting a face card

Total number of face cards = 12

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{12}{52} = \frac{3}{13}$$

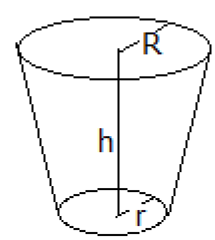
(iii) Probability of queen of diamonds

Number of queens of diamond = 1

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{52}$$

SECTION – D

29. Here, R = 28 cm and r = 21 cm, we need to find h.



Volume of frustum = 28.49 L = 28.49×1000 cm^3 = 28490 cm^3

Now, Volume of frustum = $\frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$

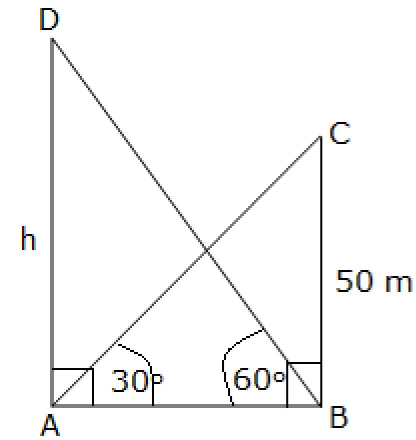
$$\Rightarrow \frac{22h}{7 \times 3}\left(28^2 + 28 \times 21 + 21^2\right) = 28490$$

$$\Rightarrow \frac{22}{21} h \times 1813 = 28490$$

$$\Rightarrow h = \frac{28490 \times 21}{22 \times 1813} = 15 \text{ cm}$$

Hence the height of bucket is 15 cm.

30. Let the height of hill is h.



In right triangle ABC,

$$\frac{50}{AB} = \tan 30^\circ \Rightarrow \frac{50}{AB} = \frac{1}{\sqrt{3}} \Rightarrow AB = 50\sqrt{3}$$

In right triangle ABD,

$$\frac{h}{AB} = \tan 60^\circ \Rightarrow \frac{h}{AB} = \sqrt{3} \Rightarrow h = \sqrt{3}AB$$

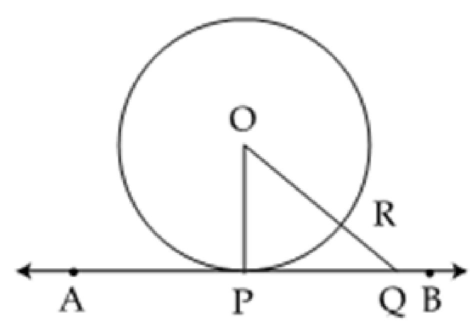
$$\Rightarrow h = \sqrt{3}\left(50\sqrt{3}\right) = 150 \text{ m}$$

Hence the height of hill is 150 m.

31. Given: AB is a tangent to a circle with centre O.

To prove: OP is perpendicular to AB.

Construction: Take a point Q on AB and join OQ.



Proof: Since Q is a point on the tangent AB, other than the point of contact P, so Q will be outside the circle.

Let OQ intersect the circle at R.

Now OQ = OR + RQ

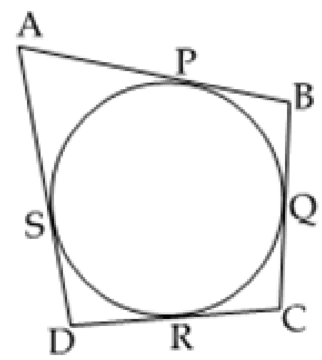
$\Rightarrow$ OQ > OR $\Rightarrow$ OQ > OP [as OR = OP]

$\Rightarrow$ OP < OQ

Thus OP is shorter than any other segment among all and the shortest length is the perpendicular from O on AB.

$\therefore$ OP $\perp$ AB. Hence proved.

OR



Let ABCD be a quadrilateral, circumscribing a circle.

Since the tangents drawn to the circle from an external point are equal, we have

AP = AS ... (1)

PB = BQ ... (2)

RC = QC ... (3)

DR = DS ... (4)

Adding, (1), (2), (3) and (4), we get

AP + PB + RC + DR = AS + BQ + QC + DS

(AP + PB) + (DR + RC) = (AS + SD) + (BQ + QC)

AB + CD = AD + BC.

32. Total cost of books = Rs 80

Let the number of books = x

So the cost of each book = Rs $\frac{80}{x}$

Cost of each book if he buy 4 more book = Rs $\frac{80}{x+4}$

As per given in question:

$$\frac{80}{x} - \frac{80}{x+4} = 1$$
$$\Rightarrow \frac{80x + 320 - 80x}{x(x+4)} = 1$$
$$\Rightarrow \frac{320}{x^2 + 4x} = 1$$
$$\Rightarrow x^2 + 4x - 320 = 0$$
$$\Rightarrow (x+20)(x-16) = 0$$
$$\Rightarrow x = -20, 16$$

Since number of books cannot be negative,

So the number of books he brought is 16.

OR

Let the first number be x then the second number be 9 – x as the sum of both numbers is 9.

Now the sum of their reciprocal is $\frac{1}{2}$, therefore

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$
$$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$$
$$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$$
$$\Rightarrow 18=9x-x^2$$
$$\Rightarrow x^2-9x+18=0$$
$$\Rightarrow (x-6)(x-3)=0$$
$$\Rightarrow x=6,3$$

If x = 6 then other number is 3.

And, if x = 3 then other number is 6.

Hence numbers are 3 and 6.

33. Given: $S_{20}=-240$ and $a=7$

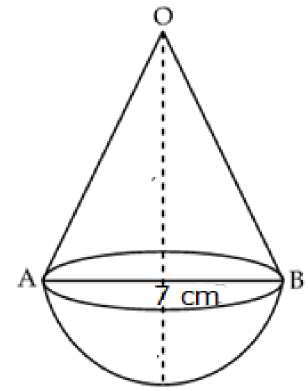
Consider, $S_{20}=-240$

$$\Rightarrow \frac{20}{2}(2\times 7+19d)=-240 \qquad \left[\because S_n=\frac{n}{2}[2a+(n-1)d]\right]$$
$$\Rightarrow 10(14+19d)=-240$$
$$\Rightarrow 14+19d=-24$$
$$\Rightarrow 19d=-38$$
$$\Rightarrow d=-2$$

Now, $a_{24}=a+23d=7+23\times -2=-39$

Hence, $a_{24}=-39$

34. Radius of hemi-sphere = 7 cm



Radius of cone = 7 cm

Height of cone = diameter = 14 cm

Volume of solid = Volume of cone + Volume of Hemi-sphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi r^2 (h + 2r)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 49(14 + 14)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 49 \times 28$$
$$= \frac{22 \times 7 \times 28}{3} = \frac{4312}{3} \text{ cm}^3$$

Radius of cylinder = Radius of cone = r = 6 cm

Height of the cylinder = Height of the cone = h = 8 cm

Slant height of the cone= l $= \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm

Total surface area of the remaining solid

= Curved Surface Area of the Cylinder + Area of the Base of the Cylinder + Curved Surface Area of the Cone

$$(2\pi rh + \pi r^2 + \pi rl)$$
$$= 2x\frac{22}{7}x6x7 + \frac{22}{7}x6^2 + \frac{22}{7}x6x\sqrt{85}$$
$$= 264 + \frac{792}{7} + \frac{132}{7}\sqrt{85}$$
$$= 377.1 + \frac{132}{7}\sqrt{85}\text{cm}^2$$