Matrices

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{ when } i \neq j \\ 0, \text{ when } i = j \end{cases}$, then A^2 is

[*CBSE Sample Paper 2021 (Term-1)*]

(*a*) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is [*CBSE 2021-22 (Term-1)*]

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{\pi}{3}$ (*c*) π (*d*) $\frac{3\pi}{2}$

3. If $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$, then value of $a + b - c + 2d$ is

[*NCERT Exemplar*] [*CBSE Sample Paper 2021 (Term-1)*]

(*a*) 8 (*b*) 10 (*c*) 4 (*d*) − 8

4. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

[*CBSE 2021-22 (Term-1)*]

(*a*) 9 (*b*) 27 (*c*) 81 (*d*) 512

5. If for a square matrix A, $A^2 - 3A + I = 0$ and $A^{-1} = xA + yI$ then the value of $x + y$ is

[*CBSE 2023 (65/1/1)*]

(*a*) − 2 (*b*) 2 (*c*) 3 (*d*) − 3

6. If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to [CBSE 2023 (65/2/1)]

(a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$

7. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to [CBSE 2023 (65/1/1)]

(a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

8. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, Then A^{2023} is equal to [CBSE 2023 (65/2/1)]

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

9. If order of matrix A is 2×3, of matrix B is 3×2, and of matrix C is 3×3, then which one of the following is not defined. [CBSE 2022 (65/1/4) (Term-1)]

(a) $C(A + B')$ (b) $C(A + B')'$ (c) BAC (d) $CB + A'$

10. If A and B are matrices of same order, then $(AB' - BA')$ is a [NCERT Exemplar]

(a) skew-symmetric matrix (b) null matrix

(c) symmetric matrix (d) unit matrix

11. If P is a 3×3 matrix such that $P' = 2P + I$, where P' is the transpose of P, then [CBSE 2022 (Term-1)]

(a) $P = I$ (b) $P = -I$ (c) $P = 2I$ (d) $P = -2I$

12. A matrix $A = [a_{ij}]_{3\times3}$ is defined by [CBSE 2021-22 (65/2/4) (Term-1)]

$$a_{ij} = \begin{cases} 2i + 3j, & i < j \\ 5, & i = j \\ 3i - 2j, & i > j \end{cases}$$

The number of elements in A which are more than 5 is

(a) 3 (b) 4 (c) 5 (d) 6

13. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are

(a) $-6, -12, -18$ (b) $-6, -4, -9$ (c) $-6, 4, 9$ (d) $-6, 12, 18$

14. If A is square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to [NCERT Exemplar]

(a) A (b) $I - A$ (c) $I + A$ (d) $3A$

15. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then $(A - 2I)(A - 3I)$ is equal to [CBSE 2022 (65/1/4) (Term-1)]

(a) A (b) I (c) $5I$ (d) O

16. If $A=\begin{bmatrix}2 & -1 & 3\\ -4 & 5 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & -2\\ 1 & 5\end{bmatrix}$, then [NCERT Exemplar]

(a) only AB is defined (b) only BA is defined

(c) AB and BA both are defined (d) AB and BA both are not defined.

17. Given that $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ and $A^2=3I$, then [CBSE Sample Paper 2022 (Term-1)]

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$ (c) $3-\alpha^2-\beta\gamma=0$ (d) $3+\alpha^2+\beta\gamma=0$

18. If $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and $(3I+4A)(3I-4A)=x^2I$, then the value(s) of x is/are [CBSE 2023 (65/1/1)]

(a) $\pm\sqrt{7}$ (b) 0 (c) ± 5 (d) 25

19. Given that matrices A and B are of order $3\times n$ and $m\times 5$ respectively, then the order of matrix $C=5A+3B$ is [CBSE Sample Paper 2022 (Term-1)]

(a) 3×5 and $m=n$ (b) 3×5 (c) 3×3 (d) 5×5

20. If $A=\begin{bmatrix}1 & -1 & 1\\ 1 & -1 & 1\\ 1 & -1 & 1\end{bmatrix}$, then $A^5-A^4-A^3+A^2$ is equal to [CBSE 2021-22 (Term-1)]

(a) $2A$ (b) $3A$ (c) $4A$ (d) O

21. If for a square matrix A, $A^2-A+I=O$, then A^{-1} equals [CBSE 2023 (65/5/1)]

(a) A (b) $A+I$ (c) $I-A$ (d) $A-I$

22. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A=B^2$, then x equals [CBSE 2023 (65/5/1)]

(a) ± 1 (b) -1 (c) 1 (d) 2

23. If $A=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$, then $B'A'$ is equal to [CBSE 2023 (65/3/2)]

(a) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

24. A and B are skew-symmetric matrices of same order. AB is symmetric, if [CBSE 2023 (65/3/2)]

(a) $AB=O$ (b) $AB=-BA$ (c) $AB=BA$ (d) $BA=O$

25. For what value of $x\in\left[0,\frac{\pi}{2}\right]$, is $A+A'=\sqrt{3}\,I$, where $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$? [CBSE 2023 (65/3/2)]

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{2}$

Answers

1. (d)	2. (b)	3. (a)	4. (d)	5. (b)	6. (b)	7. (b)
8. (c)	9. (a)	10. (a)	11. (b)	12. (b)	13. (b)	14. (a)
15. (d)	16. (c)	17. (c)	18. (c)	19. (b)	20. (d)	21. (c)
22. (c)	23. (b)	24. (c)	25. (b)			

Solutions of Selected Multiple Choice Questions

1. Given, $a_{ij} = \begin{cases} 1, \text{ when } i \neq j \\ 0, \text{ when } i = j \end{cases}$

$\Rightarrow \quad A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \Rightarrow \quad A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore$ Option (*d*) is correct.

2. We have, $A + A' = I \quad \Rightarrow \quad \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = I$

$\Rightarrow \quad \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad 2\cos\alpha = 1 \quad \Rightarrow \quad \cos\alpha = \frac{1}{2}$

$\Rightarrow \quad \cos\alpha = \cos\frac{\pi}{3} \quad \Rightarrow \quad \alpha = \frac{\pi}{3}$

$\therefore$ Option (*b*) is correct.

3. $\left.\begin{aligned} 2a + b &= 4 \\ a - 2b &= -3 \\ 5c - d &= 11 \\ 4c + 3d &= 24 \end{aligned}\right\} \Rightarrow \begin{aligned} a &= 1 \\ b &= 2 \\ c &= 3 \\ d &= 4 \end{aligned}$

$\Rightarrow \quad a + b - c + 2d = 8$

$\therefore$ Option (*a*) is correct.

4. Total number of possible matrices of order 3×3 with each entry 2 or 0 is 2^9 *i.e.*, 512.

$\therefore$ Option (*d*) is correct.

5. Given, $A^2 - 3A + I = 0$

$\Rightarrow \quad A^2.A^{-1} - 3A.A^{-1} + IA^{-1} = 0$ (Multiplying both sides by A^{-1})

$\Rightarrow \quad A - 3I + A^{-1} = 0$

$\Rightarrow \quad A^{-1} = -A + 3I$

$\Rightarrow \quad xA + yI = -A + 3I$ (Given $A^{-1} = xA + yI$)

$\Rightarrow \quad x = -1, y = 3$ (On equating)

$\Rightarrow \quad x + y = -1 + 3 = 2$

$\therefore$ Option (*b*) is correct.

6. Let $A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$

$\Rightarrow \quad A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

Since $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where Q is skew-symmetric matrix

$\Rightarrow \quad A = P + Q = \frac{1}{2}\{(A + A^T) + (A - A^T)\}$

$\Rightarrow \quad Q = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

$\Rightarrow \quad Q = \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

$\therefore$ Option (*b*) is correct.

7. Given matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$

Also, $2A + B = 0 \Rightarrow 2\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix} + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix} + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow B = -\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

$\therefore$ Option (*b*) is correct.

8. Given matrix $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \quad A^{2023} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ Option (*c*) is correct.

9. We have matrix A of order 2×3, B of order 3×2.

$\Rightarrow B'$ is of order 2×3 and C is of order 3×3.

$\therefore$ A and B' both have same order.

$\Rightarrow (A + B')$ is of order 2×3.

Now, matrix C has order 3×3 and $(A + B')$ has order 2×3.

Therefore, $C(A + B')$ is not defined because number of columns of C is not equal to number of rows of $(A + B')$.

$\therefore$ Option (*a*) is correct.

11. We have, $P' = 2P + I$ ...(*i*)

$\therefore (P')' = (2P + I)'$ (Taking transpose on both sides)

$\Rightarrow \quad P = 2P' + I$ ... (*ii*)

From (*i*) and (*ii*), we have

$P = 2(2P + I) + I \Rightarrow P = 4P + 2I + I \Rightarrow -3P = 3I \Rightarrow P = -I$

$\therefore$ Option (*b*) is correct.

12. $\because \quad A = [a_{ij}]_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$\therefore \quad a_{11} = 5, \quad a_{12} = 8, \quad a_{13} = 11$

$a_{21} = 4, \quad a_{22} = 5, \quad a_{23} = 13$

$a_{31} = 7, \quad a_{32} = 5, \quad a_{33} = 5$

$\therefore \quad A = \begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$

Here, number of elements in A which are greater than 5 are 4.

$\therefore$ Option (*b*) is correct.

13. $kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow \quad -4k = 24,\ 3a = 2k,\ 2b = 3k \quad \Rightarrow \quad k = -6, a = -4$ and $b = -9$

$\therefore$ Option (*b*) is correct.

14. We have, $A^2 = I$

$\therefore (A-I)^3 + (A+I)^3 - 7A = \{(A-I) + (A+I)\}\{(A-I)^2 + (A+I)^2 - (A-I)(A+I)\}] - 7A$

$[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

$= [(2A)\{A^2 + I^2 - 2AI + A^2 + I^2 + 2AI - (A^2 - I^2)\}] - 7A$

$= 2A[I + I^2 + I + I^2 - I + I^2] - 7A \qquad [\because A^2 = I]$

$= 2A[5I - I] - 7A \qquad [\because I^2 = I]$

$= 8AI - 7A = 8A - 7A \qquad [\because A = AI]$

$= A$

$\therefore$ Option (*a*) is correct.

15. Given, $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$\therefore \ A - 2I = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$

and $A - 3I = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$

Now, $(A - 2I)(A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore$ Option (*d*) is correct.

17. $A^2 = 3I$

$\Rightarrow \quad \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow \quad \alpha^2 + \beta\gamma = 3 \quad \Rightarrow \quad 3 - \alpha^2 - \beta\gamma = 0$

$\therefore$ Option (*c*) is correct.

18. Given, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\therefore \quad 3I - 4A = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 4\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

and $3I + 4A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$

$\therefore \ (3I + 4A)(3I - 4A) = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 9 + 16 & -12 + 12 \\ -12 + 12 & 16 + 9 \end{bmatrix}$

$= \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} = 25\,I$

$\Rightarrow \quad x^2 I = 25\, I \quad \Rightarrow x^2 = 25 \qquad \Rightarrow x = \pm 5$

$\therefore$ Option (c) is correct.

19. $C = 3A + 5B$ is defined if $n = 5$ and $m = 3$ and order of C will be 3×5.

$\therefore$ Option (b) is correct.

20. Given, $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

$\Rightarrow \quad A^2 = A$

Similarly, $A^3 = A$, $A^4 = A$ and $A^5 = A$

$\Rightarrow \quad A^5 - A^4 - A^3 + A^2 = A - A - A + A = O$

$\therefore$ Option (d) is correct.

21. $A^2 - A + I = O \Rightarrow I = A - A^2$

Pre multiplying the above equation by A^{-1}, we get

$A^{-1} I = A^{-1}(A - A^2) = A^{-1}A - A^{-1}A^2$

$\Rightarrow \quad A^{-1} = I - (A^{-1}A)A = I - IA = I - A$

$\therefore$ Option (c) is correct.

22. $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}, B^2 = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$

$\because A = B^2 \quad \Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$

$\Rightarrow x^2 = 1$ and $x + 1 = 2 \quad \Rightarrow \qquad x = \pm 1$ and $x = 1$

$\because x = 1$ satisfies both. $\Rightarrow \qquad x = 1$

$\therefore$ Option (c) is correct.

23. Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow B'A' = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}' \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}' = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$\therefore$ Option (b) is correct.

24. Given A and B are skew-symmetric matrices.

$\therefore \quad A = -A^T$ and $B = -B^T$ ...(i)

Now, $(AB)^T = B^T . A^T = -B \times (-A)$ (From (i))

$= BA$

For AB matrix to be symmetric $BA = AB$.

$\therefore$ Option (c) is correct.

25. Given, $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$\Rightarrow \qquad A' = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \quad \Rightarrow \quad A + A' = \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix}$

$\Rightarrow \quad A + A' = \sqrt{3} I$ (Given)

$\Rightarrow \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \sqrt{3} I \quad \Rightarrow \quad 2\cos x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \sqrt{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad 2\cos x = \sqrt{3} \qquad \Rightarrow \quad \cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow \quad x = \frac{\pi}{6}$

$\therefore$ Option (*b*) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A):** A matrix $A = [1 \;\; 2 \;\; 0 \;\; 3]$ is a row matrix of order 1×4.

   Reason (R): A matrix having one row and any number of column is called a row matrix.

2. **Assertion (A):** If $\begin{bmatrix} x^2 - 4x & x^2 \\ x^2 & x^3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -x+2 & 1 \end{bmatrix}$, then the value of $x = 1$.

   Reason (R): Two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ of same order $m \times n$ are equal, if $a_{ij} = b_{ij}$ for all $i = 1, 2, 3, ...m$ and $j = 1, 2, 3, ... n$.

3. **Assertion (A):** If A and B are symmetric matrices of same order then $AB - BA$ is also a symmetric matrix.

   Reason (R): Any square matrix A is said to be skew-symmetric matrix if $A = -A^T$, where A^T is the transpose of matrix A.

4. **Assertion (A):** If A is a square matrix of order 3×3, and 2 is any scalar then the value of $|2A| = 8|A|$.

   Reason (R): If k is a scalar and A is a square matrix of order $n \times n$. Then $|kA| = k^{n-1}|A|$.

5. **Assertion (A):** If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, than $A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.

   Reason (R): If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, than $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

Answers

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*a*)

Solutions of Assertion-Reason Questions

1. We have, $A = [1 \;\; 2 \;\; 0 \;\; 3]$ matrix which has one row and four columns.

   Therefore, it is a row matrix.

   Clearly, both Assertion (A) and Reason (R) are True and Reason (R) is the correct explanation of Assertion (A).

   $\therefore$ Option (*a*) is correct.

2. We have, $x^2 - 4x = -3 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x-3) - 1(x-3) = 0 \Rightarrow (x-1)(x-3) = 0 \Rightarrow x = 1, 3$

and, $x^2 = 1 \Rightarrow x = \pm 1$

Also, $x^2 = -x + 2 \Rightarrow x^2 + x - 2 = 0 \Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x+2) - 1(x+2) = 0 \Rightarrow (x+2)(x-1) = 0 \Rightarrow x = 1, -2$

and, $x^3 = 1 \Rightarrow x^3 - 1 = 0 \Rightarrow (x-1)(x^2 + x + 1) = 0$

$\Rightarrow x = 1, x^2 + x + 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{-3}}{2} \Rightarrow x = \frac{-1 \pm \sqrt{3}i}{2} = w, w^2$

$\therefore x = 1, w, w^2$

So, common value of $x = 1$.

Clearly, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

$\therefore$ Option (*a*) is correct.

3. We have, $AB - BA = P$ (Let)

$$\therefore \quad P^T = (AB - BA)^T = (AB)^T - (BA)^T$$
$$= B^T A^T - A^T B^T = -(A^T B^T - B^T A^T)$$
$$\Rightarrow \quad P^T = -(AB - BA) = -P$$
$$\Rightarrow \quad P = -P^T$$

$\therefore$ P is skew-symmetric matrix.

$\Rightarrow$ $AB - BA$ is skew-symmetric matrix.

Clearly, Assertion (A) is false and Reason (R) is true.

$\therefore$ Option (*d*) is correct.

4. As we know that if k is any scalar and A be any square matrix of order $n \times n$ then

$|kA| = k^n . |A|$

$\therefore |2A| = 2^3 . |A|$ ($\because$ A is of order 3×3 in Assertion (A))

$= 8|A|$

Clearly, Assertion (A) is true and Reason (R) is false.

$\therefore$ Option (*c*) is correct.

5. Clearly, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

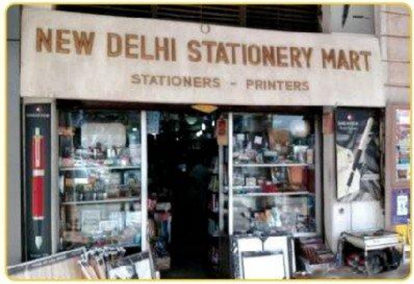
$\therefore$ Option (*a*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**

A manufacturer produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below



Market	Products (in numbers)		
	Pencil	**Eraser**	**Sharpener**
A	10,000	2,000	18,000
B	6,000	20,000	8,000

If the unit sale price of Pencil, Eraser and Sharpener are ₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively, and unit cost of the above three commodities are ₹ 2.00, ₹1.00 and ₹ 0.50 respectively.

[*CBSE Question Bank*]

(*i*) Find the total revenue of market *A*.

(*ii*) (*a*) Find the total revenue of market *B*.

OR

(*ii*) (*b*) Find the cost incurred in market *A*.

(*iii*) Find the profit in market *A* and *B*.

Sol. (*i*) Total revenue for market $A = [10,000 \;\; 2000 \;\; 18000]\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$= 10,000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$

$= ₹46000$

(*ii*) (*a*) Total revenue for market $B = [6,000 \;\; 20,000 \;\; 8,000]\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$= 6,000 \times 2.50 + 20,000 \times 1.50 + 8,000 \times 1.00$

$= ₹53000$

OR

(*ii*) (*b*) Cost incurred in market $A = [10,000 \;\; 2,000 \;\; 18,000]\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

$= 10,000 \times 2.00 + 2,000 \times 1.00 + 18,000 \times 0.50$

$= ₹31000$

(*iii*) We have,

Cost incurred in market $B = [6,000 \;\; 20,000 \;\; 8,000]\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

$= 6,000 \times 2.00 + 20,000 \times 1.00 + 8,000 \times 0.50$

$= 12,000 + 20,000 + 4,000$

$= ₹36,000$

$\therefore$ Profit in market $A = ₹46,000 - ₹31,000 = ₹15,000$

and Profit in market $B = ₹53,000 - ₹36,000 = ₹17,000$

2. Read the following passage and answer the following questions.

Amit, Biraj and Chirag were given the task of creating a square matrix of order 2. Below are the matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

(*i*) Find the sum of the matrices A, B and C, $A + (B + C)$.

(*ii*) Evaluate $(A^T)^T$.

(*iii*) (*a*) Find the matrix $AC - BC$.

OR

(*iii*) (*b*) Find the matrix of $(a + b)B$ when $a = 4$ and $b = -2$.

Sol. (*i*) We have,

$$A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \left\{\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}\right\} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A + (B + C) = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$$

(*ii*) We have,

$$A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad (A^T)^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

(*iii*) (*a*) We have,

$$AC - BC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

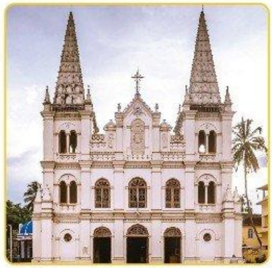
OR

(*iii*) (*b*) We have

$$(a + b)B = (4 - 2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = 2\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

3. Read the following passage and answer the following questions.

Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹25, ₹100 and ₹50 each respectively. The numbers of articles sold are given as




School/Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

[*CBSE Question Bank*]

(*i*) What is the total amount of money collected by all three schools DPS, CVC and KVS?

(*ii*) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

Sol. (*i*) Amount of money (in ₹) collected by all schools

$$= \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix}$$

$$= \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

Total amount of money collected by all three schools

= ₹ (7000 + 6125 + 7875) = ₹21,000

(*ii*) After interchanging the number of handmade fans and plates,

We have

School/Article	DPS	CVC	KVS
Handmade fans	20	30	40
Mats	50	40	50
Plates	40	25	35

Amount of money collected by all schools

$$= \begin{bmatrix} 20 & 50 & 40 \\ 30 & 40 & 25 \\ 40 & 50 & 35 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 20 \times 25 + 50 \times 100 + 40 \times 50 \\ 30 \times 25 + 40 \times 100 + 25 \times 50 \\ 40 \times 25 + 50 \times 100 + 35 \times 50 \end{bmatrix}$$

$$= \begin{bmatrix} 7500 \\ 6000 \\ 7750 \end{bmatrix}$$

∴ Total amount = ₹ (7500 + 6000 + 7750)

= ₹ 21,250

CONCEPTUAL QUESTIONS

1. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$. **[CBSE (AI) 2014]**

Sol.

$x - y = -1$

$2x - y = 0$

$y = 2x$

$x - 2x = -1$

$x = 1$

$\therefore y = 2$

$\therefore x + y = 3$

[Topper's Answer 2014]

2. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A. **[CBSE Delhi 2013]**

Sol. Given $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

3. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \dfrac{|i-j|}{2}$. **[CBSE Delhi 2015]**

Sol. $a_{23} = \dfrac{|2-3|}{2} = \dfrac{|-1|}{2} = \dfrac{1}{2}$

4. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix. **[CBSE Delhi 2010]**

Sol. If A is identity matrix, then $A = I_2$

$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On equating corresponding elements, we get

$\Rightarrow \quad \cos\alpha = 1, \ \sin\alpha = 0 \quad \Rightarrow \quad \alpha = 0$

5. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then find the value of k. **[CBSE Delhi 2010]**

Sol. Given: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} (1)(3)+(2)(2) & (1)(1)+(2)(5) \\ (3)(3)+(4)(2) & (3)(1)+(4)(5) \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

Equating the corresponding elements, we get

$k = 17$

6. Simplify: $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ **[CBSE Delhi 2012]**

Sol. Given: $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta.\cos\theta \\ -\sin\theta.\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta.\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta + \cos^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

7. If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix $5A - 3B$, given that it is defined. **[*CBSE Sample Paper 2021*]**

Sol. $O(A) = 3 \times n$, $O(B) = m \times 5$

$5A - 3B$ is defined if $m = 3$, $n = 5$.

i.e., $O(5A - 3B) = 3 \times 5$

8. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$. **[*CBSE (AI) 2012*]**

Sol. Given: $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Now $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

9. For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix? **[*CBSE (AI) 2013*]**

Sol. A will be skew symmetric matrix if $A = -A'$.

$$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

Equating the corresponding elements, we get $x = 2$.

10. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, find AA^T.

Sol. We have,

$$AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

11. Let A and B are matrices of order 3×2 and 2×4 respectively. Write the order of matrix (AB). **[*CBSE Delhi (C) 2017*]**

Sol. Order of $AB = [a_{ij}]_{3\times 2}\,[b_{ij}]_{2\times 4} = [c_{ij}]_{3\times 4}$ *i.e.,* order of AB is 3×4.

12. **Construct a 2 × 2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$.** [CBSE 2020 (65/3/1)]

Sol. We have, elements of the matrix are given by $a_{ij} = |(i)^2 - j|$.

$\therefore\ a_{11} = |(1)^2 - 1| = 0\ ,\ a_{21} = |(2)^2 - 1| = 3$

$a_{12} = |(1)^2 - 2| = 1\ ,\ a_{22} = |(2)^2 - 2| = 2$

$\therefore$ Matrix $A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$

13. **If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.** [CBSE 2018]

Sol.

A is a skew symmetric matrix

$\Rightarrow A' = -A$

$A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

$A' = -A$

Now, $\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$

On comparing we get,

$b = 3$

$a = -2$ Ans

[Topper's Answer 2018]

14. **Find the value of $x - y$, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.** [CBSE 2019 (65/1/2)]

Sol.

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

comparing corresponding elements of each matrix,

$2 + y = 5$ $\quad$ $2x + 2 = 8$

$y = 3$ $\quad$ $x = 3$

$x - y = 3 - 3$

$x - y = 0$

[Topper's Answer 2019]

15. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $(A^2 - 5A)$. **[CBSE 2019 (65/1/2)]**

Sol.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = AA$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$A^2 - 5A = A^2 + (-5)A$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + (-5)\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

$$A^2 - 5A = A^2 + (-5A)$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

[Topper's Answer 2019]

16. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix. **[CBSE (AI) 2014]**

Sol.

$$7A - I^3 - A^3 - 3A^2 I - 3AI^2$$

$$= 7A - I - A - 3A - 3A \quad \left[\because A^2 = A,\ A^3 = A\right]$$

$$= -I$$

$$\therefore\ 7A - (I + A)^3 = -I$$

[Topper's Answer 2014]

17. If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c. **[NCERT Exemplar]**

Sol. Let $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$

Since A is skew-symmetric matrix.

$\therefore \quad A' = -A$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & +1 \\ -c & -1 & 0 \end{bmatrix}$$

By equating corresponding elements, we get

$$a = -2, c = -3 \text{ and } b = -b \Rightarrow b = 0$$

$$\therefore \quad a = -2, b = 0 \text{ and } c = -3$$

Very Short Answer Questions

1. If $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$. [CBSE 2020 (65/2/1)]

Sol. $A^2 = \begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix}$ 1

$A^2 + I = kA \Rightarrow \begin{bmatrix} 12 & -8 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} -3k & 2k \\ k & -k \end{bmatrix}$ ½

$k = -4$ ½

[CBSE Marking Scheme 2020 (65/2/1)]

Detailed Solution:

We have,

$$A^2 = A \times A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix}$$

and $kA = k\begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -3k & 2k \\ k & -k \end{bmatrix}$

Given, $A^2 + I = kA \quad \Rightarrow \quad \begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3k & 2k \\ k & -k \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 12 & -8 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} -3k & 2k \\ k & -k \end{bmatrix}$$

$$\Rightarrow k = -4$$

2. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k. [CBSE (AI) 2013]

Sol. Given: $A^2 = kA$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \Rightarrow \quad k = 2$$

3. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ **is given to be symmetric, find values of *a* and *b*.** [*CBSE Delhi 2016*]

Sol. We have $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

$\because$ A is symmetric matrix.

$\Rightarrow \quad A^T = A \qquad \Rightarrow \quad \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

Equating the corresponding elements, we get

$2b = 3$ and $3a = -2$

$\Rightarrow \quad b = \frac{3}{2}$ and $a = -\frac{2}{3}$

4. Show that all the diagonal elements of a skew symmetric matrix are zero. [*CBSE Delhi 2017*]

Sol. Since, $A = [a_{ij}]$ is skew symmetric matrix.

$\therefore \quad A^T = -A$

$[a_{ij}]^T = -[a_{ij}] \quad \Rightarrow \quad [a_{ji}] = [-a_{ij}]$

For diagonal elements $i = j$

$\Rightarrow \quad a_{ii} = -a_{ii} \quad \Rightarrow \quad 2a_{ii} = 0 \quad \Rightarrow \quad a_{ii} = 0 \ \forall \ i$

Hence, diagonal elements of skew symmetric matrix are zero.

5. Find the value of $(x - y)$ from the matrix equation.

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$ [*CBSE 2019 (65/5/3)*]

Sol. $\begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x-3 & 10-4 \\ 14+1 & 2y-6+2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \qquad \Rightarrow \qquad \begin{bmatrix} 2x-3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

We know that two matrices of same order are equal if the corresponding entries are equal.

i.e., $2x - 3 = 7 \quad \Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = 5$

and $2y - 4 = 14 \quad \Rightarrow \quad 2y = 18 \quad \Rightarrow \quad y = 9$

$\therefore \quad x - y = 5 - 9 = -4$

6. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ **and** $BA = (b_{ij})$**, find** $b_{21} + b_{32}$. [*CBSE East 2016*]

Sol. We have, $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$\therefore \quad BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3\times 2} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2\times 3}$

$[b_{ij}] = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}_{3\times 3} \quad \Rightarrow \quad [b_{ij}] = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3\times 3}$

Now, $b_{21} = -16;\ b_{32} = -2$

$\therefore \qquad b_{21} + b_{32} = -16 - 2 = -18$

7. For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A^T$ and verify it is a symmetric matrix. **[*CBSE 2019(65/4/2)*]**

Sol. We have $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, A^T = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

$$\Rightarrow \quad A + A^T = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix} \quad \Rightarrow \quad (A + A^T)^T = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}^T = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix} = A + A^T$$

Hence, $A + A^T$ is a symmetric matrix.

8. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that $AB - BA$ is a skew-symmetric matrix. **[*CBSE 2019 (65/4/2)*]**

Sol. We have $A^T = A$ and $B^T = B$ and AB and BA are both defined.

Now $(AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T \qquad (\because (AB)^T = B^T A^T)$

$$= BA - AB = -(AB - BA)$$

$\Rightarrow AB - BA$ is a skew-symmetric matrix. Hence proved.

9. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + kI$. **[*CBSE Sample Paper*]**

Sol. $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \qquad \Rightarrow \qquad A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, \quad kI = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$A^2 - 5A = kI$

$$\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \quad \Rightarrow \quad k = -7$$

10. If $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$, then find the value of k, a and b. **[*CBSE 2019*]**

Sol.

$A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$

$kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$

But given $kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$ Equating individual terms,

$2k = -8 \qquad k = -4$

$3k = 4a$

$3 \times (-4) = 4a \qquad a = -3$

$-5k = 5b$

$b = -k = 4$

[Topper's Answer 2019]

11. **If $A = \begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix}$, then find the matrix $B'A'$.** [CBSE 2019 (65/3/3)]

Sol. $B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ 1

$= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$ 1

[CBSE Marking Scheme 2019 (65/3/3)]

12. **If $A = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}$, then show that $A^3 = A$.** [CBSE 2019 (65/3/2)]

Sol. $A^2 = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = A$ 1

$\Rightarrow \quad A^3 = A^2 \cdot A = A \cdot A = A^2 = A$ 1

[CBSE Marking Scheme 2019 (65/3/2)]

13. **Find the value of $x + y$ from the following equation:**

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$ [CBSE (AI) 2012, Bhubneshwar 2015, AI (C) 2017]

Sol. Given, $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding elements, we get

$2x + 3 = 7$ and $2y - 4 = 14 \quad \Rightarrow \quad x = \frac{7-3}{2}$ and $y = \frac{14+4}{2}$

$\Rightarrow \quad x = 2$ and $y = 9 \quad \therefore \quad x + y = 2 + 9 = 11$

Short Answer Questions

1. **If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the values of a and b.** [CBSE (F) 2015]

Sol. Here, $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$\therefore \quad A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$

$\Rightarrow \quad (A+B)^2 = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} \cdot \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} 1+a^2+2a & 0 \\ 2+2a+b+ab-4-2b & -2 \end{bmatrix} = \begin{bmatrix} a^2+2a+1 & 0 \\ 2a-b+ab-2 & 4 \end{bmatrix}$

Again $A^2+B^2=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}\cdot\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}+\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}\cdot\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$

$$=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}+\begin{bmatrix}a^2+b & a-1\\ ab-b & b+1\end{bmatrix}=\begin{bmatrix}a^2+b-1 & a-1\\ ab-b & b\end{bmatrix}$$

Given, $(A+B)^2=A^2+B^2$

$$\begin{bmatrix}a^2+2a+1 & 0\\ 2a-b+ab-2 & 4\end{bmatrix}=\begin{bmatrix}a^2+b-1 & a-1\\ ab-b & b\end{bmatrix}$$

Equating the corresponding elements, we get

$a^2+2a+1=a^2+b-1 \quad\Rightarrow\quad 2a-b=-2$ ...(*i*)

$a-1=0 \quad\Rightarrow\quad a=1$ ...(*ii*)

$2a-b+ab-2=ab-b \quad\Rightarrow\quad 2a-2=0$ ...(*iii*)

and $b=4$ ...(*iv*)

$\Rightarrow$ $a=1, b=4$ satisfy all four equations (*i*), (*ii*), (*iii*) and (*iv*).

Hence, $a=1, b=4$.

2. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that $CD-AB=O$.

[CBSE Delhi 2017]

Sol. Since A, B, C are all square matrices of order 2, and $CD-AB$ is well defined, D must be a square matrix of order 2.

Let $D=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$. Then $CD-AB=0$ gives

$$\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}-\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}=O$$

or $\begin{bmatrix}2a+5c & 2b+5d\\ 3a+8c & 3b+8d\end{bmatrix}-\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

or $\begin{bmatrix}2a+5c-3 & 2b+5d\\ 3a+8c-43 & 3b+8d-22\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

By equating the corresponding elements of matrices, we get

$2a+5c-3=0$ ...(*i*)

$3a+8c-43=0$ ...(*ii*)

$2b+5d=0$ ...(*iii*)

and $3b+8d-22=0$ ...(*iv*)

Solving (*i*) and (*ii*), we get $a=-191$, $c=77$ and solving (*iii*) and (*iv*), we get $b=-110$, $d=44$.

Therefore $D=\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$

3. For the following matrices A and B, verify that $(AB)'=B'A'$. **[CBSE (AI) 2010]**

$$A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix},\quad B=[-1,\ 2,\ 1]$$

Sol. Given: $A=\begin{bmatrix}1\\ -4\\ 3\end{bmatrix},\quad B=[-1,\ 2,\ 1]$

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} [-1 \quad 2 \quad 1] = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$B'A' = [-1 \quad 2 \quad 1]' \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \quad -4 \quad 3] = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$\therefore \quad (AB)' = B'A'$.

4. If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha) \cdot f(-\beta) = f(\alpha - \beta)$ **[*CBSE 2019 (65/2/1)*]**

Sol. Given $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \quad f(-\beta) = \begin{bmatrix} \cos(-\beta) & -\sin(-\beta) & 0 \\ \sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{LHS} = f(\alpha).f(-\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta + \sin\alpha\sin\beta & \cos\alpha\sin\beta - \sin\alpha\cos\beta & 0 \\ \sin\alpha\cos\beta - \cos\alpha\sin\beta & \sin\alpha\sin\beta + \cos\alpha\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0 \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= f(\alpha - \beta) = \text{RHS}$ Proved

5. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}. **[*CBSE (F) 2015*]**

Sol. Here, $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \quad \ldots(i)$$

Also, $$4A - 3I = 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \quad \ldots(ii)$$

From (*i*) and (*ii*), we get $A^2 = 4A - 3I$

Pre-multiplying both sides by A^{-1}

$$A^{-1} \cdot A^2 = A^{-1} \cdot (4A - 3I)$$

$$\Rightarrow \quad (A^{-1} \cdot A) \cdot A = 4\,A^{-1} \cdot A - 3\,A^{-1} \cdot I$$

$\Rightarrow \quad IA = 4I - 3A^{-1}$

$\Rightarrow \quad A = 4I - 3A^{-1} \qquad [\because AA^{-1} = I, A^{-1}I = A^{-1}]$

$\Rightarrow \quad 3A^{-1} = 4I - A$

$$\Rightarrow \quad A^{-1} = \frac{1}{3}\left(4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}\right) = \frac{1}{3}\left(\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix} - \begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}\right)$$

$$= \frac{1}{3}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix} = \begin{bmatrix}2/3 & 1/3\\1/3 & 2/3\end{bmatrix}$$

6. Let $A = \begin{bmatrix}2 & 3\\-1 & 2\end{bmatrix}$ then show that $A^2 - 4A + 7I = 0$. Using this result calculate A^5.

[*NCERT Exemplar*]

Sol. Here, $\quad A = \begin{bmatrix}2 & 3\\-1 & 2\end{bmatrix}$

$$\Rightarrow \quad A^2 = A \times A = \begin{bmatrix}2 & 3\\-1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\-1 & 2\end{bmatrix} = \begin{bmatrix}1 & 12\\-4 & 1\end{bmatrix}$$

Now, $\quad A^2 - 4A + 7I = \begin{bmatrix}1 & 12\\-4 & 1\end{bmatrix} - 4\begin{bmatrix}2 & 3\\-1 & 2\end{bmatrix} + 7\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$$= \begin{bmatrix}1 & 12\\-4 & 1\end{bmatrix} - \begin{bmatrix}8 & 12\\-4 & 8\end{bmatrix} + \begin{bmatrix}7 & 0\\0 & 7\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = O \text{ (zero matrix)}$$

$\Rightarrow \quad A^2 - 4A + 7I = 0$

$\Rightarrow \quad A^2 = 4A - 7I$

$\Rightarrow \quad A.A^2 = 4A.A - 7A.I \qquad$ [Pre multiplying by A]

$\Rightarrow \quad A^3 = 4A^2 - 7A \qquad [AI = A]$

$\Rightarrow \quad A^3 = 4(4A - 7I) - 7A \qquad$ [Putting the value of A^2]

$\Rightarrow \quad A^3 = 16A - 28I - 7A$

$\Rightarrow \quad A^3 = 9A - 28I$

$\Rightarrow \quad A.A^3 = 9A.A - 28A.I \qquad$ [Pre multiplying by A]

$\Rightarrow \quad A^4 = 9A^2 - 28A$

$\Rightarrow \quad A^4 = 9(4A - 7I) - 28A \qquad$ [Putting the value of A^2]

$\Rightarrow \quad A^4 = 8A - 63I$

$\Rightarrow \quad A.A^4 = 8A^2 - 63A \qquad$ [Pre multiplying by A]

$$\Rightarrow \quad A^5 = 8(4A - 7I) - 63A = -31A - 56I = -31\begin{bmatrix}2 & 3\\-1 & 2\end{bmatrix} - 56\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}-118 & -93\\31 & -118\end{bmatrix}$$

7. Express the following matrix as the sum of a symmetric and skew symmetric matrix, and verify your result. [*CBSE (AI) 2010*]

$$\begin{bmatrix}3 & -2 & -4\\3 & -2 & -5\\-1 & 1 & 2\end{bmatrix}$$

Sol. Let $A = \begin{bmatrix}3 & -2 & -4\\3 & -2 & -5\\-1 & 1 & 2\end{bmatrix}$

A can be expressed as

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A'), \quad ...(i) \quad \left[\because \frac{1}{2}(A + A') + \frac{1}{2}(A - A') = \frac{2A}{2} = A\right]$$

where, $A + A'$ and $A - A'$ are symmetric and skew symmetric matrices respectively.

Now, $$A + A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

Putting these values in (*i*), we get

$$A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$$

Verification:

$$\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{1}{2}-\frac{5}{2} & -\frac{5}{2}-\frac{3}{2} \\ \frac{1}{2}+\frac{5}{2} & -2+0 & -2-3 \\ -\frac{5}{2}+\frac{3}{2} & -2+3 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

8. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$. ***[CBSE 2023 (65/5/1)]***

Sol. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$\therefore A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} + \begin{bmatrix} -23 & -46 & -69 \\ -69 & 46 & -23 \\ -92 & -46 & -23 \end{bmatrix} + \begin{bmatrix} -40 & 0 & 0 \\ 0 & -40 & 0 \\ 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69+0 & -6+46-40 & 23-23+0 \\ 92-92+0 & 46-46+0 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

i.e., $A^3 - 23A - 40I = O$

Questions for Practice

Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) The product $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is equal to **[CBSE 2023 (65/4/1)]**

(*a*) $\begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$ (*b*) $\begin{bmatrix} (a+b)^2 & 0 \\ (a+b)^2 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} a^2+b^2 & 0 \\ a^2+b^2 & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

(*ii*) If $\begin{bmatrix} x-y & 2 \\ x & 5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$, then value of y is

(*a*) 1 (*b*) 3 (*c*) 2 (*d*) 5

(*iii*) If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to **[CBSE 2023 (65/4/1)]**

(*a*) I (*b*) A (*c*) $2A$ (*d*) $3I$

(*iv*) The $A = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $x = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ and $y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, then $AB + XY$ equals

[CBSE 2020 (65/4/1)]

(*a*) [28] (*b*) [24] (*c*) 28 (*d*) 24

(*v*) If $x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$, then **[CBSE 2023 (65/4/1)]**

(*a*) $x = 1, y = 2$ (*b*) $x = 2, y = 1$ (*c*) $x = 1, y = -1$ (*d*) $x = 3, y = 2$

(*vi*) If a matrix $A = [1 \quad 2 \quad 3]$, then the matrix AA' (where A' is the transpose of A) is **[CBSE 2023 (65/4/1)]**

(*a*) 14 (*b*) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (*c*) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (*d*) [14]

(*vii*) For any two matrices A and B, we have

(*a*) $AB = BA$ (*b*) $AB \neq BA$ (*c*) $AB = O$ (*d*) None of these

(*viii*) If a matrix A is both symmetric and skew symmetric, then A is necessarily a

(*a*) diagonal matrix (*b*) zero matrix (*c*) square matrix (*d*) identity matrix

Conceptual Questions

2. For a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}. **[CBSE Delhi 2011]**

3. Write the order of the product matrix. **[CBSE (F) 2011]**

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

4. From the following matrix equation, find the value of x

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$ **[CBSE (F) 2010]**

5. If $\begin{bmatrix} 3x-2y & 5 \\ x & -2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -3 & -2 \end{bmatrix}$, then find the value of y. **[CBSE (F) 2009]**

6. Write a square matrix of order 2, which is both symmetric and skew symmetric. **[CBSE (F) 2010]**

7. If matrix $A = [1\ \ 2\ \ 3]$, then write AA', where A' is the transpose of matrix A. **[CBSE Delhi 2009]**

8. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'. **[CBSE 2018]**

9. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^3 - 7A$. **[CBSE (AI) 2014]**

10. If a matrix has 5 elements, write all possible orders it can have. **[CBSE (AI) 2011]**

11. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$. **[CBSE Delhi 2015]**

12. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3. **[CBSE Central 2016]**

■ Very Short Answer Questions

13. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$. **[CBSE 2019 (65/1/1)]**

14. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the value of k, a and b. **[CBSE 2019 (65/3/1)]**

15. Express $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ as a sum of a symmetric and a skew-symmetric matrix. **[CBSE 2019 (65/3/1)]**

16. Solve the following matrix equation for x: $[x\ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$. **[CBSE Delhi 2014]**

17. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x - y)$. **[CBSE Delhi 2014]**

18. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then find $(3A - B)$. **[CBSE Guwahati 2015]**

19. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, then write the value of x. **[CBSE Delhi 2012]**

20. If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then write the value of λ. **[CBSE (AI) 2013]**

21. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then write the value of p. **[CBSE (AI) 2013]**

■ Short Answer Questions

22. Given matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $f(A)$, if $f(x) = 2x^2 - 3x + 5$.

23. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$.

24. Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

25. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$. **[CBSE (AI) 2010]**

26. Show that the elements along the main diagonal of a skew symmetric matrix are all zero. **[CBSE Sample Paper 2017]**

27. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and $(A + B)C$. Also verify that $(A + B)C = AC + BC$. **[CBSE Ajmer 2015]**

28. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated in the table:

Market	Products		
	x	y	z
I	10,000	2,000	18,000
II	6,000	20,000	8,000

If unit sale price of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively, then find the total revenue in each market, using matrices.

Answers

1. (i) (a) (ii) (a) (iii) (a) (iv) (a) (v) (b) (vi) (d) (vii) (d) (viii) (b)

2. $a_{12} = \frac{1}{2}$ 3. 3×3 4. $x = 1$ 5. -6 6. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 7. [14]

8. $a = -2, b = 3$ 9. I 10. 1×5 and 5×1 11. $\frac{1}{2}$

12. 81 13. $\begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$ 14. $k = -6, a = -4, b = -9$

15. $\begin{bmatrix} 4 & -\frac{1}{2} \\ -\frac{1}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ 16. $x = 2$ 17. 10 18. $\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$ 19. $x = 13$

20. $\lambda = 6$ 21. $p = 4$ 22. $\begin{bmatrix} 16 & 14 \\ 21 & 37 \end{bmatrix}$ 24. $\begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -\frac{3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & 0 \end{bmatrix}$

25. $\begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix}$ 27. $AC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}, BC = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}, (A+B)C = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$

28. I : ₹46,000 ; II : ₹53,000